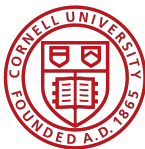


Causal Mediation in Natural Experiments

Senan Hogan-Hennessy
Economics Department, Cornell University
seh325@cornell.edu

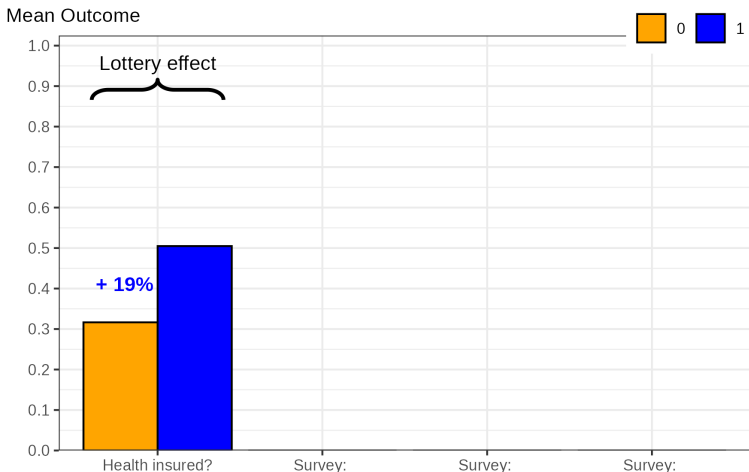


Econometric Society World Congress, Seoul
22 August 2025

Introduction: Oregon Health Insurance

In 2008, the US state Oregon started providing socialised health insurance to poor residents (Finkelstein et al, 2012).

- Over-subscribed, so random lottery assignment off a wait-list.

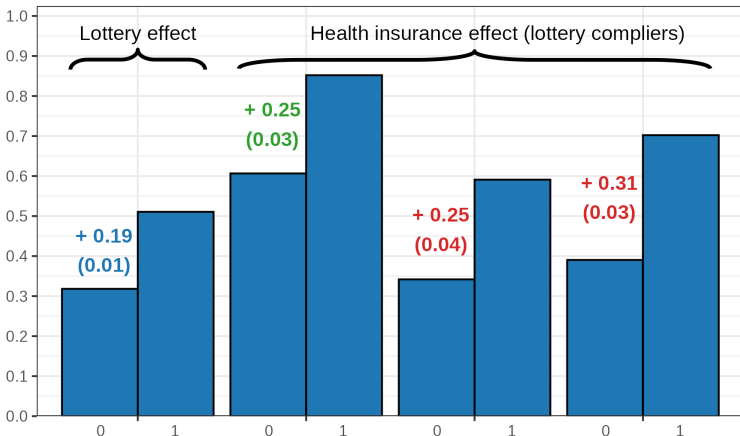


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Mean Outcome, for each $z' = 0, 1$.



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Slide describing why the effect on outcomes might not just be physical; consider less stress from being uninsured.

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Brief demonstration of what flies in applied economics, an “informal mechanism analysis.”

Introduction

This project examines Causal Mediation from an economic perspective:

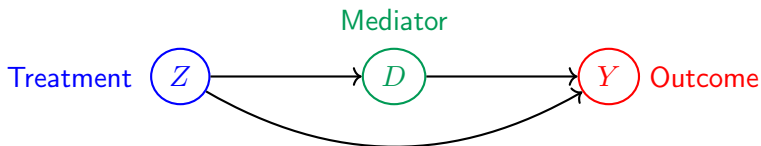
- ① Problems with conventional approach to CM (and informal mechanism analyses) in social science settings — focusing on natural experiments.
[Negative result]
- ② Recovering valid CM effects under selection-into-mediator, with modelling assumptions.
[Positive result]

Brings together ideas from two different literatures:

- **Causal mediation.**
Baron Kelly (1986), Imai Keele Yamamoto (2010), Flores Flores-Lagunes (2012), Frölich Huber (2017), Huber (2020), Kwon Roth (2024).
- **Labour theory, Selection-into-treatment, MTEs.**
Roy (1951), Heckman (1979), Heckman Honoré (1990), Vycatil (2002), Heckman Vycatil (2005), Kline Walters (2019).

Direct & Indirect Effects — Model

Consider binary **treatment** $Z_i = 0, 1$, binary **mediator** $D_i = 0, 1$, and continuous **outcome** Y_i for individuals $i = 1, \dots, N$.



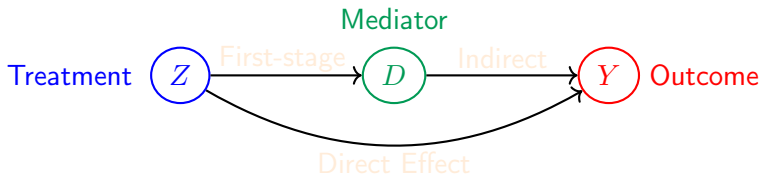
Mediator D_i is a function of Z_i . **Outcome** Y_i is a function of both Z_i, D_i .

$$D_i = \begin{cases} D_i(0), & \text{if } Z_i = 0 \\ D_i(1), & \text{if } Z_i = 1. \end{cases}$$

$$Y_i = \begin{cases} Y_i(0, D_i(0)), & \text{if } Z_i = 0 \\ Y_i(1, D_i(1)), & \text{if } Z_i = 1. \end{cases}$$

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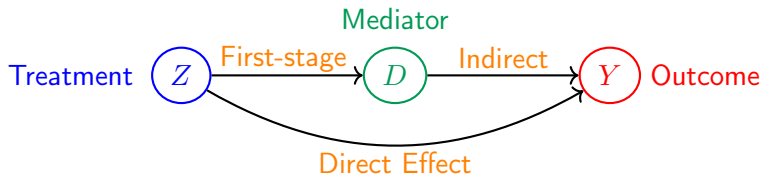
$$Z_i \perp\!\!\!\perp D_i(z), Y_i(z', d') \mid \mathbf{X}_i \text{ for } z, z', d' = 0, 1.$$

E.g., a natural experiment for Z_i disrupting open-world selection-into- Z_i

- Oregon wait-list lottery for **health insurance** (Finkelstein et al, 2012).

