Mid Semester (2022) - ITC

1 (a) $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

R in $A \times A$

$(m, n) R (p, q)$ if $m + q = n + p$

$\Rightarrow m - n = p - q$

For reflexive :- If $(x, y) R (x, y)$, then reflexive

So, $x - y = x - y$ is true.

So, R is reflexive.

For symmetric :- ~~$(x,y) R (y,x)$~~

If $(a, b) R (c, d) \Rightarrow (c, d) R (a, b)$, then symmetric

So, $a - b = c - d$

$\Rightarrow c - d = a - b \Rightarrow (c, d) R (a, b)$.

So, R is symmetric.

For transitive :-

Let $(a, b) R (c, d)$ and $(c, d) R (e, f)$

$\Rightarrow a - b = c - d$ $\qquad \Rightarrow c - d = e - f$

$\Rightarrow a - b = e - f \Rightarrow (a, b) R (e, f)$

So, R is transitive.

Hence, the Relation R is equivalence relation.

(b) To prove :- $\sqrt{5}$ is irrational. (Using contradiction)

Proof :- Let $\sqrt{5}$ be a rational number.

then, $\sqrt{5} = \frac{p}{q}$; $q \neq 0$; p, q are coprime.

$\Rightarrow q\sqrt{5} = p$

$\Rightarrow q^2 \times (\sqrt{5})^2 = p^2$

$\Rightarrow 5q^2 = p^2$ ——— (1)

So, p^2 is divisible by 5

$\Rightarrow p$ is divisible by 5

So, $p = 5c$

$\Rightarrow p^2 = 25c$

$\Rightarrow 5q^2 = 25c$ —— from equ ①

$\Rightarrow q^2 = 5c$

So, q^2 is divisible by 5

$\Rightarrow q$ is divisible by 5

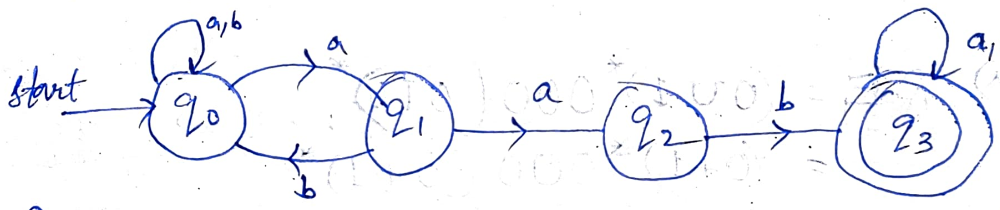
As, p and q both are divisible by 5,
they are not co-prime.

So, our assumption was wrong.

Hence, $\sqrt{5}$ is not rational number.

So, $\sqrt{5}$ is an irrational number. [proved]

(c) $\delta : Q \times \Sigma \rightarrow P(Q)$



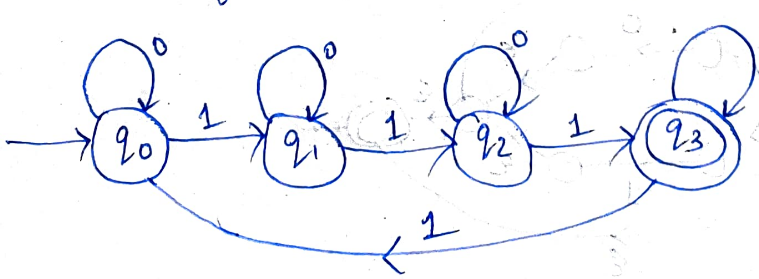
Domain of $\delta = \{(q_0, a)(q_0, b), (q_1, a), (q_1, b), (q_2, b), (q_3, a), (q_3, b)\}$

