Supervised Learning in Factor Investing: Foundations

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Agenda

Why?

Four core purposes for this course

- Gain knowledge and insights on some mainstream asset pricing results and methods
- Apply ML tools on financial datasets:
  1. what are these tools?
  2. why resort to them?
  3. how to use them?
- Understand the weaknesses of ML applied to asset management: **ML is not a magic wand**![1]
- Improve your coding skills (in R + ...!)

More broadly: ML culture/knowledge is more & more required in finance... both in buy side and sell side jobs.
My goal: that you **shine in interviews** (& get the jobs you want)!

[1] see Lopez de Prado: The 10 Reasons Most Machine Learning Funds Fail

What the course is **not** about

Other applications of ML in Finance

- Derivatives pricing & hedging
- Fraud detection
- Credit scoring

Related topics

- Alternative data use cases
- Deep mathematical developments in supervised learning + computer vision & CNN
- Algorithms in unsupervised/reinforcement learning
- NLP tools for finance (sentiment)
- ML for high frequency trading (order book dynamics)

If you like challenges: https://challengedata.ens.fr

We will focus on **insights** and **applications** for Factor Investing.

Why now?

A favorable nexus

- **Data availability** (cross-section of stocks $\sim$1000, characteristics $\sim$100, time points $\sim$100 + alt-data!) gives ML a favorable playground
- **Computational power**:
  - **Hardware**: storage & processing speed almost limitless (all major players provide cloud solutions: IBM, Google, Amazon, Microsoft + niche players)
  - **Software**: easily accessible thanks to the private sector (ML libraries funded by Google and Facebook) and to large academic research groups (INRIA, Stanford, UPenn etc.)
- **Economic grounding**: what this session is about. Computer scientists have used finance as playing field for decades (data is readily available). Applied CS is not enough: we need an economic/logical framing.

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Layout of the course

Eight sessions

1. Introduction & foundations
2. Portfolio strategies in R
3. LASSO & sparse hedging
4. Data preparation, feature engineering, labelling
5. Decision trees & extensions
6. Neural networks
7. Tuning & validating
8. Extensions: SVM, ensemble learning, interpretability + bias & backtest overfitting

Within the sessions

How this is going to work

Two (possibly three) parts

1. Theory (slides): 25%-40%
2. Practice (notebooks): 40%-70%
3. Exercises / questions (notebooks): remaining time

+ course work (ML backtest project)

One major reference: www.mlfactor.com

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References

Factor investing: the idea that (some) firm characteristics **drive** future profitability.

This topic is HUGE!

We refer to the monographs:

- **Asset Management: A Systematic Approach to Factor Investing** by Andrew Ang
- **Expected Returns: An Investor's Guide to Harvesting Market Rewards** by Antti Ilmanen

An important article on the subject is:
... and the cross section of stock returns, by Harvey et al.

+ chapter 3 in the book which we'll cover soon.

One important contribution

Fama French (1992)

Monthly portfolio sorts

The process is the following:

- Each month, assets (stocks) are sorted according to one (or two) of their **characteristics** (cap = size, B/M ratio, past return, etc.)
- Portfolios are constituted according to these sorts (e.g., quintile or decile portfolios). The allocation is usually cap-weighted or equally-weighted
- The performance of the portfolio is recorded for the month ahead, and then the portfolio composition is updated based on the new (current) values of the characteristics
- the corresponding vectors of returns can be analysed (average means, t-tests)

The results

Source: Fama French (1992).
The figures are equal to the average monthly returns of the EW-portfolios, in percent.

	Book-to-Market Portfolios										
	All	Low	2	3	4	5	6	7	8	9	High
All	1.23	0.64	0.98	1.06	1.17	1.24	1.26	1.39	1.40	1.50	1.63
Small-ME	1.47	0.70	1.14	1.20	1.43	1.56	1.51	1.70	1.71	1.82	1.92
ME-2	1.22	0.43	1.05	0.96	1.19	1.33	1.19	1.58	1.28	1.43	1.79
ME-3	1.22	0.56	0.88	1.23	0.95	1.36	1.30	1.30	1.40	1.54	1.60
ME-4	1.19	0.39	0.72	1.06	1.36	1.13	1.21	1.34	1.59	1.51	1.47
ME-5	1.24	0.88	0.65	1.08	1.47	1.13	1.43	1.44	1.26	1.52	1.49
ME-6	1.15	0.70	0.98	1.14	1.23	0.94	1.27	1.19	1.19	1.24	1.50
ME-7	1.07	0.95	1.00	0.99	0.83	0.99	1.13	0.99	1.16	1.10	1.47
ME-8	1.08	0.66	1.13	0.91	0.95	0.99	1.01	1.15	1.05	1.29	1.55
ME-9	0.95	0.44	0.89	0.92	1.00	1.05	0.93	0.82	1.11	1.04	1.22
Large-ME	0.89	0.93	0.88	0.84	0.71	0.79	0.83	0.81	0.96	0.97	1.18

The first row and first column show the trends!

