ASSIGNMENTS :- [CH-1: LOGIC & PROOFS]

$\boxed{1.1}$ 9) Let p & q be the propositions

p : you drive over 65 miles per hour.

q : you get a speeding ticket.

Write these propositions using p & q & logical connectives

a) You do not drive over 65 miles per hour.

b) You drive over 65 miles per hour, but you do not get a speeding ticket.

c) You will get a speeding ticket if you drive over 65 miles per hour.

d) If you do not drive over 65 miles per hour, then you will not get a speeding ticket.

e) Driving over 65 miles per hour is sufficient for getting a speeding ticket.

f) You get a speeding ticket, but you do not drive over 65 miles per hour.

g) Whenever you get a speeding ticket, you are driving over 65 miles per hour.

Ans) (a) $\neg p$ (b) $p \wedge \neg q$ (c) $p \rightarrow q$ (d) $\neg p \rightarrow \neg q$

(e) $p \rightarrow q$ (f) $q \wedge \neg p$ (g) $q \rightarrow p$

24) State the converse, contrapositive and inverse of each of these conditional statements.

a) If it snows tonight, then I will stay at home.

b) I go to the beach whenever it is a sunny summer day.

c) When I stay up late, it is necessary that I sleep until noon.

Ans) (a) Converse : If I will stay at home then it snows tonight

Contrapositive : If I will not stay at home then it will not snows tonight

Inverse : If it not snows tonight, then I will not stay at home.

(b) Converse : If I go to the beach then it is a sunny summer day.

Contrapositive : If I will not go to the beach then it is not a sunny summer day.

Inverse : If it is not a sunny summer day then I will not go to the beach.

(c) Converse : If I stay up late then I will sleep until noon.

Contrapositive : If I not stay up late then I will not sleep until noon.

Inverse : If I will not sleep until noon then I will also not stay up late.

36) What is the value of x after each of these statements is encountered in a computer program, if $x = 1$ before the statement is reached?

a) If $1+2=3$, then $x := x+1$

b) If $(1+1=3)$ OR $(2+2=3)$, then $x := x+1$

c) If $(2+3=5)$ AND $(3+4=7)$, then $x := x+1$

d) If $(1+1=2)$ XOR $(1+2=3)$, then $x := x+1$

e) If $x < 2$, then $x := x+1$

Ans) (a) As $1+2=3$ — True

so, $x = 2$

(b) As $(1+1=3)$ OR $(2+2=3)$ — False

so, $x = 1$

(c) As $(2+3=5)$ AND $(3+4=7)$ — True

so, $x = 2$

(d) As $(1+1=2)$ XOR $(2+2=3)$ — False

so, $x = 1$

(e) As $x < 2$ — True

so, $x = 2$.

$\boxed{1.2}$ 15) Determine whether $(\neg q \wedge (p \to q)) \to \neg p$ is a tautology.

Ans)

p	q	$\neg p$	$\neg q$	$p \to q$	$\neg q \wedge (p \to q)$	$(\neg q \wedge (p \to q)) \to$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

18) Show that $p \to q$ and $\neg q \to \neg p$ are logically equivalent.

Ans)

p	q	$\neg q$	$\neg p$	$p \to q$	$\neg q \to \neg p$
T	T	F	F	T	T
T	F	T	F	F	F
F	T	F	T	T	T
F	F	T	T	T	T

$$p \to q \equiv \neg q \to \neg p$$

19) Show that $\neg p \leftrightarrow q$ and $p \leftrightarrow \neg q$ are logically equivalent.

Ans)

p	q	$\neg p$	$\neg q$	$\neg p \leftrightarrow q$	$p \leftrightarrow \neg q$
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	F	F

$$\neg p \leftrightarrow q \equiv p \leftrightarrow \neg q$$

22) Show that $(p \to q) \wedge (p \to r)$ & $p \to (q \wedge r)$ are logically equivalent.

Ans)

p	q	r	$q \wedge r$	$p \to q$	$p \to r$	$(p \to q) \wedge (p \to r)$	$p \to (q \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	F	F	F
T	F	T	F	F	T	F	F
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	[illegible]

$(p \to q) \land (p \to r) \equiv p \to (q \land r)$

23) Show that $(p \to r) \land (q \to r)$ & $(p \lor q) \to r$ are logically equivalent.

Ans)

p	q	r	$p \lor q$	$p \to r$	$q \to r$	$(p \to r) \land (q \to r)$	$(p \lor q) \to r$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	F	T	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	F	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

$(p \to r) \land (q \to r) \equiv (p \lor q) \to r$

24) Show that $(p \to q) \lor (p \to r)$ & $p \to (q \lor r)$ are logically equivalent.

Ans)

p	q	r	$q \lor r$	$p \to q$	$p \to r$	$(p \to q) \lor (p \to r)$	$p \to (q \lor r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	F	F	F	T	T	T	T

$(p \to q) \lor (p \to r) \equiv p \to (q \lor r)$

25) Show that $(p \to r) \lor (q \to r)$ & $(p \land q) \to r$ are logically equivalent.

Ans)

p	q	r	$p \to r$	$q \to r$	$p \land q$	$(p \to r) \lor (q \to r)$	$(p \land q) \to r$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F
T	F	T	T	T	F	T	T
T	F	F	F	T	F	T	T
F	T	T	T	T	F	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

$(p \to r) \lor (q \to r) \equiv (p \land q) \to r$

29) Show that $(p \to q) \land (q \to r) \to (p \to r)$ is a tautology.

Ans)

p	q	r	$p \to q$	$q \to r$	$p \to r$	$(p \to q) \land (q \to r)$	$(p \to q) \land (q \to r) \to (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

$\boxed{1.3}$ 6) Let $N(x)$ be the statement "x has visited North Dakota", where the domain consists of the students in your school.
Express each of these quantifications in English.
(a) $\exists x\, N(x)$ (b) $\forall x\, N(x)$ (c) $\neg \exists x\, N(x)$ (d) $\exists x \neg N(x)$ (e) $\neg \forall x\, N(x)$
(f) $\forall x \neg N(x)$

Solⁿ- (a) There exist Atleast one of the student has visited North Dakota.
(b) All of the students havevisited North Dakota.
(c) Noone of the students have visited North Dakota.
(d) There exist atleast one of the student who has not visited North Dakota.
(e) Not everyone in the students have visited North Dakota
(f) All of the students have not visited North Dakota.

$\boxed{1.4}$ 2) Translate into English-
$x \in R,\ y \in R$
(a) $\exists x \forall y\, (xy = y)$
There exists some values of x for which all values of y satisfies the equation $xy = y$
(b) $\forall x \forall y\, (((x \geq 0) \land (y < 0)) \to (x - y) > 0)$
For all values of x and y such that x is non-negative and y is negative, then $x - y$ is always positive.

For all values of x and y such that, there exists some values of z for which it satisfies $x = y + z$

4) $P(x,y)$: Student x has taken class y

x : all students in your class.

y : all computer science course

(a) $\exists_x \exists_y P(x,y)$:- Some students have taken some computer science courses at your school.

(b) $\exists_x \forall_y P(x,y)$:- Some students in your class have taken all computer science courses at your school.

(c) $\forall_x \exists_y P(x,y)$:- All students in your class have taken all computer science courses at your school.

(d) $\exists_y \forall_x P(x,y)$:- Some computer science courses at your school have been taken up by all students in your class.

