

Tutorial - 4

① Independent Component Analysis
Mixing statistically ind. sources.

$$\begin{aligned}
 \text{Var. of mixture} &= \text{var}(n) = \langle (n - \langle n \rangle)^2 \rangle \\
 &= \langle n^2 \rangle - (\langle n \rangle)^2 \\
 &= \langle (\sum w_i s_i)^2 \rangle - (\langle \sum w_i s_i \rangle)^2 \\
 &= \langle (\sum w_i s_i) (\sum w_j s_j) \rangle - (\sum w_i \langle s_i \rangle) (\sum w_j \langle s_j \rangle) \\
 &= \langle \sum w_i w_j s_i s_j \rangle - \sum w_i w_j \langle s_i \rangle \langle s_j \rangle \\
 &= \sum w_i w_j (\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle) \\
 &\quad + \sum w_i w_j (\langle s_i \rangle \langle s_j \rangle - \langle s_i \rangle \langle s_j \rangle) \\
 &= \sum w_i^2 (\langle s_i s_i \rangle - \langle s_i \rangle^2) \\
 &\quad + \sum w_i w_j (\langle s_i \rangle \langle s_j \rangle - \langle s_i \rangle \langle s_j \rangle) \\
 &\quad s_i \& s_j \text{ are statistically} \\
 &\quad \text{ind. for } i \neq j \\
 &\Rightarrow \langle s_i \rangle \langle s_j \rangle - \langle s_i \rangle \langle s_j \rangle = 0 \\
 &\quad \& \text{var}(s_i) = 1
 \end{aligned}$$

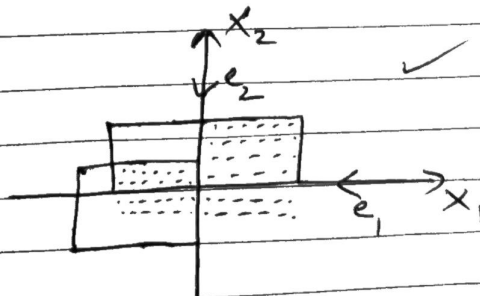
$$\therefore \text{var}(n) = \sum w_i^2$$

to guarantee that mixture has unit var,

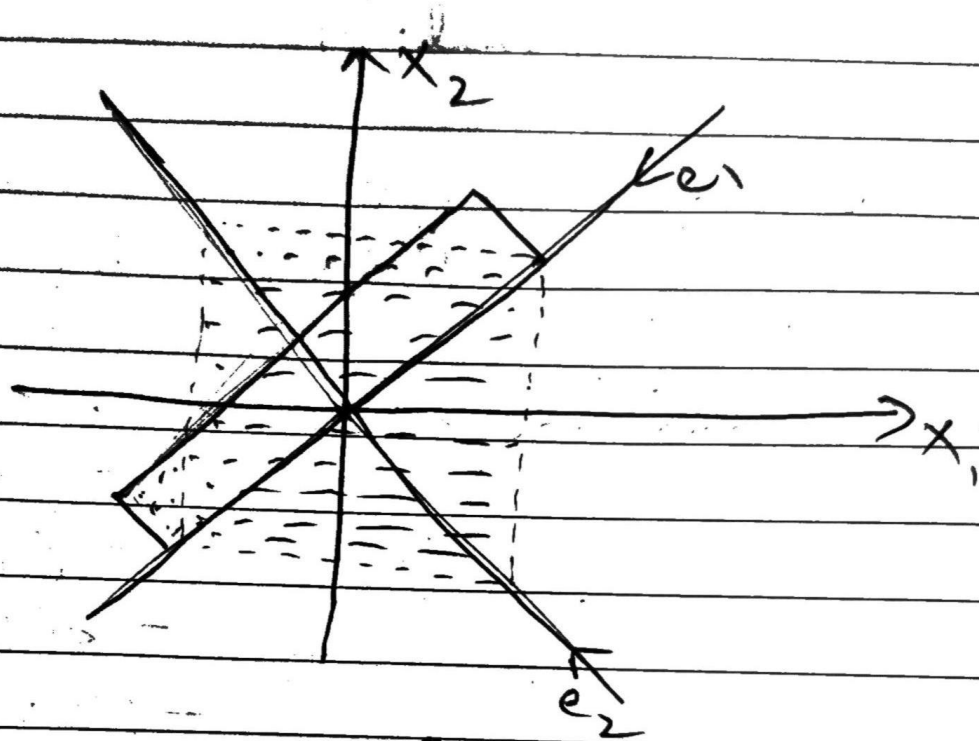
$$\text{var}(n) = 1$$

$$\Rightarrow \boxed{\sum w_i^2 = 1}$$

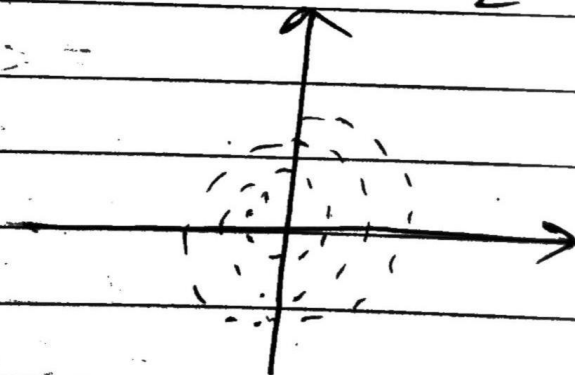
② a)



b)



c)



x can't be separated into ind. components