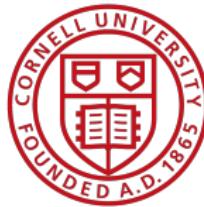


Causal Mediation in Natural Experiments

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Introduction

Motivation:

- Causal Mediation (CM) is a framework for studying mechanisms of causal effects, but is not widely used in economics
- Economists stick to vague suggestive evidence, leaving mechanisms relatively under-studied.

Contribution:

- Develop selection bias concept for CM, crystallising reasoning for economists' dismissal of these methods
- Develop a Marginal Treatment Effect (MTE) approach to credibly estimate causal channels (direct and indirect effects).

Key findings:

- Apply methods to the Oregon Health Insurance Experiment, studying mediating role of healthcare take-up
- Bring CM into economic quasi-experimental causal inference.

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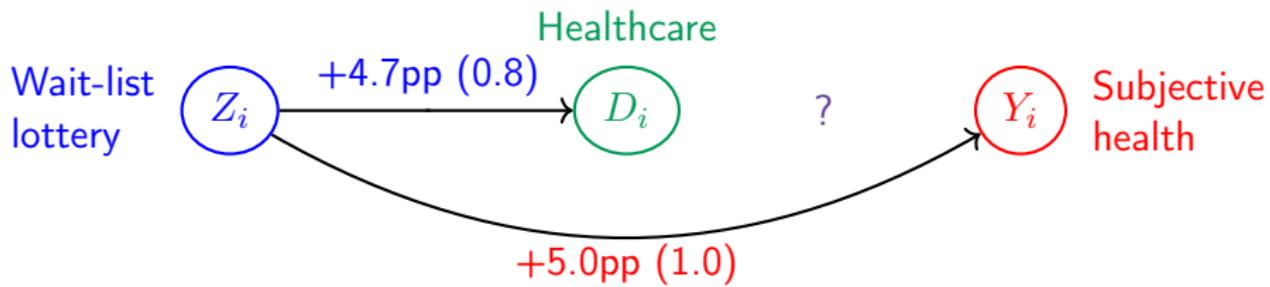
Key findings:

- Apply methods to the Oregon Health Insurance Experiment, studying mediating role of healthcare take-up
 - Bring CM into economic quasi-experimental causal inference.

1. Suggestive Evidence

In 2008, Oregon gave access to socialised health insurance by wait-list lottery (Finkelstein et al, 2012).

Figure: Model for Suggestive Evidence of a Mechanism.



Necessary but not sufficient evidence on the mediating mechanism:

- Is $D_i \rightarrow Y_i$ small, large, non-zero?
 - Can we assume this causal effect?

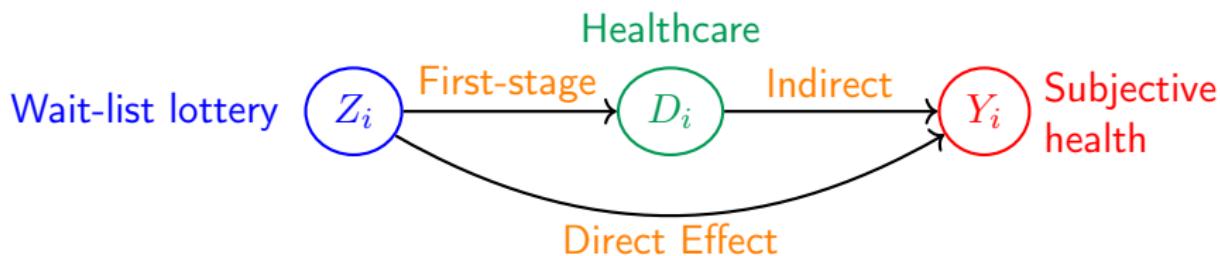
2. Causal Mediation

Causal Mediation (CM) models the entire system, giving sufficient evidence on the mediating mechanism.



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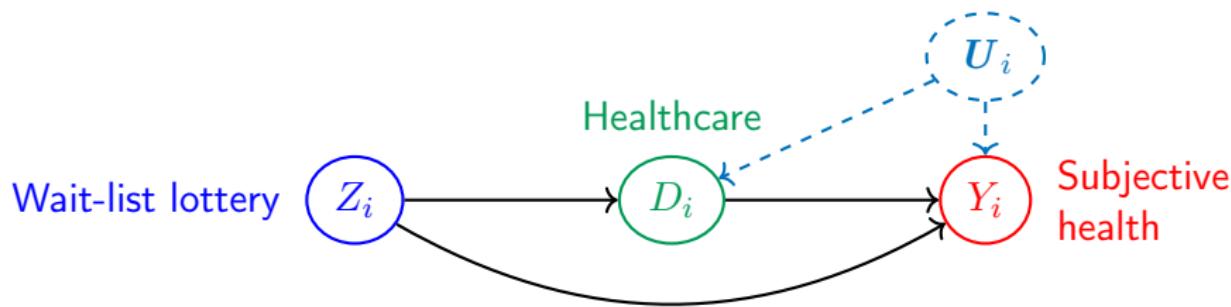
Average Direct Effect (ADE) and Average Indirect Effect (AIE):

- ADE is causal effect $Z_i \rightarrow Y_i$
 - AIE is causal effect of $D_i(Z_i) \rightarrow Y_i$.

Identification: Requires mediator D_i is quasi-randomly assigned. . . .

2. Causal Mediation — Selection Bias

People chose to visit healthcare D_i freely, not randomly assigned....



Conventional CM analyses are misleading in natural experiment settings — I derive non-parametric selection bias terms for CM.

