Group_E_Assignment02_Report

Task 1: Optimizer Performance on Non-Convex Functions

Objective

The objective of this task is to study the behavior of different optimization algorithms on non-convex functions and analyze their convergence characteristics, sensitivity to learning rate, and final solution quality.

Problem Setup

We considered two non-convex functions:

1. Rosenbrock function: f(x, y) = (1-x)^2 + 100(y - x^2)^2
2. Discontinuous function: f(x) = sin(1/x), with f(0)=0

The following optimizers were implemented from scratch in Python:

- Gradient Descent (GD)
- Stochastic Gradient Descent with Momentum
- RMSProp
- Adagrad
- Adam

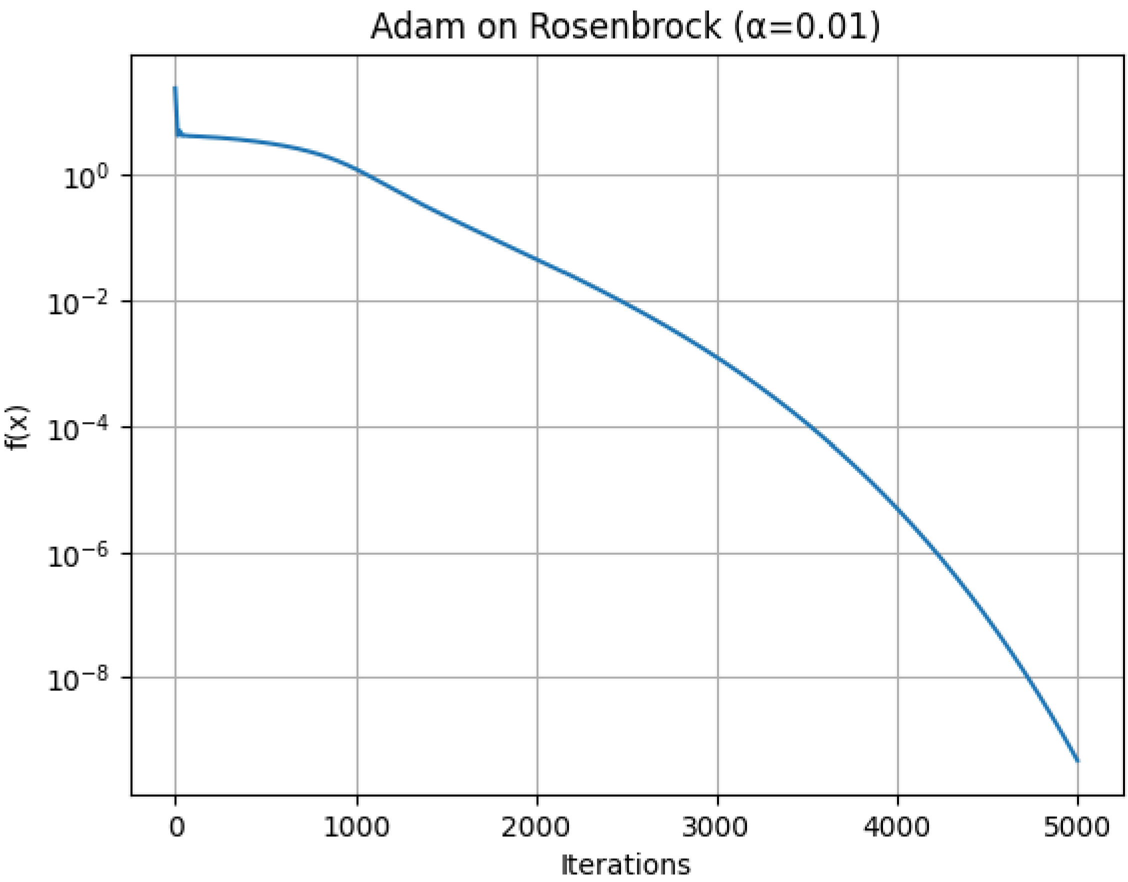
Learning rates tested: 0.01, 0.05, and 0.1.
Stopping criterion: optimization terminated when the change in function value between successive iterations fell below a predefined threshold.

