SVM

$x_1 = 1$	$y_1 = 1$
$x_2 = 3$	$y_2 = -1$

Find the output value of $z = 2$

- Use polynomial kernel

$$K(x, y) = (x^T y + 1)^d$$

- set kernel parameter, $d = 1$ (degree 1)
  - $K(x, y) = (xy + 1) \Rightarrow K(x_i, x_j) = (x_i x_j + 1)$
  - C is set to 5
- Value of α_i determination where $i = 1$ to 2, $j = 1$ to 2

$$\max\left[\sum_{i=1}^{2} \alpha_i - \frac{1}{2} \sum_{i=1}^{2} \sum_{j=1}^{2} \alpha_i \alpha_j y_i y_j (x_i x_j + 1)\right]$$

subject to the constraints

$$5 \geq \alpha_i \geq 0 \quad \text{and} \quad \sum_{i=1}^{2} \alpha_i y_i = 0$$

Here, $\max\left[\sum_{i=1}^{2} \alpha_i - \frac{1}{2} \sum_{i=1}^{2} \sum_{j=1}^{2} \alpha_i \alpha_j y_i y_j (x_i x_j + 1)\right]$

$$= \max\Big[(\alpha_1 + \alpha_2) - \frac{1}{2}\Big[\alpha_1 \alpha_1 y_1 y_1 (x_1 x_1 + 1) + \alpha_1 \alpha_2 y_1 y_2 (x_1 x_2 + 1) + \alpha_2 \alpha_1 y_2 y_1 (x_2 x_1 + 1) + \alpha_2 \alpha_2 y_2 y_2 (x_2 x_2 + 1)\Big]\Big]$$

[Put $x_1 = 1$, $x_2 = 3$, $y_1 = 1$, $y_2 = -1$]

$$= \max\left[-\alpha_1^2 + 5\alpha_1\alpha_2 - 5\alpha_2^2\right]$$

subject to $\displaystyle\sum_{i=1}^{2} \alpha_i y_i = 0$

$$\Rightarrow \alpha_1 y_1 + \alpha_2 y_2 = 0$$

$$\Rightarrow \alpha_1 \cdot 1 + \alpha_2 \cdot (-1) = 0$$

$$\Rightarrow \alpha_1 - \alpha_2 = 0$$

and $0 \leqslant \alpha_i \leqslant C$

$$\Rightarrow 0 \leqslant \alpha_i \leqslant 5$$

$\therefore$ maximize $-\alpha_1^2 + 5\alpha_1\alpha_2 - 5\alpha_2^2$

subject to $\alpha_1 - \alpha_2 = 0$

and $0 \leqslant \alpha_i \leqslant 5$

Let, $f(\alpha_1, \alpha_2) = -\alpha_1^2 + 5\alpha_1\alpha_2 - 5\alpha_2^2$

$g(\alpha_1, \alpha_2) = \alpha_1 - \alpha_2$

Assume, Lagranges multiplier, λ $[\lambda \neq 0]$

$$\frac{\partial f}{\partial \alpha_1} + \lambda \frac{\partial g}{\partial \alpha_1} = 0$$

$$\frac{\partial f}{\partial \alpha_2} + \lambda \frac{\partial g}{\partial \alpha_2} = 0$$

$$\frac{\partial f}{\partial \alpha_1} = \frac{\partial}{\partial \alpha_1}\left(-\alpha_1^2 + 5\alpha_1 \alpha_2 - 5\alpha_2^2\right) = -2\alpha_1 + 5\alpha_2$$

$$\frac{\partial g}{\partial \alpha_1} = \frac{\partial}{\partial \alpha_1}\left(\alpha_1 - \alpha_2\right) = 1$$

Again,

$$\frac{\partial f}{\partial \alpha_2} = \frac{\partial}{\partial \alpha_2}\left(-\alpha_1^2 + 5\alpha_1 \alpha_2 - 5\alpha_2^2\right) = 5\alpha_1 - 10\alpha_2$$

$$\frac{\partial g}{\partial \alpha_2} = \frac{\partial}{\partial \alpha_2}\left(\alpha_1 - \alpha_2\right) = -1$$

$$\frac{\partial f}{\partial \alpha_1} + \lambda \frac{\partial g}{\partial \alpha_1} = 0$$

$$\Rightarrow -2\alpha_1 + 5\alpha_2 + \lambda \cdot 1 = 0$$

$$\Rightarrow 2\alpha_1 - 5\alpha_2 - \lambda = 0 \quad \ldots \text{eq } (\mathrm{I})$$

$$\frac{\partial f}{\partial \alpha_2} + \lambda \frac{\partial g}{\partial \alpha_2} = 0$$

$$\Rightarrow 5\alpha_1 - 10\alpha_2 + \lambda \cdot (-1) = 0$$

$$\Rightarrow 5\alpha_1 - 10\alpha_2 - \lambda = 0 \quad \ldots \text{eq } (\mathrm{II})$$

eq (II) – eq (I) $\Rightarrow$

$$3\alpha_1 - 5\alpha_2 = 0$$

$$\Rightarrow \alpha_1 = \frac{5}{3}\alpha_2$$

from eq (I) $\Rightarrow$

$$2\alpha_1 - 5\alpha_2 - \lambda = 0$$

$$\Rightarrow 2 \cdot \frac{5}{3}\alpha_2 - 5\alpha_2 - \lambda = 0$$

$$\Rightarrow -\frac{5}{3}\alpha_2 - \lambda = 0$$

$$\Rightarrow \alpha_2 = -\frac{3}{5}\lambda$$

$$\therefore \alpha_1 = \frac{5}{3} * \left(-\frac{3}{5}\lambda\right) = -\lambda$$

