CHAPTER - 10

INVERSE TRIGONOMETRIC FUNCTIONS

Two bijective functions f(x) and g(x) defined on A such that $f \circ g\ x = x = (g \circ f)x$, then 'g' is the inverse of 'f' and it is denoted as f^{-1}. An invertible function $y = f(x) \Rightarrow x = f^{-1}(y)$.

$$x = \sin\theta \Rightarrow \theta = \sin^{-1} x;\ x = \operatorname{cosec}\theta \Rightarrow \theta = \operatorname{cosec}^{-1} x$$

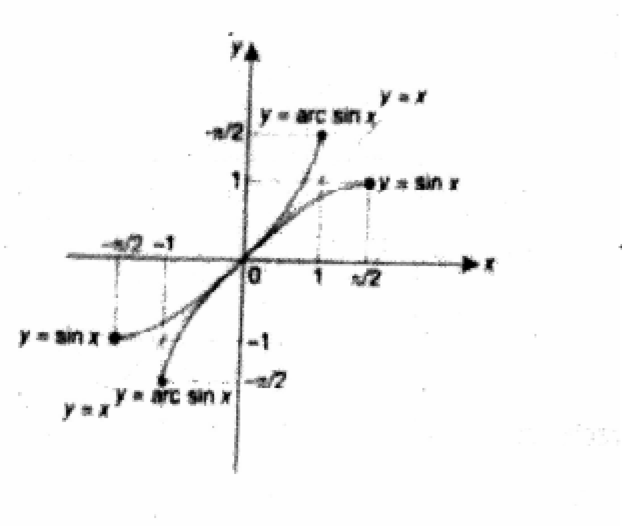
$$x = \cos\theta \Rightarrow \theta = \cos^{-1} x;\ x = \sec\theta \Rightarrow \theta = \sec^{-1} x$$

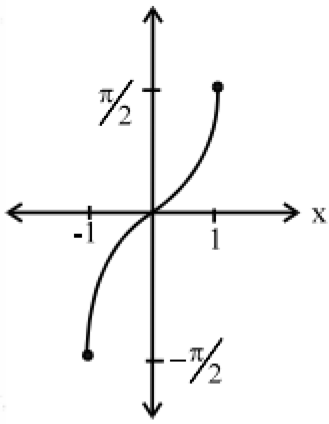
$$x = \tan\theta \Rightarrow \theta = \tan^{-1} x;\ x = \cot\theta \Rightarrow \theta = \cot^{-1} x$$

All periodic functions are manyone functions, so they are not bijective and hence they are not invertible. To define inverse trigonometric function, we forced to restrict the domain and range of them so that the restriction should be bijective.Restrictions for inverse trigonometric functions are given below.

Inverse Trigonometric function	Domain	Range
$y = \sin^{-1}x$	$[-1,1]$	$[-\pi/2, \pi/2]$
$y = \cos^{-1}x$	$[-1,1]$	$[0, \pi]$
$y = \tan^{-1}x$	R	$(-\pi/2, \pi/2)$
$y = \cot^{-1}x$	R	$(0, \pi)$
$y = \sec^{-1}x$	$R-(-1,1)$	$[0, \pi]-\{\pi/2\}$
$y=\operatorname{cosec}^{-1}x$	$R-(-1,1)$	$[-\pi/2, \pi/2]-\{0\}$

Characteristics of $f(x) = \sin^{-1} x$





1. $D_f = [-1, \ 1]$

2. $R_f = \left[-\frac{\pi}{2}, \ \frac{\pi}{2}\right]$

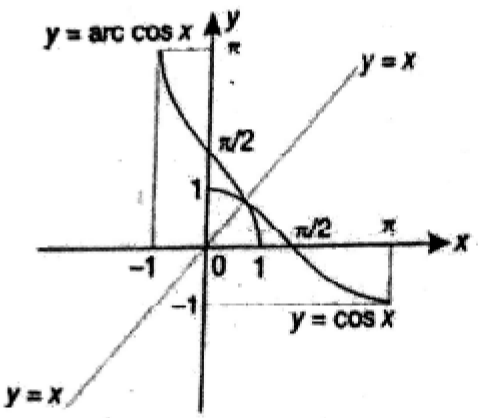
3. It is not a periodic funciton

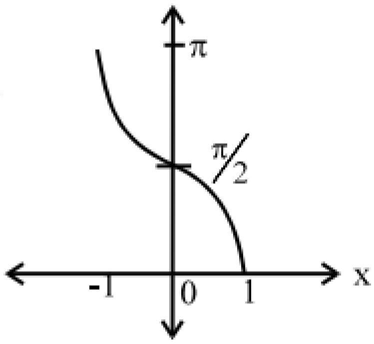
4. It is an odd function since, $\sin^{-1}(-x) = -\sin^{-1} x$