(e) $\forall_y \exists_x P(x,y)$:- All computer science courses in your school have been taken up by some students in your class.

(f) $\forall_x \forall_y P(x,y)$:- All students in your class have taken up all computer science courses at your school.

$\boxed{1.5}$ 4) What rule of inference is used —

(a) x: Kangaroos live in Australia

y: Kangaroos are marsupials

(1) $x \wedge y$ — hypothesis

(2) y — Simplification on (1)

(b) x: it is hotter than 100 degrees today.

y: pollution is dangerous.

(1) $x \vee y$

(2) $\neg x$

(3) y — Disjunctive syllogism on (1), (2).

(c) x: Linda is an excellent swimmer

y: Linda can work as a lifeguard.

(1) x

(2) $x \to y$

(3) y — Modus ponens on (1), (2)

(d) x: Steve will work at a computer company this Summer

y: Steve will be a beach bum.

(1) x

(2) $x \vee y$ — Addition on (1)

(e) x: I work all night on this homework

y: I can answer all the exercise.

z: I will understand the material.

(1) $x \to y$

(2) $y \to z$

(3) $x \to z$ — Hypothetical syllogism on (1) & (2)

6) Use rule of inference —

x: It rains

y: It is foggy

z: The sailing race will be held

a: The trophy will be awarded

b: The life saving demonstration will go on.

(1) $\neg a$ — Hypothesis

(2) $z \to a$ — Hypothesis

(3) $\neg z$ — Modus Tollens on (1) & (2)

(4) $(\neg x \vee \neg y) \to (z \wedge b)$ — Hypothesis

(5) $\neg(x \wedge y) \to (z \wedge b)$

(6) $(x \wedge y) \vee (z \wedge b)$ — By equivalence on (5)

(7) $((x \wedge y) \vee z) \wedge ((x \wedge y) \vee b)$ — Distributive law on (6)

(8) $(x \wedge y) \vee z$ — Simplification on (7)

(9) $(z \vee x) \wedge (z \vee y)$ — Distributive law on (8)

(10) $z \vee x$ — Simplification on (9)

(11) x — Disjunctive syllogism on (3) & (10)

??

8) Use rule of inference —

$P(x)$: "x is a man"

$Q(x)$: "x is an island"

(1) $\forall x\ (P(x) \rightarrow \neg Q(x))$ — Hypothesis

(2) Q (Manhattan) — Hypothesis

(3) P (Manhattan) $\rightarrow \neg Q$ (Manhattan) — (Universal Instantiation on (1))

(4) $\neg(\neg Q$ (Manhattan)) — Double negation on (2)

(5) $\neg P$ (Manhattan) — MT on (3) & (4)

$\boxed{1.6}$ 2) Use a direct proof to show that the sum of two even integers is even.

Ans) Let x, y be any even integers.

$x = 2K_1$

$y = 2K_2$

Sum $= x+y = 2K_1 + 2K_2$

$= 2(K_1+K_2)$

$= 2K$ (even), $K_1+K_2 = K \in \mathbb{Z}$

So, sum of two even integers is even.

18) Prove that if n is an integer and $3n+2$ is even, then n is even using.

(a) a proof by contrapositive.

(b) a proof by contradiction.

Ans) (a) Proof by contrapositive –

Let n is odd integer

$n = 2K+1$ (odd)

Now, $3n+2 = 3(2K+1)+2$

$= 6K+3+2 = 6K+2+2+1 = 6K+4+1$

$= 2(3K+2)+1$

$= 2t+1$ (odd), $3K+2 = t \in \mathbb{Z}$.

So, if n is an integer and $3n+2$ is even, then n is even.

(b) Proof by contradiction –

Given: n is an integer and $3n+2$ is even

To prove: n is even integer

Let us assume, n is odd integer, if possible

$n = 2K+1$

$3n+2 = 3(2K+1)+2$

$= 6K+3+2$

$= 6K+2+2+1 = 6K+4+1$

$= 2(3K+2)+1$

$= 2t+1$ (odd), $3K+2 = t \in \mathbb{Z}$

It contradicts the given statement.

So, n is an integer and $3n+2$ is even.

CHAPTER 2 : SETS :-

$\boxed{2.1}$ 8) Determine whether true or false –

(a) $\phi \in \{\phi\}$ $\longrightarrow$ True

(b) $\phi \in \{\phi, \{\phi\}\}$ $\longrightarrow$ True

(c) $\{\phi\} \in \{\phi\}$ $\longrightarrow$ False

(d) $\{\phi\} \in \{\{\phi\}\}$ $\longrightarrow$ True

(e) $\{\phi\} \subset \{\phi, \{\phi\}\}$ $\longrightarrow$ True

(f) $\{\{\phi\}\} \subset \{\phi, \{\phi\}\}$ $\longrightarrow$ True

(g) $\{\{\phi\}\} \subset \{\{\phi\}, \{\phi\}\}$ $\longrightarrow$ False

9) Determine whether true or false –

(a) $x \in \{x\}$ $\rightarrow$ True

(b) $\{x\} \subseteq \{x\}$ $\rightarrow$ True

(c) $\{x\} \in \{x\}$ $\rightarrow$ False

(d) $\{x\} \in \{\{x\}\}$ $\rightarrow$ True

(e) $\phi \subseteq \{x\}$ $\rightarrow$ True

(f) $\phi \in \{x\}$ $\rightarrow$ True

18) What is the cardinality of each of these sets?

(a) ϕ

Ans) $|A| = 0$

(b) $\{\phi\}$

Ans) $|A| = 1$

(c) $\{\phi, \{\phi\}\}$

Ans) $|A| = 2$

(d) $\{\phi, \{\phi\}, \{\phi, \{\phi\}\}\}$

Ans) $|A| = 3$

23) $A = \{a, b, c, d\}$ and $B = \{y, z\}$

(a) $A \times B = \{(a,y), (a,z), (b,y), (b,z), (c,y), (c,z), (d,y), (d,z)\}$

(b) $B \times A = \{(y,a), (y,b), (y,c), (y,d), (z,a), (z,b), (z,c), (z,d)\}$

$\boxed{2.3}$ 5) Find the range and domain of the functions -

(a) The function that assigns to each bit string the number of ones minus the number of zero.

Ans) $D = \{0, 1, \ldots\ldots, n\}$
where, n = finite

Range $= \{-n, \ldots, -1, 0, 1, \ldots, n\} = n \in Z^+$
where n = finite

(b) The function that assigns to each bit string twice the number of zeros in that string.

Ans) $D = \{0, 1, \ldots\ldots, n\}$ where, n = finite

$R = \{0, 2, \ldots\ldots, 2n\}$ where n = finite

(c) The function that assigns the number of bits left over when a bit string is split into bytes (which are blocks of 8 bits).

Ans) $D = \{0, 1, \ldots\ldots, n\}$, n = finite

Range $= \{0, 1, 2, 3, 4, 5, 6, 7\}$

(d) The function that assigns to each positive integer the largest perfect square not exceeding this integer.

Ans) $D = x \in N$

Range $= \{y \mid y^2 <= x : y \in N\}$

Range $= \{0, 1, 4, 16, \ldots\ldots, n^2\}$

12) Determine whether each of these functions from Z to Z is one-to-one.

