Practice Problem Set – Network Flow

1. Suppose you are given an $n \times n$ checkerboard with some of the squares deleted. You have a large set of dominos, just the right size to cover two squares of the checkerboard. Describe and analyze an algorithm to determine whether one tile the board with dominos – each domino must cover exactly two undeleted squares, and each undeleted square must be covered by exactly one domino.

   Your input is a 2D boolean array $DELETED_{n \times n}$ where $DELETED(x, y)$ is TRUE if and only if the square in row X and column y has been deleted. Your output is a single boolean; you do not have to compute the actual placement of dominos. In the example board shown in Figure 1 - your algorithm should return TRUE.

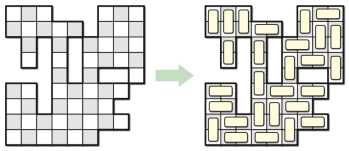

Figure 1: Covering a partial checkerboard with dominoes

2. Suppose you are given an $n \times n$ square grid with squares being colored black or white. Your input is a 2D array $Grid_{n \times n}$ such that $Grid(x, y)$ is either BLACK or WHITE. Describe and analyze an algorithm that determines whether tokens can be placed on the white squares of the grid so that

   - every token is on a white square.
   - every row of the grid contains exactly one token, and
   - every column of the grid contains exactly one token.

   Your output is single boolean.

3. Suppose you are given an $n \times n$ square grid with some of the cells being marked. Your input is a 2D array $M_{n \times n}$ such that $M(x, y)$ is TRUE if and only if the cell appearing at x-th row and y-th column is marked. A monotone path starts from the top-left cell, moves only right or down at each step, and ends at the bottom-right cell. The objective is to cover the marked cells with as few monotone paths as possible.

(a) Describe an algorithm to find a monotone path that covers the largest number of marked cells.

(b) Describe and analyze an efficient algorithm that computes the minimum number of monotone paths that covers every marked cell.

4. The island of Sodor is home to a large number of villages and towns, connected by extensive railway networks. Recently, several cases of deadly contagious disease have been reported in the village of SKARLOEY. The controller of Sodor railway authority chooses to close down certain railway stations to prevent the disease from spreading into TIDEMOUTH, his hometown. No trains can pass through a closed station. To minimize his expenses, he chooses to close down the minimum number of railway stations.

   Describe and analyze a polynomial-time algorithm that finds the minimum number of stations that must be closed so that the disease cannot spread to TIDEMOUTH.

5. The CS department of a university has a flexible curriculum with a complicated set of graduation requirements. The department offers n courses and there are m different requirements. Each requirement specifies a subset A of courses and the number of courses that must be taken from the subset. The subsets for different requirement may overlap, but each course can be used to satisfy at most one requirement. For example, there are 5 courses X_1, X_2, X_3, X_4, X_5 and $m = 2$. The requirements are

   - At least two courses can be taken from $\{X_1, X_2, X_3\}$.
   - at least two courses can be taken from $\{X_3, X_4, X_5\}$.

   A student cannot graduate by taking courses $\{X_2, X_3, X_4\}$ but can graduate by taking courses $\{X_1, X_2, X_3, X_4\}$.

   Describe and analyze a poly$(n + m)$-time algorithm that determines whether a student can graduate. The input to your algorithm is a list of n courses, a set of m requirements, and a list of courses that the student has taken.

6. Due to a large scale flooding in a region, paramedics have identified a set of n injured people distributed across the region who need to be rushed to hospitals. There are m hospitals in the region, and each of the n people need to be brought to a hospital that is within a 45 minutes driving time of their current location (so different people have different set of options right now).

   At the same time, no hospital is allowed to be overloaded by taking too many patients its way. The paramedics are in touch by the cell phone and they want to collectively work out whether they can choose a hospital for each of the injured people in such a way that the load on the hospitals is balanced; i.e. every hospital receives at most $\lceil n/k \rceil$ people.

   Design and analyze a polynomial-time algorithm that takes the given information about people's locations and determines whether every person can be brought to some hospital or not.

7. Suppose you and your friend Alanis live together with $n - 2$ other people, at a cooperative apartment. Over the next n nights each of you is supposed to cook dinner for the co-op exactly once, so that someone cooks on each of the nights.

   Of course, everyone has scheduling conflicts with some of the nights (e.g. exams, concerts, etc.), so deciding who should cook on which night becomes a tricky task. For correctness the list of people is $\{p_1, \ldots, p_n\}$ and the nights are $\{d_1, \ldots, d_n\}$; and for person p_i, there is a set of nights $S_i \subseteq \{d_1, \ldots, d_n\}$ when they are not able to cook.

   A *feasible dinner schedule* is an assignment of each person in the co-op to a different night, so that each person cooks on exactly one night, there is someone cooking on each night, and if p_i cooks on night d_j, then $d_j \notin S_i$.

   Design a polynomial-time algorithm that can decide whether there is a feasible dinner schedule for this co-operative apartment.