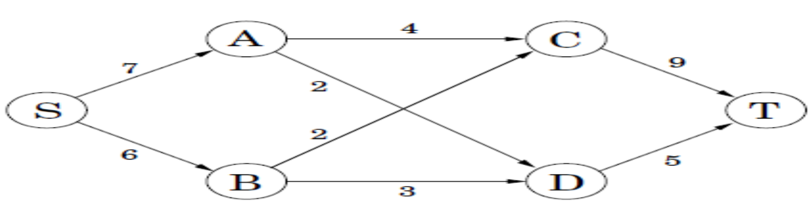
CSE 4131: ALGORITHM DESIGN 2

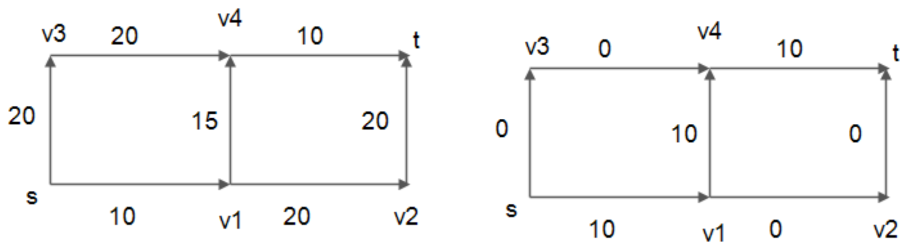
ASSIGNMENT 1:

Submission due date: 20/04/2023

- *Assignment scores/markings depend on neatness and clarity.*
- *Write your answers with enough detail about your approach and concepts used, so that the grader will be able to understand it easily. You should ALWAYS prove the correctness of your algorithms either directly or by referring to a proof in the book.*
- *The marking would be out of 100.*
- *You are allowed to use only those concepts which are covered in the lecture class till date.*
- *Plagiarized assignments will be given a zero mark.*

CO1: To understand the Network flow problem and apply it to solve different real-world problems.
CO2: -To distinguish between computationally tractable and intractable problems.
-To define and relate classP, classNP and class NPcomplete.
-To define an appropriate certificate and the verification algorithm for a given problem in NP.

Sl.No.	***Questions***	***PO***	***Level***
1.	Consider the Network flow problem with the following edge capacities, C(u, v) for edge (u, v) and flow direction also from u to v : C(s, u)=12, C(u, v)=12, C(v, t)=6, C(s, y)=13, C(y, z)=6, C(z, t)=6, C(u, w)=10, C(w, x)=5, C(x ,v)=2, C(w, v)= 2,C(y, w)=5, C(x, z)=3. Where source is s and sink is t. a) Draw the give flow network. b) Run Ford-Fulkersion Algorithm to calculate the maximum flow. c) Show each residual graph. d) If set A={x,u,y,z} and B={y,w,x,t} then what will be the capacity of the cut(A,B).	PO1,PO2,PO3	L1, L3,L6
2.	Consider the following network (the numbers are edge capacities).  a) Find the maximum flow f and a minimum cut. b) Draw the residual graph G_f (along with its edge capacities). In this residual network, mark the vertices reachable from S and the vertices from which T is reachable. c) An edge of a network is called a *bottleneck edge* if increasing its capacity results in an increase in the maximum flow. List all bottleneck edges in the above network.	PO1,PO2,PO3	L1,L2, L3

	d) Give a very simple example (containing at most four nodes) of a network which has no bottleneck edges. e) Give an efficient algorithm to identify all bottleneck edges in a network. (*Hint:* Start by running the usual network flow algorithm, and then examine the residual graph.)		
3.	Consider the following network flow $G = (V, E)$, where $V = \{s, a, b, c, d, t\}$ and capacity of the edges in E are $c(s,a) = 8, c(s,d) = 7, c(s,c) = 4, c(a,b) = 2, c(a,c) = 3, c(b,t) = 4, c(c,d) = 2, c(c,t) = 5, c(d,t) = 9$. a) List all the simple s-t augmented paths to find the Max-Flow of G. b) What is the value of the maximum flow from source s to sink t? c) List all the s-t cuts in G with their net-flow and capacity. d) Show the residual graph G_f.	PO1,PO2,PO3	L1, L3,L6
4.	Consider the flow network in Figure (1a) with a source s and a sink t. The edge capacities are shown on the edges. Figure (1b) shows an s-t flow in the same network with flow values indicated on the edges  a) A flow network with capacities on the edges b) A flow network, with current flow values shown on the edges. Figure 1: Network Flow a) Suppose the initial flow is given by Figure (1b). Construct the residual graph with respect to this initial flow. Recall that any forward/backward edges with zero capacity are not to be kept in the residual graph. b) Is there a path from s to t in the residual graph? Describe the path. What is the bottleneck b (minimum capacity of an edge) on this path? c) Suppose we push b units of flow along this path. Show the flow network with new flow values. d) Construct the residual graph with respect to the new flow. Is there a path from s to t in this residual graph? e) What is the current outgoing flow from s? Construct an s-t cut in the network whose outgoing capacity is equal to the current outgoing flow from s.	PO1,PO2,PO3	L1,L2, L3
5.	A flow network is defined by the tuple **G = (V, E, *s*, *t*)**, where **V** is the set of vertices, **E** is the set of edges, ***s*** is the source, and ***t*** is the sink. The following adjacent matrix	PO1,PO2,PO3	L2, L3