One step further: “risk factors”

Inspiration: Fama French (1993)

Long-short portfolios

- Suppose you form one portfolio that is long small firms and short big firms (SMB) and one portfolio that is long firms with high B/M ratio and short firms with low B/M ratio (HML).
- Then, you can use these (dollar-neutral) portfolios to analyse/decompose the returns of individual asset or portfolios via a linear regression:

$$r_t^i - r_t^f = \alpha + \beta^M (r_t^M - r_t^f) + \beta^{SMB} r_t^{SMB} + \beta^{HML} r_t^{HML} + \epsilon_t$$

  β^M is the exposure to the market, β^{SMB} and β^{HML} are those to the corresponding factors and α is the performance that cannot be explained by these 3 components.

The **SMB** factor is referred to as the Size factor and the **HML** one is the Value (vs Growth) factor.

More generally...

Academics and practitioners have tried to introduce new factors (i.e., based on new characteristics) in the game.

The most classical ones:

- the **Momentum** factor: the attribute here is the return between one year ago and last month (Winners minus Losers WML).
- the **Low-Vol** factor: the attribute is the volatility! (see also: low idioysinc. vol, low beta, BAB: Betting Against Beta)
- the **Profitability** factor (operating profitability: Robust Minus Weak profits) - RMW from Fama French (2015)
- the **Investment** factor (change in total asset: Conservative Minus Aggressive) - CMA also from Fama French (2015)
- the **Quality** factor (a mix!): Quality minus Junk - QMJ from Asness et al (2019)

Original factors look at the names of companies, or the number of marathons ran by the CEO! → **economic relevance**?
Nowadays, people investigate the **green** factor (ESG-based).

Identification: how to proceed?

You have to be careful!

The usual steps:

1. identify a firm **characteristic**
2. form monthly portfolios according to x-percentiles of this characteristic (e.g. five quintile portfolios or ten decile portfolios)
3. keep track of their returns over a sufficiently long period ($>$20Y)[2]
4. let's say r_t^+ and r_t^- are the returns associated with the top and bottom portfolios, perform a t-test on the series $r_t^+ - r_t^-$ (or a test for the mean on the two series - taking the difference in variance into consideration)
5. if the statistics is larger (in absolute value) to some threshold (2, 3?!), then you have yourself a factor! (more or less)

The amount of caution in such empirical design should be extreme because results are often the results of data-snooping / p-**hacking**. **Robustness checks** are compulsory!

[2] Hard for the ESG factors!

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Theoretical groundings

'*Factor investing*' can be viewed as a special case of APT.

The APT (Ross 1976)

- the asset return follows a linear model:

$$r_t^j = \alpha^j + b_1^j F_t^1 + \dots b_n^j F_t^n + \epsilon_t^j,$$

- where the F_t^k are n factors driving stock returns.

BUT! It can also be argued that it is the (raw) characteristics that matter (Daniel and Titman (1997)).[3] In an ML-driven approach, it will be easier to rely on firm attributes (factors take time to compute and depend on many degrees of freedom). Also: for predictive purposes, we will need to add a lag in the predictors (more on that later).
$\rightarrow$ **Factor analysis** is nonetheless useful because it helps understand if one attribute is valuable.

[3] E.G.: firms with low market cap can load negatively on the size factor!

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Fama-MacBeth regressions (1/3)

For a modern view on **anomaly detection**, see Baker, Luo and Taliaferro (2018) and Harvey Liu (2021).

Step one: cross-section of time-series regressions

We consider n asset returns $r_{n,t}$ and m factors f_t^m. We start by estimating n equations, each with m loadings (and a constant):

$$
\begin{aligned}
r_{1,t} &= \alpha_1 + \beta_1^1 F_t^1 + \dots + \beta_1^m F_t^m + \epsilon_{1,t}, \quad t \in (1, T) \\
&\vdots \\
r_{n,t} &= \alpha_n + \beta_n^1 F_t^1 + \dots + \beta_n^m F_t^m + \epsilon_{n,t}, \quad t \in (1, T)
\end{aligned}
$$

This gives a matrix of **loadings** $\hat{\beta}_i^j$; i relates to asset and j to factor.

