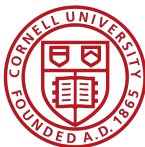


# Causal Mediation in Natural Experiments

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# Introduction

## Motivation:

- Causal Mediation (CM) is a framework for studying mechanisms of causal effects, but is not widely used in economics
- Economists stick to vague suggestive evidence, leaving mechanisms relatively under-studied.

## Contribution:

- Develop selection bias concept for CM, crystallising reasoning for economists dismissal of these methods
- Develop a Marginal Treatment Effect (MTE) approach to credibly estimate causal channels (direct and indirect effects).

## Key findings:

- Apply methods to the Oregon Health Insurance Experiment, studying mediating role of healthcare take-up
- Bring CM into economic quasi-experimental causal inference.

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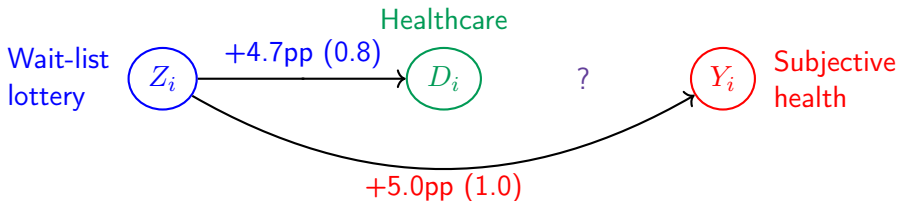
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# 1. Suggestive Evidence

In 2008, Oregon gave access to socialised health insurance by wait-list lottery (Finkelstein et al, 2012).

**Figure:** Model for Suggestive Evidence of a Mechanism.



Necessary but not sufficient evidence on the mediating mechanism:

- Is  $D_i \rightarrow Y_i$  small, large, non-zero?
- Can we assume this causal effect?

## 2. Causal Mediation

Causal Mediation (CM) models the entire system, giving sufficient evidence on the mediating mechanism.



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Average Direct Effect (ADE) and Average Indirect Effect (AIE):

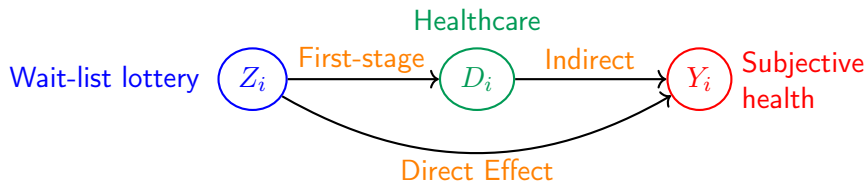
- ADE is causal effect  $Z_i \rightarrow Y_i$
- AIE is causal effect of  $D_i(Z_i) \rightarrow Y_i$ .

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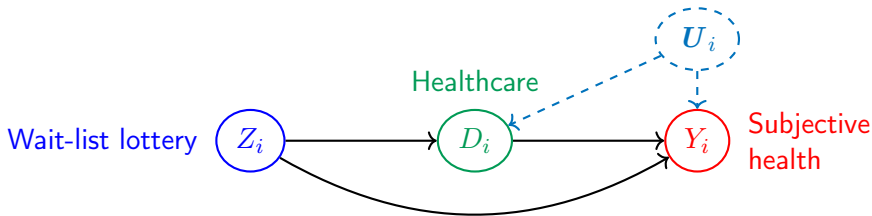
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**Identification:** Requires mediator  $D_i$  is quasi-randomly assigned. . . .

## 2. Causal Mediation — Selection Bias

People chose to visit **healthcare**  $D_i$  freely, not randomly assigned...



Conventional CM analyses are misleading in natural experiment settings — I derive non-parametric selection bias terms for CM.

