

• Running time of the algorithm.

The Ford-Fulkerson's algorithm runs in time $O(V^2 E)$ where V is the number of vertices and E is the number of edges. Observe that the value of maximum flow is $\sum_{j=1}^m f_j$ and for every $j = 1, \dots, m$, $f_j \leq u_j$. Therefore, value of the maximum flow $\leq \sum_{j=1}^m u_j \leq m \cdot \max_j u_j$. The number of vertices is $n+2$ and the number of edges is $O(m)$. Hence, it follows that the running time of the Ford-Fulkerson's algorithm is $O(n^2 m \cdot \max_j u_j)$. Finally, the construction of the graph takes $O(m)$ time. Hence, the running time of the algorithm is $O(n^2 m^2)$.

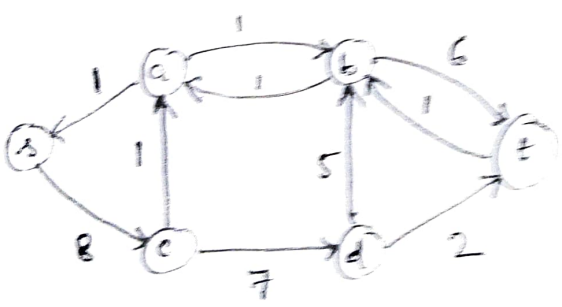
8 (70 Marks) Consider the following flow network. The numbers denote the capacities of the edges. Illustrate the execution of Ford-Fulkerson's Algorithm to compute a maximum flow from s to t in this network. Your illustration must show step by step execution of the Ford-Fulkerson's Algorithm, particularly the changes in the residual graph and the flow value at every edge at each step.



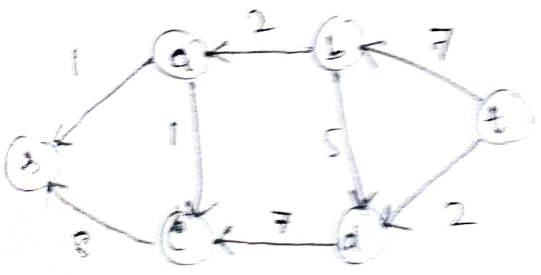
Figure 1: Flow network

All edges take
unit capacity
Flow value = Capacity
for all edges

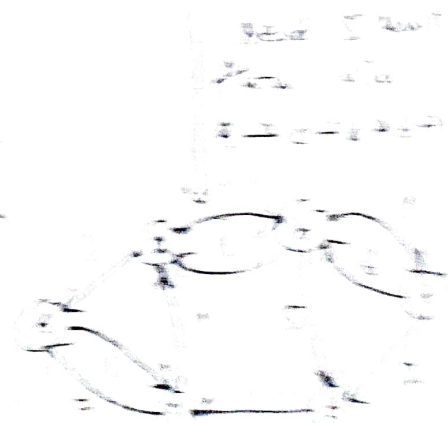
(1 unit)
Send flow via path $s \rightarrow a \rightarrow b \rightarrow t$



Send 2 units
flow via
 $s \rightarrow c \rightarrow d \rightarrow t$



Send 1 unit
flow via
 $s \rightarrow c \rightarrow d \rightarrow t$



(5
+2) marks for showing stage by stage
residual graph.

(2
+1) marks for mentioning flow value (max flow)
and ~~and~~ final value of flow for every edge