The $\hat{\beta}_i^j$ characterise the exposure of asset i to the factor j. The sign indicates the direction of the co-movement and the associated t-stat indicates whether the relationship is statistically significant.

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Fama-MacBeth regressions (2/3)

Technical side note: if we write, for a fixed t, $\boldsymbol{r}_t = \boldsymbol{F}_t \beta_t + \epsilon_t$ (including a constant factor),

Then the OLS estimator - assuming it's well defined - is

$$\hat{\beta}_t = (\boldsymbol{F}_t' \boldsymbol{F}_t)^{-1} \boldsymbol{F}_t' \boldsymbol{r}_t$$

which means that estimated coefficients are portfolio returns!
This is the spirit of the second part of the procedure.

Fama-MacBeth regressions (3/3)

Step two: time-series of cross-sectional regressions

Given the $\hat{\beta}_i^j$, estimate, for each date t,

$$r_{i,1} = \kappa^1 + \gamma_1^1 \hat{\beta}_i^1 + \dots + \gamma_1^m \hat{\beta}_i^m + \epsilon_i^1, \quad i \in (1, n)$$

$$\vdots$$

$$r_{i,T} = \kappa^T + \gamma_T^1 \hat{\beta}_i^1 + \dots + \gamma_T^m \hat{\beta}_i^m + \epsilon_i^T t, \quad i \in (1, n)$$

This gives a matrix of estimated coefficients $\hat{\gamma}_t^j$. The premium of factor j is estimated as the average of the $\hat{\gamma}_t^j$ over all $t = 1, \dots, T$. Assuming a large number of observations, the classical t-stat is

$$t_j = \frac{\frac{1}{T}\sum_{t=1}^T \hat{\gamma}_t^j}{\hat{\sigma}_m/\sqrt{T}},$$

where $\hat{\sigma}_m$ is the standard deviation of the $\hat{\gamma}_t^j$.

=> tells if the premium is strongly positive or negative in the **long run**!

Factor competitions

Academics spend a **LOT** of time trying to figure out new factors - or ways to test if factors are truly *factors*.

Singling out the *best* ones

We assume m factors F_t^m. We run m regressions: each factor is regressed against all other factors:

$$
\begin{aligned}
F_t^1 &= \alpha^1 && +\beta_2^1 F_t^2 + \beta_3^1 F_t^3 + && \dots + \epsilon_t^1 \\
F_t^2 &= \alpha^2 + \beta_1^2 F_t^1 && + \beta_3^2 F_t^3 + && \dots + \epsilon_t^2 \\
F_t^3 &= \alpha^3 + \beta_1^3 F_t^1 && +\beta_2^3 F_t^2 + && \dots + \epsilon_t^3 \\
&\vdots
\end{aligned}
$$

If the estimated $\hat{\alpha}^j$ is significantly different from zero, it means that factor j fails to be explained by the other factors. If $\hat{\alpha}^j$ is not statistically different from zero, then is **redundant** because it can be captured by **exposures** to the other factors.

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Other asset classes

Less clear...

It's all a matter of specificities/characteristics

- **Fixed income**: credit rating, bond size & maturity, duration, convexity
- **FX**: country and liquidity risk
- **Commodities**: price-based only (includes futures)
- **Exotic classes**: real-estate, art, wine, crypto, NFTs $\Rightarrow$???

Academic research is scarcer compared to equity. The framework and the empirical results are not so well established.

A common factor across all classes is **momentum** (it only requires prices, which are most of the time available).

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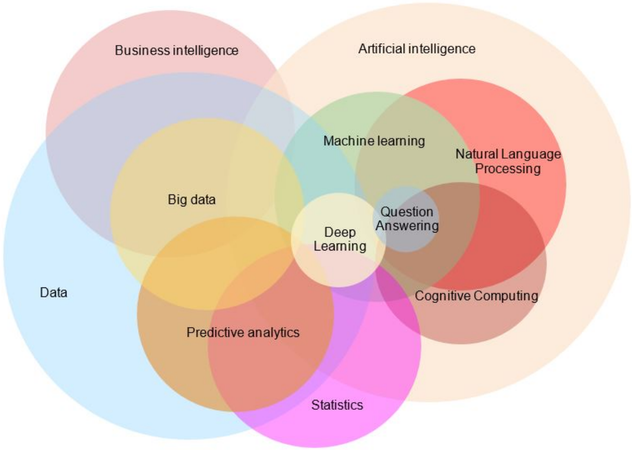
Where is it located? (1/2)

According to **IBM**...