- ADE:  $\text{CM Estimand} = \text{ADE} + (\text{Selection Bias} + \text{Group difference bias})$
- AIE:  $\text{CM Estimand} = \text{AIE} + (\text{Selection Bias} + \text{Group difference bias})$

► ADE biases

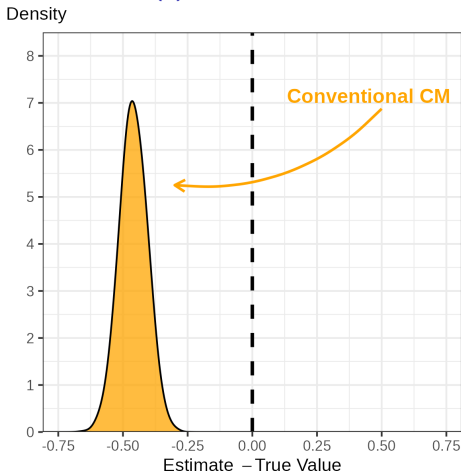
► AIE biases



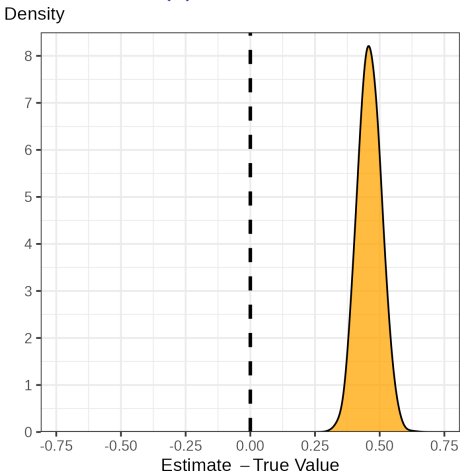
## 2. Causal Mediation — Selection Bias

Simulations with cost-benefit selection-into- $D_i$ , CM estimates are biased.

(a)  $\widehat{ADE} - ADE$ .



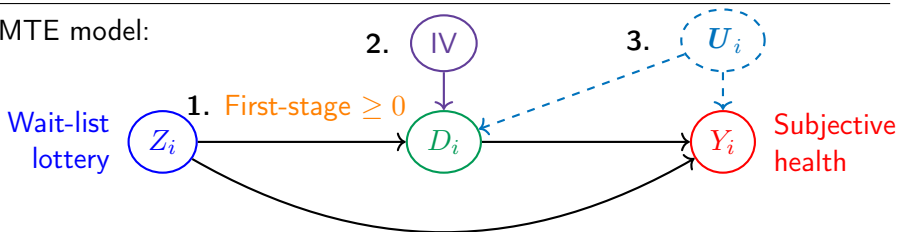
(b)  $\widehat{AIE} - AIE$ .



### 3. MTE Model

Conventional CM methods do not identify ADE + AIE in a natural experiment setting, so I build a Marginal Treatment Effect (MTE) model.

MTE model:



MTE assumptions:

- 1 Mediator monotonicity
- 2 IV for mediator take-up cost
- 3 Selection on mediator benefits.

**Intuition:** model  $U_i$  via mediator MTE to identify ADE + AIE.

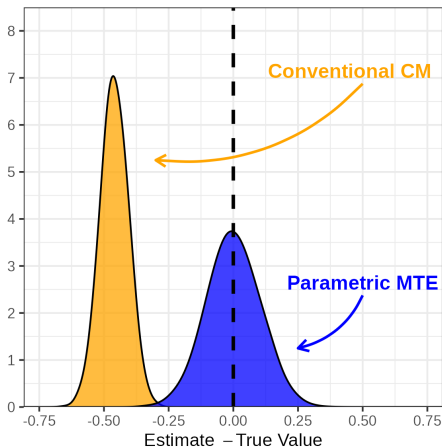
### 3. MTE Model

Simulation with cost-benefit selection-into- $D_i$ , MTE-model corrects for selection bias.

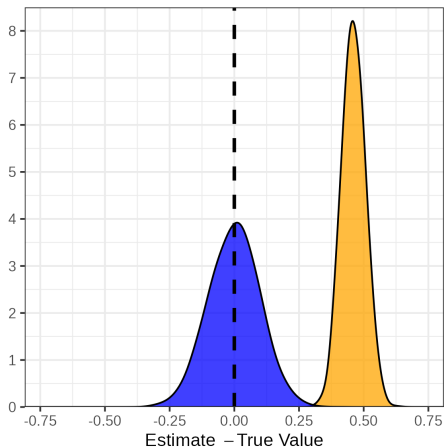
(a)  $\widehat{ADE} - ADE$ .