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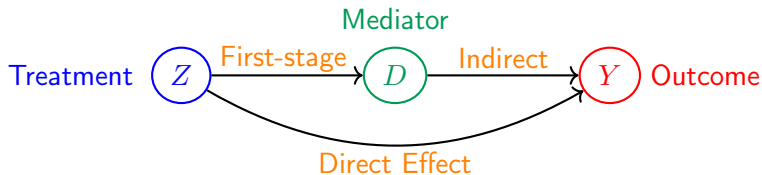
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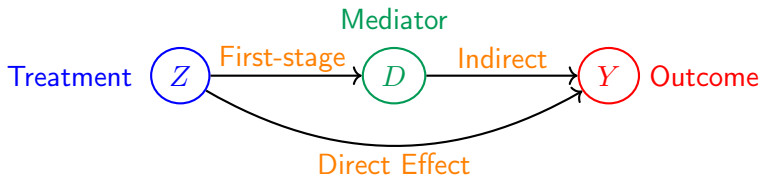
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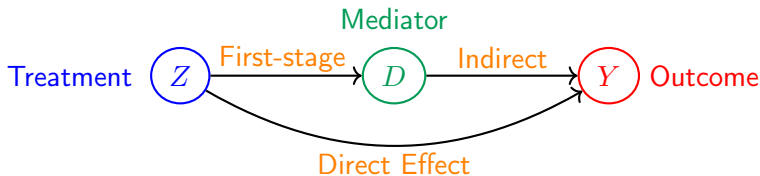
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$$\text{ATE: } \mathbb{E}[Y_i(1, D_i(1)) - Y_i(0, D_i(0))] = \mathbb{E}[Y_i \mid Z_i = 1] - \mathbb{E}[Y_i \mid Z_i = 0]$$

$$\text{Average first-stage: } \mathbb{E}[D_i(1) - D_i(0)] = \mathbb{E}[D_i \mid Z_i = 1] - \mathbb{E}[D_i \mid Z_i = 0]$$

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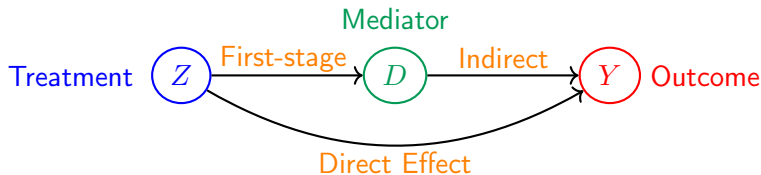
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First-stage and ATE answer important questions:

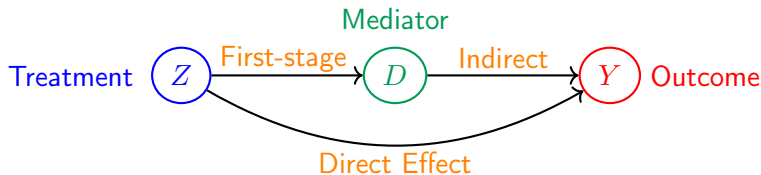
- Did socialised health insurance increase hospital use, and improve health? (Finkelstein et al, 2012).

Unanswered questions about the mechanism(s):

- Did health benefits come from using health care more? Health gains from reduced uncertainty — i.e., insurance?
- Is health insurance more about the health or more about the insurance?

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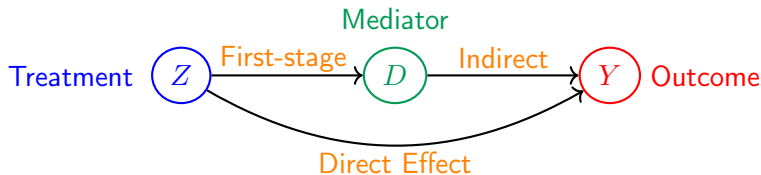
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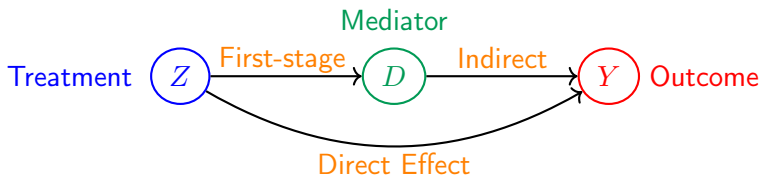
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Average Direct Effect (ADE): $\mathbb{E} \left[Y_i \left(\mathbf{1}, D_i(Z_i) \right) - Y_i \left(\mathbf{0}, D_i(Z_i) \right) \right]$

- ADE is causal effect $Z \rightarrow Y$, blocking the indirect D path.

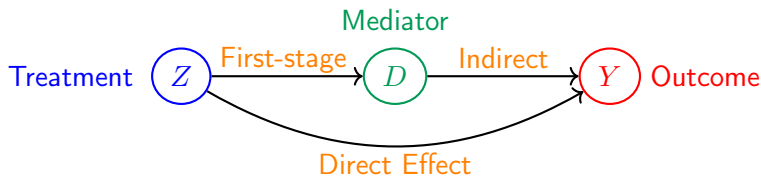
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- AIE is causal effect of $D(Z) \rightarrow Y$, blocking the direct Z path.¹

¹Note: AIE = fraction of $D(Z)$ compliers \times average effect $D \rightarrow Y$ among compliers.

