Text Classification 2 / Neural Network Basics

CSE 5525: Foundations of Speech and Language Processing

https://shocheen.github.io/cse-5525-spring-2026/

THE OHIO STATE UNIVERSITY

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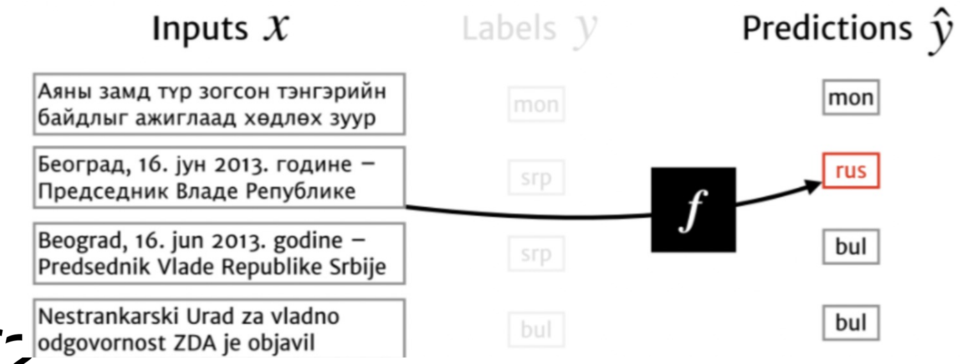
Slide Credits: Yulia Tsvetkov, Yoav Artzi, Greg Durrett

Logistics

- HW1 is due one week from today.
  - Have you started working on it?
  - Have you read the instructions / explored the code or dataset?
  - Any questions?

- Final Projects
  - A 1-2 page proposal for the final project will be due late February.
  - Please start forming your teams.
  - We will post example projects / default project details later this week, please talk to me or Abraham for advice on your project if you are going the custom route.


Inputs x
Labels y
Predictions $\hat{y}$
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Beograd, 16. jun 2013. godine – Predsednik Vlade Republike Srbije
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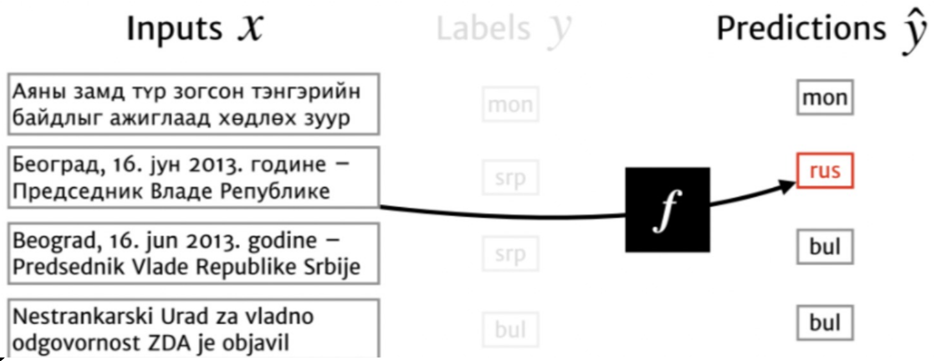
Recap: Text Classification

1. How do we evaluate our classifier *f*?
   - (Keyword for this section: … evaluation)

2. How do we “digest” text into a form usable by a function?
   - (Keywords for this section: features, feature extraction, feature selection, representations)

3. What kinds of strategies might we use to create our function *f*?
   - (Keyword for this section: models)

Recap: Text Classification



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Binary classification in logistic regression

- Given a series of input/output pairs:
  - $(x^{(i)}, y^{(i)})$

- For each observation $x^{(i)}$
  - We represent $x^{(i)}$ by a feature vector $\{x_1, x_2, \ldots, x_n\}$
  - We compute an output: a predicted class $\hat{y}^{(i)} \in \{0,1\}$

Features in logistic regression

- For feature $x_i \in \{x_1, x_2, \ldots, x_n\}$, weight $w_i \in \{w_1, w_2, \ldots, w_n\}$ tells us how important is x_i
  - x_i = "review contains 'awesome'": w_i = +10
  - x_j = "review contains horrible": w_j = -10
  - x_k = "review contains 'mediocre'": w_k = -2

How to do classification

- For each feature x_i, weight w_i tells us importance of x_i
  - (Plus we'll have a bias b)
  - We'll sum up all the weighted features and the bias

$$z = \left(\sum_{i=1}^{n} w_i x_i\right) + b$$

$$z = w \cdot x + b$$

If this sum is high, we say $y=1$; if low, then $y=0$

Formalizing “sum is high”

- We’d like a principled classifier that gives us a probability

- We want a model that can tell us:
    - $p(y=1|x;\theta)$
    - $p(y=0|x;\theta)$

The problem: z isn't a probability, it's just a number!

- z ranges from $-\infty$ to ∞

$$z = w \cdot x + b$$

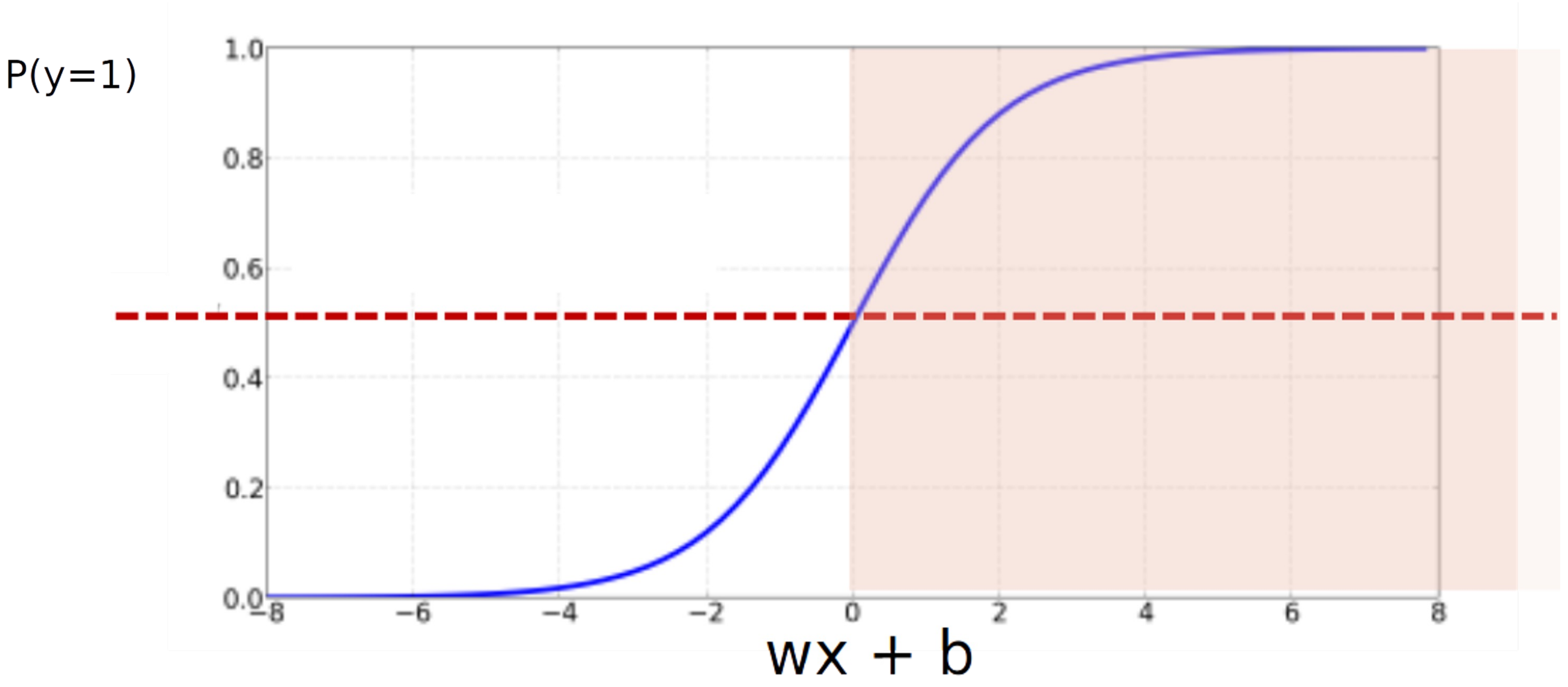
- Solution: use a function of z that goes from 0 to 1

"sigmoid" or
"logistic" function

$$y = \sigma(z) = \frac{1}{1+e^{-z}} = \frac{1}{1+\exp(-z)}$$

The probabilistic classifier

$$P(y=1) = \sigma(w \cdot x + b)$$
$$= \frac{1}{1+\exp(-(w \cdot x + b))}$$



Where do the weights (W) come from?

- Supervised classification:
  - At training time, we know the correct label y (either 0 or 1) for each x.
  - But what the system produces at inference time is an estimate $\hat{y}$

- We want to set w and b to minimize the **distance** between our estimate $\hat{y}^{(i)}$ and the true $y^{(i)}$
  - We need a distance estimator: a **loss function** or an objective function
  - We need an **optimization algorithm** to update w and b to minimize the loss

Learning components in LR

A loss function:

- **cross-entropy loss**

An optimization algorithm:

- **gradient descent**

Loss function: the distance between ŷ and y

We want to know how far is the classifier output ŷ $= \sigma(w \cdot x + b)$

from the true output: y [= either 0 or 1]

We'll call this difference: *L*(ŷ,y) = how much ŷ differs from the true y

Training Objective: Maximize the Likelihood of the Training Data.

We choose the parameters w,b that maximize

- the probability (aka likelihood)
- of the true y labels in the training data
- given the observations x

Training Data

- Our training data (also known as the training corpus) is a list of input/output pairs:
  - D = $[(x^{(1)}, y^{(1)}), \dots, (x^{(N)}, y^{(N)})]$
  - Each $x^{(i)}$ is a document (or a paragraph or a sentence) --- piece of text. Also called an observation.
  - Each $y^{(i)}$ is a label (0 or 1 in case of binary classification)

Deriving the objective for a single observation x

Goal: maximize likelihood of the correct label under the model

The predicted probability for class 1 is ŷ.

If the correct label is 1, then the likelihood is ŷ.
If the correct label is 0, then the likelihood is 1-ŷ

We can express the likelihood from our classifier:

$$p(y|x) = \hat{y}^{y}\,(1-\hat{y})^{1-y}$$

Deriving the objective for a single observation x

Goal: maximize likelihood

$$p(y|x) = \hat{y}^y (1-\hat{y})^{1-y}$$

Noting:

if $y=1$, this simplifies to $\hat{y}$
if $y=0$, this simplifies to $1 - \hat{y}$

Deriving the objective for a single observation x

Goal: maximize likelihood

$$p(y|x) = \hat{y}^y (1-\hat{y})^{1-y}$$

Now take the log of both sides (mathematically handy)
Maximize:

$$\begin{aligned}\log p(y|x) &= \log\left[\hat{y}^y (1-\hat{y})^{1-y}\right] \\ &= y\log\hat{y} + (1-y)\log(1-\hat{y})\end{aligned}$$

Deriving the objective for a single observation x

Goal: maximize likelihood

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$$\begin{aligned} \log p(y|x) &= \log \left[\hat{y}^y (1-\hat{y})^{1-y}\right] \\ &= y \log \hat{y} + (1-y)\log(1-\hat{y}) \end{aligned}$$

Whatever values maximize log p(y|x) will also maximize p(y|x)

Deriving the objective for a single observation x

Goal: maximize probability of the correct label $p(y|x)$

Maximize:
$$\begin{aligned}\log p(y|x) &= \log\left[\hat{y}^{y}\,(1-\hat{y})^{1-y}\right]\\ &= y\log\hat{y}+(1-y)\log(1-\hat{y})\end{aligned}$$

Now flip sign to turn this into a loss: something to minimize

Deriving the objective for a single observation x

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Now flip sign to turn this into a loss: something to minimize

Minimize:

$$L_{\mathrm{CE}}(\hat{y},y) = -\log p(y|x) \;=\; -[y\log\hat{y} + (1-y)\log(1-\hat{y})]$$

Deriving the objective for a single observation x

Goal: maximize probability of the correct label p(y|x)

Maximize:

$$\begin{aligned} \log p(y|x) &= \log \left[\hat{y}^{y}\,(1-\hat{y})^{1-y}\right] \\ &= y\log\hat{y} + (1-y)\log(1-\hat{y}) \end{aligned}$$

Now flip sign to turn this into a **cross-entropy loss**: something to minimize

Minimize:

$$L_{\mathrm{CE}}(\hat{y},y) = -\log p(y|x) \;=\; -[y\log\hat{y} + (1-y)\log(1-\hat{y})]$$

Deriving cross-entropy loss for a single observation x

Goal: maximize probability of the correct label $p(y|x)$

Maximize:
$$\begin{aligned}\log p(y|x) &= \log\left[\hat{y}^{y}\,(1-\hat{y})^{1-y}\right]\\ &= y\log\hat{y}+(1-y)\log(1-\hat{y})\end{aligned}$$

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Minimize:
$$L_{\mathrm{CE}}(\hat{y},y) = -\log p(y|x) \;=\; -[y\log\hat{y}+(1-y)\log(1-\hat{y})]$$

Or, plug in definition of $\hat{y} = \sigma(w{\cdot}x{+}b)$

$$\boxed{L_{\mathrm{CE}}(\hat{y},y) \;=\; -[y\log\sigma(\mathbf{w}\cdot\mathbf{x}+b)+(1-y)\log(1-\sigma(\mathbf{w}\cdot\mathbf{x}+b))]}$$

Our goal: minimize the loss

Let's make explicit that the loss function is parameterized by weights $\theta=(w,b)$

- And we'll represent $\hat{y}$ as $f(x;\theta)$ to make the dependence on θ more obvious

We want the weights that minimize the loss, averaged over all examples:

$$\hat{\theta} = \underset{\theta}{\operatorname{argmin}} \frac{1}{m} \sum_{i=1}^{m} \underbrace{L_{\mathrm{CE}}(f(x^{(i)};\theta),y^{(i)})}$$

$$L_{\mathrm{CE}}(\hat{y},y)$$

Let's see how it works for our sentiment example

We want loss to be:

- smaller if the model estimate $\hat{y}$ is close to correct
- bigger if model is confused

Let's first suppose the true label of this is $y=1$ (positive)

It's hokey . There are virtually no surprises , and the writing is second-rate . So why was it so enjoyable ? For one thing , the cast is great . Another nice touch is the music . I was overcome with the urge to get off the couch and start dancing . It sucked me in , and it'll do the same to you .