M describes the flow network **G** with the capacities associated with each of the edges. For example, **M[*s*, *v₁*]** = 7, where 7 is the capacity.

M	***s***	v_1	v_2	v_3	v_4	v_5	***t***
s	0	7	6	5	0	0	0
v_1	0	0	0	1	2	0	0
v_2	0	0	0	3	0	9	0
v_3	0	0	0	0	5	0	3
v_4	0	0	0	0	0	0	6
v_5	0	0	0	0	0	0	8
t	0	0	0	0	0	0	0

a) Illustrate the execution of the Ford-Fulkerson algorithm in the flow network **G** to find out the value of the maximum flow f_m. [Show the residual graphs at every stage]. The Ford-Fulkerson algorithm eventually terminates if edge capacities are rational numbers. TRUE or FALSE. Justify your answer.

b) We know if the flow network **G** does not have an augmenting path with respect to the flow f_m, then f_m is the maximum flow, also there is a min-cut χ of **G** such that $|f_m| = cap(\chi)$. Find out the min-cut set from **G** to compute $cap(\chi)$.

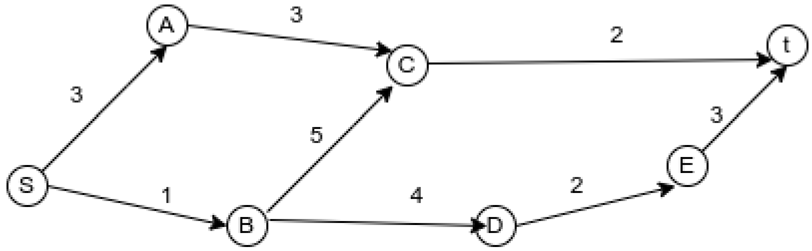
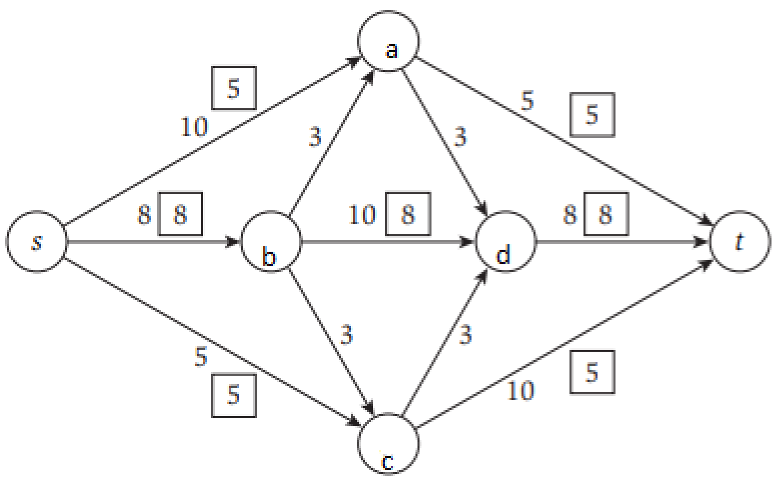
c) For a **(S, T)** cut where **S = {s, v_1, v_2, v_3, v_5}** and **T = {v_4, t}**, find out the flow and capacity across the given **(S, T)** cut.

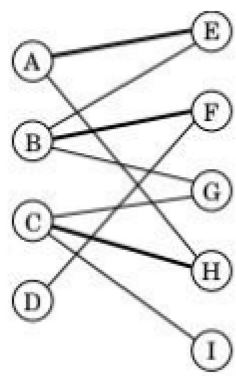
6. Consider the following network flow G=(V,E), where $V = \{s, a, b, c, d, t\}$ and capacity of the edges in E are $c(s,a) = 18, c(s,b) = 17, c(a,b) = 14, c(b,a) = 12, c(a,c) = 13, c(d,t) = 14, c(c,d) = 12, c(c,t) = 5, c(c,t) = 19, c(b,d) = 10, c(b,c) = 7, c(d,b) = 9$. Where s and t are the source and sink node respectively.

