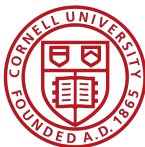


Causal Mediation in Natural Experiments

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Introduction

Motivation:

- Causal Mediation (CM) is a framework for studying mechanisms of causal effects, but is not widely used in economics
- Economists stick to vague suggestive evidence, leaving mechanisms relatively under-studied.

Contribution:

- Develop selection bias concept for CM, crystallising reasoning for economists dismissal of these methods
- Develop a Marginal Treatment Effect (MTE) approach to credibly estimate causal channels (direct and indirect effects).

Key findings:

- Apply methods to the Oregon Health Insurance Experiment, studying mediating role of healthcare take-up
- Bring CM into economic quasi-experimental causal inference.

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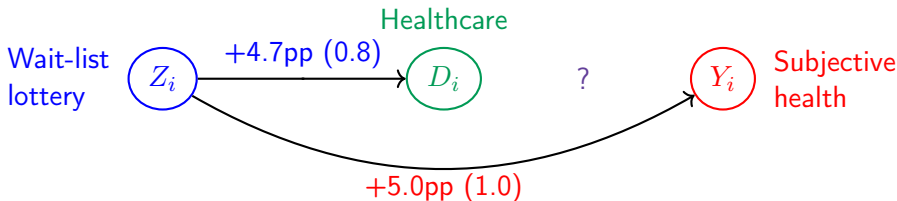
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- Bring CM into economic quasi-experimental causal inference.

1. Suggestive Evidence

In 2008, Oregon gave access to socialised health insurance by wait-list lottery (Finkelstein et al, 2012).

Figure: Model for Suggestive Evidence of a Mechanism.



Necessary but not sufficient evidence on the mediating mechanism:

- Is $D_i \rightarrow Y_i$ small, large, non-zero?
- Can we assume this causal effect?

2. Causal Mediation

Causal Mediation (CM) models the entire system, giving sufficient evidence on the mediating mechanism.



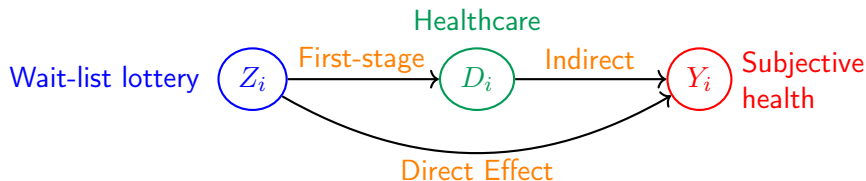
Average Direct Effect (ADE) and Average Indirect Effect (AIE):

- ADE is causal effect $Z_i \rightarrow Y_i$
- AIE is causal effect of $D_i(Z_i) \rightarrow Y_i$.

Identification: Requires mediator D_i is quasi-randomly assigned. . .

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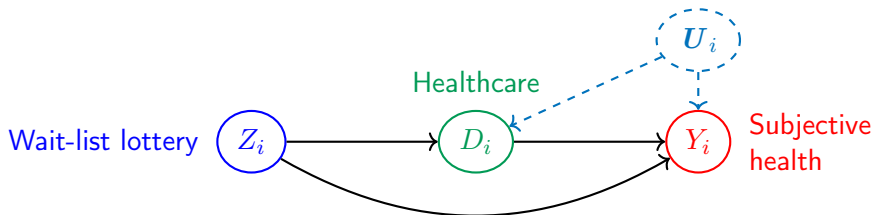
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Identification: Requires mediator D_i is quasi-randomly assigned. . . .

2. Causal Mediation — Selection Bias

People chose to visit **healthcare** D_i freely, not randomly assigned...



Conventional CM analyses are misleading in natural experiment settings —
I derive non-parametric selection bias terms for CM.

