11.3 Astrophysical Fluid Dynamics

Fluid Dynamics 101
by
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Why Study Fluid Dynamics?

- Fundamentally, physical interactions on many scales, from blood flow within vesicles, to plasma flows on the scales of clusters of galaxies are dictated by collisions between atomic particles. For a hundred years now, we have known that these interactions are governed by quantum mechanics on the microscopic scale. But how do we realistically model these flows on macroscopic scales?
- If we take a classical, deterministic approach to these problems, we can solve a range of macroscopic problems accurately, provided we can describe the large number of colliding particles as a fluid.

When is it a fluid? Size scale...

- If interactions are dictated by collisions between atoms, then how often do those interactions occur, and on what physical length scales?

- The probability of an interaction depends on the **cross-section** σ: the physical interaction area that one particle presents to another - and the **number-density** n of particles that are interacting.

- A simple dimensional analysis suggests that the length scale for the interaction, called the **mean free path**, is just: $\ell = 1 / (n\sigma)$.
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  If the length scale is *larger* than the mean free path $\ell = 1 / (n\sigma)$, and we can ignore other types of interactions (like photons scattering off atomic particles), then this is a fluid.

When is it a fluid? Time scale...

- How often the particles collide also depends on their mutual velocity. If we assume that the ensemble of particles has a characteristic speed v, then the average time between collisions is $\tau = \ell / w$.
- What is this characteristic speed? If the system is in equilibrium, then the velocities of the particles are given by the Maxwell-Boltzmann distribution. We won't concern ourselves with details. For now, let's say that the velocity distribution is a bell curve centered around the average velocity w, characterized by the temperature T.

When is it a fluid? Ideal Gas Law

- So now we have a bunch of gas with a temperature T and number density n. If we can determine the average mass of the particles, which we'll call μ, then the mass density: $\varrho = \mu n$ and the pressure of the ideal gas is: $P = nRT = \varrho RT / \mu$. R is the gas constant from your high-school chemistry class.

- The pressure is a measure of the force per unit area - think of the force a warm gas exerts on the wall of a container. But the pressure is also directly related to the energy density - the amount of internal energy in the gas per unit volume. The exact scaling between internal energy and pressure depends on the nature of the ideal gas, and the thermodynamics process being considered (isobaric, isothermal, adiabatic, etc.)

Fluid Dynamics

- So far we've been discussing our fluid as a static ensemble of particles with mass density ϱ, temperature T, and pressure P. If we push on the particles, for example with gravity, then the gas will move. This is the dynamic part.

- We apply Newton's laws to the problem. After a time step Δt, a blob of gas will move a certain distance. A blob of gas with density ϱ_0, temperature T_0, and pressure P_0 is moved a distance $v\Delta t$ and now has density ϱ_1, temperature T_1, and pressure P_1.

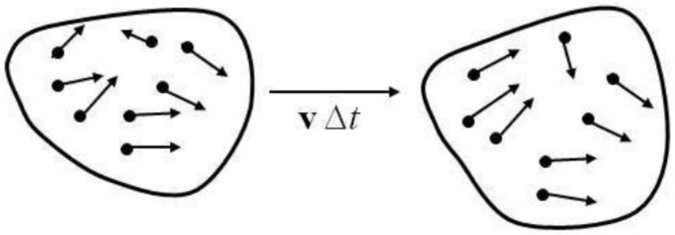

Fig. 1.2 from Chapter 1 of [Numerical PDE Techniques for Scientists and Engineers] by Dinshaw Balsara

Fluid Dynamics - Almost there...

- Earlier we expressed the internal thermal velocity of one particle as $\boldsymbol{w}$ and the velocity of a blob of particles (the bulk velocity) as $\boldsymbol{v}$. Thus the total velocity vector of any particle is $\boldsymbol{u} = \boldsymbol{v} + \boldsymbol{w}$. Without going into detail, applying Boltzmann's ideas about ideal gases to Newton's equations of motion, allows us to write down three equations for the time evolution of the fluid density, the momentum density, and the energy density.
- These equations express the fact that collisions, which lie at the heart of these interactions conserve mass, momentum, and energy. These can be expressed as partial differential equations (PDEs).
- The result is three partial differential equations in space and time:
  - The continuity equation (conserves mass)
  - The momentum equation (conserves momentum)
  - The energy equation (accounts for energy)

Fluid Dynamics - Almost there...

- Historically, this problem was first formalized by Leonhard Euler in 1757 by ignoring the effects of fluid viscosity and thermal conductivity. The PDEs are now known as the Euler Equations.

- In 1823, Claude-Louis Navier published a memoir on the motions of fluids which could account for friction (dissipation of energy) in fluids by using Laplace’s newly formulated idea of molecular forces. Those equations were formalized as PDEs and are now known as the Navier-Stokes equations. The Euler equations are a specific case of the Navier-Stokes equations. The enterprise of fluid dynamics has been to solve the Navier-Stokes equations with certain boundary conditions using a range of analytical and numerical techniques.

- It’s interesting to note that there is no mathematical proof that solutions for the Navier-Stokes equations always exist.

The PLUTO Code for Astrophysical Gas Dynamics

Fast forward to 2020, in this lesson, we will use a publicly available code to simulate fluid dynamical simulations and visualize those simulations in VisIt.

PLUTO is freely-distributed software for the numerical solution of mixed hyperbolic/parabolic systems of partial differential equations (conservation laws) targeting high Mach number flows in astrophysical fluid dynamics.

PLUTO is a finite volume code: it averages physical quantities like density and velocity within a volume element inside a mesh (and defines electric and magnetic fields and other variables) at the faces and sides of each volume element.

PLUTO Activities

To explore fluid dynamics, the rest of this lesson will ask students to complete two activities. Each will take approximately 30 minutes.

1. Download and install PLUTO. Run a first MPI simulation: 11.3 Activity 1
2. Run a Rayleigh-Taylor instability simulation in 2D: 11.3 Activity 2

Further Reading

For instructors and advanced students who wish to dive into the mathematics of numerical fluid dynamics:

Chapter 1 of Numerical PDE Techniques for Scientists and Engineers by Dinshaw Balsara

PDF: Balsara Chapter 1

PLUTO: A NUMERICAL CODE FOR COMPUTATIONAL ASTROPHYSICS by Andrea Mignone