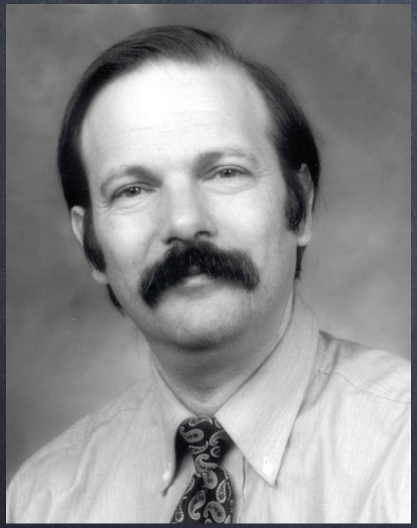
SFWRENG 3BB4:

Software Design III — Concurrent System Design

A Foreword on Logic

A Very Brief History

- The word trivial has a very interesting etymology. It originally referred to the trivium, the three fundamental curriculae: grammar, rhetorics, and logic.
- Mastery of the trivium was considered essential before one could move to the quadrivium, consisting of: arithmetic, geometry, music, and astronomy.
- Why was logic considered to be fundamental? To answer this question, we must make precise what we mean by logic?



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Computer scientist
2000 Gödel Prize
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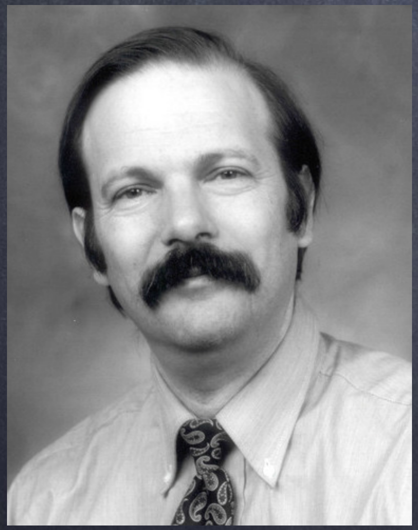

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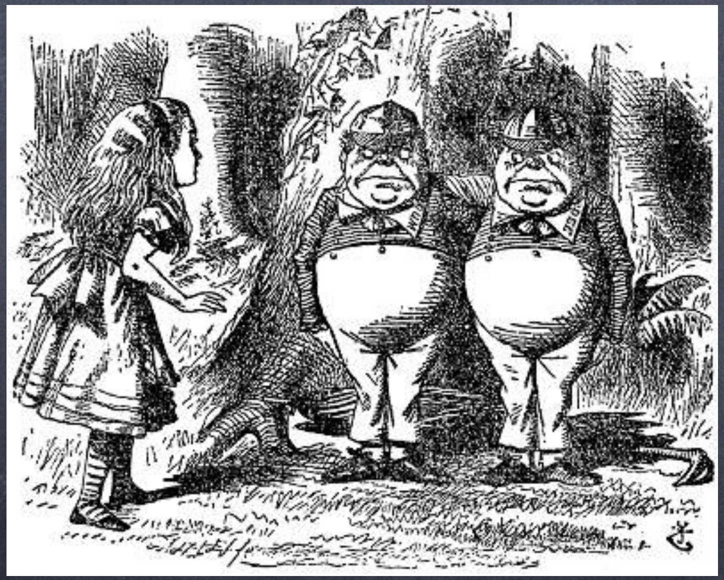
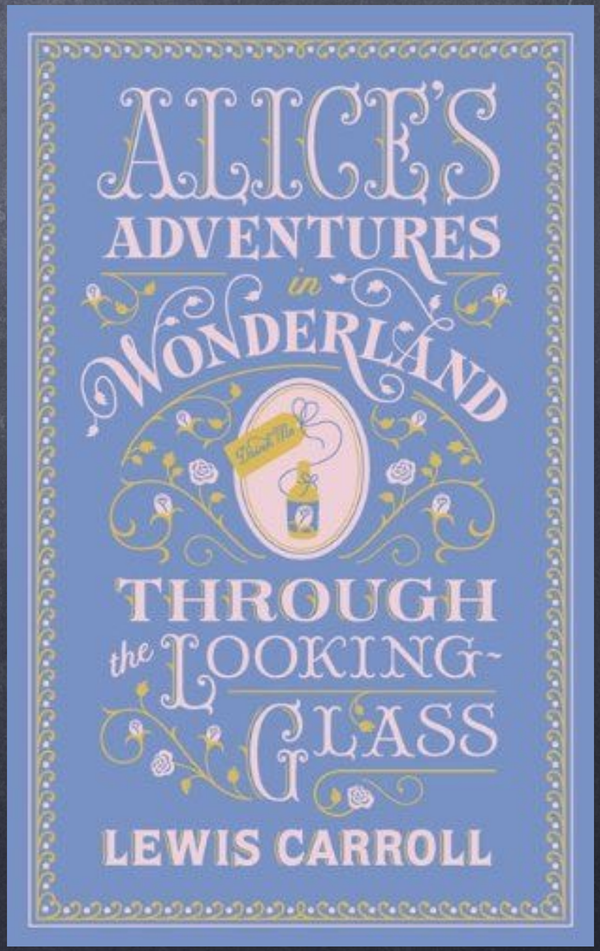