Range of $\delta = \{q_0, q_1, q_2, q_3\}$

2.(a) $\Sigma = \{0, 1\}$

$L = \{w \mid w$ has no. of 1's equal to $3 \% 4\}$

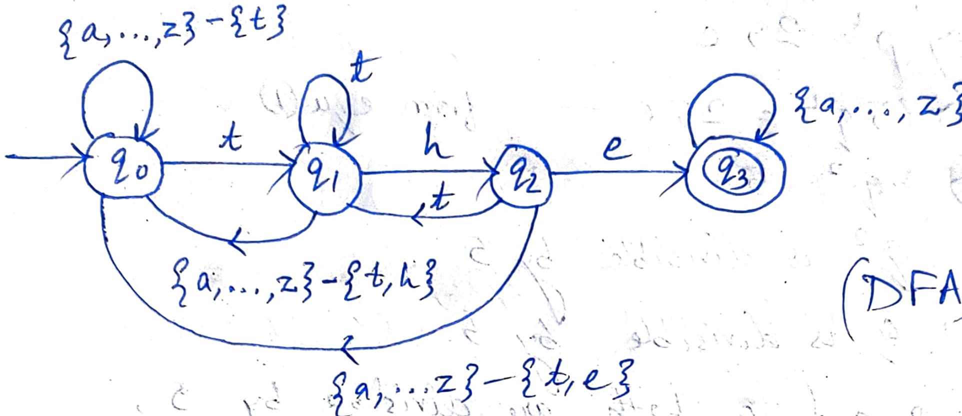
So, no. of 1's can be $\{3, 7, 11, \ldots\}$



(DFA)

(b) $\Sigma = \{a, b, c, \ldots, z\}$

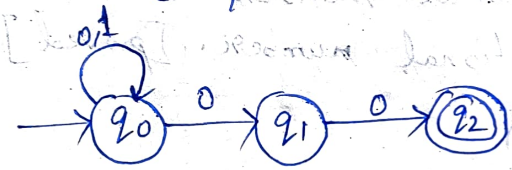
$L = \{w \mid w$ has substring 'the'$\}$



(DFA)

(c) $\Sigma = \{0, 1\}$

$L = \{w \mid w$ ends with $00\}$



(NFA)

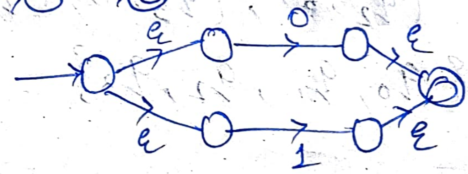
3.(a) $R.E = (0 \cup 1)^* 000 (0 \cup 1)^*$