(b)  $\widehat{AIE} - AIE$ .

Density



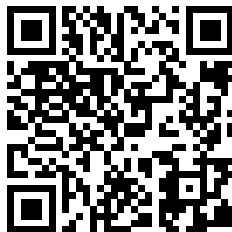
Density



## Conclusion

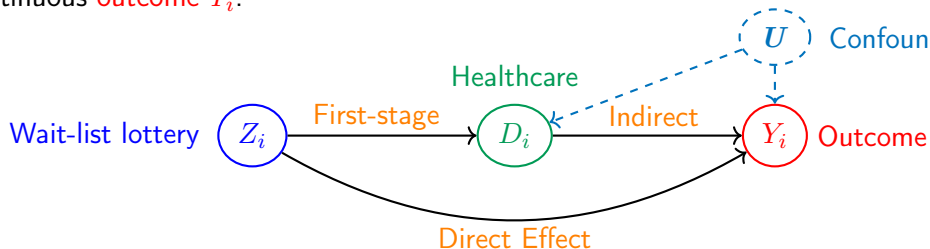
- ① Conventional CM has selection bias in observational (economic) settings
- ② MTE-based model avoids selection bias
- ③ Meaningful indirect (healthcare) and direct (psychological) effects in Oregon Health Insurance Experiment, with wide confidence intervals showing true uncertainty in mechanism explanations.

Access the paper on my website:



# Appendix: CM Guiding Model

Consider binary **treatment**  $Z_i = 0, 1$ , binary **mediator**  $D_i = 0, 1$ , and continuous **outcome**  $Y_i$ .



Average Direct Effect (ADE):  $\mathbb{E} \left[ Y_i \left( \mathbf{1}, D_i(Z_i) \right) - Y_i \left( \mathbf{0}, D_i(Z_i) \right) \right]$

- ADE is causal effect  $Z \rightarrow Y$ , blocking the indirect  $D_i$  path.

Average Indirect Effect (AIE):  $\mathbb{E} \left[ Y_i \left( Z_i, \mathbf{D_i(1)} \right) - Y_i \left( Z_i, \mathbf{D_i(0)} \right) \right]$

- AIE is causal effect of  $D_i(Z_i) \rightarrow Y_i$ , blocking the direct  $Z_i$  path.

# Group Difference — ADE

CM effects contaminated by (less interpretable) bias terms.

$$\text{CM Estimand} = \text{ADEM} + \text{Selection Bias}$$

$$\begin{aligned} & \underbrace{\mathbb{E}_{D_i} \left[ \mathbb{E} [Y_i | Z_i = 1, D_i] - \mathbb{E} [Y_i | Z_i = 0, D_i] \right]}_{\text{Estimand, Direct Effect}} \\ &= \underbrace{\mathbb{E}_{D_i=d'} \left[ \mathbb{E} [Y_i(1, D_i(Z_i)) - Y_i(0, D_i(Z_i)) | D_i(1) = d'] \right]}_{\text{Average Direct Effect on Mediator (ADEM) take-up — i.e., } D_i(1) \text{ weighted}} \\ & \quad + \underbrace{\mathbb{E}_{D_i} \left[ \mathbb{E} [Y_i(0, D_i(Z_i)) | D_i(1) = d'] - \mathbb{E} [Y_i(0, D_i(Z_i)) | D_i(0) = d'] \right]}_{\text{Selection Bias}} \end{aligned}$$

The weighted ADE you get here is a positive weighted sum of local ADEs, but with policy irrelevant weights  $D_i(1) = d'$ .