(a) $f(n) = n - 1$

Ans) Let $f(n_1) = f(n_2)$

$\Rightarrow n_1 - 1 = n_2 - 1$

$\Rightarrow n_1 = n_2$

So, it is one-to-one.

(b) $f(n) = n^2 + 1$

Ans) Let $f(n_1) = f(n_2)$

$\Rightarrow n_1^2 + \cancel{1} = n_2^2 + \cancel{1}$

$\Rightarrow n_1^2 = n_2^2$

$\Rightarrow n_1 \neq n_2$

So, it is not one-one.

(c) $f(n) = n^3$

Ans) Let $f(n_1) = f(n_2)$

$\Rightarrow n_1^3 = n_2^3$

$\Rightarrow n_1^3 - n_2^3 = 0$

$\Rightarrow (n_1 - n_2)(n_1^2 + n_2^2 + 2n_1n_2) = 0$

$\Rightarrow n_1 = n_2$

It is one-one.

(d) $f(n) = \lceil n/2 \rceil$

Ans) Let, $n_1 = \frac{1}{4}$

$n_2 = \frac{1}{2}$

$\lceil n_1/2 \rceil = \lceil n_2/2 \rceil$

$\lceil 1/8 \rceil = \lceil 1/4 \rceil$

$\Rightarrow 1 = 1$

$\because n_1 \neq n_2$

$\therefore$ It is not one-one.

15) Determine whether the func^c $f: \mathbb{Z}\times\mathbb{Z} \to \mathbb{Z}$ is onto if

(a) $f(m,n) = m+n$

Ans) $\forall y \in \mathbb{Z}$

$\exists (m,n) \in \mathbb{Z}\times\mathbb{Z}$

such that $f(m,n) = m+n$ $\therefore$ It is onto.

(b) $f(m,n) = m^2+n^2$

Ans) $\exists (m,n) \in \mathbb{Z}\times\mathbb{Z}$

s.t. $m^2 - 4 = 1$

It is not onto.

(c) $f(m,n) = m$

Ans) $y = m$

$\forall y \in \mathbb{Z}$

$\exists (m,n) \in \mathbb{Z}\times\mathbb{Z}$

such that $f(m,n) = m$

$\therefore$ It is onto.

(d) $f(m,n) = |n|$

Ans) $y = |n|$

$\forall y \in \mathbb{Z}$

$\nexists (m,n) \in \mathbb{Z}\times\mathbb{Z}$

Range does not contain negative number.

(e) $f(m,n) = m-n$

Ans) $y = m-n$

$\forall y \in \mathbb{Z}$

$\exists (m,n) \in \mathbb{Z}\times\mathbb{Z}$

s.t. $f(m,n) = m-n$

$\therefore$ It is onto.

19) Determine whether the functions are bijective from R to R.

(a) $f(x) = -3x+4$

Ans) One - One

$f(x_1) = f(x_2)$

$\Rightarrow -3x_1 + 4 = -3x_2 + 4$

$\Rightarrow -3x_1 = -3x_2$

$\Rightarrow x_1 = x_2$

$\therefore$ It is one-one.

Onto

$f(x) = -3x+4$

$\Rightarrow y = -3x+4$

$\Rightarrow -3x = y-4$

$\Rightarrow x = \dfrac{4-y}{3}$

$\therefore$ It is onto.

$\therefore$ Hence, it is bijective.

(b) $f(x) = x^2+1$

Ans) One - One

$f(x_1) = f(x_2)$

$\Rightarrow x_1^2+1 = x_2^2+1$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1 \neq x_2$

$\therefore$ It is not one-one

Onto

$f(x) = x^2+1$

$\Rightarrow y = x^2+1$

$\Rightarrow x^2 = y-1$

$\Rightarrow x = \pm\sqrt{y-1}$

$\therefore$ It is not onto.

$\therefore$ Hence, it is not bijective.

(c) $f(x) = x^3$

Ans) One - One

$f(x_1) = f(x_2)$

$\Rightarrow x_1^3 = x_2^3$

$\Rightarrow x_1 = x_2$

$\therefore$ It is one-one

Onto

$f(x) = x^3$

$y = x^3$

$x = \sqrt[3]{y}$

$\therefore$ It is onto.

$\therefore$ Hence, it is bijective.

(d) $f(x) = (x^2+1)(x^2+2)$

Ans) One - One

$f(x_1) = f(x_2)$

$\Rightarrow (x_1^2+1)(x_1^2+2) = (x_2^2+1)(x_2^2+2)$

$\Rightarrow x_1^4 + 3x_1^2 + 2 = x_2^4 + 3x_2^2 + 2$

Onto

$f(x) = (x^2+1)(x^2+2)$

$\Rightarrow y = x^4 + 3x^2 + 2$

$\therefore$ It is not onto.

$\Rightarrow x_1 + 3x_1 = x_2 + 3x_2$

$\Rightarrow x_1^4 - x_2^4 = 3(x_2^2 - x_1^2)$

$\therefore$ It is not one-one.

$\therefore$ It is not bijective.

$\boxed{2.5}$ 2) What is the term a_8 of the sequence $\{a_n\}$ —

(a) 2^{n-1}? (b) 7?

(c) $1+(-1)^n$? (d) $-(-2)^n$?

Ans) (a) 2^{n-1} (a_n)

$a_8 = 2^{8-1} = 2^7$

(b) 7 (a_n)

$a_8 = 7$

(c) $1+(-1)^n$ (a_n)

$a_8 = 1+(-1)^8 = 1+1 = 2$

(d) $-(-2)^n$ (a_n)

