SML

MidSem, Winter 2024

Q1. Consider the derivation of PCA. It first centralizes the data by removing the mean, then computes covariance matrix. Then the PCA matrix is simply the eignevectors of this covariance matrix in decreasing order of eigenvalues. Suppose we change the overall formulation in the following manner-

- We do not centralize the data, that is the mean is not removed.
- Instead of assuming the principal component vectors to be unit magnitude, we assume that the projected mean has a unit magnitude, that is the squared l_2 norm of the projected mean is 1.
- We still maximize the variance of the projected samples.

Assuming the data matrix to be $X \in \mathbb{R}^{d\times N}$, compute single projection vector $u \in \mathbb{R}^d$ which satisfies the above formulation. You need to express the answer in terms of eigenvector equation involving terms of covariance and mean of X. [3]

Q2. Similar to the above question, let us reformulate FDA. Consider a two category case. Let the data matrices be $X_1 \in \mathbb{R}^{d\times N}$ and $X_2 \in \mathbb{R}^{d\times N}$. Let the FDA vector be $w \in \mathbb{R}^d$.

- Instead of maximizing the distance between projected means, let the distance between projected means is set to 1.
- We maximize the scatter of the projected samples. Note here we consider all samples from both categories. This step is similar to what we do in PCA.
- We minimize the within class scatter.

a. Give the objective function in terms of w that is to be maximized or minimized. [1.5]

b. Find w. You can express the answer in terms of generalized eigenvalue problem. [1.5]

Q3. Find the decision boundary by considering the discriminant functions for the following -

$$p(x|\omega_1) = \Pi_{i=1}^{d} \theta_i^{x_i}(1-\theta_i)^{1-x_i}\theta_i^{\alpha_i-1}(1-\theta_i)^{\beta_i-1}/Z_i$$
$$p(x|\omega_2) = \Pi_{i=1}^{d} \phi_i^{x_i}(1-\phi_i)^{1-x_i}\phi_i^{\alpha_i-1}(1-\phi_i)^{\beta_i-1}/Z_i$$

Assume unequal priors. Z_i is a function of α_i and β_i. Express the answer in $w^\top x + b = 0$. [3]

Q4. Find the PCA matrix U for data matrix

$$X = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$$

Consider data matrix in the form of $d \times N$. For covariance, use unbiased estimate. [3]

Q5. Find MAP estimate of θ_i, where θ_i is the probability of x_i taking a value of 1. Following is a Bernoulli distribution for d dimensions, $p(x) = \Pi_{i=1}^{d} \theta_i^{x_i}(1-\theta_i)^{1-x_i}$, where x is a d dimensional vector and its elements are binary. Consider the following conjugate prior $\theta_i^{\alpha_i - 1}(1-\theta_i)^{\beta_i - 1}/Z_i$. Assume n i.i.d. samples. [3]