"Contrariwise," continued Tweedledee, "if it was so, it might be; and if it were so, it would be; but as it isn't, it ain't. That's logic."

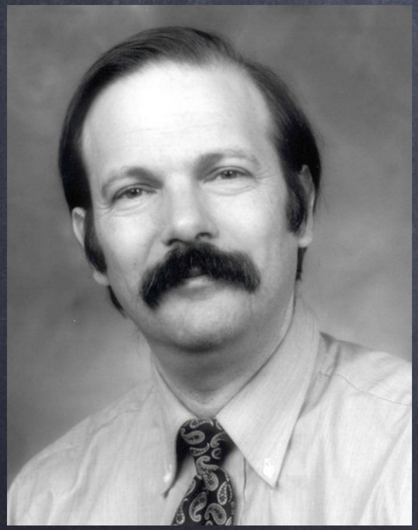


A Foreword on Logic

A Very Brief History

- More solemn attempts to define logic include, but are not restricted to:
    1. The ability to determine correct answers in a standardized manner.
    2. The study of formal inference.
    3. A sequence of verified statements.
    4. Reasoning, as opposed to intuition.
    5. The deduction of statements from a set of statements.
- All in all, the origins of what we now know as logic can be traced back to the Sophists, who engaged in formal debates sought to devise an objective set of rules to determine who was right.
- In brief, logic deals with a set of rules for reasoning and arguing. And as such, it deals with a fundamental problem in intellectual pursuits:

How to distinguish what is true from what is false,
what is right from what is wrong.



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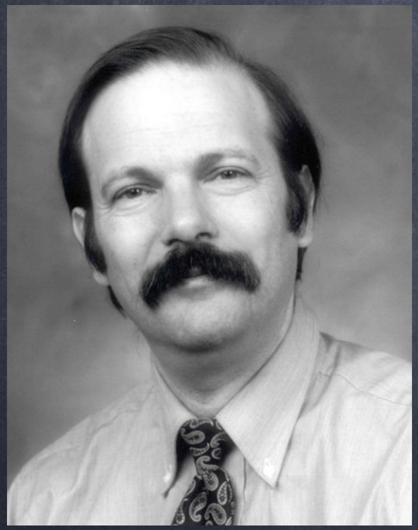
A Foreword on Logic

Logic and CS and SE

- Logic has been called "the calculus of computer science".
- The argument is that logic plays a fundamental role in computer science (similar to that played by calculus in the physical sciences and traditional engineering disciplines).
- Indeed, logic is an important topic in various areas of CS and SE:

1. Architecture (logic gates).
2. Software engineering (specification and verification).
3. Programming languages (semantics, logic programming).
4. Databases (relational algebra and SQL).
5. Artificial intelligence.
6. Automatic theorem proving.
7. Algorithms (complexity and expressiveness).
8. Theory of computation (general notions of computability).



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A Review of Classical Propositional Logic

Overview

- Classical Propositional Logic (CPL for short) is a branch of logic concerned with the study of propositions.
- Propositions are assertions about a state of affair and are either true, or false.
- Propositions are either atomic, or composed by other propositions with the use of logical connectives.
- The truth value of propositions depend on the truth value of their components.
- Typical examples of propositions are:
  1. Follow the gold path and you will reach the exit.
  2. Follow the marble path and you will be lost.
  3. If the stones path takes you to the exit,
     then also the marble road takes you to the exit.
  4. The guardian of the gold street is a liar.
- CPL allows us to express propositions and to reason about them.