5. It is a strictly increasing function

6. It is a one one function

Characteristics of $f(x) = \cos^{-1} x$





1. $D_f = [-1, \ 1]$

2. R_f $[0, \ \pi]$

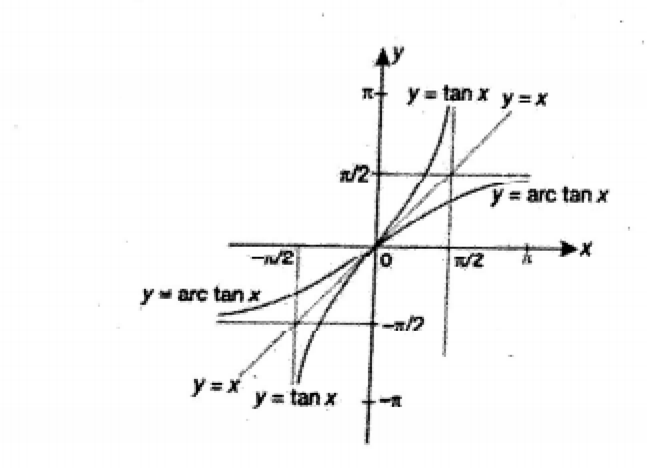
3. It is not a periodic funciton

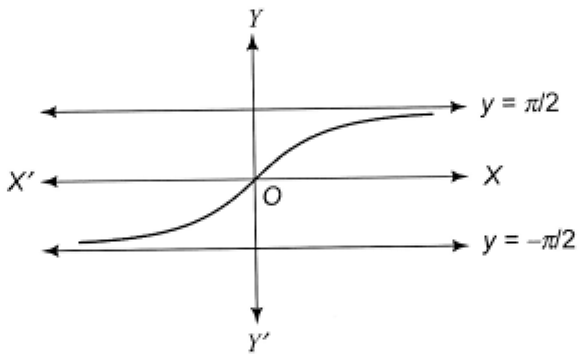
4. It is neither even nor odd function since, $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$

5. It is a strictly decreasing function

6. It is a one one function

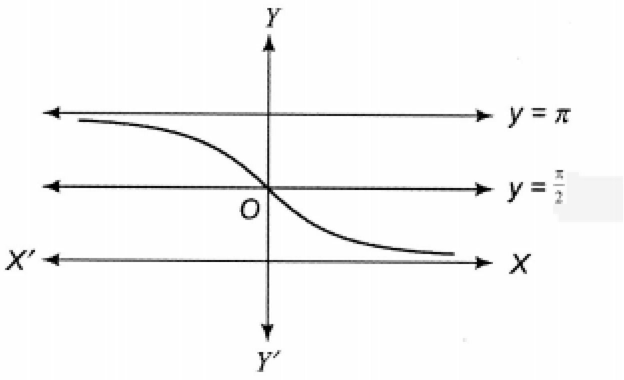
Characteristics of $f(x) = \tan^{-1} x$





1. $D_f = R$
2. $R_f = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
3. It is not a periodic funciton
4. It is an odd function since, $\tan^{-1}(-x) = -\tan^{-1} x$
5. It is a strictly increasing function
6. It is a one one function

Characteristics of $f(x) = \cot^{-1} x$

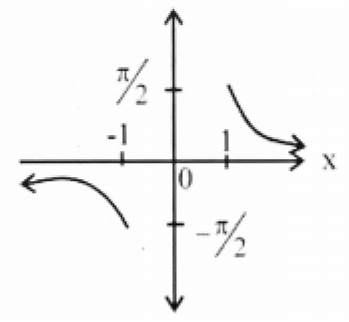


1. $D_f = R$
2. $R_f = (0, \pi)$
3. It is not a periodic funciton
4. It is neither even nor odd function since, $\cot^{-1}(-x) = \pi - \cot^{-1} x$
5. It is a strictly decreasing function

6. It is a one one function

Characteristics of $f(x)=\operatorname{cosec}^{-1}x$

	Domain	Range
$y=\operatorname{cosec}^{-1}x$	$\|x\|\geq 1$ or $R-(-1,1)$ or $(-\infty,-1]\cup[1,\infty)$	$\left[\frac{-\pi}{2},\frac{\pi}{2}\right]-\{0\}$



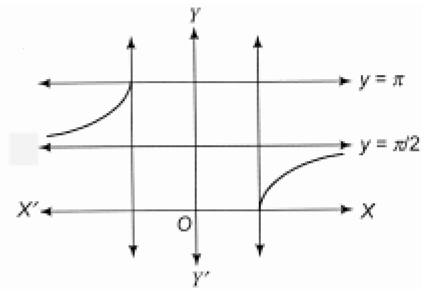
1. $D_f=(-\infty,-1]\cup[1,\infty)$

2. $R_f=\left[-\frac{\pi}{2},\ \frac{\pi}{2}\right]-\{0\}$

3. It is an odd function, since $\operatorname{cosec}^{-1}(-x)=-\operatorname{cosec}^{-1}(x)$
4. It is a non periodic function
5. It is a one one function
6. It is a strictly decreasing function with respect to its domain

Characteristics of $f(x)=\sec^{-1}x$

	Domain	Range
$y=\sec^{-1}x$	$\|x\|\geq 1$ or $R-(-1,1)$ or $(-\infty,-1]\cup[1,\infty)$	$[0,\pi]-\left\{\frac{\pi}{2}\right\}$



1. $D_f=(-\infty\quad -1]\cup[1,\quad \infty]$

2. $R_f=[0,\quad \pi]-\left\{\frac{\pi}{2}\right\}$

3. It is neither an even function nor an odd odd function, since $\sec^{-1}(-x)=\pi-\sec^{-1}(x)$

4. It is a non periodic function

5. It is a one one function

6. It is strictly increasing function with respect to its domain

II

I) $\sin^{-1}(-x)=-\sin^{-1}x, x\in[-1,1]$ $\qquad$ $\cos^{-1}(-x)=\pi-\cos^{-1}x,\ x\in[-1,1]$

$\tan^{-1}(-x)=-\tan^{-1}x,\ x\in R$ $\qquad$ $\sec^{-1}(-x)=\pi-\sec^{-1}x,\ |x|\geq 1$

$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x, |x| \geq 1$ $\qquad$ $\cot^{-1}(-x) = \pi - \cot^{-1}x,\ x \in R$

II) $\operatorname{cosec}^{-1}x = \sin^{-1}\left(\frac{1}{x}\right); |x| \geq 1$ $\qquad$ $\operatorname{cosec}^{-1} f(x) = \sin^{-1}\frac{1}{f(x)} \quad |f(x)| \geq 1$