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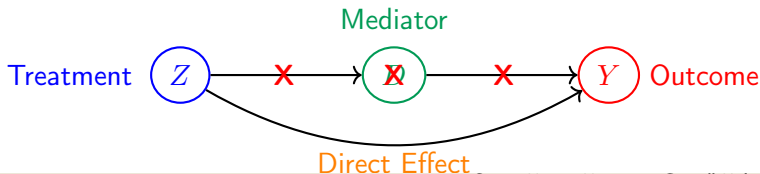
Sequential ignorability (SI, Imai Keele Yamamoto 2010):

Assume mediator D_i is *also* ignorable, conditional on \mathbf{X}_i and Z_i realisation

$$D_i \perp\!\!\!\perp Y_i(z', d') \mid \mathbf{X}_i, Z_i = z', \text{ for } z', d' = 0, 1.$$

If SI holds then ADE and AIE are identified by two-stage regression:

$$\text{ADE} = \mathbb{E}_{D_i, \mathbf{X}_i} \left[\underbrace{\mathbb{E}[Y_i \mid Z_i = 1, D_i, \mathbf{X}_i] - \mathbb{E}[Y_i \mid Z_i = 0, D_i, \mathbf{X}_i]}_{\text{Second-stage regression, } Y_i \text{ on } Z_i \text{ holding } D_i, \mathbf{X}_i \text{ constant}} \right]$$



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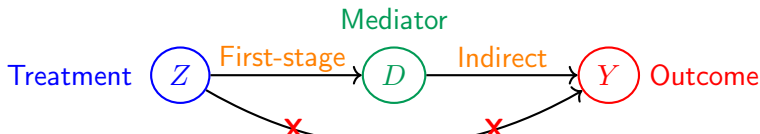
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E.g., OLS simultaneous regression (Imai Keele Yamamoto, 2010):

$$Z_i \leftarrow \text{Treatment} \quad \text{First-stage: } D_i = \phi + \pi Z_i + \psi'_1 \mathbf{X}_i + U_i$$

$$D_i \leftarrow \text{Mediator} \quad \text{Second-stage: } Y_i = \alpha + \beta D_i + \gamma Z_i + \delta Z_i D_i + \psi'_2 \mathbf{X}_i + \varepsilon_i$$

$$Y_i \leftarrow \text{Outcome} \quad \implies \text{ADE} = \gamma + \delta \mathbb{E}[D_i]$$

$$\text{AIE} = \pi (\beta + \delta \mathbb{E}[Z_i])$$

i.e., a regression decomposition.

Other estimation methods do the same decomposition, avoiding linearity assumptions (see Huber 2020 for an overview).

Direct & Indirect Effects — Selection

⇒ Great, we can use the Imai Keele Yamamoto (2010) approach to CM in all our respective applied projects.

⇒ Learn the mechanism pathways in causal research → big gain!

Before we import these methods to applied/labour economics and observational research, interrogate the **SI** assumption.

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Would this assumption hold true in settings economists study?

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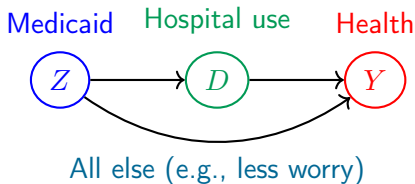
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Direct & Indirect Effects — Selection

Oregon health insurance experiment (Finkelstein+ 2012).



SI in practice:

$$D_i \perp\!\!\!\perp Y_i(z', d') \mid \mathbf{X}_i, Z_i = z', \text{ for } z', d' = 0, 1.$$

- 1 Medicaid assigned randomly (ensured by studying the 2008 Oregon wait-list lottery).
- 2 Healthcare usage is quasi-random, conditional on Medicaid assignment Z_i and demographics \mathbf{X}_i .

Direct & Indirect Effects — Selection

SI: Hospital usage is quasi-random, conditional on Medicaid assignment Z_i and demographics X_i .

Consider the case **individuals go to the hospital** to maximise health.

$$D_i(z') = \mathbb{1} \left\{ \underbrace{Y_i(z', 1) - Y_i(z', 0)}_{\text{Benefits}} \geq \underbrace{C_i}_{\text{Costs}} \right\}, \quad \text{for } z' = 0, 1.$$

i.e., Roy (1951) selection into D_i .

Theorem: If selection is Roy-style, and benefits are not 100% explained by Z_i, X_i , then **SI** does not hold.

Proof sketch: suppose D_i is ignorable \implies selection-into- D_i is explained 100% by $\{C_i, Z_i, X_i\}$, while unobserved benefits explain 0%.

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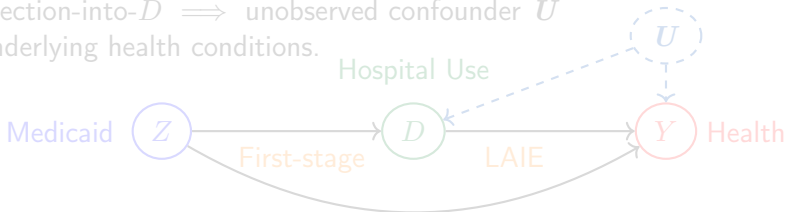
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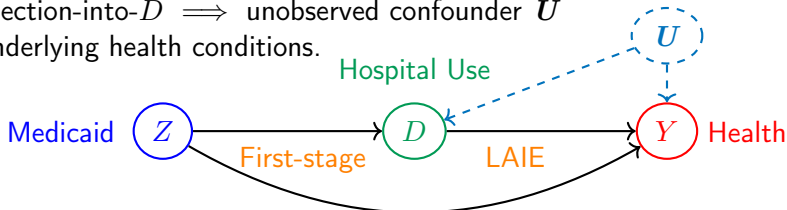
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Direct & Indirect Effects — Selection

In practice, the only way to believe the **SI** assumption (selection-on-observables) is to study a case with another natural experiment for D_i — in addition to the one that guaranteed Z_i is ignorable.

