Math 153 - Lab 2

1. We are interested in estimating a Binomial p. The scientist we are working with tells us that her best guess is that $p = .1$, but also that she is 90 percent sure that p is no bigger than .2. We recall that the mean of a Beta(α, β) is given as $\frac{\alpha}{\alpha+\beta}$. We actually derived this value. The probability that a Beta RV is less than any value $x \in (0,1)$, i.e. $F(x)$, can not be computed analytically. Luckily, R will numerically approximate this value via *pbeta(x,a,b)*.

a. Final appropriate values of α and β that encode the scientist's expert knowledge.

b. Write a function that takes in a best guess and a probability bound and returns values of α and β that correspond to the inputted beliefs.

2. Consider the prior that you found in 1(a). It is not fair to say that the true value of p is random with this (or any other) distribution, we are simply using this distribution to encode our belief about p. Consider instead that over your career as a statistician, you will do lots of data analyses regarding lots of different problems. Additionally, suppose that your priors are *accurate*. Over your lifetime, you will be asked to estimate a binomial p many times, and many of those times you will use the prior you just found. To say that we are able to accurately choose a prior is to say that if we look at the actual value of p for all the times we used this prior, and drew a histogram of them, it would match this prior. Let's think about how we did over our lifetime.

a. Compute the MSE over the lifetime of estimation problems using our accurately chosen prior, and compare this to the MSE of the frequentist MLE. Do this for several sample sizes.

b. Using the approximation formula for the median of the Beta, compare the posterior median to the posterior mean and the MLE based on mean absolute error (MAE) (a different way to think about performance of an estimator) under the same framework as (a).

3. We had the relationship that $\hat{p}_B = \lambda \hat{p}_f + (1-\lambda)\mu$ where $\lambda = \frac{n}{n+\psi}$, $\hat{p}_B$ was the posterior mean, $\hat{p}_f$ the sample proportion (and frequentist estimator), $\psi = \alpha + \beta$ and μ was the prior mean. We also saw that the Bayesian estimators (mean, median, mode) are all asymptotically equivalent to the MLE.

a. Create a graphic that shows the posterior density changing as the data becomes more numerous. Starting with a prior distribution, plot several posterior densities based on fixed values of $\hat{p}_f$, but based on larger and larger sample sizes.
(I would do this by evaluating some mesh of points (seq(0,1,.01)) via *pbeta* and then using the *lines* command to draw these on the same plot. *lines* has a color argument! *?lines*) Write a short paragraph explaining the graphic and the idea that it is illustrating.

b. Repeat (a), but do it for a bad choice of prior.