

# SML

## Practice

Q1. We derived NP criteria using Gaussians. Now consider the case where a matching is needed, for instance matching faces to determine a criminal, matching unique ID number such as Aadhar, finding a transmitted radio signal in a noisy environment. In all these cases, there are two possibilities, a match or no match, that is two categories. Suppose when there is no match this class is  $\omega_1$ , and when match happens it is class  $\omega_1$ . One can have several samples and obtain the distribution of these two classes. Let  $p(x|\omega_1)$  is standard Gaussian and  $p(x|\omega_2)$  is exponential pdf  $\theta e^{-\theta x}$  for  $x \geq 0$ , and 0 otherwise,  $\theta = 0.5$ . Suppose the error rate for the case when observed sample  $x$  belong to  $\omega_1$  but is classified as  $\omega_2$  is  $E_1 = 0.1$ . Find the error rate when observed sample  $x$  belongs to  $\omega_2$  but is classified as  $\omega_1$ .

Q2. Consider following data matrices for two classes:

$$X_1 = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

$$X_2 = \begin{pmatrix} 1 & -1 & 1 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

Here,  $X \in R^{d \times N}$  form. Find the class of sample  $x = [1, 1, -1, 1]^\top$ .

Q3. Derive MLE for arbitrary distribution  $p(x) = \mathcal{N}(\mu, 1) \exp(\mu)$ , that is the distribution is product of Gaussian with variance 1, and exponential pdf with parameter  $\mu$ .