

AND THE MEASUREMENT OF SHORT MEAN LIVES

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§ 1. Coincidence circuits

Coincidence circuits are used to investigate time relationships between nuclear events. They are the instruments of time spectroscopy, a kind of spectroscopy apart from the energy or momentum spectroscopy that is normally brought to mind by the title of this book. In addition, energy spectroscopy itself may be carried out by time-of-flight measurements.

A simple twofold coincidence circuit element gives an output pulse whenever two input pulses are 'simultaneous' within a certain resolving time, but gives no output when only one of the input pulses is present. If the circuit element detects the overlap of two pulses each of length τ_0 , then the resolving time can be quoted as $2\tau_0$, because a coincidence can be detected if one pulse lies within $\pm\tau_0$ of the other. If there are two sources of uncorrelated pulses of rates n_1 and n_2 per second, the expected rate of chance coincidences is easily seen to be $2\tau_0 n_1 n_2$ per second. If the recorded coincidence rate differs from the expected chance rate, a correlation is established between the two sources of pulses. Both for the suppression of chance coincidences, and for the measurement of short time intervals, it is desirable to have the coincidence resolving time as short as possible. Coincidence circuits of multiplicity higher than two are in frequent use, but will not be discussed here.

Coincidence circuit elements for pulses that are standardized in amplitude are almost trivially easy to design. We have only to add the two pulse trains, not necessarily linearly, and use some biased device to detect pulses larger than the single pulses. Digital computers use coincidence elements in profusion, working on standardized pulses; they are usually called AND circuits or AND gates. We will confine ourselves to coincidence circuits that will work on pulses of variable size, such as are usually delivered by nuclear radiation detectors.

The parallel, or Rossi, type of coincidence circuit element¹ is shown in Fig. 1a. Negative input pulses are applied to the grids of two vacuum tubes, of sufficient amplitude to cut off the standing currents, i , in the tubes. The positive output at the anode can never exceed Ri for a single input pulse; this amplitude limiting is the central feature of the circuit. Simultaneous input pulses cause an output up to $2Ri$. The biased diode D allows only those pulses greater than Ri to pass, thus selecting the coincidence events. If R is large, the vacuum tubes may be 'bottomed', and the amplitude ratio of double to single events will be much greater than 2; this was true of the original Rossi circuit. For high speed, however, R must be small so as to give the

¹ B. Rossi, Nature 125 (1930) 636.

anode circuit a short time constant. We then regard the two vacuum tubes as current limiters, R as the device in which the two limited currents are added, and D as the coincidence detector. The circuit is similar to a pair of switches in parallel, both of which must be fully opened if the current is to be completely interrupted. The fast Rossi circuit should be recognized by its circuit functions (limit, add, discriminate), rather than by actual circuit details.

The speed of a coincidence circuit must be preserved up to the element that makes the decision between single and coincidence events (diode D in Fig. 1a).

Other forms of parallel coincidence element may be made with transistors or diodes replacing vacuum tubes. In Fig. 1b, two PNP transistors replace the pentodes in the Rossi circuit. Very clean discrimination between singles and coincidences can be had by operating the transistors bottomed, or saturated, but charge storage effects may then cause an objectionable loss of speed. In saturated operation the diode D may not be necessary, but some mild discrimination must still be present to reject the small 'singles' pulses. The polarity of the whole circuit may be reversed by using NPN

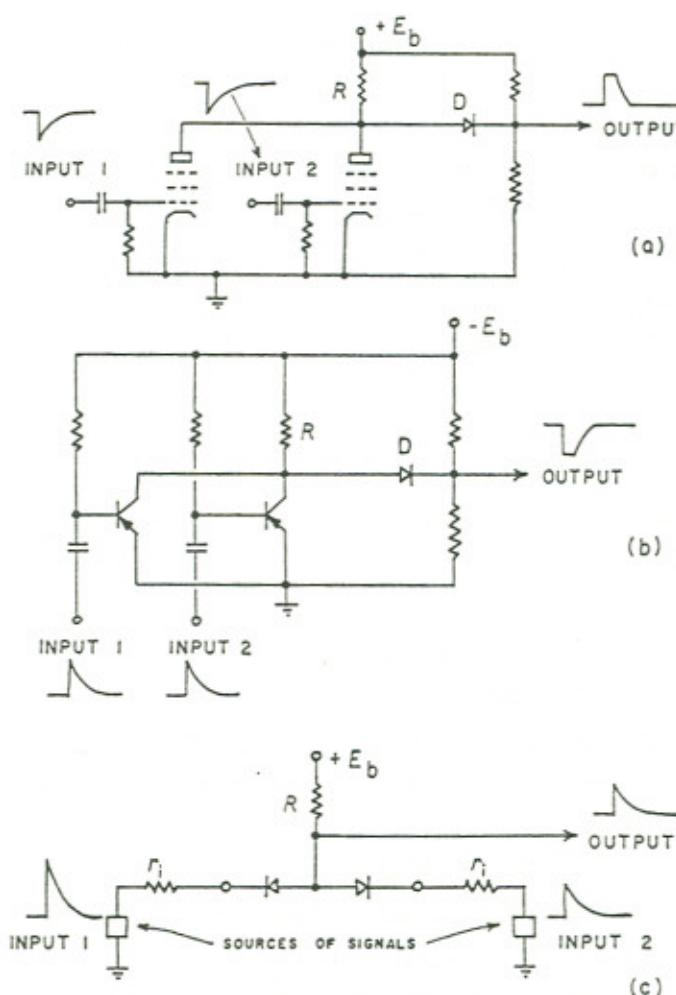
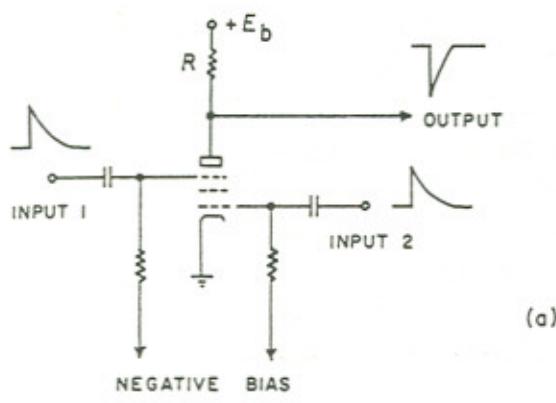


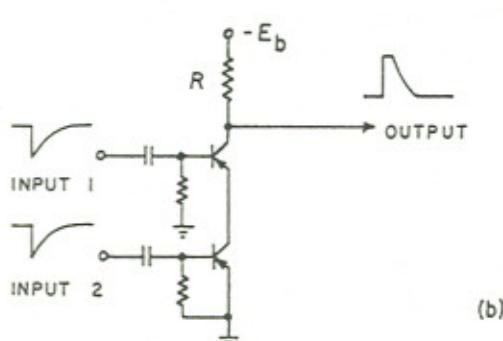
Fig. 1. The fast Rossi, or parallel, coincidence circuit in three forms, using (a) vacuum tubes, (b) transistors, and (c) diodes

transistors instead of PNP. In Fig. 1c, the two diodes each carry a current $E_b/2R$, which flows to ground through the diode and signal source impedance r_i , assumed much smaller than R . A positive pulse from one source can at most shift the current entirely to the other diode, with a resultant small output $E_b r_i / 2R$. Positive pulses from both sources cause an output approximately equal to the smaller of the two signals, which can be many times greater than the single effect. Again some device for mild discrimination against the singles (not shown in Fig. 1c) must be present. This diode circuit may easily be reversed in polarity.

The series coincidence circuit element, first used by Bothe², resembles a pair of switches in series, both of which must be closed if any current is to flow. Fig. 2a shows a multigrid vacuum tube with two control grids, both of them biased negatively so as



(a)



(b)

Fig. 2. The Bothe, or series, coincidence circuit in two forms, using (a) a multigrid vacuum tube, and (b) a pair of transistors

to cut off the flow of current. Positive pulses applied to either grid alone cannot cause the current to flow, but simultaneous positive input pulses of sufficient size to overcome the bias will give an output pulse at the anode. Separate vacuum tubes can be used instead of one multigrid tube, though at least one of the input pulses would then have to have a larger amplitude. Transistors are better than tubes in this respect, because a transistor can pass collector current with less than a volt of collector supply voltage. Among multigrid tubes, there are many pentagrid converters and similar tubes that can be used. The 6AS6 tube has a sensitive suppressor grid in addition to its

² W. Bothe, Z. Fysik 59 (1930) 1.

normal control grid, and the 6BN6 gated beam tube has also been used. In very high speed work, electron transit time effects have to be taken into account, but need not impair the circuit action. Fig. 2b shows a transistor series coincidence circuit. The transistors are initially cut off because the bases are not being supplied with current. Negative pulses at both bases are necessary to make current flow. If these pulses are large enough, the collector may go to saturation, as illustrated by the waveform in Fig. 2b. Charge storage effects may have to be taken into account even if saturation is not reached, because of the low collector voltage on the lower transistor. As usual, the circuit may be reversed in polarity by using NPN transistors.

