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## CHAPTER - 3

# QUADRATIC EQUATIONS

### 1. Quadratic Expression

A polynomial of degree two in the form  $ax^2 + bx + c$  ( $a \neq 0$ ) is called a quadratic expression in  $x$

### 2. Quadratic Equation

A second degree polynomial equated to zero is a quadratic equation. A quadratic equation

$ax^2 + bx + c = 0$  ( $a \neq 0$ ) has two roots, given by

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ \& } \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

### 3. General methods of solving a Quadratic Equation

#### Factorization Method

Let  $ax^2 + bx + c = a(x - \alpha)(x - \beta) = 0$

Then  $x = \alpha$  and  $x = \beta$  will satisfy the given equation

#### Formula method

Quadratic equation  $ax^2 + bx + c = 0$  ( $a \neq 0$ ) has two roots, given by

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ \& } \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

### 4. Sum and product of roots in terms of coefficient in the equation

Consider  $ax^2 + bx + c = 0$  ( $a \neq 0$ )  $\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

$$\text{sum of roots} = \alpha + \beta = -\frac{b}{a}$$

$$\text{product of roots} = \alpha\beta = \frac{c}{a}$$

**5. Formation of an equation with given roots**

A quadratic equation whose roots are  $\alpha$  and  $\beta$  is given by  $(x - \alpha)(x - \beta) = 0$

$$\therefore x^2 - \alpha x - \beta x + \alpha\beta = 0$$

$$\therefore x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

i.e.  $x^2 - (\text{sum of roots})x + \text{product of roots} = 0$

$$\therefore x^2 - Sx + P = 0$$

**6. Relation between roots and coefficients**

If roots of quadratic equation  $ax^2 + bx + c = 0$  ( $a \neq 0$ ) are  $\alpha$  and  $\beta$  then

$$(i) (\alpha - \beta) = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta} = \pm \frac{\sqrt{b^2 - 4ac}}{a}$$

$$ii) \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \frac{b^2 - 2ac}{a^2}$$

$$iii) \alpha^2 - \beta^2 = (\alpha + \beta)\sqrt{(\alpha + \beta)^2 - 4\alpha\beta} = -\frac{b\sqrt{b^2 - 4ac}}{a^2}$$

$$iv) \alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = \frac{b(b^2 - 3ac)}{a^3}$$

$$v) \alpha^3 - \beta^3 = (\alpha - \beta)^3 + 3\alpha\beta(\alpha - \beta) = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta} \{(\alpha + \beta)^2 - \alpha\beta\} = \frac{(b^2 - ac)\sqrt{b^2 - 4ac}}{a^3}$$

$$vi) \alpha^4 + \beta^4 = \{(\alpha + \beta)^2 - 2\alpha\beta\}^2 - 2\alpha^2\beta^2 = \left(\frac{b^2 - 2ac}{a^2}\right)^2 - 2\frac{c^2}{a^2}$$

$$vii) \alpha^4 - \beta^4 = (\alpha^2 - \beta^2)(\alpha^2 + \beta^2) = \frac{-b(b^2 - 2ac)\sqrt{b^2 - 4ac}}{a^4}$$

$$viii) \alpha^2 + \alpha\beta + \beta^2 = (\alpha + \beta)^2 - \alpha\beta$$

$$ix) \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$x) \alpha^2\beta + \beta^2\alpha = \alpha\beta(\alpha + \beta)$$

$$\text{xi) } \left(\frac{\alpha}{\beta}\right)^2 + \left(\frac{\beta}{\alpha}\right)^2 = \frac{\alpha^4 + \beta^4}{\alpha^2\beta^2} = \frac{(\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2}{\alpha^2\beta^2}$$

## 7. Nature of roots

The term  $b^2 - 4ac$  is called discriminant of the equation. It is denoted by  $\Delta$  or  $D$

(i) Suppose  $a, b, c \in \mathbb{R}$  and  $a \neq 0$  then

(a) If  $D > 0 \Rightarrow$  roots are real and unequal

(b) If  $D = 0 \Rightarrow$  roots are real and equal and each equal to  $-b/2a$

(c) If  $D < 0 \Rightarrow$  roots are imaginary and unequal or complex conjugate