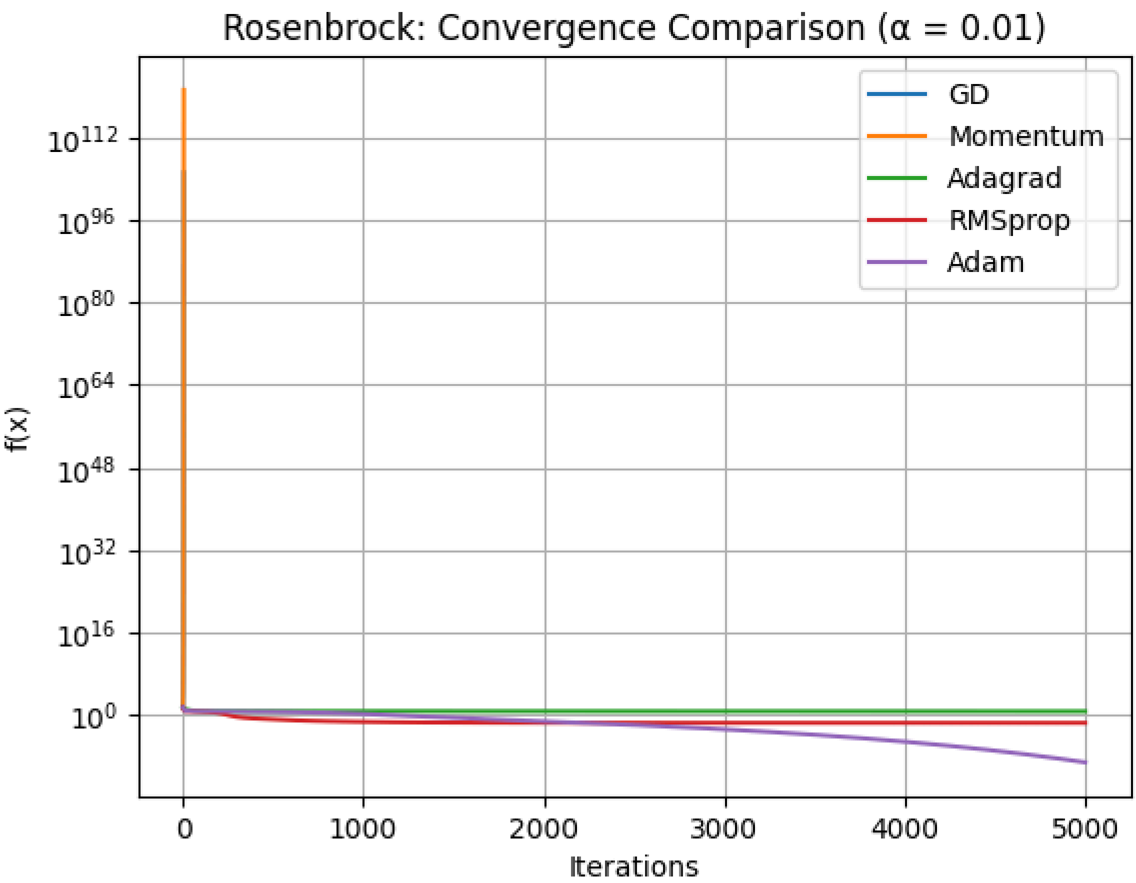
Methodology

For each optimizer and learning rate:

- Gradients were computed analytically.
- Parameters were updated iteratively.
- Convergence behavior (loss vs. iterations) was recorded.
- Final optimized values of x (and y where applicable) were reported.
- Time taken for convergence was measured.

Results and Analysis





- Compare speed of convergence across optimizers.

  Relative convergence speed (from fastest to slowest, in general):

1. **Adam — Fastest overall**
   - Typically shows the **quickest drop in loss in early iterations**.
   - Handles non-convex landscapes well due to adaptive moment estimates.
2. **RMSProp — Very fast**
   - Usually converges almost as quickly as Adam.
   - More stable than plain SGD on rugged loss surfaces.
3. **SGD with Momentum — Moderate speed**
   - Faster than vanilla Gradient Descent because it builds velocity and helps escape shallow regions and local minima.
   - Performs well but still sensitive to learning rate choice.
4. **Adagrad — Fast initially, slow later**
   - Often reduces loss quickly in early iterations.
   - Becomes very slow later due to continuously shrinking effective learning rate.
5. **Vanilla Gradient Descent — Slowest**
   - Requires the most iterations to converge.
   - Highly sensitive to learning rate and often struggles with non-convex functions like Rosenbrock.

- Discuss sensitivity to learning rate.

  Impact of learning rate on convergence:

- **α = 0.01 (small learning rate):**
  - Generally **stable but slow convergence**.
  - Loss decreases smoothly but requires many iterations to reach the optimum.
  - Less risk of divergence, even for difficult functions like Rosenbrock.
- **α = 0.05 (moderate learning rate):**
  - Often provides the **best balance between speed and stability**.
  - Typically leads to faster convergence without major oscillations.
- **α = 0.1 (large learning rate):**
  - Can cause **faster initial progress** but risks **oscillations or divergence**, especially for Gradient Descent and Momentum-based SGD.
  - More likely to overshoot the minimum in highly curved regions of the loss surface.

Optimizer robustness to learning rate:

- **Adam and RMSProp** are **less sensitive** to learning rate changes due to adaptive scaling of updates.
- **Vanilla GD and Momentum SGD** are **highly sensitive**, requiring careful tuning of α.
- Overall, the results highlight that choosing an appropriate learning rate is crucial, especially for non-convex functions with multiple local minima.

Task 2: Linear Regression Using Multi-Layer Neural Network (From Scratch)

Dataset

Boston Housing Dataset was used with two features:

- RM (average number of rooms per dwelling)
- CRIM (per capita crime rate by town)
  Target variable: MEDV (median value of homes).