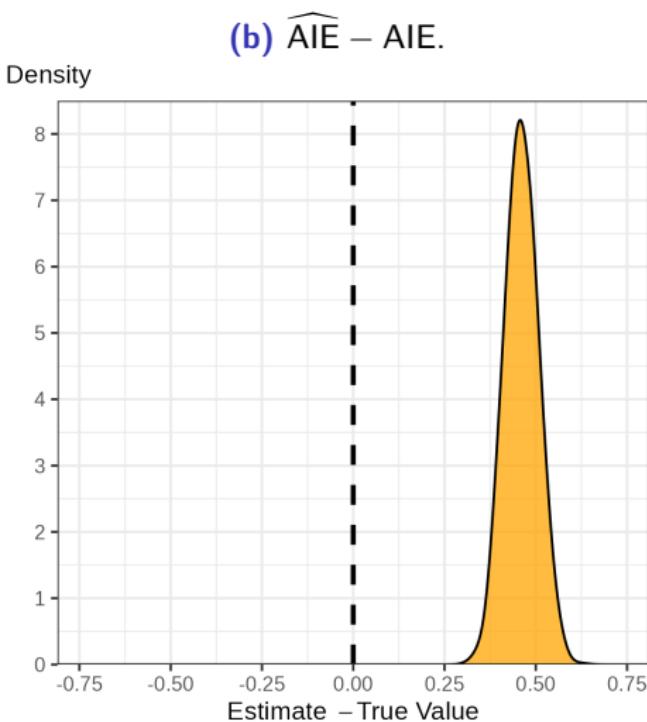
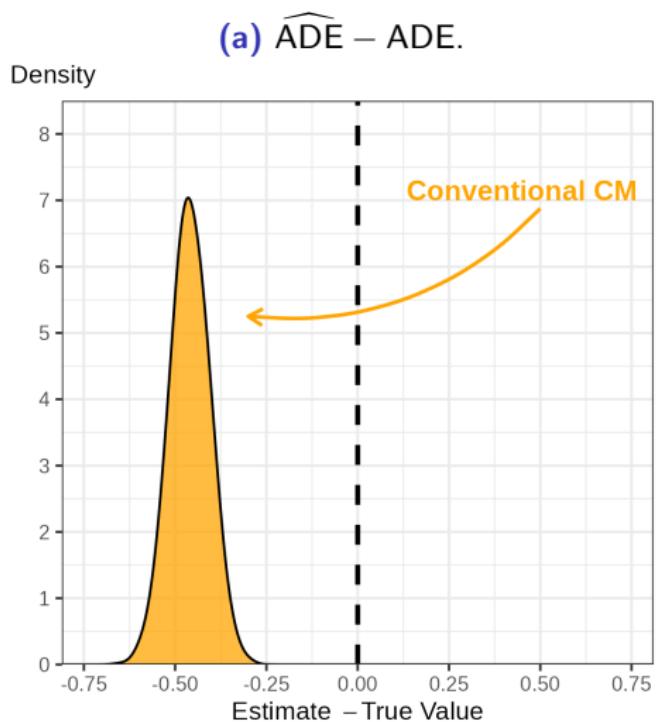
- ADE: CM Estimand = ADE + (Selection Bias + Group difference bias)
 - AIE: CM Estimand = AIE + (Selection Bias + Group difference bias)

► ADE biases

► AIE biases

2. Causal Mediation — Selection Bias

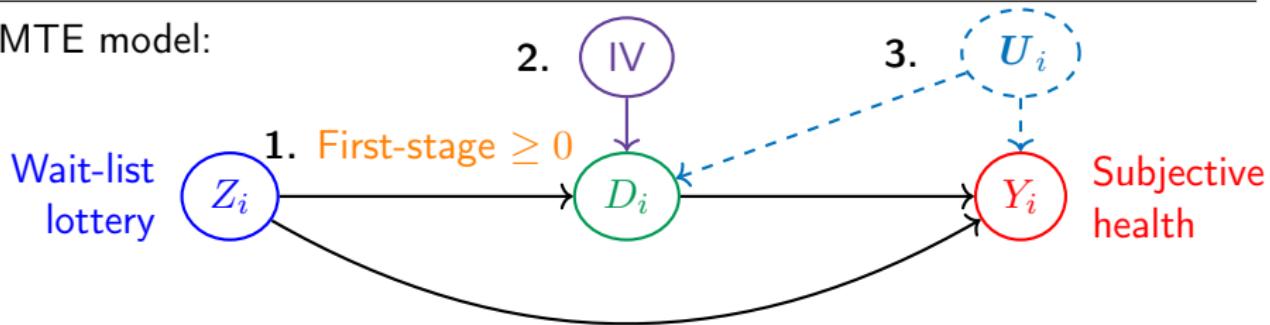
Simulations with cost-benefit selection-into- D_i , CM estimates are biased.



3. MTE Model

Conventional CM methods do not identify ADE + AIE in a natural experiment setting, so I build a Marginal Treatment Effect (MTE) model.

MTE model:



MTE assumptions:

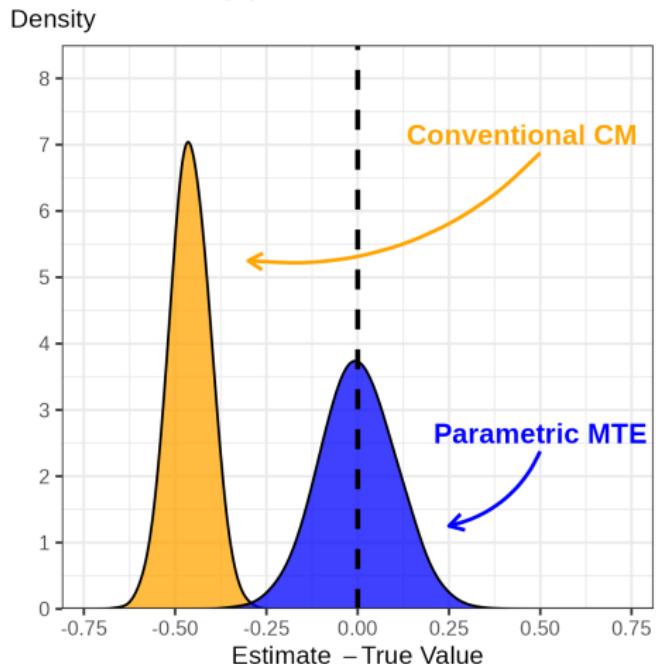
- ① Mediator monotonicity
 - ② IV for mediator take-up cost
 - ③ Selection on mediator benefits.