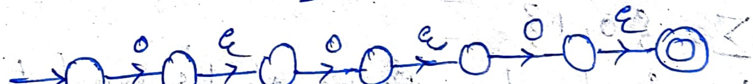
$= (0+1)^* 000 (0+1)^*$

0 :- 

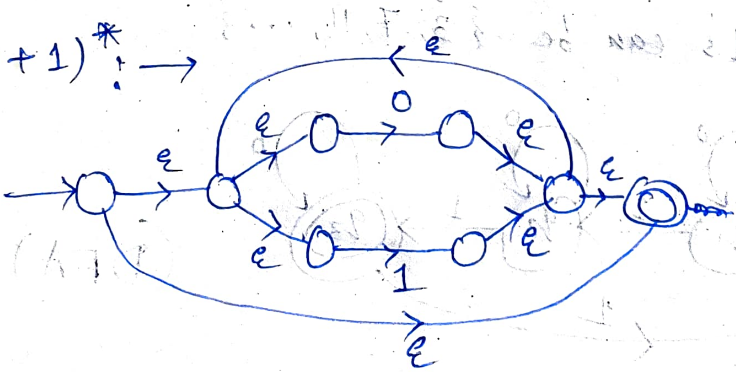
ε :- 

1 :- 

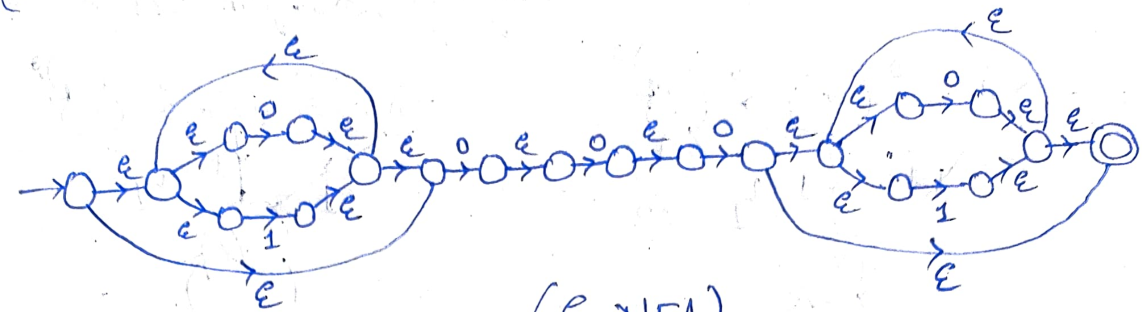
0+1 :- 

000 :- 

$(0+1)^*$:-

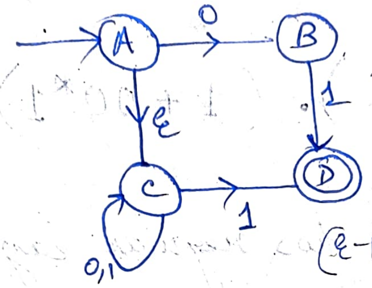


$(0+1)^*000(0+1)^*$:-

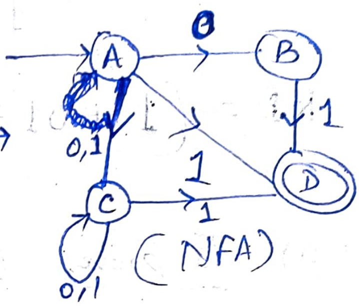


(ε-NFA)

(b) Given :-



(ε-NFA)

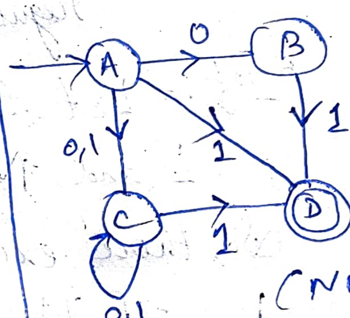


(NFA)

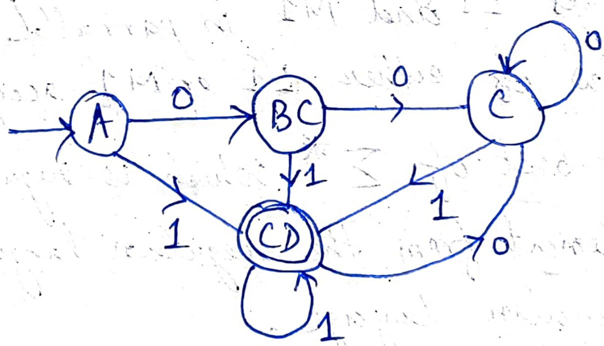
δ :-

	0	1	0	1
[A]	B,C	C,D	BC	CD
[BC]	ϕC	DCD	C	CD
[C]	C	CD	C	CD
[CD]	Cϕ	CDϕ	C	CD
[B]	ϕ	D	–	D
[D]	ϕ	ϕ		

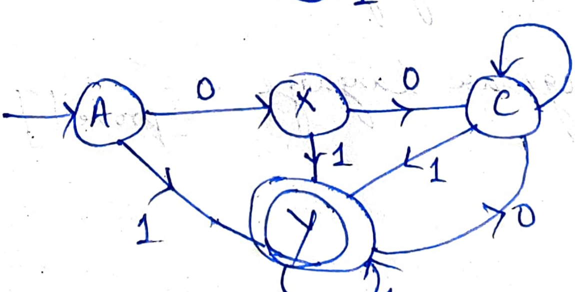
ignored



(NFA)

