CS711A: Introduction to Game Theory and Mechanism Design **Sep-Nov 2020**

Project Report: Efficient Learning and Grading Mechanism

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Abstract

We propose a cheat-proof mechanism that can be employed in online educational settings to curb unfair means and promote learning. It uses a probabilistic estimate of cheating by studying the similarity in response profile obtained from students, and efficient allocation of grades from the standard grading on a curve technique. Eventually, the model uses a master Groves mechanism which incorporates these preliminaries, and builds on top of them to enforce truthfulness of students. It also ensures an efficient and fair allocation.

1 Introduction

Academic dishonesty has been a big problem [KNR+00, RL07, Arh09] for scholars long before the onset of internet-based education [AB99]. With distance education increasing, it has posed an even bigger challenge to the education system [GSG20, Eat20, GSKK20]. The standard measures [WH03, EIR08, DL95] to curb cheating like live invigilation are not scalable for an online examination. Online learning has promoted cheating behaviors among students as they can resort to plagiarising from internet sources during exams. These practices have reduced the reliability of distant learning platforms. During the pandemic of COVID-19, universities moved online. Evaluating the students' performance is posed a severe educational challenge to the faculty [Bow03, SW06]. Finding a solution to prevent cheating in online assessment methods can significantly raise the quality of internet-based education and can promote open courses.

1.1 Related work

In the past academic dishonesty has been understood through the lens of criminology literature. These frameworks conceptualize students as delinquents and have been ineffective in curbing cheating. Recent developments have led researchers to apply the game theory approach to students' cheating behaviors [DiP10]. In game theory, the instructor is a planner who has a set of strategies to use. We can model the payoffs to incorporate different aspects of the exam to minimize cheating [DiP10]. The beginning of internet-based education and distance learning promotes more cheating behaviors among students [SS10] and the offline methods used by scholars are ineffective. Students are easily able to plagiarise through other internet sources during online exams. Researchers have suggested incorporating additional assessment methods like student group projects and online collaboration [SS10, WTL+15]. However, these assessment strategies neither prevent cheating in other online evaluations nor accurately test an individual student's performance.

[NHC16] deals with cheating in massive open online courses (MOOC) using multiple existences. This involves using a fake account to acquire the correct answers by making guesses, while another account is used to submit the correct answers. Mechanisms that logically limit this cheating practice employ asynchronous exams where answers are revealed only after the assignments are due and use algorithmic generation of assessment questions to receive randomly varying questions, each with different solutions [NHC16]. However, cheating through other means still prevails [MBT12, KY15] and pose a severe threat to online certification's trustworthiness. Another solution to prevent cheating in online exams introduces cyclic questions randomization strategy [NGL+20]. This strategy limits the time to answer each question and, at the same time, gives different questions to different students. However, time management is a critical aspect of the student's evaluation, but this strategy makes it highly restrictive.

Item response theory (IRT) is a psychometric paradigm of testing based on the relationship between item (question) parameters, a student's ability, and probable correctness of the corresponding response. Several works have reviewed the compatibility of this theory with grading. The standard tests like GRE and TOEFL make use of IRT in assessing students. [KD82] studied the feasibility of IRT assumptions for GRE and evaluated the premises and effects of the 3-parameter model in IRT. They also assessed the stability of item parameter estimates across varying samples. They found IRT true score equating to be the most suitable among equi-percentile equating, linear equating, and IRT true score equating. Though there were some challenges from the heterogeneous quantitative section, we cannot refute that the other methods would not face similar issues. [WR91] concluded that the best data-fit for the TOEFL exam was found for the three-parameter model (3PL) when compared with the modified one-parameter model (M1PL) and modified two-parameter model (M2PL). The efficiency of IRT has also been tested on test performances of students in the real world. [ZKMA08] in their work on student ranking for objective type questions compare rankings obtained from Classical Test Theory (CTT) and IRT. CTT takes into account the performance of a student on a test-by-test basis compared to per-item analysis in IRT. The test comprises 80 multiple choice questions, and they selected 400 students randomly from private and public schools in District Malakand of Pakistan. They found IRT to be useful as a discriminator in cases of equal raw scores. [HTB16] compared traditional assessment methods to a model using IRT followed by K-means to classify students' rank. They used the English testing responses of 1111 examinees for the entrance examination to Gifted school - Hanoi National University of Education to conclude that there were not many differences in CTT and IRT results. Still, the latter produced more precise student-level distribution and classified student ability better. However, there have been no studies that considered a dishonest scenario where peer cheating might be involved.