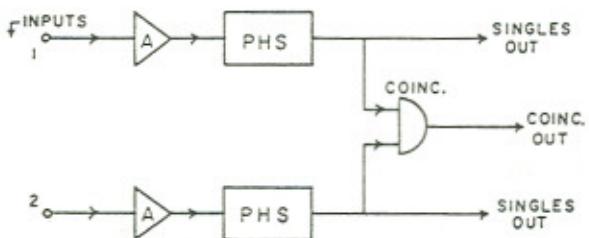
Many other kinds of coincidence circuit element have been used, some of them highly ingenious. There is a class of nonlinear bridge coincidence circuit in which, for example, one pulse produces no output because the circuit is balanced. The other pulse cannot reach the output, but unbalances the bridge for the first pulse; in this way an output is produced only when both pulses are present simultaneously. The main disadvantage is that nonlinear bridges are hard to keep in balance over a large range of pulse amplitudes, so that a large single pulse may produce a false output. Still other arrangements use more than one coincidence circuit with differential time delays (chronotrons, differential coincidence circuits), or make use of oscilloscope tubes with crossed deflections, etc. Generally speaking, the exact details of the coincidence circuit do not matter very much, and nearly any of the types we have mentioned will serve. In practice a majority of circuits are of the parallel type, and most of the remainder are of the series type.

Many reviews of coincidence circuitry and short time measurements have been published. Reference 3 lists some of them, where references to further work may be found. Much excellent coincidence equipment remains undescribed in published papers, or appears only in institutional reports. A number of new solid state devices are seeing service in coincidence circuits; the most important are tunnel diodes and avalanche transistors.

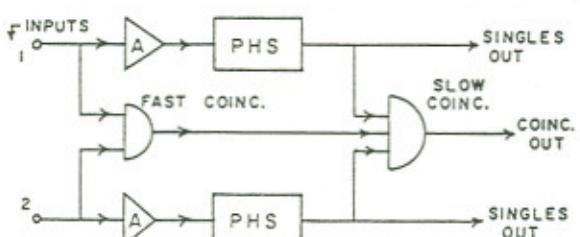
A complete coincidence circuit for two sources of pulses will have to provide for pulse amplitude selection for each pulse train, as well as for time selection (i.e. the coincidence function). A straightforward coincidence assembly for this purpose is shown in Fig. 3a. The outputs of the two counters are passed through amplifiers (A)

³ R. E. Bell, Ann. Rev. Nucl. Sci. 4 (1954) 93;
S. de Benedetti and R. W. Findley, Encyclopedia of Physics 45 (ed. S. Flügge; Springer, Berlin, 1958) 222;
S. Devons, Nuclear Spectroscopy, part A (ed. F. Ajzenberg-Selove; Academic Press, New York, 1960) 512;
E. Bashandy, Nucl. Instr. and Meth. 12 (1961) 227;
Electromagnetic lifetimes and properties of nuclear states (Gatlinburg conference proceedings) (ed. P. H. Stelson; National Academy of Sciences-National Research Council, Washington, Pub. no. 974, 1962);
R. L. Chase, Nuclear Pulse Spectroscopy (McGraw-Hill, New York, 1961) 148;
M. Bonitz, Nucl. Instr. and Meth. 22 (1963) 238. This review contains extensive references to time analyzers of all types.

and pulse height selectors (PHS), and impressed on a coincidence element. This simple assembly is excellent for many purposes, but has the defect that the delays imposed by the linear amplifiers and pulse height selectors prevent the use of a very fast coincidence element. In Fig. 3b we show how this defect can be overcome by performing the coincidence function on pulses unselected as to amplitude, and the amplitude selection on pulses unselected as to time, and then selecting events that satisfy both criteria with a slow coincidence element at the end. Fast amplifiers may be inserted ahead of



(a)



(b)

Fig. 3. (a) A straightforward coincidence assembly with provision for pulse height selection (PHS) after amplification (A) of the pulses from the two counters, 1 and 2. (b) A fast-slow coincidence assembly that performs the same functions as (a), but allows the use of a short resolving time

the fast coincidence circuit if necessary; they need not be particularly linear or stable. This 'fast-slow' principle, first described by Bell and Petch⁴, is now universally used in complex coincidence assemblies. Various inserted delays may be required in either the fast or the slow channels; they are not shown in Fig. 3. The slow circuit output may be used to open the gate of a multichannel analyzer, or to perform further coincidence functions.

A few words must be said about anticoincidence circuits, in which we have an output pulse from one counter except when a cancelling pulse is present from a second counter. Most of the preceding discussion applies to these devices, and the circuits given in Figs. 1 and 2 can be converted to anticoincidence by changing the dc bias and reversing the pulse polarity at the cancelling pulse input. The special difficulty with anticoincidence work is that the cancelling pulse must begin before the signal pulse and finish after. These precise timing requirements are best satisfied at slow speed, and

⁴ R. E. Bell and H. E. Petch, Phys. Rev. 76 (1949) 1409.

therefore a fast-slow system should be used. The assembly of Fig. 3b is changed to an anticoincidence circuit if slow output 2 and the fast coincidence together suppress output 1. Anticoincidence work is easier than coincidence work in the senses that the highest speed is seldom required, and that a few chance anticoincidences seldom matter very much.

There is another whole class of circuits, called linear gate circuits, in which the passage of a signal pulse is permitted only in the presence of a gating pulse. The distinction between this kind of circuit and an ordinary coincidence circuit is that the linear gate circuit preserves the amplitude of the signal pulse, whereas the coincidence circuit merely indicates its presence or absence. We will not consider these circuits further. They are discussed by de Benedetti and Findley³.

§ 2. Measurement of short mean lives of excited states

In what follows we review the main direct methods of measuring short nuclear mean lives*, i.e. those too short to be measured by human or mechanical means, say $< 10^{-4}$ sec. Nuclear mean lives that are short by this definition occur practically only for excited states decaying by electromagnetic transitions (gamma and internal conversion). The object of all the measurements is the transition probability or inverse mean life. Chapter XXI below deals with methods of measuring the widths Γ of nuclear excited states, which are then related to mean lives by the uncertainty relation $\Gamma\tau = \hbar$. Transition probabilities can also be measured by the Coulomb excitation process, covered in Chapter XII. The various methods are complementary in that methods for measuring mean lives work best for mean lives that are not too short, while methods for measuring widths work best for widths that are not too small, i.e. mean lives that are not too long. The region of overlap between the methods usually lies at between 10^{-10} and 10^{-11} sec.

The experimental mean life of an excited state is the inverse sum of all the transition probabilities by which it can de-excite. If our object is the mean life for some particular mode of de-excitation, the experimental mean life must be corrected accordingly. In particular if an excited state de-excites via a particular γ -ray with branching ratio f and total conversion coefficient α , the γ -ray partial mean life τ_γ is related to the experimental mean life τ_{exp} by $\tau_\gamma = \tau_{\text{exp}}(1 + \alpha)/f$. The branching ratio f may include the M1/E2 mixing ratio, if, for example, an M1 partial mean life is desired.

The most numerous nuclear γ -transitions are those of M1 or E2 character, and except for a few E1 transitions, these have the shortest mean lives. In Fig. 4 we show some rough mean life estimates for M1 and E2 radiations for the cases $Z = 10$ and $Z = 82$. The curves have been computed from the single particle estimates of Chapter XV with the following modifications, based on experience. Since E2 transition probabilities are usually found to be enhanced over the single particle estimates by factors ranging from unity to 100, the single particle E2 mean lives have been divided by 10.

* Throughout what follows we use the mean life, τ , rather than the half life. The loose word lifetime is avoided.

For similar reasons the M1 single particle mean lives have been multiplied by 100. The curves plotted in Fig. 4 should be used only as very rough guides, of course. The figure also shows a broken line dividing the mean life region presently accessible to time measurements from that accessible to width measurements. The division has been shown at 2×10^{-11} sec, but this dividing region is rough, and the two kinds of

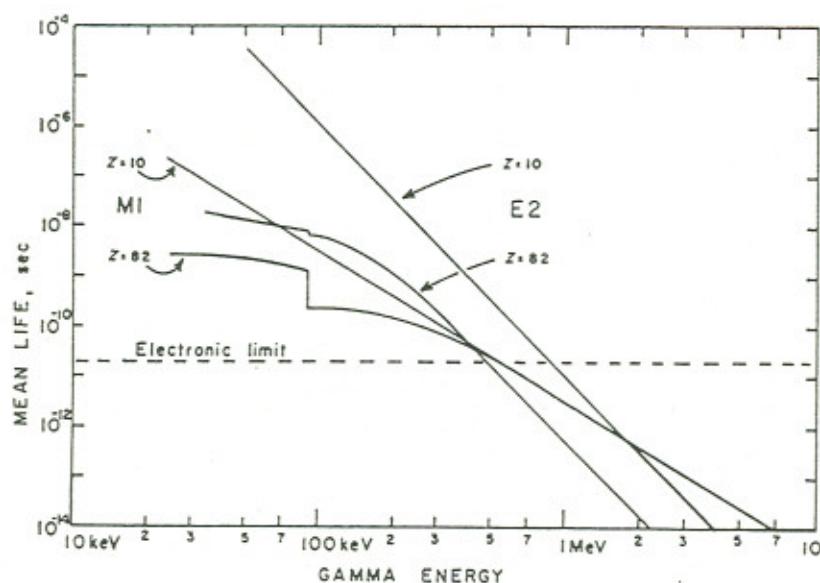


Fig. 4. Estimated mean lives of M1 and E2 γ -transitions as functions of energy, based on the single particle estimates of Chapter XV, modified as described in the text. Two values of Z are shown (10 and 82), and the internal conversion effect is included

measurements overlap to some extent. Fig. 4 shows that there is strong impetus to extend the time measurements to the shortest possible mean lives, and much of the discussion in this chapter will be directed to that end.