$a_8 = -(-2)^8 = -256$

4) What are the terms a_0, a_1, a_2 and a_3 of $\{a_n\}$ sequence —

(a) $(-2)^n$

Ans) $a_n = (-2)^n$

$a_0 = (-2)^0 = 1$

$a_1 = (-2)^1 = -2$

$a_2 = (-2)^2 = 4$

$a_3 = (-2)^3 = -8$

(b) 3

Ans) $a_n = 3$

$a_0 = 3$

$a_1 = 3$

$a_2 = 3$

$a_3 = 3$

(c) $a_n = 7+4^n$

$a_0 = 7+4^0 = 8$

$a_1 = 7+4^1 = 11$

$a_2 = 7+4^2 = 23$

$a_3 = 7+4^3 = 71$

(d) $a_n = 2^n+(-2)^n$

$a_0 = 2^0+(-2)^0 = 2$

$a_1 = 2^1+(-2)^1 = 0$

$a_2 = 2^2+(-2)^2 = 8$

$a_3 = 2^3+(-2)^3 = 0$

18) Compute each of these —

(a) $\sum_{i=1}^{3}\sum_{j=1}^{2}(i-j)$

Ans) $\sum_{i=1}^{3}\left\{\sum_{j=1}^{2}(i-j)\right\}$

$= \sum_{i=1}^{3}(i-1+i-2)$

$= \sum_{i=1}^{3}(2i-3)$

$= (2\times1-3+2\times2-3+2\times3-3)$

$= 2-3+4-3+6-3 = 12-9 = 3$

(b) $\sum_{i=0}^{3}\sum_{j=0}^{2}(3i+2j)$

Ans) $\sum_{i=0}^{3}\left\{\sum_{j=0}^{2}(3i+2j)\right\}$

$= \sum_{i=0}^{3}(3i+3i+2+3i+4)$

$= \sum_{i=0}^{3}(9i+6)$

$= 6+9+6+18+6+27+6$

$= 78$

(c) $\sum_{i=1}^{3}\sum_{j=0}^{2} j$

Ans) $\sum_{i=1}^{3}\left\{\sum_{j=0}^{2} j\right\}$

$= \sum_{i=1} (0+2+2) = \sum_{i=1} 3$

$= 3+3+3 = 9$

(d) $\sum_{i=0}^{2} \sum_{j=0}^{3} (i^2 - j^3)$

Ans) $\sum_{i=0}^{2} \left\{ \sum_{j=0}^{3} (i^2 - j^3) \right\}$

$= \sum_{i=0}^{2} (i^2 + i^2 - 1 + i^2 - 8 + i^2 - 27)$

$= \sum_{i=0}^{2} (4i^2 - 36)$

$= -36 + 4 - 36 + 16 - 36$

$= 20 - 108 = -88$

CHAPTER 3: ALGORITHM :-

$\boxed{3.1}$ 14) (a) Linear Search:-

{1, 3, 4, 5, 6, 8, 9, 11}

$i := 1$ $(n = 8, x = 7)$

while $(1 \leq 8$ and $x \neq 1)$

$i := 2$

while $(2 \leq 8$ and $x \neq 3)$

$i := 3$

while $(3 \leq 8$ and $x \neq 4)$

$i := 4$

while $(4 \leq 8$ and $x \neq 5)$

$i := 5$

while $(5 \leq 8$ and $x \neq 6)$

$i := 6$

while $(6 \leq 8$ and $x \neq 8)$

$i := 7$

while $(7 \leq 8$ and $x \neq 9)$

$i := 8$

while $(8 \leq 8$ and $x \neq 11)$

~~If $i \leq n$, then location := i~~
~~else location := 0~~ location is 0.

$i := i + 1$

while $(9 \leq 8$ & $x \neq a?)$

If $9 \leq 8$, (false)

else, location is 0.

(b) Binary Search:-

{1, 3, 4, 5, 6, 8, 9, 11}

$i = 1, j = 8, x = 7$

while $1 < 8$

$m = \left[\frac{1+8}{2}\right] = 4$

while $5 < 8$,

$m = \left[\frac{5+8}{2}\right] = 6$

$i = 7, j = 8$

while $7 < 8$.

$m = \left[\frac{7+8}{2}\right] = 8$

$i = 8, j = 8$

$i = j$

end

location of 7, is 0.

35) Use the bubble sort –

{3, 1, 5, 7, 4}

Ans) 1st pass

3	1	1	1	1
1	3	3	3	3
5	5	5	5	5
7	7	7	7	4
4	4	4	4	$\boxed{7}$

2nd pass

1	1	1	1
3	3	3	3
5	5	5	4
4	4	4	$\boxed{5}$
$\boxed{7}$	$\boxed{7}$	$\boxed{7}$	$\boxed{7}$

3rd pass

1	1	1
3	3	3
4	4	$\boxed{4}$
$\boxed{5}$	$\boxed{5}$	$\boxed{5}$
$\boxed{7}$	$\boxed{7}$	$\boxed{7}$

4th pass

1	1
3	$\boxed{3}$
$\boxed{4}$	$\boxed{4}$
$\boxed{5}$	$\boxed{5}$
$\boxed{7}$	$\boxed{7}$

Final list

1
3
4
5
7

34) Use the insertion sort —

$\{3, 1, 5, 7, 4\}$

(I) 3, 1, 5, 7, 4

As $1 < 3$, so —

(II) 1, 3, 5, 7, 4

As $5 > 3$, $5 > 1$, so —

(III) 1, 3, 5, 7, 4

As $7 > 5$, $7 > 3$, $7 > 3$, so —

(IV) 1, 3, 5, 7, 4

As $4 > 1$, $4 > 3$, but $5 > 4$, $7 > 4$, so —

(V) $\boxed{1, 3, 4, 5, 7}$

$\boxed{3.4}$ 17. Evaluate —

a) 13 mod 3

$a = b + mk$

$= 13 + 3 \times -4$

$= 1$

b) -97 mod 11

$a = b + mk$

$= -97 + 11 \times 9$

$= 2$

c) 155 mod 19

$a = b + mk$

$= 155 + 19 \times -8$

$= 3$

d) -221 mod 23

$a = b + mk$

$= -221 + 23 \times 10$

$= 9$

$\boxed{3.5}$ 11. So, for satisfying the condition of relatively prime, it should get —

$\gcd(a, b) = 1$

So, the positive integers less than 30 which are relatively prime to 30 is —

1, 7, 11, 13, 17, 19, 23, 29

13. (a) 11, 15, 19

$\gcd(11, 15) = 1$

$11 = 1^1 . 11^1$

$15 = 1^1 . 3^1 . 5^1$

$\gcd(11, 19) = 1$

$19 = 1^1 . 19^1$

$11 = 1^1 . 11^1$

$\gcd(15, 19) = 1$

$15 = 1^1 . 3^1 . 5^1$

$19 = 1^1 . 19^1$

∴ Yes, this is pairwise relatively prime.

(b) 14, 15, 21.

$\gcd(14, 15) = 1$

$\gcd(14, 21) = 1^1 \times 7^1 = 7$

$\gcd(15, 21) = 1^1 \times 3^1 = 3$

$14 = 1^1 \times 2^1 \times 7^1$

$15 = 1^1 \times 3^1 \times 5^1$

$21 = 1^1 \times 3^1 \times 7^1$

∴ No, this is not pairwise relatively prime.

(c) 12, 17, 31, 37

$\gcd(12, 17) = 1$

$\gcd(12, 31) = 1$

$\gcd(12, 37) = 1$

$\gcd(17, 31) = 1$

$\gcd(17, 37) = 1$

$\gcd(31, 37) = 1$

$12 = 1^1 . 2^2 . 3^1$

$17 = 1^1 . 17^1$

$31 = 1^1 . 31^1$

$37 = 1^1 . 31^1$

∴ Yes, this is pairwise relatively prime.

(d) 7, 8, 9, 11

$\gcd(7, 8) = 1$

$\gcd(7, 9) = 1$

$\gcd(8, 9) = 1$

$\gcd(8, 11) = 1$

$\gcd(9, 11) = 1$

$7 = 1^1 \cdot 7^1$

$8 = 1^1 \cdot 2^3$

$9 = 1^1 \cdot 3^2$

$11 = 1^1 \cdot 11^1$

$\therefore$ Yes, this is pairwise relatively prime.