Some Basic Definitions

Definition (Alphabet of CPL) The alphabet of CPL consists of:

1. A set L of logical symbols, i.e., the set $\{\neg, \wedge, \vee, \rightarrow\}$.
2. A set $\{p_i\}_{i\in\omega}$ of propositional symbols.
3. A set A of auxiliary symbols, i.e, the set $\{(,)\}$.

Definition (Language of CPL) The language of CPL is the smallest set $\mathcal{L}$ s.t.:

1. $\{p_i\}_{i\in\omega} \cup \{\top, \bot\} \subseteq \mathcal{L}$.
2. If $\{\varphi, \psi\} \subseteq \mathcal{L}$, then,

   $\neg\varphi \in \mathcal{L}$, $\varphi \wedge \psi \in \mathcal{L}$, $\varphi \vee \psi \in \mathcal{L}$, and $\varphi \rightarrow \psi \in \mathcal{L}$.

The elements of $\mathcal{L}$ are called propositions or formulæ.

A Review of Classical Propositional Logic

Some Basic Definitions

Definition (Interpretation) An interpretation is a function $I : \{p_i\}_{i \in \omega} \to \{1, 0\}$ s.t.: $I(\top) = 1$ and $I(\bot) = 0$.

Definition (Satisfaction) An interpretation I satisfies a proposition φ, denoted $I \models \varphi$, according to the following rules:

1. $I \models p_i$ iff $I(\varphi) = 1$.
2. $I \models \neg\varphi$ iff $I \not\models \varphi$.
3. $I \models \varphi \wedge \psi$ iff $I \models \varphi$ and $I \models \psi$.
4. $I \models \varphi \vee \psi$ iff $I \models \varphi$ or $I \models \psi$.
5. $I \models \varphi \to \psi$ iff $I \models \neg\varphi$ or $I \models \psi$.

Definition (Consequence) A proposition φ is a consequence of a set of propositions Γ, denoted by $\Gamma \models \varphi$, iff there is no interpretation I such that $I \models \psi$ for all $\psi \in \Gamma \cup \{\neg\varphi\}$.

A Review of Classical Propositional Logic

Example

A story tells that, following a treasure map, Blackbeard found two chests in a cave in an inhabited island.

He knew that each of the chests either contained a treasure or a fatal trap.