⇒ consider this group bias, noting difference from true ADE. [▶ Back](#)

# Selection Bias — Direct Effect

CM Effects + contaminating bias.

$$\text{CM Estimand} = \text{ADE} + \left( \text{Selection Bias} + \text{Group difference bias} \right)$$

► Model

$$\begin{aligned} & \underbrace{\mathbb{E}_{D_i=d'} \left[ \mathbb{E} [Y_i \mid Z_i = 1, D_i = d'] - \mathbb{E} [Y_i \mid Z_i = 0, D_i = d'] \right]}_{\text{Estimand, Direct Effect}} \\ &= \underbrace{\mathbb{E} [Y_i(1, D_i(Z_i)) - Y_i(0, D_i(Z_i))]}_{\text{Average Direct Effect}} \\ &+ \underbrace{\mathbb{E}_{D_i=d'} \left[ \mathbb{E} [Y_i(0, D_i(Z_i)) \mid D_i(1) = d'] - \mathbb{E} [Y_i(0, D_i(Z_i)) \mid D_i(0) = d'] \right]}_{\text{Selection Bias}} \\ &+ \underbrace{\mathbb{E}_{D_i=d'} \left[ \left( 1 - \Pr(D_i(1) = d') \right) \right.}_{\text{Group difference bias}} \\ &\quad \times \left. \left( \mathbb{E} [Y_i(1, D_i(Z_i)) - Y_i(0, D_i(Z_i)) \mid D_i(1) = 1 - d'] \right. \right. \\ &\quad \left. \left. - \mathbb{E} [Y_i(1, D_i(Z_i)) - Y_i(0, D_i(Z_i)) \mid D_i(0) = d'] \right) \right]}_{\text{Group-diff}} \end{aligned}$$

# Group Difference — AIE

CM effects contaminated by (less interpretable) bias terms.

$$\text{CM Estimand} = \text{AIEM} + \left( \text{Selection Bias} + \text{Group difference bias} \right)$$

$$\begin{aligned} & \underbrace{\mathbb{E}_{Z_i} \left[ \left( \mathbb{E}[D_i | Z_i = 1] - \mathbb{E}[D_i | Z_i = 0] \right) \times \left( \mathbb{E}[Y_i | Z_i, D_i = 1] - \mathbb{E}[Y_i | Z_i, D_i = 0] \right) \right]}_{\text{Estimand, Indirect Effect}} \\ &= \underbrace{\mathbb{E} \left[ Y_i(Z_i, D_i(1)) - Y_i(Z_i, D_i(0)) \mid D_i = 1 \right]}_{\text{Average Indirect Effect on Mediated (AIEM) — i.e., } D_i = 1 \text{ weighted}} \\ &+ \underbrace{\pi \left( \mathbb{E}[Y_i(Z_i, 0) \mid D_i = 1] - \mathbb{E}[Y_i(Z_i, 0) \mid D_i = 0] \right)}_{\text{Selection Bias}} \\ &+ \underbrace{\pi \left[ \left( \frac{1 - \Pr(D_i(1) = 1, D_i(0) = 0)}{\Pr(D_i(1) = 1, D_i(0) = 0)} \right) \left( \mathbb{E}[Y_i(Z_i, 1) - Y_i(Z_i, 0) \mid D_i(1) = 0 \text{ or } D_i(0) = 1] - \mathbb{E}[Y_i(Z_i, 1) - Y_i(Z_i, 0)] \right) \right]}_{\text{Groups difference Bias}} \end{aligned}$$

The weighted AIE you get here is not a positive weighted sum of local AIEs, because the AIE is only about  $D(Z)$  compliers. [▶ Model](#).

⇒ consider this group bias, noting difference from true AIE.



# Selection Bias — Indirect Effect

CM Effects + contaminating bias, where  $\bar{\pi} = \Pr(D_i(0) \neq D_i(1))$ .

$$\text{CM Estimand} = \text{AIE} + \left( \text{Selection Bias} + \text{Group difference bias} \right) \quad \text{Model}$$

