Advanced Physics Lab SS19

Experiment: Nuclear spin

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Abstract

Contents

1 Theory

The contents of this chapter are, if not otherwise specified, derived from the guide to the experiment [1]

1.1 Spin and nuclear spin

The spin or intrinsic angular momentum of a elementary particle is an intrinsic property of particles from the family of the fermions. Members of this family, such as protons, neutrons and electrons all have a spin of $s = \frac{1}{2}$. The spin can be explained semi classically, as rotation of the particle around its own 'centre of mass', with fixed frequency and variable axis of rotation. However, this illustration only makes sense in finite-size particles, of course. Just as with the angular momentum, not all three spin components can be defined at the same time, but only the amount and projection on a freely selectable 'quantization axis'. The possible spin quantum numbers are

$$\left|\vec{S}\right| = \hbar\sqrt{S\left(S+1\right)}$$

with $S = 0, \frac{1}{2}, 1, ...$ and Planck's constant $\hbar$. Atomic nuclei are also assigned a spin, the nuclear spin, which is defined with the nuclear spin number I, analogue to the spin:

$$\left|\vec{I}\right| = \hbar\sqrt{I\left(I+1\right)}$$

The nuclear spin number is also quantified in its direction. Analogue to the electron spin, the projection of the nuclear spin can also assume certain states as , e.g. with the z-axis as the quantization axis $I_z = m_I\hbar$ with $-I \leq m_I \leq +I$. In total there would be $2I + 1$ different states for I_z. Protons or the nucleus of ^{19}F both have a nuclear spin number of $I = \frac{1}{2}$. So both have only two possible states: $m_I = \pm\frac{1}{2}$. They can only align parallel or antiparallel with the quantization axis in the experiment.

1.2 Magnetic momentum

The spin of a quantum mechanical particle is connected to a magnetic dipole momentum $\vec{\mu}$, the ratio of both is described as the gyromagnetic ratio γ.

$$\vec{\mu} = \gamma\vec{I} \qquad \text{with} \quad \gamma = \frac{g_I\mu_K}{\hbar}$$

The constant g_I is the nuclear g factor, which is to be calculated during the exam. g_I has no dimension and is unique for each nucleus. The second constant μ_K is the nuclear magneton, which is computed analogue to the Bohr magneton:

$$\mu_K = \frac{e\hbar}{2m_p}$$

The difference between those two is that for the Bohr magneton the elecron mass is used and for the nuclear magneton the proton mass. In the ground state of atomic nuclei, the nucleons are arranged according to the Pauli principle so that each orbital is occupied by two protons or neutrons of opposite spins. If now a eu-nucleus (with an even number of protons and an uneven number of neutrons) or if an ue-nucleus (where the even and uneven nucleons are reversed) is present, an unpaired nucleon remains. This leads to an half-digit total spin. For a uu-nucleus two unpaired nucleons remain resulting in an integer total spin. In a ee-nucleus all nucleons are paired, therefore the total spin is zero. Examples for ee-nuclei are $^{16}_{8}$O and $^{12}_{6}$C. Therefore it is possible to measure the spin of hydrogen $I = \frac{1}{2}$ utilizing glycol ($C_2H_6O_2$) and water (H_2O) samples. For the $^{9}_{19}$F nucleus with 9 protons and 10 neutrons the total spin is also $I = \frac{1}{2}$.

1.3 Interaction with magnetic fields and radiation (nuclear magnetic resonance)

Classically the the energy of a magnetic dipole moment $\hat{\mu}$ in a magnetic field B is described by the equation 1.

$$E = -\hat{\mu} \cdot B \tag{1}$$

If the magnetic field goes in the z direction this can be written in quantum mechanics like in equation 2 and is called Zeeman-splitting.

$$E = -\mu_K g_I m_I B_x \tag{2}$$

Here our energy niveaus are degenerated if there is no outer magnetic field, with the magnetic field the levels spits up depending on the quantum number m_j. In figure 1 we see this splitting up under the influence of the magnetic field. The difference energy ΔE between attached m_j can be written as follows:

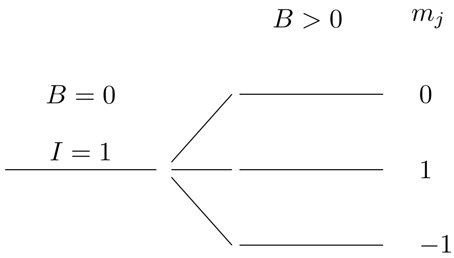

Figure 1: Zeeman splitting for $I = 1$

$$\Delta E = g_I \mu_K B \tag{3}$$

This amount of energy needs to be absorbed or emitted for the spin to change its direction. This can happen through photons or by interaction with a 'Strahlungsfeld'. Since a certain amount of energy is needed it happens only at certain frequencies. This so called resonance frequency is given by:

$$\nu = \frac{\Delta E}{h} = \frac{g_I \mu_K B}{h} = \frac{\gamma B}{2\pi} \tag{4}$$

If a spin absorbs energy of the 'Strahlungsfeld' and changes into a higher level the intensity of 'Strahlungsfeld' decreases which is measurable.

2 Relaxation Effects

In thermal equilibrium the occupation number are Boltzmann distributed. The probability of a state depending on energy and temperature is given through:

$$p_i = \frac{e^{-E_i/kT}}{Z} \tag{5}$$

Here k is the Boltzmann constant and Z is the canonical partition function of all the states in the system. The probability p_i can also be given by:

$$p_i = \frac{N_i}{N}$$

With that the relationship between two states 1 and 2 is given through eq.6

$$\frac{N_1}{N_2} = e^{-\frac{E_1 - E_2}{kT}} = e^{-\frac{\Delta E}{kT}} \tag{6}$$

That means that there will always be more particles in the lower state than in an upper one. That would mean that the occupation numbers should equalize and with that the measurable effect. This is not happening because of so called relaxation effects. There are two major relaxation effects:

1. Spin-Lattice Relaxation: Here the exited nucleus give their energy to the lattice structure of the molecule. This energy is lost to the 'Strahlungsfeld'.

2. Spin-Spin Relaxation: One nucleus creates a magnetic field at the another nucleus which shifts the outer magnetic increasing or decreasing it. This leads to an increase in the width of the absorption line.

3 Hall Sensor

The Hall sensor is used to measure magnetic fields. It uses the Hall effect. The effect happens to electrons in a cable effected by a outer magnetic field. Here the electrons are pushed under the Lorenz force $\vec{F}_L = q \cdot (\vec{v} \times \vec{B})$to the side of the cable till certain voltage is reached which counters the Lorenz force. That means we have $F_L = F_E$ This voltage U_{Hall} can be measured. Equation 7 gives a way to calculate the magnetic force acting on the cable.

$$U_{Hall} = H\frac{IB}{d} \tag{7}$$

- H: Is the Hall constant $\frac{1}{ne}$ with n the electric charge density and e the charge of an electron.
- I: The current.
- B: The magnetic field.
- d: The width of the cable.

3.1 Lock-in Amplifier

The lock-in method is used to make small signal visible inside of huge noise. To do this the main signal will be multiplied with a reference signal and integrated with a low pass filter and amplified. That way only the part of the signal will pass through at which the signal is expected.

4 Method of Measuring

4.1 Measuring the Magnetic Field

To measure the magnetic field the earlier discussed Hall sensor is used. It is put onto a long rod which a cm scale. That way it can be put into the magnetic field and measure it for certain depths. That way it can be checked if the field is Homogen since the magnetic field value will stay the same.

4.2 Measurement of the Resonance Frequency

To measure the resonance frequency the constant field method is used. Here the 'Strahlungsfeld' of the NMR (Nuclear Magnetic Resonance) Oscillator is set at a constant frequency and the magnetic field is changed with a wave to check for the correct resonance frequency. If this frequency is hit the Spins will change and the 'Strahlungsfeld will lose energy which will be seen in the amplitude. For the magnetic field two electromagnets are used which have to smaller ones for the variance of the magnetic field. To measure the correct frequency two different methods are used.

The first one uses a sinus wave to change the magnetic field. That means that the correct frequency will be hit two times each period and with this two absorption lines. To find the exact resonance frequency the minima need to be equidistant, sine at this point they will be at the 'Nulldurchgang' of the modulated magnetic field. At this point the correct frequency to the set magnetic field is found. The experimental setup for this part can be seen in figure **??**

For the second part the lock-in method should be used since it is more precise duo to its lower background noise. Instead of the former absorption curve this method gives the differentiated curve. For the modulation of the magnetic field the superposition of a sinus and a sawtooth is used. Here the sawtooth is used mainly for the variance of the field while the sinus is used to create the differentiated signal since it has a similar frequency to the reference signal. The former minima of the absorption curve will now be the 'Nulldurchgang' of the signal. The moment the 'Nulldurchgang' of both measured signals overlap the correct resonance frequency is hit. Examples of both signals are shown in figure **??** and the setup is given in figure **??**.

5 Conduct of the Experiment

In the beginning of the experiment it had to be checked if the magnetic field inside the setup is homogeneously distributed. For this a Hall-Sensor was used with which the field strength can be measured. Here by the sensor was slowly put into the field and depending on place the strength of the field was recorded. After conforming the uniformity of the field, a position in the middle of the homogeneous part was chosen to place the probes into.

With this set the experiment was set up like in figure **??** described. The measurement was started with Glycol at a depth of 2 cm and a constant magnetic field of 425 mT. With this set, the corresponding frequency of the nuclear magnetic resonance (NMR) oscillator was set by looking for the absorption peaks in the oscilloscope. After learning that it would be easier to find the rough position with a fixed frequency it became much easier to locate the peaks. The fine positioning was still done by modifying the frequency. By setting all peaks equidistant to one another the correct resonance frequency could be found. After this two underground samples were made, one without the H_1 probe inside the field and by setting wrong combinations of magnetic field and oscillation. The H_1 probe was used for all background as well as for the calibration between the change in frequency and the change in position on the oscilloscope. The reason for using H_1 is that it has the least amount of disturbances. It is so to speak our standard candle. With the position - frequency calibration done the actually measurements were started anew. For different combinations of magnetic field and frequency with equidistant absorption lines, the CSV files were taken. After doing this for H_1 and Glycol the Hall-Sensor broke and another one had to be used. This one had the problem that it was strongly influenced by the temperature. This showed by the slowly decrease in measured field strength. The first value measured around the depth of 2 cm was noted.

After finishing this way the ^{19}F probe the Lock in Method was used to decrease the background noise. After building the setup of figure **??** a suitable resonance frequency was chosen. Here a slow shifting of the absorption peaks was noted, most likely duo to the heating of the magnets. After waiting for them to be warmed up the shifting stopped and the first measurement with the Lock in Method could be started. Here a new calibration of position and frequency was made by shifting the positions of the differentiated absorption signal.

Duo to some problems with the measurement of the magnetic field some measurements to the influence of the Hall-Sensor were made. One with the sawtooth voltage and one without it. With these and the voltage and ampere used to create the magnetic field, the accuracy of measurement with the new Hall-Sensor can be determined more closely.

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References

[1] *Versuchsanleitung: Fortgeschrittenen Praktikum Teil 1 Kernspin.*

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