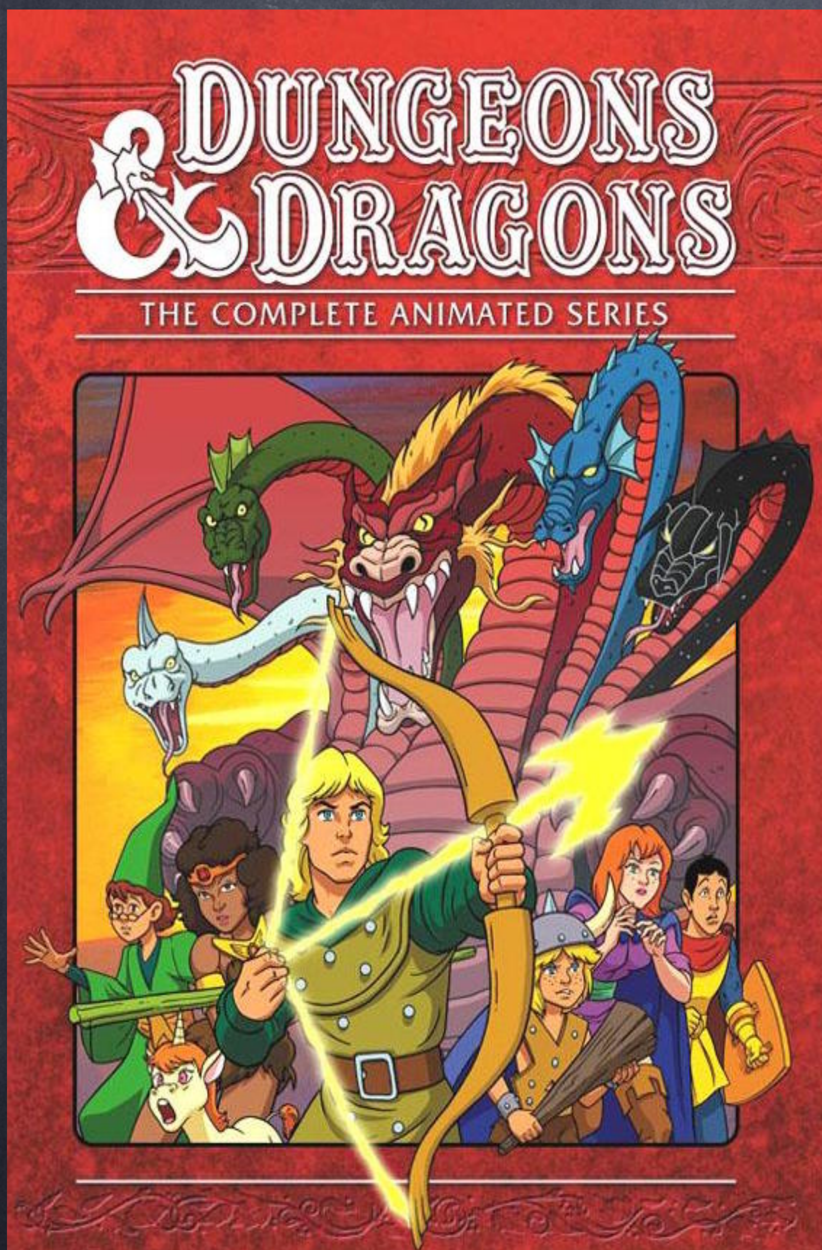
On one it was written: "At least one of these chests contains a treasure."

On the other it was written: "In the other chest there is a fatal trap."

Assuming only that either both inscriptions are true, or both inscriptions are false, The pirate took the chest that contained the treasure and left the island. How did he know which chest to choose?

A Review of Classical Propositional Logic

Example The show focused on the adventures of a group of friends who are sucked into the realm of Dungeons & Dragons by taking a magical dark ride on an amusement park.



The main goal of the heroes is to find a way home.

The setting: The heroes find themselves in front of three doors. Behind one of the doors is a path to freedom. Behind the other two, however, is an evil dragon. Opening a door to the dragon means certain death. On each door, there is an inscription.

``Home is behind this door''

``Home is not behind this door''

``Home is not behind the door in the middle''

If at least one of the statements on the doors is true and at least one of them is false, is there a door leading the group home?