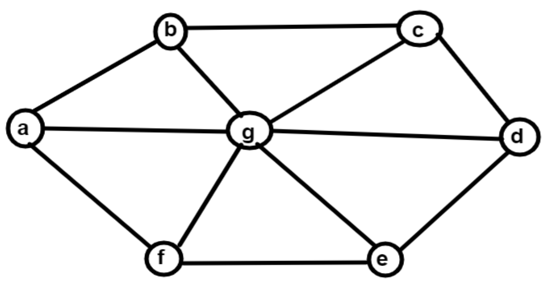
a) Draw the above flow network.
b) Find all possible cut set of the given graph.
c) Find the minimum cut set of the graph.

PO1,PO2,PO3

L2,L3, L6

	d) Prove that minimum cut set is the maximum flow of the above graph.		
7.	 a) Find the maximum flow from s to t in the above flow network. b) Find a cut which has capacity of the same value above (a). c) Find the cut edges, flow of the min cut, and list the nodes on the source side of this min cut.	PO1,PO2,PO3	L2, L3, L4
8.	In the given flow network at some instance the s-t flow has been computed. Each edge is associated with a label to represent capacity and numbers in boxes to represent amount of flow sent on it. Edges having label without boxed numbers have no flow on them.  a) Find out a cut and show that net flow across the cut is same as the value of the flow in the network. b) Is the current flow, max flow of the network? If yes find the minimum s-t cut in the flow network. If no, carryout the steps to find the minimum s-t cut in the flow network.	PO1,PO2,PO3	L2, L3, L4
9.	In a flow network G = (V, E), a condemning edge is defined as an edge in which a reduction in capacity leads to a reduction in maximum flow. Additionally, a congestion edge increases the maximum flow in the net-work when its capacity is increased. Either prove this statement or use a counterexample to disprove it “All condemning edges are also congestion edges”.	PO1,PO2,PO3	L2, L3

10.	Let G = (V1 ∪ V2, E) be a bipartite graph (so each edge has one endpoint in V1 and one endpoint inV2), and let M ∈ E be a matching in the graph (that is, a set of edges that don't touch). A vertex is said to be covered by M if it is the endpoint of one of the edges in M. An alternating path is a path of odd length that starts and ends with a non-covered vertex, and whose edges alternate between M and E − M. a) In the bipartite graph below Figure-1, a matching M is shown in bold. Find an alternating path. b) Prove that a matching M is maximal if and only if there does not exist an alternating path with respect to it. c) Design an algorithm that finds an alternating path in O(|V | + |E|) time using a variant of breadth-first search. d) Give a direct O(|V|×|E|) algorithm for finding a maximal matching in a bipartite graph.  Figure-1	PO1,PO2,PO3	L2, L3
11.	Prove that 2-SAT problem is in P.	PO1,PO2,PO4	L2, L3
12.	Cold Water Dispenser: BMC Bhubaneswar wants to put cold water dispensing units at several road junction points throughout the city during the summer. It wants to put as less as possible units covering all roads. a) Formulate the scenario as a computational problem and specify the Input and Expected Output. b) Propose steps to find a solution for the said problem. c) Whether the steps can lead to a polynomial time algorithm? Justify your answer.	PO1,PO2,PO4	L2, L3
13.	Consider an instance of the Boolean formula. $\varphi(x) =((x_1 \wedge \overline{x_2}) \vee (x_2 \wedge \overline{x_3})) \vee \overline{(\overline{x_3} \rightarrow x_4)} \vee (\overline{(\overline{x_2} \rightarrow x_5)}))$ a) Reduce this formula to the corresponding 3-CNF SAT instance. b) Show that the reduction occurred in polynomial time.	PO1,PO2,PO4	L2, L3