$\sec^{-1}x = \cos^{-1}\left(\frac{1}{x}\right);\ |x| \geq 1$ $\qquad$ $\sec^{-1} f(x) = \cos^{-1}\frac{1}{f(x)} \quad |f(x)| \geq 1$

$$\cot^{-1}x = \begin{cases} \tan^{-1}\left(\frac{1}{x}\right) & x > 0 \\ \pi + \tan^{-1}\frac{1}{x} & x < 0 \end{cases} \qquad \cot^{-1} f(x) = \begin{cases} \tan^{-1}\frac{1}{f(x)} & f(x) > 0 \\ \pi + \tan^{-1} f(x) & f(x) < 0 \end{cases}$$

Conversion property

I) Conversions of one inverse trigonometric function into another one.

a) For $x \in (0,1)$

$$\sin^{-1}x = \cos^{-1}\sqrt{1-x^2} \qquad \cos^{-1}x = \sin^{-1}\sqrt{1-x^2}$$

$$\sin^{-1}x = \tan^{-1}\frac{x}{\sqrt{1-x^2}} \qquad \cos^{-1}x = \tan^{-1}\frac{\sqrt{1-x^2}}{x}$$

$$\sin^{-1}x = \sec^{-1}\frac{1}{\sqrt{1-x^2}} \qquad \cos^{-1}x = \cot^{-1}\frac{x}{\sqrt{1-x^2}}$$

$$\sin^{-1}x = \cot^{-1}\frac{\sqrt{1-x^2}}{x} \qquad \cos^{-1}x = \sec^{-1}\frac{1}{x}$$

$$\sin^{-1}x = \operatorname{cosec}^{-1}\frac{1}{x} \qquad \cos^{-1}x = \operatorname{cosec}^{-1}\frac{1}{\sqrt{1-x^2}}$$

b) For $x \in (0,\infty)$

$$\tan^{-1}x = \sin^{-1}\frac{x}{\sqrt{1+x^2}} \qquad \cot^{-1}x = \sin^{-1}\frac{1}{\sqrt{x^2+1}}$$

$$\tan^{-1}x = \cos^{-1}\frac{1}{\sqrt{1+x^2}} \qquad \cot^{-1}x = \cos^{-1}\frac{x}{\sqrt{x^2+1}}$$

$$\tan^{-1}x = \cot^{-1}\frac{1}{x}, x \qquad \cot^{-1}x = \tan^{-1}\frac{1}{x}$$

$$\tan^{-1}x = \operatorname{cosec}^{-1}\frac{\sqrt{1+x^2}}{x} \qquad \cot^{-1}x = \sec^{-1}\frac{\sqrt{x^2+1}}{x}$$

$$\tan^{-1}x = \sec^{-1}\sqrt{1+x^2} \qquad \cot^{-1}x = \operatorname{cosec}^{-1}\sqrt{x^2+1}$$

$$\sin^{-1}x+\sin^{-1}y=\begin{cases}\sin^{-1}\left[x\sqrt{1-y^2}+y\sqrt{1-x^2}\right]; \text{if } x,y\in[-1,1] \text{ and } x^2+y^2\le 1 \text{ or} \\ \qquad\qquad \text{if } x,y\in[-1,1], xy<0 \text{ and } x^2+y^2\ge 1 \\ \pi-\sin^{-1}\left[x\sqrt{1-y^2}+y\sqrt{1-x^2}\right] \text{ if } 0<x,y\le 1 \text{ and } x^2+y^2>1 \\ -\pi-\sin^{-1}\left\{x\sqrt{1-y^2}+y\sqrt{1-x^2}\right\} \text{ if } -1\le x,y<0 \text{ and } x^2+y^2>1\end{cases}$$

$$\sin^{-1}x-\sin^{-1}y=\begin{cases}\sin^{-1}\left[x\sqrt{1-y^2}-x\sqrt{1-x^2}\right] \text{ if } x,y\in[-1,1] \text{ and } x^2+y^2\le 1 \text{ or if } xy>0\ x^2+y^2>1 \\ \pi-\sin^{-1}\left[x\sqrt{1-y^2}-y\sqrt{1-x^2}\right]; \text{if } 0<x\le 1, -1\le y<0 \text{ and } x^2+y^2>1 \\ -\pi-\sin^{-1}\left[x\sqrt{1-y^2}-y\sqrt{1-x^2}\right] \text{if } -1\le x<0, 0<y\le 1 \text{ and } x^2+y^2>1\end{cases}$$

$$\cos^{-1}x+\cos^{-1}y=\begin{cases}\cos^{-1}\left\{xy-\sqrt{1-x^2}\sqrt{1-y^2}\right\}; \text{ if } x,y\in[-1,1] \text{and } x+y\ge 0 \\ 2\pi-\cos^{-1}\left\{xy-\sqrt{1-x^2}\sqrt{1-y^2}\right\}; \text{ if } x,y\in[-1,1] \text{and } x+y<0\end{cases}$$

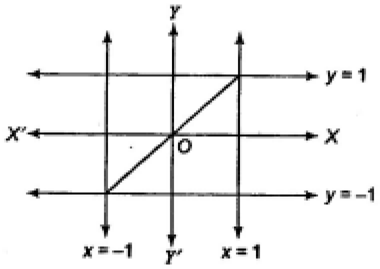
$$\cos^{-1}x-\cos^{-1}y=\begin{cases}\cos^{-1}\left\{xy+\sqrt{1-x^2}\sqrt{1-y^2}\right\}; \text{if } x,y\in[-1,1] \text{and } x\le y \\ -\cos^{-1}\left\{xy+\sqrt{1-x^2}\sqrt{1-y^2}\right\}; \text{ if } x\in[0,1] \text{and } y\in[-1,0]\end{cases}$$

$$\tan^{-1}x+\tan^{-1}y=\begin{cases}\tan^{-1}\left(\dfrac{x+y}{1-xy}\right) \text{ if } xy<1, x>0, y>0 \\ \pi+\tan^{-1}\left(\dfrac{x+y}{1-xy}\right) \text{ if } xy>1, x>0, y>0 \\ -\pi+\tan^{-1}\left(\dfrac{x+y}{1-xy}\right) \text{ if } xy>1, x<0, y<0\end{cases}$$

$$\tan^{-1}x-\tan^{-1}y=\begin{cases}\tan^{-1}\left(\dfrac{x-y}{1+xy}\right); \text{ if } xy>-1 \\ \pi+\tan^{-1}\left(\dfrac{x-y}{1+xy}\right); \text{ if } xy<-1, x>0, y<0 \\ -\pi+\tan^{-1}\left(\dfrac{x-y}{1+xy}\right); \text{ if } xy<-1, x<0, y>0\end{cases}$$

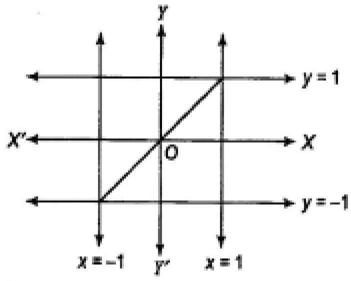
III

(i) Graph of $y=\sin\left(\sin^{-1}x\right)$



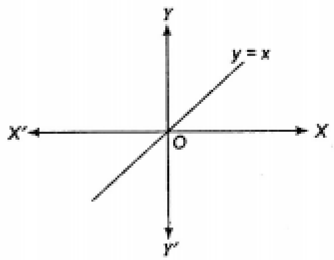
1. $D_f=[-1,1]$
2. $R_f=[-1,1]$
3. It is a non-periodic function.

(ii) Graph of $y=\cos\left(\cos^{-1}x\right)$



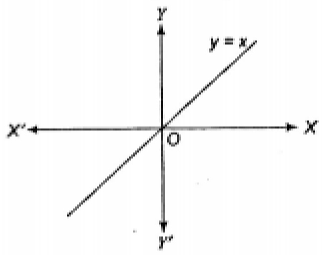
1. $D_f=[-1,1]$
2. $R_f=[-1,1]$
3. It is a non-periodic function.

(iii) Graph of $y=\tan\left(\tan^{-1}x\right)$



1. $D_f=R$
2. $R_f=R$
3. It is a non-periodic function.

(iv) Graph of $y=\cot\left(\cot^{-1}x\right)$

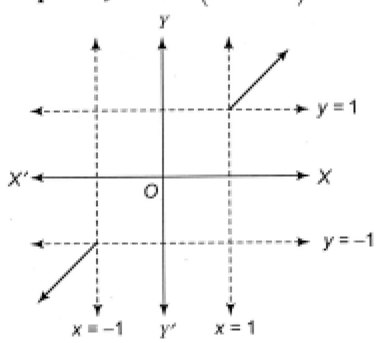


$D_f=R$

$R_f=R$

It is a non-periodic function.

(v) Graph of $y=\operatorname{cosec}\left(\operatorname{cosec}^{-1}x\right)$

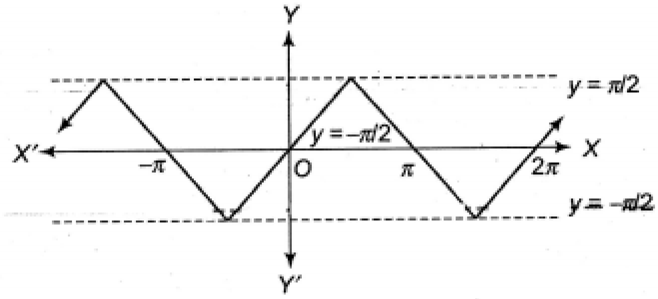


1. $D_f=(-\infty,-1]\cup[1,\infty)$
2. $R_f=(-\infty,-1]\cup[1,\infty)$
3. It is a non-periodic function.

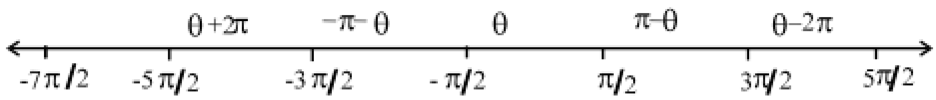
(i) A function $f : R \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is defined as $f(x) = \sin^{-1}(\sin x)$

1. $D_f = R$
2. $R_f = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
3. It is an odd function.
4. It is a periodic function with period 2π
5. $\sin^{-1}(\sin x) = \begin{cases} x & : -\frac{\pi}{2} \le x \le \frac{\pi}{2} \\ \pi - x & : \frac{\pi}{2} \le x \le \frac{3\pi}{2} \\ x - 2\pi & : \frac{3\pi}{2} \le x \le \frac{5\pi}{2} \\ -\pi - x & : -\frac{3\pi}{2} \le x \le -\frac{\pi}{2} \end{cases}$

Graph of $f(x) = \sin^{-1}(\sin x)$



$y = \sin^{-1}(\sin\theta), \theta \in R$



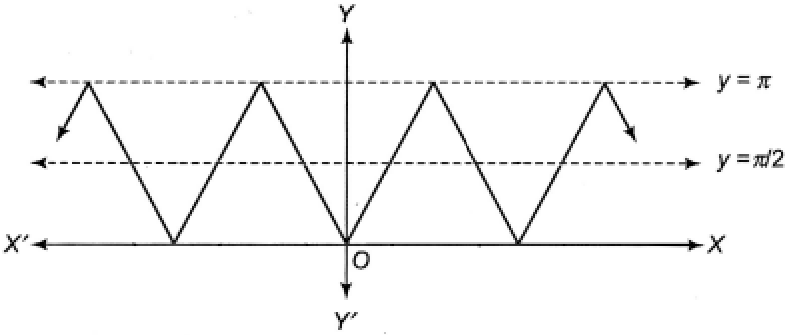
$\sin^{-1}(\sin m\theta) = (-1)^n (m\theta - n\pi)$; $\frac{(2n-1)\pi}{2m} < \theta \le \frac{(2n+1)\pi}{2m}$