Intuition: model U_i via mediator MTE to identify ADE + AIE.

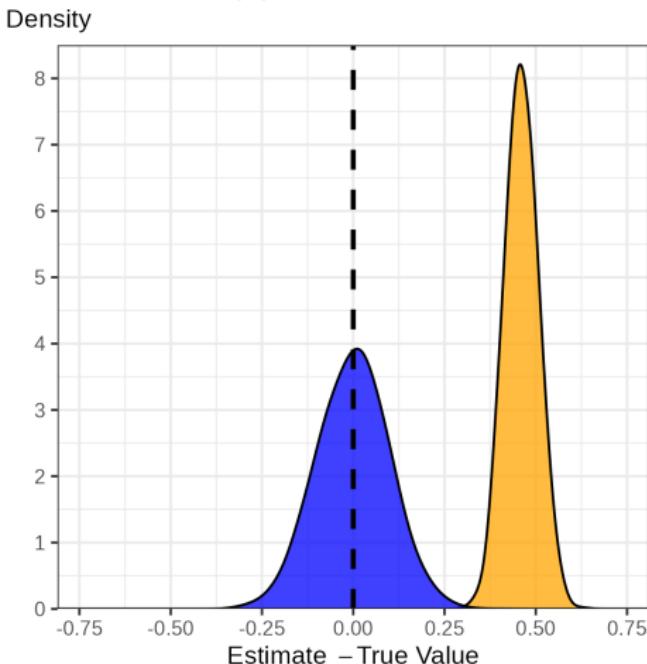
3. MTE Model

Simulation with cost-benefit selection-into- D_i , MTE-model corrects for selection bias.

(a) $\widehat{\text{ADE}} - \text{ADE}$.



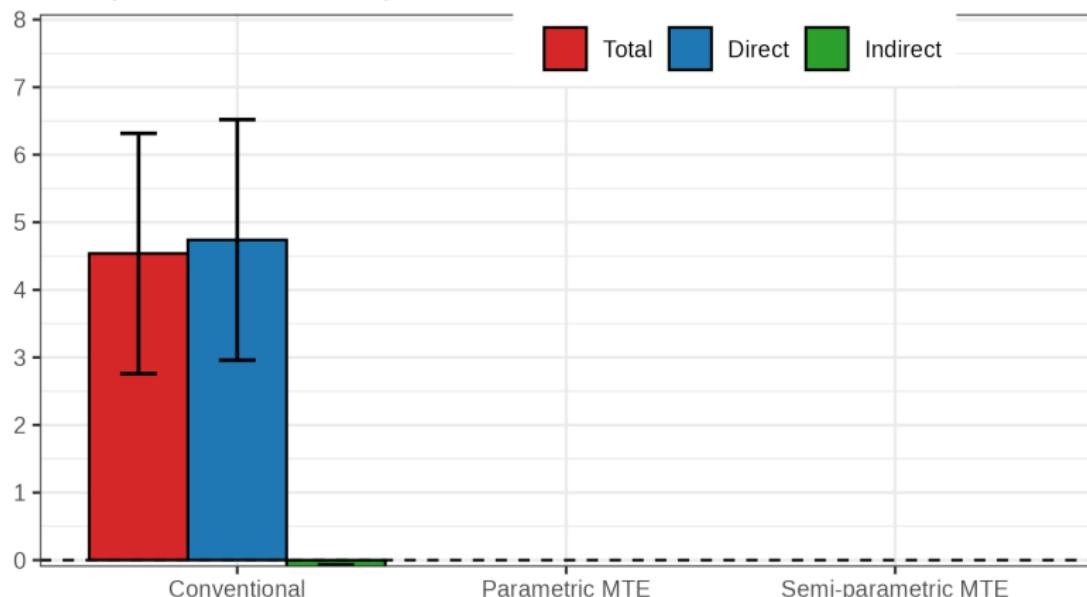
(b) $\widehat{\text{AIE}} - \text{AIE}$.



4. Returning to Oregon

Conventional CM estimates lottery effects as mostly direct, ≈ 0 healthcare.

