

13.12.12 regression

{ Stock market forecast
 self driving car
 recommendation

example: $f(x_{cp}, x_s, x_{hp}, x_h, x_w) = y$

step 1: model: a set of function

$$y = b + w \cdot x_{cp} \Rightarrow \text{Linear model}$$

$$y = b + \sum w_i x_i$$

\uparrow bias \uparrow weight

← feature

step 2: goodness of function

training data: $(x^1, \hat{y}^1) \dots (x^n, \hat{y}^n)$

loss function (L): $L(f) = L(w, b)$

$$= \sum_{i=1}^n (\hat{y}_i - (b + w \cdot x_i))^2$$

(w, b) 作自变量的
 系数, 函数为点

step 3: Best function $f^* = \arg \min_f L(f)$

gradient descent, 处理任意 $L(f)$ 可微

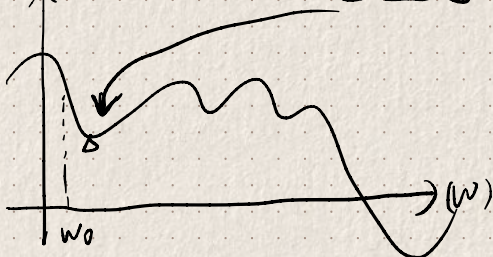
gradient descent

1. 随机选取初始点 w^0

2. 算微分 $\frac{dL}{dw}|_{w=w^0}$ $\begin{cases} \text{negative} \rightarrow \text{increase } w \\ \text{positive} \rightarrow \text{decrease } w \end{cases}$

$$w^1 = w^0 - \underset{\substack{\downarrow \text{学习率}}}{\eta} \frac{dL}{dw}|_{w=w^0} \quad \downarrow L \text{ 变小}$$

$L(w)$ 不断重复 直到 local optimal (局部)



(对于 w, b , 需计算两个偏微分)

$$\nabla L = \begin{bmatrix} \frac{\partial L}{\partial w} \\ \frac{\partial L}{\partial b} \end{bmatrix} \text{gradient} \quad \begin{array}{l} \text{此时 Loss } L(w, b) \\ \text{是椭圆等高线,} \\ \text{箭头线表线方向} \end{array}$$

线性模型中: Loss 是没有局部最优值的 convex

$$\frac{\partial L}{\partial w} = \sum 2(y^n - (b + wx^n))(-x^n)$$

$$\frac{\partial L}{\partial b} = \sum 2(y^n - (b + wx^n))(-1)$$

generalization 外推 test data

$$\text{loss}(\text{test}) > \text{loss}(\text{train})$$

selecting another model (step 1) ↓

$$y = b + w_1 x_{cp} + w_2 (x_{cp})^2$$

$$y = b + w_1 x_{cp} + w_2 (x_{cp})^2 + w_3 (x_{cp})^3$$

... ..

⇒ 模型复杂后，测试集误差可能会非常大
overfitting

⇒ 引入正则特征，优选模型

$f(x)$ 真值 $\text{int}(\text{bool})$ 「冲激函数？」

regularization 正则

$$L = \sum (y - \hat{y})^2 + \boxed{\lambda \sum (w_i)^2}$$

参数值接近0的函数更平滑（斜率小）

（平滑度取水平线，Loss大）