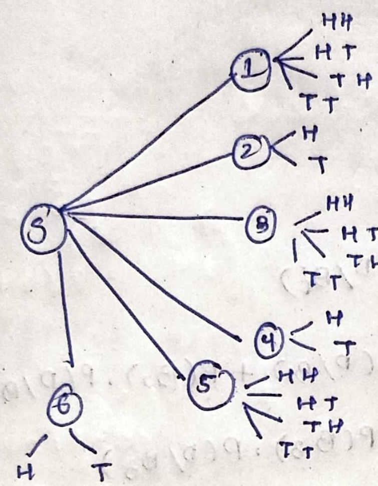
2018 - PS (Solved)

Q.1) a) Sample space

$S = \{1HH, 1HT, 1TH, 1TT, 2H, 2T, 3HH, 3HT, 3TH, 3TT, 4H, 4T, 5HH, 5HT, 5TH, 5TT, 6H, 6T\}$



b) 7 - mathematic books

3 - physics

2 - chemistry

Total 3 books to be selected.

$P(\text{2 maths books selected}) = \frac{{}^7C_2}{{}^{12}C_3} = \frac{21}{220}$

$= 0.095$

c) if the fun^n is a density fun^n then

$$\int_0^1 f(x)\,dx = 1$$

$$\Rightarrow \int_0^1 3x^{-4}\,dx = 3\int_0^1 x^{-4}\,dx$$

$$= 3\left[\frac{x^{-3}}{-3}\right]_0^1$$

$$= [x^{-3}]_0^1 = 1$$

2) a)

Given

$P(B_1) = 0.3$

$P(B_2) = 45\% = 0.45$

$P(B_3) = 0.25$

$P(D/B_1) = 0.02$

$P(D/B_2) = 0.03$

$P(D/B_3) = 0.02$

$$P(D) = \sum_{i=1}^{3} P(B_i) \cdot P(D/B_i)$$

$$= P(B_1) \cdot P(D/B_1) + P(B_2) \cdot P(D/B_2) + P(B_3) \cdot P(D/B_3)$$

$$= 0.3(0.02) + (0.45)(0.03) + (0.025)(0.02)$$

$$= 0.0245.$$

b) $P(B_3/D) = \dfrac{P(B_3 \cap D)}{P(D)} = \dfrac{P(B_3) \cdot (D/B_3)}{P(D)}$

$$= \frac{(0.25)(0.02)}{0.0245} = 0.2040.$$

c) As it is a probability distribution

so, $\sum_{\text{all } x} f(x) = 1$ (discrete)

$f(0) + f(1) + f(2) + f(3) = 1$

$4c + 5c + 8c + 13c = 1$

$30c = 1$

$c = 1/30.$

3) a)

$P(x,y)$	X				
y	0	1	2	3	Total
0	0(0)	c(1/30)	2c(1/15)	3c(1/10)	6c
1	c(1/30)	2c(1/15)	3c(1/10)	4c(4/30)	10c
2	2c(1/15)	3c(1/10)	4c(4/30)	5c(1/6)	14c
Total	3c	6c	9c	12c	30c

So as we know

$30c = 1$

$c = 1/30$

b) marginal distribution of X

X	P(x)
0	1/10
1	1/5
2	1/15
3	1/10

y	P(y)
0	1/5
1	1/3
2	14/30

c) $P(x>2, y\leq 1) = \sum_{y\leq 1} F(3,0) + F(3,1)$

$= \frac{1}{10} + \frac{4}{30} = \frac{3+4}{30} = 7/30$

4) a)

$P(H) = \frac{3}{4}$ (Given in Quest^n)

$P(T) = 1/4$

let T be the random variable for no. of toice

$P(T=0) = P(HH) = \frac{3}{4} \times \frac{3}{4} = 9/16$

$P(T=1) = P(HT, TH) = \frac{3}{4} \times \frac{1}{4} + \frac{3}{4} \times \frac{1}{4} = \frac{6}{16}$

$P(T=2) = P(TT) = \frac{1}{4} \times \frac{1}{4} = 1/16$

b) $f(x) = \begin{cases} \frac{8}{x^3}, & x > 2 \\ 0, & \text{otherwise.} \end{cases}$

$Z = 2X + 1$.