Let's see if this works for our sentiment example

True value is y=1 (positive). How well is our model doing?

$$
\begin{aligned}
p(+|x) = P(Y=1|x) &= \sigma(w \cdot x + b) \\
&= \sigma([2.5, -5.0, -1.2, 0.5, 2.0, 0.7] \cdot [3, 2, 1, 3, 0, 4.19] + 0.1) \\
&= \sigma(.833) \\
&= 0.70
\end{aligned}
$$

Pretty well!

Let's see if this works for our sentiment example

True value is $y=1$ (positive). How well is our model doing?

$$\begin{aligned} p(+|x) = P(Y=1|x) &= \sigma(w \cdot x + b) \\ &= \sigma([2.5, -5.0, -1.2, 0.5, 2.0, 0.7] \cdot [3, 2, 1, 3, 0, 4.19] + 0.1) \\ &= \sigma(.833) \\ &= 0.70 \end{aligned}$$

Pretty well! What's the loss?

$$\begin{aligned} L_{CE}(\hat{y}, y) = & \quad -[y \log \sigma(\mathbf{w} \cdot \mathbf{x} + b) + (1-y) \log(1 - \sigma(\mathbf{w} \cdot \mathbf{x} + b))] \\ = & \quad -[\log \sigma(\mathbf{w} \cdot \mathbf{x} + b)] \\ = & \quad -\log(.70) \\ = & \quad .36 \end{aligned}$$

Let's see if this works for our sentiment example

Suppose the true value instead was y=0 (negative).

$$\begin{aligned} p(-|x) = P(Y=0|x) &= 1-\sigma(w\cdot x+b) \\ &= 0.30 \end{aligned}$$

Let's see if this works for our sentiment example

Suppose the true value instead was $y=0$ (negative).

$$\begin{aligned} p(-|x) = P(Y=0|x) &= 1-\sigma(w\cdot x+b) \\ &= 0.30 \end{aligned}$$

What's the loss?

$$\begin{aligned} L_{\mathrm{CE}}(\hat{y},y) &= -[y\log\sigma(\mathbf{w}\cdot\mathbf{x}+b)+(1-y)\log(1-\sigma(\mathbf{w}\cdot\mathbf{x}+b))] \\ &= -[\log(1-\sigma(\mathbf{w}\cdot\mathbf{x}+b))] \\ &= -\log(.30) \\ &= 1.2 \end{aligned}$$

Let's see if this works for our sentiment example

The loss when the model was right (if true y=1)

$$\begin{aligned} L_{\text{CE}}(\hat{y},y) &= -[y\log\sigma(\mathbf{w}\cdot\mathbf{x}+b)+(1-y)\log(1-\sigma(\mathbf{w}\cdot\mathbf{x}+b))] \\ &= -[\log\sigma(\mathbf{w}\cdot\mathbf{x}+b)] \\ &= -\log(.70) \\ &= .36 \end{aligned}$$

The loss when the model was wrong (if true y=0)

$$\begin{aligned} L_{\text{CE}}(\hat{y},y) &= -[y\log\sigma(\mathbf{w}\cdot\mathbf{x}+b)+(1-y)\log(1-\sigma(\mathbf{w}\cdot\mathbf{x}+b))] \\ &= -[\log(1-\sigma(\mathbf{w}\cdot\mathbf{x}+b))] \\ &= -\log(.30) \\ &= 1.2 \end{aligned}$$

Sure enough, loss was bigger when model was wrong!

Learning components

A loss function:

- **cross-entropy loss**

An optimization algorithm:

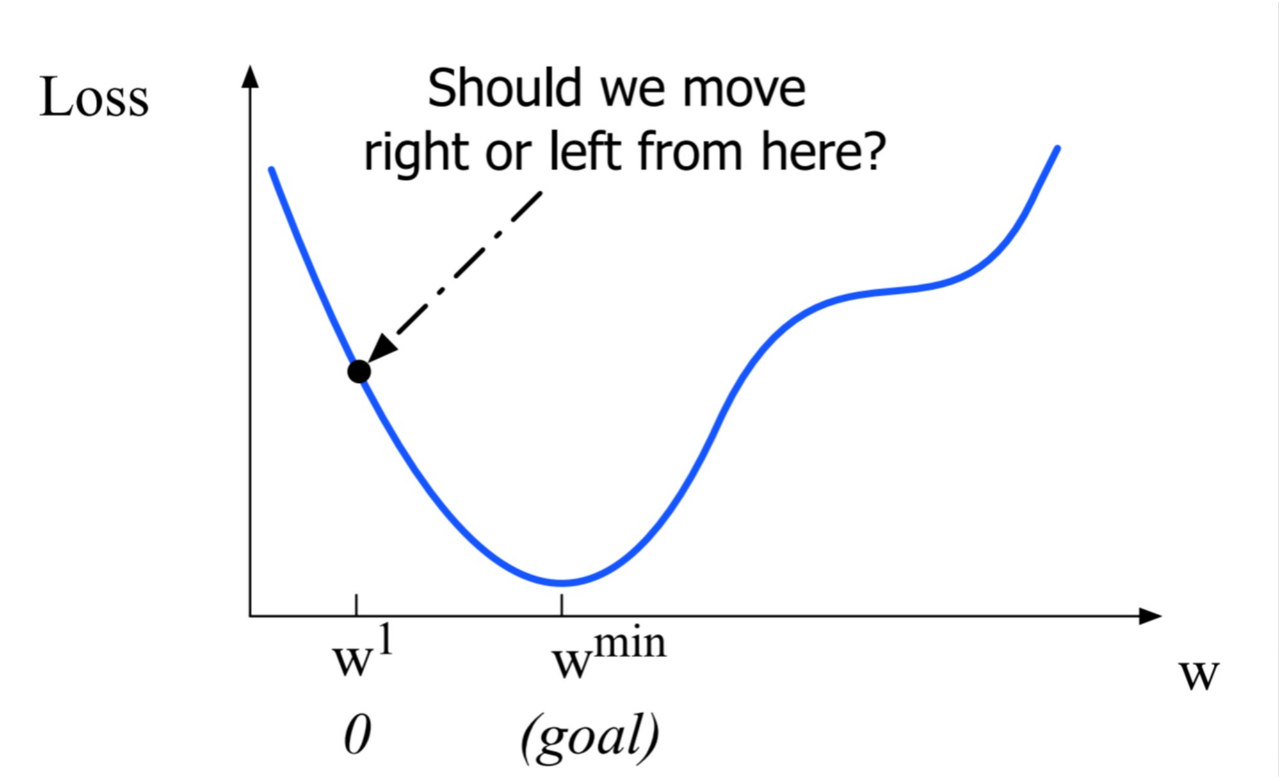
- **gradient descent**

Gradient Descent

- Gradient Descent algorithm
  - is used to optimize the weights for a machine learning model

Let's first visualize for a single scalar w

Q: Given current w, should we make it bigger or smaller?
A: Move w in the reverse direction from the slope of the function



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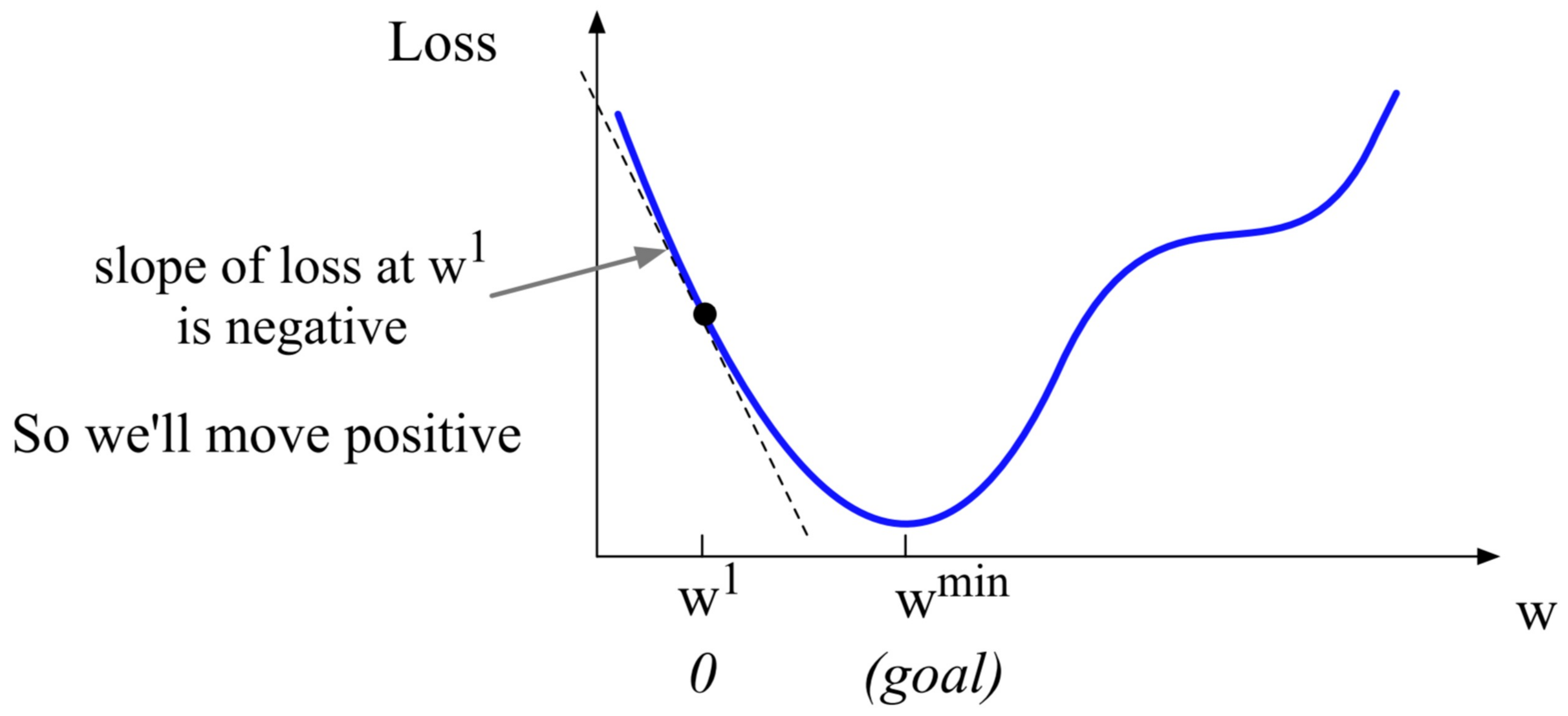

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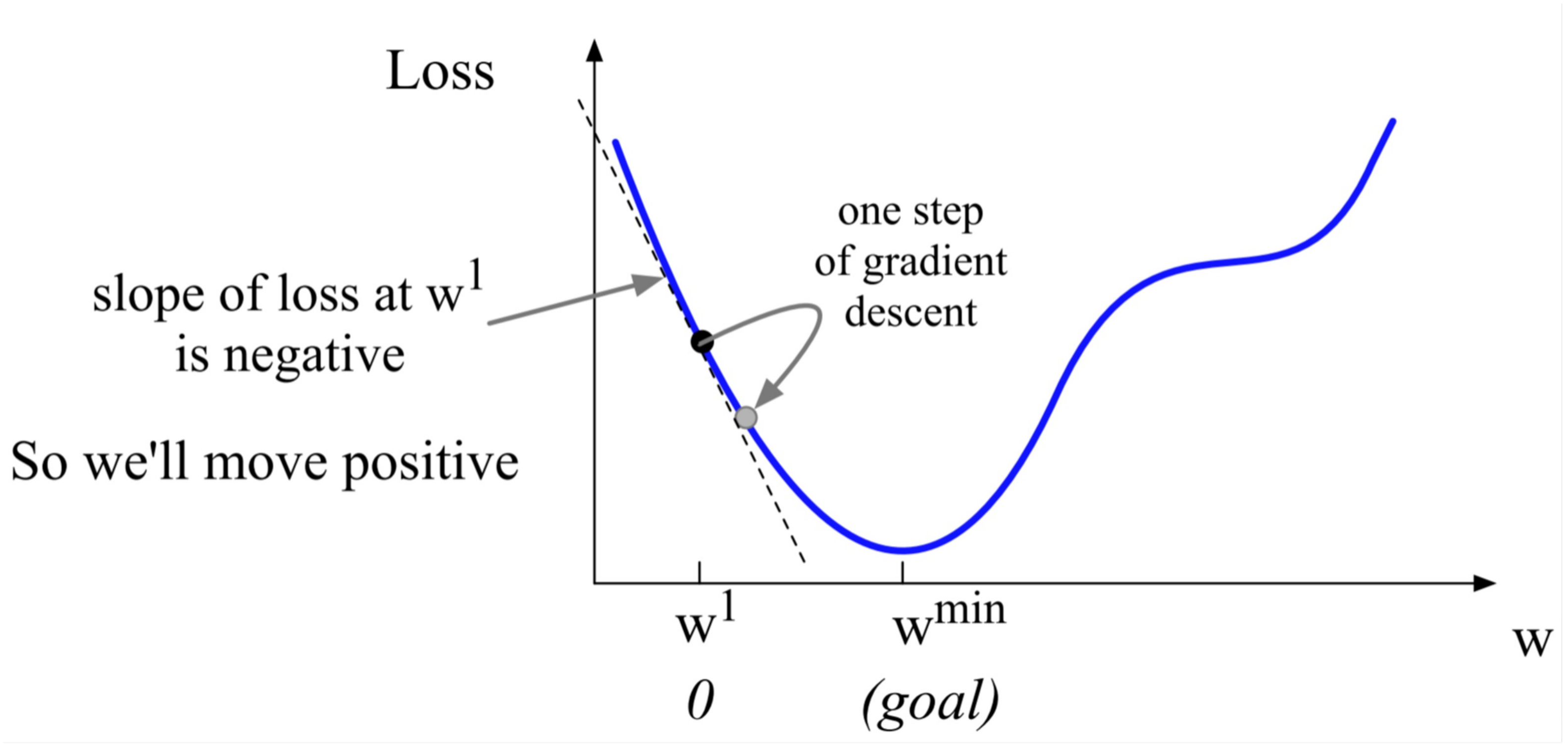

Loss
slope of loss at w^1 is negative
So we'll move positive
one step of gradient descent
w^1
0
w^{min}
(goal)
w

Our goal: minimize the loss

For logistic regression, loss function is **convex**

- A convex function has just one minimum
- Gradient descent starting from any point is guaranteed to find the minimum
  - (Loss for neural networks is non-convex)

Gradients

The **gradient** of a function is a vector pointing in the direction of the greatest increase in a function.