- (a) Cells in a lab → **SI** believable. (b) People choosing healthcare → **SI** not.

Direct & Indirect Effects — Selection Bias

- What happens if you go ahead and estimate CM anyway?
 - Would this be problematic?
 - Estimating causal effects with an unobserved confounder is usually bad. . .
-

Definition: Selection bias (Heckman Ichimura Smith Todd, 1998).

Estimating $D \rightarrow Y$, if D not ignorable:

$$\begin{aligned} & \mathbb{E}[Y_i | D_i = 1] - \mathbb{E}[Y_i | D_i = 0] \\ &= \text{ATT} \\ &+ \underbrace{\left(\mathbb{E}[Y_i(., 0) | D_i = 1] - \mathbb{E}[Y_i(., 0) | D_i = 0] \right)}_{\text{Selection Bias}}. \end{aligned}$$

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Direct & Indirect Effects — Selection Bias

⇒ CM Effects have this same flavour, causal effects contaminated by (less interpretable) bias terms. ► Model

$$\text{CM Estimand} = \text{ADE} + \left(\text{Selection Bias} + \text{Group difference bias} \right)$$

$$\begin{aligned} & \underbrace{\mathbb{E}_{D_i=d'} \left[\mathbb{E} [Y_i \mid Z_i = 1, D_i = d'] - \mathbb{E} [Y_i \mid Z_i = 0, D_i = d'] \right]}_{\text{Estimand, Direct Effect}} \\ &= \underbrace{\mathbb{E} [Y_i(1, D_i(Z_i)) - Y_i(0, D_i(Z_i))]}_{\text{Average Direct Effect}} \\ &+ \underbrace{\mathbb{E}_{D_i=d'} \left[\mathbb{E} [Y_i(0, D_i(Z_i)) \mid D_i(1) = d'] - \mathbb{E} [Y_i(0, D_i(Z_i)) \mid D_i(0) = d'] \right]}_{\text{Selection Bias}} \\ &+ \underbrace{\mathbb{E}_{D_i=d'} \left[\left(1 - \Pr(D_i(1) = d') \right) \right.}_{\text{Group difference bias}} \\ &\quad \left. \times \left(\mathbb{E} [Y_i(1, D_i(Z_i)) - Y_i(0, D_i(Z_i)) \mid D_i(1) = 1 - d'] \right. \right. \\ &\quad \left. \left. - \mathbb{E} [Y_i(1, D_i(Z_i)) - Y_i(0, D_i(Z_i)) \mid D_i(0) = d'] \right) \right]} \end{aligned}$$

Direct & Indirect Effects — Selection Bias

⇒ CM Effects have this same flavour, causal effects contaminated by (less interpretable) bias terms. ▶ Model Put $\pi = \Pr(D_i(1) = 1, D_i(0) = 0)$.

$$\text{CM Estimand} = \text{AIE} + \left(\text{Selection Bias} + \text{Group difference bias} \right)$$

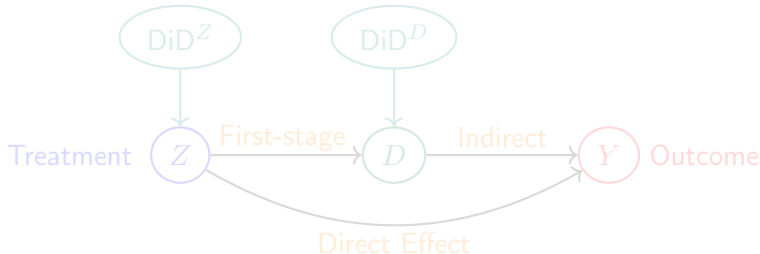
$$\begin{aligned} & \underbrace{\mathbb{E}_{Z_i} \left[\left(\mathbb{E}[D_i | Z_i = 1] - \mathbb{E}[D_i | Z_i = 0] \right) \times \left(\mathbb{E}[Y_i | Z_i, D_i = 1] - \mathbb{E}[Y_i | Z_i, D_i = 0] \right) \right]}_{\text{Estimand, Indirect Effect}} \\ &= \underbrace{\mathbb{E}[Y_i(Z_i, D_i(1)) - Y_i(Z_i, D_i(0))]}_{\text{Average Indirect Effect}} \\ &+ \underbrace{\pi \left(\mathbb{E}[Y_i(Z_i, 0) | D_i = 1] - \mathbb{E}[Y_i(Z_i, 0) | D_i = 0] \right)}_{\text{Selection Bias}} \\ &+ \pi \left[\left(1 - \Pr(D_i = 1) \right) \left(\mathbb{E}[Y_i(Z_i, 1) - Y_i(Z_i, 0) | D_i = 1] \right. \right. \\ &\quad \left. \left. + \left(\frac{1 - \Pr(D_i(1) = 1, D_i(0) = 0)}{\Pr(D_i(1) = 1, D_i(0) = 0)} \right) \left(\mathbb{E}[Y_i(Z_i, 1) - Y_i(Z_i, 0) | D_i(1) = 0 \text{ or } D_i(0) = 1] \right. \right. \right. \right. \\ &\quad \left. \left. \left. - \mathbb{E}[Y_i(Z_i, 1) - Y_i(Z_i, 0) | D_i = 0] \right) \right] \right] \end{aligned}$$

Identification Under Selection

That was a long way of giving negative results. Is there any hope?

If you can use a two-way research design, then please do!

Figure: Two-way Diff-in-Diff (see Deuchert Huber Schelker, 2019).