$$\begin{aligned} & \underbrace{\mathbb{E}_{Z_i} \left[ \left( \mathbb{E}[D_i | Z_i = 1] - \mathbb{E}[D_i | Z_i = 0] \right) \times \left( \mathbb{E}[Y_i | Z_i, D_i = 1] - \mathbb{E}[Y_i | Z_i, D_i = 0] \right) \right]}_{\text{Estimand, Indirect Effect}} \\ &= \underbrace{\mathbb{E}[Y_i(Z_i, D_i(1)) - Y_i(Z_i, D_i(0))]}_{\text{Average Indirect Effect}} \\ &+ \underbrace{\bar{\pi} \left( \mathbb{E}[Y_i(Z_i, 0) | D_i = 1] - \mathbb{E}[Y_i(Z_i, 0) | D_i = 0] \right)}_{\text{Selection Bias}} \\ &+ \bar{\pi} \left[ \begin{aligned} & \left( 1 - \Pr(D_i = 1) \right) \left( \mathbb{E}[Y_i(Z_i, 1) - Y_i(Z_i, 0) | D_i = 1] \right. \\ & \quad \left. - \mathbb{E}[Y_i(Z_i, 1) - Y_i(Z_i, 0) | D_i = 0] \right) \\ & + \left( \frac{1 - \Pr(D_i(1) = 1, D_i(0) = 0)}{\Pr(D_i(1) = 1, D_i(0) = 0)} \right) \left( \mathbb{E}[Y_i(Z_i, 1) - Y_i(Z_i, 0) | D_i(Z_i) \neq Z_i] \right. \\ & \quad \left. - \mathbb{E}[Y_i(Z_i, 1) - Y_i(Z_i, 0)] \right) \end{aligned} \right] \end{aligned}$$

Groups difference Bias    ▶ Group-diff

## Semi-parametric Control Functions

Semi-parametric specifications for the CFs  $\lambda_0, \lambda_1$  bring some complications to estimating the AIE.

$$\mathbb{E}[Y_i | Z_i, D_i = 0, \mathbf{X}_i] = \alpha + \gamma Z_i + \varphi(\mathbf{X}_i) + \rho_0 \lambda_0(\pi(Z_i; \mathbf{X}_i)),$$

$$\mathbb{E}[Y_i | Z_i, D_i = 1, \mathbf{X}_i] = (\alpha + \beta) + (\gamma + \delta) Z_i + \varphi(\mathbf{X}_i) + \rho_1 \lambda_1(\pi(Z_i; \mathbf{X}_i))$$

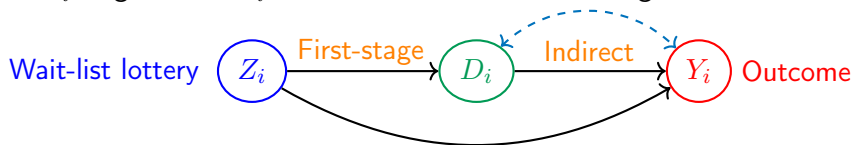
Intercepts,  $\alpha, (\alpha + \beta)$ , and relevance parameters  $\rho_0, \rho_1$  are not separately identified from the CFs  $\lambda_0(\cdot), \lambda_1(\cdot)$  so CF extrapolation term  $(\rho_1 - \rho_0)\Gamma(\pi(0; \mathbf{X}_i), \pi(1; \mathbf{X}_i))$  is not directly identified or estimable.

These problems can be avoided by estimating the AIE using its relation to the ATE,  $\widehat{\text{AIE}}^{\text{CF}} =$

$$\widehat{\text{ATE}} - (1 - \bar{Z}) \underbrace{\left( \frac{1}{N} \sum_{i=1}^N \hat{\gamma} + \hat{\delta} \hat{\pi}(1; \mathbf{X}_i) \right)}_{\widehat{\text{ADE}} \text{ given } Z_i=1} - \bar{Z} \underbrace{\left( \frac{1}{N} \sum_{i=1}^N \hat{\gamma} + \hat{\delta} \hat{\pi}(0; \mathbf{X}_i) \right)}_{\widehat{\text{ADE}} \text{ given } Z_i=0}.$$

## Appendix: CM with Selection

Suppose  $Z_i$  is ignorable,  $D_i$  is not, so we have the following causal model.



