1. Suppose that $f(n) = \Theta(g(n))$. Assume that both functions increase without limit.
   (a) Must it be true that $\log f(n) = \Theta(\log g(n))$? Prove or disprove.
   (b) Must it be true that $2^{f(n)} = \Theta(2^{g(n)})$? Prove or disprove.

Answer:

(a)

Since $f(n) = \Theta(g(n))$, there exists constant c_1, c_2 and n_0 such that
$c_1 \cdot g(n) \le f(n) \le c_2 \cdot g(n), \forall n \ge n_0,$
then $log(c_1 \cdot g(n)) \le log(f(n)) \le log(c_2 \cdot g(n)),$
$log(c_1) + log(g(n)) \le log(f(n)) \le log(c_2) + log(g(n)),$
since $log(c_1)$ and $log(c_2)$ are constants,
$\log f(n) = \Theta(\log g(n))$.

(b)

2. (a) Suppose you have a function of two variables, n and k; for instance $h(n, k)$.
If you are told that $h(n, k) = O(n + k)$, what should that mean mathematically?

(b) Let $f(n) = O(n)$ and $g(n) = O(n)$. Let c be a positive constant.
Prove or disprove that $f(n) + c \cdot g(k) = O(n + k)$.

Answer:

(a)

(b)

3. Let $f(n) = \sum_{y=1}^{n} (n^6 \cdot y^{23})$.

   Find a simple $g(n)$ such that $f(n) = \Theta(g(n))$, by proving that $f(n) = O(g(n))$, and that $f(n) = \Omega(g(n))$.

   Don't use induction / substitution, or calculus, or any fancy formulas.
   Just exaggerate and simplify for big-O, then underestimate and simplify for Ω.

   Answer: