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Measurement of the Index of Refraction of Glass

Background:
We have developed a new type of glass and wish to know its index of refraction. After light hits the glass at incident angle alpha, it travels through the glass at some other angle beta. Assuming the light is initially traveling through air, Snell's law tells us that these variables can be characterized by the following relationship:

$$sin(\beta) = \frac{sin(\alpha)}{n_{glass}}$$

where n_{glass} is a property of the glass, the index of refraction. To determine this quantity, measurements have been taken of both alpha and beta. This data is stored in the file refractionData.txt. Light was shined on the glass at six incident angles, from 10º to 60º in 10º steps. For each incident angle alpha, beta, the angle of refraction, was measured 16 times. This report details the analysis and presentation of the data, which can be found in the file lab2Braier.py.

Part 1: Histogram
The data were initially contained in a single file. I separated the data by index of refraction and plotted each of these datasets in a histogram. The result is a figure with six histograms, each representing the spread of beta values corresponding to a given alpha. Alpha and beta are both given in degrees. The histogram is shown below. Note that the distribution for beta shifts to the right as alpha increases.

Part 2: Table of Measurements
I printed the measurements and some important values to the iPython console in a readable table. The first column contains alpha, the incident angle, in degrees. The second column contains sin(alpha). Each alpha was converted to radians by multiplying by pi/180, and the sine of the resultant value was taken. The third column contains a mean beta value, obtained by averaging all sixteen beta values for a given alpha. Beta is given in degrees. The fourth column contains sin(beta), obtained through the same process as sin(alpha). It is important to note that sin(beta) is the sine of the average of beta, not the average of sin(beta). Mathematically, these are slightly different. Finally, the fifth column contains the uncertainty in sin(beta). For each alpha, all 16 beta measurements were passed into the sine function and the standard deviation of the results was measured. However, sigma(sin(beta)) represents the uncertainty in the mean, so the standard deviation was then divided by the square root of the number of data points—4, since there are 16 data points. Note that alpha is taken to be a known quantity, not a measurement, and as such has no uncertainty. The table is pictured below.

Part 3: Snell's Law Fit

The next portion of the data analysis involves fitting the data to a line according to Snell's Law (see Background). The values of sin(alpha) and sin(beta) shown above were plotted against each other, with sigma(sin(beta)) used as error bars on beta. On this graph, the x-axis is sin(alpha) and the y-axis is sin(beta). The result appears linear with y-intercept 0, as predicted by Snell's law. A linear fit was conducted using curve_fit to determine the value of n_{glass}. It was found to be $n_{glass} = 1.49 \pm 0.02$, which is printed to the console. I used this best-fit value to create a model for sin(beta) as a function of sin(alpha). I then plotted the model on the same graph as the data. This graph is the one on top in figure 2. I conducted a one-parameter chi-squared fit on the model. I calculated chi-squared using the formula

$$\chi^2 = \sum\left(\frac{sin(beta) - sin(alpha)/n_{glass}}{sigma(sin(beta))}\right)^2$$

where the summation is calculated over the set of alphas and all variable names represent the same values shown in Table 1. The dataset has 6-1=5 degrees of freedom because there are 6 data points and 1 fitting parameter. The chi-squared value and the number of degrees of freedom were used to obtain a P-value, and all three of these numbers were printed to the console (Figure 3).

Part 4: Chi-squared as a Function of Index of Refraction

The final portion of the analysis involves calculating the chi-squared values that correspond to different indices of refraction. Each value of n_{glass} results in a different model with a different chi-squared value. We expect our best fit for n_{glass}, 1.49, to correspond to the least possible chi-squared. I calculated chi-squared for different indices of refraction using the same process described above in Part 3. I plugged in 51 values between 1.42 and 1.55. This range was chosen because the resultant curve is roughly symmetric, with its minimum in the middle. I experimented with using larger ranges, but the chi-squared values grow quickly far away from n=1.49, making it difficult to see the behavior of the graph near the minimum. In Part 3, I found a value for n_{glass} with an uncertainty, sigma. The best fit value is marked by a dashed black vertical line, with more vertical lines to the left and right which are one sigma away. The vertical line in the middle should coincide with the minimum calculated chi-squared. The two on the sides should coincide with one more than the minimum chi-squared. To check this, I plotted a horizontal line at the minimum chi-squared and another horizontal line exactly one unit above it. The resultant graph is the one on the bottom of Figure 3. The dashed lines intersect as expected.