Then this system has the following random coefficient equations:

$$D_i = \phi + \pi Z_i + \varphi(\mathbf{X}_i) + U_i$$

$$Y_i = \alpha + \beta D_i + \gamma Z_i + \delta Z_i D_i + \zeta(\mathbf{X}_i) + \underbrace{(1 - D_i) U_{0,i} + D_i U_{1,i}}_{\text{Correlated error term}}$$

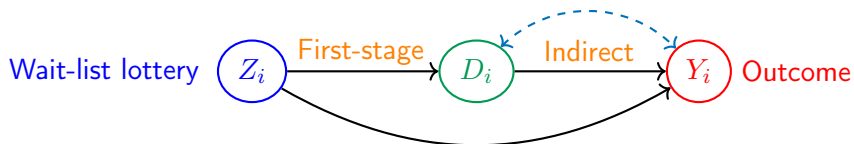
where  $\beta, \gamma, \delta$  are functions of  $\mu_{d'}(z'; \mathbf{X}_i)$ .

$$\text{ADE} = \mathbb{E}[\gamma + \delta D_i], \quad \text{AIE} = \mathbb{E}\left[\pi(\beta + \delta Z_i + \tilde{U}_i)\right]$$

with  $\tilde{U}_i = \mathbb{E}[U_{1,i} - U_{0,i} | \mathbf{X}_i, D_i(0) \neq D_i(1)]$  unobserved complier gains.

## Appendix: CM with Selection

Suppose  $Z_i$  is ignorable,  $D_i$  is not, so we have the following causal model.



Main problem, second-stage is not identified:

$$\begin{aligned} \mathbb{E}[Y_i | Z_i, D_i, \mathbf{X}_i] &= \alpha + \beta D_i + \gamma Z_i + \delta Z_i D_i + \varphi(\mathbf{X}_i) \\ &+ (1 - D_i) \mathbb{E}[U_{0,i} | D_i = 0, \mathbf{X}_i] \\ &+ \underbrace{D_i \mathbb{E}[U_{1,i} | D_i = 1, \mathbf{X}_i]}_{\text{Unobserved } D_i \text{ confounding.}} \end{aligned}$$

**Identification intuition:** Identify second-stage via MTE control function.

## Appendix: CM with Selection — Identification

Assume:

- ① Mediator monotonicity,  $\Pr(D_i(0) \leq D_i(1) | \mathbf{X}_i) = 1$   
 $\implies D_i(z') = \mathbb{1}\{U_i \leq \pi(z'; \mathbf{X}_i)\}, \text{ for } z' = 0, 1 \text{ (Vycatil 2002).}$
- ② Selection on mediator benefits,  $\text{Cov}(U_i, U_{0,i}), \text{Cov}(U_i, U_{1,i}) \neq 0$   
 $\implies$  First-stage take-up informs second-stage confounding.
- ③ There is an IV for the mediator,  $\mathbf{X}_i^{\text{IV}}$  among control variables  $\mathbf{X}_i$ .  
 $\implies \pi(Z_i; \mathbf{X}_i) = \Pr(D_i = 1 | Z_i, \mathbf{X}_i)$  is separately identified.

---

**Proposition:**

$$\begin{aligned} & \mathbb{E}[Y_i(z', 1) - Y_i(z', 0) | Z_i = z', \mathbf{X}_i, U_i = p'] \\ &= \beta + \delta z' + \mathbb{E}[U_{1,i} - U_{0,i} | \mathbf{X}_i, U_i = p'], \quad \text{for } p' \in (0, 1). \end{aligned}$$

## Appendix: CM with Selection — Identification

The marginal effect has corresponding Control Functions (CFs), describing unobserved selection-into- $D_i$ ,

$$\rho_0 \lambda_0(p') = \mathbb{E} [U_{0,i} \mid p' \leq U_i], \quad \rho_1 \lambda_1(p') = \mathbb{E} [U_{1,i} \mid U_i \leq p'] .$$

These CFs restore second-stage identification, by extrapolating from  $\mathbf{X}_i^{IV}$  compliers to  $D_i(Z_i)$  mediator compliers,

$$\begin{aligned} \mathbb{E} [Y_i \mid Z_i, D_i, \mathbf{X}_i] &= \alpha + \beta D_i + \gamma Z_i + \delta Z_i D_i + \varphi(\mathbf{X}_i) \\ &\quad + \underbrace{\rho_0 (1 - D_i) \lambda_0(\pi(Z_i; \mathbf{X}_i)) + \rho_1 D_i \lambda_1(\pi(Z_i; \mathbf{X}_i))}_{\text{CF adjustment.}} \end{aligned}$$

