

# Complexity classes

upper bounds

- $O(1)$  denotes constant running time
- $O(\log n)$  denotes logarithmic running time
- $O(n)$  denotes linear running time
- $O(n \log n)$  denotes log-linear running time
- $O(n^c)$  denotes polynomial running time ( $c$  is a constant)
- $O(c^n)$  denotes exponential running time ( $c$  is a constant being raised to a power based on size of input)

↓ complexity increases

# Constant complexity

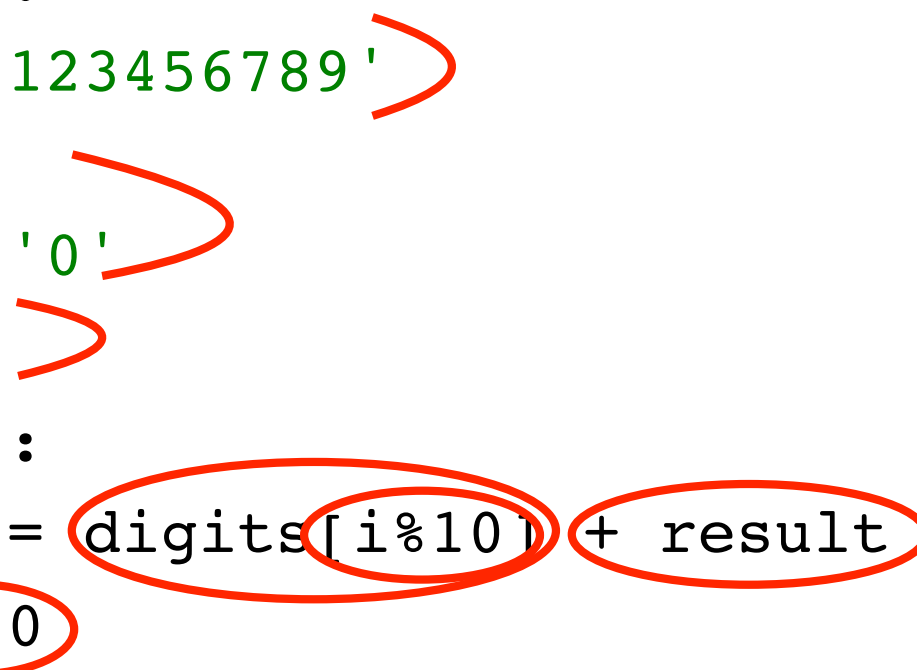
- Complexity independent of inputs
- Very few interesting algorithms in this class, but can often have pieces that fit this class
- Can have loops or recursive calls, but number of iterations or calls independent of size of input

# Logarithmic complexity

- Complexity grows as log of size of one of its inputs
- Example:
  - Bisection search
  - Binary search of a list

# Logarithmic complexity

```
def intToStr(i):  
    digits = '0123456789'  
    if i == 0:  
        return '0'  
    result = ''  
    while i > 0:  
        result = digits[i%10] + result  
        i = i/10  
    return result
```



# Logarithmic complexity

```
def intToStr(i):  
    digits = '0123456789'  
    if i == 0:  
        return '0'  
    result = ''  
    while i > 0:  
        result = digits[i%10] + result  
        i = i//10  
    return result
```

- Only have to look at loop as no function calls
- Within while loop constant number of steps <sup>6</sup>
- How many times through loop?
  - How many times can one divide i by 10?
  - $O(\log(i))$

# Linear complexity

- Searching a list in order to see if an element is present
- Add characters of a string, assumed to be composed of decimal digits

```
def addDigits(s):  
    val = 0  
    for c in s:  
        val += int(c)  
    return val
```

- $O(\text{len}(s))$

# Linear complexity

- Complexity can depend on number of recursive calls

```
def fact(n):  
    if n == 1:  
        return 1  
    else:  
        return n * fact(n-1)
```

- Number of recursive calls?
  - Fact(n), then fact(n-1), etc. until get to fact(1)
  - Complexity of each call is constant
  - $O(n)$

# Log-linear complexity

- Many practical algorithms are log-linear
- Very commonly used log-linear algorithm is merge sort
- Will return to this