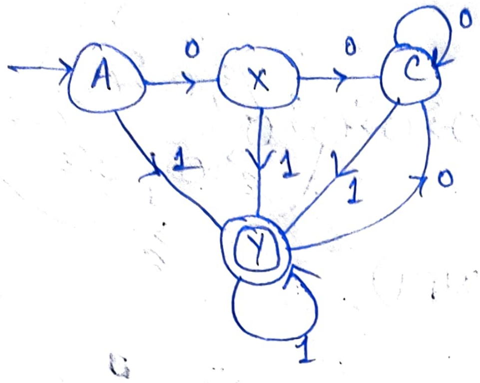


We can, replace BC with X and CD with Y

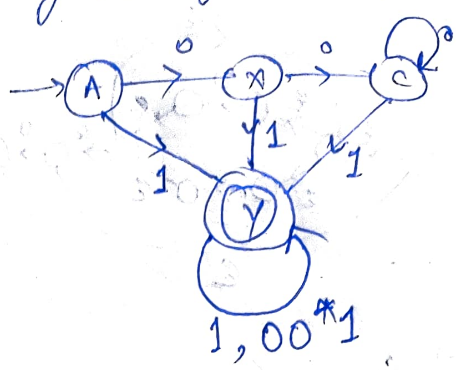


(DFA)

(c) The DFA accepts the language, L from R.E;



$\longrightarrow$



$$R.E = (1 + 01 + 000^*1).(1 + 00^*1)^*$$

4. (a) <u>To prove:</u>- Union of two regular language is a regular language

proof:- Let there are two regular languages, L and M.

So, there exists a DFA satisfying L, M.

$L_1 = (Q1, \Sigma, \delta 1, q_0 1, F1) \rightarrow$ satisfying L

$M_1 = (Q2, \Sigma, \delta 2, q_0 2, F2) \rightarrow$ satisfying M

We construct a new DFA, N which ~~rec~~ recognizes $L \cup M$ by taking cross product of L_1, M_1.

Now, N simulates $L1$ and $M1$ in parrallel and accepts a string w if either $L1$ or $M1$ accepts w.

Now, w is a subset of Σ^* which is regular.

As, w is the element from the regular language then, N is a regular langauge.

Hence, $L \cup M$ is regular language. [proved].

(b) Language, $A = \{0^n 1^n 2^n \mid n \geq 0\}$

To prove :- A is not regular language

Proof :- (proof by contradiction)

Let A be a regular language

Then, according to the pumping lemma;

$\exists\, xyz \in A$, such that $|y| > 0$, $|xy| \leq p$, p is pumping length

then, for each $i \geq 0$,

$xy^iz \in A$.

Let string $s = 012$; $s \in A$.

now, $x = 0$, $y = 1$, $z = 2$.

taking $i = 2$; new string $= xy^2z$
$= 0112 \notin A$

So, it doesn't satisfy the pumping lemma.

Hence, A is not a regular language [proved]

(c) $E \to E - T \mid T$

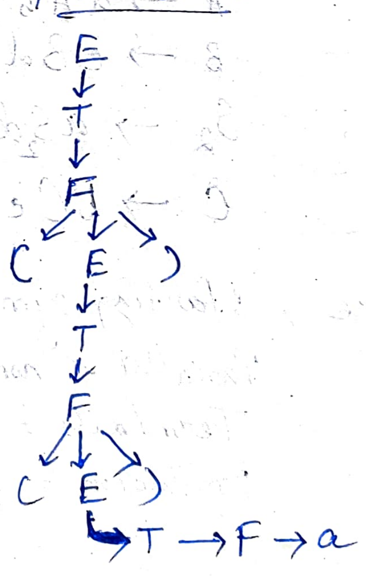
$T \to T/F \mid F$

$F \to (E) \mid a$

For string "((a))" :-

$E \to T$
$\to F$
$\to (E)$
$\to (T)$
$\to (F)$
$\to ((E))$
$\to ((T))$
$\to ((F))$
$\to ((a))$