(ii) $\cos^{-1}(\cos x)$:

A function $f: R \to [0, \pi]$ is defined as $f(x) = \cos^{-1}(\cos x)$

Graph of $f(x) = \cos^{-1}(\cos x)$:

1. $D_f = R$
2. $R_f = [0, \pi]$
3. It is even function.
4. It is periodic function with period 2π.
5. $\cos^{-1}(\cos x) = \begin{cases} x & : 0 \le x \le \pi \\ 2\pi - x & : \pi \le x \le 2\pi \\ x - 2\pi & : 2\pi \le x \le 3\pi \\ -x & : -\pi \le x \le 0 \end{cases}$



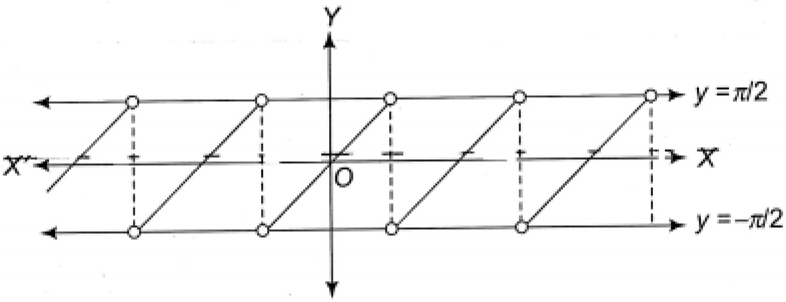
$$\cos^{-1}(\cos m\theta) = \begin{cases} m\theta - n\pi, & \frac{n\pi}{m} < \theta \le \frac{(n+1)\pi}{m}, \quad \text{n is even} \\ (n+1)\pi - m\theta; & \frac{n\pi}{m} < \theta \le \frac{(n+1)\pi}{m}, \text{n is ood} \end{cases}$$

(iii) $\tan^{-1}(\tan x)$:

A function $f: R - (2n+1)\frac{\pi}{2} \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ is defined as $f(x) = \tan^{-1}(\tan x)$

Graph of $f(x) = \tan^{-1}(\tan x)$:

1. $D_f = R - (2n+1)\frac{\pi}{2}, n \in I$
2. $R_f = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
3. It is an odd function.
4. It is a periodic function with period π
5. $\tan^{-1}(\tan x) = \begin{cases} x & : \frac{-\pi}{2} < x < \frac{\pi}{2} \\ x - \pi & : \frac{\pi}{2} < x < \frac{3\pi}{2} \\ x - 2\pi & : \frac{\pi}{2} < x < \frac{3\pi}{2} \\ x + \pi & : \frac{-3\pi}{2} < x < \frac{-\pi}{2} \end{cases}$

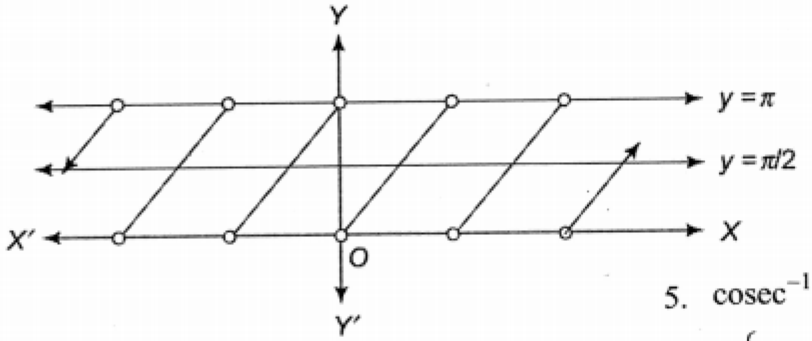


(iv) $\cot^{-1}(\cot x)$:

A function $f : R-(n\pi) \to (0, \pi)$ is defined

as $f(x) = \cot^{-1}(\cot x)$

Graph of $f(x) = \cot^{-1}(\cot x)$:



1. $D_f = R - n\pi, \quad n \in I$
2. $R_f = (0, \pi)$
3. It is neither even nor odd function.
4. It is a periodic function with period π.
5. $\operatorname{cosec}^{-1}(\operatorname{cosec} x)$

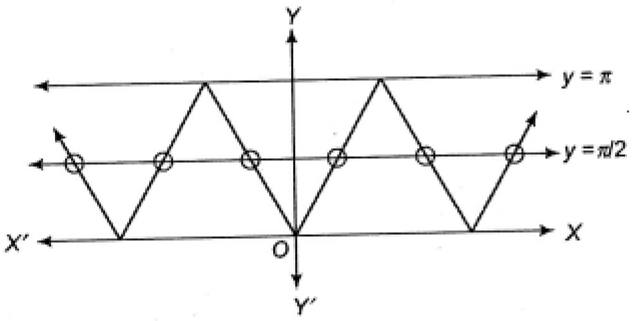
$$= \begin{cases} x & : \frac{-\pi}{2} \le x \le \frac{\pi}{2} \\ \pi - x & : \frac{\pi}{2} \le x \le \frac{3\pi}{2} \\ x - 2\pi & : \frac{3\pi}{2} \le x \le \frac{5\pi}{2} \\ -x - \pi & : \frac{-3\pi}{2} \le x \le \frac{-\pi}{2} \end{cases}$$