Preprocessing

- Features were normalized.
- Data was split into 80% training and 20% testing.

Model Architecture

- Input Layer: 2 neurons
- Hidden Layer 1: 5 neurons (ReLU activation)

- Hidden Layer 2: 3 neurons (ReLU activation)
- Output Layer: 1 neuron (linear activation)

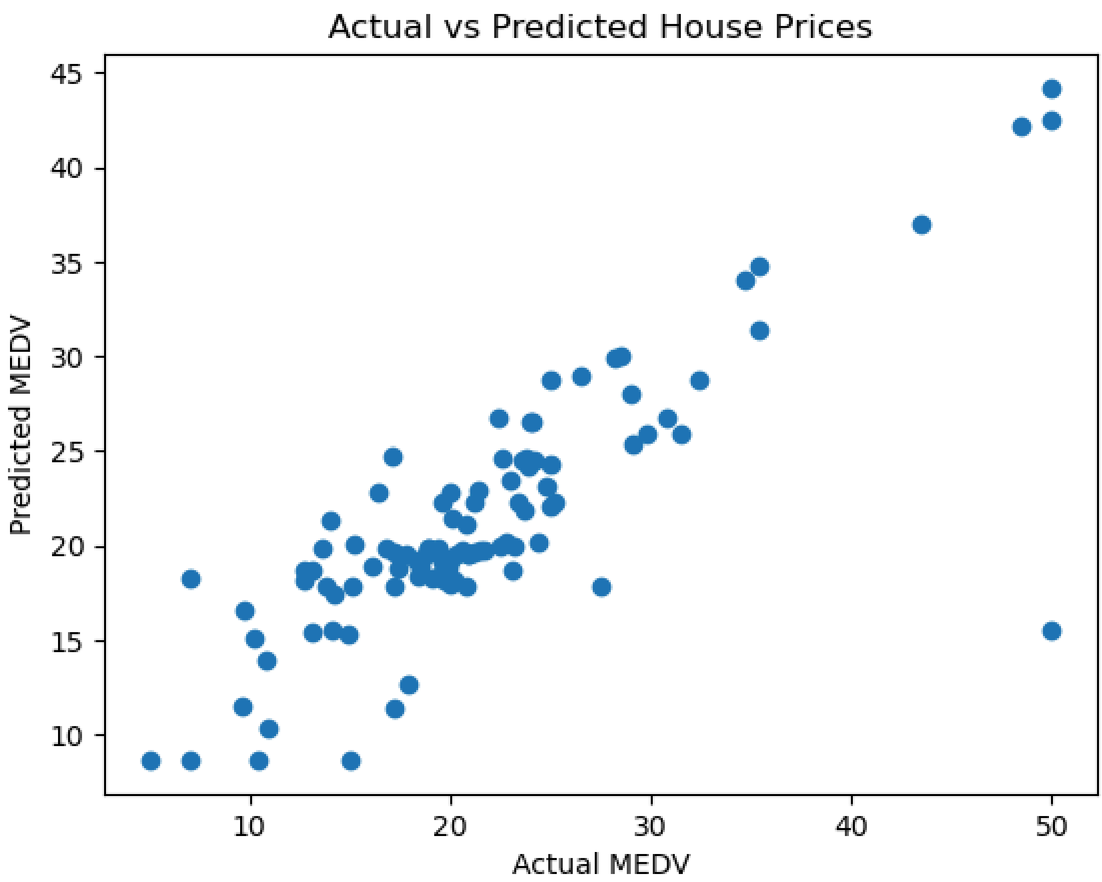
Training Setup

Optimizers implemented:

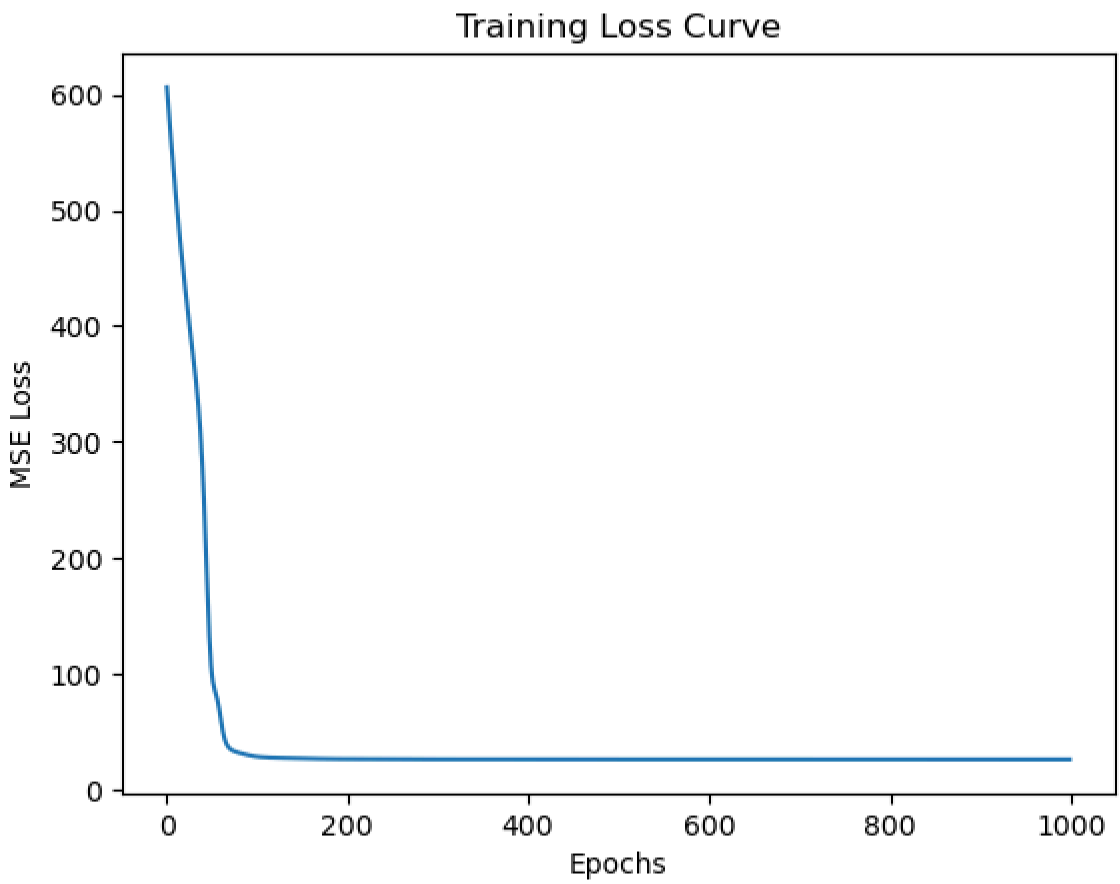
- Basic Gradient Descent
- Momentum
- Adam
  Learning rates tested: 0.01 and 0.001
  Epochs: 1000
  Loss function: Mean Squared Error (MSE)

Evaluation

- Test MSE reported
  Test Mean Squared Error: `24.40887101467112`
- Plot: Predicted vs Actual values



- Loss vs Epoch plot



Bonus Experiments

- Added a third hidden layer with 2 neurons and analyzed impact.
- Implemented L2 regularization and discussed effect on overfitting.

Task 3: Multi-class Classification Using FCNN

Dataset (Custom)

Two datasets were used:

1. Linearly separable dataset (3 classes, 2D)
2. Non-linearly separable dataset (2 or 3 classes, 2D)

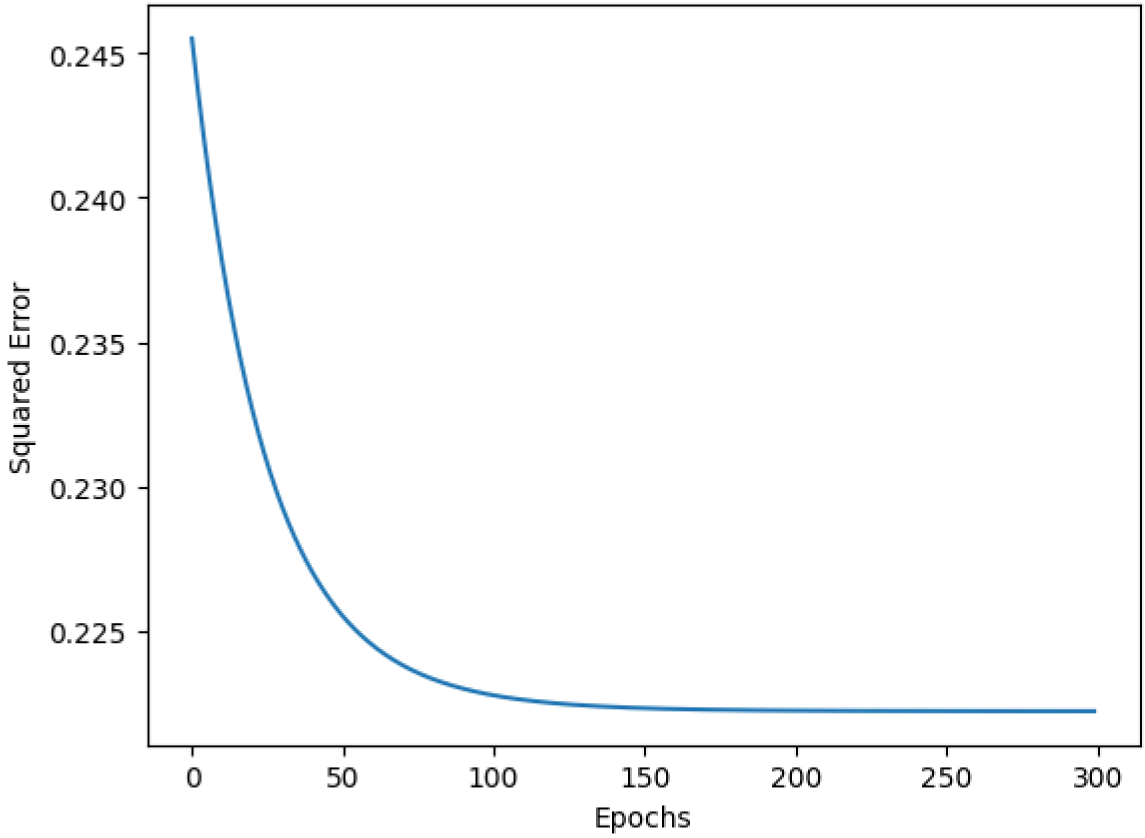
Each class was split into:

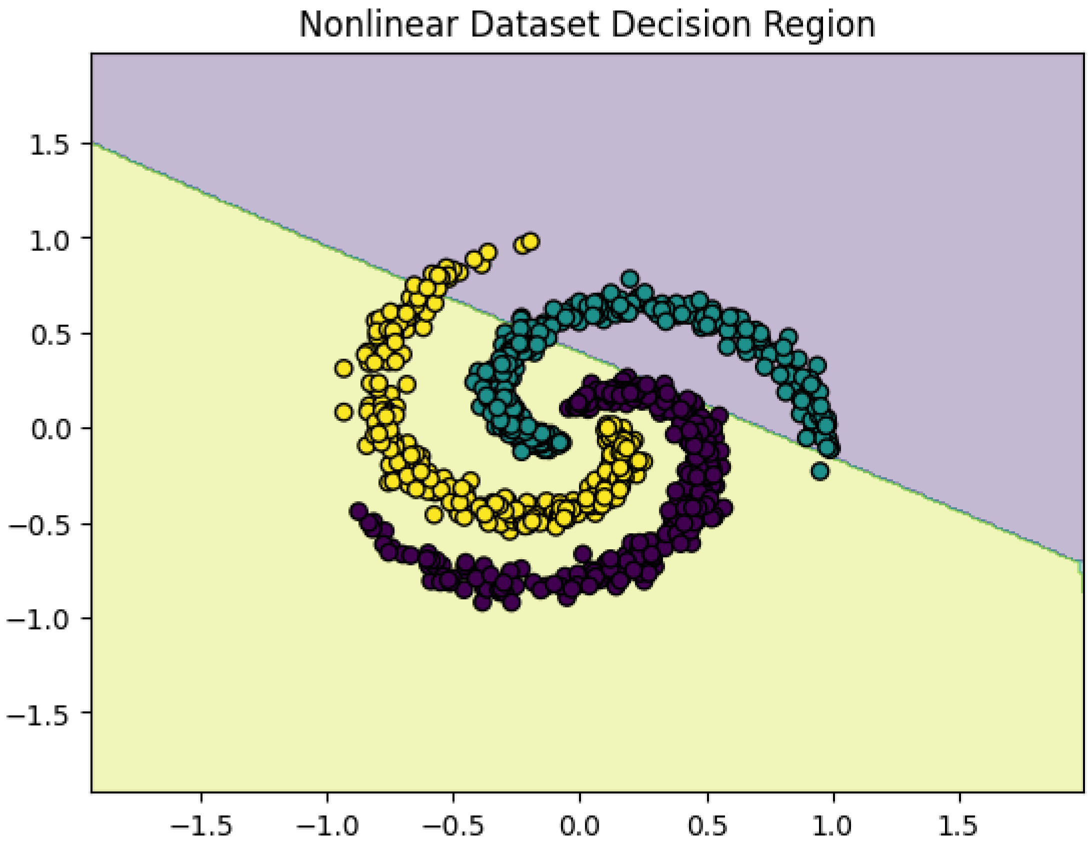
- 60% Training
- 20% Validation
- 20% Testing