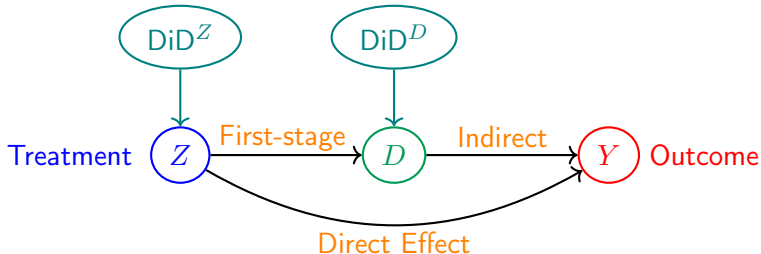
Note: assumes common trends across complier groups, identifies ADE + AIE local to complier groups.

Identification Under Selection

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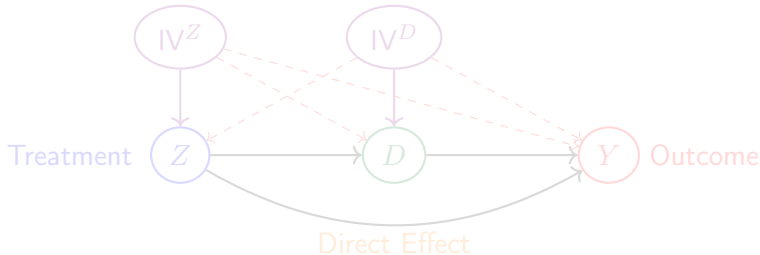
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Figure: Two-way IV (see Frölich Huber, 2017).



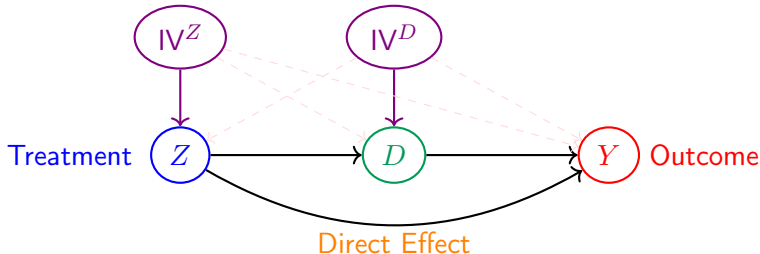
Note: two-way exclusion restriction, identifies ADE + AIE local to overlapping complier groups. Also avoid 2SLS (see Kim 2025)!

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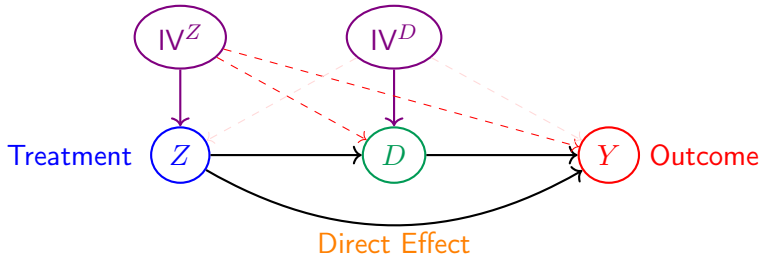
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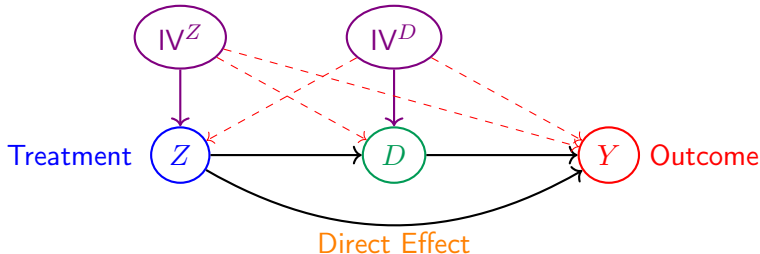
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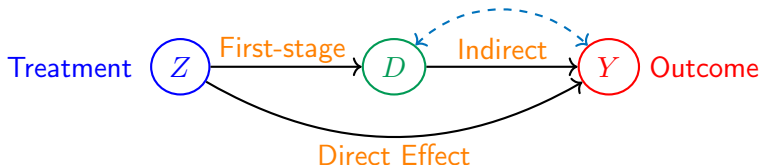
What about the mainstream case, with research design for only Z ?
How do economists do causal effects in these systems?

- 1 Estimate the ATE, and call it a day.
 - 2 (optional) Present suggestive evidence of mechanisms. ... [► Suggestive](#)
-

New: Control Function solution to identification.

Identification with a Control Function

Suppose Z is ignorable, D is not, so we have the following causal model.



Write outcomes as sum of means and mean-zero errors, $U_{D_i,i}$.

$$Y_i(Z_i, 0) = \mathbb{E} [Y_i(Z_i, 0) \mid \mathbf{X}_i] + U_{0,i}, \quad Y_i(Z_i, 1) = \mathbb{E} [Y_i(Z_i, 1) \mid \mathbf{X}_i] + U_{1,i}.$$

Then this system has the following regression equations:

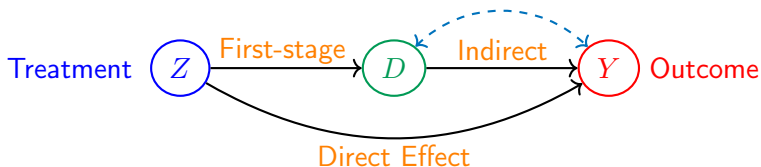
$$D_i = \phi + \pi Z_i + \varphi(\mathbf{X}_i) + U_i$$

$$Y_i = \alpha + \beta D_i + \gamma Z_i + \delta Z_i D_i + \zeta(\mathbf{X}_i) + \underbrace{(1 - D_i) U_{0,i} + D_i U_{1,i}}_{\text{Correlated error term.}}$$

Where $\beta, \gamma, \delta, \pi$ comprise the ADE and AIE.