(vi) $\sec^{-1}(\sec x)$: A function

$f : R-(2n+1)\frac{\pi}{2} \to [0, \pi] - \left\{\frac{\pi}{2}\right\}$ is

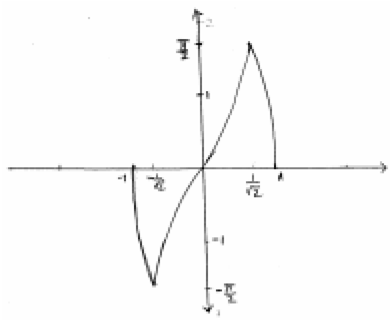
defined as $f(x) = \sec^{-1}(\sec x)$

Graph of $f(x) = \sec^{-1}(\sec x)$

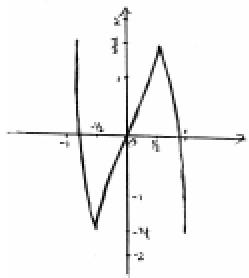


1. $D_f = R - (2n+1)\frac{\pi}{2}, n \in I$
2. $R_f = [0, \pi] - \left\{\frac{\pi}{2}\right\}$
3. It is neither even nor odd function.
4. It is a periodic function with period 2π
5. $\sec^{-1}(\sec x) = \begin{cases} x & : 0 \le x \le \pi \\ 2\pi - x & : \pi \le x \le 2\pi \\ x - 2\pi & : 2\pi \le x \le 3\pi \\ -x & : -\pi \le x \le 0 \end{cases}$

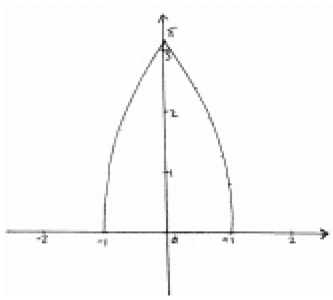
$$\sin^{-1}2x\sqrt{1-x^2}=\begin{cases}-\pi-2\sin^{-1}x, & \text{if } -1\le x<-\frac{1}{\sqrt{2}}\\ 2\sin^{-1}x, & -\frac{1}{\sqrt{2}}\le x<\frac{1}{\sqrt{2}}\\ \pi-2\sin^{-1}x, & \frac{1}{\sqrt{2}}<x\le 1\end{cases}$$



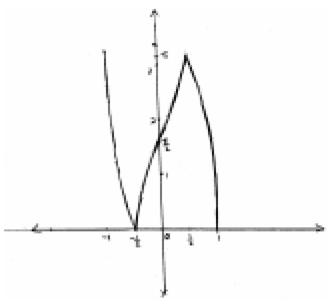
$$\sin^{-1}(3x-4x^3)=\begin{cases}-\pi-3\sin^{-1}x; -1\le x<-\frac{1}{2}\\ 3\sin^{-1}x; -\frac{1}{2}\le x<\frac{1}{2}\\ \pi-3\sin^{-1}x; \frac{1}{2}\le x\le 1\end{cases}$$



$$\cos^{-1}\left(2x^2-1\right)=\begin{cases}2\pi-2\cos^{-1}x & -1\le x<0\\ 2\cos^{-1}x & 0\le x\le 1\end{cases}$$



$$\cos^{-1}\left(4x^3-3x\right)=\begin{cases}-2\pi+3\cos^{-1}x; -1\le x\le \frac{-1}{2}\\ 2\pi-3\cos^{-1}x; -\frac{1}{2}<x\le\frac{1}{2}\\ 3\cos^{-1}x; \frac{1}{2}<x\le 1\end{cases}$$



Note: Need not be duscuss all graph. But few of them must be disussed A

PART I - (JEEMAIN)

SECTION - I - Straight objective type questions

1. The principal value of $\sin^{-1}\left(\sin\frac{5\pi}{3}\right)$ is

(1) $\frac{5\pi}{3}$ (2) $-\frac{5\pi}{3}$ (3) $-\frac{\pi}{3}$ (4) $\frac{4\pi}{3}$

2. The value of $\cot\left(\sum_{n=1}^{50}\tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right)$ is

1) $\frac{26}{25}$ 2) $\frac{25}{26}$ 3) $\frac{50}{51}$ 4) $\frac{52}{51}$

3. The range of value of p for which the equation $\sin\cos^{-1}(\cos(\tan^{-1}x)) = p$ has a solution is

A) $\left(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ B) $[0,1)$ C) $\left(\frac{1}{\sqrt{2}},1\right)$ D) $(-1,1)$

4. $2\sin^{-1}\frac{3}{5}+\cos^{-1}\frac{24}{25} =$

(1) $\frac{\pi}{2}$ (2) $\frac{2\pi}{3}$ (3) $\frac{5\pi}{3}$ (4) None of these

5. The value of $\sin\left(2\tan^{-1}\left(\frac{1}{3}\right)\right)+\cos(\tan^{-1}2\sqrt{2}) =$

(1) $\frac{16}{15}$ (2) $\frac{14}{15}$ (3) $\frac{12}{15}$ (4) $\frac{11}{15}$

6. If $\cos^{-1}x > \sin^{-1}x$ then x lies in the interval

1) $\left(\frac{1}{2},1\right]$ 2) $(0,1]$ 3) $\left[-1,\frac{1}{\sqrt{2}}\right)$ 4) $[-1,1]$

7. Considering only the principle values, if $\tan(\cos^{-1}x) = \sin\left(\cot^{-1}\frac{1}{2}\right)$, then x =

(1) $\pm\frac{5}{3}$ (2) $\frac{\sqrt{5}}{3}$ (3) $\pm\frac{5}{\sqrt{3}}$ (4) None of these