This adjusted second-stage re-identifies the ADE and AIE,

$$\text{ADE} = \mathbb{E} [\gamma + \delta D_i], \quad \text{AIE} = \mathbb{E} \left[ \bar{\pi} \left( \beta + \delta Z_i + \underbrace{(\rho_1 - \rho_0) \Gamma(\pi(0; \mathbf{X}_i), \pi(1; \mathbf{X}_i))}_{\text{Mediator compliers extrapolation.}} \right) \right]$$

## Appendix: CM with Selection — Estimation

Will explain how estimation works, with simulation evidence.

- ① Random treatment  $Z_i \sim \text{Binom}(0.5)$ , for  $n = 5,000$ .
- ②  $(U_{0,i}, U_{1,i}) \sim \text{BivariateNormal}(0, 0, \sigma_0, \sigma_1, \rho)$ , Costs  $C_i \sim N(0, 0.5)$ .

Roy **selection-into- $D_i$** , with constant partial effects + interaction term.

$$D_i(z') = \mathbb{1} \{C_i \leq Y_i(z', 1) - Y_i(z', 0)\},$$

$$Y_i(z', d') = (z' + d' + z'd') + U_{d'} \quad \text{for } z', d' = 0, 1.$$

Following the previous, these data have the following first and second-stage equations, where  $\mathbf{X}_i^{\text{IV}}$  is an additive cost IV:

$$D_i = \mathbb{1} \left\{ C_i - (U_{1,i} - U_{0,i}) \leq Z_i - \mathbf{X}_i^{\text{IV}} \right\}$$

$$Y_i = Z_i + D_i + Z_i D_i + (1 - D_i) U_{0,i} + D_i U_{1,i}.$$

$\implies$  unobserved confounding by  $\text{BivariateNormal}(U_{0,i}, U_{1,i})$ .

## Appendix: CM with Selection — Estimation

Errors are normal, so system is Heckman (1979) selection model.

CFs are the inverse Mills ratio, with  $\phi(\cdot)$  normal pdf and  $\Phi(\cdot)$  normal cdf,

$$\lambda_0(p') = \frac{\phi(-\Phi^{-1}(p'))}{\Phi(-\Phi^{-1}(p'))}, \quad \lambda_1(p') = \frac{\phi(\Phi^{-1}(p'))}{\Phi(\Phi^{-1}(p'))}, \quad \text{for } p' \in (0, 1).$$

### Parametric Estimation Recipe:

- ① Estimate first-stage  $\pi(Z_i; \mathbf{X}_i)$  with probit, including  $\mathbf{X}_i^{\text{IV}}$ .
- ② Include  $\lambda_0, \lambda_1$  CFs in second-stage OLS estimation.
- ③ Compose CM estimates from two-stage plug-in estimates.

→ Same as conventional CM estimates (two-stages), with CFs added.

$$\widehat{\text{ADE}} = \mathbb{E} \left[ \widehat{\gamma} + \widehat{\delta} D_i \right], \quad \widehat{\text{AIE}} = \mathbb{E} \left[ \widehat{\pi} \left( \widehat{\beta} + \widehat{\delta} Z_i + \underbrace{(\widehat{\rho}_1 - \widehat{\rho}_0) \Gamma(\widehat{\pi}(0; \mathbf{X}_i), \widehat{\pi}(1; \mathbf{X}_i))}_{\text{Mediator compliers extrapolation.}} \right) \right]$$



## Appendix: CM with Selection — Estimation

If errors are not normal, then CFs do not have a known form, so semi-parametrically estimate them (e.g., splines).

$$\mathbb{E}[Y_i | Z_i, D_i = 0, \mathbf{X}_i] = \alpha + \gamma Z_i + \varphi(\mathbf{X}_i) + \rho_0 \lambda_0(\pi(Z_i; \mathbf{X}_i)),$$

$$\mathbb{E}[Y_i | Z_i, D_i = 1, \mathbf{X}_i] = (\alpha + \beta) + (\gamma + \delta) Z_i + \varphi(\mathbf{X}_i) + \rho_1 \lambda_1(\pi(Z_i; \mathbf{X}_i))$$