$\therefore\ \alpha_1 = -\lambda$

$\alpha_2 = -\frac{3}{5}\lambda$

if $\lambda = -c$ $\quad \alpha_1 = -(-5) = 5$ $\quad$ Since $\alpha_1 - \alpha_2 = 0$ $\Rightarrow \alpha_2 = 5$

$= -5,$ $\quad \alpha_2 = -\frac{3}{5}\lambda$ $\quad$ Since $\alpha_1 - \alpha_2 = 0$ $\Rightarrow \alpha_1 = 3$

$= -\frac{3}{5}(-5)$

$= 3$

$$\boxed{\begin{aligned}\alpha_1 &= 5, 3\\ \alpha_2 &= 5, 3\end{aligned}}$$ solution

Additional

Again

From eq (11) $\Rightarrow$

$5\alpha_1 - 10\alpha_2 - \lambda = 0$

$\Rightarrow 5 . \frac{5}{3}\alpha_2 - 10\alpha_2 - \lambda = 0$

$\Rightarrow \frac{25}{3}\alpha_2 - 10\alpha_2 - \lambda = 0$

$\Rightarrow -\frac{5}{3}\alpha_2 - \lambda = 0$

$\Rightarrow \alpha_2 = -\frac{3}{5}\lambda$ $\quad$ where $\alpha_1 = \frac{5}{3}\alpha_2$

$= \frac{5}{3} * \left(-\frac{3}{5}\lambda\right)$

$= -\lambda$

For $\lambda = -c$, $\alpha_1 = -\lambda = c = 5$

$\alpha_2 = -\frac{3}{5}\lambda = -\frac{3}{5} * (-5)$

$= 3$

$$\boxed{\begin{aligned}\alpha_1 &= 5\\ \alpha_2 &= 3\end{aligned}}$$ solution

data, $x=2$,

$\cancel{f(x) = -4x+5}$

$\cancel{\Rightarrow f(x) = -4 \times 2 + 5}$

$\cancel{= -8+5}$

$\cancel{= -3}$

~~= Negative class~~

~~$\therefore$ for, $x=2$, output class = -1~~

$f(\alpha_1, \alpha_2) =$

$f(\alpha_1, \alpha_2) = -\alpha_1^2 + 5\alpha_1\alpha_2 - 5\alpha_2^2$

$= -\alpha_1^2 + 5\alpha_1^2 - 5\alpha_1^2 \qquad [\alpha_1 = \alpha_2]$

$= -\alpha_1^2$

$= -5^2 \qquad [\alpha_1 = 5]$

$= -25$

Again, $f(\alpha_1, \alpha_2) = -\alpha_1^2 + 5\alpha_1\alpha_2 - 5\alpha_2^2$

$= -\alpha_1^2$

$= -9 \qquad [\alpha_1 = 3]$

Here, maximum value is -9 for $\alpha_1 = 3$

So, $\alpha_1 = 3$, $\alpha_2 = 3$

$$\boxed{\begin{array}{l} \alpha_1 = 3 \\ \alpha_2 = 3 \end{array}}$$

Testing

$x_1 = 1 \quad y_1 = 1 \quad \alpha_1 = 3$

$x_2 = 3 \quad y_2 = -1 \quad \alpha_2 = 3$

Discreminant function,

$$f(z) = \sum_{j=1}^{s} \alpha_{t_j} y_{t_j} k(x_{t_j} \cdot z) + b$$

$$= \sum_{j=1}^{2} \alpha_{t_j} y_{t_j} k(x_{t_j} \cdot z) + b$$

$$= \alpha_1 y_1 (x_1 z + 1) + \alpha_2 y_2 (x_2 z + 1) + b$$

$$= 3 * 1 (1 \cdot z + 1) + 3 * (-1)(3z + 1) + b$$

$$= 3z + \cancel{3} - 9z - \cancel{3} + b$$

$$= -6z + b$$

if $z = x_1 = 1$

$$f(x_1) = -6x_1 + b$$

$$\Rightarrow y_1 = -6 * 1 + b$$

$$\Rightarrow 1 = -6 + b$$

$$\Rightarrow b = 7$$

$\therefore$ Discreminant function, $f(z) = -6z + 7$

· New data, $z = 2$, $f(z) = -6z + 7$

$$= -6 * 2 + 7$$

$$= -12 + 7$$

$$= -5 = \text{Negative class}$$

For test data 2, output class $= -1$