Ex. one root                  other root

$$\alpha + i\beta \qquad \alpha - i\beta$$

(ii) Suppose  $a, b, c \in \mathbb{Q}$ ,  $a \neq 0$  then

(a) If  $D > 0$  &  $D$  is perfect square

$\Rightarrow$  roots are unequal & rational

(b) If  $D > 0$  &  $D$  is not perfect square

$\Rightarrow$  roots are irrational & unequal

Ex. One root                  Other root

$$a + \sqrt{\beta} \qquad a - \sqrt{\beta}$$

## 8. Roots under particular case

For the quadratic equation  $ax^2 + bx + c = 0$

i) If  $b = 0 \Rightarrow \alpha + \beta = 0 \Rightarrow$  roots are of equal magnitude but of opposite sign

ii) If  $c = 0 \Rightarrow \alpha\beta = 0 \Rightarrow$  one root is zero, other is  $-b/a$

iii) If  $b = c = 0 \Rightarrow \alpha + \beta = \alpha\beta = 0 \Rightarrow$  both roots are zero

iv) If  $a = c \Rightarrow$  roots are reciprocal to each other

v) If sign of  $a \neq$  sign of  $c \Rightarrow$  both roots of opposite signs ( $D > 0$ )

vi) If sign of  $a =$  sign of  $b =$  sign of  $c \Rightarrow$  both roots are negative ( $D > 0$ )

vii) If sign of  $a =$  sign of  $c \neq$  sign of  $b \Rightarrow$  both roots are positive ( $D > 0$ )

viii) If sign of  $a =$  sign of  $b \neq$  sign of  $c \Rightarrow$  greater root in magnitude is negative ( $D > 0$ )

ix) If sign of  $b =$  sign of  $c \neq$  sign of  $a \Rightarrow$  greater root in magnitude is positive

x) If  $a+b+c = 0 \Rightarrow$  one root is 1 and second root is  $c/a$

xi) If  $a = b = c = 0$  then equation will become an identity and will be satisfied by every value of  $x$

## 9. Equation in terms of the roots of another equation

If  $\alpha, \beta$  are roots of the equation  $ax^2 + bx + c = 0$

then the equation whose roots are

$$(i) -\alpha, -\beta \Rightarrow ax^2 - bx + c = 0 \text{ (replace } x \text{ by } -x)$$

$$(ii) \frac{1}{\alpha}, \frac{1}{\beta} \Rightarrow cx^2 + bx + a = 0 \text{ (replace } x \text{ by } 1/x)$$

$$(iii) \alpha^n, \beta^n; n \in \mathbb{N} \Rightarrow a(x^{1/n})^2 + b(x^{1/n}) + c = 0 \text{ (replace } x \text{ by } x^{1/n})$$

$$(iv) k\alpha, k\beta \Rightarrow ax^2 + kbx + k^2c = 0 \text{ (replace } x \text{ by } x/k)$$

$$(v) k + \alpha, k + \beta \Rightarrow a(x - k)^2 + b(x - k) + c = 0 \text{ (replace } x \text{ by } x - k)$$

$$(vi) \frac{\alpha}{k}, \frac{\beta}{k} \Rightarrow k^2ax^2 + kbx + c = 0 \text{ (replace } x \text{ by } kx)$$

$$(vii) \alpha^{1/n}, \beta^{1/n}; n \in \mathbb{N} \Rightarrow a(x^n)^2 + b(x^n) + c = 0 \text{ (replace } x \text{ by } x^n)$$

Note : This method can also be applied to higher degree equations

## 10. Theory of equations

If  $\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n$  are the roots of the equation

$$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_{n-1}x + a_n = 0$$

where  $a_0, a_1, a_2, \dots, a_n$  are all real,  $a_0 \neq 0$  then

$$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_{n-1}x + a_n$$

$$= a_0(x - \alpha_1)(x - \alpha_2)(x - \alpha_3) \dots (x - \alpha_n)$$

$$= a_0(x^n - x^n)(\sum \alpha_1) + x^{n-1}(\sum \alpha_1\alpha_2) + \dots + (-1)^n \alpha_1\alpha_2\alpha_3 \dots \alpha_n$$

