[Solution] Quiz - 2 (A)

Name:____________________________ Roll:_________________ Sign:_____________

Instructions

1. You have 30 minutes to answer all the questions in the space provided in the question paper.
2. Switch off your electronic devices and put them in your bag or pocket.
3. Please read all questions carefully before writing your answers. If you have any questions, do not discuss them with your neighbours; raise your hand we will come to you.

All The Best!

1. Let X be a positive random variable and 'a' be a positive constant. Which of the following statement is 'incorrect' for computing an upper bound on Pr(X ≥ a)?
   - ☐ It cannot be bounded if Var[X] is known to be a fixed value and E[X] is unknown.
   - ☐ It can be bounded if only $E[X^2]$ is known to be a fixed value.
   - ☐ It can be bounded if Var[X] and E[X] are known to be fixed values.
   - ☐ **It cannot be bounded if only $E[X^2]$ is known to be a fixed value.**

2. Consider you have a biased coin. You toss it for a pair of 2 times and observed (HH, HT). Let θ be the probability of getting heads. From the following options, which is the most likely θ_{MLE}?

☐ 0.3	☐ 0.5	☐ **0.7**	☐ 0.9

Solution: For every p ∈ {0.3, 0.5, 0.7, 0.9} compute Likelihood, $L(p) = p^3 \cdot (1-p)^1$. Since, Likelihood is maximum at p = 0.7 so, $\theta_{MLE} = 0.7$.

3. Consider an unbiased die of 6 sides. Let X be the value that we see when the die is rolled. As, we know that E[X] = 3.5, so with at least what probability a roll will return 1,2,3 or 4 (Use Markov's inequality)?

☐ 1/3	☐ **0.3**	☐ 2/3	☐ 0.4

4. Let $X_1, X_2, \ldots, X_n$ be independently and identically distributed (i.i.d.) random variables, where the expectation and variance of each variables are 0.3 and 0.2 respectively, (i.e., for every $1 \le i \le n$ $E[X_i] = 0.3$ and $Var(X_i) = 0.2$). Let , $Y = \frac{1}{n}\sum_{i=1}^{n} X_i$ then what is the minimum required value of n such that Y is between 0.2 and 0.4 with at least 0.9 probability?

Solution: Here E[Y] = E[$\frac{1}{n}\sum_{i=1}^{n} X_i$] = 0.3. Further, the Var(Y) = 0.2/n.

We require P(Y ∈ (0.2,0.4)) ≥ 0.9 or equivalently P(Y ≤ 0.2 or Y ≥ 0.4) ≤ 0.1.

Rewrite the event, (Y ≤ 0.2 or Y ≥ 0.4) as E_1: |Y – E[Y]| ≥ 0.1. Now, applying chebyshev's inequality we get,

$P(|Y - E[Y]| \ge 0.1) \le 0.2/(n \cdot 0.1^2)$. By simple algebra we need n ≥ 200, to bound its probability by at most 0.1.