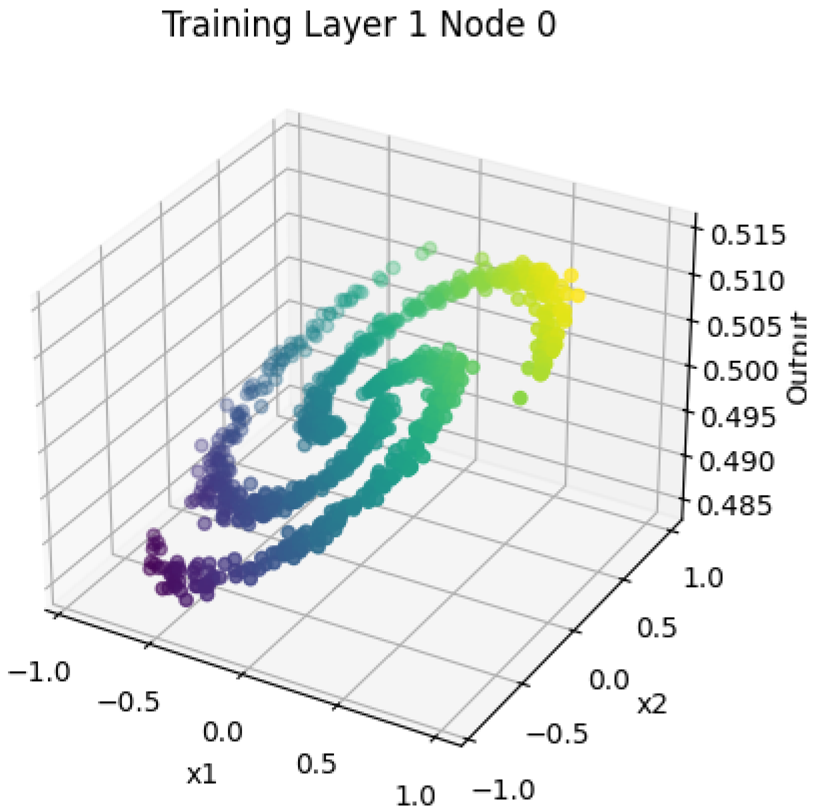
Model

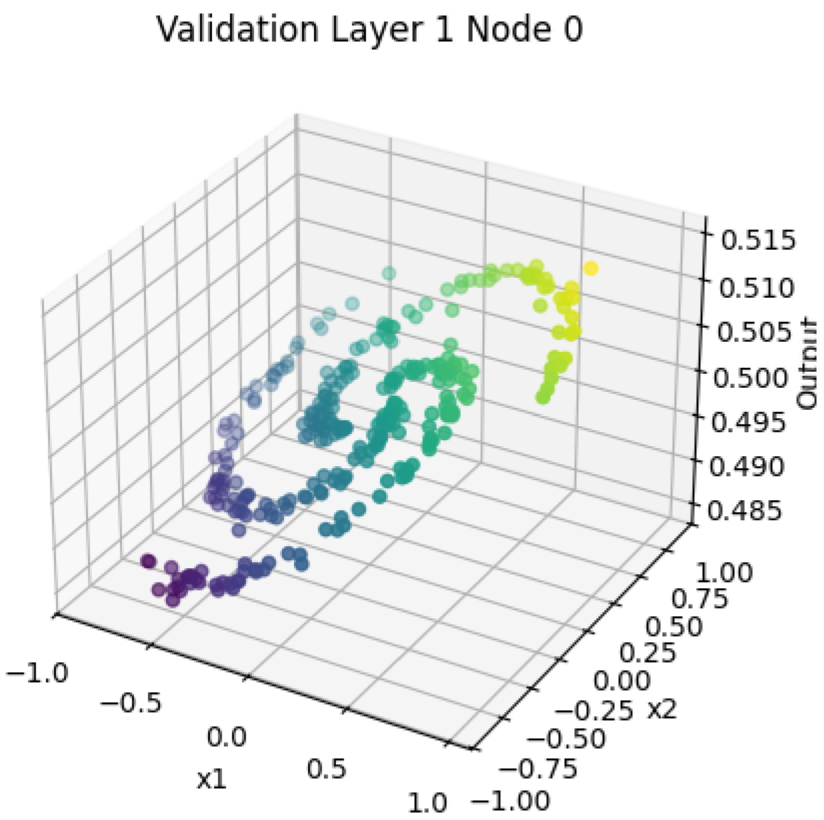
- Dataset 1: FCNN with 1 hidden layer
- Dataset 2: FCNN with 2 hidden layers
- Loss: Squared error
- Training: Stochastic Gradient Descent (SGD)

Results (Best Architecture)




Nonlinear Dataset Decision Region
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5


Training Layer 1 Node 0
0.515
0.510
0.505
0.500
0.495
0.490
0.485
Output
−1.0
−0.5
0.0
0.5
1.0
x1
1.0
0.5
0.0
−0.5
−1.0
x2



- Validation Confusion matrix:

```
[[ 1  0 99]
 [44  0 56]
 [ 2  0 98]]
```

- Validation Accuracy: `0.33`
- Test Confusion matrix:

```
[[  0   0 100]
 [ 47   0  53]
 [  8   0  92]]
```

- Test Accuracy: `0.30666666666666664`

Task 4: FCNN on MNIST (Optimizer Comparison)

Dataset (MNIST)

Subset of MNIST with 5 chosen classes.

- Image size: 28×28 → flattened to 784-dimensional vector.