A Primer on Linear Temporal Logic

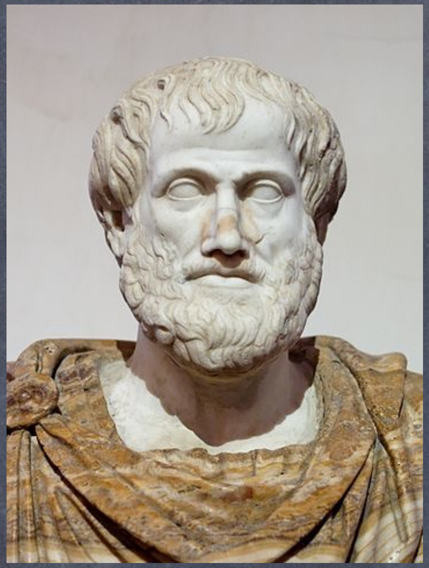
The "shortcomings" of CPL

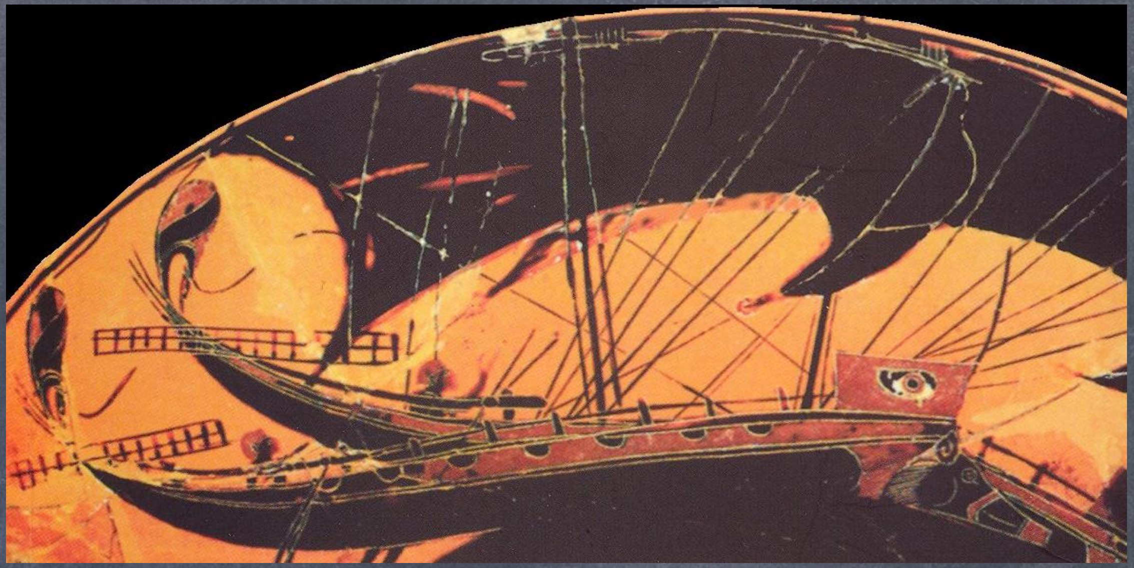
- Classical Propositional Logic (CPL for short) is a branch of logic concerned with the study of propositions. As such, CPL allows us to express propositions and to reason about them.
- Propositions are assertions about a state of affair and are either true, or false in a "fixed" world (interpretation).
- This "fixed" status of truth in CPL makes it inadequate for reasoning about propositions involving notions of time (tense).

A Primer on Linear Temporal Logic

Example

- The Problem of Future Contingents (Aristotle's Sea Battle)





- Suppose that a sea-battle will not be fought tomorrow, then it was also true yesterday that it will not be fought. But all past truths are necessary truths. Therefore, it is not possible that the battle will be fought

- In general, if something will not be the case, it is not possible for it to be the case. This conflicts with the idea of our own free choice

A Primer on Linear Temporal Logic

Example

- The Surprise Exam Paradox
- A Mac professor, worried that his class is not working consistently through the term, choosing instead to “cram” on the night before an exam announces that there will be a surprise exam on the work covered this week the coming week.
- The students, not knowing which day the quiz is on, do not have the option of staying up the night before. They must work consistently.
- One student, however, quickly realizes that the test cannot be on Friday.
- The reasoning is as follows: Suppose it is Thursday night, and the quiz has not happened yet, then the student will know to stay up all night and cram, which defeats the purpose of the surprise exam. Thus, the quiz will not happen on Friday. Similar arguments allow the student to conclude that the exam cannot happen on Thursday, Wednesday, Tuesday, and Monday, and therefore, that the surprise exam cannot happen at all.
- So far, so good. On Tuesday, the teacher hands out the surprise quiz. The student is, frankly, surprised.

