SHOBHIT VERMA
2018130062

Page No. Date

Tutorial - 5
Evaluation & Measurement of Hypothesis Testing

① $H_0: P = 0.7$
$H_a: P \neq 0.7$
Level of significance $(\alpha) = 0.1$
test stat: binomial var. with $p = 0.7$, $n = 15$

$X = 8$ & $np = 15 \times 0.7 = 10.5$
$\therefore P = 2P(X \leq 8 \text{ when } p = 0.7)$
$= 2 \sum_{x=0}^{8} b(x, 15, 0.7)$

$= 2 \times 0.1311$
$= 0.2622$
$\because P > 0.1$, we do not reject H_0.
Hence, there is insufficient reason to doubt the builder's claim.

② $H_0: P = 0.6$
$H_a: P > 0.6$
Given: $x = 70$, $n = 100$, $p = 0.6$, $\alpha = 0.05$

$$z = \frac{x - np_0}{\sqrt{np_0q_0}}$$
$$= \frac{70 - 100 \times 0.6}{\sqrt{100 \times 0.6 \times 0.4}} = 2.04$$

$P = P(Z > 2.04)$
$\Rightarrow P = 0.0207$
$\because P < \alpha$, we reject H_0. Thus, the new drug is superior.

Page No.
Date

(3) Let p_1 be proportion of Mumbai voters
" p_2 " " " surrounding area residents.

$$\alpha = 0.05$$

$$\hat{p}_1 = \frac{120}{200} = 0.6$$

$$\hat{p}_2 = \frac{240}{500} = 0.48$$

$$\hat{p}_p = \frac{120 + 240}{200 + 500} = 0.514$$

$H_0: p_1 \le p_2$
$H_a: p_1 > p_2$

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}_p (1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\Rightarrow z = \frac{\cancel{0.5}\ 0.6 - 0.48}{\sqrt{(0.514)(1-0.514)\left(\frac{1}{200} + \frac{1}{500}\right)}}$$

$$\Rightarrow z = 2.869$$

$$P(z > \cancel{0}\,2.869) = 0.0044$$

$\because$ $p < \alpha$, the null hypothesis is rejected. Thus, the prop. of Mumbai voters favouring the proposal is higher than the prop. of surrounding area voters.

(4) a) $H_0: p = 0.2$ critical region is in
$H_a: p > 0.2$ right-tail

Page No.
Date

b) $H_0: \mu = 3$ critical region is in both tails
$H_a: \mu \neq 3$

c) $H_0: p = 0.15$ critical region is in left ~~both~~ tails
$H_a: p \leq 0.15$

d) $H_0: \mu = 500$ critical region is in left tail
$H_a: \mu > 500$

e) $H_0: \mu = 15$ critical region is in both tails
$H_a: \mu \neq 15$

(3) Let μ_1 = mean popn. 'robustness' of Comp. A
μ_2 = mean popn. 'robustness' of Comp. B

$H_0: \mu_1 = \mu_2$
$H_a: \mu_1 \neq \mu_2$
Given: $\alpha = 0.05$

$$\bar{x}_1 = \frac{1}{n_1} \sum_{i=1}^{n_1} x_{1i} = \frac{9.3+8.8+6.8+8.7+8.5+6.7+8+6.5}{10}$$

$$\Rightarrow \bar{x}_1 = 7.95$$

$$\bar{x}_2 = \frac{1}{n_2} \sum_{i=1}^{n_2} x_{2i} = \frac{11+9.8+9.9+10.2+10.1+9.7+11+11.1+10.2}{10}$$

$$\Rightarrow \bar{x}_2 = 10.26$$

$$s_1^2 = \frac{1}{(n_1-1)} \sum_{i=1}^{n_1} \left(x_{1i}^2 - n_1 \bar{x}_1^2\right) = \frac{10.865}{9}$$

$$\Rightarrow s_1^2 = 1.207$$

$$s_2^2 = \frac{1}{(n_2-1)} \left(\sum_{i=1}^{n_2} x_{2i}^2 - n_2 \bar{x}_2^2\right) = \frac{2.924}{9}$$

$\Rightarrow s_2^2 = 0.325$

$\because$ Sample var. are very diff., we can't assume popn. variances are equal. Thus, we will use the unpooled t-test.

$$v = \frac{\left(\left(\frac{s_1^2}{n_1}\right) + \left(\frac{s_2^2}{n_2}\right)\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{(n_1 - 1)} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{(n_2 - 1)}}$$

$$\Rightarrow v = \frac{(1.207/10 + 0.325/10)^2}{\frac{(1.207/10)^2}{9} + \frac{(0.325/10)^2}{9}}$$

$$\Rightarrow v = 10.3 \approx 10$$

Now, $t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$

For this example, $(\mu_1 - \mu_2) = 0$

$$\therefore t = \frac{(7.95 - 10.26) - 0}{\sqrt{\frac{1.207}{10} + \frac{0.325}{10}}} = -5.9$$

Now, critical value of t with $\alpha = 0.05$ & $v = 10$ d.o.f. is

$$t_{0.025, 10} = 4.587$$

Now, since $t < -t_{\alpha/2, v}$ the null hypothesis is rejected.