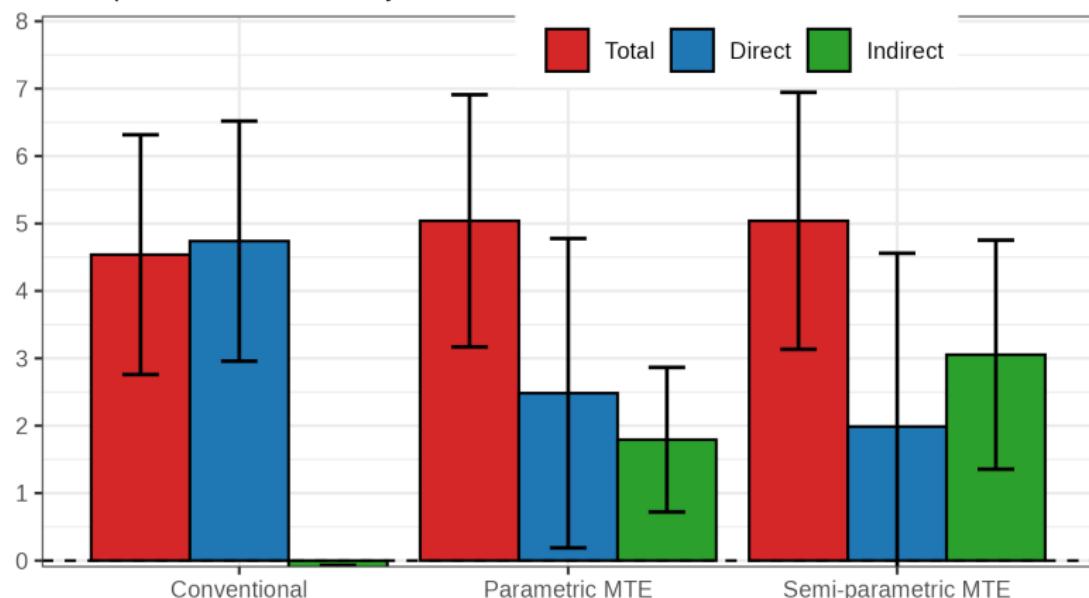
Estimate, percent effect on subjective health



4. Returning to Oregon

Using my MTE model, with regular healthcare location as an excluded IV, restores indirect effect through increased **healthcare visitation**.

Estimate, percent effect on subjective health



Conclusion

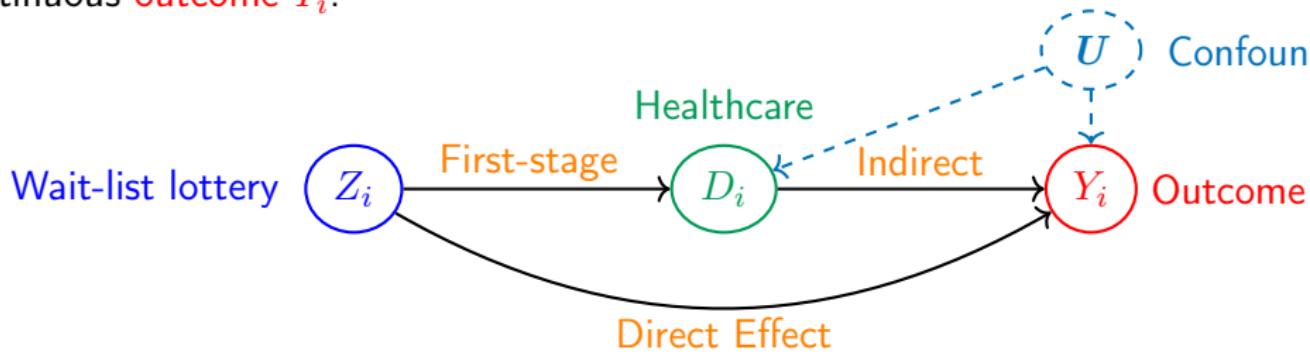
- ① Conventional CM has selection bias in observational (economic) settings
 - ② MTE-based model avoids selection bias
 - ③ Meaningful indirect (healthcare) and direct (psychological) effects in Oregon Health Insurance Experiment, with wide confidence intervals.

Access the paper on my website:



Appendix: CM Guiding Model

Consider binary treatment $Z_i = 0, 1$, binary mediator $D_i = 0, 1$, and continuous outcome Y_i .



Average Direct Effect (ADE) : $\mathbb{E} \left[Y_i(1, D_i(Z_i)) - Y_i(0, D_i(Z_i)) \right]$

- ADE is causal effect $Z \rightarrow Y$, blocking the indirect D_i path.

Average Indirect Effect (AIE): $\mathbb{E} \left[Y_i(Z_i, D_i(1)) - Y_i(Z_i, D_i(0)) \right]$

- AIE is causal effect of $D_i(Z_i) \rightarrow Y_i$, blocking the direct Z_i path.

Group Difference — ADE

CM effects contaminated by (less interpretable) bias terms.

CM Estimand \equiv ADEM + Selection Bias

$$\begin{aligned}
 & \underbrace{\mathbb{E}_{D_i} \left[\mathbb{E}[Y_i | Z_i = 1, D_i] - \mathbb{E}[Y_i | Z_i = 0, D_i] \right]}_{\text{Estimand, Direct Effect}} \\
 &= \underbrace{\mathbb{E}_{D_i=d'} \left[\mathbb{E}[Y_i(1, D_i(Z_i)) - Y_i(0, D_i(Z_i)) | D_i(1) = d'] \right]}_{\text{Average Direct Effect on Mediator (ADEM) take-up — i.e., } D_i(1) \text{ weighted}} \\
 &+ \underbrace{\mathbb{E}_{D_i} \left[\mathbb{E}[Y_i(0, D_i(Z_i)) | D_i(1) = d'] - \mathbb{E}[Y_i(0, D_i(Z_i)) | D_i(0) = d'] \right]}_{\text{Selection Bias}}
 \end{aligned}$$

The weighted ADE you get here is a positive weighted sum of local ADEs, but with policy irrelevant weights $D_i(1) = d'$.

⇒ consider this group bias, noting difference from true ADE.

