

Q1) CHALLENGE ACCEPTED (CAKEWALK)

topic: RECURSION

Odyssey has concluded, but no one can forget it, particularly the performance by Shankar–Ehsaan–Loy. All ADA students are feeling melancholic. Udit, the most generous individual in our college, has come up with a challenge to reignite the enthusiasm for ADA among students. Udit always remains alone due to his obsession with subsequences, and his challenge is evidence of it.

Challenge: You are given an array with m elements. Udit asks you to determine whether a subsequence exists in that array where the sum of elements of that subsequence is a multiple of n . However, as you all know, Udit is the most generous person in our college. He **removes any one element** from the array that you don't know. Your task is to check whether the **remaining array** formed by removing "any one" element has a **subsequence whose sum** is a multiple of n (sarcasm spotted). Here, the selected subsequence must contain at least one element.

A subsequence of an array A , denoted as B , can have none, some, or all elements of array A . For example, if $A=\{1,2,3\}$, then its subsequences are $\{\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}$, and $\{1,2,3\}$. (According to the question, null subsequence is not allowed to use)

If a subsequence exists from the remaining array after removing any one element whose sum is divisible by n , then print "YES"; otherwise, print "NO".

Input: t (number of test cases), m (number of elements in the array), n (check whether the subsequence sum is divisible by n), arr (array with m elements)

Output: "YES" or "NO"

Test Case:

$t=1$
 $m=4, n=4$
 $arr=\{1, 2, -2, -1\}$

Output = "YES"

Explanation:

If we delete 1: Subsequence: $2 + (-2) = 0$

If we delete 2: Subsequence: $1 + (-1) = 0$

If we delete -2: Subsequence: $1 + (-1) = 0$

1If we delete 1: Subsequence: $2 + (-2) = 0$

We are getting a subsequence divisible by k, no matter which element is deleted; therefore the answer is 'YES'

SOLUTION:

Pseudocode:

```
help(m,n,arr,skip,j,sum,flag):
```

```
    // base cases
```

```
    If (sum divisible by n and flag==1):
```

```
        return TRUE
```

```
    If j==m:    // array ends
```

```
        return FALSE
```

```
    If j==skip:
```

```
        return help(m,n,arr,skip,j+1,sum,flag)
```

```
    // recurrence relation
```

```
    If (help(m,n,arr,skip,j+1,sum+arr[j],TRUE)==TRUE or
```

```
help(m,n,arr,skip,j+1,sum,flag)==TRUE):
```

```
        return TRUE
```

```
    Else: return FALSE
```

```
CHALLENGEACCEPTED(m,n,arr):
```

```
    flag=TRUE
```

```
    For (i: 1 to m):
```

```
        If help(m,n,arr,i,0,0,0)==FALSE:
```

```
            flag=FALSE
```

```
            break
```

```
    If flag==TRUE:
```

```
        print("YES")
```

```
    Else:
```

```
        print("NO")
```

Time Complexity: exponential:- $O(2^m)$

Code for Reference:

```
#include<stdio.h>

int help(int n,long long int arr[],int skip,int j,long long sum,int k,int null){
    if(sum%k==0 && null==1){
        return 1;
    }
    if(j==n){
        return 0;
    }
    if(j==skip){
        return help(n,arr,skip,j+1,sum,k,null);
    }
    int x1=help(n,arr,skip,j+1,sum+arr[j],k,1);
    int x2=help(n,arr,skip,j+1,sum,k,null);
    if(x1==1 || x2==1){
        return 1;
    }
    else{
        return 0;
    }
}

int main(){
    int t;
    scanf("%d",&t);
    for(int i=0;i<t;i++){
        int n;
        scanf("%d",&n);
        int k;
        scanf("%d",&k);
        long long int arr[n];
```

```

        for(int j=0;j<n;j++){
            scanf("%lld",&arr[j]);
        }
        int flag=1;
        for(int j=0;j<n;j++){
            int x=help(n,arr,j,0,0,k,0);
            if(x==0){
                flag=0;
                break;
            }
        }
        if(flag==1){
            printf("YES");
        }
        else{
            printf("NO");
        }
        printf("\n");
    }
}

```

Q2) THE BATTLE OF HOGWARTS (EASY-MEDIUM)

topic: BINARY SEARCH

The battle of Hogwarts has finally begun, and it is Harry's team vs Voldemort's Army.