Identification with a Control Function

Suppose Z is ignorable, D is not, so we have the following causal model.



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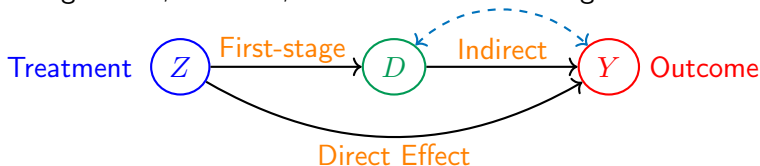
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Where $\beta, \gamma, \delta, \pi$ comprise the ADE and AIE.

Control Function intuition: Identify second-stage (despite correlated error term), to get ADE + AIE.

Identification with a Control Function

Suppose Z is ignorable, D is not, so we have the following causal model.



Note: Roy selection has first- and second-stage errors correlated.

$$D_i = \mathbb{1} \left\{ Z_i(\delta + \beta) + (1 - Z_i)\beta \geq C_i - \left(U_{1,i} - U_{0,i} \right) \right\}$$

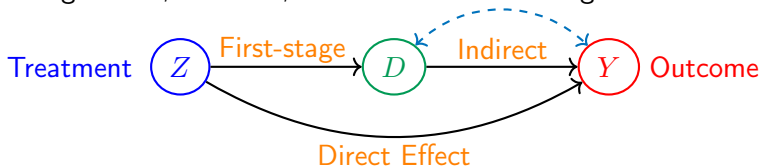
$$Y_i = \alpha + \beta D_i + \gamma Z_i + \delta Z_i D_i + \zeta(\mathbf{X}_i) + \underbrace{(1 - D_i) U_{0,i} + D_i U_{1,i}}_{\text{Correlated error term}}$$

where C_i are costs of taking D_i .

Control Function intuition: use first-stage errors to purge second-stage correlated errors.

Identification with a Control Function

Suppose Z is ignorable, D is not, so we have the following causal model.



Heckman (1979) Control Function, assumptions:

- Mediator monotonicity, $\Pr(D_i(1) \geq D_i(0) \mid \mathbf{X}_i) = 1$

$$\implies D_i(z') = \mathbb{1} \{ \mu(z'; \mathbf{X}_i) \geq U_i \}.$$

- First-stage errors inform second-stage errors,

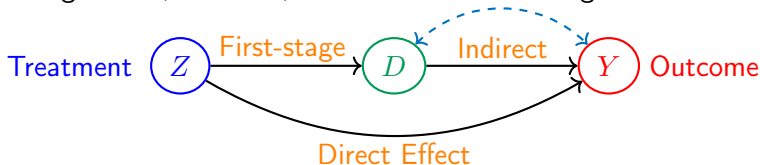
$$\text{Cov} \left[U_i, (1 - D_i) U_{0,i} + D_i U_{1,i} \right] \neq 0.$$

- Error-term distribution, $U_i, U_{0,i}, U_{1,i} \sim \text{TriNormal}(\mathbf{M}, \mathbf{\Sigma})$.

\implies identify second-stage, and thus ADE + AIE.

Identification with a Control Function

Suppose Z is ignorable, D is not, so we have the following causal model.



Heckman (1979) Control Function, in operation:

- 1 Back out Control Function (CF) in first-stage (probit, normal errors),

$$\hat{K}_i = D_i - \hat{\mathbb{E}}[D_i | Z_i, \mathbf{X}_i].$$

- 2 Include Mills ratio CF in OLS estimates of the second-stage,

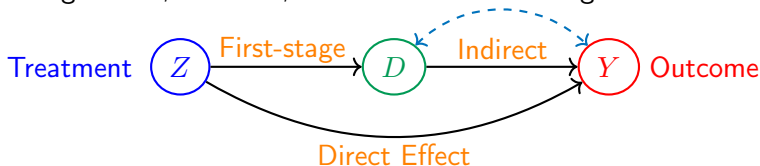
$$Y_i = \alpha + \beta D_i + \gamma Z_i + \delta Z_i D_i + \zeta' \mathbf{X}_i + \underbrace{(1 - D_i) \lambda(-\hat{K}_i) + D_i \lambda(\hat{K}_i)}_{\text{CF correction, } \lambda(\cdot) \text{ inv Mills ratio.}} + \varepsilon_i$$

- 3 Compose estimates from second-stage,

$$\widehat{ADE} = \hat{\alpha} + \hat{\delta} \mathbb{E}[D_i] \quad \widehat{AIE} = \hat{\pi} \left(\hat{\beta} + \hat{\delta} \mathbb{E}[Z_i] + \mathbb{E} \left[\lambda(-\hat{K}_i) - \lambda(\hat{K}_i) \right] \right)$$

Identification with a Control Function

Suppose Z is ignorable, D is not, so we have the following causal model.



Semi-parametric control function (Newey Imbens 2012), assumptions:

- 1 Mediator monotonicity, $\Pr(D_i(1) \geq D_i(0) \mid \mathbf{X}_i) = 1$

$$\implies D_i(z') = \mathbb{1} \{ \mu(z'; \mathbf{X}_i) \geq U_i \}.$$

- 2 First-stage errors inform second-stage errors,

$$\text{Cov} \left[U_i, (1 - D_i) U_{0,i} + D_i U_{1,i} \right] \neq 0.$$

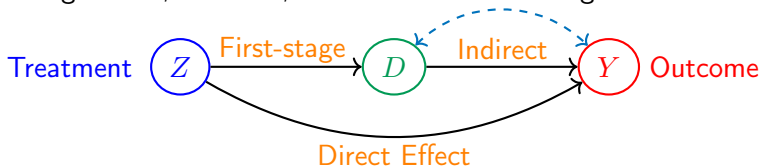
- 3 Valid instrument \mathbf{X}_i^{IV} for D_i , to separate CF functional form.