Now comparing the coefficients from above identity then

$$\sum \alpha_1 = \frac{a_1}{a_0}$$

$$\sum \alpha_1\alpha_2 = \frac{a_2}{a_0}; \sum \alpha_1\alpha_2\alpha_3 = \frac{a_3}{a_0}, \dots$$

$$\sum \alpha_1\alpha_2\alpha_3 \dots \alpha_n = (-1)^n \frac{a_n}{a_0}$$

eg. If  $\alpha, \beta, \gamma$  are the roots of  $ax^3 + bx^2 + cx + d = 0$

$$\text{then } \alpha + \beta + \gamma = \frac{b}{a}, \beta\gamma + \gamma\alpha + \alpha\beta = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{-d}{a}$$

### 11. Condition for common roots

Let quadratic equations are  $a_1x^2 + b_1x + c_1 = a_2x^2 + b_2x + c_2 = 0$

i) If only one root is common

$$\frac{\alpha^2}{b_1c_2 - b_2c_1} = \frac{\alpha}{a_2c_1 - a_1c_2} = \frac{1}{a_1b_2 - a_2b_1}$$

ii) If both roots are common:  $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\therefore$  The condition for only one root common is

$$(c_1a_2 - c_2a_1)^2 = (b_1c_2 - b_2c_1)(a_1b_2 - a_2b_1)$$

Note : Two different quadratic equation with rational coefficient cannot have single common root which is complex or irrational, as imagine and surd roots always occur in pair

### 12. Graph of Quadratic expression

consider  $y = ax^2 + bx + c$

$$y = a \left[ \left( x + \frac{b}{2a} \right)^2 - \frac{D}{4a^2} \right] \dots\dots(1)$$

$$\Rightarrow \left( x + \frac{b}{2a} \right)^2 = \frac{1}{a} \left( y + \frac{D}{4a} \right)$$

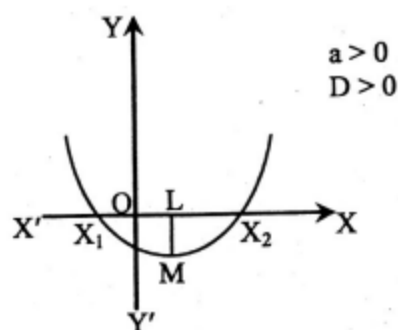
It is a parabola of the form  $X^2 = 4AY$ , with vertex  $\left( \frac{-b}{2a}, \frac{-D}{4a} \right)$

Where  $D = b^2 - 4ac$  is the discriminant of the quadratic equation  $ax^2 + bx + c = 0$

Clearly graph of a Quadratic equation is a parabola of the form  $X^2 = 4AY$ . If  $a > 0$  then the shape of the parabola is concave upward and if  $a < 0$  then the shape of the parabola is concave downwards. Possible graphs of a quadratic expressions are given below:

### Case-I When $a > 0$

i) If  $D > 0$

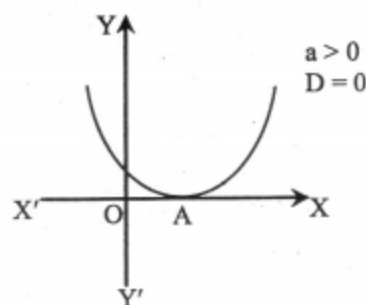


Roots are real and different ( $X_1$  and  $X_2$ )

$$\text{Min. value } LM = \frac{4ac - b^2}{4a} \text{ at } x = OL = \frac{-b}{2a}$$

$y$  is positive for all  $x$  outside interval  $[x_1, x_2]$  and is negative for all  $x$  inside  $(x_1, x_2)$

ii) If  $D = 0$

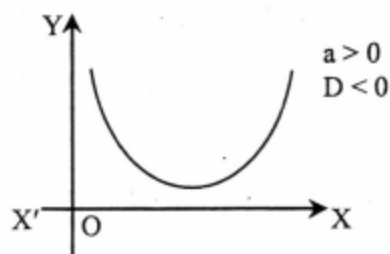


Roots are equal (OA)