Voldemort has divided his army into b bases (spread throughout Hogwarts). Each base has a defensive power d , and w innocent wizards which it is keeping hostage and whom they threaten to kill, if Harry doesn't surrender.

Seeing this, Harry also decided to divide his team into n teams, each with a certain attacking power a . A team can attack all bases which have a defensive power less than or equal to its attacking power. If a team attacks a base it saves all the wizards in that base.

Harry is still undecided which team to send out first, so he would like to know for each team what is the maximum amount of wizards it can save.(independently of all others).

Testcase:

n=5 , b=4

a= [1 3 5 2 4] (attacking powers)

d, w (defensive powers and wizards at a base) - 4 bases in total

0 1

4 2

2 8

9 4

ans= [1 9 11 9 11]

The first team can only attack the first base

The second team can attack the first and third bases

The third team can attack the first, second and third bases (1 + 2 + 8= 11) and so on

Solution:

Sort the bases in increasing order of their defensive strength.

Then, compute the prefix sum of the wizards (that is, **prefix[j]** would hold the sum of wizards of all the bases from **0...j**).

Finally, for each team, perform a binary search over the prefix sum and the bases arrays to compute how much wizards it could save.

Overall Time complexity: $O(s \log b + b \log b)$.

Code for Reference:

```
#include <bits/stdc++.h>
using namespace std;
typedef long long int ll;
#define pb push_back

int main()
{
    ios_base::sync_with_stdio(false);
    int n,b,d,w;
    cin>>n>>b;
```

```

vector<int> powers(n,0);
vector<pair<int,int>> bases(b+1,{0,0});
vector<int> p_sum(b+1,0);

for(int i=0;i<n;i++)
{
    cin>>powers[i];
}

for (int i=1;i<=b;i++)
{
    cin>>d>>w;
    bases[i]={d,w};
}
sort(bases.begin(),bases.end());
vector<int> defences(b+1,0);
for (int i=0;i<=b;i++)
{
    defences[i]=bases[i].first;
}

for(int i=1;i<=b;i++)
{
    p_sum[i]=p_sum[i-1]+bases[i].second;
}

for (int i=0;i<n;i++)
{
    int x=lower_bound(defences.begin(),defences.end(),powers[i])
    x=x-defences.begin(); // finding the index
    if (defences[x]!=powers[i]) x--;
    cout<<p_sum[x]<<" ";
}
cout<<endl;
}

```

```

int f2 = count(s.begin(),s.begin()+mid,c)           // Count occurrences of c in l
ans2+=(s.size()/2 - f2)
return min(ans1,ans2)                               //Return the minimum of the two computations

```

This will return the minimum energy required to do the task at hand.

Overall Time complexity- $O(n \log n)$

Sample Test Cases:

$n=8$, $s="cdbbaaaa"$

In this case, the answer will be 0. This is because it is already the perfect string for the character 'a'. Explanation-

- the second half of the string ("aaaa") consists of only the character 'a'.
- the first half of the string ("cdbb") is a '*b*'-good string, because:
 - the second half of the string ("bb") consists of only the character 'b'
 - the first half of the string ("cd") is a '*c*'-good string, because:
 - the first half of the string ("c") consists of only the character 'c';
 - the second half of the string ("d") is the 'd'-good string.

$n=8$, $s="ceaaaabb"$

In this case, the answer will be 4.

There will be several ways to do this. Here is one of them:

- we make sure that the first half of the string ("ceaa") consists of only character 'a'. This increases our answer by 2 and converts the string into "aaaa". ($ans=2$)
- in the second half ("aabb"), we further divide the string:
 - the right half of the string ("bb") is already perfect for character 'b'.
 - the left half of the string ("aa") can be made perfect for character 'c' in the following manner:
 - the right half ("a") will convert to "c". ($ans=3$)
 - the left half will convert to "d". (or vice versa) ($ans=4$)

Thus, in 4 steps, we convert it to a perfect string for letter 'a'.