- ADE: $\text{CM Estimand} = \text{ADE} + (\text{Selection Bias} + \text{Group difference bias})$
- AIE: $\text{CM Estimand} = \text{AIE} + (\text{Selection Bias} + \text{Group difference bias})$

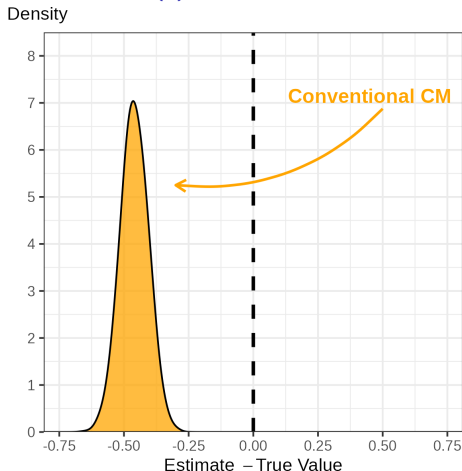
► ADE biases

► AIE biases

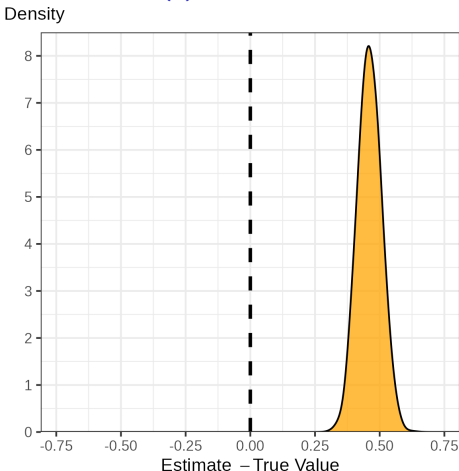
2. Causal Mediation — Selection Bias

Simulations with cost-benefit selection-into- D_i , CM estimates are biased.

(a) $\widehat{ADE} - ADE$.



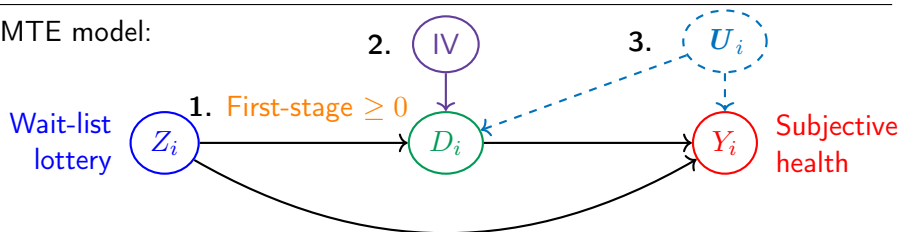
(b) $\widehat{AIE} - AIE$.



3. MTE Model

Conventional CM methods do not identify ADE + AIE in a natural experiment setting, so I build a Marginal Treatment Effect (MTE) model.

MTE model:



MTE assumptions:

- ① Mediator monotonicity
- ② IV for mediator take-up cost
- ③ Selection on mediator benefits.

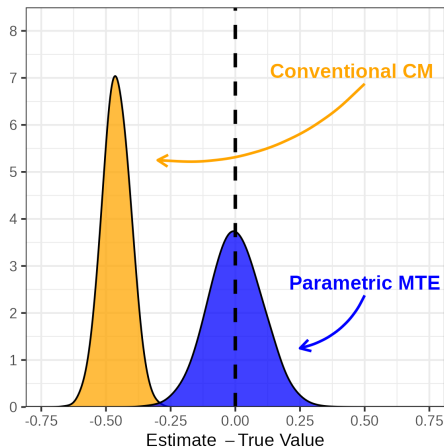
Intuition: model U_i via mediator MTE to identify ADE + AIE.

3. MTE Model

Simulation with cost-benefit selection-into- D_i , MTE-model corrects for selection bias.

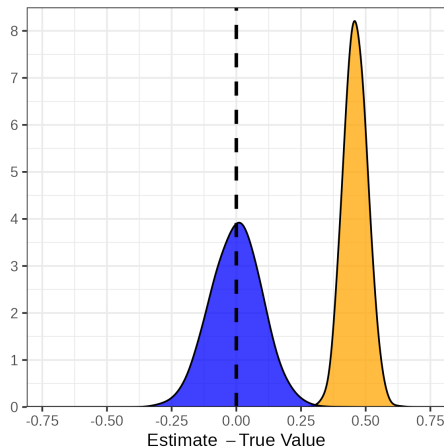
(a) $\widehat{ADE} - ADE$.

Density



(b) $\widehat{AIE} - AIE$.