8. If $\cos^{-1}p+\cos^{-1}q+\cos^{-1}r = \pi$ then $p^2+q^2+r^2+2pqr =$

(1) 3 (2) 1 (3) 2 (4) −1

9. $\tan^{-1}\left[\dfrac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right] =$

(1) $\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x^2$ (2) $\frac{\pi}{4}+\cos^{-1}x^2$ (3) $\frac{\pi}{4}+\frac{1}{2}\cos^{-1}x$ (4) $\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x^2$

10. $\sum\limits_{m=1}^{n}\tan^{-1}\left(\dfrac{2m}{m^4+m^2+2}\right)$ is equal to

(1) $\tan^{-1}\left(\dfrac{n^2+n}{n^2+n+2}\right)$ (2) $\tan^{-1}\left(\dfrac{n^2-n}{n^2-n+2}\right)$ (3) $\tan^{-1}\left(\dfrac{n^2+n+2}{n^2+n}\right)$ (4) None of these

11. The number of real solutions of $\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$ is

(1) Zero (2) One (3) Two (4) Infinite

12. $2\tan^{-1}\left[\sqrt{\dfrac{a-b}{a+b}}\tan\dfrac{\theta}{2}\right] =$

(1) $\cos^{-1}\left(\dfrac{a\cos\theta+b}{a+b\cos\theta}\right)$ (2) $\cos^{-1}\left(\dfrac{a+b\cos\theta}{a\cos\theta+b}\right)$

(3) $\cos^{-1}\left(\dfrac{a\cos\theta}{a+b\cos\theta}\right)$ (4) $\cos^{-1}\left(\dfrac{a\cos+b\theta}{a+b\cos\theta}\right)$

13. $2\tan^{-1}(\cos x)=\tan^{-1}(\operatorname{cosec}^2 x)$, then x =

(1) $\frac{\pi}{2}$ (2) π (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{3}$

14. If we consider only the principle values of the inverse trigonometric functions then the value of $\tan\left(\cos^{-1}\dfrac{1}{5\sqrt{2}}-\sin^{-1}\dfrac{4}{\sqrt{17}}\right)$ is

(1) $\frac{\sqrt{29}}{3}$ (2) $\frac{29}{3}$ (3) $\frac{\sqrt{3}}{29}$ (4) $\frac{3}{29}$

15. If $\cos^{-1}\left(\dfrac{x}{a}\right)+\cos^{-1}\left(\dfrac{y}{b}\right)=\alpha$, then $\dfrac{x^2}{a^2}-\dfrac{2xy}{ab}\cos\alpha+\dfrac{y^2}{b^2}=$

(1) $\sin^2\alpha$ (2) $\cos^2\alpha$ (3) $\tan^2\alpha$ (d) $\cot^2\alpha$

16. If $a_1, a_2, a_3, \ldots, a_n$ is an A.P. with common difference d then

$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\ldots+\tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)\right]=$$

(1) $\frac{(n-1)d}{a_1+a_n}$ (2) $\frac{(n-1)d}{1+a_1a_n}$ (3) $\frac{nd}{1+a_1a_n}$ (4) $\frac{a_n-a_1}{a_n+a_1}$

17. Number of solutions of the equation $\cos^{-1}(1-x)-2\cos^{-1}x=\frac{\pi}{2}$ *is*

A) 3 B) 2 C) 1 D) 0

18. The value of $\cot\left(\sum_{n=1}^{19}\cot^{-1}\left(1+\sum_{p=1}^{n}2p\right)\right)$ is:

(1) $\frac{22}{23}$ (2) $\frac{23}{22}$ (3) $\frac{21}{19}$ (4) $\frac{19}{21}$

19. Assertion & Reasoning

(1) If both Statement-I and Statement-II are true and the reason is the correct explanation of the statement-I.

(2) If both Statement-I and Statement-II are true but reason is not the correct explanation of the statement-I.

(3) If Statement-I is true but Statement-II is false.

(4) If Statement-I is false but Statement-II is true.

Statement-I: If $\tan^{-1}x+\tan^{-1}y=\frac{\pi}{4}-\tan^{-1}z$ and $x+y+z=1$, then arithmetic mean of odd powers of x, y, z is equal to 1/3

Statement-II: For any x, y, z we have $xyz-xy-yz-zx+x+y+z=1+(x-1)(y-1)(z-1)$.

20. S_1: $\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{1}{7}=\frac{\pi}{4}$

S_2: $\tan^{-1}\frac{x}{y}+\tan^{-1}\left(\frac{y-x}{y+x}\right)=\frac{\pi}{4}$ $\quad x>0, y>0$

1) Statement 1 and 2 are correct and 2 is the correct explanation of 1

2) Statement 1 and 2 are correct and 2 is not the correct explanaiton of 1

3) Statement 1 is true and 2 false

4) Statement 1 false and 2 true

SECTION - II

Numerical Type Questions

21. $\sec^2(\tan^{-1} 2) + \text{cosec}^2(\cot^{-1} 3) =$

22. If $\cot^{-1}\left(\frac{n^2 - 10n + 26}{2\sqrt{3}}\right) > \frac{\pi}{6}, n \in N$, then the minimum value of n is

23. If $\sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right] = \frac{m}{n}$, then m + n =

24. If $\tan\left[\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\right] = \frac{m}{n}$, then m + n =

25. Number of integral solutions of the equation $\sin^{-1}(\sin x) = \cos^{-1}(\cos x)$ *in* $[0, 5\pi]$ is

PART - II (JEE ADVANCED)

SECTION - III (Only one correct option type)

26. Sum of infinite terms of the series $\cot^{-1}\left(1^2 + \frac{3}{4}\right) + \cot^{-1}\left(2^2 + \frac{3}{4}\right) + \cot^{-1}\left(3^2 + \frac{3}{4}\right) + \ldots$ is