► MTEs

\implies identifies second-stage, ADE + AIE (w.out error dist assumption).

Identification with a Control Function

Suppose Z is ignorable, D is not, so we have the following causal model.



Semi-parametric control function (Newey Imbens 2012), in operation:

- 1 Back out Control Function (CF) in first-stage (semi/non-parametric), with IV \mathbf{X}_i^{IV} ,

$$\hat{K}_i = D_i - \hat{\mathbb{E}} \left[D_i \mid Z_i, \mathbf{X}_i^{\text{IV}}, \mathbf{X}_i \right].$$

- 2 Include semi-parametric CF in OLS estimates of the second-stage,

$$Y_i = \alpha + \beta D_i + \gamma Z_i + \delta Z_i D_i + \underbrace{\zeta' \mathbf{X}_i + (1 - D_i) \lambda_0 \left(-\hat{K}_i \right) + D_i \lambda_1 \left(\hat{K}_i \right)}_{\text{CF correction, } \lambda_0(\cdot), \lambda_1(\cdot) \text{ splines.}} + \varepsilon_i$$

- 3 Compose estimates from second-stage,

$$\widehat{\text{ADE}} = \hat{\alpha} + \hat{\delta} \mathbb{E} [D_i] \quad \widehat{\text{AIE}} = \hat{\pi} \left(\hat{\beta} + \hat{\delta} \mathbb{E} [Z_i] + \mathbb{E} \left[\hat{\lambda}_0 \left(\hat{K}_i \right) - \hat{\lambda}_1 \left(-\hat{K}_i \right) \right] \right)$$

Simulation Evidence

Simulation with trivariate normal errors + unobserved costs, $N = 10,000$.

① Random treatment $Z_i \sim \text{Binom}(0.5)$

② $(U_{0,i}, U_{1,i}) \sim \text{BivariateNormal}(0, 0, \sigma_0, \sigma_1, \rho)$, Costs $C_i \sim N(0, 0.5)$.

Roy selection-into- D_i , with constant partial effects + interaction term.

$$D_i(z') = \mathbb{1} \{Y_i(z', 1) - Y_i(z', 0) \geq C_i\},$$

$$Y_i(z', d') = (z' + d' + z'd') + U_{d'} \quad \text{for } z', d' = 0, 1.$$

Following the previous, these data have the following first and second-stage equations, where X_i^{IV} is an additive cost IV:

$$D_i = \mathbb{1} \left\{ Z_i - X_i^{\text{IV}} \geq C_i - (U_{1,i} - U_{0,i}) \right\}$$

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Simulation Evidence

Simulation with Roy selection, BivariateNormal errors + unobserved costs.

Figure: Simulated Distribution of CM Effect Estimates from 10,000 DGPs.

(a) ADE.

(b) AIE.

Simulation Evidence

Simulation with Roy selection, trivariate normal errors, unobserved costs.

Figure: Point Estimates of CM Effects, OLS versus Control Function, varying ρ values with $\sigma_0 = 1, \sigma_1 = 2$ fixed.

(a) ADE.

(b) AIE.

Conclusion

Overarching goals:

- ① Ward economists away from using CM methods unabashedly.
→ Noted problems in the most popular methods for CM effects, pertinent for economic applications.
- ② CM methods away from ignorability assumptions, inappropriate for economics (+ social science) settings.
→ Methods valid when selection-into-treatment theory relevant.

Work-in-progress part of LWIPS:

- Connect the control function approach to MTE methods ► MTEs
- Large sample properties + analytical SEs
- Use this approach to estimate direct and indirect effects of genetics and education (companion paper)
- (eventually) *R* package for selection-adjusted CM effects, by Heckman model and IV-assisted CF/MTE.

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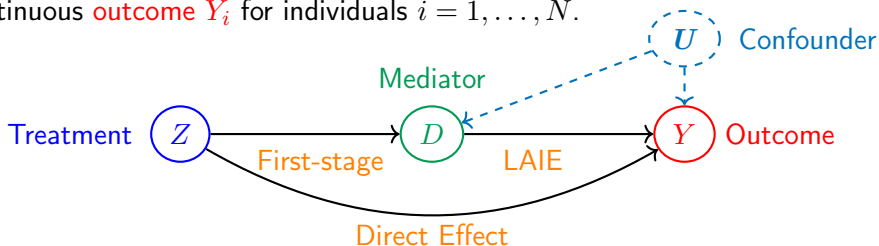
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Appendix: CM Guiding Model

Consider binary **treatment** $Z_i = 0, 1$, binary **mediator** $D_i = 0, 1$, and continuous **outcome** Y_i for individuals $i = 1, \dots, N$.



Average Direct Effect (ADE): $\mathbb{E} \left[Y_i \left(\mathbf{1}, D_i(Z_i) \right) - Y_i \left(\mathbf{0}, D_i(Z_i) \right) \right]$

- ADE is causal effect $Z \rightarrow Y$, blocking the indirect D path.

Average Indirect Effect (AIE): $\mathbb{E} \left[Y_i \left(Z_i, \mathbf{D}_i(1) \right) - Y_i \left(Z_i, \mathbf{D}_i(0) \right) \right]$

- AIE is causal effect of $D(Z) \rightarrow Y$, blocking the direct Z path.²

²Note: AIE = fraction of $D(Z)$ compliers \times average effect $D \rightarrow Y$ among compliers.

