Quiz-2 (ADA-2024)

February 14, 2024

Roll Number: **Section:**

1. Which of the following is true?

(A) While solivng original problem Dynamic programming divides problem into non-overlapping subproblems.

(B) Divide and conquer divides a problem into subproblems while dynamic programming does not divide into subproblems.

(C) Dynamic programming does not always guarantee an optimal solution.

(D) While solivng original problem Dynamic programming divides problem into overlapping subproblems.

Option (D) is correct.

2. Let T be a node weighted rooted tree that is not necessarily a binary tree. The weight function is $\mathsf{wt} : V(T) \to \mathbb{R}^+$. Moreover, let the root of the tree be r. Also, for every node x, let T_x denotes the subtree of T rooted at x. Let $\mathsf{MVC}(x, 0)$ denotes the total weight of a minimum weighted vertex cover of T_x that does not contain x and $\mathsf{MVC}(x, 1)$ denotes the total weight of a minimum weighted vertex cover of T_x that does contains x. Let Y denotes the set of all children of x. Clearly, $Y = \emptyset$ if x is a leaf and $Y \neq \emptyset$ otherwise. Which of the following statements is correct?

(A) The subproblem $\mathsf{MVC}(r, 1)$ returns the value of an optimal solution for the tree T.

(B) If $Y \neq \emptyset$, then $\mathsf{MVC}(x, 1) = \mathsf{wt}(x) + \sum_{y \in Y} \mathsf{MVC}(y, 0)$.

(C) If $Y \neq \emptyset$, then $\mathsf{MVC}(x, 0) = \sum_{y \in Y} \mathsf{MVC}(y, 1)$.

(D) If $Y \neq \emptyset$, then $\mathsf{MVC}(x, 0) = \mathsf{wt}(x) + \sum_{y \in Y} \mathsf{MVC}(y, 0)$.

Option (C) is correct

3. Suppose that there are n weeks, a plan is specified by a 'low stress job', a 'medium stress job', and a 'high stress job' or 'none' in each of the weeks. For any $i \geq 3$, if a high stress job is chosen for the week i, then no job can be chosen in the week $i - 1$ and $i - 2$. If a medium stress job is chosen for the week $i \geq 2$, then no job can be chosen at the week $i - 1$. However, if a low stress job is chosen for the week i, then both the high stress, medium stress, and low stress job can be chosen for the week $i - 1$. In the week 1, a high stress job, or a medium stress job, or a low stress job both can be chosen. Similarly, in the second week, a high stress job can be chosen if no job was chosen in the first week. A low stress job gives revenue a_i, a medium stress job gives revenue b_i, and a high stress job gives a revenue of c_i. The objective is to design an algorithm that maximizes the revenue.

Let $\mathsf{REVENUE}(k)$ denotes the maximum possible revenue in the weeks $\{1, 2, \ldots, k\}$. Which of the following options is correct?

(A) For every $k \geq 4$, the value $\mathsf{REVENUE}(k) = \max \begin{cases} b_k + \mathsf{REVENUE}(k-1) \\ c_k + \mathsf{REVENUE}(k-2) \\ a_k + \mathsf{REVENUE}(k-3) \end{cases}$

(B) For every $k \geq 4$, the value $\mathsf{REVENUE}(k) = \max \begin{cases} c_k + \mathsf{REVENUE}(k-1) \\ a_k + \mathsf{REVENUE}(k-2) \\ b_k + \mathsf{REVENUE}(k-3) \end{cases}$

(C) For every $k \geq 4$, the value $\mathsf{REVENUE}(k) = \max \begin{cases} b_k + \mathsf{REVENUE}(k-1) \\ a_k + \mathsf{REVENUE}(k-2) \\ c_k + \mathsf{REVENUE}(k-3) \end{cases}$

(D) For every $k \geq 4$, the value $\mathsf{REVENUE}(k) = \max \begin{cases} a_k + \mathsf{REVENUE}(k-1) \\ b_k + \mathsf{REVENUE}(k-2) \\ c_k + \mathsf{REVENUE}(k-3) \end{cases}$

Option (D) is correct.

4. Which of the following is not true?

(A) Memoization makes recursive function calls to solve subproblems.

(B) Table-Filling starts from the simplest subproblem and gradually reaches up to the original problem.

(C) Unlike memoization, table-Filling method does not use solutions of subproblem while solving the original problem.

(D) Memoization starts from the original problem and gradually breaks it down into subproblems.

Option (C) is correct

5. Suppose that an equipment manufacturing company manufactures s_i units in the i-th week. Each week's production has to be shipped by the end of that week. Every week, one of the two shipping agents A and B are involved in shipping that week's production and they charge in the following:

- Company A charges a rupees per week but will only ship for a block of three consecutive weeks.
- Company B charges b rupees per unit.

The objective is to find the minimum cost of a schedule. Let $\mathsf{Cost}(k)$ denotes the minimum cost of any schedule for the weeks $\{1, 2, \ldots, k\}$. Which of the following options is correct?

(A) For all $k \geq 4$, $\mathsf{Cost}(k) = \min \begin{cases} 3a + \mathsf{Cost}(k-3) \\ bs_k + \mathsf{Cost}(k-1) \end{cases}$

(B) For all $k \geq 4$, $\mathsf{Cost}(k) = \min \begin{cases} 3a + \mathsf{Cost}(k-2) \\ bs_k + \mathsf{Cost}(k-1) \end{cases}$

(C) For all $k \geq 4$, $\mathsf{Cost}(k) = \min \begin{cases} as_k + \mathsf{Cost}(k-1) \\ 3b + \mathsf{Cost}(k-3) \end{cases}$

(D) For all $k \geq 4$, $\mathsf{Cost}(k) = \min \begin{cases} as_k + \mathsf{Cost}(k-1) \\ 4b + \mathsf{Cost}(k-3) \end{cases}$

Option (A) is correct