As we know,

$E(2x+1) = \int_2^\infty g(x) f(x) dx$

$= \int_2^\infty (2x+1) \frac{8}{x^3} dx$

$= \int_2^\infty \frac{16}{x^2} + \frac{8}{x^3} dx = 8\left[-\frac{2}{x} - \frac{1}{2x^2}\right]_2^\infty$

$= 8\left[\frac{-2}{\infty} - \frac{1}{2(\infty)} - \left[\frac{-2}{2} - \frac{1}{8}\right]\right]$

$= 8\left(1 + \frac{1}{8}\right) = 9$

continued on last page.

c) $\mu = 10$

$\sigma^2 = 4 \Rightarrow \sigma = 2$

$P(5 < X < 15)$

According to chebyshev' theorem

$P(\mu - k\sigma < X < \mu + k\sigma) \geq 1 - \frac{1}{k^2}$

$\mu - k\sigma \Rightarrow 10 - 2k = 5$

$-2k = -5$

$k = 5/2$

$\mu + k\sigma = 15$

$10 + 2k = 15$

$2k = 5$

$k = 5/2$

$k = 5/2$

$P(5 < x < 15) \geq 1 - \frac{1}{(\frac{5}{2})^2} = 1 - \frac{4}{25} = \frac{21}{25}$..

5) a) $X \rightarrow$ no. of field mice per acre.

$\mu_x = 12$

(follows poison distribution)

$$P(x) = \frac{e^{-12} \cdot 12^{x}}{x!} \quad (0, 1, 2, \ldots, 60)$$

$$P(X < 7) = \sum_{x=0}^{6} P(x)$$

$\Rightarrow$ calculate

$= 0.0458$.

b) Let x be the no. of games for team A to win the series

$X \rightarrow$ follows negative binomial distributn

$k = 4,\ P = 0.55,\ q = 0.45$

$$b^*(x) = {}^{x-1}C_3\,(0.55)^4\,(0.45)^{x-4}, \quad x = 4, 5, 6, 7$$

$$b^*(6) = {}^5C_3\,(0.55)^4\,(0.45)^2$$

$= 10 \times 0.091 \times 0.2025$

$= 0.1842$

c) X follows a cont. uniform distributn

so, $P(x) = \frac{1}{(5-1)} = 1/4$.

so,

$$P(2.5 > x \leq 4) = \int_{2.5}^{4} P(x)\,dx$$

$$= \int_{2.5}^{4} \frac{1}{4}\,dx = \frac{4 - 2.5}{4} = 0.375.$$

6) a)

Let $x \rightarrow$ no. of defectives in the 100 items

$\mu = np =$

$\sigma^2 = npq \Rightarrow \sigma = \sqrt{npq}$

$p = 10/100 = 0.1$

$q = 0.9..$

$np = 100 \times 0.1 = 10$

$\sigma = \sqrt{npq} = \sqrt{100 \times 0.1 \times 0.9} = 3.$

$P(X < 8) =$ we have to use the formula

$$P(X < x) = P\left(z < \frac{x - 0.5 - np}{\sqrt{npq}}\right)$$

$$\Rightarrow P(X < 8) = P\left(z < \frac{8 - 0.5 - 10}{3}\right)$$

$= P(z < -0.83)$

$= 0.2033$

7) a) According to Poison's distribution

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

Acc. to Moment generating fun^n

$$M_X(t) = E(e^{xt}) = \sum_{all\,x} e^{xt} f(x).$$

$$M_X(t) = \sum_{x=0}^{\infty} \frac{\lambda^x e^{-\lambda}}{x!} e^{xt}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{(\lambda e^t)^x}{x!}$$

$= e^{-\lambda} e^{\lambda e^t}$

$= e^{\lambda(e^t - 1)}$

c) ~~Step-1~~ $\sigma = 40$ hrs

$\bar{x} = 780$ hrs

$n = 30$

Step - 1

$100(1-\alpha) = 96$

$\alpha = 0.04$

$\alpha/2 = 0.02$.