### Semi-parametric Estimation Recipe:

- ① Estimate first-stage  $\pi(Z_i; \mathbf{X}_i)$ , including  $\mathbf{X}_i^{IV}$ .
- ② Estimate second-stage separately for  $D_i = 0$  and  $D_i = 1$ , with regressors  $\lambda_0(p')$ ,  $\lambda_1(p')$ , semi-parametric in  $\hat{\pi}(Z_i; \mathbf{X}_i)$ .
- ③ Compose CM estimates from two-stage plug-in estimates.

→ Same as conventional CM estimates, with semi-parametric CFs. ▶ CFs.

$$\widehat{\text{ADE}} = \mathbb{E}[\hat{\gamma} + \hat{\delta} D_i], \quad \widehat{\text{AIE}} = \mathbb{E}\left[\hat{\pi}\left(\hat{\beta} + \hat{\delta} Z_i + (\hat{\rho}_1 - \hat{\rho}_0) \Gamma(\hat{\pi}(0; \mathbf{X}_i), \hat{\pi}(1; \mathbf{X}_i))\right)\right]$$

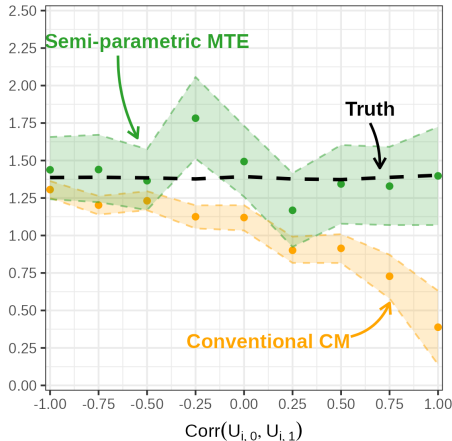
# Appendix: CM with Selection — Estimation

**Figure:** CF Adjusted Estimates Work with Different Error Term Parameters.

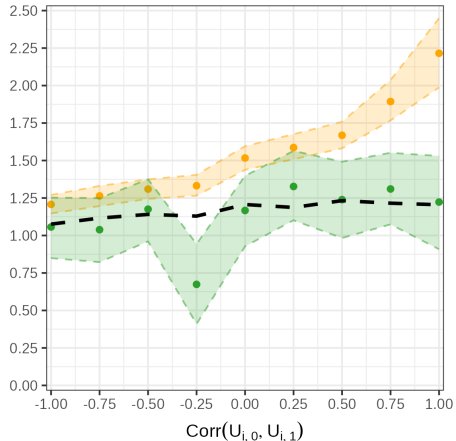
(a) ADE.

(b) AIE.

Estimate



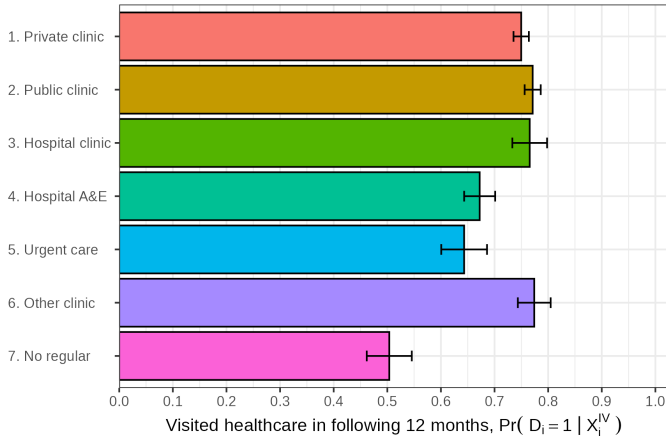
Estimate



# Appendix: OHIE IV

IV first-stage F stat. is 124, for all categories (minus base).

Usual Healthcare Location



Structural estimate of mediator compliers'  $D_i \rightarrow Y_i$  is +32.9pp (4.4).