Selection Bias — Direct Effect

CM Effects + contaminating bias.

$$\text{CM Estimand} = \text{ADE} + (\text{Selection Bias} + \text{Group difference bias})$$

► Model

$$\begin{aligned}
& \underbrace{\mathbb{E}_{D_i=d'} \left[\mathbb{E} [Y_i \mid Z_i = 1, D_i = d'] - \mathbb{E} [Y_i \mid Z_i = 0, D_i = d'] \right]}_{\text{Estimand, Direct Effect}} \\
&= \underbrace{\mathbb{E} [Y_i(1, D_i(Z_i)) - Y_i(0, D_i(Z_i))]}_{\text{Average Direct Effect}} \\
&\quad + \underbrace{\mathbb{E}_{D_i=d'} \left[\mathbb{E} [Y_i(0, D_i(Z_i)) \mid D_i(1) = d'] - \mathbb{E} [Y_i(0, D_i(Z_i)) \mid D_i(0) = d'] \right]}_{\text{Selection Bias}} \\
&\quad + \underbrace{\mathbb{E}_{D_i=d'} \left[\left(1 - \Pr(D_i(1) = d') \right) \times \left(\mathbb{E} [Y_i(1, D_i(Z_i)) - Y_i(0, D_i(Z_i)) \mid D_i(1) = 1 - d'] - \mathbb{E} [Y_i(1, D_i(Z_i)) - Y_i(0, D_i(Z_i)) \mid D_i(0) = d'] \right) \right]}_{\text{Group-diff bias}}
\end{aligned}$$

Group Difference — AIE

CM effects contaminated by (less interpretable) bias terms.

$$\text{CM Estimand} = \text{AIEM} + (\text{Selection Bias} + \text{Group difference bias})$$

$$\underbrace{\mathbb{E}_{Z_i} \left[\left(\mathbb{E}[D_i | Z_i = 1] - \mathbb{E}[D_i | Z_i = 0] \right) \times \left(\mathbb{E}[Y_i | Z_i, D_i = 1] - \mathbb{E}[Y_i | Z_i, D_i = 0] \right) \right]}_{\text{Estimand, Indirect Effect}}$$

$$= \mathbb{E} [Y_i(Z_i, D_i(1)) - Y_i(Z_i, D_i(0)) \mid D_i = 1]$$

Average Indirect Effect on Mediated (AIEM) — i.e., $D_i = 1$ weighted

$$+ \underbrace{\bar{\pi} \left(\mathbb{E} [Y_i(Z_i, 0) | D_i = 1] - \mathbb{E} [Y_i(Z_i, 0) | D_i = 0] \right)}_{\text{Selection Bias}}$$

$$+ \bar{\pi} \underbrace{\left[\left(\frac{1 - \Pr(D_i(1) = 1, D_i(0) = 0)}{\Pr(D_i(1) = 1, D_i(0) = 0)} \right) \left(\mathbb{E}[Y_i(Z_i, 1) - Y_i(Z_i, 0) | D_i(1) = 0 \text{ or } D_i(0) = 1] \right) - \mathbb{E}[Y_i(Z_i, 1) - Y_i(Z_i, 0)] \right]}_{\text{Groups difference Bias}}$$

The weighted AIE you get here is not a positive weighted sum of local AIEs, because the AIE is only about $D(Z)$ compliers. ▶ Model .

→ consider this group bias, noting difference from true AIF.

▶ Back

Selection Bias — Indirect Effect

CM Effects + contaminating bias, where $\bar{\pi} = \Pr(D_i(0) \neq D_i(1))$.

$$\text{CM Estimand} = \text{AIE} + (\text{Selection Bias} + \text{Group difference bias})$$

$$\underbrace{\mathbb{E}_{Z_i} \left[\left(\mathbb{E}[D_i | Z_i = 1] - \mathbb{E}[D_i | Z_i = 0] \right) \times \left(\mathbb{E}[Y_i | Z_i, D_i = 1] - \mathbb{E}[Y_i | Z_i, D_i = 0] \right) \right]}_{\text{Estimand, Indirect Effect}}$$

$$= \underbrace{\mathbb{E} [Y_i(Z_i, D_i(1)) - Y_i(Z_i, D_i(0))]}_{\text{Average Indirect Effect}}$$

$$+ \underbrace{\bar{\pi} \left(\mathbb{E}[Y_i(Z_i, 0) | D_i = 1] - \mathbb{E}[Y_i(Z_i, 0) | D_i = 0] \right)}_{\text{Selection Bias}}$$

$$+ \bar{\pi} \left[\left(1 - \Pr(D_i = 1)\right) \begin{pmatrix} \mathbb{E}[Y_i(Z_i, 1) - Y_i(Z_i, 0) | D_i = 1] \\ -\mathbb{E}[Y_i(Z_i, 1) - Y_i(Z_i, 0) | D_i = 0] \end{pmatrix} + \left(\frac{1 - \Pr(D_i(1) = 1, D_i(0) = 0)}{\Pr(D_i(1) = 1, D_i(0) = 0)}\right) \begin{pmatrix} \mathbb{E}[Y_i(Z_i, 1) - Y_i(Z_i, 0) | D_i(Z_i) \neq Z_i] \\ -\mathbb{E}[Y_i(Z_i, 1) - Y_i(Z_i, 0)] \end{pmatrix} \right]$$

Groups difference Bias ▶ Group-diff

Semi-parametric Control Functions

Semi-parametric specifications for the CFs λ_0, λ_1 bring some complications to estimating the AIE.

$$\mathbb{E}[Y_i | Z_i, D_i = 0, \mathbf{X}_i] = \alpha + \gamma Z_i + \varphi(\mathbf{X}_i) + \rho_0 \lambda_0(\pi(Z_i; \mathbf{X}_i)),$$

$$\mathbb{E}[Y_i | Z_i, D_i = 1, \mathbf{X}_i] = (\alpha + \beta) + (\gamma + \delta) Z_i + \varphi(\mathbf{X}_i) + \rho_1 \lambda_1(\pi(Z_i; \mathbf{X}_i))$$

