

# Types of Probability Density Function

There are several types of continuous distributions thus, different probability density functions are used. Some important continuous distributions are given below:

- Normal Distribution
- Standard Normal Distribution
- Student - t Distribution
- Chi-Square Distribution
- Continuous Uniform Distribution

The next section covers the probability density function formula for these distributions.

# Types of Probability Density Functions



$$\text{Normal Distribution: } f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$\text{Standard Normal Distribution: } f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$\text{Student-t Distribution: } f(x) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{v\pi}\Gamma\left(\frac{v}{2}\right)} \left(1 + \frac{x^2}{v}\right)^{-\frac{v+1}{2}}$$

$$\text{Chi-Square Distribution: } f(x) = \frac{x^{\frac{k}{2}-1} e^{-\frac{x}{2}}}{2^{\frac{k}{2}} \Gamma\left(\frac{k}{2}\right)}$$

$$\text{Continuous Uniform Distribution: } f(x) = \frac{1}{b-a}$$

## Normal Distribution Probability Density Function

A random variable that follows a [normal distribution](#) is denoted as  $X \sim N(\mu, \sigma^2)$ .

Here,  $\mu$  is the [mean](#) and  $\sigma^2$  is the [variance](#) and they form the parameters of the normal distribution. The graph of a normal distribution is a bell curve and is symmetric about the mean. The formula for the type of probability density function for a normal distribution is given below:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

## Standard Normal Distribution Probability Density Function

When a continuous random variable,  $X$ , follows a normal distribution such that the mean equals 0 and the standard deviation is equal to 1 then such a probability distribution is known as a standard normal distribution. It is denoted as  $X \sim N(0,1)$ . The formula for the probability density function of a standard normal distribution is given as follows:

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

### Student T Distribution Probability Density Function

A continuous random variable following a student t distribution is denoted as  $X \sim t(v)$ , where  $v$  denotes the degrees of freedom. A [student t distribution](#) is used when the sample size is very small and a normal distribution cannot be used. The probability density function of a student t distribution is given as follows:

$$f(x) = \frac{\Gamma(v/2)}{\Gamma(v/2) \sqrt{v\pi}} \left(1 + \frac{x^2}{v}\right)^{-v/2}$$

Here,  $\Gamma$  represents the gamma function

### Chi-Squared Distribution Probability Density Function

Chi squared distribution is widely used for hypothesis testing. It can be defined as the sum of squares of  $k$  independent standard normal variables. It is denoted as  $X \sim \chi^2(k)$ . The type of probability density function used for a chi squared distribution is given as follows:

$$f(x) = \frac{1}{2^{k/2} \Gamma(k/2)} x^{k/2-1} e^{-x/2}, x > 0.$$

$$f(x) = 0, \text{ otherwise}$$

### Continuous Uniform Distribution Probability Density Function

A [uniform distribution](#) is used to describe a random experiment such that the outcome lies between two values. The notation is given as  $X \sim U(a, b)$ . The probability density function when  $x$  lies between  $a$  and  $b$  is given as follows:

$$f(x) = \frac{1}{(b - a)}$$