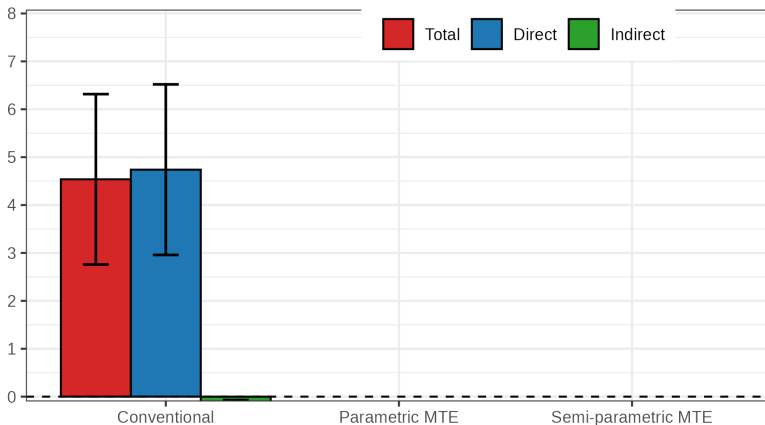
Density



4. Returning to Oregon

Conventional CM estimates lottery effects as mostly direct, ≈ 0 healthcare.

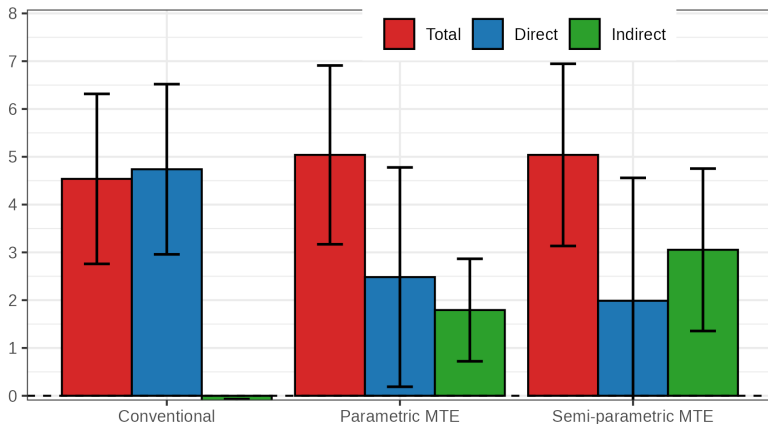
Estimate, percent effect on subjective health



4. Returning to Oregon

Using my MTE model, with regular healthcare location as an excluded IV, restores indirect effect through increased **healthcare visitation**.

Estimate, percent effect on subjective health



Conclusion

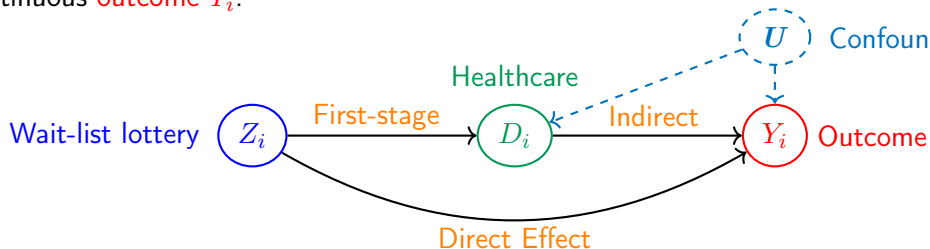
- ① Conventional CM has selection bias in observational (economic) settings
- ② MTE-based model avoids selection bias
- ③ Meaningful indirect (healthcare) and direct (psychological) effects in Oregon Health Insurance Experiment, with wide confidence intervals.

Access the paper on my website:



Appendix: CM Guiding Model

Consider binary **treatment** $Z_i = 0, 1$, binary **mediator** $D_i = 0, 1$, and continuous **outcome** Y_i .



Average Direct Effect (ADE): $\mathbb{E} \left[Y_i \left(\mathbf{1}, D_i(Z_i) \right) - Y_i \left(\mathbf{0}, D_i(Z_i) \right) \right]$

- ADE is causal effect $Z \rightarrow Y$, blocking the indirect D_i path.

Average Indirect Effect (AIE): $\mathbb{E} \left[Y_i \left(Z_i, \mathbf{D_i(1)} \right) - Y_i \left(Z_i, \mathbf{D_i(0)} \right) \right]$

- AIE is causal effect of $D_i(Z_i) \rightarrow Y_i$, blocking the direct Z_i path.