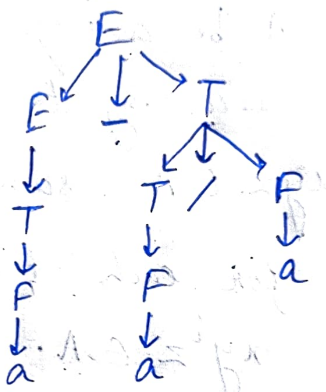
Parse tree :-



For string "a − a/a" :-

$E \rightarrow E - T$
$\rightarrow T - T$
$\rightarrow F - T$
$\rightarrow a - T$
$\rightarrow a - T/F$
$\rightarrow a - F/F$
$\rightarrow a - a/F$
$\rightarrow a - a/a$

<u>parse tree</u> :-



5. (a) $L = \{a^n b^n c^m d^m \mid n, m \geq 0\} \cup \{a^n b^m c^m d^n \mid n, m \geq 0\}$

$L = L1 \cup L2$

$L1 = \{a^n b^n c^m d^m \mid n, m \geq 0\}$

$L2 = \{a^n b^m c^m d^n \mid n, m \geq 0\}$

L1 :- $S \rightarrow AB$
$A \rightarrow aAb \mid \epsilon$
$B \rightarrow cBd \mid \epsilon$

L2 :- $S \rightarrow aSd \mid A \mid \epsilon$
$A \rightarrow bAc \mid \epsilon$

L :- $S_0 \rightarrow S_1 \mid S_2$
$S_1 \rightarrow AB$
$A \rightarrow aAb \mid \epsilon$
$B \rightarrow cBd \mid \epsilon$
$S_2 \rightarrow aS_2d \mid C \mid \epsilon$
$C \rightarrow bCc \mid \epsilon$

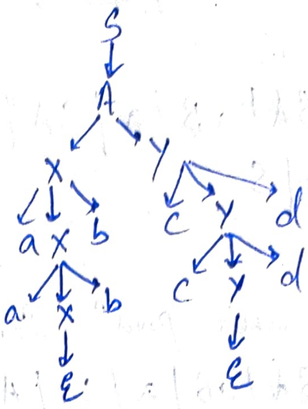
$\rightarrow$

$S \rightarrow A \mid B$
$A \rightarrow XY$
$X \rightarrow aXb \mid \epsilon$
$Y \rightarrow cYd \mid \epsilon$
$B \rightarrow aBd \mid Z \mid \epsilon$
$Z \rightarrow bZc \mid \epsilon$

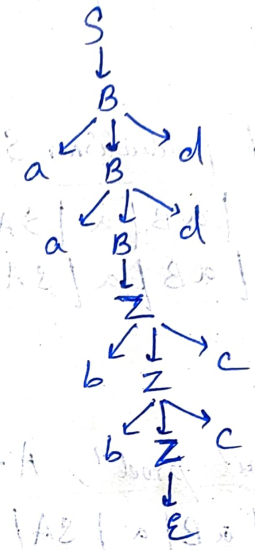
Here, starting symbol = S
Variables or non-terminals = $\{S, A, X, Y, B, Z\}$
Terminals = $\{a, b, c, d, \epsilon\}$
Production = $\{S \rightarrow A, S \rightarrow B, A \rightarrow XY, X \rightarrow aXb, X \rightarrow \epsilon, Y \rightarrow cYd, Y \rightarrow \epsilon, B \rightarrow aBd, B \rightarrow Z, B \rightarrow \epsilon, Z \rightarrow bZc, Z \rightarrow \epsilon\}$

(b) $w = aabbccdd$

$S \rightarrow A$
$\rightarrow XY$
$\rightarrow aXbY$
$\rightarrow aaXbbY$
$\rightarrow aa\varepsilon bbY$
$\rightarrow aabbcYd$
$\rightarrow aabbccYdd$
$\rightarrow aabbcc\varepsilon dd$
$\rightarrow aabbccdd$



$S \rightarrow B$
$\rightarrow aBd$
$\rightarrow aaBdd$
$\rightarrow aaZdd$
$\rightarrow aabZcdd$
$\rightarrow aabbZccdd$
$\rightarrow aabb\varepsilon ccdd$
$\rightarrow aabbccdd$



Since, this string has more than one derivation & parse tree, this grammar is ambigeous. (Ans)