(A) $\pi/4$ (B) $\tan^{-1} 2$ (C) $\tan^{-1} 3$ (D) $\tan^{-1} 4$

27. If $a\sin^{-1} x - b\cos^{-1} x = c$, then the value of $a\sin^{-1} x + b\cos^{-1} x$ (whenever exists) is equal to

(A) 0 (B) $\frac{\pi ab + c(b-a)}{a+b}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi ab + c(a-b)}{a+b}$

28. The number of solutions of the equation $\sin^{-1}\left[x^2 + \frac{1}{3}\right] + \cos^{-1}\left[x^2 - \frac{2}{3}\right] = x^2$, for $x \in [-1,1]$, and $[x]$ denotes the greatest integer less than or equal to x, is

A) 2 B) 0 C) 4 D) Infinite

Passage-II:

For $x, y, z, t \in R$, $\sin^{-1} x + \cos^{-1} y + \sec^{-1} z \geq t^2 - \sqrt{2\pi}t + 3\pi$

29. The value of x+y+z is equal to

A) 1 B) 0 C) 2 D) -1

30. The principal value of $\cos^{-1}\left(\cos 5t^2\right)$ is

A) $\frac{3\pi}{2}$ B) $\frac{\pi}{2}$ C) $\frac{\pi}{3}$ D) $\frac{2\pi}{3}$

31. The value of $\cos^{-1}(\min\{x,y,z\})$ is

A) 0 B) $\frac{\pi}{2}$ C) π D) $\frac{\pi}{3}$

SECTION - IV (More than one correct answer)

32. The domain of the function $\cos^{-1}\left(\frac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi}\right)$ is

A) $R-\left\{-\frac{1}{2},\frac{1}{2}\right\}$

B) $(-\infty,-1]\cup[1,\infty)\cup\{0\}$

C) $\left(-\infty,\frac{-1}{2}\right]\cup\left(\frac{1}{2},\infty\right)\cup\{0\}$

D) $\left(-\infty,\frac{-1}{\sqrt{2}}\right)\cup\left(\frac{1}{\sqrt{2}},\infty\right)\cup\{0\}$

33. The values of x satisfying $\sin^{-1}x+\sin^{-1}(1-x)=\cos^{-1}x$ is/are

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2

34. The solution of the equation $\sin[2\cos^{-1}\{\cot(2\tan^{-1}x)\}]=0$ are

(A) ±1 (B) $1\pm\sqrt{2}$ (C) $-1\pm\sqrt{2}$ (D) $\pm\sqrt{2}$

35. For any positive integer n, let $S_n:(0,\infty)\to\mathbb{R}$ be defined by

$$S_n(x)=\sum_{k=1}^{n}\cot^{-1}\left(\frac{1+k(k+1)x^2}{x}\right),$$

where for any $x\in\mathbb{R}, \cot^{-1}x\in(0,\pi)$ and $\tan^{-1}(x)\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$. Then which of the following statements is (are) TRUE?

A) $S_{10}(x)=\frac{\pi}{2}-\tan^{-1}\left(\frac{1+11x^2}{10x}\right)$, for all $x>0$

B) $\lim_{n\to\infty}\cot(S_n(x))=x$, for all $x>0$

C) The equation $S_3(x)=\frac{\pi}{4}$ has a root in $(0,\infty)$

D) $\tan(S_n(x))\le\frac{1}{2}$, for all $n\ge1$ and $x>0$

SECTION - V (Numerical Type)

36. The solution set of inequality $\left(\cot^{-1}x\right)\left(\tan^{-1}x\right)+\left(2-\frac{\pi}{2}\right)\cot^{-1}x-3\tan^{-1}x-3\left(2-\frac{\pi}{2}\right)>0$ is (a,b) then the value of $\cot^{-1}a+\cot^{-1}b$ is

37. If $0<\cos^{-1}x<1$ and $1+\sin\left(\cos^{-1}x\right)+\sin^{2}\left(\cos^{-1}x\right)+\sin^{3}\left(\cos^{-1}x\right)+....\infty=2$ then the value of $12x^{2}$ is

38. If $r=x+y+z$ and $\tan^{-1}\sqrt{\frac{xr}{yz}}+\tan^{-1}\sqrt{\frac{yr}{zx}}+\tan^{-1}\sqrt{\frac{zr}{xy}}=k\pi$ then the value of k is

39. Let $f(x)=(\text{arc}\tan x)^{3}+(\text{arc}\cot x)^{3}$. If the range of $f(x)$ is $[a, b)$ then the value of $\frac{b}{7a}$ is

SECTION VI - (Matrix match type)

40.

Column-I	Column-II
A) $\left(\sin^{-1}x\right)^{2}+\left(\sin^{-1}y\right)^{2}=\frac{\pi^{2}}{2}\Rightarrow x^{3}+y^{3}=$	(P) 1
B) $\left(\cos^{-1}x\right)^{2}+\left(\cos^{-1}y\right)^{2}=2\pi^{2}\Rightarrow x^{5}+y^{5}$	(Q) -2
C) $\left(\sin^{-1}x\right)^{2}\left(\cos^{-1}y\right)^{2}=\frac{\pi^{4}}{4}\Rightarrow\lvert x-y\rvert$	(R) 0
D) $\lvert\sin^{-1}x-\sin^{-1}y\rvert=\pi\Rightarrow x^{y}$	(S) 2

A) A-Q,R,S,B-Q, C-R,S,D-P
B) A-P,R,S,B-Q, C-R,S,D-P
C) A-Q,R,S,B-Q, C-S,D-P
D) A-Q,R,S,B-p, C-R,S,D-P