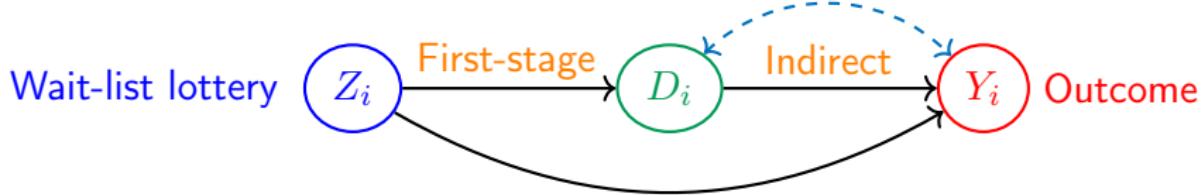
Intercepts, α , $(\alpha + \beta)$, and relevance parameters ρ_0, ρ_1 are not separately identified from the CFs $\lambda_0(\cdot), \lambda_1(\cdot)$ so CF extrapolation term $(\rho_1 - \rho_0)\Gamma(\pi(0; \mathbf{X}_i), \pi(1; \mathbf{X}_i))$ is not directly identified or estimable.

These problems can be avoided by estimating the AIE using its relation to the ATE, $\widehat{\text{AIE}}^{\text{CF}} =$

$$\widehat{\text{ATE}} - (1 - \bar{Z}) \underbrace{\left(\frac{1}{N} \sum_{i=1}^N \widehat{\gamma} + \widehat{\delta} \widehat{\pi}(1; \mathbf{X}_i) \right)}_{\widehat{\text{ADE}} \text{ given } Z_i=1} - \bar{Z} \underbrace{\left(\frac{1}{N} \sum_{i=1}^N \widehat{\gamma} + \widehat{\delta} \widehat{\pi}(0; \mathbf{X}_i) \right)}_{\widehat{\text{ADE}} \text{ given } Z_i=0}.$$

Appendix: CM with Selection

Suppose Z_i is ignorable, D_i is not, so we have the following causal model.



Then this system has the following random coefficient equations:

$$D_i = \phi + \bar{\pi} Z_i + \varphi(\mathbf{X}_i) + U_i$$

$$Y_i = \alpha + \beta D_i + \gamma Z_i + \delta Z_i D_i + \zeta(\mathbf{X}_i) + \underbrace{(1 - D_i) U_{0,i} + D_i U_{1,i}}_{\text{Correlated error term}}$$

where β, γ, δ are functions of $\mu_{d'}(z'; \mathbf{X}_i)$.

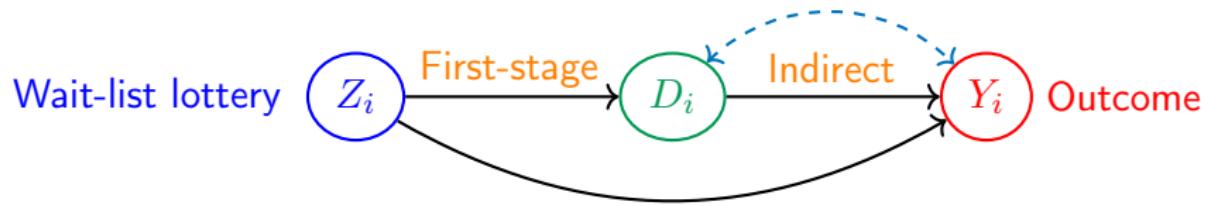
Correlated error term

$$\text{ADE} = \mathbb{E} [\gamma + \delta D_i], \quad \text{AIE} = \mathbb{E} \left[\bar{\pi} (\beta + \delta Z_i + \tilde{U}_i) \right]$$

with $\tilde{U}_i = \mathbb{E} [U_{1,i} - U_{0,i} | \mathbf{X}_i, D_i(0) \neq D_i(1)]$ unobserved complier gains.

Appendix: CM with Selection

Suppose Z_i is ignorable, D_i is not, so we have the following causal model.



Main problem, second-stage is not identified:

$$\begin{aligned} \mathbb{E}[Y_i | Z_i, D_i, \mathbf{X}_i] &= \alpha + \beta D_i + \gamma Z_i + \delta Z_i D_i + \varphi(\mathbf{X}_i) \\ &+ (1 - D_i) \mathbb{E}[U_{0,i} | D_i = 0, \mathbf{X}_i] \\ &+ D_i \mathbb{E}[U_{1,i} | D_i = 1, \mathbf{X}_i] \end{aligned}$$

Unobserved D_i confounding.

Identification intuition: Identify second-stage via MTE control function.

Appendix: CM with Selection — Identification

Assume:

- ① Mediator monotonicity, $\Pr(D_i(0) \leq D_i(1) | \mathbf{X}_i) = 1$

$$\implies D_i(z') = \mathbb{1}\{U_i \leq \pi(z'; \mathbf{X}_i)\}, \quad \text{for } z' = 0, 1 \text{ (Vycatil 2002).}$$

- ② Selection on mediator benefits, $\text{Cov}(U_i, U_{0,i}), \text{Cov}(U_i, U_{1,i}) \neq 0$

\implies First-stage take-up informs second-stage confounding.

- ③ There is an IV for the mediator, \mathbf{X}_i^{IV} among control variables \mathbf{X}_i .

$\implies \pi(Z_i; \mathbf{X}_i) = \Pr(D_i = 1 | Z_i, \mathbf{X}_i)$ is separately identified.