(c) $S \rightarrow ASA \mid aB$
$A \rightarrow B \mid S$
$B \rightarrow b \mid \varepsilon$

Step-1 :- $S_0 \rightarrow S$
$S \rightarrow ASA \mid aB$
$A \rightarrow B \mid S$
$B \rightarrow b \mid \varepsilon$

Step-2 :- Eliminate null prodn, $B \rightarrow \varepsilon$:-
$S_0 \rightarrow S$
$S \rightarrow ASA \mid aB \mid a$
$A \rightarrow B \mid S \mid \varepsilon$
$B \rightarrow b$

Eliminate null prodn, $A \to \varepsilon$:-

$S_0 \to S$

$S \to ASA \mid aB \mid a \mid SA \mid AS \mid S$

$A \to B \mid S$

$B \to b$

Eliminate unit prodn ; $S_0 \to S$:-

$S_0 \to ASA \mid aB \mid a \mid SA \mid AS \mid S$

$S \to ASA \mid aB \mid a \mid SA \mid AS \mid S$

$A \to B \mid S$

$B \to b$

Eliminate unit productions, $S_0 \to S$, $S \to S$:-

$S_0 \to ASA \mid aB \mid a \mid SA \mid AS$

$S \to ASA \mid aB \mid a \mid SA \mid AS$

$A \to B \mid S$

$B \to b$

Eliminate unit prodn, $A \to B$, $A \to S$:-

$S_0 \to ASA \mid aB \mid a \mid SA \mid AS$

$S \to ASA \mid aB \mid a \mid SA \mid AS$

$A \to b \mid ASA \mid aB \mid a \mid SA \mid AS$

$B \to b$

Step-3 :- Eliminate, $S_0 \to aB$, $S \to aB$, $A \to aB$:-

$S_0 \to ASA \mid XB \mid SA \mid AS \mid a$ (∵ Replacing 'a' with X)

$X \to a$ (adding)

$S \to ASA \mid XB \mid SA \mid AS \mid a$

$A \to ASA \mid XB \mid SA \mid AS \mid a \mid b$

$B \to b$

$X \to a$

Step-4 :- Eliminate prodn with more than two non-terminal on RHS :-

(Taking 'AS' as M, adding $M \rightarrow AS$)

$S_0 \rightarrow MA \mid XB \mid SA \mid AS \mid a$

$S \rightarrow MA \mid XB \mid SA \mid AS \mid a$

$A \rightarrow MA \mid XB \mid SA \mid AS \mid a \mid b$

$B \rightarrow b$

$X \rightarrow a$

$M \rightarrow AS$

This is the Chomsky Normal Form of given CFG.

15/12/22

MID-SEMESTER EXAMINATION, December-2022
Theory of Computation (CSE3031)

Programme: B.Tech. **Semester: V**
Full Marks: 30 **Time: 2 Hours**

Subject/Course Learning Outcome	*Taxonomy Level	Ques. Nos.	Marks
Able to enhance/develop the ability to understand and conduct mathematical proofs for computation and algorithms.	L1, L2	1. a),b),c)	6
Able to design and analyze finite automata, and regular expression for describing regular languages.	L1, L3, L5, L6	2. a),b),c), 3. a),b),c) 4. b)	14
Design and analyze pushdown automata, and context-free grammars.	L1, L2, L3, L5	4. c) 5. a),b),c)	8
Design and analyze Turing machines.			
Enhance the ability to understand the decidability criteria of various computational problems.			
Demonstrate the understanding of key notions, such as algorithm, computability and complexity through problem solving.	L1,L6	4. a)	2

*Bloom's taxonomy levels: Knowledge (L1), Comprehension (L2), Application (L3), Analysis (L4), Evaluation (L5), Creation (L6)

Answer all questions. Each question carries equal mark.