Gradient Descent: Find the gradient of the loss function at the current point and move in the **opposite** direction.

How much do we move in that direction?

- The value of the gradient (slope in our example) $\frac{d}{dw}L(f(x;w),y)$
  - weighted by a learning rate η
- Higher learning rate means move w faster

$$w^{t+1} = w^t - \eta \frac{d}{dw} L(f(x;w),y)$$

Now let's consider N dimensions

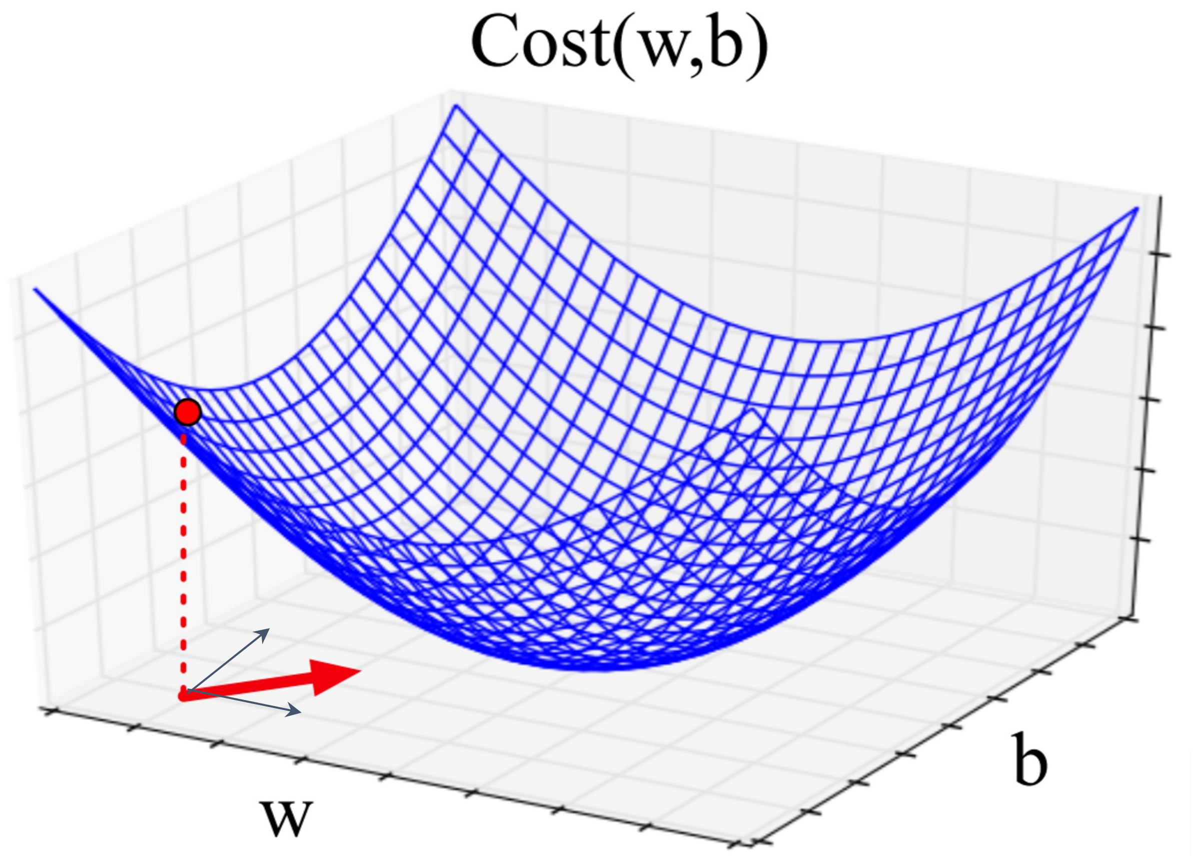
We want to know where in the N-dimensional space (of the N parameters that make up θ) we should move.

The gradient is now a vector; it expresses the directional components of the sharpest slope along each of the N dimensions.

Imagine 2 dimensions, w and b

Visualizing the gradient vector at the red point

It has two dimensions shown in the x-y plane



Real gradients

Are much longer; lots and lots of weights

For each dimension w_i the gradient component i tells us the slope with respect to that variable.

- "How much would a small change in w_i influence the total loss function L?"
- We express the slope as a partial derivative ∂

The gradient is then defined as a vector of these partials.

The gradient

We'll represent $\hat{y}$ as $f(x;\theta)$ to make the dependence on θ more obvious:

$$\nabla_\theta L(f(x;\theta),y)) = \begin{bmatrix} \frac{\partial}{\partial w_1} L(f(x;\theta),y) \\ \frac{\partial}{\partial w_2} L(f(x;\theta),y) \\ \vdots \\ \frac{\partial}{\partial w_n} L(f(x;\theta),y) \end{bmatrix}$$

The final equation for updating θ based on the gradient is thus:

$$\theta_{t+1} = \theta_t - \eta \nabla L(f(x;\theta),y)$$

What are these partial derivatives for logistic regression?

The loss function

$$L_{\mathrm{CE}}(\hat{y},y) \;=\; -\left[y\log\sigma(w\cdot x+b)+(1-y)\log\left(1-\sigma(w\cdot x+b)\right)\right]$$

The elegant derivative of this function

$$\frac{\partial L_{\mathrm{CE}}(\hat{y},y)}{\partial w_j} \;=\; [\sigma(w\cdot x+b)-y]x_j$$

$$= \;(\hat{y}-y)\mathbf{x}_j$$

Gradient Descent Algorithm: Summary

1. Given a dataset D = [(x, y)], a model with weights w=(w1, … wn), and a loss function L(D, w).
2. Initialize w randomly
3. Compute gradient of L, ∇L
   Update $w \leftarrow w - \eta \nabla L$
4. Repeat 3
   - until **convergence**

Hyperparameters

The learning rate η is a **hyperparameter**

- too high: the learner will take big steps and overshoot
- too low: the learner will take too long

Hyperparameters:

- Briefly, a special kind of parameter for an ML model
- Instead of being learned by algorithm from supervision (like regular parameters), they are chosen by algorithm designer.

Mini-batch training

Gradient descent computes the loss over the entire dataset. That can be slow.

$$\hat{\theta} = \underset{\theta}{\operatorname{argmin}} \frac{1}{m} \sum_{i=1}^{m} L_{\mathrm{CE}}(f(x^{(i)};\theta), y^{(i)})$$

More common to compute gradient over batches of training instances.

Mini-batch training: m random examples at every gradient step.

Also known as stochastic gradient descent

Overfitting

A model that perfectly match the training data has a problem.

It will also **overfit** to the data, modeling noise

- A random word that perfectly predicts y (it happens to only occur in one class) will get a very high weight.
  - E.g. if most Chris Nolan movies are reviewed positively, a model might learn to associate its mention with positive rating.
- Failing to perform well (or generalize) to a test set without this word.

Overfitting = exploiting “accidental” correlations in training data.
A good model should be able to **generalize**

Regularization

A general term for solutions for overfitting

One common method: Add a **regularization** term R(θ) to the loss function

$$\hat{\theta} = \underset{\theta}{\operatorname{argmin}} \frac{1}{m} \sum_{i=1}^{m} L_{\mathrm{CE}}(f(x^{(i)};\theta), y^{(i)}) + \alpha R(\theta)$$

Idea: choose an R(θ) that penalizes large weights

- fitting the data well with lots of big weights not as good as fitting the data a little less well, with small weights

L2 regularization (ridge regression)

The sum of the squares of the weights

$$R(\theta) \;=\; ||\theta||_2^2 = \sum_{j=1}^{n} \theta_j^2$$

L2 regularized objective function:

$$\hat{\theta} \;=\; \underset{\theta}{\operatorname{argmin}} \frac{1}{m} \sum_{i=1}^{m} L_{\mathrm{CE}}(f(x^{(i)};\theta), y^{(i)}) + \alpha \sum_{j=1}^{n} \theta_j^2$$

Multinomial Logistic Regression

Classification into more than 2 classes.

If >2 classes we use **multinomial logistic regression**
= Softmax regression
= Multinomial logit
= (defunct names : Maximum entropy modeling or MaxEnt)

Multinomial Logistic Regression

In binary classification, we have a set of weights w, one corresponding to each feature.

In N-class classification, we define a set of weights for each class, w_y: n weights for each feature

Multinomial Logistic Regression

- Binary: convert the features to a score (real number), and apply sigmoid
  - Score: $\mathrm{z} = \ w \cdot x + b$
  - Probability of y=1: $\sigma(z)$

- N-class: convert the features in N scores (also called logits):
  - Scores: [$w_1 \cdot x + b, w_2 \cdot x + b$, …] = [$z_1, z_2, \ldots$.]
  - Convert to probabilities using a "softmax" function – N-dimensional generalization of sigmoid.

Sigmoid → softmax

$$\frac{1}{1+e^{-z}} \longrightarrow \frac{e^{z_j}}{\sum_{c=1}^{k} e^{z_c}}$$

Softmax gives you a vector (whose values sum up to 1)

The **softmax** function

- Turns a vector $z = [z_1, z_2, \ldots, z_k]$ of k arbitrary values (logits) into probabilities

$$\text{softmax}(z) = \left[\frac{\exp(z_1)}{\sum_{i=1}^{k} \exp(z_i)}, \frac{\exp(z_2)}{\sum_{i=1}^{k} \exp(z_i)}, \ldots, \frac{\exp(z_k)}{\sum_{i=1}^{k} \exp(z_i)} \right]$$

- The denominator $\sum_{i=1}^{k} e^{z_i}$ is used to normalize all the values into probabilities

Summary

- Loss function to train a logistic regression classifier: Cross Entropy
- Optimization algorithm to find the weights of the classifier
  - Gradient descent (or stochastic gradient descent)
  - Can lead to overfitting, adding regularization helps.
- Logistic regression can be extended to multiple classes by
  - Creating weights for each class
  - using a softmax function instead of sigmoid.

Neural Nets Basics

Neural Networks

A Little Bit of History

- Neural network algorithms date to the 1980s, and design trace their origin to the 1950s
    - Originally inspired by early neuroscience
- Historically slow, complex, and unwieldy
- Now: term is abstract enough to encompass almost any model – but useful!
- Dramatic shift started around 2013-15 away from linear, convex (like logistic regression) to *neural networks* (non-linear architecture, non-convex)

Why neural networks?

- Linear models like logistic regression require hand-designing features.
    - Requires knowledge of the task, domain, language.
    - Time consuming

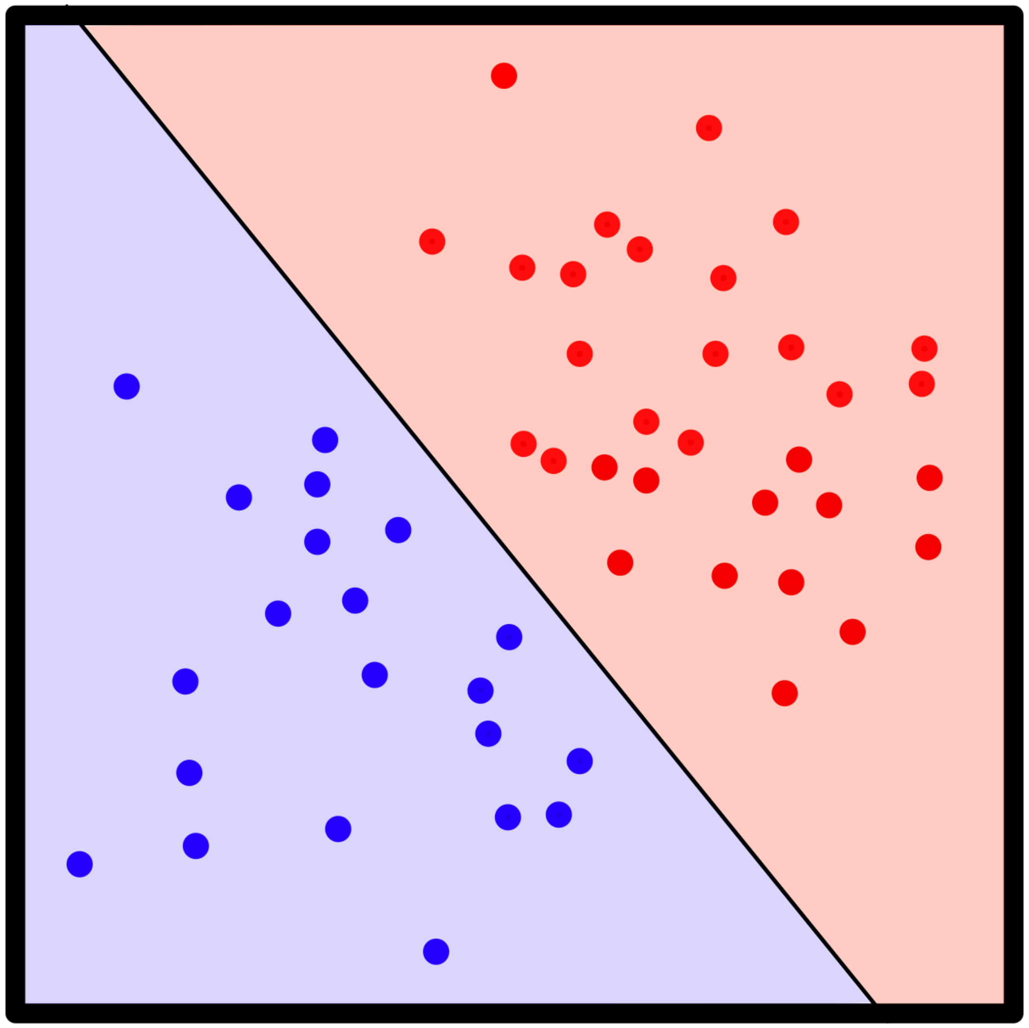
- Linear models assume the classes are linearly separable given the features.

