Introduction to Discrete Mathematics

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Importance in Computer Science

- Provides a foundation for programming logic and algorithms.
- Essential in cryptography, artificial intelligence, and data structures.
- Used in networking, security, and computational theory.

Boolean Logic

Definition: Boolean logic is a form of algebra in which values are true or false.

Basic Operations:

- AND ($A \wedge B$)
- OR ($A \vee B$)
- NOT ($\neg A$)

Example: Truth Table for AND Operation:

A	B	$A \wedge B$
0	0	0
0	1	0
1	0	0
1	1	1

Set Theory

Definition: Set theory deals with collections of objects.

Operations:

- Union ($A \cup B$)
- Intersection ($A \cap B$)
- Difference ($A - B$)

Example: If $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$:

$$A \cup B = \{1, 2, 3, 4\}, \quad A \cap B = \{2, 3\}, \quad A - B = \{1\}$$

Set Theory Overview

Definition: A set is a collection of distinct objects.

- **Example:** A = $\{1,3,4,7,8,9\}$, B = $\{1,2,3,4,5\}$, C = $\{1,3\}$
- **Subset Notation:** $C \subseteq AandC \subseteq B$

Explanation:

- **Definition:** A set C is a subset of A if every element of C is also in A.
- **Example:** C = $\{1,3\}$ is a subset of A = $\{1,3,4,7,8,9\}$
- **Mathematical Notation:** C $\subseteq$ A

Properties of Subsets

- If $C \subseteq A and C \subseteq B$, then C is contained in both sets.
- If $A \subseteq B and B \subseteq C, then A \subseteq C$ (Transitivity).
- The empty set $(\emptyset)$ is a subset of every set.

Graph Theory

Definition: Graph theory deals with nodes and edges representing relationships.

Example Graph:

$$V = \{A, B, C\}, \quad E = \{(A, B), (B, C), (C, A)\}$$

Applications: Network routing, social network analysis, shortest path algorithms.

Combinatorics

Definition: Combinatorics is the study of counting and arrangements.
Formula: Number of ways to arrange n distinct objects:

$$n! = n \times (n-1) \times \ldots \times 1$$

Example: Arranging 5 books:

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

Number Theory

Definition: Number theory deals with properties of integers.
Example: Finding the Greatest Common Divisor (GCD):

$$\text{GCD}(48, 18) = 6$$

Types of Mathematical Statements

Mathematical statements are used in logical reasoning in discrete mathematics. The main types of statements include:

- **Propositions** - Statements that are either true or false.
- **Predicates** - Statements with variables that become propositions when values are assigned.
- **Quantified Statements** - Statements using quantifiers such as $\forall$ (for all) and $\exists$ (there exists).

Propositions

A proposition is a declarative sentence that is either true or false, but not both. **Examples:**

- "The sky is blue." (True or False based on conditions)
- "$2 + 2 = 4$" (Always True)
- "All prime numbers are odd." (False, since 2 is even)

Predicates

A predicate contains one or more variables and becomes a proposition when values are assigned. **Example:**

- $P(x)$: "x is greater than 5"
- $P(7)$ is True, $P(3)$ is False

Quantified Statements

Quantified statements express conditions for all or some elements in a set. **Types of Quantifiers:**

- **Universal Quantifier** ($\forall$): "For all x, ..."
- **Existential Quantifier** ($\exists$): "There exists an x such that..."

Example:

$$\forall x(x > 0 \rightarrow x^2 > 0) \quad \text{(For all x, if x is positive, then x squared is positiv}$$

Logical Operations

Logical operations allow us to combine and manipulate propositions. **Fundamental Operations:**

- **Negation (NOT)** - $\neg p$: Reverses truth value.
- **Conjunction (AND)** - $p \wedge q$: True if both are true.
- **Disjunction (OR)** - $p \vee q$: True if at least one is true.
- **Implication (IF-THEN)** - $p \rightarrow q$: "If p, then q."
- **Biconditional (IFF)** - $p \leftrightarrow q$: "p if and only if q."

Truth Tables

Truth tables display all possible truth values for logical operations.

Example: Truth Table for $p \rightarrow q$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Logical Equivalences

Logical equivalences ensure equivalent logical expressions.
Important Laws:

- **Commutative Laws:** $p \wedge q \equiv q \wedge p$, $p \vee q \equiv q \vee p$
- **Associative Laws:** $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- **Distributive Laws:** $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
- **De Morgan's Laws:** $\neg(p \wedge q) \equiv \neg p \vee \neg q$, $\neg(p \vee q) \equiv \neg p \wedge \neg q$

Set Operations

Set operations allow manipulation of elements in different sets.

Basic Set Operations:

- **Union** ($A \cup B$): Elements in A or B or both.
- **Intersection** ($A \cap B$): Elements in both A and B.
- **Complement** (A^c): Elements not in A.
- **Set Difference** ($A - B$): Elements in A but not in B.

Example:

$$A = \{1, 2, 3\}, \quad B = \{3, 4, 5\}$$

$$A \cup B = \{1, 2, 3, 4, 5\}, \quad A \cap B = \{3\}, \quad A - B = \{1, 2\}$$

Mathematical Statement

A **Mathematical Statement** is a declarative sentence that can be either True or False , but not both simultaneously.

- Statements form the basis for mathematical reasoning.
- They help develop complex logical expressions.

Atomic Statements

An Atomic Statement is a simple, indivisible statement that cannot be broken into smaller components.

Examples:

- "The moon is made of cheese." (False)
- "42 is a perfect square." (True)
- "$3 + 7 = 12$" (False)

Non-Statements

Not all sentences qualify as statements. Some are commands or questions and do not hold a truth value.

Examples:

- "Would you like some cake?"
- "Go to your room!"

Molecular Statements and Logical Connectives

Molecular Statements are created by combining Atomic Statements using logical connectives.

Logical Connectives:

- **Conjunction** ($\wedge$) - "and"
- **Disjunction** ($\vee$) - "or"
- **Implication** ($\rightarrow$) - "if...then..."
- **Biconditional** ($\leftrightarrow$) - "if and only if"
- **Negation** ($\neg$) - "not"

Truth Tables for Logical Connectives

Truth tables help analyze logical expressions based on atomic statements.

Truth Table for Conjunction ($\wedge$)

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Truth Table for Disjunction ($\vee$)

A disjunction $P \vee Q$ is true if at least one of P or Q is true.

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Truth Table for Implication ($\rightarrow$)

An implication $P \rightarrow Q$ is false only if P is true and Q is false.

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Truth Table for Biconditional ($\leftrightarrow$)

A biconditional $P \leftrightarrow Q$ is true when both have the same truth value.

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

Negation ($\neg$)

Negation reverses the truth value of a statement.

P	$\neg P$
T	F
F	T

Examples of Molecular Statements

Example 1:

- **Statement:** "If Bob gets a 90 on the final, then Bob will pass the class."
- **Implication:** $P \to Q$
- **Condition:** False if Bob gets a 90 but does not pass.

Example 2:

- **Statement:** "If $1 = 1$, then most horses have 4 legs."
- **Implication:** True since both statements are logically correct.

Exercise

- List the elements of each set where N $= 1, 2, 3, \ldots.$
  - $A = x \in N | 3 < x < 9$
  - $B = x \in N | xis\ even, x < 11$
  - Aconsists of the positive integers between 3 and 9;