A Primer on Linear Temporal Logic

Example

- The Surprise
- A Mac prof … ently through the term, ch … xam announces t … vered this week the co
- The students … ve the option of staying u
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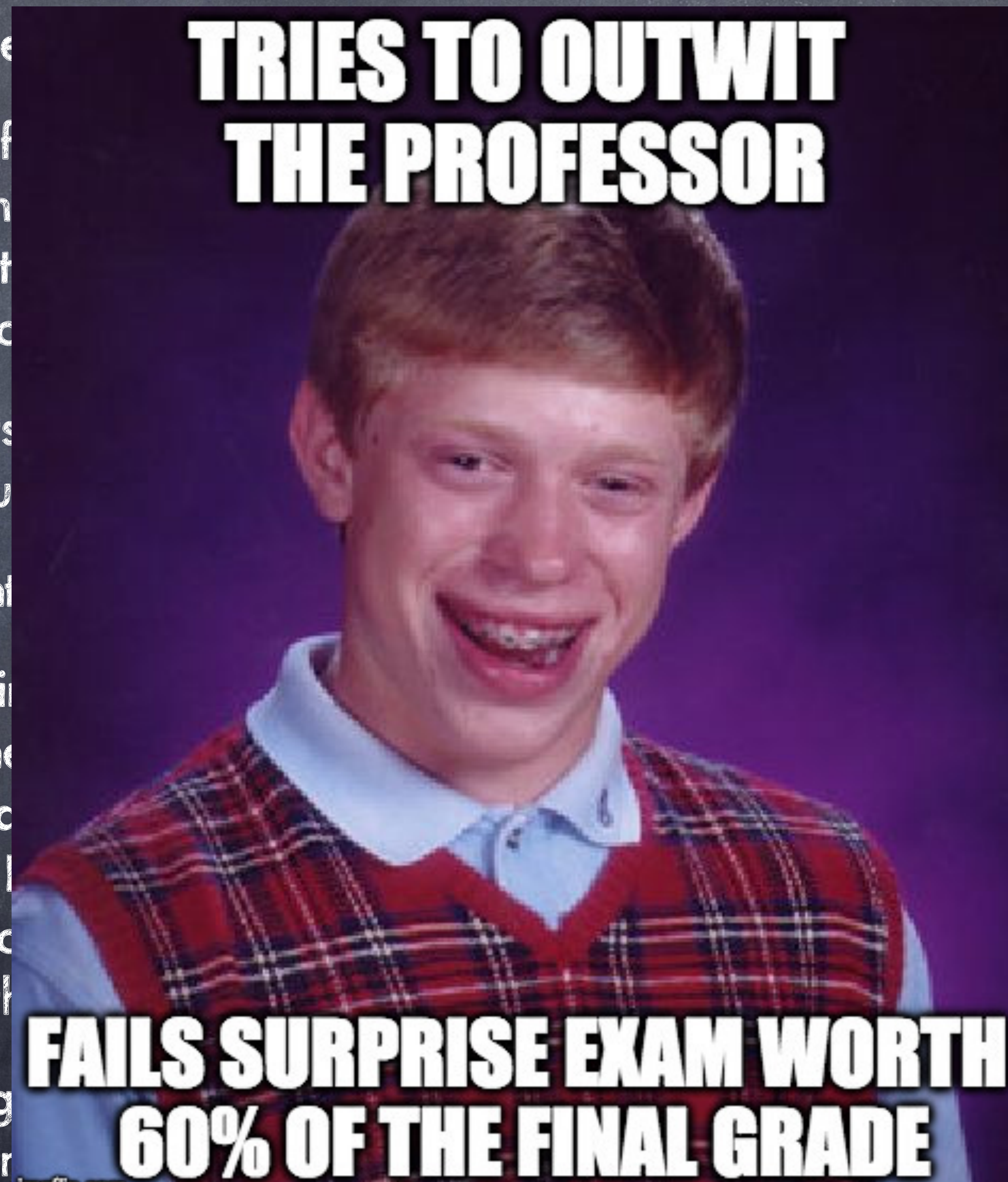

A Primer on Linear Temporal Logic

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Temporal Logics (and LTL)

- Temporal Logic (TL for short) is a branch of logic concerned with the study of propositions qualified in terms of time.
- Linear Temporal Logic (LTL for short) is a sub-branch of TP concerned with the study of propositions qualified in terms of time based on a linear time perspective.
- Typical propositions qualified in terms of time are:

    1. A battle will be fought tomorrow.
    2. I will start studying right after I finish watching this episode of TWD.

Some Basic Definitions

Definition (Alphabet of LTL) The alphabet of LTL consists of:

1. A set L of classical logical symbols, i.e., the set $\{\neg, \wedge, \vee, \rightarrow, \top, \bot\}$.
2. A set T of temporal logical symbols, i.e., the set $\{\mathbf{G}, \mathbf{F}, \mathbf{X}, \mathbf{U}\}$.
3. A set $\{p_i\}_{i\in\omega}$ of propositional symbols.
4. A set A of auxiliary symbols, i.e, the set $\{(,)\}$.

