Tests of Hypothesis (Chapter - 10)

Statistical hypothesis - The statistical hypothesis is a conclusion to the basis of incomplete information about one or more population.

Null hypothesis

A hypothesis we wish to test is known as Null hypothesis. It is denoted by H_0

Alternative hypothesis

The rejection of Null hypothesis leads to ~~the~~ accept the alternative hypothesis. It is denoted by H_1

Type I error

	H_0 is null hypothesis	H_1 is alternative hypothesis
	H_0 is true	H_0 is false
Do not reject H_0	True decision	Type - II error
Reject H_0	Type I error	Correct decision

The rejection of null hypothesis H_0 which is true is known as Type I error

The ~~null~~ or non rejection of null hypothesis which is false is known as Type II error

• The probability of Type-I and Type-II error

$P(\text{Type-I error}) = \alpha$

$P(\text{Type-II error}) = \beta$

Note : The type of error can be minimized by increasing the number of sample size.

~~Ex 10.2 A real estate~~

10.2 A sociologist is concerned about the effectiveness of a training course designed to get more drivers to use seat belts in automobiles

a) What hypothesis is she testing if she commits a type I error by erroneously concluding that the training course is ineffective?

b) What hypothesis is she testing if she commits a type-II error by erroneously concluding that the training course is effective?

Soln a) Conclusion : Training course is ineffective

H_1 : Training course is ineffective

H_0 : Training course is effective

b) Conclusion : Training course is effective

H_0 : Training course is effective

H_1 : Training course is ineffective

10.1 Suppose that an allergist wishes to test the hypothesis that at least 30 % of the public is allergic to some cheese products. Explain how the allergist could commit

(a) A type-I error

(b) A type-II error

Soln (a) Conclude that fewer than 30 % of the public are allergic to some cheese products, when, in fact 30 % or more are allergic

(b) Conclude that atleast 30 % of the public are allergic to some cheese products when, in fact fewer than 30 % are allergic

Testing of Hypothesis

Step 1 : State the null Hypothesis H_0 and alternative hypothesis H_1

Step 2 : Choose a significance level α

Step 3 : Choose an appropriate statistical list and establish critical region based on α

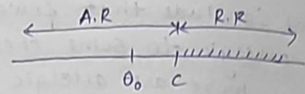
Step 4 : Reject H_0 if the computed test statistics is in the critical region

Step 5: Draw conclusion

Statistical test

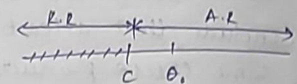
① for mean μ with known variance (Z-test)
② for mean μ with unknown variance (T-test)
③ for variance (Chi-square tests)

For critical region

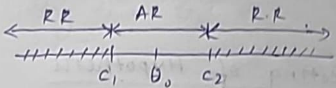


one sided tests $H_0 : \theta = \theta_0$

Case I: $H_1 : \theta > \theta_0$



Case II: $H_1 : \theta < \theta_0$



Case III: $H_1 : \theta \neq \theta_0$

Two sided tests

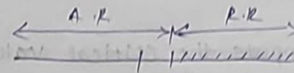
AR: Acceptance ~~Rate~~ Region
RR: Rejection Region

H_0	value of the statistics	H_1	critical Region
$\mu = \mu_0$	$Z = \dfrac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$, σ known	$\mu < \mu_0$	$Z < -Z_\alpha$
		$\mu > \mu_0$	$Z > Z_\alpha$
		$\mu \neq \mu_0$	$Z < -Z_{\frac{\alpha}{2}}$ or $Z > Z_{\frac{\alpha}{2}}$

Ex 10.3 A random sample of 100 recorded deaths in the United States during the past year showed an avg. life span of 71.8 yrs. Assuming a population standard deviation of 8.9 yrs, does this seem to indicate that the mean life span today is greater than 70 years? Use a 0.05 level significance