§ 3. The delayed coincidence method

This method is the most broadly applicable one for measuring short nuclear mean lives, as opposed to widths. Consider that the excited state whose mean life we wish to measure has been formed by some preceding decay or nuclear reaction. Let the preceding event provide us with a 'preceding' pulse, and let the radiations by which the excited state decays to lower states provide us with a 'delayed' pulse. The distribution of time delays between the two kinds of pulses is of the form $\exp(-t/\tau)$, where τ is the desired mean life, modified by any timing uncertainties in the two pulses. We can measure this distribution of time delays in three ways, the first of which is now only of historical interest.

(1) In the *integral* delayed coincidence method, the preceding pulses are extended to a known, variable duration T , while the delayed pulses are kept relatively short. The two trains of pulses are impressed on a suitable coincidence element, and the coincidence counting rate is measured as T is changed. At any one value of T , the

coincidence counting rate is the integral of the desired time distribution from 0 to T . Subtraction of readings at successive values of T then reproduces the original time distribution. This integral method, sometimes called the method of variation of resolving time, appears to have started with the work of Feather and Dunworth⁵, and ended with that of Jelley⁶ on the 6.0 μ sec mean life of Po^{213} .

(2) In the *differential* delayed coincidence method, the preceding and delayed pulses are kept fixed in length, but the preceding pulses are delayed by a time x before being impressed on a coincidence circuit. We thus detect those delayed pulses which originally occurred later than preceding pulses by a time x , the range of delays near x over which the pulses are detected being the resolving time of the coincidence circuit. If the time resolution of the apparatus is short compared with the mean life being measured, the form of the measured curve of counting rate versus x is $\exp(-x/\tau)$, and the desired result is measured directly. The statistical accuracy of this method is superior to that of the integral method for the same reasons that single channel pulse height analysis is superior to differentiation of an integral bias curve.

The first recognizable use of this classical delayed coincidence method was by Jacobsen⁷ in 1934 using two moving-iron oscilloscopes. The method was used in modern form by Jacobsen and Sigurgeirsson⁸ in 1943, using Geiger counters and a resolving time of 75 μ sec. By 1948 de Benedetti and McGowan⁹ had surveyed over 60 radioactive species, finding measurable mean lives in the μ sec domain in several of them. Fig. 5 shows their result for the mean life of the 615 keV excited state of Ta^{181} . The preceding radiation was β -particles leading to the excited state, and the delayed radiation was a 481 keV γ -ray, the detectors for both being Geiger counters. The curve of counting rate versus inserted delay time x resembles an ordinary radioactive decay curve, and the mean life is taken from it in the usual way.

The necessary delays may be inserted either by electronic delay circuits (univibrators, phantastrons, etc.), by digital circuits that count the pulses of a clock oscillator, or by variable delay lines. In the more accessible time regions the first two are useful, but generally speaking only the last is suitable when resolving times near the limit of technique are in use. Digital instruments for measuring time intervals are available commercially, and laboratory oscilloscopes contain accurate, wide-range analogue time-base circuits.

The introduction of scintillation counters with fast organic phosphors led to rapid progress towards shorter resolving times and hence the measurement of shorter mean lives. Fig. 6 illustrates the measurement of the mean life of the 84 keV excited state of Yb^{170} by Graham *et al.*¹⁰ in 1952. They used the coincidence circuit of Bell *et al.*¹¹, in-

⁵ N. Feather and J. V. Dunworth, Proc. Roy. Soc. (London) A168 (1938) 52.

⁶ J. V. Jelley, Can. J. Research A26 (1948) 255.

⁷ J. C. Jacobsen, Nature 128 (1931) 185; 133 (1934) 565.

⁸ J. C. Jacobsen and Th. Sigurgeirsson, Mat.-Fys. Medd. Dan. Vid. Selsk. 20 (1943) no. 11.

⁹ S. de Benedetti and F. K. McGowan, Phys. Rev. 74 (1948) 728.

¹⁰ Graham, Wolfson and Bell, Can. J. Phys. 30 (1952) 459.

¹¹ Bell, Graham and Petch, Can. J. Phys. 30 (1952) 35.

serting the necessary delays by means of a variable coaxial delay cable. Fig. 6 also shows the time resolution curve for their apparatus, made by measuring a source of prompt coincidences, i.e. a comparison source having negligible τ . Their result

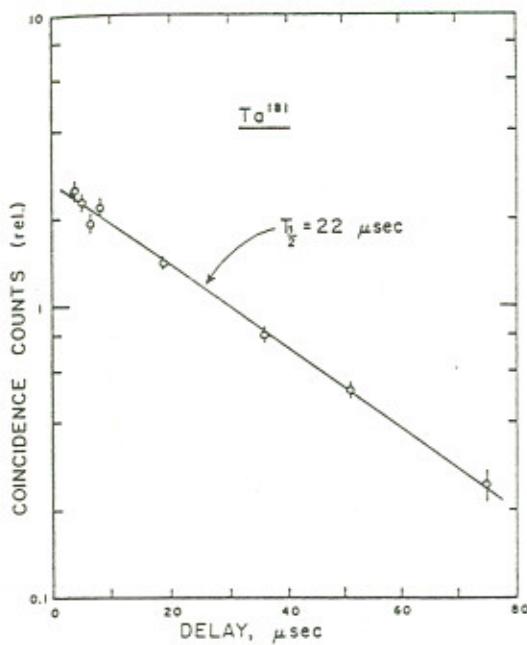


Fig. 5. Decay curve for the 615 keV excited state of Ta^{181} recorded by de Benedetti and McGowan⁹, using a single channel apparatus with electronic delays. Their result for the half life corresponds to 32 μ sec for the mean life, a value now thought to be about 15% too high

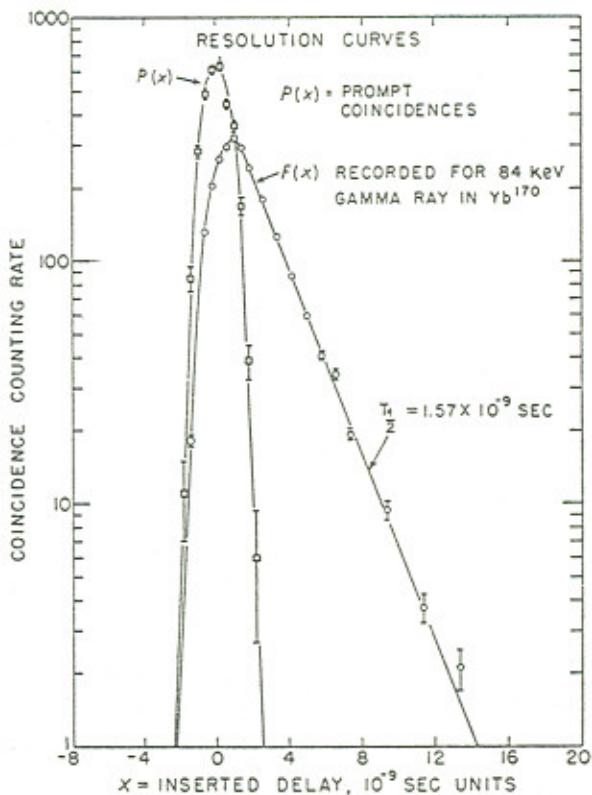


Fig. 6. Decay curve for the 84 keV excited state of Yb^{170} , recorded by Graham *et al.*¹⁰, using a single channel apparatus with coaxial cable delays

corresponds to $\tau = (2.27 \pm 0.07)$ nsec. This example is one of the earliest measurements of the mean life of what is now known as the first rotational state of an even-even deformed nucleus (see Chapter X).

