Chapter-04 Mathematical Expectation

Mean for discrete Random variable:

$$\mu = E(X) = \sum_{x} x f(x)$$

Mean for continuous case:

$$\mu = E(X) = \int_{-\infty}^{\infty} x f(x)\, dx$$

variance of X: σ^2

variance for discrete Random variable:

$$\sigma^2 = \sum (x-\mu)^2 f(x)$$

variance for continuous case:

$$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$$

$f(x, y) \to$ given

$$\mu_X = E(X) = \sum_{x} x \cdot g(x)$$

$$\mu_Y = E(Y) = \sum_{y} y \cdot h(y)$$

$$E(g(x, y)) = \sum_{x} \sum_{y} g(x, y)\, f(x, y)$$

Two random variable X and Y are independent the expecta of XY:

$$E(XY) = E(X)\,E(Y)$$

Chebyshev's theorem) :

$$\{P(\mu - k\sigma < X < \mu + k\sigma) \geq 1 - \frac{1}{k^2}\}$$

$$\downarrow$$

$$P(-k\sigma < X - \mu < k\sigma) \geq 1 - \frac{1}{k^2}$$

$$\{P(|X - \mu| < k\sigma) \geq 1 - \frac{1}{k^2}\}$$

$$1 - P(|X - \mu| < k\sigma) \leq \frac{1}{k^2}$$

$$\{P(|X - \mu| \geq k\sigma) \leq \frac{1}{k^2}\}$$

Chapter - 5 Binomial Distribution

binomial distribution:

$$b(x, n, p) = {}^{n}C_{x}\, p^{x} q^{n-x}$$

Multinomial distribution:

$$f(x_1, x_2, \ldots x_k; p_1, p_2 \ldots p_k; n) = \frac{n!}{x_1! \cdots x_k!} \times p_1^{x_1} \cdot p_2^{x_2} \cdots p_k^{x_k}$$

Hypergeometric distribution:

$$h(x, N, n, k) = \frac{{}^{k}C_{x}\ {}^{N-k}C_{n-x}}{{}^{N}C_{n}}$$

Multivariate hypergeometric distribution

$$f(x_1, x_2, \ldots x_k; a_1, a_2 \ldots a_k; N; n) = \frac{{}^{a_1}C_{x_1}\ {}^{a_2}C_{x_2} \cdots {}^{a_k}C_{x_n}}{{}^{N}C_{n}}$$

Negative binomial distribution

$$b^*(x, k, p) = {}^{x-1}C_{k-1}\, p^{k} q^{x-k}$$

Geometric distribution:

$$g(x,p) = pq^{x-1}$$

Normal distribution:

$$n(x,\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad ; -\infty < x < \infty$$

Standard normal distribution:

$$Z = \frac{X-\mu}{\sigma}$$

Normal approx to binary:

$$\mu = np \qquad \sigma^2 = npq$$

$$Z = \frac{X-np}{\sqrt{npq}}$$

$$P(X \leq x) = P\left(Z \leq \frac{x+0.5-\mu}{\sigma}\right)$$

$$P(X < x) = P\left(Z < \frac{x-0.5-\mu}{\sigma}\right)$$

chapter-03 Random variable and probability distribution

discrete Probability distribution:

→ $f(x) \geq 0$

→ $\sum_{x} f(x) = 1$

→ $P(X = x) = f(x)$

commutative distribution function:

$$f(x) = P(X < x) = \sum_{t \leq x} f(t)$$

$$P(a < X < b) = F(b) - F(a)$$

continuous Probability distribution:

→ cond^n 1:

$f(x) \geq 0 \;, \forall x \in R$

→ cond^n 2:

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

→ cond^n 3:

$$P(a < x < b) = P(a \leq x < b)$$
$$= P(a < x \leq b)$$
$$= P(a \leq x < b)$$
$$= \int_a^b f(x)\, dx = F(b)$$

commutative distribution function:

$$F(x) = P(X \leq x)$$

$$F(x) = \int_{-\infty}^{x} f(t)\, dt$$

Joint Probability distribution:

(1) For discrete case:

→ $f(x, y) \geq 0$

→ $\sum_x \sum_y f(x, y) = 1$

→ $P(X = x, Y = y) = f(x, y)$

(2) For continuous case:

→ $f(x, y) \geq 0$

→ $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)\, dx\, dy = 1$

→ $P(a < x < b, c < y < d) = \int_a^b \int_c^d f(x, y)\, dy\, dx$

Marginal distribution: (discrete case)