1.2 Brief overview of the report

We introduce the requisite background and the formal model is Section 2. We validate and show why our model works in Section 3. We present graphical validations for our method in Section 4 and conclude this report in Section 5.

2 Formal model of the problem

2.1 Item Response Theory

We model the chances a student i of ability β_i has for solving a question j, of difficulty δ_j and sensitivity c_j, correctly using a three parameter logistic model [Ras93]. In a multiple choice question (MCQ) setup having r_j possible answers, the chances of random guessing is g_j. This parameter can be set to $\frac{1}{r_j}$ or be statistically determined [BA81]. Let, this probability of student i getting question j correct be $\pi_1(\beta_i, \delta_j, c_j, g_j)$. Using this we can compute the probability $\pi_{2\phi}(\beta_i, \delta_j, c_j, g_j, l_{j1}, l_{j2}, \ldots, l_{j(r_j-1)})$ (Section A) that student i chooses an incorrect option ϕ for question j. Our objective is to determine if two people have cheated in an exam. It is not possible to give a deterministic answer for the same. Instead we propose to give probabilistic framework to detect cheating. We formulate the probability that two students with abilities β_i and β_k cheated on the question j by $\psi(\beta_i, \beta_k; \delta_j, c_j, g_j, r_j, \mathcal{L}_j)$. We defer the exact explations of the notations used here to Section A.

2.2 Cheating Estimation using Information Theory

We formalize the chances that two students collaborated (cheated) to solve a question using item response theory. However, two answer scripts being similar doesn't imply cheating. To account for randomness in the answers given by the students, we compute the dual total correlation [TS78] between the answers for the students. For a pair of sets of random variables, the dual total correlation is given by the ratio of the mutual information [Sha01] and joint entropy [KK00, MMK03, Cov99]. For our problem, we represent the mutual information and joint entropy for two students i and k with $\mathcal{I}(i,k)$ and $\mathcal{H}(i,k)$ respectively. We defer the exact computation of these quantities to Section A. The dual total correlation $\mathcal{D}(i,k)$ is given by $\frac{\mathcal{I}(i,k)}{\mathcal{H}(i,k)}$. Let, the cheating coefficient for student $i \in [N]$ be $\mathcal{C}_i$. We define this coefficient below as the summation of the dual total correlation and defer the formal reasoning behind this to Section 3.

$$\mathcal{C}_i = \sum_{\substack{k=1 \\ k \neq i}}^{N} \mathcal{D}(i,k) - \sum_{k=1}^{N} \sum_{\substack{j=1 \\ j \neq k}}^{N} \mathcal{D}(j,k)$$

2.3 Ability Updation

The item response theory(IRT) uses maximum likelihood to update (estimate) a student's ability (Section B). We additionally use a factor of cheating coefficient computed in Section 2.2 to incorporate possible dishonesty in exams.

$$\beta_{s+1,j} = \beta_{s,j} + \frac{1}{\mathcal{C}_j} \cdot \frac{\sum_{i=1}^{N} c_i \cdot [u_i - P_i(\beta_s)]}{\sum_{i=1}^{N} c_i^2 P_i(\beta_{s,j}) Q_i(\beta_{s,j})}$$

where $\beta_{s,j}$ represents the ability of a student at iteration s for an exam j

c_i is the discrimination parameter of item i, i = 1, 2, ...N

u_i is the response made by the student to item i: correct - 1, incorrect - 0

$P_i(\beta_{s,j})$ is the probability of correct response to item i at ability level β_j within iteration s: $\pi_1(\beta_j, \delta_i, c_i, g_i)$

$Q_i(\beta_{s,j}) = 1 - P_i(\beta_{s,j})$

$\mathcal{C}_j$ is the cheating coefficient

2.4 Mechanism

In this subsection, we discuss the proposed cheat-proof mechanism which allocates grades optimally and promotes learning:

- Students appear for the online exam and provide their response profiles, $\mathcal{R}$ which basically denotes the marked answers in the questions. Each student has an intrinsic ability value $\mathcal{A}$. We define the type of student i by $\theta_i = (\mathcal{A}_i, \mathcal{R}_i)$.
- Using the response profile, estimated students ability and question difficulty, the planner (basically the professor) computes the cheating coefficient, $\mathcal{C}$ for all the students.
- Marks($\mathcal{M}$) for the students are obtained from the response profiles using the marking scheme. In this step, Relative Performance Scores(RPS) are assigned to the students. Instead of assigning letter grades in this step, cumulative RPS scores are considered at the end for letter grade allocation. The marks are fit to a normal distribution and RPS is taken as the z-value ((mark-avg)/std) corresponding to the marks. This is grading by curve fitting technique [Mid33] and is methodically equivalent to choosing RPS values which maximise the summation of product of marks and RPS with the constraint that RPS values lie on a normal distribution with mean 0 and std 1.
- The valuation of students is the product of obtained marks(representing actual learning in the exam) and RPS score(relative performance in the exam).
- Now, the time for the next exam is shortened by a factor of cheating coefficient(which is normalised between 1 and 2) as penalty to elicit truthful responses.
- Update ability for the students for the next exam.
- After all the exams are over, cumulative RPS scores are used to assign letter grades to the students. Since sum of normal distributions is also a normal distribution, summation of RPS scores also conforms to grading on a curve methodology.

3 Main results

We propose that the mechanism discussed in Section 2.4 enforces truthful reporting of response profile, optimal allocation of grades, and ensures students' voluntary participation in such a cheat-proof setting. It also validates the formulation of the cheating coefficient from IRT.

The typeset and corresponding marks of students are represented by θ_i and $\mathcal{M}_i$ respectively. After obtaining students' types and marks, the planner allocates RPS scores by curve fitting [Sch73].

$$\theta_i = (\mathcal{A}_i, \mathcal{R}_i)$$
$$\mathcal{M}_i = func(\mathcal{R}_i)$$
$$\mathcal{RPS}_1^*, \ldots, \mathcal{RPS}_N^* = \operatorname*{arg\,max}_{\mathcal{RPS}_1, \ldots, \mathcal{RPS}_N \in \mathcal{N}(0,1)} \sum_{i=1}^{N} \mathcal{M}_i \times \mathcal{RPS}_i$$

The payment incurred by the student comprises of the effort/work component to get the obtained marks and RPS values and the penalty component in the form of time reduction in the next exam. The well known quasi-linear utility formulation [SLB08] is used to denote the net payoff of students. Penalising overstating is essential to elicit truthful behaviour since only dictatorial mechanisms attain truthfulness in absence of penalty [Sat75, Vle09]. The valuation and quasi-linear utility for the student are represented by:

$$v_i(\theta_i, (RPS, p)) = \mathcal{M}_i \times RPS_i$$
$$u_i(\theta_i, (RPS, p)) = \mathcal{M}_i \times RPS_i - p_i$$

Let us consider the Groves payment structure [Gro73] which is given by:

$$p_i = h_i(\theta_{-i}) - \sum_{\substack{k=1 \\ k \neq i}}^{N} v_k(rps(\theta), \theta_k)$$

Substituting the h function with the following specific function:

$$h_i(\theta_{-i}) = g_i(\theta_{-i}) - \sum_{\substack{k=1 \\ k \neq i}}^{N} \sum_{\substack{j=1 \\ j \neq k,i}}^{N} \frac{\mathcal{I}(k,j)}{\mathcal{H}(k,j)}$$

Since the pair wise sum of the ratio of information and entropy for all students except student i is essentially the cheating coefficient of the student, we can write the payment function as:

$$p_i = g_i(\theta_{-i}) - \sum_{\substack{k=1 \\ k \neq i}}^{N} v_i(rps(\theta), \theta_k) + C_i$$

Here, since $\mathcal{C}_i$ is an established measure of cheating, the second term represents the penalty component of the payment. The first term gives us an estimate of payment in the form of effort to achieve the marks and RPS values. Substituting the g function with Clarke's pivotal term [Cla71], we get an estimate of the marginal contribution of the student which can be used to provide feedback about relative effort and effort difference in absence of cheating to the student.

Since the Groves payment structure incorporates cheating coefficient in it, it validates the formulation of cheating coefficient from IRT and its usage as penalty.

Theorem 1 *The proposed mechanism is dominant strategy incentive compatible and allocatively efficient.*

Proof: Since the used mechanism is a Groves mechanism(Groves payment rule structure for payment and allocative efficiency from social welfare maximisation for allocation), the proof follows directly from the standard Groves theorem proof [Gro73]. ■

This result is particularly important since other mechanisms which exhibit allocative efficiency can be designed, grading by using effective marks(marks/cheating coefficient) is an example which eradicates the benefit from cheating in grading, but enforcing truthfulness is important to maximise learning and ensuring that marks reflect the learning which lead to a fair division in grades.