(3) The *multichannel* delayed coincidence method gives the whole of the delay distribution in a single measurement. For the longer times, digital timing apparatus is the most useful. Many commercial multichannel pulse amplitude analyzers either possess, or can be provided with, inputs for use as multichannel time analyzers; indeed, most such analyzers are basically time analyzers, their use as pulse amplitude analyzers being accomplished by a conversion from amplitude to time at their inputs. The large multichannel time analyzers used in neutron time-of-flight work are of the type required for mean life measurements. Digital devices driven by accurate clock oscillators are capable of extreme systematic accuracy, and are usable down to times of perhaps 0.1 μ sec per channel. Fig. 7 shows a decay curve measured for the 14.4 keV excited state of Fe^{57} by Clark (private communication), using a multichannel digital

analyzer with channel times of $0.1 \mu\text{sec}$. In this case the preceding radiation, a 123 keV γ -ray, gated on a 10 Mc/s clock oscillator, and the delayed radiation, the well-known 14.4 keV γ -ray of Fe^{57} , stopped it. Each coincidence event thus provided its own address in a multichannel recording device. Clark's result was $\tau = (140.2 \pm 0.3) \text{nsec}$,

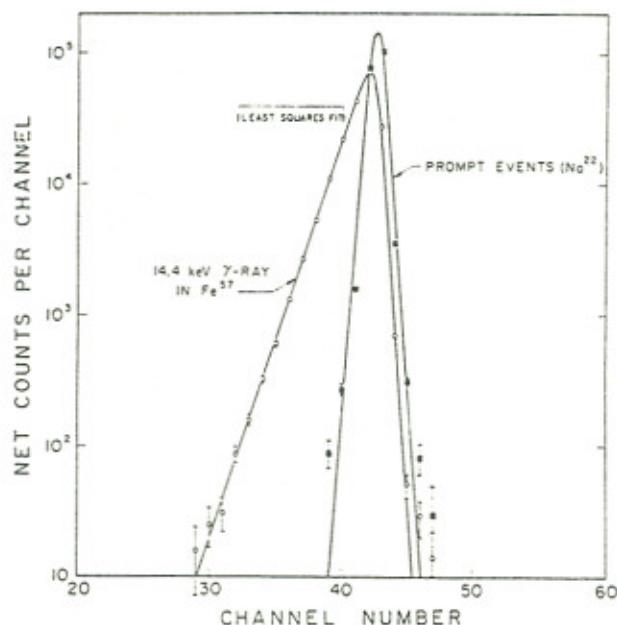


Fig. 7. Decay curve for the 14.4 keV excited state of Fe^{57} , recorded by Clark (unpublished) with a multichannel digital apparatus. Clark's result is $\tau = (140.2 \pm 0.3) \text{nsec}$, found by a least squares fit to the region of the curve indicated in the diagram

based on many runs like that of Fig. 7. Another example of a mean life measured precisely with digital analyzers is that of the positive muon, $2.21 \pm 0.005 \mu\text{sec}$, first measured accurately by W. E. Bell and Hincks¹², and most recently by Reiter *et al.*¹³, in good agreement. Both the Fe^{57} and the muon mean lives need to be known accurately for comparison with other data, but in most cases an accuracy of a few percent in a mean life is quite sufficient.

When mean lives shorter than about $0.1 \mu\text{sec}$ are at stake, the digital device gives way to the time-to-amplitude converter (TAC). In this device, the time interval between the preceding pulse and the delayed pulse is converted to a pulse whose amplitude is proportional to the interval. A multichannel amplitude analyzer of any suitable type can then be used to record the time interval distribution, or time spectrum. (Note that there is no direct connection between the time intervals being measured and the recording speed of the multichannel analyzer.) The time spectrum is on a relative scale only, and requires a calibration if it is to yield absolute times. Since we are dealing with an analogue method, the systematic accuracy is lower than it is in a

¹² W. E. Bell and E. P. Hincks, Phys. Rev. 88 (1952) 168.

¹³ Reiter, Romanowski, Sutton and Chidley, Phys. Rev. Lett. 5 (1960) 22.

digital device. The method is capable of extreme speed, however, and the ultimate limit is to be found in timing uncertainties in radiation detectors rather than in the TAC circuit itself.

The use of the TAC goes back at least as far as 1942, when Rossi and Nereson¹⁴ used one in the first laboratory measurement of the mean life of the muon. The first use of a TAC circuit in the nanosecond region is due to Fraser and Milton¹⁵ in 1953. Further developments were made by Weber, Johnstone, and Cranberg¹⁶. Both of these TAC circuits used the principle of having the preceding pulse turn on a constant current, and the delayed pulse turn it off again (the start pulse-stop pulse system). The constant current flowing into an integrating capacitor produces a voltage pulse proportional to the time interval. This scheme is straightforward, but has the following disadvantage. In most coincidence experiments, the single counting rates greatly exceed the coincidence rate. Thus in most cases the preceding pulse starts the current, but no delayed pulse arrives to stop it, and the maximum output pulse height is reached. The desired coincidence events then occur as a small number of small pulses among a much larger number of large pulses.

A better system, in many ways, is to shape the two pulses to standard size and shape, and allow them to overlap in time. The duration of overlap is linearly proportional to the relative delay of the two pulses, and the constant current is turned on only during the overlap time. If there is no near coincidence, no current flows, and no output pulse appears. The desired pulses now appear alone on the pulse amplitude scale. Many groups of workers have used TAC circuits of this kind, among them Jones and Warren¹⁷, Green and Bell¹⁸, Gorodetzky *et al.*¹⁹, and Schwartzschild and Kane²⁰. The earlier versions usually used pentode limiters for pulse equalization and a 6BN6 gated beam tube for controlling the constant current. Later one finds the 6BN6 replaced by a simple biased diode or triode, or a double-diode current switch. Some workers have used transistor limiters, or fast tunnel-diode discriminators in place of limiters. The exact electronic details do not seem very important, because the limiting factors in performance lie elsewhere.

The advantage of the multichannel method is the same as that of a multichannel pulse amplitude analyzer over a single channel analyzer. The whole time spectrum is measured in one observation. Drifts in the apparatus affect all time channels alike, and source decay corrections become either trivially easy or wholly unnecessary. In the case of the TAC circuit there is one disadvantage. Since time intervals are represented by pulse heights, a drift in amplification or zero position in the time-

¹⁴ B. Rossi and N. Nereson, Phys. Rev. 62 (1942) 417.

¹⁵ J. S. Fraser and J. C. D. Milton, Chalk River progress report PR-P-20 (1953); Nucl. Instr. and Meth. 2 (1958) 275.

¹⁶ Weber, Johnstone and Cranberg, Rev. Sci. Instr. 27 (1956) 166.

¹⁷ G. Jones and J. B. Warren, J. Sci. Instr. 33 (1956) 429.

¹⁸ R. E. Green and R. E. Bell, Nucl. Instr. and Meth. 3 (1958) 127.

¹⁹ Gorodetzky, Richert, Manquenouille and Knipper, Nucl. Instr. and Meth. 7 (1960) 50.

²⁰ A. Schwartzschild and J. V. Kane, Phys. Rev. 122 (1961) 854.

analogue circuitry is the equivalent of a time shift or a change of time scale. Extra precautions must therefore be observed whenever one is either working near the limit of technique, or striving for high accuracy; special stabilizing devices may be necessary (see § 7 below).

Fig. 8 shows an example of the measurement of a short mean life in Ra²²⁶ by means of a TAC apparatus, taken from the work of Bell *et al.*²¹. The preceding radiation was α -particles from Th²³⁰ leading to the 68 keV excited state of Ra²²⁶, and the delayed radiation was conversion electrons from the 68 keV transition to the ground state.

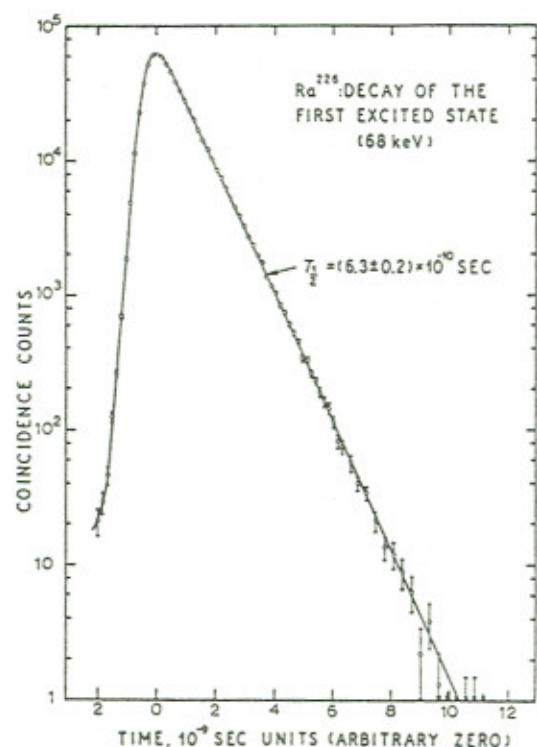


Fig. 8. Decay curve for the 68 keV excited state of Ra²²⁶, recorded by Bell *et al.*²¹, using an analogue multichannel apparatus (TAC)

The mean life of the 68 keV excited state was found to be $(9.1 \pm 0.3) \times 10^{-10}$ sec. The statistical accuracy in this measurement is excellent, and the quoted error reflects only systematic uncertainties.

§ 4. Physical limits on time resolution

A fundamental limit on the time resolution obtainable is furnished by the dimensions of the apparatus and the velocities of the radiations concerned.

The total transit time of the pulse through a typical photomultiplier is 300×10^{-10} sec, of protons through a Van de Graaf generator perhaps 1500×10^{-10} sec, and of 50 keV electrons through a β -spectrometer possibly 50×10^{-10} sec. Clearly an appreciable spread in any of these times, caused by either finite dimensions of apparatus, or velocity spreads, or path length spreads, can degrade the time resolution so as to

²¹ Bell, Bjørnholm and Severiens, Mat-Fys. Medd. Dan. Vid. Selsk. 32 (1960) no. 12.

obscure a lifetime in the 10^{-10} second region. We expect therefore that care will be needed in measuring mean lives near 10^{-10} seconds, and that the measurements may become impossibly difficult around 10^{-11} seconds, or not far below.