Proposition:

$$\begin{aligned} & \mathbb{E}[Y_i(z', 1) - Y_i(z', 0) | Z_i = z', \mathbf{X}_i, U_i = p'] \\ &= \beta + \delta z' + \mathbb{E}[U_{1,i} - U_{0,i} | \mathbf{X}_i, U_i = p'], \quad \text{for } p' \in (0, 1). \end{aligned}$$

Appendix: CM with Selection — Identification

The marginal effect has corresponding Control Functions (CFs), describing unobserved selection-into- D_i ,

$$\rho_0 \lambda_0(p') = \mathbb{E} [U_{0,i} \mid p' \leq U_i], \quad \rho_1 \lambda_1(p') = \mathbb{E} [U_{1,i} \mid U_i \leq p'].$$

These CFs restore second-stage identification, by extrapolating from \mathbf{X}_i^{IV} compliers to $D_i(Z_i)$ mediator compliers,

$$\begin{aligned} \mathbb{E} [Y_i \mid Z_i, D_i, \mathbf{X}_i] &= \alpha + \beta D_i + \gamma Z_i + \delta Z_i D_i + \varphi(\mathbf{X}_i) \\ &\quad + \underbrace{\rho_0 (1 - D_i) \lambda_0(\pi(Z_i; \mathbf{X}_i)) + \rho_1 D_i \lambda_1(\pi(Z_i; \mathbf{X}_i))}_{\text{CF adjustment.}} \end{aligned}$$

This adjusted second-stage re-identifies the ADE and AIE,

$$\text{ADE} = \mathbb{E} [\gamma + \delta D_i], \quad \text{AIE} = \mathbb{E} \left[\bar{\pi} \left(\beta + \delta Z_i + \underbrace{(\rho_1 - \rho_0) \Gamma(\pi(0; \mathbf{X}_i), \pi(1; \mathbf{X}_i))}_{\text{Mediator compliers extrapolation.}} \right) \right]$$

Appendix: CM with Selection — Estimation

Will explain how estimation works, with simulation evidence.

- ① Random treatment $Z_i \sim \text{Binom}(0.5)$, for $n = 5,000$.
- ② $(U_{0,i}, U_{1,i}) \sim \text{BivariateNormal}(0, 0, \sigma_0, \sigma_1, \rho)$, Costs $C_i \sim N(0, 0.5)$.

Roy selection-into- D_i , with constant partial effects + interaction term.

$$D_i(z') = \mathbb{1} \left\{ C_i \leq Y_i(z', 1) - Y_i(z', 0) \right\},$$

$$Y_i(z', d') = (z' + d' + z'd') + U_{d'} \quad \text{for } z', d' = 0, 1.$$

Following the previous, these data have the following first and second-stage equations, where X_i^{IV} is an additive cost IV:

$$D_i = \mathbb{1} \left\{ C_i - \left(U_{1,i} - U_{0,i} \right) \leq Z_i - X_i^{\text{IV}} \right\}$$

$$Y_i = Z_i + D_i + Z_i D_i + (1 - D_i) U_{0,i} + D_i U_{1,i}.$$

\implies unobserved confounding by BivariateNormal $(U_{0,i}, U_{1,i})$.

Appendix: CM with Selection — Estimation

Errors are normal, so system is Heckman (1979) selection model.

CFs are the inverse Mills ratio, with $\phi(\cdot)$ normal pdf and $\Phi(\cdot)$ normal cdf,

$$\lambda_0(p') = \frac{\phi(-\Phi^{-1}(p'))}{\Phi(-\Phi^{-1}(p'))}, \quad \lambda_1(p') = \frac{\phi(\Phi^{-1}(p'))}{\Phi(\Phi^{-1}(p'))}, \quad \text{for } p' \in (0, 1).$$

Parametric Estimation Recipe:

- ① Estimate first-stage $\pi(Z_i; \mathbf{X}_i)$ with probit, including \mathbf{X}_i^{IV} .
- ② Include λ_0, λ_1 CFs in second-stage OLS estimation.
- ③ Compose CM estimates from two-stage plug-in estimates.

→ Same as conventional CM estimates (two-stages), with CFs added.

$$\widehat{\text{ADE}} = \mathbb{E} \left[\widehat{\gamma} + \widehat{\delta} D_i \right], \quad \widehat{\text{AIE}} = \mathbb{E} \left[\widehat{\pi} \left(\widehat{\beta} + \widehat{\delta} Z_i + \underbrace{(\widehat{\rho}_1 - \widehat{\rho}_0) \Gamma(\widehat{\pi}(0; \mathbf{X}_i), \widehat{\pi}(1; \mathbf{X}_i))}_{\text{Mediator compliers extrapolation.}} \right) \right]$$

Appendix: CM with Selection — Estimation

If errors are not normal, then CFs do not have a known form, so semi-parametrically estimate them (e.g., splines).

$$\mathbb{E}[Y_i | Z_i, D_i = 0, \mathbf{X}_i] = \alpha + \gamma Z_i + \varphi(\mathbf{X}_i) + \rho_0 \lambda_0(\pi(Z_i; \mathbf{X}_i)),$$

$$\mathbb{E}[Y_i | Z_i, D_i = 1, \mathbf{X}_i] = (\alpha + \beta) + (\gamma + \delta)Z_i + \varphi(\mathbf{X}_i) + \rho_1 \lambda_1(\pi(Z_i; \mathbf{X}_i))$$

Semi-parametric Estimation Recipe:

- ① Estimate first-stage $\pi(Z_i; \mathbf{X}_i)$, including \mathbf{X}_i^{IV} .
- ② Estimate second-stage separately for $D_i = 0$ and $D_i = 1$, with regressors $\lambda_0(p'), \lambda_1(p')$, semi-parametric in $\hat{\pi}(Z_i; \mathbf{X}_i)$.
- ③ Compose CM estimates from two-stage plug-in estimates.

