SML

Assignment

Q1. Consider two Cauchy distributions in one dimension

$$p(x|\omega_i) = \frac{1}{\pi b}\frac{1}{1+(\frac{x-a_i}{b})^2}, \quad i = 1,2$$

Assume $P(\omega_1) = P(\omega_2)$.

Find the total probability of error. Note you need to first obtain decision boundary using $p(\omega_1|x) = p(\omega_2|x)$. Then determine the regions where error occurs and then use $p(error) = \int_x p(error|x)p(x)dx$. Plot the the conditional likelihoods, $p(x|\omega_i)p(\omega_i)$, and mark the regions where error will occur. This shall be rough hand-drawn sketch. As $p(x)$ is same when equating posteriors, we can simply use $p(x|\omega_i)p(\omega_i)$. [1]

Q2. Compute the unbiased covariance matrix: [0.5]

$$X = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

Here, $X \in R^{d\times N}$ form.

Q3.a. In multi-category case, probability of error $p(error)$ is given as 1-$p(correct)$, where $p(correct)$ is the probability of being correct. Consider a case of 3 classes or categories. Draw a rough sketch of $p(x|\omega_i)p(\omega_i)$ $\forall i = 1,2,3$. Give an expression for $p(error)$. Assume equi-probable priors for simplicity. [1]

b. Mark the regions if the three conditional likelihoods are Gaussians $p(x|\omega_i)$ $N(\mu_i, 1)$, $\mu_1 = -1, \mu_2 = 0, \mu_3 = 1$. Find the $p(error)$ in terms of CDF of standard distribution. [1]

Q4. Find the likelihood ratio test for following Cauchy pdf:

$$p(x|\omega_i) = \frac{1}{\pi b}\frac{1}{1+(\frac{x-a_i}{b})^2}, \quad i = 1,2$$

Assume $P(\omega_1) = P(\omega_2)$ and 0-1 loss. [1]