Group Difference — ADE

CM effects contaminated by (less interpretable) bias terms.

$$\text{CM Estimand} = \text{ADEM} + \text{Selection Bias}$$

$$\begin{aligned} & \underbrace{\mathbb{E}_{D_i} \left[\mathbb{E} [Y_i \mid Z_i = 1, D_i] - \mathbb{E} [Y_i \mid Z_i = 0, D_i] \right]}_{\text{Estimand, Direct Effect}} \\ &= \underbrace{\mathbb{E}_{D_i=d'} \left[\mathbb{E} [Y_i(1, D_i(Z_i)) - Y_i(0, D_i(Z_i)) \mid D_i(1) = d'] \right]}_{\text{Average Direct Effect on Mediator (ADEM) take-up — i.e., } D_i(1) \text{ weighted}} \\ & \quad + \underbrace{\mathbb{E}_{D_i} \left[\mathbb{E} [Y_i(0, D_i(Z_i)) \mid D_i(1) = d'] - \mathbb{E} [Y_i(0, D_i(Z_i)) \mid D_i(0) = d'] \right]}_{\text{Selection Bias}} \end{aligned}$$

The weighted ADE you get here is a positive weighted sum of local ADEs, but with policy irrelevant weights $D_i(1) = d'$.

⇒ consider this group bias, noting difference from true ADE. [▶ Back](#)

Selection Bias — Direct Effect

CM Effects + contaminating bias.

$$\text{CM Estimand} = \text{ADE} + \left(\text{Selection Bias} + \text{Group difference bias} \right)$$

► Model

$$\begin{aligned} & \underbrace{\mathbb{E}_{D_i=d'} \left[\mathbb{E} [Y_i \mid Z_i = 1, D_i = d'] - \mathbb{E} [Y_i \mid Z_i = 0, D_i = d'] \right]}_{\text{Estimand, Direct Effect}} \\ &= \underbrace{\mathbb{E} [Y_i(1, D_i(Z_i)) - Y_i(0, D_i(Z_i))]}_{\text{Average Direct Effect}} \\ &+ \underbrace{\mathbb{E}_{D_i=d'} \left[\mathbb{E} [Y_i(0, D_i(Z_i)) \mid D_i(1) = d'] - \mathbb{E} [Y_i(0, D_i(Z_i)) \mid D_i(0) = d'] \right]}_{\text{Selection Bias}} \\ &+ \underbrace{\mathbb{E}_{D_i=d'} \left[\left(1 - \Pr(D_i(1) = d') \right) \right.}_{\text{Group difference bias}} \\ &\quad \times \left. \left(\mathbb{E} [Y_i(1, D_i(Z_i)) - Y_i(0, D_i(Z_i)) \mid D_i(1) = 1 - d'] \right) \right. \\ &\quad \left. - \mathbb{E} [Y_i(1, D_i(Z_i)) - Y_i(0, D_i(Z_i)) \mid D_i(0) = d'] \right) \left. \right]}_{\text{Group-diff}} \end{aligned}$$

Group Difference — AIE

CM effects contaminated by (less interpretable) bias terms.

$$\text{CM Estimand} = \text{AIEM} + \left(\text{Selection Bias} + \text{Group difference bias} \right)$$

$$\begin{aligned} & \underbrace{\mathbb{E}_{Z_i} \left[\left(\mathbb{E}[D_i | Z_i = 1] - \mathbb{E}[D_i | Z_i = 0] \right) \times \left(\mathbb{E}[Y_i | Z_i, D_i = 1] - \mathbb{E}[Y_i | Z_i, D_i = 0] \right) \right]}_{\text{Estimand, Indirect Effect}} \\ &= \underbrace{\mathbb{E} \left[Y_i(Z_i, D_i(1)) - Y_i(Z_i, D_i(0)) \mid D_i = 1 \right]}_{\text{Average Indirect Effect on Mediated (AIEM) — i.e., } D_i = 1 \text{ weighted}} \\ &+ \underbrace{\pi \left(\mathbb{E}[Y_i(Z_i, 0) \mid D_i = 1] - \mathbb{E}[Y_i(Z_i, 0) \mid D_i = 0] \right)}_{\text{Selection Bias}} \\ &+ \underbrace{\pi \left[\left(\frac{1 - \Pr(D_i(1) = 1, D_i(0) = 0)}{\Pr(D_i(1) = 1, D_i(0) = 0)} \right) \left(\mathbb{E}[Y_i(Z_i, 1) - Y_i(Z_i, 0) \mid D_i(1) = 0 \text{ or } D_i(0) = 1] - \mathbb{E}[Y_i(Z_i, 1) - Y_i(Z_i, 0)] \right) \right]}_{\text{Groups difference Bias}} \end{aligned}$$

The weighted AIE you get here is not a positive weighted sum of local AIEs, because the AIE is only about $D(Z)$ compliers. [► Model](#).