→ Marginal distribution along x:

$$g(x) = \sum_{y} f(x, y)$$

→ Marginal distribution along Y:

$$h(y) = \sum_{x} f(x, y)$$

continuous case:

→ Marginal distribution along x:

$$g(x) = \int_{-\infty}^{\infty} f(x, y)\, dy$$

→ Marginal distribution along Y:

$$h(y) = \int_{-\infty}^{\infty} f(x, y)\, dx$$

condn distribution (y given $x = x$)

$$f(y|x) = \frac{f(x, y)}{g(x)}, \quad g(x) > 0$$

condn distribution of (x given $Y = y$)

$$f(x|y) = \frac{f(x, y)}{h(y)}, \quad h(y) > 0$$

→ discrete case:

$$\sum_{a < x < b} f(x|y)$$

→ continuous case:

$$P(a < x\ b|y) = \int_{a}^{b} f(x|y)\, dx$$

Independent Random variable:

$$f(x, y) = g(x)\, h(y)$$

chapter-02 probability of an Event

A : Event

S : sample space.

$$P(A) = \frac{|A|}{|S|}$$

$$P(\phi) = \frac{|\phi|}{|S|} = 0$$

$$P(S) = \frac{|S|}{|S|} = 1$$

↓

Largest set

Condn probability: Event A given B $P(A|B)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) > 0$$

→ Event B given A $P(B|A)$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}, \quad P(A) > 0$$

$$P(A|B) = P(A)$$

$$P(B|A) = P(B)$$

Independent event: two event A and B are indepen-dent iff and only if $P(A|B) = P(A)$, $P(B|A) = P(B)$

A and B are independent: $P(A \cap B) = P(A) \times P(B)$

$$P(A|B) = \frac{P(A\cap B)}{P(B)}, \quad P(B)>0 \quad \text{— (I)}$$

$$P(B|A) = \frac{P(A\cap B)}{P(A)}, \quad P(A)>0 \quad \text{— (II)}$$

as, two event A and B are independent $P(A|B) = P(A)$
$P(B|A) = P(B)$

from (I)

$$P(A|B) = P(A) = \frac{P(A\cap B)}{P(B)}$$

$$\boxed{P(A\cap B) = P(A)\,P(B)}$$

from (II) $P(B|A) = P(B) = \frac{P(A\cap B)}{P(A)}$

$$\boxed{P(A\cap B) = P(A)\,P(B)}$$

Theorem: ~~P(A'~~ Prove that if 2 Events A and B are independent then their complements A' and B' are also independent.

LHS

$$P(A'\cap B') = P((A\cup B)')$$
$$= 1 - P(A\cup B)$$
$$= 1 - [P(A) + P(B) - P(A\cap B)]$$
$$= 1 - P(A) - P(B) + P(A)P(B)$$
$$= 1 - P(A) - P(B)[1 - P(A)]$$
$$[1-P(A)][1-P(B)]$$
$$P(A')\,P(B') \text{ RHS}$$

Theorem of total probability

if the events $B_1, B_2 \ldots B_k$ constitute a partition of a sample space 'S' such that $P(B_i) \neq 0$, $\{i = 1, 2, \ldots k\}$

for any event A of S : $\boxed{P(A) = \sum_{i=1}^{k} P(B_i \cap A)}$

$$= \sum_{i=1}^{k} P(B_i)\,P(A|B_i)$$

Baye's Rule: If the ~~element~~ events $B_1, B_2, \cdots, B_K$ constitute a partition of a sample space 'S' such that

$P(B_i) \neq 0 \; \{i = 1, 2, \cdots k\}$

$P(A) \neq 0$

$$P(B_r \mid A) = \frac{P(B_r \cap A)}{P(A)}$$

$$= \frac{P(B_r)\, P(A \mid B_r)}{\sum_{i=1}^{K} P(B_i)\, P(A \mid B_i)}$$

chapter - 01

Sample mean: $\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i = \frac{1}{x} (x_1 + x_2 + \cdots x_n)$

sample median:

$$\tilde{x} = \begin{cases} x\left(\frac{n+1}{2}\right), & n \text{ is odd} \\ \frac{1}{2}\left(x_{\frac{n}{2}} + x\left(\frac{n}{2}+1\right)\right) \end{cases}$$

Sample variance:

$$S^2 = \frac{1}{n-1} \left[\sum_{i=1}^{n} (x_i - \bar{x})^2 \right], \text{where } \bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$\frac{1}{n-1} \left[(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots (x_n - \bar{x})^2 \right]$$

Sample standard deviation: $S = +\sqrt{S^2}$