$\boxed{3.6}$ 2) (a) 321

$321 = 2 \cdot 160 + 1$
$160 = 2 \cdot 80 + 0$
$80 = 2 \cdot 40 + 0$
$40 = 2 \cdot 20 + 0$
$20 = 2 \cdot 10 + 0$
$10 = 2 \cdot 5 + 0$
$5 = 2 \cdot 2 + 1$
$2 = 2 \cdot 1 + 0$
$1 = 2 \cdot \boxed{0} + 1$

$(321)_{10} = (101000001)_2$

(b) 1023

$1023 = 2 \cdot (511) + 1$
$511 = 2 \cdot 255 + 1$
$255 = 2 \cdot 127 + 1$
$127 = 2 \cdot 63 + 1$
$63 = 2 \cdot 31 + 1$
$31 = 2 \cdot 15 + 1$
$15 = 2 \cdot 7 + 1$
$7 = 2 \cdot 3 + 1$
$3 = 2 \cdot 1 + 1$
$1 = 2 \cdot \boxed{0} + 1$

$(1023)_{10} = (11111\ 11111)_2$

(c) 100632

$100632 = 2 \cdot 50316 + 0$
$50316 = 2 \cdot 25158 + 0$
$25158 = 2 \cdot 12579 + 0$
$12579 = 2 \cdot 6289 + 1$
$6289 = 2 \cdot 3144 + 1$
$3144 = 2 \cdot 1572 + 0$
$1572 = 2 \cdot 786 + 0$
$786 = 2 \cdot 393 + 0$
$393 = 2 \cdot 196 + 1$
$196 = 2 \cdot 98 + 0$
$98 = 2 \cdot 49 + 0$
$49 = 2 \cdot 24 + 1$
$24 = 2 \cdot 12 + 0$
$12 = 2 \cdot 6 + 0$
$6 = 2 \cdot 3 + 0$
$3 = 2 \cdot 1 + 1$
$1 = 2 \cdot \boxed{0} + 1$

$(100632)_{10} = (11000\ 1001000\ 11000)_2$

3. (a) 11111

$\begin{matrix} 1 & 1 & 1 & 1 & 1 \\ 4 & 3 & 2 & 1 & 0 \end{matrix}$

$= 2^4 + 2^3 + 2^2 + 2^1 + 2^0 = 31$

(b) 10 0000 0001

$\begin{matrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \end{matrix}$

$2^9 + 2^0 = 513$

(c) ~~11 1011 1110~~ 1 0101 0101

$\begin{matrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \end{matrix}$

$= 2^8 + 2^6 + 2^4 + 2^2 + 2^0 = 341$

(d) 110 1001 0001 0000

$\begin{matrix} 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 14 & 13 & 12 & 11 & 10 & 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \end{matrix}$

$= 2^{14} + 2^{13} + 2^{12} + 2^{8} + 2^{4} = 26896$

20\. $11^{644} \bmod 645$

$b = 1,\ m = 645,\ n = 644 = (1010000100)_2$

$x := 1$, power $= 11 \bmod 645$

$i = 0$ to 9

$i = 0:\ a_0 = 0,\ x = 1$, power $= 11^2 \bmod 645 = 121$

$i = 1;\ a_1 = 0,\ x = 1$, power $= 121^2 \bmod 645 = 14641 \bmod 645 = 451$

$i = 2;\ a_2 = 1,\ x = 1 \times 451 \bmod 645 = 451$, power $= 451^2 \bmod 645$
$= 203401 \bmod 645 = 226$

$i = 3;\ a_3 = 0,\ x = 451$, power $= 226^2 \bmod 645 = 51076 \bmod 645 = 121$

$i = 4;\ a_4 = 0,\ x = 451$, power $= 121^2 \bmod 645 = 451$

$i = 5;\ a_5 = 0,\ x = 451$, power $= 451^2 \bmod 645 = 226$

$i = 6;\ a_6 = 0,\ x = 451$, power $= 226^2 \bmod 645 = 121$

$i = 7;\ a_7 = 1,\ x = (451 \times 121) \bmod 645 = 54571 \bmod 645 = 391$,
power $= 121^2 \bmod 645 = 451$

$i = 8;\ a_8 = 0,\ x = 391$, power $= 451^2 \bmod 645 = 226$

$i = 9;\ a_9 = 1,\ x = (391 \times 226) \bmod 645 = 88366 \bmod 645 = 1$,
power $= 226^2 \bmod 645 = 121$

$\therefore x = b^n \bmod m$

$1 = b^n \bmod m$

$\Rightarrow 1 \equiv 11^{644} \bmod 645$

22\. $123^{1001} \bmod 101$

$n = 1001 = (1111101001)_2$

$b = 123,\ m = 101$

power $= 123 \bmod 101 = 22$

$x := 1$

$i = 0;\ a_0 = 1,\ x = (1 \times 22) \bmod 101 = 22$, power $= 22^2 \bmod 101 = 80$

$i = 1;\ a_1 = 0,\ x = 22$, power $= 80^2 \bmod 101 = 37$

$i = 2;\ a_2 = 0,\ x = 22$, power $= 37^2 \bmod 101 = 56$

$i = 3;\ a_3 = 1,\ x = (22 \times 56) \bmod 101 = 20$, power $= 56^2 \bmod 101 = 5$

$i = 4;\ a_4 = 0,\ x = 20$, power $= 5^2 \bmod 101 = 25$

$i = 5;\ a_5 = 1,\ x = (20 \times 25) \bmod 101 = 96$, power $= 25^2 \bmod 101 = 19$

$i = 6;\ a_6 = 1,\ x = (96 \times 19) \bmod 101 = 6$, power $= 19^2 \bmod 101 = 58$

$i = 7;\ a_7 = 1,\ x = (6 \times 58) \bmod 101 = 45$, power $= 58^2 \bmod 101 = 31$

$i = 8;\ a_8 = 1,\ x = (45 \times 31) \bmod 101$, power $= 31^2 \bmod 101 = 52$

$i = 9;\ a_9 = 1,\ x = (82 \times 52) \bmod 101 = 22$, power $= 52^2 \bmod 101 =$

$\therefore x = b^n \bmod m$

$\Rightarrow 22 = 123^{1001} \bmod 101$

23\. (a) $\gcd(12, 18)$

$18 = 12 \times 1 + 6$

$12 = 6 \times 2 + 0$

$\gcd = 6$

(b) $\gcd(111, 201)$

$201 = 111 \times 1 + 90$

$111 = 90 \times 1 + 21$

$90 = 21 \times 4 + 6$

$21 = 6 \times 3 + 3$

$6 = 3 \times 2 + 0$

$\gcd = 3$

(c) $\gcd(1001, 1331)$

$1331 = 1001 \times 1 + 330$

$1001 = 330 \times 3 + 11$

$330 = 11 \times 30 + 0$

$\gcd = 11$

(d) $\gcd(12345, 54321)$

$54321 = 12345 \times 4 + 4941$

$12345 = 4941 \times 2 + 2463$

$4941 = 2463 \times 2 + 15$

$2463 = 15 \times 164 + 3$

$15 = 3 \times 5 + 0$

$\gcd = 3$

(e) gcd (1000, 5040)

$5040 = 1000 \times 5 + 40$

$1000 = 40 \times 25 + 0$

gcd = 40

(f) gcd (9888, 6060)