⇒ consider this group bias, noting difference from true AIE. [► Back](#)

Selection Bias — Indirect Effect

CM Effects + contaminating bias, where $\bar{\pi} = \Pr(D_i(0) \neq D_i(1))$.

$$\text{CM Estimand} = \text{AIE} + \left(\text{Selection Bias} + \text{Group difference bias} \right) \quad \text{► Model}$$

$$\begin{aligned} & \underbrace{\mathbb{E}_{Z_i} \left[\left(\mathbb{E}[D_i | Z_i = 1] - \mathbb{E}[D_i | Z_i = 0] \right) \times \left(\mathbb{E}[Y_i | Z_i, D_i = 1] - \mathbb{E}[Y_i | Z_i, D_i = 0] \right) \right]}_{\text{Estimand, Indirect Effect}} \\ &= \underbrace{\mathbb{E}[Y_i(Z_i, D_i(1)) - Y_i(Z_i, D_i(0))]}_{\text{Average Indirect Effect}} \\ &+ \underbrace{\bar{\pi} \left(\mathbb{E}[Y_i(Z_i, 0) | D_i = 1] - \mathbb{E}[Y_i(Z_i, 0) | D_i = 0] \right)}_{\text{Selection Bias}} \\ &+ \bar{\pi} \left[\begin{aligned} & \left(1 - \Pr(D_i = 1) \right) \left(\mathbb{E}[Y_i(Z_i, 1) - Y_i(Z_i, 0) | D_i = 1] \right. \\ & \quad \left. - \mathbb{E}[Y_i(Z_i, 1) - Y_i(Z_i, 0) | D_i = 0] \right) \\ & + \left(\frac{1 - \Pr(D_i(1) = 1, D_i(0) = 0)}{\Pr(D_i(1) = 1, D_i(0) = 0)} \right) \left(\mathbb{E}[Y_i(Z_i, 1) - Y_i(Z_i, 0) | D_i(Z_i) \neq Z_i] \right. \\ & \quad \left. - \mathbb{E}[Y_i(Z_i, 1) - Y_i(Z_i, 0)] \right) \end{aligned} \right] \end{aligned}$$

Groups difference Bias ► Group-diff

Semi-parametric Control Functions

Semi-parametric specifications for the CFs λ_0, λ_1 bring some complications to estimating the AIE.

$$\mathbb{E}[Y_i | Z_i, D_i = 0, \mathbf{X}_i] = \alpha + \gamma Z_i + \varphi(\mathbf{X}_i) + \rho_0 \lambda_0(\pi(Z_i; \mathbf{X}_i)),$$

$$\mathbb{E}[Y_i | Z_i, D_i = 1, \mathbf{X}_i] = (\alpha + \beta) + (\gamma + \delta) Z_i + \varphi(\mathbf{X}_i) + \rho_1 \lambda_1(\pi(Z_i; \mathbf{X}_i))$$

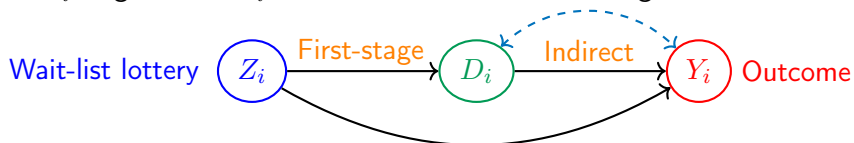
Intercepts, $\alpha, (\alpha + \beta)$, and relevance parameters ρ_0, ρ_1 are not separately identified from the CFs $\lambda_0(\cdot), \lambda_1(\cdot)$ so CF extrapolation term $(\rho_1 - \rho_0)\Gamma(\pi(0; \mathbf{X}_i), \pi(1; \mathbf{X}_i))$ is not directly identified or estimable.

