① For $\forall n \geq 1$ & $n, K \in R^+$, $\left\lceil \frac{n}{K} \right\rceil = \left\lfloor \frac{n+K-1}{K} \right\rfloor$

basic step:

$n = 1$.

$$\left\lceil \frac{1}{K} \right\rceil = \left\lfloor \frac{\cancel{1}+K-\cancel{1}}{K} \right\rfloor \Rightarrow 1 = 1$$

$\underline{LHS = RHS}$.

Inductive step:

lets assume. for $n = K$., it is true.

$$\left\lceil \frac{K}{K} \right\rceil = \left\lfloor \frac{K+K-1}{K} \right\rfloor \Rightarrow 1 = \left\lfloor \frac{2K-1}{K} \right\rfloor$$

$$= 2 - \left\lceil \frac{1}{K} \right\rceil = 2 - \underline{1} = 1$$

Prove for $n = K+1$,

$$\left\lceil \frac{K+1}{K} \right\rceil = \left\lfloor \frac{K+\cancel{1}+K-\cancel{1}}{K} \right\rfloor$$

$$1 + \left\lceil \frac{1}{K} \right\rceil = \left\lfloor \frac{2\cancel{K}}{\cancel{K}} \right\rfloor \Rightarrow 1+1 = 2.$$

$\underline{LHS = RHS}$.

hence proved for $\underline{n = K+1}$,

②

For every non-negative integer. n

$$\sum_{i=0}^{n} 3^i = \frac{3^{n+1}-1}{2}.$$

basic step:

for $n = 0$

$$3^0 = \frac{3^{0+1}-1}{2} \Rightarrow 1 = \frac{3-1}{2} \Rightarrow \underline{LHS = RHS}.$$

inductive step:

assume. for $n = K$, it is true.

$$\sum_{i=0}^{K} 3^i = \frac{3^{K+1}-1}{2}.$$

Prove for, $n = K+1$

$$\sum_{i=0}^{K+1} 3^i = \frac{3^{(K+1)+1}-1}{2}.$$

$$LHS = \sum_{i=0}^{K} 3^i + 3^{K+1}$$

$$= \frac{3^{K+1}-1}{2} + 3^{K+1} = \frac{3^{K+1}-1+2(3^{K+1})}{2}.$$

$$= \frac{3(3^{K+1})-1}{2}.$$

$$= \frac{3^{(K+1)+1}-1}{2} = RHS.$$

hence proved for $n = K+1$.

(3)

basic step:

$1 = 2^0$, which is a sum of distinct power of 2.

induction hypothesis:

let j be a natural no., $j \leq K$ can be written as the sum of distinct powers of 2.

inductive step:

we need to show $K+1$ can be written as sum of distinct powers of two.

$K+1$ can be odd or even.

Case I: if $K+1$ is even, then $\frac{(K+1)}{2}$ is an even natural no. less than K.

So, by induction hypothesis.

there exist distinct power $0 < P_1 < P_2 < \dots P_n$ &

$$\frac{(K+1)}{2} = 2^{P_1} + 2^{P_2} + 2^{P_3} + \dots 2^{P_n}$$

$$(K+1) = 2^{P_1+1} + 2^{P_2+1} + 2^{P_3+1} + \dots 2^{P_n+1}$$

thus, K+1 is a sum of distinct powers of 2.

Case II: If K+1 is odd, then K is even, so we can express K in sum of distinct powers of 2. and.
where $0 < P_1 < P_2 < P_3 \dots < P_n$.

$$K = 2^{P_1} + 2^{P_2} + 2^{P_3} + \dots 2^{P_n}.$$

$$K+1 = 2^{P_1} + 2^{P_2} + 2^{P_3} + \dots 2^{P_n} + 2^0$$

thus, K+1 is a sum of distinct powers of 2.

Since, K+1 is proved for both cases, all natural no. can be represented as the sum of distinct power of 2.

(4) Let n be the number of internal nodes in a full binary tree, & let T be that Tree.

basic step:
for n = 1, internal node there can only be 2 leaves
So the statement holds for n = 1;

Inductive:
Assume that for T containg K internal nodes, full binary tree has K+1 leaves.
Prove for: n = K+1
Adding 1 internal node to full binary tree will bring 2 leaves.
∵ The new internal node was a leaf node before, after adding a new node the no. of leaf nodes increases by 1 i.e., K+2 leaf nodes.
hence proved.

(5)

basic step:

For $n = 3$

3 is divisible by 3

$\therefore$ The statement holds true for $n = 3$

Inductive step:

Let the statement be true for an arbitrary number $n = abc$.

$$\therefore n = 10^2 \times a + 10^1 \times b + 10^0 \times c$$
$$= (99+1)a + (9+1)b + c$$
$$= 99a + 9b + a + b + c$$

Now, n is divisible by 3.

we get $\frac{n}{3} = 33a + 3b + \frac{a+b+c}{3}$

This shows that a number is divisible by 3 only when the sum of its digits is divisible by 3.