$9888 = 6060 \times 1 + 3828$

$6060 = 3828 \times 1 + 2232$

$3828 = 2232 \times 1 + 1596$

$2232 = 1596 \times 1 + 636$

$1596 = 636 \times 2 + 324$

$636 = 324 \times 1 + 312$

$324 = 312 \times 1 + 12$

$312 = 12 \times 25 + 12$

$12 = 12 \times 1 + 0$

gcd = 12

$\boxed{3.7}$ 1) (a) 10, 11

gcd (10, 11) = 1

$1 = 10(-1) + 11(1)$

(b) 21, 44

gcd (21, 44) = 1

$44 = 21 \times 2 + 2$

$21 = 2 \times 10 + 1$

$2 = 1 \times 2 + 0$

gcd (21, 44) = 1

$= 21 - 10 \times 2$

$= 21 + (-10) \times 2$

$= 21 + (-10) \times (44 - 2.21)$

$= 21 + (-10) \times 44 + 20.21$

$= 21.(21) + (-10).44$

(c) 36, 48

$48 = 36 \times 1 + 12$

$36 = 12 \times 3 + 0$

gcd (36, 48) = 12

$= 48 - 36 \times 1$

$= 48(1) + (-1)36$

(d) 34, 55

$55 = 34 \times 1 + 21$

$34 = 21 \times 1 + 13$

$21 = 13 \times 1 + 8$

$13 = 8 \times 1 + 5$

$8 = 5 \times 1 + 3$

$5 = 3 \times 1 + 2$

$3 = 2 \times 1 + 1$

$2 = 1 \times 2 + 0$

gcd (34, 55) = 1

$= 1 \times 3 - 2 \times 1$

$= 1 \times 3 - (5 - 3 \times 1) \times 1$

$= 3 \times 1 + (-1)(5 - 1 \times 3)$

$= 3 \times 1 - 1 \times 5 + 1 \times 3$

$= 2 \times 3 - 1 \times 5$

$= 2 \times (8 - 5 \times 1) + (-1) \times 5$

$= 2 \times 8 + (-3) \times 5$

$= 2 \times 8 + (-3)(13 - 8 \times 1)$

$= 5 \times 8 + -3 \times 13$

$= 5 \times (21 - 13 \times 1) + (-3)\, 13$

$= 5 \times 21 + (-8) \times 13$

$= 5 \times 21 + (-8)(34 - 21 \times 1)$

$= 13 \times 21 + (-8).34$

$= 13 \times (55 - 34 \times 1) + (-8).34$

$= 13(55) + (-21).34$

$= 34(-21) + 55(13)$

(e) 117, 213

$213 = 117 \times 1 + 96$

$117 = 96 \times 1 + 21$

$96 = 21 \times 4 + 12$

$12 = 1 \times 9 + 3$

$9 = 3 \times 3 + 0$

$\gcd(117, 213) = 3$

$= 12 - 9 \times 1$

$= 12 - (21 - 12)$

$= 12 \times 2 - 21 \times 1$

$= (96 - 21 \times 4) \times 2 - 21$

$= 96 \times 2 - 21 \times 8 - 21$

$= 96 \times 2 - (117 - 96) \times 9$

$= 96 \times 2 - 117 \times 9 + 96 \times 9$

$= 96 \times 11 - 117 \times 9$

$= (213 - 117) \times 11 - 117 \times 9$

$= 213 \times 11 - 117 \times 20$

$= 213(11) + 117(-20)$

(f) $(0, 223)$

$223 = 1 \times 0 + 223$

$0 = 0 \times 223 + 0$

$\gcd = 223$

$= 1 \times 0 + 223 \times 1$

g) $123, 2347$

$2347 = 123 \times 19 + 10$

$123 = 12 \times 10 + 3$

$10 = 3 \times 3 + 1$

$3 = 3 \times 1 + 0$

$\gcd(123, 2347) = 1$

$= 10 - 3 \times 3$

$= 10 - 3 \times (123 - 12 \times 10)$

$= 37 \times 10 + (-3) \times 123$

$= 37 \times (2347 - 123 \times 19) + (-3) \times 123$

$= 37 \times 2347 + (-706) 123$

(h) $3454, 4666$

$4666 = 1 \times 3454 + 1212$

$3454 = 1212 \times 2 + 1030$

$1212 = 1030 \times 1 + 182$

$1030 = 182 \times 5 + 120$

$182 = 120 \times 1 + 62$

$120 = 62 \times 1 + 58$

$62 = 58 \times 1 + 4$

$58 = 14 \times 4 + 2$

$4 = 2 \times 2 + 0$

$\gcd(3454, 4666) = 2$

$= 58 - 14 \times 4$

$= 1 \times 58 + (-14) \times 4$

$= 1 \times 58 + (-14) \times (62 - 1 \times 58)$

$= 15 \times 58 + (-14) \times 62$

$= 15(120 - 1 \times 62) + (-14) \times 62$

$= 15 \times 120 + (-29) \times 62$

$= 15 \times 120 + (-29)(182 - 1 \times 120)$

$= 15 \times 120 + 29 \times 120 + (-29)(182)$

$= 44(1030 - 5 \times 182) + (-29) 182$

$= 44 \times 1030 + (-249) \times 182$

$= 44 \times 1030 + (-249)(1212 - 1 \times 1030)$

$= 293(1030) + (-249) \times 182$

$= 293(3454 - 2 \times 1212) + (-249) 121$

$= 293 \times 3454 + (-835) \times 1212$

$= 293 \times 3454 + (-835)(4666 - 1 \times 34$

$= (1128) \times 3454 + (-835) 4666$

(i) $9999, 11111$

$11111 = 9999 \times 1 + 1112$

$9999 = 8 \times 1112 + 1103$

$1112 = 1 \times 1103 + 9$

$1103 = 122 \times 9 + 5$

$9 = 5 \times 1 + 4$

$5 = 1 \times 4 + 1$

$4 = 4 \times 1 + 0$

$\gcd(9999, 11111) = 1$

$= 5 - 4\times 1$

$= 5 - (9 - 5\times 1)1$

$= 5\times 2 - 9\times 1$

$= (1103 - 122\times 9)\times 2 - 9\times 1$

$= 1103(2) - 244\times 9 - 9\times 1 = 1103(2) - (245)\times 9$

$= 1103(2) - 245\times(1112 - 1103\times 1)$

$= 1103(2) - 245\times(1112) + 245(1103)$

$= 1103(247) - 245(1112)$

$= (9999 - 1112\times 8)\,247 - 245(1112)$

$= 9999\times(247) - 1112(2221)$

$= 9999(247) - (11111 - 9999\times 1)(2221)$

$= 9999(2468) + 11111(-2221)$

3) Show that –

15 is an inverse of 7 modulo 26.

Solⁿ– $ab = 15.7 = 105$

$= 26.4 + 1$

$= 1 \bmod 26$

So, 15 is an inverse of 7 mod 26.

5) Inverse – 4 modulo 9

$9 = 4.2 + 1$

$\gcd(4,9) = 1$

$= 9 - 2.4$

$= 9(1) + 4(-2)$

This shows -2 is an inverse of 4 modulo 9.