These problems can be avoided by estimating the AIE using its relation to the ATE, $\widehat{\text{AIE}}^{\text{CF}} =$

$$\widehat{\text{ATE}} - (1 - \bar{Z}) \underbrace{\left(\frac{1}{N} \sum_{i=1}^N \hat{\gamma} + \hat{\delta} \hat{\pi}(1; \mathbf{X}_i) \right)}_{\widehat{\text{ADE}} \text{ given } Z_i=1} - \bar{Z} \underbrace{\left(\frac{1}{N} \sum_{i=1}^N \hat{\gamma} + \hat{\delta} \hat{\pi}(0; \mathbf{X}_i) \right)}_{\widehat{\text{ADE}} \text{ given } Z_i=0}.$$

Appendix: CM with Selection

Suppose Z_i is ignorable, D_i is not, so we have the following causal model.



Then this system has the following random coefficient equations:

$$D_i = \phi + \pi Z_i + \varphi(\mathbf{X}_i) + U_i$$

$$Y_i = \alpha + \beta D_i + \gamma Z_i + \delta Z_i D_i + \zeta(\mathbf{X}_i) + \underbrace{(1 - D_i) U_{0,i} + D_i U_{1,i}}_{\text{Correlated error term}}$$

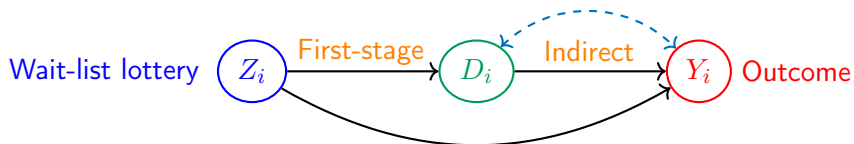
where β, γ, δ are functions of $\mu_{d'}(z'; \mathbf{X}_i)$.

$$\text{ADE} = \mathbb{E}[\gamma + \delta D_i], \quad \text{AIE} = \mathbb{E}\left[\pi(\beta + \delta Z_i + \tilde{U}_i)\right]$$

with $\tilde{U}_i = \mathbb{E}[U_{1,i} - U_{0,i} | \mathbf{X}_i, D_i(0) \neq D_i(1)]$ unobserved complier gains.

Appendix: CM with Selection

Suppose Z_i is ignorable, D_i is not, so we have the following causal model.



Main problem, second-stage is not identified:

$$\begin{aligned}\mathbb{E}[Y_i | Z_i, D_i, \mathbf{X}_i] &= \alpha + \beta D_i + \gamma Z_i + \delta Z_i D_i + \varphi(\mathbf{X}_i) \\ &+ (1 - D_i) \mathbb{E}[U_{0,i} | D_i = 0, \mathbf{X}_i] \\ &+ \underbrace{D_i \mathbb{E}[U_{1,i} | D_i = 1, \mathbf{X}_i]}_{\text{Unobserved } D_i \text{ confounding.}}\end{aligned}$$

Identification intuition: Identify second-stage via MTE control function.

Appendix: CM with Selection — Identification

Assume:

- ① Mediator monotonicity, $\Pr(D_i(0) \leq D_i(1) | \mathbf{X}_i) = 1$
 $\implies D_i(z') = \mathbb{1}\{U_i \leq \pi(z'; \mathbf{X}_i)\}, \text{ for } z' = 0, 1 \text{ (Vycatil 2002).}$
- ② Selection on mediator benefits, $\text{Cov}(U_i, U_{0,i}), \text{Cov}(U_i, U_{1,i}) \neq 0$
 \implies First-stage take-up informs second-stage confounding.
- ③ There is an IV for the mediator, \mathbf{X}_i^{IV} among control variables \mathbf{X}_i .
 $\implies \pi(Z_i; \mathbf{X}_i) = \Pr(D_i = 1 | Z_i, \mathbf{X}_i)$ is separately identified.

Proposition:

$$\begin{aligned} & \mathbb{E}[Y_i(z', 1) - Y_i(z', 0) | Z_i = z', \mathbf{X}_i, U_i = p'] \\ &= \beta + \delta z' + \mathbb{E}[U_{1,i} - U_{0,i} | \mathbf{X}_i, U_i = p'], \quad \text{for } p' \in (0, 1). \end{aligned}$$

Appendix: CM with Selection — Identification

The marginal effect has corresponding Control Functions (CFs), describing unobserved selection-into- D_i ,

$$\rho_0 \lambda_0(p') = \mathbb{E} [U_{0,i} \mid p' \leq U_i], \quad \rho_1 \lambda_1(p') = \mathbb{E} [U_{1,i} \mid U_i \leq p'] .$$