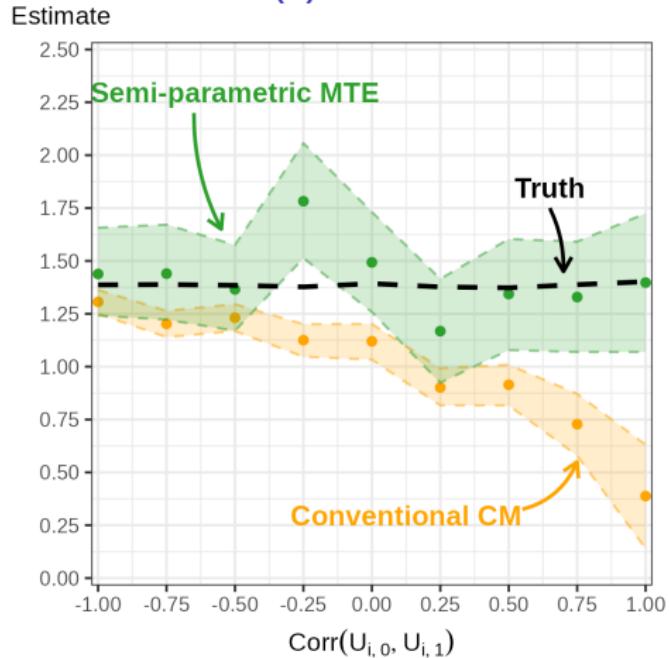
→ Same as conventional CM estimates, with semi-parametric CFs. CFs.

$$\widehat{\text{ADE}} = \mathbb{E}[\widehat{\gamma} + \widehat{\delta}D_i], \quad \widehat{\text{AIE}} = \mathbb{E}\left[\widehat{\pi}\left(\widehat{\beta} + \widehat{\delta}Z_i + (\widehat{\rho}_1 - \widehat{\rho}_0)\Gamma(\widehat{\pi}(0; \mathbf{X}_i), \widehat{\pi}(1; \mathbf{X}_i))\right)\right]$$

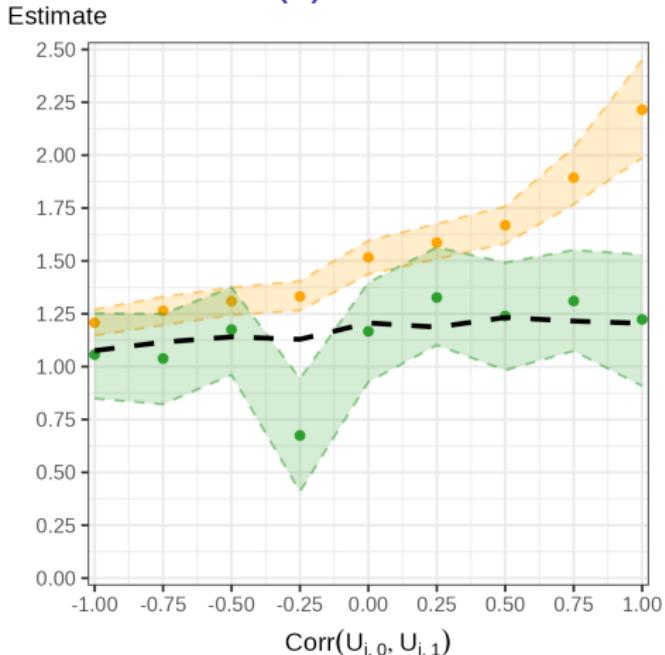
Appendix: CM with Selection — Estimation

Figure: CF Adjusted Estimates Work with Different Error Term Parameters.

(a) ADE.



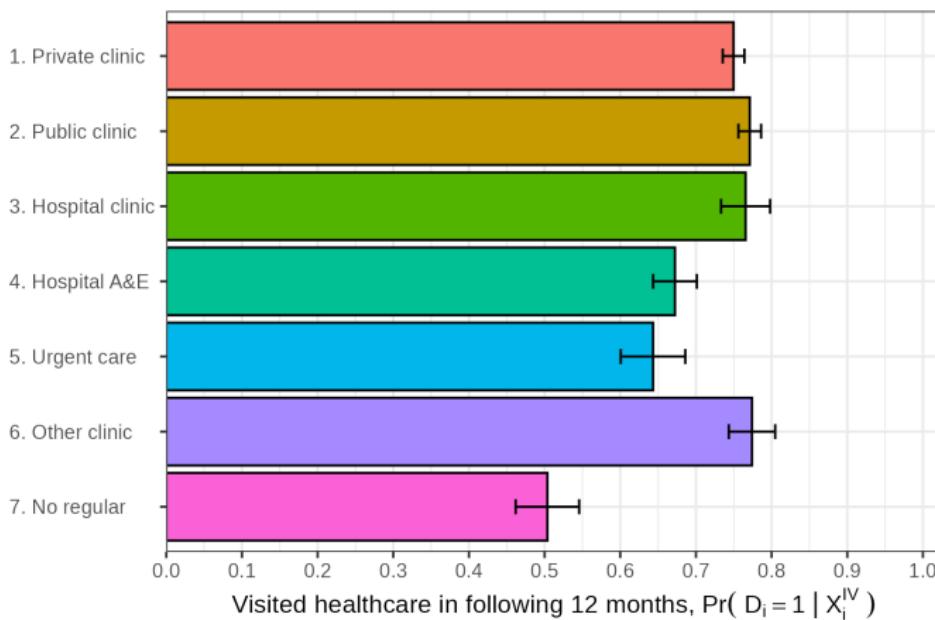
(b) AIE.



Appendix: OHIE IV

IV first-stage F stat. is 124, for all categories (minus base).

Usual Healthcare Location



Structural estimate of mediator compliers' $D_i \rightarrow Y_i$ is +32.9pp (4.4).