Another kind of limitation appears in the pulsed accelerator method. This variant of the classical delayed-coincidence method uses a pulsed accelerator to form the excited state, and a counter to detect the delayed radiation. The preceding pulse is then an electrical pulse synchronized with the accelerator beam pulse, and the measurement proceeds as before. The techniques for pulsing an accelerator form a subject of their own. The most obvious way is by sweeping a beam across a slit, using a high frequency deflecting voltage. Years ago Mobley²² showed that a properly designed deflecting magnet can compress the resulting beam burst in time; a slanted target can also assist in this compression. More recently Fowler and Good²³ have shown that the degree of time compression obtainable depends on the original quality of the beam (spread in energy and angle), and that introducing the time compression inevitably involves a degradation of the other qualities of the beam. The relations they present resemble a kind of uncertainty principle. In general the time resolution with a pulsed accelerator is no better than that in the ordinary delayed coincidence method, but there are other advantages. The main ones are: (1) the coincidence rate and the ratio of true to chance coincidence rates may be very favorable because in effect the preceding counter has been endowed with 100% efficiency, and (2) excited states can often be reached by bombardment that cannot be excited in radioactive decay. Pulsed accelerators are also essential in many time-of-flight spectroscopy experiments.

§ 5. Statistical limits on coincidence measurements

Statistical limits for mean life measurements occur both at very short times and at very long times. Consider a source of N disintegrations per second giving a pair of cascade radiations 1 and 2; we are measuring the mean life τ of the state intermediate between the radiations 1 and 2. Suppose one counter detects only radiation 1, with efficiency $\varepsilon_1 (\ll 1)$, and the other counter detects only radiation 2, with efficiency $\varepsilon_2 (\ll 1)$. Then the single counting rates and the total true coincidence rate are respectively

$$N_1 = N\varepsilon_1, \quad N_2 = N\varepsilon_2, \quad N_{12} = N\varepsilon_1\varepsilon_2.$$

If we are using a multichannel time analyzer, the coincidence counting rate in the most favorable channel will be roughly $N_{12}\tau^{-1}\Delta$, where Δ is the time width of each channel, assumed to be at least a few times smaller than τ . If we are to observe the decay over at least one mean life, the final channel will record a rate of only $N_{12}(e\tau)^{-1}\Delta$, where e is the base of natural logarithms. The chance coincidence rate, the same for all channels, is at least $\Delta N_1 N_2$. Thus in the least favorable channel the ratio of true to chance coincidence rates will be no better than $(e\tau N)^{-1}$. This result shows that if τ is large, N must be correspondingly small in order to preserve any chosen true-to-chance

²² R. C. Mobley, Phys. Rev. 88 (1952) 360;
Cranberg, Fernald, Hahn and Shrader, Nucl. Instr. and Meth. 12 (1961) 335.

²³ T. K. Fowler and W. M. Good, Nucl. Instr. and Meth. 7 (1960) 245.

coincidence ratio. With N reduced for this reason, all the counting rates are low, and the mean life experiment must either have a duration proportional to the mean life being measured, or sacrifice statistical accuracy. The estimates just given are optimistic because either or both of the counters could be detecting radiations other than those called 1 and 2 above, thus raising the chance coincidence rate without raising the true rate, and perhaps mixing in some coincidence events of another mean life. On the other hand if either counter has an efficiency approaching unity, the chance coincidence rate is greatly reduced. High efficiency may be achieved in α - or β -counters of suitable type, and is automatic when the preceding pulse is provided by a pulsed accelerator. We always find this high efficiency feature present in experimental measurements of mean lives longer than a few tens of microseconds.

In the above we have assumed that the time resolution of the apparatus was sufficiently good. At the short end of the time scale, timing uncertainties in the counters and auxiliary apparatus introduce an effective time spread that may be comparable with τ or larger. The time resolution curve of the apparatus for prompt coincidences (i.e. coincidences where τ is negligible) has a full width W at half maximum, which is a measure of this time spread. When τ/W is small, means must be found to detect the small unsymmetrical broadening and shifting of the resolution curve due to the presence of τ . On general statistical grounds, one might expect that about $(W/\tau)^2$ counts in the resolution curve would detect the presence of τ , and several times that number would give a fair measure of the magnitude of τ . Thus if $W = 5 \times 10^{-10}$ sec, we might hope to detect the presence of $\tau = 5 \times 10^{-12}$ sec if 10^4 coincidence events are recorded, and to form a rough quantitative idea of this τ , if, say, 10^5 coincidences are recorded. Actually, however, systematic limitations are entering strongly at this point, and the difficulty in measuring the mean life may well be more systematic than statistical. A practical limit for the delayed coincidence method is a few times 10^{-11} sec, varying of course with the particular case.

The advantage of having W small is very evident, and the causes of this width are themselves statistical. We will examine them semi-quantitatively for the case of scintillation counters with fast organic phosphors. Assume that the incident radiation causes a light flash that decays exponentially with a mean life τ_p characteristic of the phosphor, and that this light flash produces exactly R photoelectrons at the photocathode of the photomultiplier. Each photoelectron starts a multiplicative cascade that traverses the photomultiplier, arriving at the output with time uncertainty σ , tacitly assumed to be the standard deviation of a Gaussian distribution. The parameters τ_p , R , and σ are not very accurately known for actual scintillation counters. Rough values would be $\tau_p = 4$ nsec for a plastic phosphor, and $R = 600$ for 1 MeV electrons absorbed in it. A 5 cm end-window photomultiplier with curved cathode might have $\sigma = 1$ ns, whereas earlier models with flat cathodes would be more than twice as bad. For phosphors of ordinary size, the spread in time caused by light collection over finite dimensions is usually negligible. The photoelectrons leaving the cathode are individuals, but the photomultiplier output response to each one is usually more than 1 nsec wide, so that it may not be very meaningful to talk of individual photoelectrons

at the output of the photomultiplier; nevertheless we will do so, because simple arguments can then be used to give useful if not highly accurate results.

If σ is negligible, the output response from the light flash is R individual photo-electron pulses distributed exponentially over a time characterized by τ_p . The initial rate of photoelectron pulses is R/τ_p , and one guesses informally that both the mean and the standard deviation of the time of arrival of the first photoelectron are τ_p/R . This was proved by Post and Schiff²⁴ in 1950, and is accurate to the extent that $R \gg 1$. In our numerical example τ_p/R is less than 10^{-11} sec.

If the effect of σ is now included, the total response is spread over a time of the order of $\sqrt{(\sigma^2 + \tau_p^2)}$, not very different from τ_p . Now, however, the initial rate of photoelectron pulses is much smaller than R/τ_p because we are working on the skirt of the assumed Gaussian shape for the photomultiplier time spread. The standard deviation in the time of arrival of the first photoelectron pulse will be of the order of $\sqrt{[\sigma^2 + (\tau_p/R)^2]}$. By a rough argument first given by Morton²⁵, and much elaborated by Gatti and Svelto²⁵, the standard deviation of the arrival time of the n th photo-electron should be, for $n \ll R$,

$$\sigma_n = \sqrt{[\sigma^2/n + n(\tau_p/R)^2]} .$$

Thus if the coincidence circuit 'triggers on the n th photoelectron', it should produce a prompt time resolution curve of approximately Gaussian shape (if $n \gg 1$) with standard deviation $\sigma_n\sqrt{2}$, the factor $\sqrt{2}$ entering because there are two scintillation counters in the circuit. If the circuit responds to the average time of arrival of the first n photoelectrons, rather than to the exact time of the n th one, the time spread is reduced by a factor of about $\sqrt{3}$, as may be shown in an elementary way. The full width at half maximum, W , is connected with σ_n by

$$W = 2\sigma_n \sqrt{\left[\frac{2}{3} \ln 2\right]} = 1.92\sigma_n .$$

Now minimizing σ_n with respect to n , we find that the minimum occurs for $n/R = \sigma/\tau_p$, and then W_{\min} is equal to $2.7\sqrt{(\sigma\tau_p/R)}$. For the numerical parameters already introduced as examples, we finally get $W = 2.7 \times 10^{-10}/\sqrt{E}$ sec, where E is the electron energy (MeV) lost in the phosphor. The maximum Compton electron energy for Co⁶⁰ γ -rays is about 1.0 MeV, so that $W = 2.7 \times 10^{-10}$ sec should apply to γ - γ coincidences from Co⁶⁰ with selection of pulses near the maximum size. A curve recorded by Weaver and Bell (unpublished) under these conditions is shown in Fig. 9, and gives $W = 2.4 \times 10^{-10}$ sec, in satisfactory agreement with the estimates above. The slope of the side of a Gaussian plotted on semi-logarithmic paper is not a constant, but corresponds to an instrumental mean life lying between $0.20W$ and $0.14W$, if measured on the part of the curve lying between 0.1 and 0.01 of the peak. The instrumental mean

²⁴ R. F. Post and L. I. Schiff, Phys. Rev. 80 (1950) 1113.

²⁵ G. A. Morton, Nucleonics 10, no. 2 (1952) 39;
E. Gatti and V. Svelto, Nucl. Instr. and Meth. 4 (1959) 189.

lives marked in Fig. 9 lie around 4.5×10^{-11} sec, as expected, and represent the limit of mean life that can be measured by straightforward slope measurement with this particular apparatus at this energy.