4 Simulations

In this section, we visualize the variation of various parameters we propose in the previous sections. Figure 1 shows how the probability π_1 of answering a question correctly varies with ability β_i of the student i and the difficulty δ_j of the question j. Figure 2 shows how the cheating coefficients vary if we change the marks and ability of one student for a 2 student MCQ examination. Figure 3 shows that we don't penalize low ability students to a high degree if they score decent marks on an exam of difficulty higher than their ability.

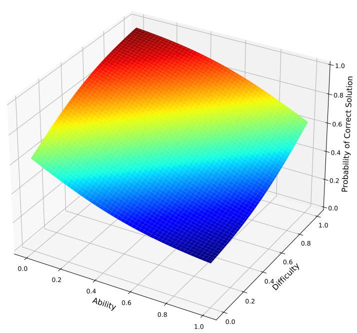

Figure 1: **3-parameter Logistic IRT Model** Probability of solving a question correctly is parameterized by the ability of the student and the question difficulty. The jacobian of the surface is given by the sensitivity parameter. The minimum probability is given by the guess chance.

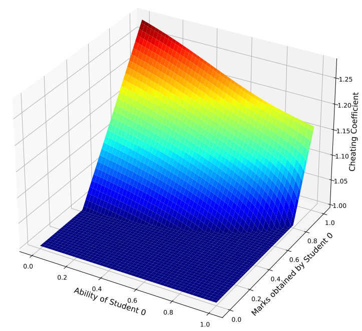

Figure 2: **Cheating Coefficient** In a 2 student exam of difficulty *1.0*, we vary the ability and marks obtained by Student 0. Student 1 always scores *100* marks and has an ability of *1.0*. We successfully detect that Student 0 cheated when (s)he has scored a high marks despite having a lower ability.

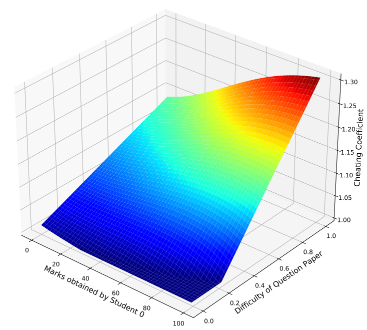

Figure 3: **Cheating Coefficient vs Exam Difficulty** Keeping the ability of student fixed to low value, if we increase both the marks obtained by the student and the difficulty of the exam, the cheating coefficient goes up. Students having low ability and moderately high scores are not penalized heavily.

5 Summary and Discussions

The need for an enactable cheating disincentivizing mechanism and the lack of adequate focus towards it in online settings is discussed in the first sections followed by our proposed mechanism and its preliminaries in Section 2. Section 3 establishes why the mechanism works by theoretically showing it to be a Groves mechanism and validating our formulation of cheating coefficient from IRT. In Section 4, we present the results of our simulation on various general and dummy settings to illustrate that the cheating coefficient is indeed a good measure of cheating and is justified in its use as penalty in the mechanism. An experiment to test the mechanism in non-course setting is not practically possible since that requires providing incentives to get a good grade and cheat, which even if loosely possible, carries various complications in the current online domain. A good future direction of this work is to test it in an educational course setting and making it compatible so as to integrate it with online learning platforms.

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Appendices

A Cheating Estimation using IRT and Information Theory

In this subsection, we provide the exact mathematical preliminaries used for computing the cheating coefficient. We compute the probability $\pi_1(\beta_i, \delta_j, c_j, g_j)$ of student i getting question j correct as:

$$\pi_1(\beta_i, \delta_j, c_j, g_j) = g_j + \frac{1 - g_j}{1 + e^{-c_j(\beta_i - \delta_j)}}$$