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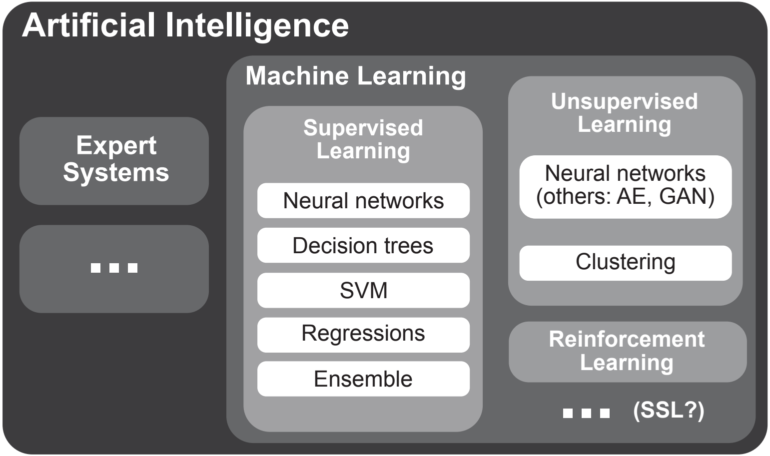
Where is it located? (2/2)

The way I see things (fewer colours).



Natural Language Processing is related to several rounded rectangles.

The AI paradigm shift

Expert vs Data

Also: **symbolists** vs **connexionists**.

- Expert systems (popular in the 1990s) are rule based (e.g.: if that then that)
- Given the complexity of tasks, rules quickly become too limited
- An alternative route is to consider massive amounts of data from which to learn patterns (and hence infer rules)
  $\rightarrow$ connexionism (neural networks mostly)

François Chollet summarises this as:

- **Expert system**: Rules + Data $\rightarrow$ Answers
- **Supervised Learning**: Answers + Data $\rightarrow$ Rules

+ Reinforcement learning & unsupervised methods / SSL.

How does supervized learning work? (1/2)

A primer

Consider a large dataset (e.g., rectangular). You want to be able to understand (and then forecast) one column (y) as a function of the others (x). You can always represent the problem as follows:

$$y_i = f\left(x_i^1, x_i^2, \dots, x_i^n\right) + \epsilon_i,$$

where i denotes the occurrence number and ϵ_i the related error. Two essential (and related) questions in ML are:

1. How do I find/choose f?
2. Will the performance of my model (f) change if I test it on new data?

(in **unsupervised learning**, the above expression does not hold: the machine learns on its own because there is no y.)

The link to asset management

Characteristics!

As we saw previously, the firms' **characteristics** are likely to impact their future performance. This impact is probably nonlinear and time-varying. A very flexible way to evaluate this impact is to consider the above model:

$$y_i = f\left(x_i^1, x_i^2, \dots, x_i^n\right) + \epsilon_i,$$

where y is a proxy for future performance (many choices are possible, e.g., horizon) and the x^j are a set of characteristics. The degrees of freedom are numerous; a shortlist:

1. the set of characteristics
2. the investment set (data availability)
3. the family of functions f: which ML tool?
4. data preprocessing: labelling and feature engineering

In short: factors (**characteristics**) $\leftrightarrow$ **features** (inputs)!

Example

The typical **factor dataset** looks like that:

	Tick	Date	Close	Vol_1M	Mkt_Cap	P2B	D2E	Prof_Marg	ESG_rank	Forward_return
	<chr>	<date>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
1	F	2021-01-29	10.5	45.0	41893.	1.37	529.	-7.75	44.2	0.111
2	DIS	2021-01-29	168.	27.8	305105.	3.56	59.4	0.105	67.5	0.124
3	D	2021-01-29	72.9	19.0	59465.	2.48	142.	19.4	43.1	-0.0627
4	CVX	2021-01-29	84.0	30.3	164011.	1.23	33.7	-2.68	34.9	0.190
5	CVS	2021-01-29	71.6	21.6	93784.	1.35	122.	1.40	40	-0.0491
6	C	2021-01-29	58.0	40.2	120733.	0.671	327.	20.9	46.9	0.136
7	BAC	2021-01-29	29.6	32.9	256497.	1.03	196.	27.2	58.5	0.171
8	AAPL	2021-01-29	132.	35.3	2215357.	33.5	169.	25.8	79.9	-0.0797

So, basically, the aim is to explain $y =$ the forward return (i.e., predict the return) using $x =$ the other columns (the predictors)
NOTE: obviously, Tick and Date are not predictors...