Group Difference — ADE

CM effects contaminated by (less interpretable) bias terms.

$$\text{CM Estimand} = \text{ADEM} + \text{Selection Bias}$$

$$\begin{aligned}
& \underbrace{\mathbb{E}_{D_i} \left[\mathbb{E} [Y_i \mid Z_i = 1, D_i] - \mathbb{E} [Y_i \mid Z_i = 0, D_i] \right]}_{\text{Estimand, Direct Effect}} \\
&= \underbrace{\mathbb{E}_{D_i=d'} \left[\mathbb{E} [Y_i(1, D_i(Z_i)) - Y_i(0, D_i(Z_i)) \mid D_i(1) = d'] \right]}_{\text{Average Direct Effect on Mediator (ADEM) take-up — i.e., } D_i(1) \text{ weighted}} \\
&\quad + \underbrace{\mathbb{E}_{D_i} \left[\mathbb{E} [Y_i(0, D_i(Z_i)) \mid D_i(1) = d'] - \mathbb{E} [Y_i(0, D_i(Z_i)) \mid D_i(0) = d'] \right]}_{\text{Selection Bias}}
\end{aligned}$$

The weighted ADE you get here is a positive weighted sum of local ADEs, but with policy irrelevant weights $D_i(1) = d'$.

⇒ consider this group bias, noting difference from true ADE. [▶ Back](#)

Group Difference — AIE

CM effects contaminated by (less interpretable) bias terms.

$$\text{CM Estimand} = \text{AIEM} + \left(\text{Selection Bias} + \text{Group difference bias} \right)$$

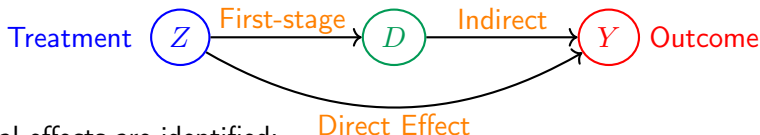
$$\begin{aligned}
& \underbrace{\mathbb{E}_{Z_i} \left[\left(\mathbb{E}[D_i | Z_i = 1] - \mathbb{E}[D_i | Z_i = 0] \right) \times \left(\mathbb{E}[Y_i | Z_i, D_i = 1] - \mathbb{E}[Y_i | Z_i, D_i = 0] \right) \right]}_{\text{Estimand, Indirect Effect}} \\
&= \underbrace{\mathbb{E} \left[Y_i(Z_i, D_i(1)) - Y_i(Z_i, D_i(0)) \mid D_i = 1 \right]}_{\text{Average Indirect Effect on Mediated (AIEM) — i.e., } D_i = 1 \text{ weighted}} \\
&+ \underbrace{\pi \left(\mathbb{E}[Y_i(Z_i, 0) \mid D_i = 1] - \mathbb{E}[Y_i(Z_i, 0) \mid D_i = 0] \right)}_{\text{Selection Bias}} \\
&+ \underbrace{\pi \left[\left(\frac{1 - \Pr(D_i(1) = 1, D_i(0) = 0)}{\Pr(D_i(1) = 1, D_i(0) = 0)} \right) \left(\mathbb{E}[Y_i(Z_i, 1) - Y_i(Z_i, 0) \mid D_i(1) = 0 \text{ or } D_i(0) = 1] - \mathbb{E}[Y_i(Z_i, 1) - Y_i(Z_i, 0)] \right) \right]}_{\text{Groups difference Bias}}
\end{aligned}$$

The weighted AIE you get here is not a positive weighted sum of local AIEs, because the AIE is only about $D(Z)$ compliers. [▶ Model](#).

⇒ consider this group bias, noting difference from true AIE. [▶ Back](#)

Appendix: Suggestive Evidence of Mechanisms

How empirical economists currently give evidence for mechanisms/mediators in causal effects.



Two causal effects are identified:

$$\text{ATE: } \mathbb{E} [Y_i(1, D_i(1)) - Y_i(0, D_i(0))] = \mathbb{E} [Y_i | Z_i = 1] - \mathbb{E} [Y_i | Z_i = 0]$$

$$\text{Average first-stage: } \mathbb{E} [D_i(1) - D_i(0)] = \mathbb{E} [D_i | Z_i = 1] - \mathbb{E} [D_i | Z_i = 0]$$

⇒ Show results of these two effects and assume indirect effect is positive, constant → suggestive evidence of mechanisms!

See Blackwell Matthew Ruofan Opacic (2024) for this in full, and a partial identification approach to avoid its unrealistic assumptions.

Appendix: Connection to MTEs

The ADE is fine to estimate with a Control Function/CF, but AIE refers to mediator benefits only among mediator compliers.

$$\text{AIE} = \mathbb{E} [D_i(1) \neq D_i(0)] \mathbb{E} [Y_i(Z_i, 1) - Y_i(Z_i, 0) \mid D_i(1) \neq D_i(0)].$$

Outline of MTE approach to identifying AIE:

- ① Mediator monotonicity has a Control Function for D_i (Vykatil 2002).

$$D_i(z') = \mathbb{1} \{ \mu(z'; \mathbf{X}_i) \geq U_i \}$$

- ② Identify Marginal Indirect Effect (MIE), with instrument by LIV.

$$\mathbb{E} [Y_i(Z_i, 1) - Y_i(Z_i, 0) \mid U_i = u']$$

- ③ AIE among compliers is an integral of the MIE (Mogstad Santos Torgovitsky, 2017).

$$\int \mathbb{E} [Y_i(Z_i, 1) - Y_i(Z_i, 0) \mid U_i = u'] dF_W(u'),$$

$$\text{for } W = \left\{ i \mid D_i(1) = 1, D_i(0) = 0 \right\}.$$