Neural Networks

The Promise

- Non-neural ML works well because of human-designed representations and input features
- ML becomes just optimizing weights
- **Representation learning** attempts to automatically learn good features and representations
- **Deep learning** attempts to learn multiple levels of representation of increasing complexity/abstraction

Linear models assume separability

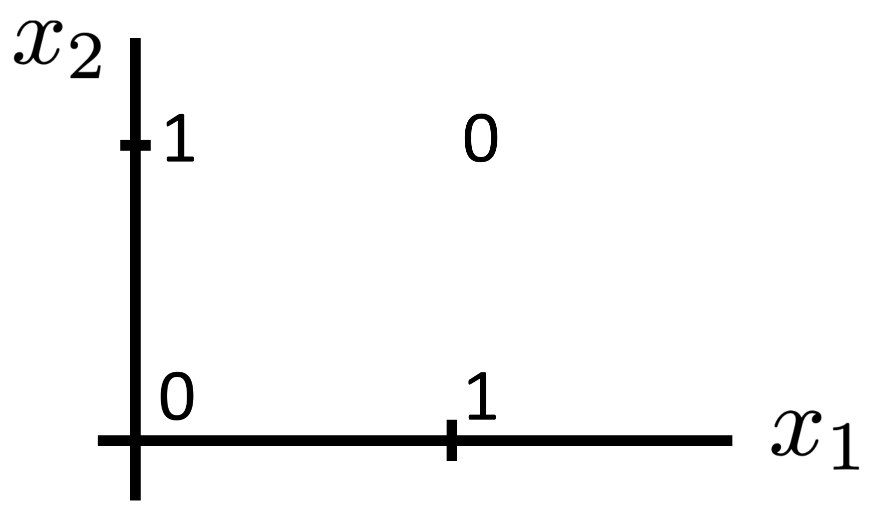


Neural Networks: XOR

- Let's see how we can use neural nets to learn a simple nonlinear function

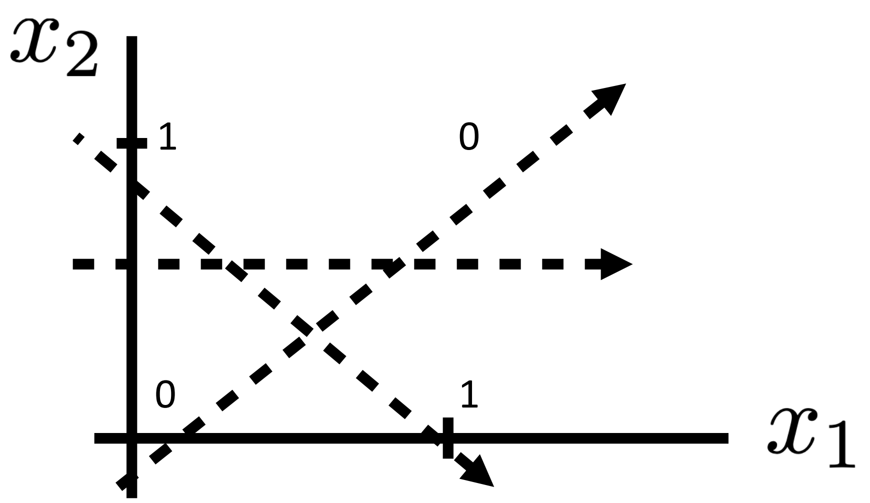
- Inputs $x_1, \ x_2$

- Output y



x_1	x_2	$y = x_1 \text{ XOR } x_2$
0	0	0
0	1	1
1	0	1
1	1	0

Neural Networks: XOR



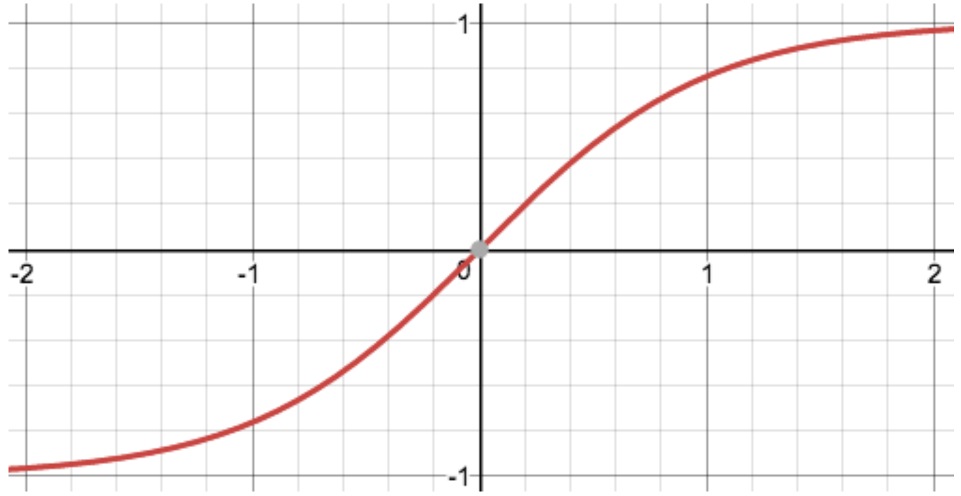
$$y = a_1 x_1 + a_2 x_2$$ X

$$y = a_1 x_1 + a_2 x_2 + a_3 \tanh(x_1 + x_2)$$ ✓

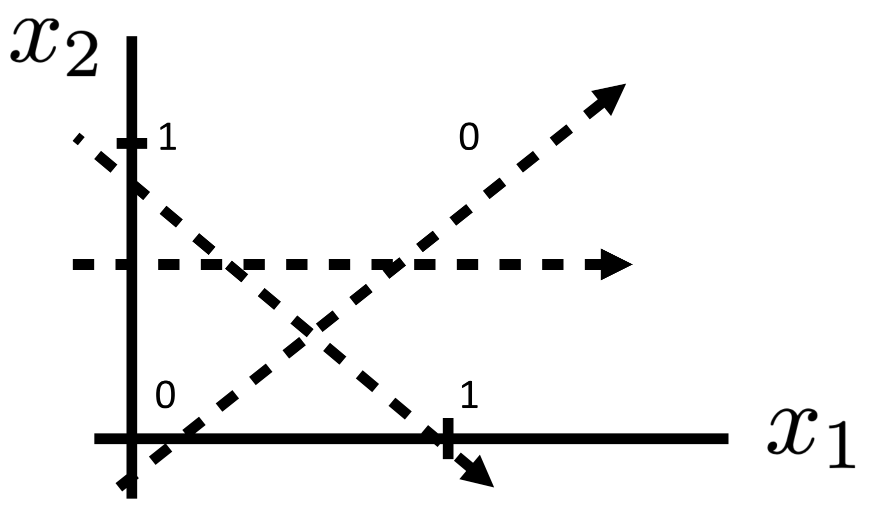
"or"

x_1	x_2	$x_1 \text{ XOR } x_2$
0	0	0
0	1	1
1	0	1
1	1	0

(looks like action potential in neuron)



Neural Networks: XOR

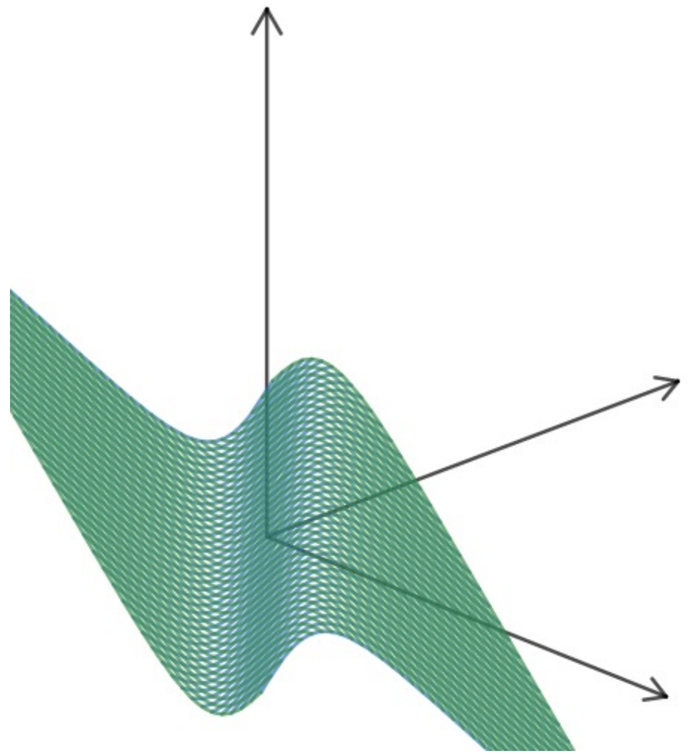


$y = a_1 x_1 + a_2 x_2$ ✗

$y = a_1 x_1 + a_2 x_2 + a_3 \tanh(x_1 + x_2)$ ✓

$y = -x_1 - x_2 + 2 \tanh(x_1 + x_2)$

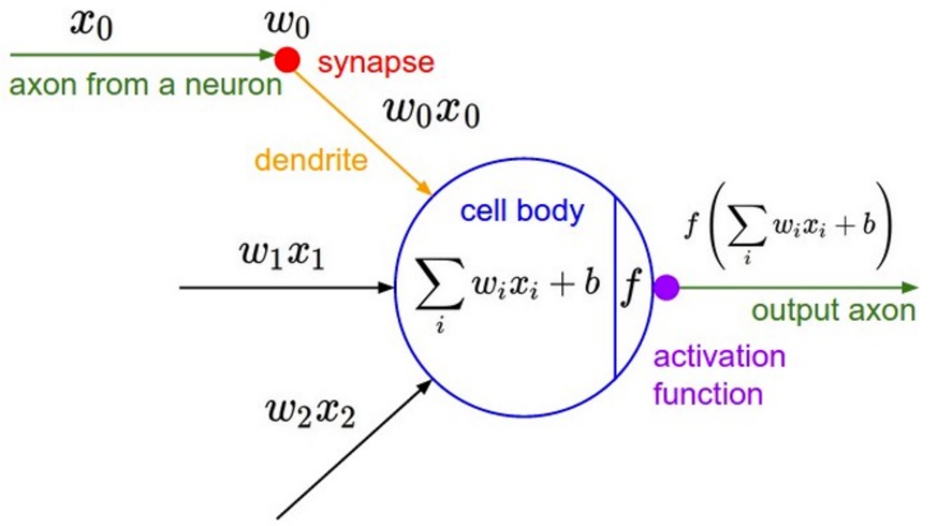
x_1	x_2	x_1 XOR x_2
0	0	0
0	1	1
1	0	1
1	1	0



Building Blocks

The Neuron

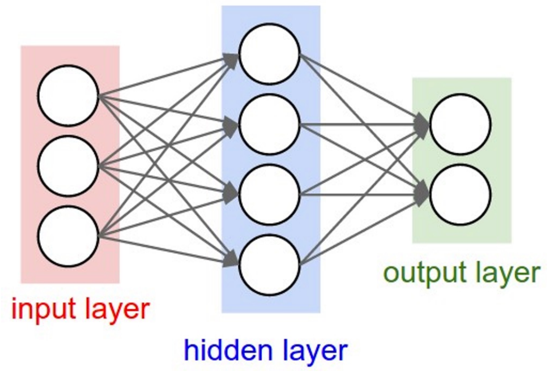
- Neural networks traditionally come with their own terminology baggage
  - Some of it is less common in more recent work
- Parameters:
  - Inputs: x_i
  - Weights: w_i and b
  - Activation function f
- If we drop the activation function, reminds you of something?