Step 2

$P(Z \le Z_{\alpha/2}) = P(Z < 0.02)$

$= 1 - 0.02 = 0.98$

Step - 3 $\quad Z_{0.02} = 2.06$

$\bar{x} = 780$ hrs

Step - 4

$k = \frac{\sigma}{\sqrt{n}} Z_{\alpha/2}$

$= \frac{40}{\sqrt{30}} \times 2.06 = 15.04$

Step - 5

CONF 96% $\{ 780 - 15.04 \le \mu \le 780 + 15.04 \}$

$= \{ 765 \le \mu \le 795 \}$

8) $\bar{u} = 5.5$

$\bar{x} = 5.2$ $\quad H_0 : \mu = 5.5$ ounces

$n = 64$ $\quad H_1 : \mu < 5.5$

$\alpha = 0.05$

$\sigma = 0.24$ $\quad Z_0 = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{5.23 - 5.5}{\frac{0.24}{\sqrt{64}}} = -9.$

Now, $P(Z < c) = 0.05$

$c = -1.64$.

∴ hypothesis rejected.

b) $H_0 : \sigma = 1.5$

$H_1 : \sigma > 1.5$

right sided test.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

$\alpha = 0.05$

to find the critical value.

we have to solve

$P(\chi^2 > c) = 0.05$

at 24 dof.

$c = 36.415$

right side part (region) of the critical value will be rejected

c) $$\chi_0^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

$$= \frac{24 \times (2.03)^2}{(1.15)^2} = 74.78.$$

we reject the hypothesis.

$\therefore$ machine is out of control

9) a)

x	y	$(x-\bar{x})$	$(y-\bar{y})$	$(x-\bar{x})(y-\bar{y})$	$(x-\bar{x})^2$
77	82	-1.55	8.89	-13.78	2.4025
50	66	-28.55	-7.11	202.99	815.1025
71	78	-7.55	4.89	-36.92	57.0025
72	34	-6.55	-39.11	256.17	42.9025
81	47	2.45	-26.11	-63.97	6.00
94	85	15.45	11.89	183.70	238.70
96	99	17.45	25.89	451.78	304.50
99	99	20.45	25.89	529.46	418.2025
67	68	-11.55	-5.11	59.02	133.40

$\bar{x} = \frac{707}{9} = 78.55$

$\bar{y} = \frac{658}{9} = 73.11$

$(x - \bar{x})(y - \bar{y}) = 1568.445$

$(x - \bar{x})^2 = 2018.22$

co-relat^n co-efficient,

$r = \frac{S_{xy}}{S_x \cdot S_y}$

$S_{xy} = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})$

$= \frac{1}{8} \times 1568.445$

$= 196.055$

$S_x^2 = \frac{1}{8} \times \sum_{i=1}^{9} (x_i - x)^2$

$= \frac{1}{8} \times 2018.22 = 252.2775$

$S_x = \sqrt{252.77} = 15.89.$

similarity

$S_y^2 = 481.8025$

$S_y = 21.95$

$r = \frac{196.0}{15.89 \times 21.95} = 0.561$

b) let regress^n line

$y = a + bx$

$b = \frac{\sum_{i=1}^{m} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - x)^2}$

$= \frac{1568.445}{2018.22} = 0.77$

b)

H_0 : presence /absence of hypertension is independent of smoking habbits

H_1 : " of hypertens^n is not " of habits.

$\alpha = 0.05$

$P(\chi^2 > c) = 0.05$

with 2 dof

$c = 5.991$

critical region

6) b) $\alpha = 2$

$\beta = 1$

To find $P(1.8 < x < 2.4)$

$$P(1.8 < X < 2.4) = \int_{1.8}^{2.4} \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}$$

$$= \int_{1.8}^{2.4} x e^{-x}$$

$$= 0.1544.$$

4) b) continued

$$\sigma_z^2 = E\{[z-\mu]^2\}$$

$$= E[[(2x+1)-9]^2]$$

$$= \int_2^{\infty} (2x-8)^2 \frac{8}{x^3} dx$$

$$= 8\int_2^{\infty} (4x^2+64-32x) \tfrac{1}{x^3} dx$$

$$= 8 \int_{2}^{\infty} \left(\frac{4}{x} + \frac{64}{x^3} - \frac{32}{x^2} \right) dx$$

$$= 8 \left[4 - \frac{64}{2x^2} + \frac{32}{x} \right]_{2}^{\infty}$$

$$= 8 \left[4 - \frac{64}{16} + \frac{32}{2} \right]$$

$$= 8 \times 16 = 128.$$