Question 5 (10 marks)**.**

(a) (7 marks) For each $j \in \{0, \ldots, n\}$, let $p_j(x)$ be a nonzero polynomial of degree j. Show that the $\{p_0(x), \ldots, p_n(x)\}$ is a linearly independent subset of $\mathbb{P}_n$ (the vector space of all polynomials with real coefficients which have degree at most n).

Alternative Solution.

Let $S = \{p_0(x), \ldots, p_n(x)\}$.

Suppose if possible that S is linearly dependent.

By Proposition titled "Characterization of Linearly Dependent Sets" covered in class, there exists $j > 0$ such that p_j is a linear combination of $p_0, \ldots, p_{j-1}$.

But this is impossible because p_j is a polynomial of degree j, whereas a linear combination of polynomials of degree less than j cannot possibly be a polynomial of degree j.

Therefore the set S must be linearly independent.

Rubric.

- 1 mark for using the technique of proof by contradiction
- 3 marks for either using the Characterization of linearly independent sets or using any other reasonable argument to justify that one of the polynomials may be written as a linear combination of the others
- 3 marks for reasoning (the argument should be clear and obvious) that this leads to a contradiction.