Part 5: Figures and Tables

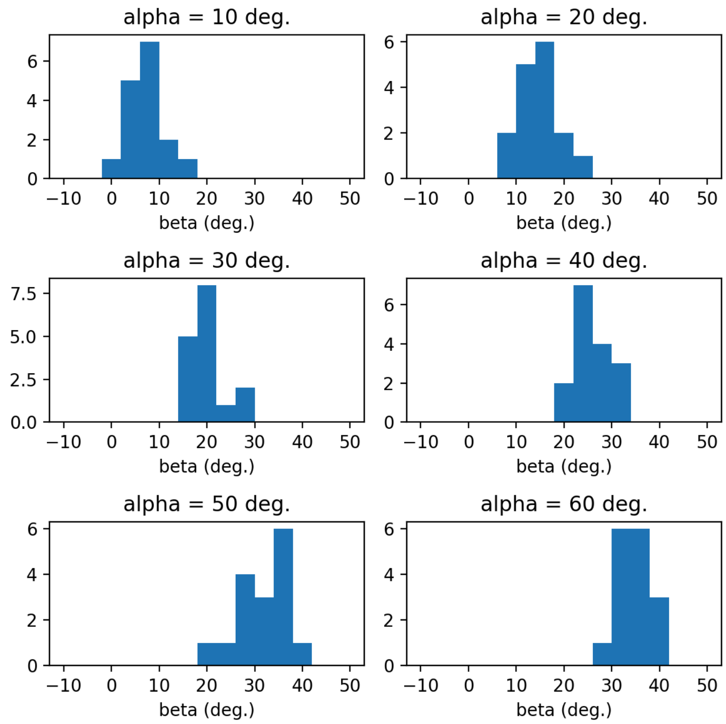

Figure 1.

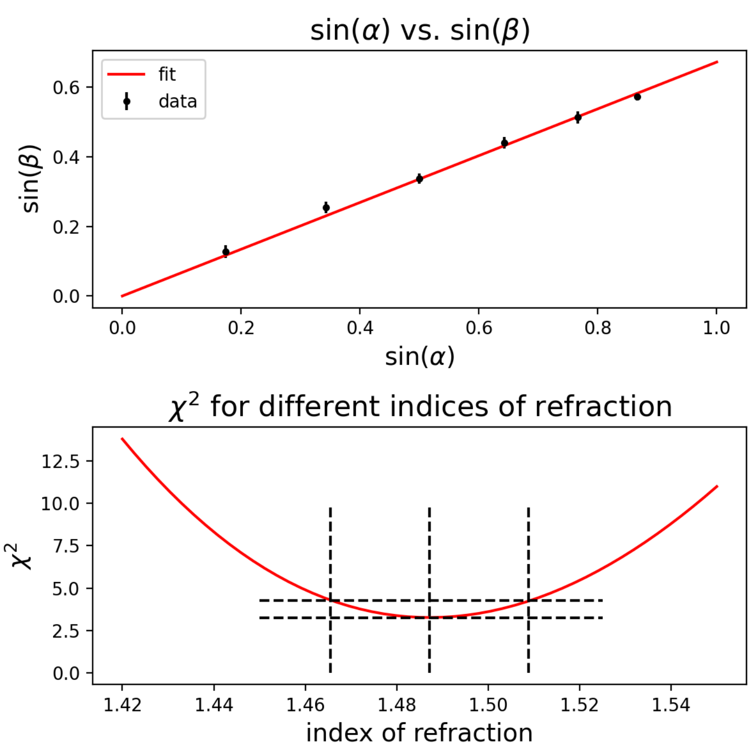

Figure 2.

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chisq=3.2646, dof=5, pval=0.6593
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Figure 3.

alpha	sin(alpha)	beta	sin(beta)	sigma(sin(beta))
10	0.174	7.306	0.127	0.018
20	0.342	14.769	0.255	0.017
30	0.500	19.669	0.337	0.015
40	0.643	26.131	0.440	0.016
50	0.766	30.919	0.514	0.018
60	0.866	34.994	0.573	0.012

Table 1.