$-2 \bmod 9 = 7 \bmod 9$ – (also)

11) Congruence –

$4x \equiv 5 \bmod 9$

$\Rightarrow 9 \mid (4x - 5)$

$x = -1$

$\therefore \boxed{x = -1 + 9K}, K \in \mathbb{Z}$

CHAPTER 4: INDUCTION AND RECURSION :-

$\boxed{4.1}$ 8) $P(n) = 2 - 2.7 + 2.7^2 - \ldots\ldots + 2(-7)^n = (1 - (-7)^{n+1})/4$

Basic Step: $n = 0$

$\{P(0) \Rightarrow 2(-7)^0 = 2\times 1 = 2\} \rightarrow P(0) = 2$ (simply)

$2 = \dfrac{1 - (-7)^1}{4} = \dfrac{1+7}{4} = \dfrac{8}{4} = 2$

Induction Step: Let $P(K)$ be true

$P(K) = 2 - 2.7 + 2.7^2 - \ldots\ldots + 2(-7)^K = \cancel{2+(-7)}\,(1 - (-7)^{K+1})/4$

T.P :- $P(K+1) = 2 - 2.7 + 2.7^2 - \ldots\ldots + 2(-7)^{K+1}\ \cancel{2(-7)^{K+2}}$

$= (1 - (-7)^{K+2})/4$

$LHS = \dfrac{1 - (-7)^{K+1}}{4} + 2(-7)^{K+1}$

$= \dfrac{1 - (-7)^{K+1} + 8(-7)^{K+1}}{4}$

$= \dfrac{1 - [(-7)^{K+1}(1 - 8)]}{4}$

$= \dfrac{1 - [(-7)(-7)^{K+1}]}{4}$

$= \dfrac{1 - [(-7)^{K+2}]}{4} = \dfrac{1 - (-7)^{K+2}}{4} = RHS$

$\therefore$ By PMI, $P(n)$ is true for all non-negative integers.

21) **Basic Step:** $n = 5$

$2^n = 2^5 = 32$

$n^2 = 5^2 = 25$

$32 > 25$

$\Rightarrow P(5)$ is true

Induction Step: Let $P(K)$ be true $\forall\ K > 4$

$\Rightarrow 2^K > K^2$

To prove: $P(K+1)$ is true

$P(K+1) = 2^{K+1}$

$= 2.2^K$

$= 2^K + 2^K$

$\Rightarrow 2^K + 2^K > K^2 + K^2$ (from induction step)

$\Rightarrow 2^K + 2^K > K^2 + K.K$

$\Rightarrow 2^{K+1} > K^2 + 4K$ $(\because K > 4)$

$\Rightarrow 2^{K+1} > K^2 + 2K + 1$ $(\because 4K > 2K+1$ when $K > 4)$

$\Rightarrow 2^{K+1} > (K+1)^2$

$\therefore P(K+1)$ is true

By principle of mathematical induction $\forall\ n > 4$; $P(n)$ holds true.

33) Basic step: $n=0$

$n^5 - n = 0$

$\therefore P(0)$ is true as 0 divisible by 5

Induction step: Let $P(K)$ be true.

$\Rightarrow 5 \mid (K^5 - K)$

$\Rightarrow 5 \mid K^5$ and $5 \mid K$

To prove: $P(K+1)$ is true.

$P(K+1) = (K+1)^5 - (K+1)$

$= K^5 + 5K^4 + 10K^3 + 10K^2 + 5K - 1 - K - 1$

$= K^5 - K + 5K^4 + 10K^3 + 10K^2 + 5K$

$= (K^5 - K) + 5(K^4 + 2K^3 + 2K^2 + K)$

Now, $5 \mid K^5 - K$ from induction step.

and $5(K^4 + 2K^3 + 2K^2 + K)$ is also divisible by 5, thus $P(K+1)$ is also divisible by 5.

$\Rightarrow P(K+1)$ is true

By PMI, $\forall\ n \geq 0$, $P(n)$ holds true.

34) Basic step: $n=0$

$n^3 - n = 0$ and 6 divides it.

$\therefore P(0)$ is true.

Induction step: Let $P(K)$ be true

$\Rightarrow 6 \mid K^3 - K$

To prove: $P(K+1)$ is true

$P(K+1) = (K+1)^3 - (K+1)$

$= K^3 + 3K^2 + 3K + 1 - K - 1$

$= (K^3 - K) + 3K(K+1)$

Now, 6 divides $(K^3 - K)$ and the term $3K(K+1)$ is clearly divisible by 3, but since it contains a product of 2 consecutive no.'s it must be divisible by 2 as well,

$\therefore 6 \mid 3K(K+1)$

$\Rightarrow P(K+1)$ is true, $\forall\ n \geq 0$, By PMI

$\boxed{4.3}$ 8) (a) $a_n = 4n - 2$

$a_1 = 4(1) - 2 = 4 - 2 = 2$

$a_2 = 4(2) - 2 = 6 = a_1 + 4$

$a_3 = 4(3) - 2 = 10 = a_2 + 4$

$\vdots$

$a_n = a_{n-1} + 4$

(b) $a_n = 1 + (-1)^n$

$a_1 = 1 - 1 = 0$

$a_2 = 2 = 2$

$a_3 = 0 = a_1$

$a_4 = 1+1 = 2 = a_2$

$\vdots$

$a_n = a_{n-2}$, where $n \geqslant 3$

(c) $a_n = n(n+1)$

$a_1 = 1.2 = 2$

$a_2 = 2.3 = 6 = a_1 + 2\times2$

$a_3 = 3.4 = 12 = a_2 + 2\times3$

$\vdots$

$a_n = a_{n-1} + 2.n$, where $n \geqslant 2$

(d) $a_n = n^2$

$a_1 = 1$

$a_2 = 4 \quad = a_1 + 2\times2 - 1$

$a_3 = 9 \quad = a_2 + 2\times3 - 1$

$\vdots$

$a_n = a_n + 2n - 1$, where $n \geqslant 2$

CHAPTER 5: COUNTING :-

$\boxed{5.1}$ (a) $|A| = 900$, where $A \in \mathbb{Z}^+$, $100 \leq A \leq 999$

$d = 7$

n_7 (integers divisible by 7) $= \frac{|A|}{d} = 128.57$

$\simeq 128$

(b) $|A| = 900$

odd no.s $= |A| -$ no. of int divisible by 2

$= 900 - \lfloor \frac{900}{2} \rfloor$

$= 900 - 450 = 450$

(c) First digit = 9 ways

2nd ~~digit~~ digits = 1 ways

3rd ~~digits~~ digits = 1 way

$\Rightarrow$ 9 integers have same three decimal digits.

(d) no. of int not divisible by $4 = |A| -$ no. of int div by 4.

$= 900 - \lfloor \frac{900}{4} \rfloor$

$= 900 - 225$

$= 675$

(e) No. of int divisible by 3 or 4 $= n_{3 \text{ or } 4}$

" " " " by $3 = n_3$

" " " " by $4 = n_4$

$\therefore n_{3 \text{ or } 4} = n_3 + n_4 - n_{3 \text{ and } 4}$ or

$n_{3 \text{ or } 4} = n_3 + n_4 - n_{12}$

$n_3 = \lfloor \frac{900}{3} \rfloor = 300$

$n_4 = \lfloor \frac{900}{4} \rfloor = 225$

$n_{12} = \lfloor \frac{900}{12} \rfloor = 75$

$n_{4 \text{ or } 3} = 300 + 225 - 75$

$= 450$

(f) $n_{\text{not }(3 \text{ or } 4)} = |A| - n_{3 \text{ or } 4}$

$= 900 - 450$

$= 450$

(g) $n_{3, \text{not } 4} = n_3 - n_{12} = 300 - 75 = 225$

(h) $n_{3 \text{ and } 4} = n_{12} = 75$

27) First letter = 26 ways

Second letter = 26 ways

First digit = 10 ways

Second digit = 10 ways

Third digit = 10 ways

Fourth digit = 10 ways

$\therefore$ 1st case $= 26^2 \times 10^4 = 6,760,000$

First digit = 10 ways

Second digit = 10 ways

∴ 2 case = 10 × 26' = 45,697,600

∴ Total liscence plate made = 52,457,600 ways

$\boxed{5.3}$ 12) a) Exactly 3 1's $= {}^{12}C_3$

$= 220$

b) at most four 1's $= {}^{12}C_3 + {}^{12}C_2 + {}^{12}C_1 + {}^{12}C_0$

$= 220 + 66 + 12 + 1$

$= 299$

c) at least three 1's $= 2^{12} - {}^{12}C_0 - {}^{12}C_1 - {}^{12}C_2$

$= 2^{12} - 1 - 12 - 66$

$= 4017$

d) an equal no. s of 0's and 1's = 6 1's and 6 0's

$= {}^{12}C_6$

$= \frac{12!}{6!\,6!} = 924$

24) 10 women can be arranged in 10! ways or ${}^{10}P_{10}$

_W_W_W_W_W_W_W_W_W_W_

~~No. of~~ So, the 6 men can be arranged as –

$= {}^{11}P_6$

∴ No. of ways for 10 women & 6 men to stand in a line so that no two men stand next to each other is –