Experience has shown that the best resolving time is indeed attained at around $n/R \approx \sigma/\tau_p \approx 0.25$ for fast organic phosphors and fast photomultipliers, that is, that the best time resolution comes from a circuit actuated by approximately the first 25 %

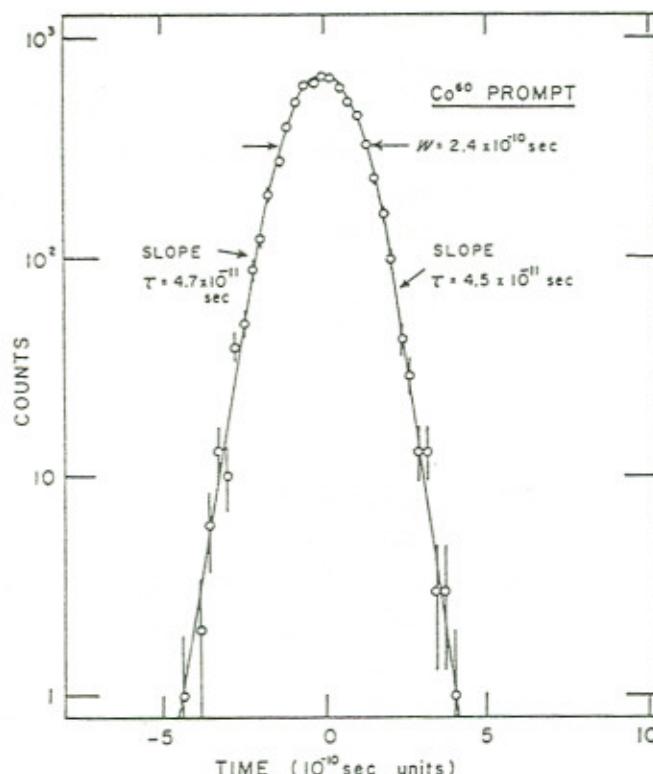


Fig. 9. Prompt time resolution curve for Co^{60} $\gamma\gamma$ coincidences recorded by Weaver and Bell (unpublished). (NATON-136 plastic phosphors, CBS7817 photomultipliers)

of the integrated output pulse. The other predictions of the foregoing (that the resolution width at optimum adjustment varies as the square root of σ , τ , and inverse R) are roughly borne out. For sodium iodide phosphors, for example, τ_p is 2.5×10^{-7} sec and R is at least 4000 electrons at 1 MeV. Then for 511 keV $\gamma\gamma$ coincidences (annihilation radiation), a resolving time of around $W = 2 \times 10^{-9}$ should be possible at small n/R , and has been observed. We see that photocathode sensitivity and good light collection are as important in time resolution (through the factor R) as in pulse height resolution.

Solid state detectors with thin depletion layers have about the same collection times as photomultipliers with fast organic phosphors (a few nsec), but the number of electrons collected is perhaps 500 times greater. These detectors therefore have an intrinsic time resolving power about $\sqrt{500}$ times better than scintillation counters. Technical difficulties of amplification and noise have so far prevented the effective use

of solid state counters for fast coincidence work. Another difficulty is that thin solid state counters are unsuitable as γ -ray detectors.

§ 6. Evaluation of delayed coincidence experiments

The ideal delayed coincidence experiment consists of a measurement of the time spectrum of the delayed radiation, and a comparison measurement of a prompt radiation of exactly similar properties except for its mean life. We call the time distribution of the delayed radiation $F(t)$, and that of the prompt radiation $P(t)$; with the two distributions normalized to equal areas, we have

$$F(t) = \tau^{-1} \int_0^\infty e^{-t'/\tau} P(t' - t) dt'$$

for the case where all the radiation detected by the 'delayed' counter has mean life τ . Then, as shown by Bay²⁶ and Newton²⁷, the following facts are true:

(a) $F(t)$ and $P(t)$ intersect at the maximum of $F(t)$; see Figs. 6 and 7, for example. This is a useful check on the consistency of an experiment. In single channel experiments we would use x , the inserted delay time, in place of t .

(b) $F(t)$ falls off exponentially with mean life τ in the region where $F(t) \gg P(t)$. This straight-line portion of $F(t)$ on a logarithmic plot yields τ directly by inspection, as in an ordinary radioactive decay curve; see Figs. 6, 7, and 8, for example. Standard numerical methods such as least squares fitting or Peierls's method²⁸ may be used. The mean life is said to be measured by the slope method, in this case.

(c) The centroid of $F(t)$ lies at a time τ later than the centroid of $P(t)$. An evaluation of this 'centroid shift' is statistically the best determination of τ ; unhappily it may also be subject to instrumental drifts, as well as to uncertainties about how to measure a strictly applicable prompt curve. The centroid is the normalized first moment, and there is a whole series of moment relations that may be written

$$M_r(F) = \sum_{k=0}^r \frac{r!}{k!(r-k)!} M_{r-k}(P) M_k(w),$$

where M_r stands for the normalized r th moment, for example

$$M_r(F) = \frac{\sum_t t^r F(t)}{\sum_t F(t)},$$

and w is the radioactive decay distribution $w(t) = \tau^{-1} \exp(-t/\tau)$. These moment relations have been applied to cases where the delayed curve is complex by Birk *et al.*²⁹. A particular case is the normalized third moment of the delayed curve about its own

²⁶ Z. Bay, Phys. Rev. 77 (1950) 419.

²⁷ T. D. Newton, Phys. Rev. 78 (1950) 490.

²⁸ R. D. Evans, The Atomic Nucleus (McGraw-Hill, New York, 1955) 812.

²⁹ Birk, Goldring and Wolfson, Phys. Rev. 116 (1959) 730;
Birk, Blaugrund, Goldring, Skurnik and Sokolowski, Phys. Rev. 126 (1962) 726.

centroid, which Weaver and Bell³⁰ showed was equal to $2\tau^3$ if the prompt curve is symmetrical. In this way τ may be determined with only sketchy knowledge of the prompt curve, but the statistical accuracy is inferior. (Any moment of a measured curve is of course easily calculated from the experimental counting rates at different delays, using the definition of M_r given above.) Either the first moment (centroid) or third moment methods may be used in the self-comparison method¹¹, in which the rôles of the two counters are interchanged. The centroid shift is then 2τ , and the third moment difference is $4\tau^3$. The method is attractive because the real effect is doubled while many systematic errors are cancelled. Care must be taken, however, to ensure that the counters have interchanged their rôles in a genuinely symmetrical way; when this is done, the limiting accuracy may approach 10^{-11} sec.

An excellent discussion of the feasibility of a coincidence measurement, and of its evaluation, is given by de Benedetti and Findley³. A basic reference for evaluation of delayed coincidence experiments is the paper of Bell *et al.*¹¹.

§ 7. A fast time-to-amplitude converter

Fig. 10 is the diagram of an actual fast TAC circuit in use at McGill University. The prompt resolution curve of Fig. 9 was recorded with this unit. In the diagram, the counters are RCA 6342A, CBS 7817, or 56AVP photomultipliers with plastic phos-

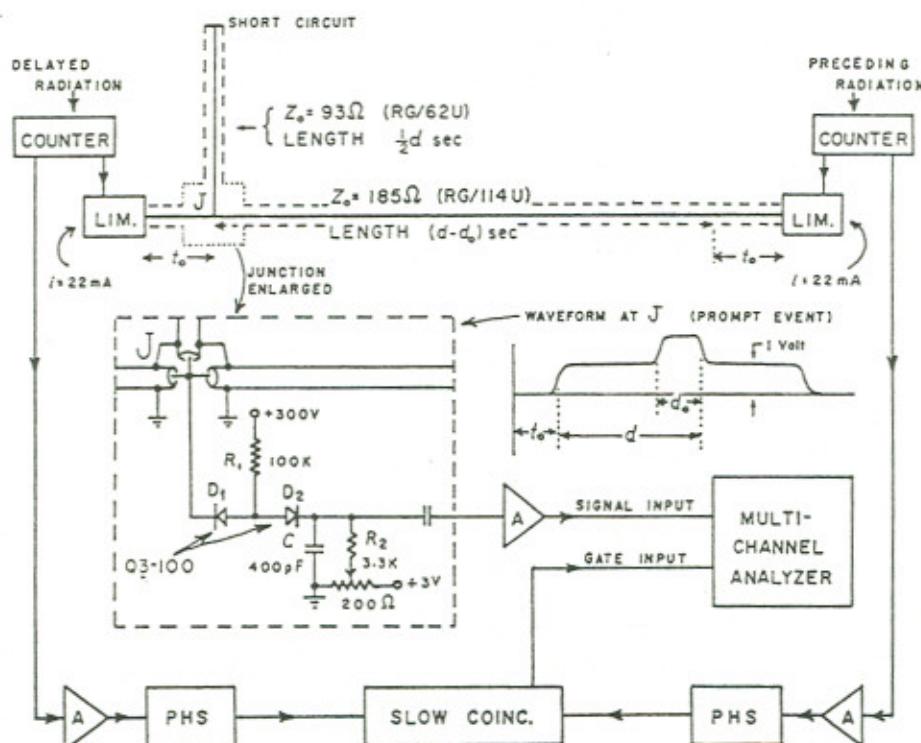


Fig. 10. Diagram of the fast time-to-amplitude converter at McGill University with which the curve of Fig. 9 was recorded. The action of the circuit is described in the text

³⁰ R. S. Weaver and R. E. Bell, Nucl. Instr. and Meth. 9 (1960) 149.

phors, and the limiters are 404A or E186F (7737) pentodes carrying 22 mA. This standing current is cut off by negative pulses from the photomultiplier anodes. These pulses are adjusted to be about 8 V in amplitude, about four times the size required to cut off the limiters. Thus the effective time at which the limiters are cut off is some average over the time of arrival of the first 25% of the anode pulse, as discussed in the preceding section. The anode loads of the photomultipliers ($20 \text{ k}\Omega$, not shown in Fig. 10) give the anode pulses a decay time constant of the order of $0.5 \mu\text{sec}$, during which time the limiters remain cut off. The plates of the pentode limiters have 180Ω load resistors (not shown) that correctly terminate the $Z_0 = 185 \Omega$ coaxial cables in Fig. 10. Upon arrival at the junction J, where the total impedance is $185/4 = 46 \Omega$, the wave from a limiter has a steep front, a flat top, a duration of the order of $0.5 \mu\text{sec}$, and an amplitude of 1 volt positive. It is clipped by the short-circuited line to a duration $d \text{ sec}$. Simultaneous pulses from the 'delayed' and 'preceding' limiters each have a length d , and overlap in time by an amount d_0 , as shown in the waveform in Fig. 10. The overlap time is increased by any delay time, t , in the 'delayed' radiation, up to $t = d - d_0$; then the overlap starts to decrease again.

