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Tutorial - 6

1. a) $P(H) = \lambda$
$P(T) = 1-\lambda$
$P(\text{first head at } k+1 \text{ toss}) = (1-\lambda)^k \lambda$

b) Let M be the no. of tosses to get first head.
Let $S = E[M]$
As tosses are ind. & eqn. is additive,
$S = \lambda \times 1 + (1-\lambda)(S+1)$
$\Rightarrow \lambda S = 1$
$\Rightarrow S = 1/\lambda$

2. X: random var.
a) To prove: $var(X) = E[X^2] - (E[X])^2$
$LHS = var(X) = E[(X - E[X])^2]$
$= E[X^2 + (E[X])^2 - 2X E[X]]$
$= E[X]^2 + (E[X])^2 - 2(E[X])^2$
$= E[X]^2 - ~~2~~ (E[X])^2$
$(\because E(X E[X]) = (E[X])^2)$
$\Rightarrow LHS = RHS$
Hence proved.

b) $E(X) = 0$, $E[X^2] = 1$
(i) $var(X) = E[X^2] - (E[X])^2$
$= 1$

(ii) $Y = a + bX$
$\rightarrow E[Y^2] = E[(a+bX)^2] = E[a^2 + b^2X^2 + 2abX]$
$= E[a^2] + E[b^2X^2] +$
$2E[abX]$

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$\Rightarrow E[Y^2] = a^2 + 2ab\, \cancel{E[X]}^{\,0} + b^2\, \cancel{E[X^2]}^{\,1}$
$= a^2 + b^2$

$\rightarrow E[Y] = E[a+bx] = E[a] + E[bx]$
$= a + b\, \cancel{E[X]}^{\,0}$
$= a$

$\rightarrow \text{var}(Y) = E[Y^2] - [E(Y)]^2$
$= (a^2+b^2) - a^2$
$= b^2$

3. $A \rightarrow$ Aku predicts given horse is winning horse
$\sim A \rightarrow$ " " " " is not " "
$B \rightarrow$ Event that given horse wins
$\sim B \rightarrow$ " " " " does not win

a) Given a horse, prob. it wins is
$P(B) = P(B, A) + P(B, \sim A)$
$= P(B|A)\,P(A) + P(B|\sim A)\,P(\sim A)$
$= 0.99 \times 10^{-5} + (1-0.99)(1-10^{-5})$
$\Rightarrow P(B) = 1.99 \times 10^{-5}$

b) Prob. that Aku predicts black beauty is winning is

$P(A|B) = \dfrac{P(A,B)}{P(B)} = \dfrac{P(A|B)\,P(B)}{P(B)}$
$= \dfrac{0.99 \times 10^{-5}}{1.99 \times 10^{-5}}$
$\Rightarrow P(A|B) = \underline{\underline{0.499}}$