Soln: $\mu = 100$, $\bar{x} = 71.8$
$\sigma = 8.9$, $\alpha = 0.05$

$H_1 : \mu > 70$
$H_0 : \mu = 70$



$$Z = \frac{\bar{x} - \mu_0}{\left(\frac{\sigma}{\sqrt{n}}\right)}, \quad \mu_0 = 70$$

$$= \frac{71.8 - 70}{\left(\frac{8.9}{10}\right)} = 2.02$$

$C = 1.645$

$P(Z < Z_\alpha) = 1 - \alpha$

$C = Z_\alpha$

∴ the calculated value of Z lies in the rejection region so the null hypothesis is rejection

The avg. life of U.S people is more than 70 years

Testing of ~~mean & ai~~

Testing of Hypothesis for mean μ with unknown variance

Step 1: State the null hypothesis H_0 and alternate hypothesis H_1

Step 2. Choose a significance level (α)

Step 3. By T-Test, $t = \dfrac{\bar{x} - \mu_0}{\left(\frac{s}{\sqrt{n}}\right)}$, $\bar{x}$ = sample mean

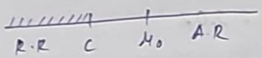
s = sample standard deviation

n = sample size

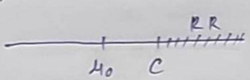
Step 4. Compute the critical value C from t-distribution table with $(n-1)$ degree of freedom.

Case I: H_0: Null hypothesis — critical region

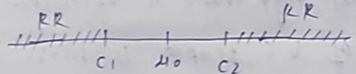
If H_1: $\mu < \mu_0$, $t < -t_\alpha$



Case II: If H_1: $\mu > \mu_0$, $t > t_\alpha$



Case III: If H_1: $\mu \neq \mu_0$, $t < -t_{\frac{\alpha}{2}}$ or $t > t_{\frac{\alpha}{2}}$



Step 4: Draw the conclusion

Ex 10.5 The Edison electric Institute has published figures on the no. kwh used annually by various home appliances. It is claimed that a vacuum cleaner uses an avg. of 46 kwh per yr. If a random sample of 12 homes included in a planned study indicates that vacuum cleaners use an avg. of 42 kwh per year with a standard deviation of 11.9 kwh, does this suggest at the 0.05 level of significance that vacuum cleaners use, on avg., less than 46 kwh annually? Assume the popn. of kwh is to be normal

Soln: μ_0: $\mu = 46$ $n = 12$, $\alpha = 0.05$

μ_1: $\mu < 46$ $\bar{x} = 42$, $s = 11.9$

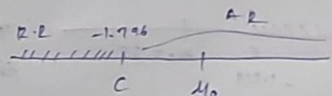
Since the problem about testing the mean with unknown variance we can use t-distribution.

$$t = \frac{\bar{x} - \mu_0}{\left(\frac{s}{\sqrt{n}}\right)} = \frac{42 - 46}{\left(\frac{11.9}{\sqrt{12}}\right)} = -1.16$$

Critical point (C)

$\nu = n - 1 = 11$

$C = -1.796$



$\therefore$ μ_0 is accepted

$\therefore$ conclude that the avg. use of home appliance is greater than 46 hrs.

10.29 Past experiences indicate that the time req. for high school seniors to complete a standardized test is a normal random variable with a mean of 35 mins. If a random sample of 20 high school seniors took an avg. of 33.1 mins to complete this test with a standard deviation of 4.3 mins, test the hypothesis, at the 0.05 level of significance, that $\mu = 35$ mins against the alternative that $\mu < 35$ mins.

Sol^n^ $H_0 : \mu = 35$ $\quad n = 20$, $s = 4.3$

$H_1 : \mu < 35$ $\quad \bar{x} = 33.1$ $\quad \alpha = 0.05$

$\because$ ~~the testing~~ Apply t-distribution as it is mean with unknown variance

$$t = \frac{\bar{x} - \mu_0}{\left(\frac{s}{\sqrt{n}}\right)} = \frac{33.1 - 35}{\left(\frac{4.3}{\sqrt{20}}\right)} = -1.98$$

Critical point (C).