$= {}^{11}P_6 \times {}^{10}P_{10}$

$= 55440 \times 3{,}628{,}800$ ways.

34) Department contains –

Men = 10

Women = 15

∴ No. of ways to form a committee with 6 members if it must have more women than men is

$= {}^{15}C_6 \times {}^{10}C_0 + {}^{15}C_5 \times {}^{10}C_1 + {}^{15}C_4 \times {}^{10}C_2$

$= \frac{15!}{6!\,9!} \times \frac{10!}{0!\,10!} + \frac{15!}{5!\,10!} \times \frac{10!}{1!\,9!} + \frac{15!}{4!\,11!} \times \frac{10!}{8!\,2!}$

$= 5005 + 30030 + 61425$

$= 96460$

$\boxed{5.4}$ 2) a) $(x+y)^5$ ← by combinatorial reasoning.

$(x+y)^5 = (x+y)(x+y)(x+y)(x+y)(x+y)$

$= {}^5C_0\, x^5 + {}^5C_1\, x^4 y + {}^5C_2\, x^3 y^2 + {}^5C_3\, x^2 y^3 + {}^5C_4\, x y^4 + {}^5C_5\, y^5$

$= \frac{5!}{0!\,5!} x^5 + \frac{5!}{1!\,4!} x^4 y + \frac{5!}{2!\,3!} x^3 y^2 + \frac{5!}{3!\,2!} x^2 y^3 + \frac{5!}{4!\,1!} x y^4 + \frac{5!}{5!\,0!} y^5$

$= x^5 + 5x^4 y + 10 x^3 y^2 + 10 x^2 y^3 + 5 x y^4 + y^5$

(b) $(x+y)^5$ – by Binomial th^m.

$= \binom{5}{0} x^5 y^0 + \binom{5}{1} x^4 y^1 + \binom{5}{2} x^3 y^2 + \binom{5}{3} x^2 y^3 + \binom{5}{4} x^1 y^4 + \binom{5}{5} x^0 y^5$

$= x^5 + 5x^4 y + 10x^3 y^2 + 10x^2 y^3 + 5xy^4 + y^5$

7. $(2-x)^{19}$

Coefficient of x^9 –

$(2-x)^{19} = \binom{19}{r} 2^{19-r} \cdot (-x)^r$

$= \binom{19}{9} 2^{19-9} \cdot (-x)^9$

$= (-{}^{19}C_9 \cdot 2^{10}) \cdot x^9$

∴ Coefficient of x^9 in the expansion $(2-x)^{19}$ –

$= -46189.$

CHAPTER 7 : RELATIONS :-

$\boxed{7.1}$ 7. $(x, y) \in R$

(a) $x \neq y$

→ Symmetric

(b) $xy \geq 1$

→ Symmetric and transitive

(c) $x = y+1$ or $x = y-1$

→ Symmetric

(d) $x \equiv y \pmod 7$

→ Reflexive, symmetric, transitive

(e) x is a multiple of y

→ Reflexive, transitive

(f) x and y are both negative or both non-negative

→ Reflexive, symmetric, transitive

(g) $x = y^2$

→ Antisymmetric

(h) $x \geq y^2$

→ Antisymmetric, transitive.

25. $R = \{(a,b) \mid a \text{ divides } b\}$

(a) R^{-1}

$R^{-1} = \{(a,b) \mid b \text{ divides } a\}$

(b) $\bar{R}$

$\bar{R} = \{(a,b) \mid a \text{ does not divide } b\}$

7. (a) $x \neq y$

→ R is symmetric as $(1,2) \in R \Rightarrow (2,1) \in R$

(b) $xy \geq 1$

→ R is symmetric because if $xy \geq 1$, then $yx = xy \geq 1$

→ R is transitive as if $xy \geq 1$ and $yz \geq 1$, then x & y and y & z are two non-zero integers of same sign. So x and z are non-zero integers of the same sign $\Rightarrow xz \geq 1$

(c) $x = y+1$ or $x = y-1$

→ R is symmetric as for $x = y+1$; $y = x-1$ and for $x = y-1$; $y = x+1$

(d) $x \equiv y \pmod 7$

→ R is reflexive as $x \equiv x \pmod 7$ is true for any integer x.

→ R is symmetric as if $x \equiv y \pmod 7$ then $y \equiv x \pmod 7$

(e) x is a multiple of y.

→ R is reflexive as x is always a multiple of itself.

→ R is transitive as if x is a multiple of y and y is a multiple of z, then x is also a multiple of z.

(f) x and y are both negative or both non-negative.

→ R is reflexive as x always has the same sign as itself.

→ R is also symmetric as if x and y are both negative or both non-negative then y and x are also ~~about~~ both negative or both non-negative.

→ R is also transitive as for x and y to be (-ve) or non-negative or non-negative and y and z to be (-ve) or non-negative. x and z must be -ve or non-negative.

(g) $x = y^2$

→ It is Antisymmetric.

(h) $x \geq y^2$

→ R is transitive as if $x \geq y^2$ and $y \geq z^2$, then

$$x \geq y^2 \geq y \geq z^2$$

→ R is also antisymmetric.

$\boxed{7.3}$ 10. (a) $\{(a,b) \mid a \leq b\}$

$$= 1000 + \frac{1000(999)}{2}$$

$$= 500{,}500$$

(b) $\{(a,b) \mid a = b \pm 1\}$

Subdiagonals will be 1 in the matrix (below and above)

$i = j + 1, \ m_{ij} = 1$

So, no. of 1 = 999 + 999

= 1998.

(c) $\{(a,b) \mid a+b = 1000\}$

Total element = 10,00,000

a + b = 1000

No. of ~~element~~ entries = 999

(d) $\{(a,b) \mid a+b \leq 1001\}$

Non-zero elements are –

$1000 + 999 + 998 + \ldots\ldots + 1$

$= \frac{1000 \times 1001}{2} = 500500$

(e) $\{(a,b) \mid a \neq 0\}$

Here $a \neq 0$ as 0 is not in A

$A = \{1, 2, 3, \ldots\ldots, 1000\}$

So, $a \in A$.

Thus, all 10,00,000 entries in the matrix are one.

15. $M_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$R^2 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

$R^3 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ $\quad \because M_{R^3} = M_{R^2} \odot M_R$

$R^4 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ $\quad \because M_{R^4} = M_{R^3} \odot M_R$