Definition (Language of LTL) The language of LTL is the smallest set $\mathcal{L}$ s.t.:

1. $\{p_i\}_{i\in\omega} \cup \{\top, \bot\} \subseteq \mathcal{L}$.
2. If $\{\varphi, \psi\} \subseteq \mathcal{L}$, then,

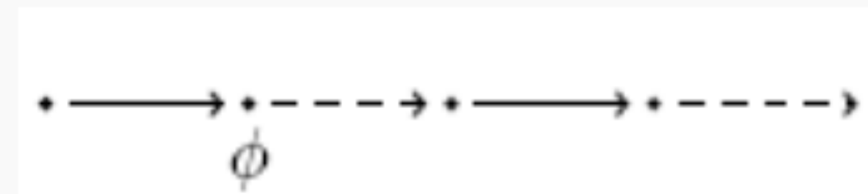
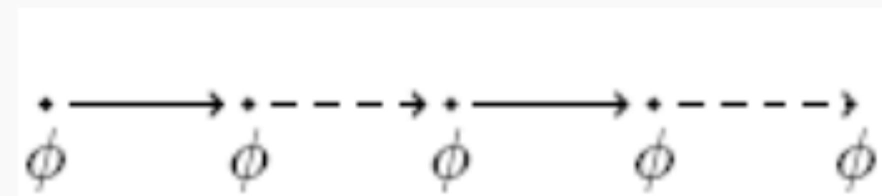
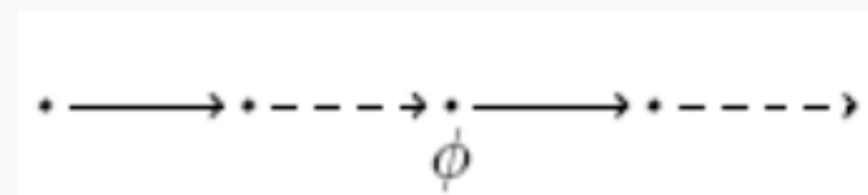
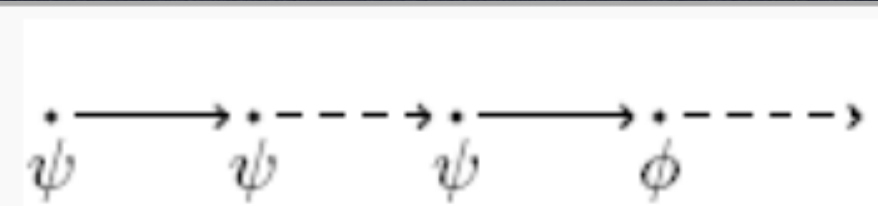
   $\neg\varphi \in \mathcal{L}$, $\varphi \wedge \psi \in \mathcal{L}$, $\varphi \vee \psi \in \mathcal{L}$, and $\varphi \rightarrow \psi \in \mathcal{L}$,

   $\mathbf{G}(\varphi) \in \mathcal{L}$, $\mathbf{F}(\varphi) \in \mathcal{L}$, $\mathbf{X}(\varphi) \in \mathcal{L}$, and $\varphi\, \mathbf{U}\, \psi \in \mathcal{L}$.

The elements of $\mathcal{L}$ are called LTL formulæ

A Primer on Linear Temporal Logic

Some Basic Definitions

Textual	Explanation	Diagram
X ϕ	ne**X**t: ϕ has to hold at the next state.	
G ϕ	**G**lobally: ϕ has to hold on the entire subsequent path.	
F ϕ	**F**inally: ϕ eventually has to hold (somewhere on the subsequent path).	
ψ **U** ϕ	**U**ntil: ψ has to hold *at least* until ϕ, which holds at the current or a future position.	

A Primer on Linear Temporal Logic

Example

- Online Shopping.

1. Once an order is placed, it is eventually processed.
2. An order is shipped immediately after it is processed.
3. A shipped order is eventually delivered.
4. It is not possible for an order to be placed and never delivered.

Language	**Formalization**
$pl =$ "An order is placed"	$(1)\ pl \rightarrow \mathbf{F}(pr)$
$pr =$ "The order is processed"	$(2)\ pr \rightarrow \mathbf{X}(sh)$
$sh =$ "The order is shipped"	$(3)\ sh \rightarrow \mathbf{F}(\mathbf{G}(dl))$
$dl =$ "The order is placed"	$(4)\ \neg(pl \wedge \mathbf{G}(\neg dl)$

Logical Properties

Reading Material

Chapter 14 of
Magee and Kramer Concurrency: State, Models, and Java Programming.