$v = 20 - 1 = 19$

$c = -1.729$

$t = -1.98$

c $\quad \mu_0 = 35$

-1.729

The value of t lies in the rejection region.

$\rightarrow$ H_0 is rejected

$\therefore$ conclude that

10.23 Test the hypothesis that the avg. content of containers of a particular lubricant is 10 liters if the contents of a random sample of 10 containers are 10.2, 9.7, 10.1, 10.3, 10.1, 9.8, 9.9, 10.4, 10.3, and 9.8 litres. Use a 0.01 level of significance and assume that the distribution of contents is normal.

Sol^n^ $\alpha = 0.01$, $n = 10$

$H_0 : \mu = 10$

$H_1 :$

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{10.2 + 9.7 + 10.1 + 10.3 + 10.1 + 9.8 + 9.9 + 10.4 + 10.3 + 9.8}{10}$$

$= 10.06$

$$s = \frac{1}{n(n-1)} \Sigma$$

Testing of hypothesis for variance

Step 1: State null hypothesis H_0 and alternative hypothesis H_1. $H: \sigma^2 < \sigma_0^2$, $\sigma^2 > \sigma_0$, $\sigma^2 \neq \sigma_0^2$

Step 2: Choose a significance level α

Step 3: The statistical test is Chi-squared test. Find the value of critical point from Chi squared distribution with $\nu = n-1$, degree of freedom.

Step 4: Compute $\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$

where n is sample size and s^2 sample variance

Step 5: Critical Region

If $H_1: \sigma^2 < \sigma_0^2$, critical region is $\chi^2 < \chi^2_{1-\alpha}$

If $H_1: \sigma^2 > \sigma_0^2$, critical region is $\chi^2 > \chi^2_{\alpha}$

If $H_1: \sigma^2 \neq \sigma_0^2$, the critical region is

$\chi^2 < \chi^2_{1-\frac{\alpha}{2}}$ or $\chi^2 > \chi^2_{\frac{\alpha}{2}}$

Step 6: Draw conclusion.

Ex 10.12 A manufacturer of car batteries claims that the life of the company's batteries is approximately normally distributed with a standard deviation equal to 0.9 year. If a random sample of 10 of these batteries has a standard deviation of 1.2 yrs, do you think that $\sigma > 0.9$ year? Use a 0.05 level of significance.

Soln $\sigma = 0.9$, $n = 10$, $s = 1.2$,

$\sigma > 0.9$, $\alpha = 0.05$.

$H_0: \sigma^2 = \sigma_0^2 = (0.9)^2 = 0.81$

$H_1: \sigma^2 > 0.81$

chi squared distribution, $\nu = n-1 = 9$, $\alpha = 0.05$

$c = 16.919 \Rightarrow c = \chi^2_{\alpha}$

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{9 \times (1.2)^2}{0.81} = 16$$

$\therefore$ critical region is $\chi^2 > \chi^2_{\alpha}$

$\chi^2 > 16.919$

$\therefore$ The computed value χ^2 which is 16 does not lies in the critical region, so the null hypothesis is ~~rejected~~ accepted.

conclu

10.68 Past experience indicate that the time req. for high school seniors to complete a standardized test is a normal random variable with a standard deviation of 6 minutes. Test the hypothesis that $\sigma = 6$ against the alternative that $\sigma < 6$ of a random sample of the test times of 20 high school seniors has a standard deviation $s = 4.51$. We a 0.05 level of significance

Sol: $H_0 : \sigma^2 = 36$ $\sigma_0^2 = 36$, $n = 20$, $s = 4.51$, $\alpha = 0.05$
$H_1 : \sigma^2 < 36$ $\nu = n - 1 = 19$,

$$\chi^2_{1-\alpha} = \chi^2_{0.95} = 10.12$$

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{19 \times (4.51)^2}{3.6} = 10.75$$

Critical region

$$\chi^2 < \chi^2_{1-\alpha}$$
$$\chi^2 < 10.12$$

$\Rightarrow$ Null hypothesis is accepted, so alternative hypothesis is rejected

conclude that σ^2 is not less than 36

10.71 A soft drink dispensing machine is said to be out of control if the variance of the contents exceeds 1.15 decilitres. If a random sample of 25 drinks from this machine has a variance of 2.03 decilitres, does this indicate the 0.05 level of significance that the machine is out of control? Assume that the contents are approximately normally distributed.

Sol: $\sigma^2 = 1.15$, $n = 25$
$s^2 = 2.03$, $\alpha = 0.05$
$H_0 : \sigma^2 = \sigma_0^2 = 1.15$
$\sigma^2 > 1.15$

$\nu = n - 1 = 24$

$\chi^2_\alpha = 36.415 = c =$ critical point

$$\chi^2 = \frac{24 \times 2.03}{1.15} = 42.36$$

Goodness of Fit Test

Step 1: State the null hypothesis H_0 and alternative hypothesis H_1

Step 2: Choose a significance level α

Step 3: Compute $\chi^2 = \sum_{i=1}^{k} \frac{(o_i - e_i)^2}{e_i}$

where k is the number of cells

Step 4: Find critical region which is

$$\chi^2 > \chi^2_\alpha$$

where $\nu = k-1$ degree of freedom

Step 5: Draw the conclusion.

Q- Suppose that a die is rolled 120 times. The observed and expected values are given in the following table

Face	1	2	3	4	5	6
Observed	20	22	17	18	19	24
Expected	20	20	20	20	20	20

Test the hypothesis at 0.05 significance level. The die is fair.

Sol^n H_0 : The die is fair (Null hypothesis)

H_1 : The die is not fair (Alternative hypothesis)

$\alpha = 0.05$

$$\chi^2 = \sum_{i=1}^{k=6} \frac{(o_i - e_i)^2}{e_i}$$

$$= \frac{0 + 4 + 9 + 4 + 1 + 16}{20}$$

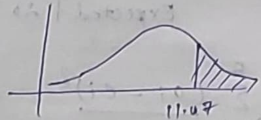
$$= \frac{34}{20} = 1.7$$

Critical region

$\chi^2 > \chi^2_\alpha$ $\quad \nu = k-1 = 5$

$\Rightarrow \chi^2 > \chi^2_{0.05}$

$\Rightarrow \chi^2 > 11.07$



$\because$ The computed value of χ^2 doesn't lie in critical region

$\Rightarrow$ The null hypothesis is accepted

Conclude that the die is a fair die.

10.80 The grades in a statistics course for a particular semester were as follows:

Grade	A	B	C	D	F
f	14	18	32	20	16

Test the hypothesis at 0.05 level of significance, that the distribution of grades is uniform

Sol^n: H_0 : Distribution of grade is uniform

H_1 : Distribution of grade is non-uniform

$\alpha = 0.05$, $K = 5$

Grade	A	B	C	D	F
observed	14	18	32	20	16
Expected	20	20	20	20	20

$$\chi^2 = \sum_{i=1}^{5} \frac{(o_i - e_i)^2}{e_i}$$

$$= \frac{36 + 4 + 64 + 10 + 16}{20}$$

$$= \frac{120}{20} = 10$$

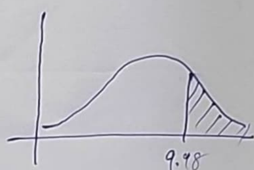
Critical Region

$\nu = K - 1 = 4$

$\chi^2 > \chi^2_\alpha$

$\Rightarrow \chi^2 > \chi^2_{0.05}$

$\Rightarrow \chi^2 > 9.48$



$\because$ The computed value of χ^2 falls in the critical region

$\Rightarrow$ The null hypothesis is rejected

Conclude that the distribution of grade is non-uniform.

10.83 A coin is thrown until a head occurs and the number X of tosses recorded. After repeating the experiment

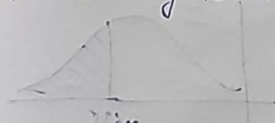
10.87 A random sample of 90 adults is classified acc. to gender and the no. of hrs of television watched during a week:

	Gender	
	Male	Female
over 25 hrs	15	29
under 25 hrs	27	19

use a 0.01 significance level and test the hypothesis that the time spent watching television is independent of whether the viewer is male or female.