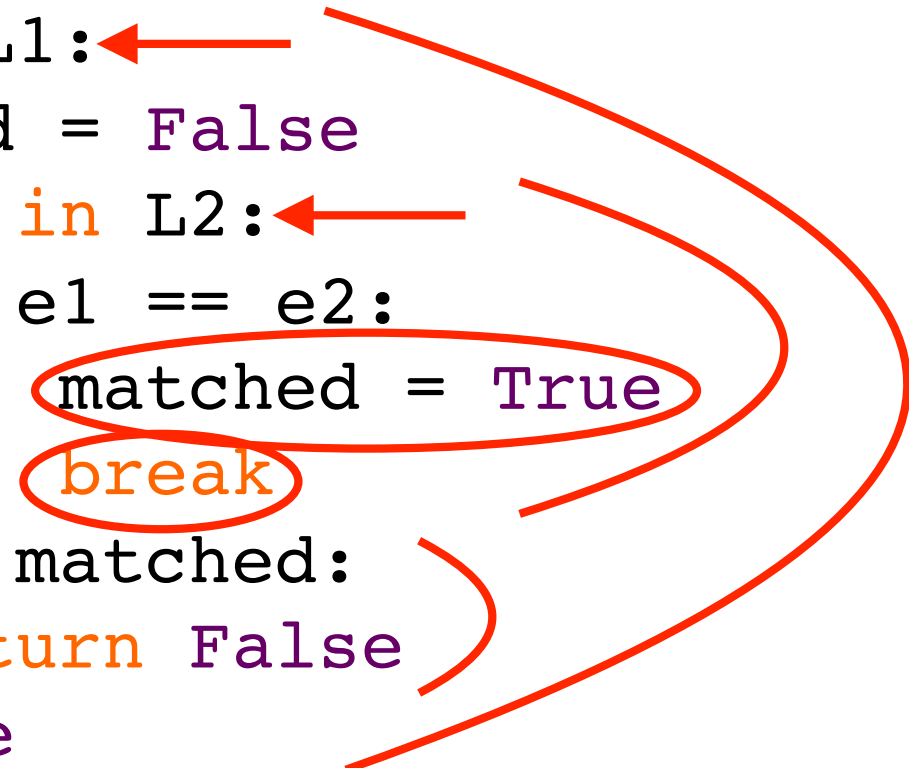


# Polynomial complexity

- Most common polynomial algorithms are quadratic, i.e., complexity grows with square of size of input
- Commonly occurs when we have nested loops or recursive function calls

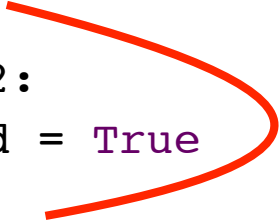
# Quadratic complexity

```
def isSubset(L1, L2):  
    for e1 in L1:←  
        matched = False  
        for e2 in L2:←  
            if e1 == e2:  
                matched = True  
                break  
        if not matched:  
            return False  
    return True
```



# Quadratic complexity

```
def isSubset(L1, L2):  
    for e1 in L1:  
        matched = False  
        for e2 in L2:  
            if e1 == e2:  
                matched = True  
                break  
        if not matched:  
            return False  
    return True
```



- Outer loop executed  $\text{len}(L1)$  times
- Each iteration will execute inner loop up to  $\text{len}(L2)$  times
- $O(\text{len}(L1) * \text{len}(L2))$
- Worst case when  $L1$  and  $L2$  same length, none of elements of  $L1$  in  $L2$
- $O(\text{len}(L1)^2)$

# Quadratic complexity

Find intersection of two lists, return a list with each element appearing only once

```
def intersect(L1, L2):  
    tmp = []  
    for e1 in L1: ←  
        for e2 in L2: ←  
            if e1 == e2:  
                tmp.append(e1)  
    res = []  
    for e in tmp:  
        if not(e in res):  
            res.append(e)  
    return res
```

# Quadratic complexity

```
def intersect(L1, L2):  
    tmp = []  
    for e1 in L1:  
        for e2 in L2:  
            if e1 ==  
e2:  
  
        tmp.append(e1)  
        res = []  
        for e in tmp:  
            if not(e in  
res):  
  
        res.append(e)  
    return res
```

- First nested loop takes  $\text{len}(L1) * \text{len}(L2)$  steps
- Second loop takes at most  $\text{len}(L1)$  steps
- Latter term overwhelmed by former term
- $O(\text{len}(L1) * \text{len}(L2))$

# Exponential complexity

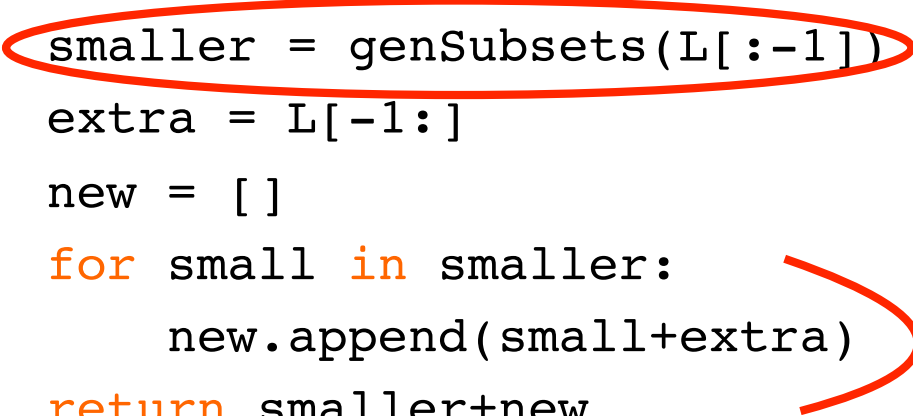
- Recursive functions where more than one recursive call for each size of problem
  - Towers of Hanoi
- Many important problems are inherently exponential
  - Unfortunate, as cost can be high
  - Will lead us to consider approximate solutions more quickly

# Exponential complexity

```
def genSubsets(L):  
    res = []  
    if len(L) == 0:  
        return [[]] #list of empty list  
    smaller = genSubsets(L[:-1])  
    # get all subsets without last element  
    extra = L[-1:]  
    # create a list of just last element  
    new = []  
    for small in smaller:  
        new.append(small+extra)  
    # for all smaller solutions, add one with last  
    # element  
    return smaller+new  
    # combine those with last element and those  
    # without
```

# Exponential complexity

```
def genSubsets(L):  
    res = []  
    if len(L) == 0:  
        return [[]]  
    smaller = genSubsets(L[:-1])  
    extra = L[-1:]  
    new = []  
    for small in smaller:  
        new.append(small+extra)  
    return smaller+new
```



- Assuming append is constant time
- Time includes time to solve smaller problem, plus time needed to make a copy of all elements in smaller problem



# Exponential complexity

```
def genSubsets(L):  
    res = []  
    if len(L) == 0:  
        return [[]]  
    smaller = genSubsets(L[:-1])  
    extra = L[-1:]  
    new = []  
    for small in smaller:  
        new.append(small+extra)  
    return smaller+new
```

- But important to think about size of smaller
- Know that for a set of size  $k$  there are  $2^k$  cases
- So to solve need  $2^{n-1} + 2^{n-2} + \dots + 2^0$  steps
- Math tells us this is  $O(2^n)$

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