Code for reference:

```
#include <bits/stdc++.h>
using namespace std;

int recur(string &s, char c){
    if (s.size()==1)
        return c!=s[0];

    int mid=(s.size())/2;
    string l=string(s.begin(),s.begin()+mid); // left half
    string r=string(s.begin()+mid,s.end()); // right half

    int ans1=recur(l,c+1);
    int ans2=recur(r,c+1);

    int f1=count(s.begin()+mid,s.end(),c);
    int f2=count(s.begin(),s.begin()+mid,c);

    ans1+=(s.size()/2-f1);
    ans2+=(s.size()/2-f2);
    return min(ans1,ans2);
}

void solve(int test_case)
{
    long long n;
    string s;
    cin >> n >> s;
    cout<<recur(s,'a')<<"\n";
}

int32_t main()
{
    long long num_test_case = 1;
    cin >> num_test_case;
    for (long long i = 1; i <= num_test_case; i++)
        solve(i);
    return 0;
}
```

Q4) POKEMON MASTER QUEST (HARD)

topic: DNC (Divide & Conquer)

Ash has set out on his journey to become a pokemon master. To choose his starter pokemon he is currently standing in professor Oak's lab . Professor Oak has n pokemon (all standing next to each other) for Ash to choose from . Ash can choose any number of pokemon to start with, however he wants to choose only that subset which is most friendly.

Each pokemon has a friendliness stat represented by a_i (a is array of length n) . To help him, Professor provides him with a list of q numbers, the i th of which is the total friendliness stat choosen by the greatest pokemon trainers of history when they started out on their journey.

For all q numbers Ash wants to know whether he can choose some pokemon with total stat = q_i . However there is a condition, he can only choose pokemon in the following way :-

- assume $mid = \lfloor \frac{\max(\text{array}) + \min(\text{array})}{2} \rfloor$, where \max and \min — are functions that find the maximum and the minimum stats . In other words, mid is the sum of the maximum and the minimum friendliness stat divided by 2 rounded down.
- Then the array of pokemon is split into two parts left and right. The left array contains all Pokemon which have stat less than or equal mid , and the right array contains all pokemon which have stat greater than mid . Pokemon in left and right keep their relative order from array.
- During the third step we choose which of the left and right arrays we want to keep. The chosen array replaces the current one, and the pokemon in the other array get angry and run away.

You need Ash to find the result of all q tests. For each test, he starts again with the same set of n Pokemon (whose stats are represented by array a).

Solution:

To begin with, you can notice that the split operation does not depend on the order of the 'a' array. So we can sort it.

Now let's build a tree of transitions from the original array to all its possible states. You can simply prove that the height of this tree does not exceed $\log(\max)$. Since $\max(\text{current } a) - \min(\text{current } a)$ after each operation of the section is reduced at least twice.

Having understood this, we can write a simple recursive search over the states (left,right). The state will describe a sub-segment of the 'a' array that is the current array. For each state, we can calculate the current amount (on the segment from left to right) and add it to any convenient collection (set).

Next, to respond to requests, we can simply look at our collected collection

Time complexity: $O(n \cdot \log n + q \log l)$ l is len of set where $l \leq n$

CODE FOR REFERENCE:

```
#include <bits/stdc++.h>
using namespace std;

typedef long long int ll;
#define pb push_back

void bisect (vector<int> &arr, set<ll> &st,int low,int high)
{
    if (low>high) return;

    int mid=(arr[low]+arr[high])/2;
    ll sum=0;
    int idx=-1;

    for (int i=low;i<=high;i++)
    {
        sum+=arr[i];
        if (arr[i]<=mid) idx=i;
    }

    st.insert(sum);
    if (idx==high) return;

    bisect(arr,st,low,idx);
    bisect(arr,st,idx+1,high);
}

int main()
{
    ios_base::sync_with_stdio(false);
    int t,n,q,s;

    cin>>t;
```

```
for(int i=0;i<t;i++)
{
    cin>>n>>q;
    vector<int> arr(n,0);
    for(int i=0;i<n;i++)
    {
        cin>>arr[i];
    }

    set <ll> st;
    sort(arr.begin(),arr.end());
    bisect(arr,st,0,n-1);

    for (int i=0;i<q;i++)
    {
        cin>>s;
        if (st.find(s)==st.end()) cout<<"No"<<endl;
        else cout<<"Yes"<<endl;
    }
}
```