The actual time-to-amplitude converting element is the double diode switch D_1, D_2 . A resistor R_1 of effectively infinite size ($100 \text{ k}\Omega$) feeds 3 mA of current through diode D_1 to ground via the short circuit on the clipping cable. The common point of the two diodes D_1D_2 sits at about 0.3 V, the forward drop of D_1 . When a 'single' pulse arrives at J, the common point D_1D_2 rises by 1 V to 1.3 V. Diode D_2 is reverse-biased by the 200Ω potentiometer to a voltage of 1.3 V, so that it remains cut off for all single pulses. In the case of an overlap pulse, the common point D_1D_2 starts rising towards 2.3 V. When the pulse reaches 1.3 V, however, current starts to flow in D_2 , and when the pulse reaches 1.6 V, the whole 3 mA has switched from D_1 to D_2 . The input to D_1 continues to rise for another 0.4 V, reverse-biasing D_1 by that amount. The 3 mA is now flowing into C , charging it at a constant rate of $7.5 \times 10^6 \text{ V/sec}$. In the McGill unit, the maximum possible overlap time, d , is set at 30 ns (d_0 is 5 ns), and in this overlap time C charges to 0.225 V. The common point D_1D_2 of course rises by the same amount, and reaches 1.825 V. At this time D_1 is still reverse-biased by 0.175 V, which serves as safety margin at full scale. The charge on C leaks off through R_2 with time constant $1.3 \mu\text{sec}$, chosen so as to be very long compared with the fast time scale. A pulse with this time decay is suitable for use in ordinary amplifiers and multichannel analyzers.

Finally the time-analogue pulse, proportional to d_0 plus the delay t of the delayed radiation, is further amplified and fed to a gated multichannel analyzer. The opening of the gate is controlled by amplifiers and pulse height selectors (PHS) in the 'fast-slow' manner. The PHS may be of any degree of complexity.

The diode switch D_1D_2 acts as a second limiter, so that the charging rate of C is independent of modest changes in the amplitudes of the overlapping pulses coming from the first limiters. The diodes must be of a fast-switching type. A simple biased diode or triode can serve almost as well for a TAC element, but does not give the second limiting action. The diode switch was suggested as a TAC element by Jones

(unpublished). With a different cable configuration, a 6BN6 tube furnishes a good degree of second limiter action^{15, 18}.

§ 8. Calibration methods and auxiliary apparatus

As shown above, the calibration factor from time to amplitude can be estimated in the design of the circuit, but in practice it is always established by a calibration procedure at the time of measurement. Several methods are available, most of them ultimately depending on the velocity of light being 30 cm/ns in vacuum.

(a) The easiest method is to observe some time spectrum at several different lengths of one of the cables marked t_0 in Fig. 10. At each change of cable length, the whole spectrum shifts by an observable amount on the pulse amplitude scale, corresponding to the change in cable length. Knowing the velocity of signals in the cable (about 26.4 cm/ns in RG-114/U), the calibration is established. The method is subject to errors from attenuation and reflections in the cables, as well as to any uncertainty in the signal velocity on the cable.

(b) A better method is to generate a train of fast pulses, perhaps with a mercury-contact pulser, and branch it through variable delays to both limiters. The pulses are coupled to the limiter grids with small capacitances, and are shaped at least roughly to resemble the photomultiplier pulses. The effect is to produce a 'line' in the time spectrum whose position is varied by the variable delays. Graham *et al.*³¹ have described such a system in some detail. They used an air-insulated delay line to eliminate velocity uncertainties. The influence of reflections is eliminated, and that of attenuation much reduced, by placing the variable delay before the limiters rather than after. The width of the observed line in the time spectrum measures the electronic resolving time of the system. This calibration apparatus can be left connected (but turned off), and may be used at any moment; it may also be left turned on for time stabilization (see below).

(c) A fundamental method consists of moving the counters far apart (more than one meter) and placing a source of annihilating positrons on the line joining them. A prompt coincidence response to the annihilation radiation will be observed in the time spectrum, whose position on the time scale depends on the location of the source along the line between the counters. The time differences are calculated from the velocity of light, and a few measurements establish the calibration. The method is slow and laborious, requires a particular experimental arrangement, and requires care to see that varying counting rates in the two counters either do not cause errors or are compensated. It has been used mainly to verify other calibration methods³¹.

(d) At a Gatlinburg conference³, de Waard described a method in which artificial pulses are supplied to the two counters with a time separation that is being continuously varied. A tuned 500 Mc/s amplifier and some auxiliary apparatus allow the time-analogue pulse to be recorded only when the pulses are separated by approximately an odd number of half periods at 500 Mc/s. The final result is a time spectrum consisting

³¹ Graham, Geiger, Bell and Barton, Nucl. Instr. and Meth. 15 (1962) 40.

of discrete lines spaced at 2 ns intervals. The method is very ingenious and rapid, but requires complex equipment and is slightly indirect.

Summarizing calibration methods, the one labelled (b) above seems the most accurate and straightforward, while (a) requires the least additional equipment.

Other pieces of auxiliary apparatus are sometimes required, under the following headings.

Double value rejection. Consideration of Fig. 10 will show that the same degree of overlap is produced by a pair of pulses with relative delay $2(d - d_0 - t)$ as with relative delay t . Each time-analogue pulse amplitude thus represents, not one, but two possible time delays. The main effect is to cause the chance coincidence rate to be $2\Delta N_1 N_2$ instead of $\Delta N_1 N_2$ (see § 5 above). When the true-to-chance ratio is favorable, this effect does not matter very much. If it is serious, it can be eliminated by another coincidence circuit that demands that t be not greater than $(d - d_0)$ if an event is to be recorded. A supervisory circuit of this kind was used by Green and Bell¹⁸ with their TAC circuit. Double-value rejection may also be done as described in the next paragraph.

Pile-up rejection. At high counting rates, a true event may be falsified in measurement by the accidental occurrence of another pulse nearby in time. The trouble may be that a limiter is just recovering from a previous pulse, or that two pulses close together cause the slow PHS to give a false answer. A pile-up detector circuit may be used to detect such cases, and ensure that they are not recorded. Bell and Jørgensen³² used such a device in their lifetime measurements. It added the trains of pulses from the two counters together, and produced a cancelling pulse whenever any two pulses occurred within 3 μ sec of each other. The device had a resolving time of 30 ns, that is, two pulses separated by less than 30 ns looked like one. If this resolving time is equal to $(d - d_0)$, a pile-up rejector also accomplishes double-value rejection (see the preceding paragraph). If the pile-up rejector had a very short resolving time, it would cause the whole time spectrum, true and chance, to be rejected. A pile-up rejector can of course be very useful in ordinary single counter work at high rates.

Pulse height compensation. To first approximation, the limiters remove the effect of the varying height of the counter pulses on their timing. There will remain a residual effect, however, by which a larger pulse appears to come sooner. Thus a larger 'preceding' pulse gives a larger time-analogue pulse, while a larger 'delayed' pulse gives a smaller time-analogue pulse. When pulse height windows of finite width are used in the PHS, these effects cause a broadening of the apparent time resolution curve. The effect can be compensated (again to first order) by adding a suitable fraction of the proportional pulse from each counter to the time analogue pulse. In the case of the 'preceding' pulses, the sign must be reversed to give a subtraction. This procedure, long envisaged but perhaps first used by Bell and Jørgensen³², does not improve the time resolution, but reduces the deterioration of time resolution due to wide pulse height windows in the two counters. It thus increases efficiency. It also removes the dependence of the

³² R. E. Bell and M. Jørgensen, Can. J. Phys. 38 (1960) 652.

centroid position on the shape of the pulse height spectrum within the pulse height windows; in some cases this may be its most important function.