Let, the $r_j - 1$ incorrect options for the question have a likelihood $\mathcal{L}_j = \{l_{j1}, l_{j2}, \ldots, l_{j(r_j-1)}\}$. Thus the chances of randomly guessing the correct answer would be $g_j = 1 - \sum_{n=1}^{r_j-1} l_{jn}$. Using this we can compute the probability $\pi_{2\phi}(\beta_i, \delta_j, c_j, g_j, l_{j1}, l_{j2}, \ldots, l_{j(r_j-1)})$ that student i chooses an incorrect option ϕ for question j as:

$$\pi_{2\phi}(\beta_i, \delta_j, c_j, g_j, r_j, \mathcal{L}_j) = \frac{l_{k\phi}}{\sum_{n=1}^{r_j-1} l_{jn}} \cdot (1 - \pi_1(\beta_i, \delta_j, c_j, g_j))$$

The chances that answers for two students i and k match for a question j is given by $\psi(\beta_i, \beta_k; \delta_j, c_j, g_j, r_j, \mathcal{L}_j)$.

$$\begin{aligned}\psi(\beta_i, \beta_k; \delta_j, c_j, g_j, r_j, \mathcal{L}_j) = {} & \pi_1(\beta_i, \delta_j, c_j, g_j) \cdot \pi_1(\beta_k, \delta_j, c_j, g_j) \\ & + \sum_{n=1}^{r_j-1} \pi_{2n}(\beta_i, \delta_j, c_j, g_j, r_j, \mathcal{L}_j) \cdot \pi_{2n}(\beta_k, \delta_j, c_j, g_j, r_j, \mathcal{L}_j)\end{aligned}$$

Next, using these probabilities we compute $\mathcal{I}(i,k)$ and $\mathcal{H}(i,k)$ as follows:

$$\begin{aligned}\mathcal{I}(i,k) = & \sum_j \mathbb{I}[\text{match}_j] \cdot \log(\psi(\beta_i, \beta_k; \delta_j, c_j, g_j, r_j, \mathcal{L}_j)) \\ & \quad + \mathbb{I}[\text{non-match}_j] \cdot \log(1 - \psi(\beta_i, \beta_k; \delta_j, c_j, g_j, r_j, \mathcal{L}_j)) \\ \mathcal{H}(i,k) = & \sum_j \psi(\beta_i, \beta_k; \delta_j, c_j, g_j, r_j, \mathcal{L}_j) \cdot \log(\psi(\beta_i, \beta_k; \delta_j, c_j, g_j, r_j, \mathcal{L}_j)) \\ & \quad + (1 - \psi(\beta_i, \beta_k; \delta_j, c_j, g_j, r_j, \mathcal{L}_j)) \cdot \log(1 - \psi(\beta_i, \beta_k; \delta_j, c_j, g_j, r_j, \mathcal{L}_j))\end{aligned}$$

B Ability Updation in IRT

The item response theory(IRT) uses maximum likelihood to update (estimate) a student's ability [Bak01]. For each exam, it follows an iteration given a prior value of ability, value of IRT parameters and student response profile. The goal is to decrease difference in response profile obtained and expected from probability calculated by using difficulty, discrimination and guess parameter. The formula below represents the equation after an iteration s for estimating the ability of a student after an exam j. The iterations are stopped when the change is less than 0.001 usually.

$$\beta_{s+1,j} = \beta_{s,j} + \frac{\sum_{i=1}^{N} c_i \cdot [u_i - P_i(\beta_s)]}{\sum_{i=1}^{N} c_i^2 P_i(\beta_{s,j}) Q_i(\beta_{s,j})}$$

where $\beta_{s,j}$ represents the ability of a student at iteration s for an exam j
c_i is the discrimination parameter of item i, i = 1, 2, ...N

u_i is the response made by the student to item i: correct - 1, incorrect - 0
$P_i(\beta_{s,j})$ is the probability of correct response to item i at ability level β_j within iteration s: $\pi_1(\beta_j, \delta_i, c_i, g_i)$
$Q_i(\beta_{s,j}) = 1 - P_i(\beta_{s,j})$

The IRT though makes an assumption of an honest scenario where no cheating takes place. The update equation uses obtained responses directly but they may have been obtained through dishonest means. In order to incorporate cheating, we introduce a factor of cheating coefficient computed in 2.2.

$$\beta_{s+1,j} = \beta_{s,j} + \frac{1}{\mathcal{C}_j} . \frac{\sum_{i=1}^{N} c_i \cdot [u_i - P_i(\beta_s)]}{\sum_{i=1}^{N} c_i^2 P_i(\beta_{s,j}) Q_i(\beta_{s,j})}$$

This factor of $\mathcal{C}_j$ ensures that the update of ability is penalised for cheating and the obtained responses are not assumed to be honest by default.