These CFs restore second-stage identification, by extrapolating from \mathbf{X}_i^{IV} compliers to $D_i(Z_i)$ mediator compliers,

$$\begin{aligned} \mathbb{E} [Y_i \mid Z_i, D_i, \mathbf{X}_i] &= \alpha + \beta D_i + \gamma Z_i + \delta Z_i D_i + \varphi(\mathbf{X}_i) \\ &\quad + \underbrace{\rho_0 (1 - D_i) \lambda_0(\pi(Z_i; \mathbf{X}_i)) + \rho_1 D_i \lambda_1(\pi(Z_i; \mathbf{X}_i))}_{\text{CF adjustment.}} \end{aligned}$$

This adjusted second-stage re-identifies the ADE and AIE,

$$\text{ADE} = \mathbb{E} [\gamma + \delta D_i], \quad \text{AIE} = \mathbb{E} \left[\bar{\pi} \left(\beta + \delta Z_i + \underbrace{(\rho_1 - \rho_0) \Gamma(\pi(0; \mathbf{X}_i), \pi(1; \mathbf{X}_i))}_{\text{Mediator compliers extrapolation.}} \right) \right]$$

Appendix: CM with Selection — Estimation

Will explain how estimation works, with simulation evidence.

- ① Random treatment $Z_i \sim \text{Binom}(0.5)$, for $n = 5,000$.
- ② $(U_{0,i}, U_{1,i}) \sim \text{BivariateNormal}(0, 0, \sigma_0, \sigma_1, \rho)$, Costs $C_i \sim N(0, 0.5)$.

Roy **selection-into- D_i** , with constant partial effects + interaction term.

$$D_i(z') = \mathbb{1} \{C_i \leq Y_i(z', 1) - Y_i(z', 0)\},$$

$$Y_i(z', d') = (z' + d' + z'd') + U_{d'} \quad \text{for } z', d' = 0, 1.$$

Following the previous, these data have the following first and second-stage equations, where \mathbf{X}_i^{IV} is an additive cost IV:

$$D_i = \mathbb{1} \left\{ C_i - (U_{1,i} - U_{0,i}) \leq Z_i - \mathbf{X}_i^{\text{IV}} \right\}$$

$$Y_i = Z_i + D_i + Z_i D_i + (1 - D_i) U_{0,i} + D_i U_{1,i}.$$

\implies unobserved confounding by BivariateNormal $(U_{0,i}, U_{1,i})$.

Appendix: CM with Selection — Estimation

Errors are normal, so system is Heckman (1979) selection model.

CFs are the inverse Mills ratio, with $\phi(\cdot)$ normal pdf and $\Phi(\cdot)$ normal cdf,

$$\lambda_0(p') = \frac{\phi(-\Phi^{-1}(p'))}{\Phi(-\Phi^{-1}(p'))}, \quad \lambda_1(p') = \frac{\phi(\Phi^{-1}(p'))}{\Phi(\Phi^{-1}(p'))}, \quad \text{for } p' \in (0, 1).$$

Parametric Estimation Recipe:

- ① Estimate first-stage $\pi(Z_i; \mathbf{X}_i)$ with probit, including \mathbf{X}_i^{IV} .
- ② Include λ_0, λ_1 CFs in second-stage OLS estimation.
- ③ Compose CM estimates from two-stage plug-in estimates.

→ Same as conventional CM estimates (two-stages), with CFs added.

$$\widehat{\text{ADE}} = \mathbb{E} \left[\widehat{\gamma} + \widehat{\delta} D_i \right], \quad \widehat{\text{AIE}} = \mathbb{E} \left[\widehat{\pi} \left(\widehat{\beta} + \widehat{\delta} Z_i + \underbrace{(\widehat{\rho}_1 - \widehat{\rho}_0) \Gamma(\widehat{\pi}(0; \mathbf{X}_i), \widehat{\pi}(1; \mathbf{X}_i))}_{\text{Mediator compliers extrapolation.}} \right) \right]$$

Appendix: CM with Selection — Estimation

If errors are not normal, then CFs do not have a known form, so semi-parametrically estimate them (e.g., splines).