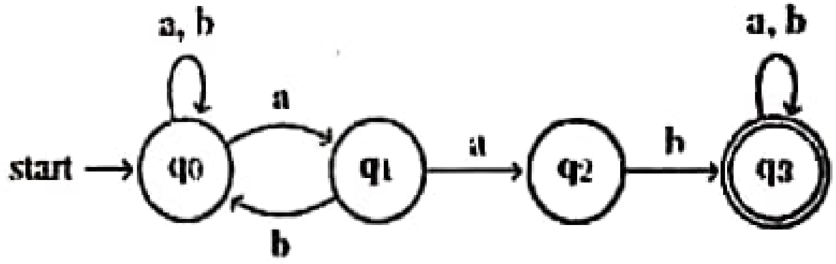
1. a) Suppose A= {1,2,3,4,5,6,7,8,9} and R is a relation on A × A is defined by (m, n) R (p, q) if m + q = n + p, then show that R is an equivalence relation. **2**

Try

b) A number is rational if it is the ratio of two integers m and n, (n ≠ 0). A number is *irrational* if it is not rational. Apply method of contradiction to prove that √5 is irrational. **2**

Try

c) The transition function for a nondeterministic finite automaton (NFA) can be defined as $\delta: Q \times \Sigma \rightarrow P(Q)$. Find the domain and range of δ for the following finite automaton **2**



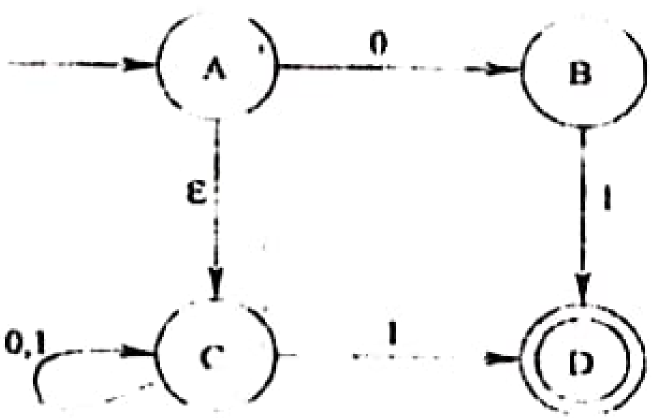
2. a) Construct a DFA accepting all strings w over $\Sigma = \{0,1\}$ such that the number of 1's in w is 3 mod 4. ≥ 3 **2**

b) Build a deterministic finite automaton which accepts a string containing "*the*" anywhere in a string of $\Sigma = \{a, b, c \ldots, z\}$. (**Hint:** "*there*" is accepted but not "*those*"). **2**

c) Illustrate the state transition diagram of NFA recognizing the language $\{w \mid w$ ends with $00\}$ with three states over the alphabet $\Sigma = \{0,1\}$ **2**

3. a) Illustrate the state transition diagram for an NFA-ε recognizing the regular expression: $(0 \cup 1)^* \, 000 \, (0 \cup 1)^*$ **2**

b) Develop an equivalent deterministic finite automaton for the following non-deterministic finite automata. **2**



c) Define the language recognized by your DFA? Your answer may be either a regular expression or an explicit description of the set. **2**

4. a) Apply method of construction to prove that union of two regular language is a regular language. **2**

b) Using pumping lemma, show that the language given below is not a regular language.

$$A = \{0^n 1^n 2^n \mid n \geq 0\}.$$

c) Consider the following CFG,

$$E \rightarrow E - T \mid T$$
$$T \rightarrow T / F \mid F$$
$$F \rightarrow (E) \mid a$$

Build the parse trees and the derivations for the strings "((a))" and "a – a / a".

5. a) Build the context free grammar for the language $L = \{a^n b^n c^m d^m \mid n, m \geq 0\} \cup \{a^n b^m c^m d^n \mid n, m \geq 0\}$

b) The grammar obtained in Q.5.a) is ambiguous or unambiguous? Justify your answer considering the string **w = aabbccdd**

c) Construct the equivalent Chomsky Normal Form (CNF) for the following CFG

$$S \rightarrow ASA \mid aB$$
$$A \rightarrow B \mid S$$
$$B \rightarrow b \mid \varepsilon$$

******************* End of Questions ******************