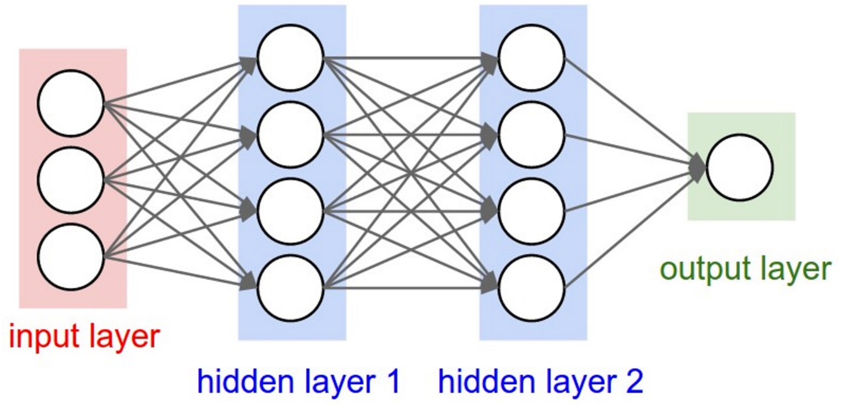


Building Blocks

Hidden Layers

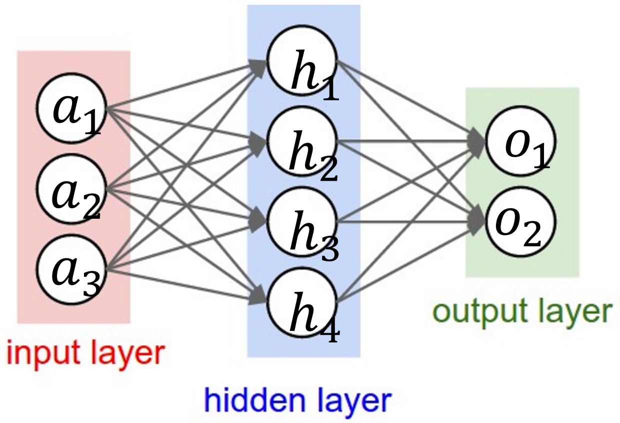
- It gets interesting when you connect and stack neurons
- This modularity is one of the greatest strengths of neural networks
- Input vs. hidden vs. output layers
- The activations of the hidden layers are the learned representation





Building Blocks

Matrix Notation

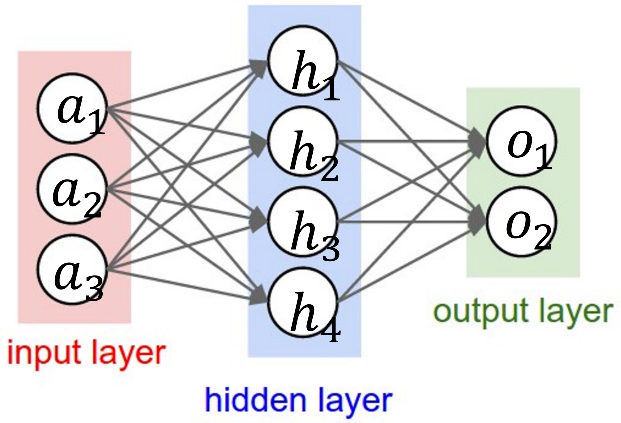


No activation/non-linearity function

Building Blocks

Matrix Notation

$$h_1 = a_1W'_{11} + a_2W'_{21} + a_3W'_{31} + b'_1$$
$$h_2 = a_1W'_{12} + a_2W'_{22} + a_3W'_{32} + b'_1$$
$$h_3 = a_1W'_{13} + a_2W'_{23} + a_3W'_{33} + b'_1$$
$$h_4 = a_1W'_{14} + a_2W'_{24} + a_3W'_{34} + b'_4$$



$$\boldsymbol{h}_{4\times 1} = \boldsymbol{W'}_{4\times 3}\boldsymbol{a}_{3\times 1} + \boldsymbol{b'}_{4\times 1}$$

$$\boldsymbol{o}_{2\times 1} = \boldsymbol{W''}_{2\times 4}\boldsymbol{h}_{4\times 1} + \boldsymbol{b''}_{2\times 1}$$

$$o_1 = h_1W''_{11} + h_2W''_{21} + h_3W''_{31} + h_4W''_{41} + b''_1$$
$$o_2 = h_1W''_{12} + h_2W''_{22} + h_3W''_{32} + h_4W''_{42} + b''_2$$

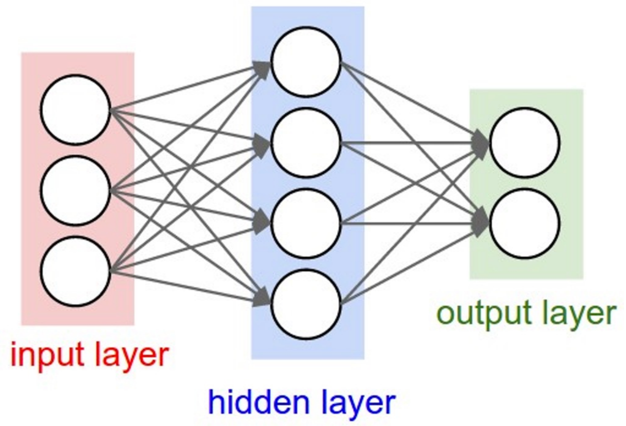
No activation/non-linearity function

Building Blocks

Activation Functions

Activation (non-linearity) function is an entry-wise function

$f : \mathbb{R} \rightarrow \mathbb{R}$



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$$\boldsymbol{h}_{4\times1} = \boldsymbol{f}(\boldsymbol{W}'_{4\times3}\boldsymbol{a}_{3\times1} + \boldsymbol{b}'_{4\times1})$$

$$\boldsymbol{o}_{2\times1} = \boldsymbol{W}''_{2\times4}\boldsymbol{h}_{4\times1} + \boldsymbol{b}''_{2\times1}$$

Building Blocks

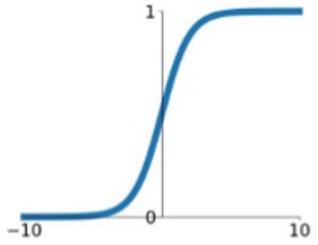
Activation Functions

Activation (non-linearity) function is an entry-wise function

$f: \mathbb{R} \rightarrow \mathbb{R}$

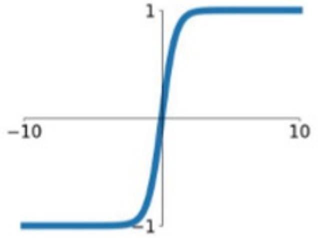
Sigmoid

$\sigma(x) = \frac{1}{1+e^{-x}}$



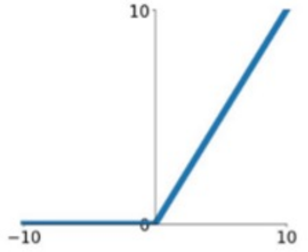
tanh

$\tanh(x)$



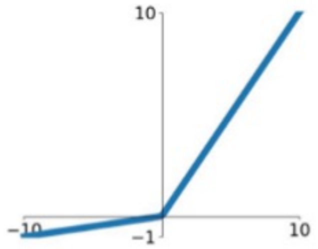
ReLU

$\max(0, x)$



Leaky ReLU

$\max(0.1x, x)$

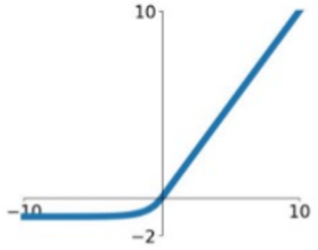


Maxout

$\max(w_1^T x + b_1, w_2^T x + b_2)$

ELU

$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



Why activation functions?

- What if we do not have activation functions

$$\boldsymbol{o} = \boldsymbol{W}''\boldsymbol{h} + \boldsymbol{b}''$$
$$\boldsymbol{o} = \boldsymbol{W}''(\boldsymbol{W}'\boldsymbol{a} + \boldsymbol{b}') + \boldsymbol{b}''$$
$$\boldsymbol{o} = \boldsymbol{W}''\boldsymbol{W}'\boldsymbol{a} + \boldsymbol{W}''\boldsymbol{b}' + \boldsymbol{b}''$$

Define $W''' = W''W'$ and $b''' = W''b' + b''$

A multi-layer linear network is the same as a 1-layer network (with some caveats)

Deep Neural Networks

$$\boldsymbol{y} = g(\mathbf{W}\boldsymbol{x} + \boldsymbol{b})$$

$$\mathbf{z} = g(\mathbf{V}\mathbf{y} + \mathbf{c})$$

$$\mathbf{z} = g(\mathbf{V}\underbrace{g(\mathbf{W}\mathbf{x} + \mathbf{b})}_{\text{output of first layer}} + \mathbf{c})$$

Taken from http://colah.github.io/posts/2014-03-NN-Manifolds-Topology/

Building Blocks of Neural NLP

One-hot Word Representations

- Neural networks take continuous vector inputs
- How can we represent text as continuous vectors?
- One-hot vectors

$$\begin{aligned} hotel &= [0 \quad 0 \quad 0 \quad \cdots 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \\ conference &= [0 \quad 0 \quad 0 \quad \cdots 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0] \end{aligned}$$

- Dimensionality: size of the vocabulary
  - Can be >10M for web-scale corpora
- Problems?

Building Blocks for Neural NLP

One-hot Word Representations

- One-hot vectors

$$\begin{aligned} hotel &= [0 \quad 0 \quad 0 \quad \cdots 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \\ conference &= [0 \quad 0 \quad 0 \quad \cdots 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0] \end{aligned}$$

- Problems?
    - Information sharing? “hotel” vs. “hotels”

Building Blocks

Word Embeddings

- Each word is represented using a dense low-dimensional vector
    - Low-dimensional << vocabulary size
- If trained well, similar words will have similar vectors
- How to train? What objective to maximize?
    - As part of task training (e.g., supervised training)
    - Pre-training (more on this later)

Training Neural Networks

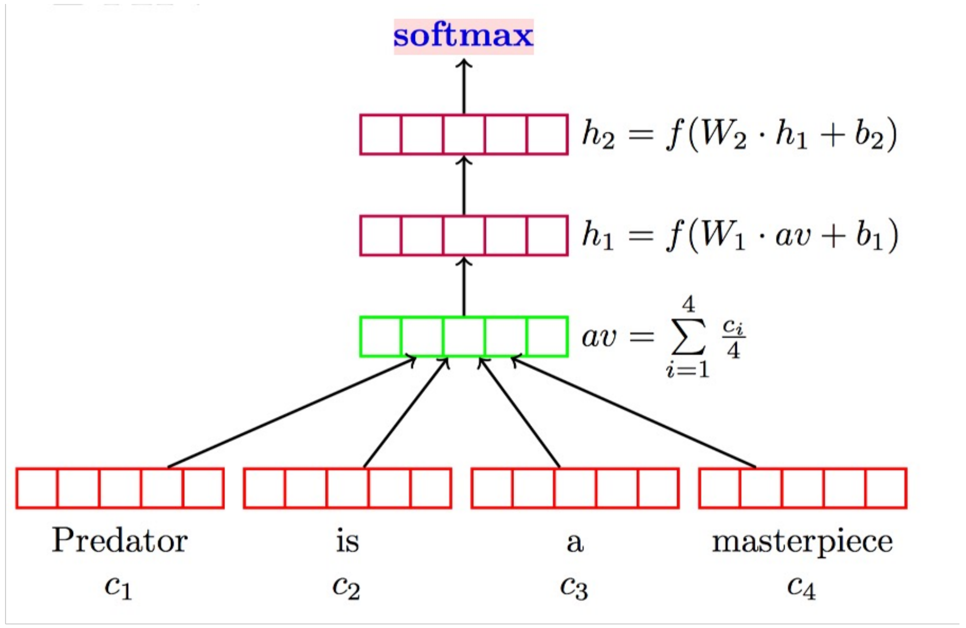
- No hidden layer $\longrightarrow$ same as logistic regression (convex, guaranteed to converge)
- With hidden layers:
    - Latent units $\longrightarrow$ not convex
    - What do we do?
        - Back-propagate the gradient
        - Based on the chain rule
        - About the same, but no guarantees

Neural Bag of Words

- One of the most basic neural models
- Example: sentiment classification
  - Input: text document
  - Classes: very positive, positive, neutral, negative, very negative
- We discussed doing this with a bag-of-words feature-based model
- What would be the neural equivalent?
  - Concatenate all vectors?
    - Problem: different documents $\rightarrow$ different input length
  - Instead: sum, average, etc.

Neural Bag of Words

Deep Averaging Networks (Iyyer et al. 2015)



softmax
$h_2 = f(W_2 \cdot h_1 + b_2)$
$h_1 = f(W_1 \cdot av + b_1)$
$av = \sum_{i=1}^{4} \frac{c_i}{4}$
Predator
is
a
masterpiece
c_1
c_2
c_3
c_4

IMDB Sentiment Analysis

BOW + linear model	88.23
NBOW DAN	89.4

Computation Graphs

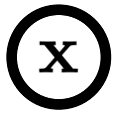
- The descriptive language of deep learning models
- Functional description of the required computation
- Can be instantiated to do two types of computation:
    - Forward computation
    - Backward computation

expression:

$$\mathbf{x}$$

graph:

A **node** is a {tensor, matrix, vector, scalar} value

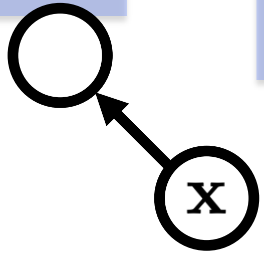


An **edge** represents a function argument (and also data dependency). They are just pointers to nodes.

A **node** with an incoming **edge** is a **function** of that edge's tail node.

A **node** knows how to compute its value and the *value of its derivative w.r.t each argument (edge) times a derivative of an arbitrary input* $\frac{\partial \mathcal{F}}{\partial f(\mathbf{u})}$.