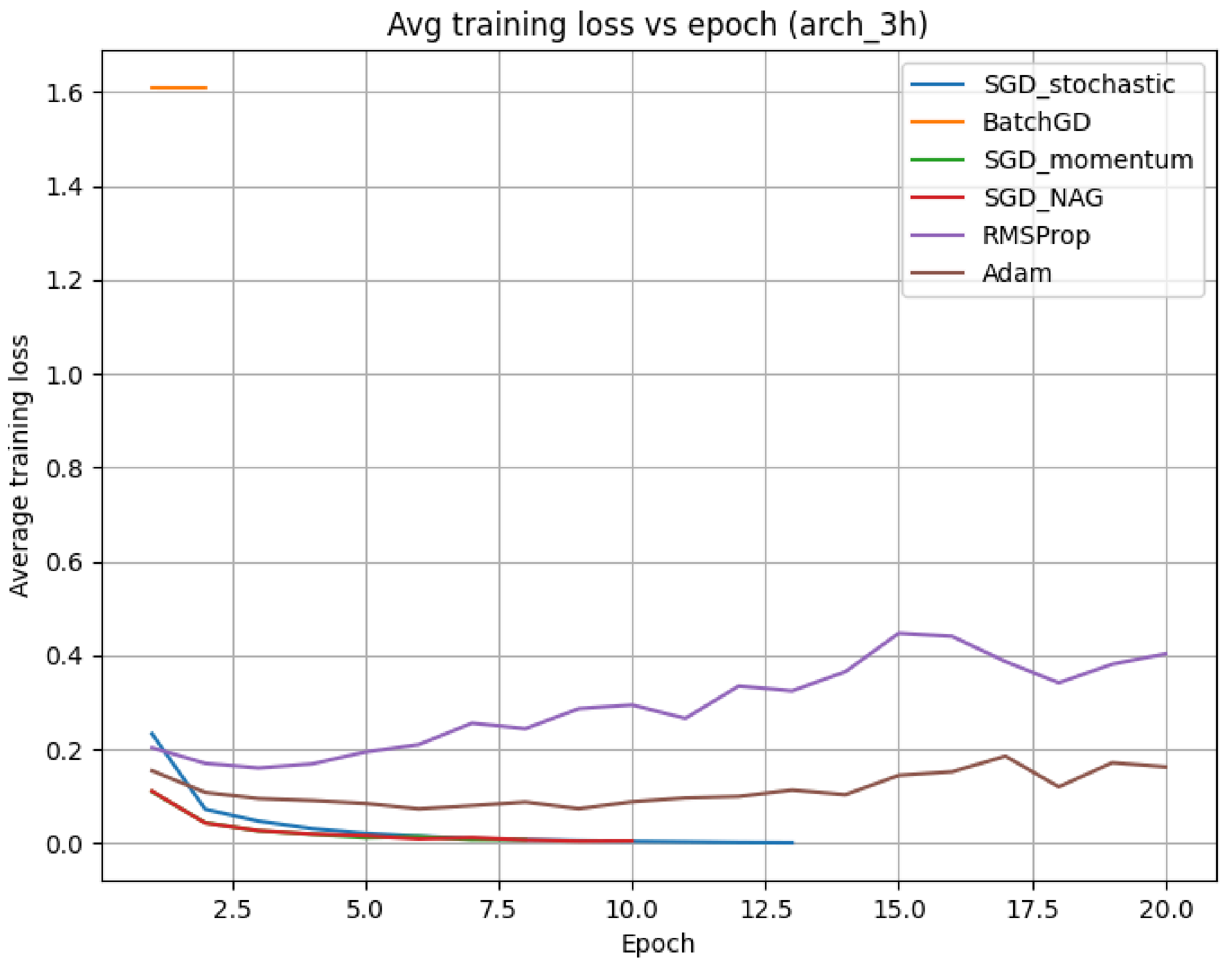
- Split: 80% training, 20% testing.

Model (PyTorch)

FCNN with 3 to 5 hidden layers.
Loss: Cross-entropy.

