SML 2024, Monsoon, Quiz 1, Dur. 1 hr 10 mins.

Q1. Consider a two-category problem. The likelihoods are given to be multivariate Gaussian. Both the classes have same covariance but different means. The means are $\mu_1 = [1, 0, 0]^\top$ and $\mu_2 = [0, 1, 0]^\top$. μ_1 and $\mu_2 \in R^d$, where $d = 3$. The covariance matrix is $\Sigma = I$, where I is the identity matrix. Find the class of sample $\mathbf{X} = [.5, 0.25, 1]^\top$ by finding the value of discriminant for each class. [2]

Q2. Using likelihood ratio test, find the class of $\mathbf{x} = [.25, .25, .5]^\top$. Assume the classes have identity covariance and equal priors. The values of $\lambda = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}$. The means are $\mu_1 = [0, 0, 0]^\top$ and $\mu_2 = [1, 1, 1]^\top$. μ_1 and $\mu_2 \in R^d$, where $d = 3$. [2]

Q3. A discriminant can be expressed in terms of likelihood and prior. Consider a case where likelihood is expressed as multivariate Gaussian, $p(\mathbf{x}|\mu_i, \Sigma) \sim N(\mu_i, I)$, for classes $i = 1, 2, .., c$ and I is identity matrix. Now let us modify this likelihood as,

$p(\mathbf{x}|\mu_i, \Sigma)p(\mu_i|\mu_0, \Sigma_0)$, where $p(\mu_i|\mu_0, \Sigma_0) \sim N(\mu_0, \Sigma_0)$. In other words, means μ_i are also random variables and are distributed according to multivariate Gaussian pdf.

Consider $p(\mathbf{x}|\mu_i, \Sigma)p(\mu_i|\mu_0, \Sigma_0)$ as the new likelihood with $\Sigma = I$ and find the discriminant assuming equal priors. Note that the distribution of mean μ_i is same across all classes. You must express the discriminant as $W^\top \mathbf{x} + b$ and remove the terms that do not impact discriminant for different classes. [2]

Q4. Consider the following

$$p(x|\omega_1) = ap(x|\mu_1, \sigma^2) + bp(x|\mu_2, \sigma^2)$$
$$p(x|\omega_2) = bp(x|\mu_1, \sigma^2) + ap(x|\mu_2, \sigma^2)$$

where $p(x|\mu_1, \sigma^2)$ is Gaussian with mean $\mu_1 = 1$, $\sigma^2 = 1$, and $p(x|\mu_2, \sigma^2)$ is Gaussian with mean $\mu_1 = -1$. $a << b$. Priors are equal. Assume that the two pdfs cross over each other only once. Find the decision boundary and total probability of error. You can express the error in the form of CDF and not necessarily give a numerical value. [2]