$f(\mathbf{u}) = \mathbf{u}^\top$



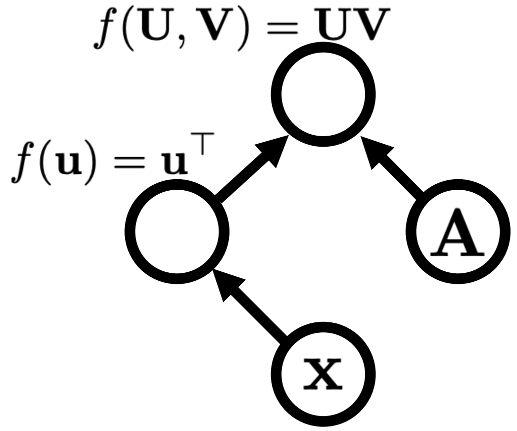
$$\frac{\partial f(\mathbf{u})}{\partial \mathbf{u}} \frac{\partial \mathcal{F}}{\partial f(\mathbf{u})} = \left(\frac{\partial \mathcal{F}}{\partial f(\mathbf{u})} \right)^\top$$

expression:

$$\mathbf{x}^\top \mathbf{A}$$

graph:

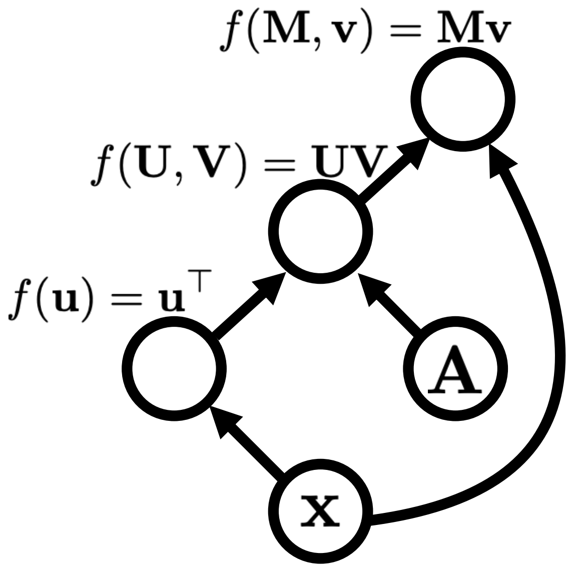
Functions can be nullary, unary, binary, … *n*-ary. Often they are unary or binary.


$f(\mathbf{U}, \mathbf{V}) = \mathbf{U}\mathbf{V}$
$f(\mathbf{u}) = \mathbf{u}^\top$
$\mathbf{A}$
$\mathbf{x}$

expression:

$$\mathbf{x}^\top \mathbf{A}\mathbf{x}$$

graph: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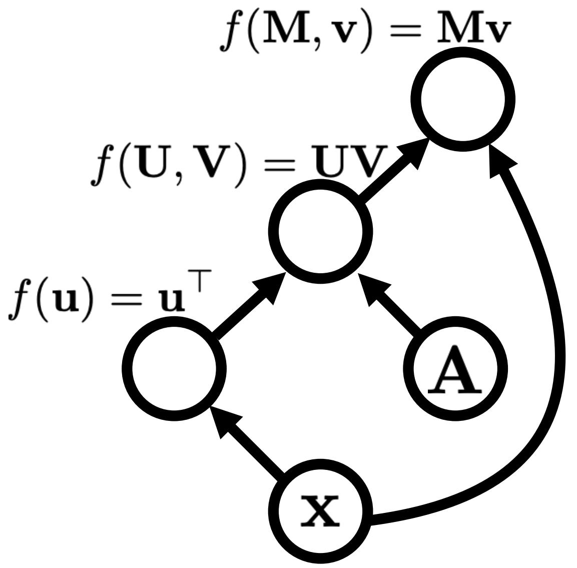
$\mathbf{A}$
$\mathbf{x}$

Computation graphs are directed and acyclic (usually)

expression:

$$\mathbf{x}^\top \mathbf{A}\mathbf{x}$$

graph:



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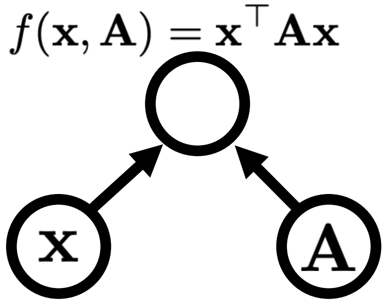


$f(\mathbf{x},\mathbf{A}) = \mathbf{x}^\top \mathbf{A}\mathbf{x}$
$\mathbf{x}$
$\mathbf{A}$

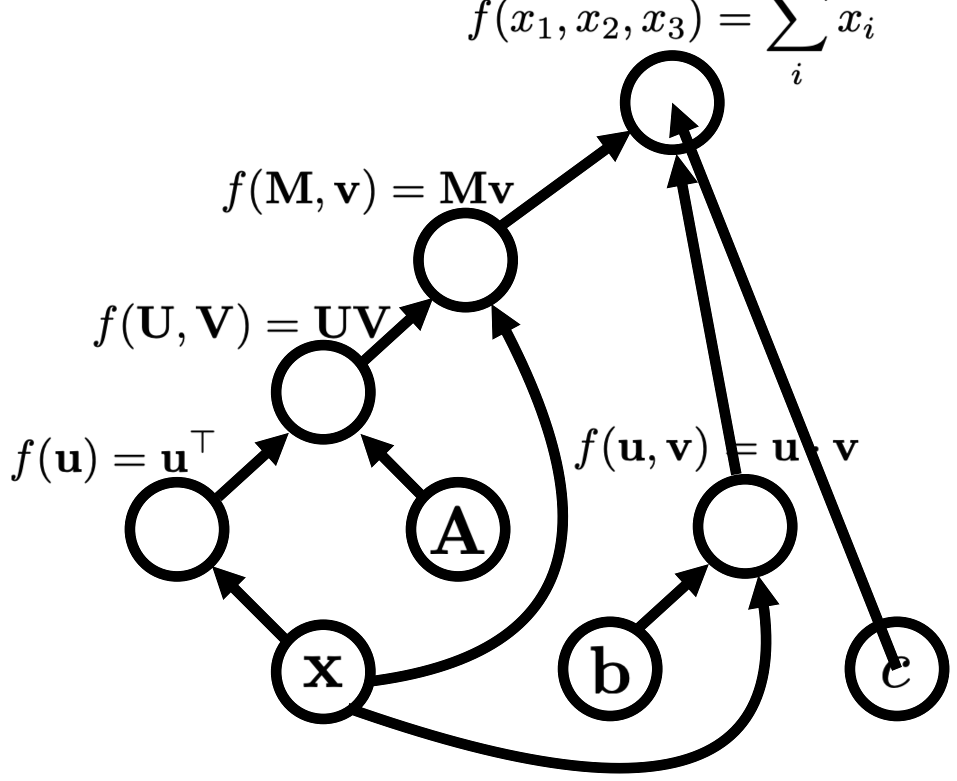
$$\frac{\partial f(\mathbf{x},\mathbf{A})}{\partial \mathbf{x}} = (\mathbf{A}^\top + \mathbf{A})\mathbf{x}$$

$$\frac{\partial f(\mathbf{x},\mathbf{A})}{\partial \mathbf{A}} = \mathbf{x}\mathbf{x}^\top$$

expression:

$$\mathbf{x}^\top \mathbf{A}\mathbf{x} + \mathbf{b} \cdot \mathbf{x} + c$$

graph: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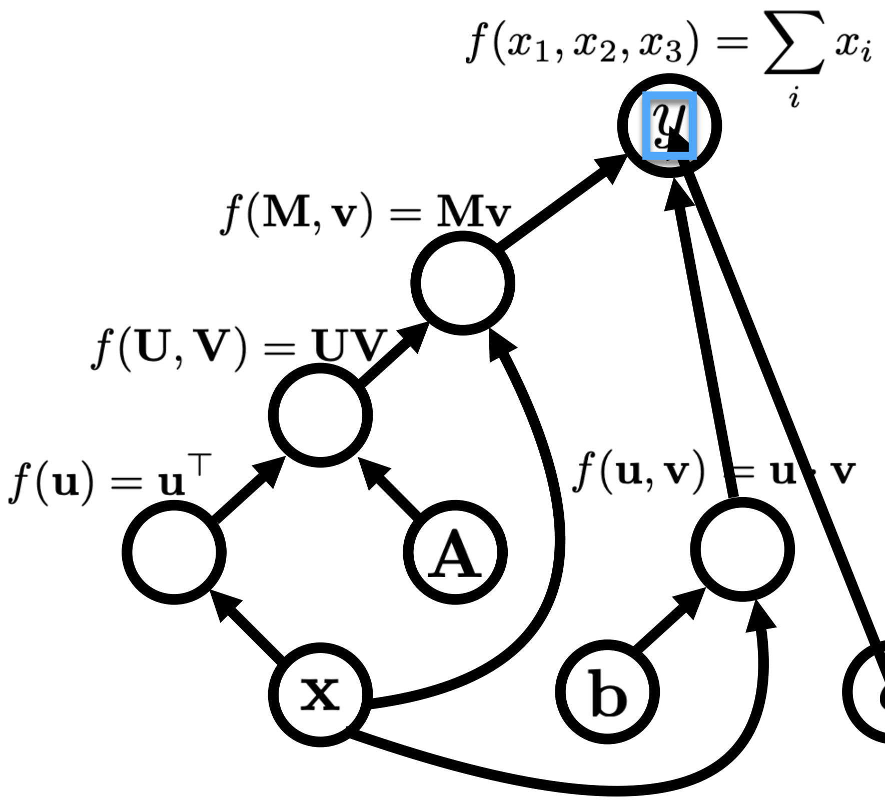
expression:

$$\boxed{y =} \mathbf{x}^\top \mathbf{A}\mathbf{x} + \mathbf{b} \cdot \mathbf{x} + c$$

graph:



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y
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$\mathbf{x}$
$\mathbf{b}$
c

variable names are just labelings of nodes.

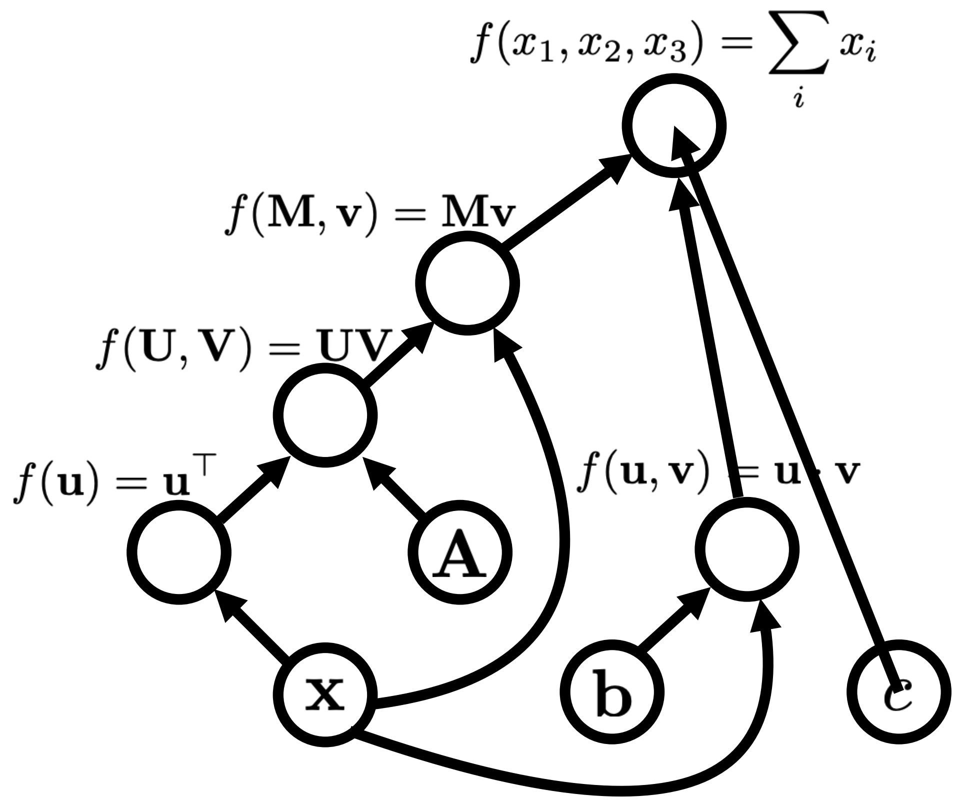
Computation Graphs

Algorithms

- **Graph construction**
- **Forward propagation**
  - Loop over nodes in topological order
    - Compute the value of the node given its inputs
  - *Given my inputs, make a prediction (or compute an "error" with respect to a "target output")*
- **Backward propagation**
  - Loop over the nodes in reverse topological order starting with a final goal node
    - Compute derivatives of final goal node value with respect to each edge's tail node
  - *How does the output change if I make a small change to the inputs?*

Forward Propagation

graph:



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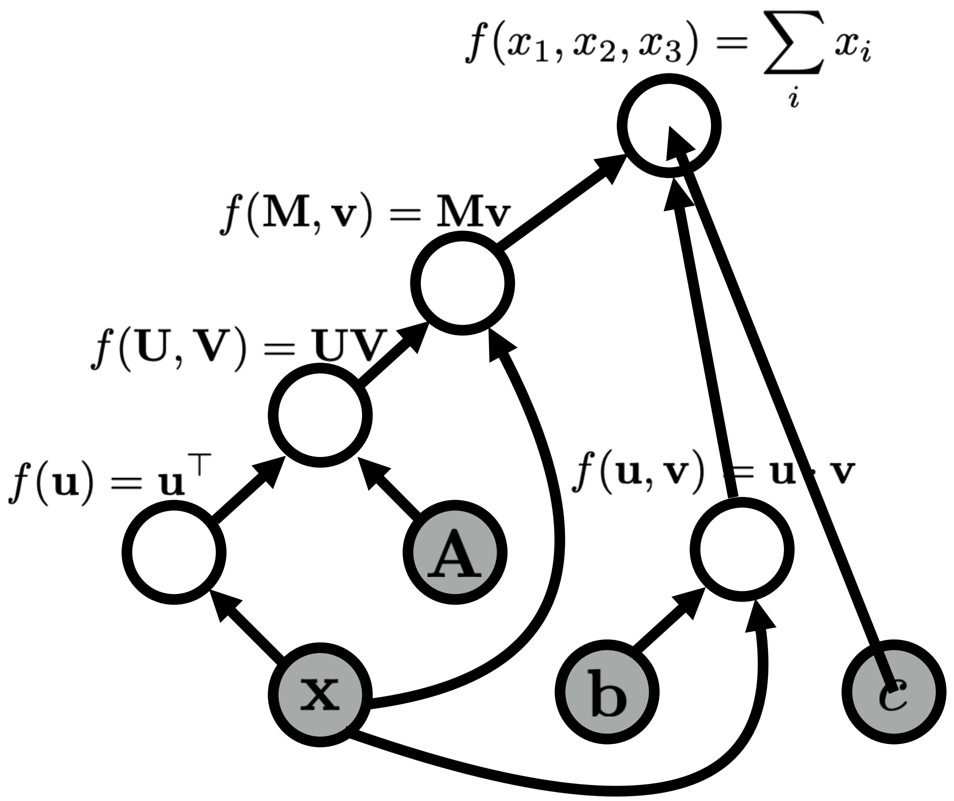
Forward Propagation

graph:



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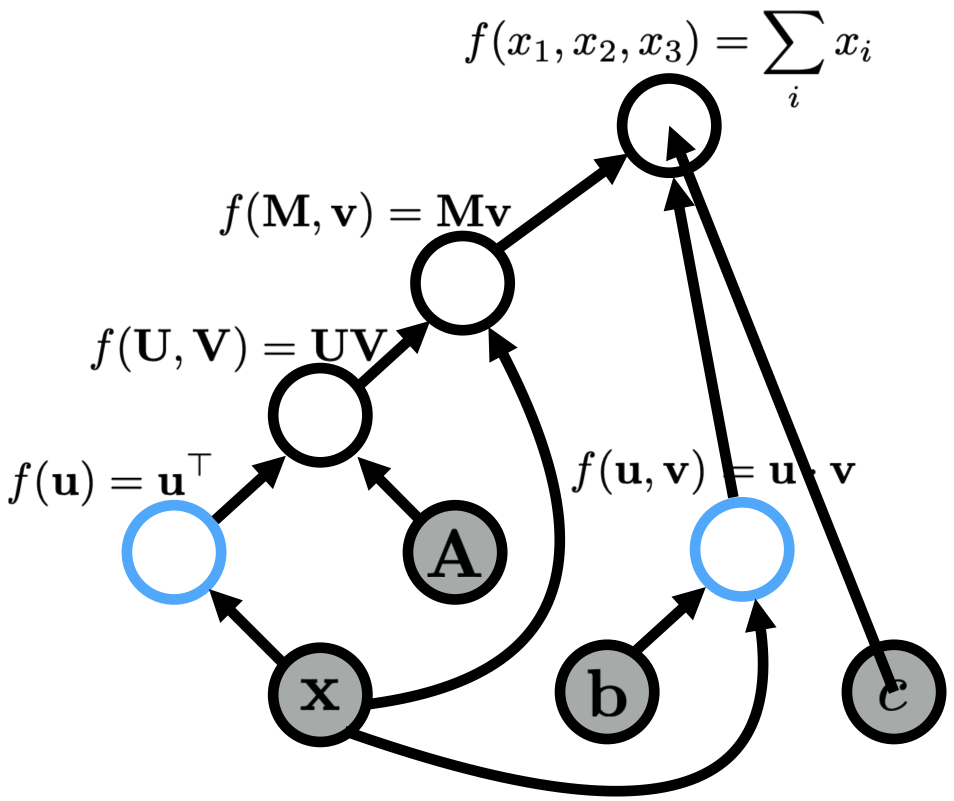
Forward Propagation

graph:



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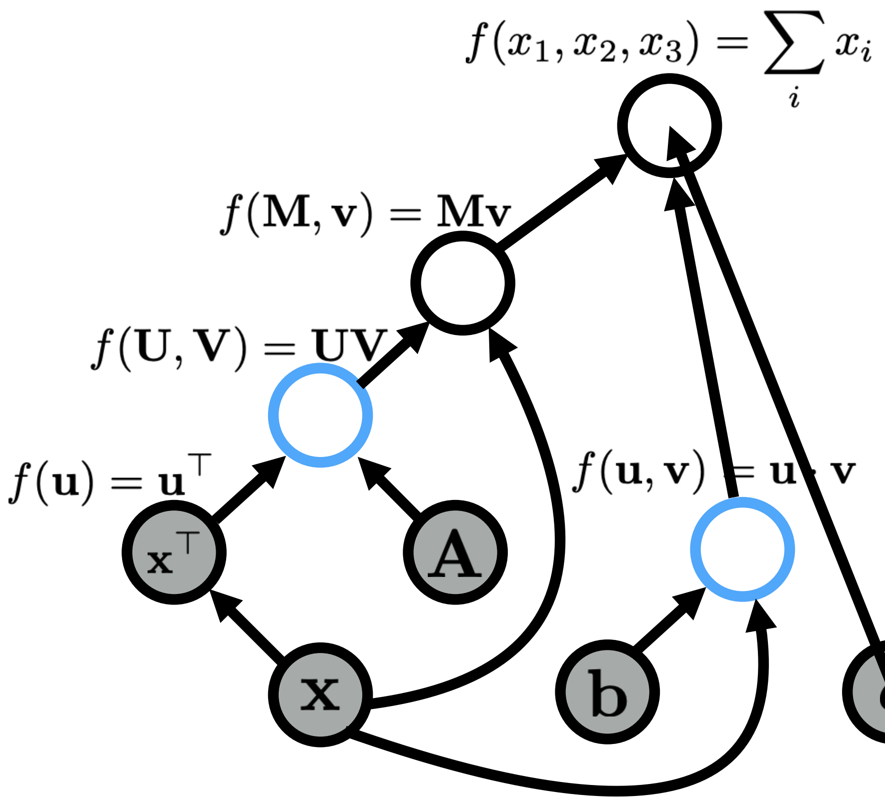
Forward Propagation

graph:



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Forward Propagation

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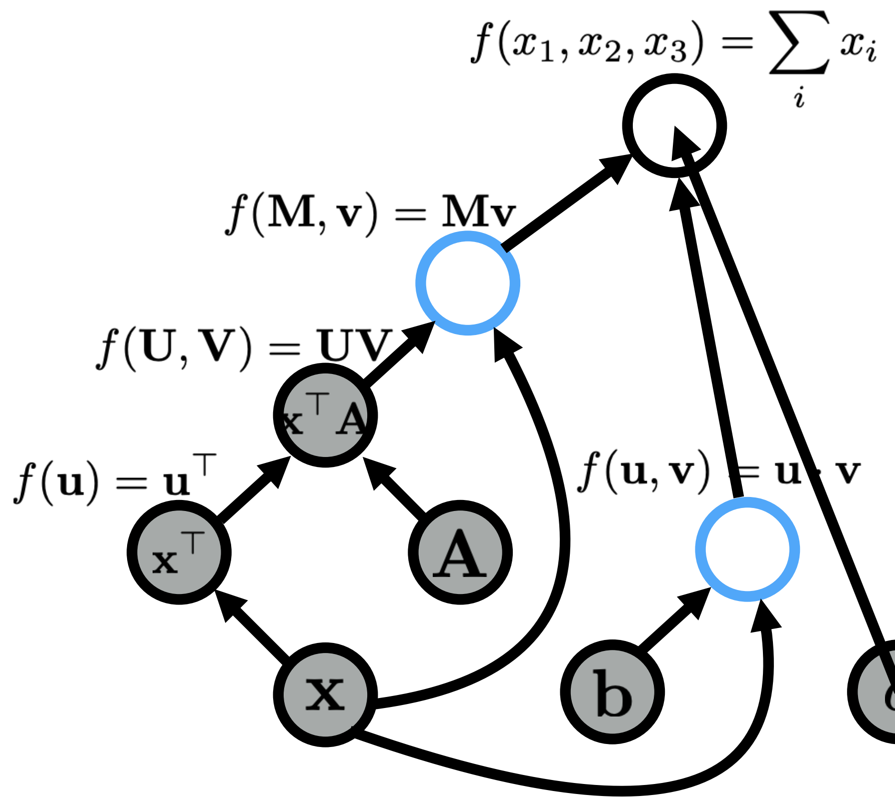
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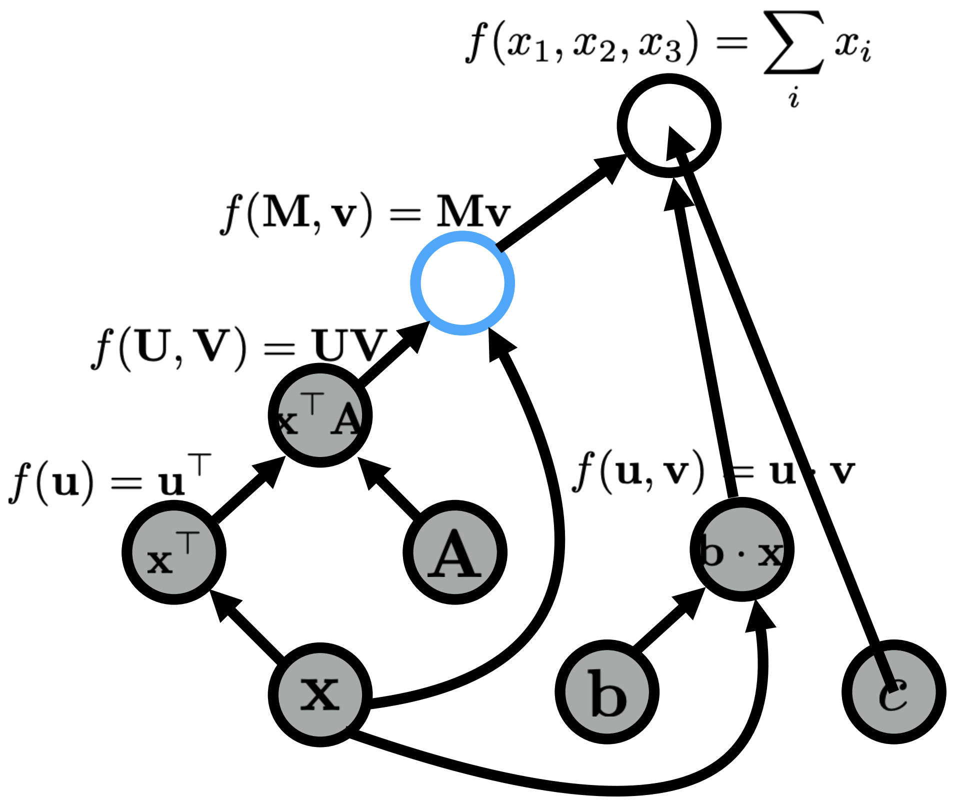
Forward Propagation

graph:



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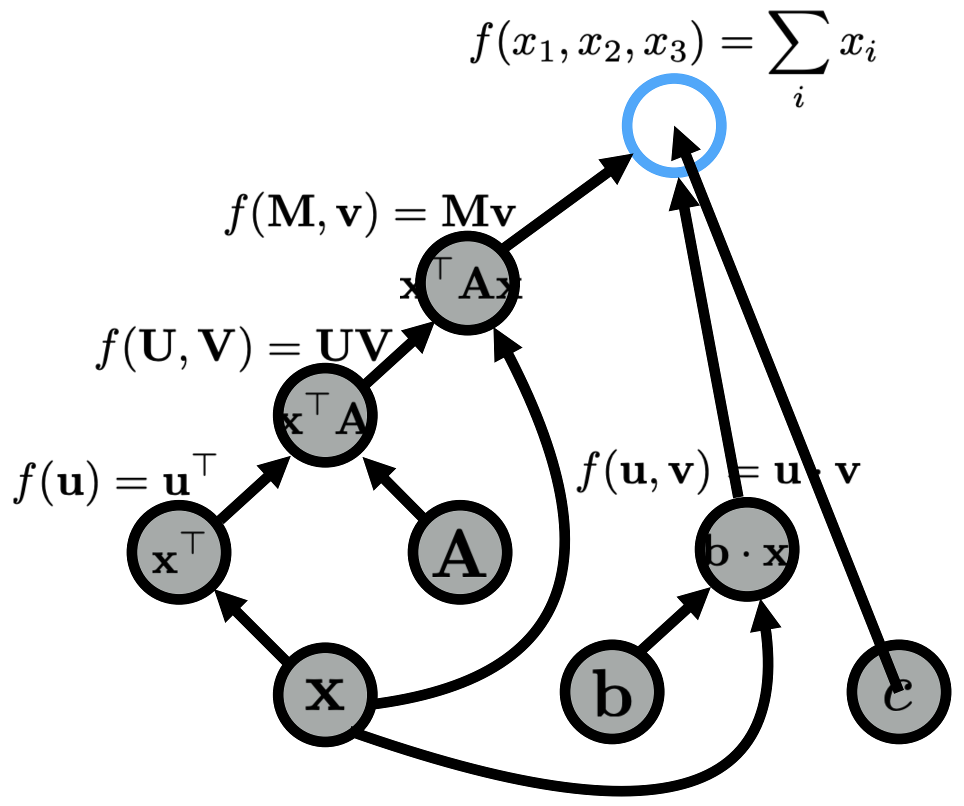
Forward Propagation

graph:



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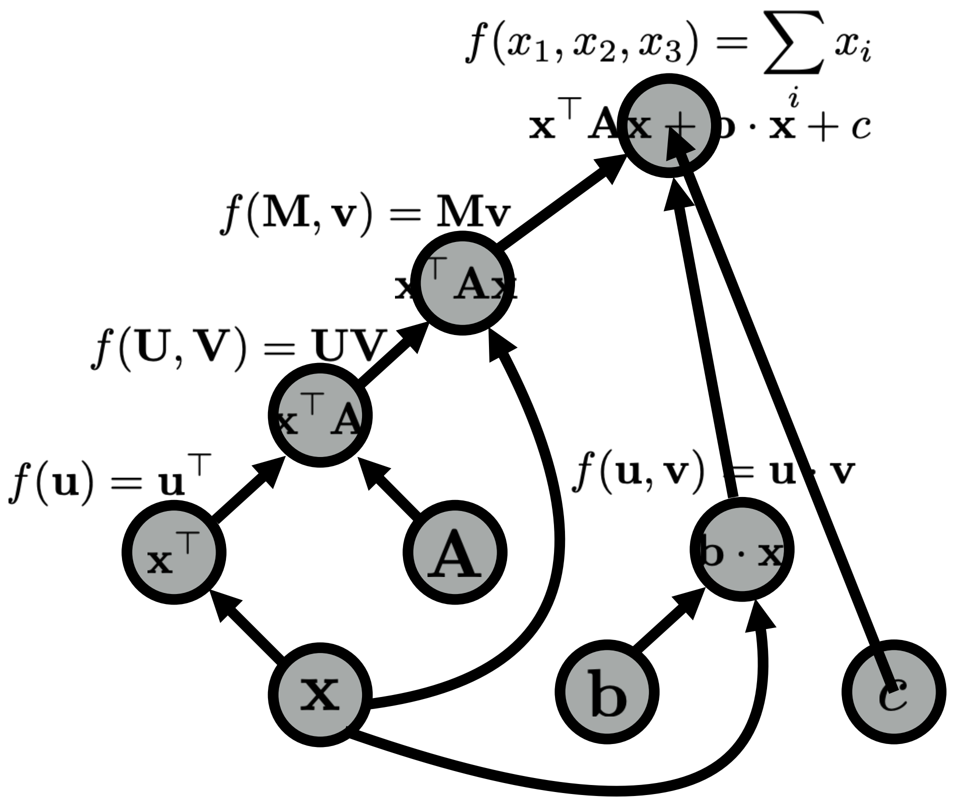
Forward Propagation

graph:



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$\mathbf{b} \cdot \mathbf{x}$
$\mathbf{x}$
$\mathbf{b}$
c

Constructing Graphs

Two Software Models

- Static declaration
  - Phase 1: define an architecture
    (maybe with some primitive flow control like loops and conditionals)
  - Phase 2: run a bunch of data through it to train the model and/or make predictions
- Dynamic declaration (a.k.a define-by-run)
  - Graph is defined implicitly (e.g., using operator overloading) as the forward computation is executed
  - Graph is constructed dynamically
  - This allows incorporating conditionals and loops into the network definitions easily

Batching

- Two senses to processing your data in batch
  - Computing gradients for more than one example at a time to update parameters during learning
  - Processing examples together to utilize all available resources
- CPU: made of a small number of cores, so can handle some amount of work in parallel
- GPU: made of thousands of small cores, so can handle a lot of work in parallel
- Process multiple examples together to use all available cores

Batching

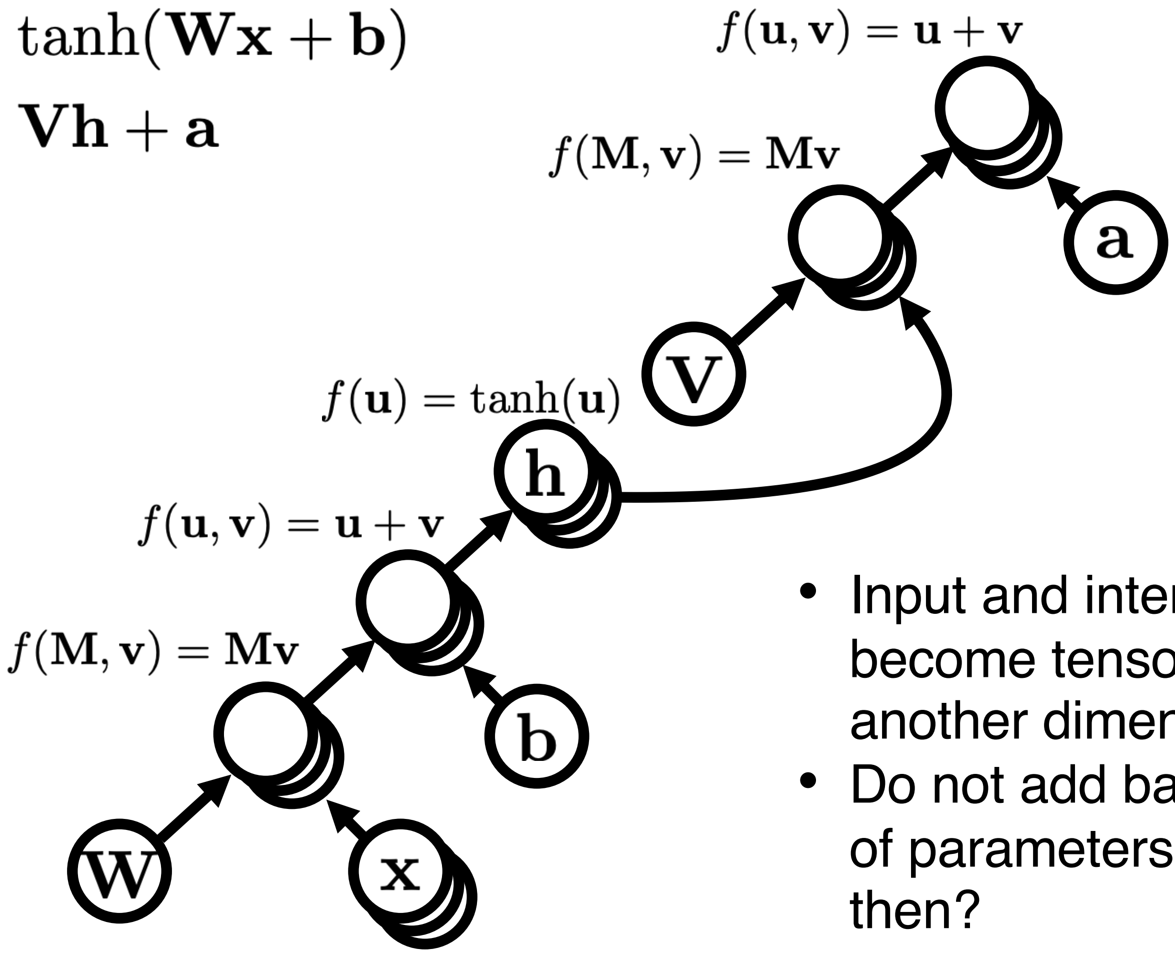
- Relatively easy when the network looks exactly the same for all examples
- More complex with language data: documents/sentences/words have different lengths
- Frameworks provide different methods to help common cases, but still require work on the developer side
- Key concept is broadcasting: https://pytorch.org/docs/stable/notes/broadcasting.html

Batching

MLP Sketch

$$\mathbf{h} = \tanh(\mathbf{W}\mathbf{x} + \mathbf{b})$$

$$\mathbf{y} = \mathbf{V}\mathbf{h} + \mathbf{a}$$



$f(\mathbf{u}, \mathbf{v}) = \mathbf{u} + \mathbf{v}$
$f(\mathbf{M}, \mathbf{v}) = \mathbf{M}\mathbf{v}$
a
V
$f(\mathbf{u}) = \tanh(\mathbf{u})$
h
$f(\mathbf{u}, \mathbf{v}) = \mathbf{u} + \mathbf{v}$
$f(\mathbf{M}, \mathbf{v}) = \mathbf{M}\mathbf{v}$
b
W
x

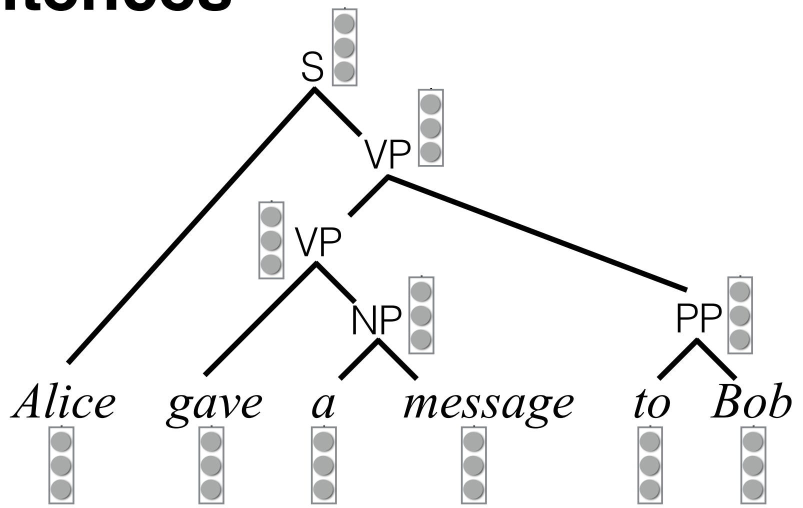
- Input and intermediate results become tensors — batch is another dimension!
- Do not add batch dimension of parameters! What happens then?

Batching

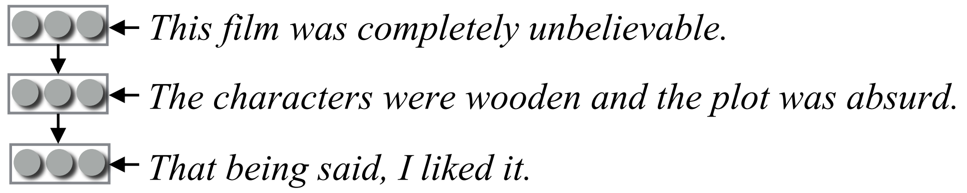
Complex Network Architectures

- Complex networks may include different parts with varying length (more about this later)
- In the extreme, it may be complex to batch complete examples this way
- But: you can still batch sub-parts across examples, so you alternate between batched and non-batched computations

Sentences



Documents


This film was completely unbelievable.
The characters were wooden and the plot was absurd.
That being said, I liked it.

Backpropagation

But what about the gradient w.r.t. `W_1`?

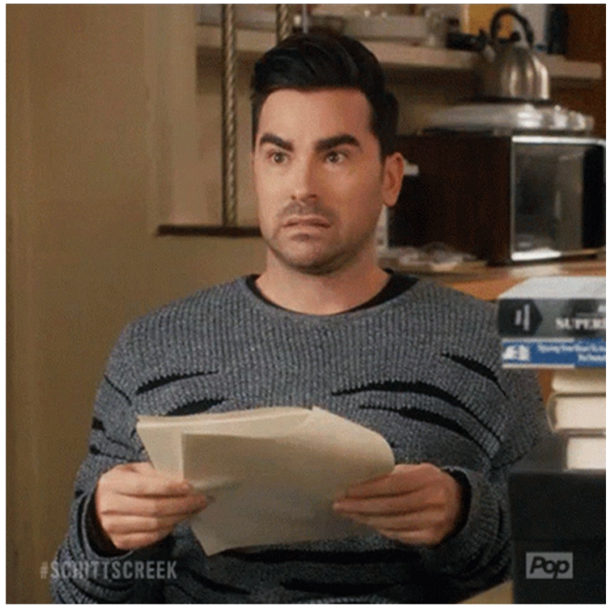
Apply the chain rule

$$\frac{\partial \mathcal{L}(x, i^*)}{\partial W_{1_{i,j}}} = \frac{\partial \mathcal{L}(x, i^*)}{\partial z} \cdot \frac{\partial z}{\partial W_{1_{i,j}}}$$

$$\frac{\partial z}{\partial W_{1_{i,j}}} = \frac{\partial g(a)}{\partial a}$$

$$a = W_1 f(x)$$

Are we going to compute derivatives ourselves every time?



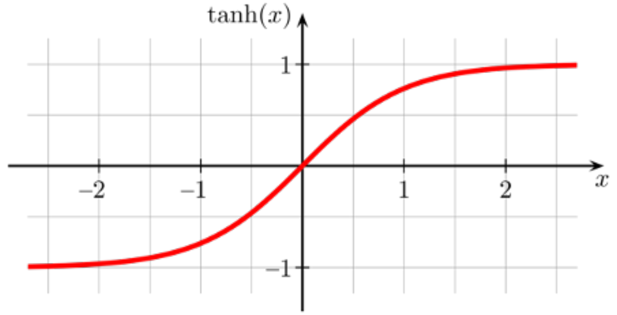
No, we will use frameworks that we will do them for us!

- Deep Learning with PyTorch: A 60 Minute Blitz

```
import torch
from torchvision.models import resnet18, ResNet18_Weights
model = resnet18(weights=ResNet18_Weights.DEFAULT)
data = torch.rand(1, 3, 64, 64)
labels = torch.rand(1, 1000)
prediction = model(data) # forward pass
loss = (prediction - labels).sum()
loss.backward() # backward pass; autograd calculates and stores the gradients for each model
parameter in the parameter's .grad attribute.
optim = torch.optim.SGD(model.parameters(), lr=1e-2, momentum=0.9)
optim.step() #gradient descent; optimizer adjusts each parameter by its gradient stored in .grad
```

Neural Networks: Practical Tips

- Select network structure appropriate for the problem
    - ○ Window vs. recurrent vs. recursive (will discuss throughout the semester)
- Parameter initialization
- Model is powerful enough?
    - ■ If not, make it larger
    - ■ Yes, so regularize, otherwise it will overfit
- Gradient checks to identify bugs
    - ○ If you build from scratch
- Know your non-linearity function and its gradient
    - ○ Example $\tanh(x)$
        - ■ $\frac{\partial}{\partial x}\tanh(x) = 1 - \tanh^2(x)$



Practical Tips

Debugging

- Verify value of initial loss when using softmax
- Perfectly fit a single example, then mini-batch, then train
- If learning fails completely, maybe gradients stuck
    - Check learning rate
    - Verify parameter initialization
    - Change non-linearity functions

Practical Tips

Avoid Overfitting

- Very expressive models, can overfit easily
  - It will look great on the training data, but everything else will be terrible
- Some potential cures ⛑️
  - Reduce model size (but not too much)
  - L1 and L2 regularization
  - Early stopping (e.g., patience)
  - Learning rate scheduling
  - Dropout (Hinton et al. 2012)
    - Randomly set 50% of inputs in each layer to 0