14.	Consider a special case of 3SAT in which all clauses have exactly three literals, and each variable appears at most three times. Show that this problem can be solved in polynomial time.	PO1,PO2,PO4	L2, L3, L4
15.	a) Reduce a 3-SAT formula to an independent set problem. (Explain with an example). b) Suppose, the 3-SAT problem has k clauses, then it is satisfiable if and only if the corresponding graph has an independent set of size k. Now, give an example to show that 3-SAT is not satisfiable if and only if the corresponding graph does not have any independent set of size k.	PO1,PO2,PO4	L2,L3 L4
16.	Consider the following instance of satisfiability: $\varphi(x) = (x_1 \vee x_2 \vee \overline{x_3}) \wedge (\overline{x_1} \vee x_3) \wedge (\overline{x_2} \vee x_3) \wedge (\overline{x_1} \vee \overline{x_2})$ a) Following the reduction method from satisfiability to clique, transform the above formula into an instance of clique for which the answer is yes iff the above formula is satisfiable. b) Find a clique of size 4 in your graph and convert it into a satisfying assignment for the formula given above.	PO1,PO2,PO4	L2,L3 L4
17.	Given below a decision Vertex-cover problem instance (G,k) as given in the figure and k=4,  a) Find the decision set-cover problem instances. (i.e $U, S_1\ to\ S_m\ and\ k$). b) Design a reduction algorithm for Vertex- Cover $\leq_P$ Set-Cover. c) Find the minimum Vertex- Cover and maximum Set-Cover.	PO1,PO2,PO4	L2,L3 L4
18.	$\varphi(x) = (x_1 \vee x_2 \vee x_3) \wedge (x_4 \vee x_5 \vee x_6) \wedge (x_7 \vee x_8 \vee x_9) \wedge (x_{10} \vee x_{11} \vee x_{12})$ $\mu(x) = (x_1 \vee x_2 \vee x_3) \wedge (\overline{x_1} \vee \overline{x_2} \vee \overline{x_3}) \wedge (x_4 \vee x_5 \vee x_6) \wedge (\overline{x_4} \vee \overline{x_5} \vee \overline{x_6})$ a) Reduce the 3-SAT formula $\varphi(x)$ in to independent set and find the maximum independent set. b) Reduce the 3-SAT formula $\mu(x)$ in to independent set and find the maximum independent set.	PO1,PO2,PO4	L2,L3 L4
19.	Let $G(V, E)$ be a directed weighted graph. A Hamiltonian path on the graph G is a path that traverse each vertex of the graph G exactly once. Design a polynomial time algorithm to detect whether a Hamiltonian path on G.	PO1,PO2,PO4	L6
20.	The department of CSE at ITER is organizing a one-week national level workshop with multiple sessions each of one hour duration. The department has asked the second-year students to contribute as volunteers for the event. The students are divided into groups of five students and at most one group will be allocated to each session of the workshop. Let's assume there are n groups $g_1, g_2, g_3, \dots, g_n$ and m	PO1,PO2,PO4	L2,L3 L4

sessions in total as $s_1, s_2, s_3, \dots, s_m$. The classes of the students are rescheduled in such a way that each group is available for some of the sessions and unavailable for others. Each session s_j must be assigned with at most one group. The maximum load given to a group cannot exceed a load limit of l_i. We need to maximize the number of sessions covered by the given groups considering their availability and without exceeding the workloads of the students.

a) Give a mathematical model of the given problem as the maximum flow problem. Explain the objective and the constraints. How can we solve it using Ford-Fulkerson algorithm?

b) Given an instance of the above problem as follows, find the assignment of the groups to the sessions if the maximum load limit for any group is two hours. Check if any session or group is remaining unassigned.

Groups	**Available in sessions**
g_1	s_1, s_2, s_4, s_6
g_2	s_1, s_3, s_5, s_7
g_3	s_2, s_4, s_6, s_7

21. A new telecom company is trying to establish its offices in a large city. The company wants to open offices at various (traffic) junctions of the road to cover maximum percentage of consumers. The local govt. authority has put a restriction that the company cannot open offices at adjacent junctions and has to leave at least two junctions' gap between two offices. Now, our goal is to find the maximum number of offices that can be opened by the company in the given city.

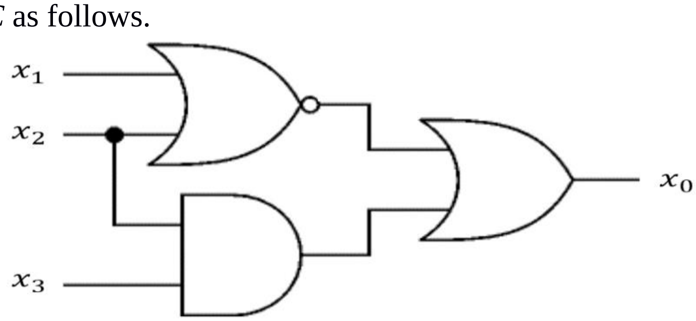
a) Define the decision version of the problem

b) Show that, the given problem is a NP-Complete problem. (Hint: Show the reduction of a NP complete problem to the given problem and justify the hardness).

PO1,PO2,PO4

L2,L3 L4

22. Given a circuit *C* as follows.



a) Convert it into an instance of CNF-SAT

b) Reduce the CNF-SAT to an instance of 3-SAT

c) Find a truth value assignment for the 3-SAT

PO1,PO2,PO4

L2,L3 L4

Submission and Grading:

Submit the hard copy of your assignment by the due date, i.e. 20.04.2023.

Part of your assignment grade comes from its "external correctness." This is based on correct output on various sample inputs.

The rest of your assignment's score comes from "internal correctness." Internal correctness includes:
1. Use of methods to minimize the number of steps.
2. Appropriate use of rules, axioms, and suitable diagrams to enhance readability of your responses.