Sol^n: H_0 : Watching television is independent of whether the view is male or female

H_1 : Watching television is dependent



Gender

	Male	Female	
over 25 yrs	15	29	44
under 25 yrs	27	19	46
	42	48	90

$E(15) = \frac{44 \times 42}{90} = 20.5$, $E(29) = \frac{44 \times 48}{90} = 23.46$

$E(27) = \frac{46 \times 42}{90} = 21.5$ $E(19) = \frac{46 \times 48}{90} = 24.5$

$$\chi^2 = \sum_{i=1}^{k=4} \frac{(o_i - e_i)^2}{e_i}$$

$$= \frac{30.25}{20.5} + \frac{30.25}{23.5} + \frac{30.25}{21.5} + \frac{30.25}{24.5}$$

$$= \cancel{1.47}\ 1.47 + 1.28 + 1.40 + 1.23$$

$$\approx 5.45$$

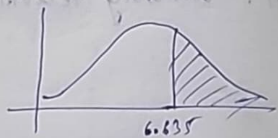
Critical region

$\chi^2 > \chi^2_\alpha$ $\quad v = (r-1) \times (c-1)$

$\quad = 1 \times 1 = 1$

$\Rightarrow \chi^2 > \chi^2_{0.01}$ $\quad \alpha = 0.01$

$\Rightarrow \chi^2 > \cancel{\chi^2}\ 6.635$



The computed value of χ^2 does not lie in the critical region

$\Rightarrow$ Null hypothesis is accepted

conclude that watching television is independent of whether the viewer is male or female.

Linear Regression and correlation (Chapter-11)

Given the samples (x_i, y_i), $i = 1, 2, 3, \ldots, n$

The regression line equation for the above data by least square method is

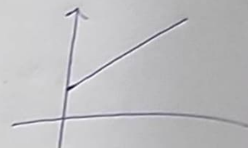
$$y = b_0 + b_1 x$$

where
$$b_1 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}$$

$$\boxed{\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i}$$

$$b_0 = \bar{y} - b_1 \bar{x}$$

are regression coefficients



correlation coefficients

Suppose (x_i, y_i) are given samples for the random variable X and Y, $i = 1, 2, \ldots, n$

The correlation coefficient analysis to measure the strength of X and Y. It can be determined by the help of correlation coefficient (r)

$$r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}}$$

$$S_{xy} = \frac{1}{n-1}\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})$$

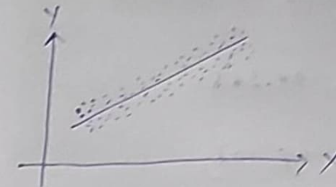
$$S_{xx} = \frac{1}{n-1}\sum_{i=1}^{n} (x_i - \bar{x})^2$$

$$S_{yy} = \frac{1}{n-1}\sum_{i=1}^{n} (y_i - \bar{y})^2$$

$$-1 \leq r \leq 1$$

If $r = 0$, X and Y are uncorelated

$r = 1$



$r = -1$

11.43 Compute and interpret the correlation coefficient for the following grades of 6 students selected at random

Mathematics grade	70	92	80	74	65	83
English grade	74	84	63	87	78	90

S_{xy}

$\bar{x} = 77.33 \quad \bar{y} = 79.33$

$$S_{xy} = (70-77.3)(74-79.33) + (92-77.33)(84-79.33) + (80-77.3)(63-79.33) + (74-77.3)(87-79.33) + (65-77.3)(78-79.33) + (83-77.3)(90-79.33)$$

$$= \frac{115.34}{5} = 23.068$$

$$S_{xx} = \frac{471.34}{5} = 94.268$$

$$S_{yy} = \frac{390.54}{5} = 78.108$$

$$r = \frac{23.068}{\sqrt{(94.268)(78.108)}} = 0.26$$