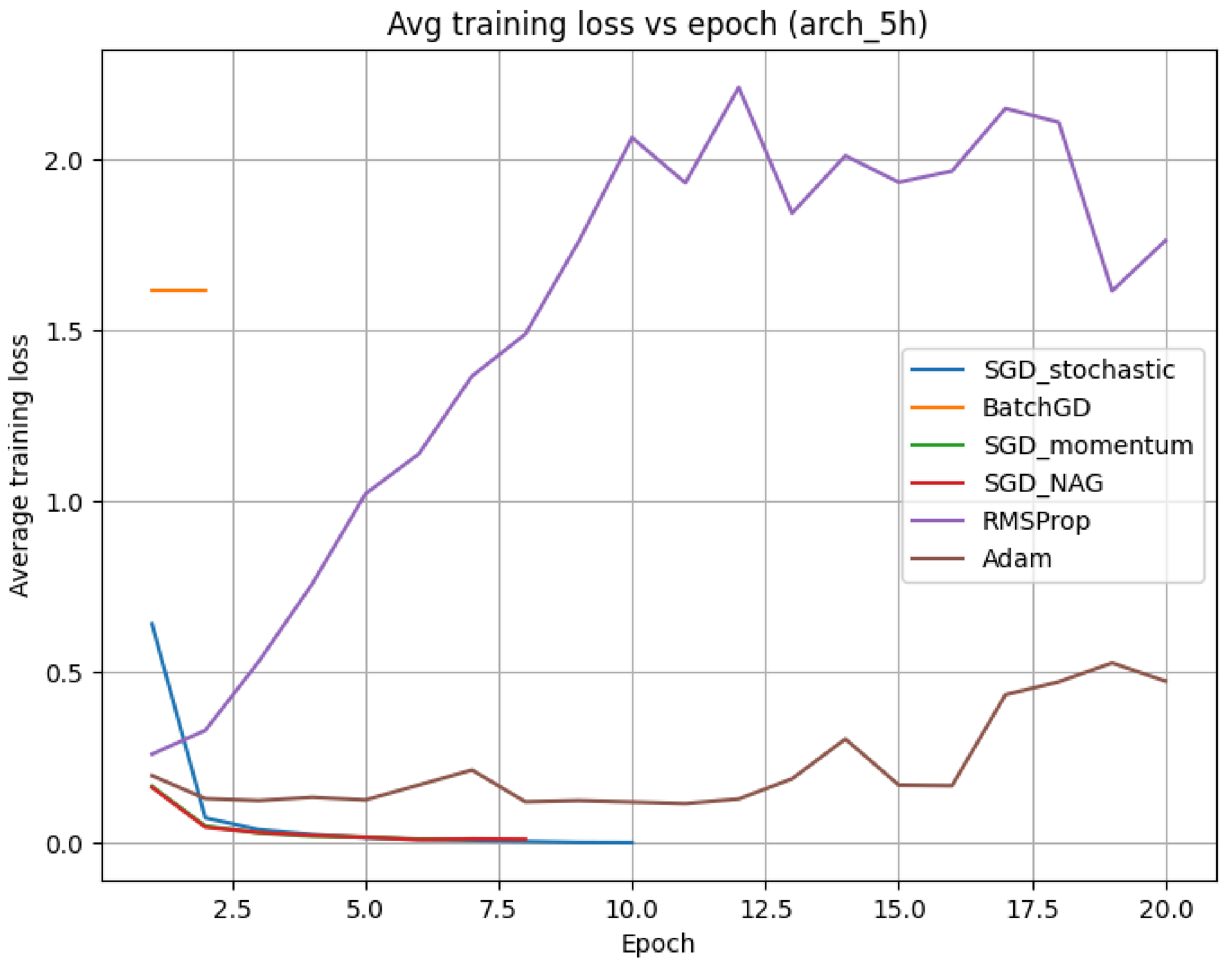
Results

- The **3-hidden-layer model** converged fastest but slightly underperformed in validation accuracy.



- The **4-hidden-layer model (arch_4h)** achieved the best balance between convergence speed and generalization, making it the optimal choice.

- The **5-hidden-layer model** did not provide significant improvement over arch_4h and in some cases showed signs of overfitting or slower convergence.



BEST RESULT

- Best Architecture: `arch_4h`
- Hidden Layer Sizes: `[512, 256, 128, 64]`
- Best Optimizer: `SGD_momentum`
- Validation Confusion Matrix:

```
[[5496,    0,    0,    1,    1],
 [   0, 6322,    0,    3,    1],
 [   0,    0, 5631,   12,    0],
 [   0,    0,    3, 5656,    0],
 [   0,    1,    6,    1, 5454]]
```

- Validation Accuracy: `0.9935637330348398`
- Test Confusion Matrix:

```
[[1399,    0,    3,    2,    1],
 [   0, 1542,    3,    1,    5],
 [   5,    2, 1325,   12,    3],
 [   1,    1,    2, 1478,    0],
 [   2,    1,    1,    1, 1357]]
```

- Training Accuracy: `0.998985588358752`

Optimizers Compared

Across all architectures (3h, 4h, and 5h), the loss curves demonstrate distinct convergence characteristics for different optimizers:

- **SGD (batch_size=1):**
  Exhibited the slowest convergence among all methods, with noisy and fluctuating loss curves, particularly in deeper architectures. This is expected due to high variance in gradient estimates.
- **Batch Gradient Descent:**
  Showed smoother loss curves than SGD but converged more slowly in terms of epochs, especially for the 5-hidden-layer architecture.
- **SGD with Momentum (0.9):**
  Provided significantly faster and more stable convergence compared to vanilla SGD. This optimizer performed best overall and was used in the final selected model.
- **NAG Momentum:**
  Demonstrated behavior similar to standard momentum but with slightly improved stability in some architectures, particularly in early epochs.
- **RMSProp:**
  Converged relatively fast in shallow architectures but showed some instability in deeper networks due to adaptive step sizes.
- **Adam:**
  Showed rapid initial convergence but in some cases plateaued earlier than momentum-based methods, leading to slightly lower validation performance compared to SGD with momentum.

Conclusion

This assignment demonstrates the impact of optimizers, network depth, and regularization on convergence and generalization. Task 1 highlights optimizer behavior on non-convex landscapes, Task 2 shows the effectiveness of multi-layer networks for regression, Task 3 explores decision boundaries in classification, and Task 4 provides a comprehensive comparison of optimizers on MNIST.