How does supervized learning work? (2/2)

A primer

- Parametrising the model with the information at our disposal!
- Usually, f will depend on some parameters $\boldsymbol{\Theta}$.
- We define a 'loss' (or error) function $L(y_i, f_{\boldsymbol{\Theta}}(\boldsymbol{x}_i))$, often $L(y_i, f_{\boldsymbol{\Theta}}(\boldsymbol{x}_i)) = (y_i - f_{\boldsymbol{\Theta}}(\boldsymbol{x}_i))^2$ when working with continuous variables.
- Machine Learning is simply finding

$$\boldsymbol{\Theta}^* = \underset{\boldsymbol{\Theta}}{\text{argmin}} \sum_{i=1}^N L(y_i, f_{\boldsymbol{\Theta}}(\boldsymbol{x}_i)),$$

  where $i = 1, \dots, N$ are the occurrence indices within the training sample.
- this ensures that the model values $f_{\boldsymbol{\Theta}}(\boldsymbol{x}_i)$ are as close as possible to the 'true'/observed values y_i.

Sometimes, the parameters will not be numbers, but 'architectural choices', in which case they are often referred to as **hyper-parameters**.

ML 101: regressions

There are limitless choices for f.

One crucial building block!

- The simplest one is the linear form:

$$f(\boldsymbol{x}_i) = \beta_0 + \sum_{k=1}^{K} \beta_k x_i^{(k)}$$

  the parameters are the betas and are usually estimated via OLS.
- Some ideas driving the regression are also behind more elaborate nonlinear tools (decision trees and neural networks).
- For more complex models, there are no closed-forms for the optimal parameters $\boldsymbol{\Theta}$.
- Often, the optimal solution can be iteratively approximated using gradient methods for instance.
- In other situations, hyper-parameter tuning will required some level of expertise.

Predictive regressions?

In a factor context

Suppose you have identified ‘factors’ f^k (not necessarily L/S portfolios) that are likely to drive future performance. As an investor, you want to be able to **predict** this performance. Let’s call it r for simplicity ($\rightarrow$ return!).
One very simple way to proceed is to estimate:

$$r_t = \alpha + \beta_1 f^1_{t-1} + \dots + \beta_n f^n_{t-1} + \epsilon_t$$

with data from the past up to the last know r_t. At time t, the f^j_t are known, thus, the conditional expectation (i.e., the forecast) is:

$$\hat{r}_{t+1} = \mathbb{E}[r_{t+1}|\boldsymbol{f}_t] = \hat{\alpha} + \hat{\beta}^1 f^1_t + \dots + \hat{\beta}^n_t f^n_t$$

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Predictive panels!

In fact... does the cross-section of stocks matter?

Another way to look at it is:

$$r_{t+1,n} = a + \sum_{k=1}^{K} b^{(k)} x_{t,n}^{(k)} + u_{t+1,n},$$

the double index in time t and firm n calls for panel-like estimations.
NOTE: a, $b^{(k)}$ do not depend on t and n. The characteristics $x_{t,n}^{(k)}$ depend on everything. For technical reasons, we can decompose the error as

$$r_{t+1,n} = \sum_{k=1}^{K} b^{(k)} x_{t,n}^{(k)} + \mu_n + e_{t+1,n},$$

where μ_n is an average firm-specific term. In this case, the constant a disappears.[4]

Factor models that rely on ML are simply generalizations of this model to non-linear and possibly penalized relationships.

[4] For more on that, see *Panel data econometrics in R: The plm package*

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Key takeaways (1/2)

Characteristics[5]

- academic research finds that characteristics/factors help understand/predict future returns
- there is no real **consensus**, except on a very small group of key features and the devil can be in the details[6] and in time variation!
- the foundation of our approach is very agnostic: characteristics drive profitability, but we do not necessarily know which ones, and if the relationships are stable through time (risk premia are notoriously time-varying)
- but we keep in mind that the inputs should make sense economically (first letters of firm names?)

[5] See Daniel & Titman (1997)
[6] Asness & Frazzini (2013)

Key takeaways (2/2)

Factors

- there are several ways to define and quantify an asset pricing factor/anomaly and test can help extract the variables that truly matter
- machine learning tools are expected to sort out the factors/features that matter **agnostically** (which has some pros & cons) - *even if the choice of inputs can/should be economically motivated (brute force data mining is not the best option)*
- nonetheless, expert guidance will probably improve results out-of-sample
- practice is key $\rightarrow$ train models & **train yourselves**!

Thank you for your attention

Any questions?

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