$$\mathbb{E}[Y_i | Z_i, D_i = 0, \mathbf{X}_i] = \alpha + \gamma Z_i + \varphi(\mathbf{X}_i) + \rho_0 \lambda_0(\pi(Z_i; \mathbf{X}_i)),$$

$$\mathbb{E}[Y_i | Z_i, D_i = 1, \mathbf{X}_i] = (\alpha + \beta) + (\gamma + \delta) Z_i + \varphi(\mathbf{X}_i) + \rho_1 \lambda_1(\pi(Z_i; \mathbf{X}_i))$$

Semi-parametric Estimation Recipe:

- 1 Estimate first-stage $\pi(Z_i; \mathbf{X}_i)$, including \mathbf{X}_i^{IV} .
- 2 Estimate second-stage separately for $D_i = 0$ and $D_i = 1$, with regressors $\lambda_0(p')$, $\lambda_1(p')$, semi-parametric in $\hat{\pi}(Z_i; \mathbf{X}_i)$.
- 3 Compose CM estimates from two-stage plug-in estimates.

→ Same as conventional CM estimates, with semi-parametric CFs. ▶ CFs.

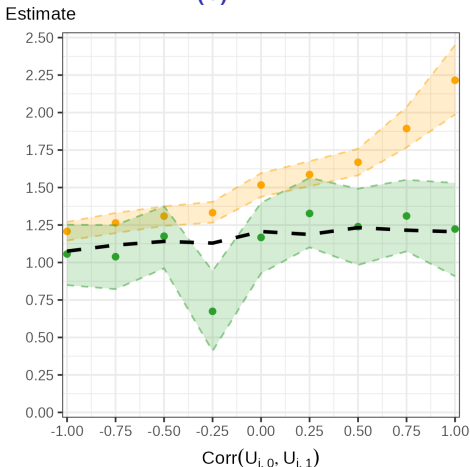
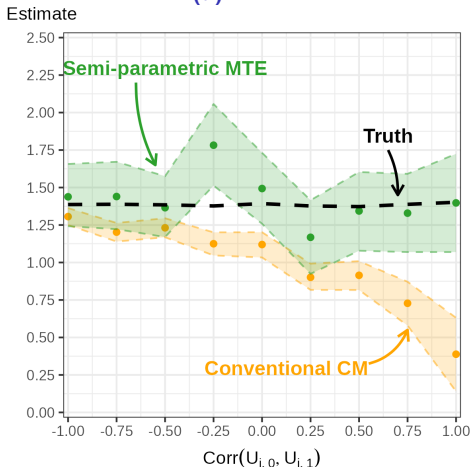
$$\widehat{\text{ADE}} = \mathbb{E}[\hat{\gamma} + \hat{\delta} D_i], \quad \widehat{\text{AIE}} = \mathbb{E}\left[\hat{\pi}\left(\hat{\beta} + \hat{\delta} Z_i + (\hat{\rho}_1 - \hat{\rho}_0) \Gamma(\hat{\pi}(0; \mathbf{X}_i), \hat{\pi}(1; \mathbf{X}_i))\right)\right]$$

Appendix: CM with Selection — Estimation

Figure: CF Adjusted Estimates Work with Different Error Term Parameters.

(a) ADE.

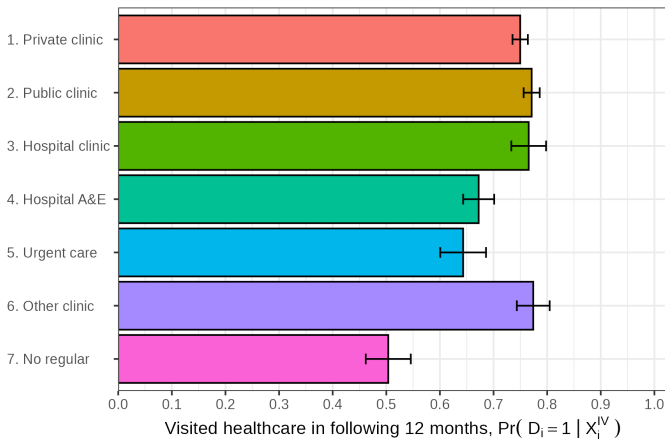
(b) AIE.



Appendix: OHIE IV

IV first-stage F stat. is 124, for all categories (minus base).

Usual Healthcare Location



Structural estimate of mediator compliers' $D_i \rightarrow Y_i$ is +32.9pp (4.4).