Time stabilization. As already remarked, a drift of gain in a TAC system is equivalent to a time drift. This drift is particularly serious in centroid-shift measurements. If a calibrator like that described under calibration method (b) is available, it can be switched on and used to stabilize the time scale. The calibrator events can be made to fall into an unoccupied part of the time spectrum, or, in a split-memory analyzer, can be routed to an unused part of the memory. The recorded calibrator events give a measure of the drifts, and a correction is made. If now a feedback system is installed that holds these events in a particular channel of the analyzer (or equivalent), the time scale is stabilized against drifts. This technique has long been used in pulse height spectroscopy (as opposed to time spectroscopy), and has also been used at McGill in certain critical time measurements (e.g. Weaver³³). It should be noted that the stabilization does not extend to the photomultipliers themselves, but covers all the rest of the system. Stabilization can be by gain change or zero shift: if two calibration lines are used, both gain and zero can be stabilized, and so on. This time-stabilization technique will undoubtedly see increasing use.

In some cases a third counter may be used to distinguish the prompt from the delayed events, and direct them to separate parts of the multichannel recorder. This idea was first published by Simms *et al.*³⁴, but had been used earlier. It removes most of the effects of time drifts, but can be used only in special cases.

§ 9. Direct timing methods

The statistical limits on resolving time discussed in § 5 place a limit on the shortest mean lives that can be measured with scintillation counters. If the timing could be accomplished before detection of the radiation by the counters, rather than afterwards, these limits could be avoided; the physical limits of § 4 would remain, of course. Direct timing can be accomplished by interaction of charged particles with high frequency electromagnetic fields.

Consider a pair of cascade γ -rays each highly converted. A pair of magnetic spectrometers may focus the two conversion lines from a single source onto the two counters of a coincidence circuit, and the mean life of the intermediate state may now be measured by delayed coincidence in the usual way. For direct timing, on the other hand, the conversion electrons are modulated in either energy or direction by high frequency electric fields, so that the electrons remain in focus only at those instants when the modulating field is passing through zero. Now only those coincidences are recorded that correspond to a delay between the two conversion electrons equal to the time between zero phases of the two modulating fields. The timing is now being done by the electromagnetic fields, and the coincidence circuit serves only to identify the events. If the phase difference between the modulating fields is varied, we are in effect

³³ R. S. Weaver, Can. J. Phys. 40 (1962) 1684.

³⁴ Simms, Benczer-Koller and Wu, Phys. Rev. 121 (1961) 1169.

performing a single channel delayed coincidence experiment. The effective resolving time is set by the frequency and amplitude of the modulating fields and the properties of the β -spectrometers, and may be several times better than that of the best coincidence circuits. Experiments of this kind go back very far in physics, but in the present context the method was proposed in the direction-modulation form by Johansson and Alvager³⁵ and in the energy-modulation form by Blaugrund³⁶. Both schemes are described by Bashandy³.

The difficulty with the method is the low coincidence counting efficiency, even in those cases where two heavily converted cascade radiations are available; counting times of days or weeks were necessary to produce the relatively few results that have been published. A major improvement was made by Blaugrund *et al.*³⁷, who substituted a pulsed accelerator for the counter of the 'preceding' β -spectrometer. The excited state whose mean life is to be measured is formed by the pulsed beam in a target at the source position of the 'delayed' β -spectrometer. Any nuclear reaction or Coulomb excitation process that will form the desired state is suitable. Blaugrund *et al.* used a 3 MeV proton electrostatic generator and a magnetic lens β -spectrometer, both modulated at a frequency of 2450 Mc/s. They reached a limiting mean life of 2.4×10^{-11} sec, about 4 times better than could have been done with the ordinary delayed coincidence 'slope' method at the energies involved. This time region is very rich in information, however, and this technique represents a valuable contribution. The same experimental group has used a pulsed beam to measure mean lives by the ordinary delayed coincidence method²⁹.

§ 10. Recoil and Doppler methods

In this section we briefly describe a number of related methods for measuring mean lives, all of them depending on the state of motion of the emitting nucleus. All the methods give rather rough results, which may nevertheless be very valuable.

The most straightforward is the direct recoil distance measurement introduced by Devons *et al.*³⁸ and Thirion and Telegdi³⁹. An accelerated beam produces a reaction in a thin target such that excited nuclei will recoil forward in vacuum with well-defined velocity, usually around 10^9 cm/sec. If the mean life is long enough, the excited nuclei will decay at a measurable distance from the target. A collimator and detector at 90° to the beam allow a scan along the beam direction, to measure the decay of the recoiling excited nuclei. (Either the target or the collimator may be moved.) The various experiments and results are summarized by Devons³. The equivalent time resolution is only moderately superior to that of conventional circuitry, because of physical limitations like those discussed in § 4 above, mainly involving the dimensions

³⁵ B. Johansson and T. Alvager, Nucl. Instr. and Meth. 3 (1958) 49; Arkiv för Fys. 17 (1960) 163.

³⁶ A. E. Blaugrund, Phys. Rev. Lett. 3 (1959) 226.

³⁷ Blaugrund, Dar and Goldring, Phys. Rev. 120 (1960) 1328.

³⁸ Devons, Hereward and Lindsay, Nature 164 (1949) 586.

³⁹ J. Thirion and V. L. Telegdi, Phys. Rev. 92 (1953) 1253.

of the collimator. Severiens and Richter⁴⁰ have used a magnetic lens that produced an effective sharpening of the collimation, and achieved limiting slopes of their prompt curve corresponding to a mean life of about 3×10^{-11} sec. This figure is about three times better than could be obtained directly, in their case.

A large jump towards shorter mean life measurements is provided by the Doppler methods. Here one detects the state of motion of a recoiling excited nucleus by means of the Doppler effect in the γ -radiation it emits. The attainable velocities usually correspond to a Doppler effect of a few percent. The stopping time, T , of the recoiling nucleus in a solid (or in vacuum followed by a solid) gives a time scale by which τ , the mean life of the excited state, may be measured. If $\tau \gg T$, the radiation is emitted from nuclei at rest, and no Doppler effect is seen; if $\tau \ll T$, the full Doppler effect is developed. For intermediate cases the average Doppler effect, measured with a detector of modest energy resolution, is a function of τ/T , and if T can be estimated, τ is measured. Usually T is 10^{-12} to 10^{-13} seconds, and τ must lie not far from this region.

The first measurement of this kind was made by Elliott and Bell⁴¹ for the 479 keV excited state of Li⁷ produced in the exoergic thermal neutron reaction $B^{10}(n, \alpha) Li^{7*}$. In this case the Doppler effect on the 479 keV γ -ray from Li^{7*} was a broadening (all directions of travel for the Li^{7*} particles equally probable), of maximum possible amount $\pm 1.6\%$ (from the Q of the reaction), detected with a secondary electron radiator and magnetic spectrometer. In the measurement, T was varied by using three different substances for stopping the Li^{7*} particles, and three different degrees of broadening were observed. The value deduced for τ was $(7.5 \pm 2.5) \times 10^{-14}$ sec, verified by Devons *et al.*⁴² using the Doppler method but producing the Li^{7*} in a different reaction. Resonance fluorescent methods give a slightly higher result, around 1.1×10^{-13} sec (Chapter XXI).

A shift is easier to measure than a broadening, especially when the detector has the low resolution of a scintillation counter. To get a Doppler shift, the direction of motion of the moving nucleus must be defined either by coincidence counters or by the use of an accelerator beam. The Doppler shift can be varied from positive through zero to negative by moving the measuring device from 0° through 90° to 180° from the direction of flight. The average shift at any angle can be changed by varying the slowing-down time of the moving nuclei, i.e. by varying the slowing-down medium. Detection is usually by scintillation counter, but magnetic spectrometers have also been used. The motion of the nucleus may be provided by natural α -decay recoil, although serious difficulties then may occur in preparing a thin enough source⁴⁰.

Some examples of the techniques involved are given in the measurement of $\tau =$ about 2×10^{-12} sec for the 40 keV level of Tl²⁰⁸ by Burde and Cohen, and Siekman and de

⁴⁰ J. C. Severiens and F. W. Richter, Nucl. Phys. 13 (1959) 239.

⁴¹ L. G. Elliott and R. E. Bell, Phys. Rev. 74 (1948) 1869 and 76 (1949) 168;

R. E. Bell, Beta- and Gamma-ray Spectroscopy (ed. K. Siegbahn; North-Holland Publ. Co., Amsterdam, 1955) 518.

Waard⁴²; the measurement of $\tau =$ about 7×10^{-13} sec for the 1.63 MeV level in Ne^{20} by Devons *et al.*⁴³; and the measurement of $\tau =$ about 8×10^{-12} sec for the 3.85 MeV level in C^{13} by Simpson *et al.*⁴⁴. More information may be found in ref. 3.

The quantities that must be measured or estimated, apart from the kinematics of the reaction, are the Doppler shift itself and the rate of slowing down of the moving nuclei. Neither of these is likely to be accurately known, so that the overall accuracy is never high. The nuclear velocity, and hence the accuracy of the Doppler shift measurement, can be somewhat increased by using heavy ion acceleration, and particularly by bombarding a light target with a heavy projectile⁴⁴. The poor knowledge of slowing-down times remains. In spite of its limited accuracy, however, the technique gives information not usually obtainable in any other way.

⁴² J. Burde and S. G. Cohen, Phys. Rev. 104 (1956) 1093;
J. G. Siekman and H. de Waard, Nucl. Phys. 8 (1958) 402.

⁴³ Devons, Manning and Towle, Proc. Phys. Soc. (London) A169 (1956) 174.

⁴⁴ Simpson, Clark and Litherland, Can. J. Phys. 40 (1962) 769.