$$\text{Min. value} = 0 \text{ at } x = OA = \frac{-b}{2a}$$

$$y > 0 \text{ for all } x \in \mathbb{R} - \left\{ \frac{-b}{2a} \right\}$$

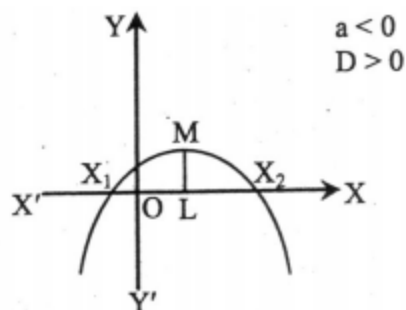
iii) If  $D < 0$



Roots are complex conjugate and  $y$  is positive for all  $x \in \mathbb{R}$

**Case -II When  $a < 0$**

(i) If  $D > 0$



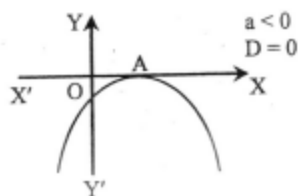
Roots are real and different ( $x_1$  and  $x_2$ )

$$\text{Max. value} = LM = \frac{4ac - b^2}{4a}$$

$$\text{at } x = OL = \frac{-b}{2a}$$

$y$  is positive for all  $x$  inside  $(x_1, x_2)$  and  $y$  is negative for all  $x$  outside  $[x_1, x_2]$

(ii) If  $D = 0$

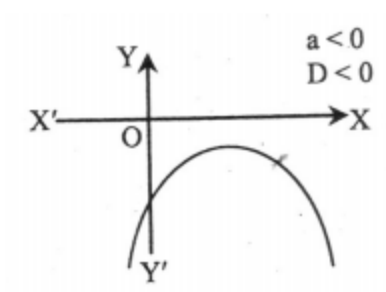


Roots are equal = OA

$$\text{Max. value} = 0 \text{ at } x = OA = \frac{-b}{2a}$$

$y$  is negative for all  $x \in \mathbb{R} - \left\{ \frac{-b}{2a} \right\}$

iii) If  $D < 0$



Roots are complex conjugate.  $y$  is negative for all  $x \in \mathbb{R}$

### 13. Position for roots of a quadratic equation $ax^2 + bx + c = 0$

i) Condition for both roots are greater than  $k$

- a)  $D > 0$       b)  $k < \frac{-b}{2a}$       c)  $af(k) > 0$

ii) Condition for both roots are less than  $K$

- A)  $D > 0$       B)  $k > -\frac{b}{2a}$       C)  $af(k) > 0$

iii) condition for  $k$  lie between the roots

- A)  $D > 0$       B)  $af(k) < 0$

iv) Condition for exactly one root lie in the interval  $(k_1, k_2)$  where  $k_1 < k_2$

- A)  $D > 0$       B)  $f(k_1)f(k_2) < 0$

v) When both roots lie in the interval  $(k_1, k_2)$  where  $k_1 < k_2$

- A)  $D > 0$       B)  $f(k_1).f(k_2) > 0$       C)  $k_1 < \frac{-b}{2a} < k_2$

### 14. Maximum and minimum value of Quadratic Expression

Consider  $ax^2 + bx + c$

i) If  $a > 0$ , quadratic expression has least value at  $x = -\frac{b}{2a}$ . This least value is

given by  $\frac{4ac - b^2}{4a} = \frac{-D}{4a}$

ii) If  $a < 0$ , quadratic expression has greatest value at  $x = -\frac{b}{2a}$ . This greatest value is given by

$\frac{4ac - b^2}{4a} = \frac{-D}{4a}$



**15. Range of an algebraic expression in this form**  $\frac{ax^2 + bx + c}{px^2 + qx + r}$

$$\text{Let } y = \frac{ax^2 + bx + c}{px^2 + qx + r} \dots\dots(1) \Rightarrow x^2(py - a) + x(qy - b) + (r - c) = 0 \dots\dots(2)$$

To find the range of the given expression, we first solve it for x

For real value of x, D of (2) should be greater than or equal to zero

$$D = (qy - b)^2 - 4(py - a)(r - c) \geq 0 \dots\dots(3) \text{ Hence we can find the range of } y$$

**16. Some important points**

i) If polynomial  $f(x)$  is divided by  $x-h$ , the remainder is  $f(h)$  (Remainder Theorem)

$$\text{ii) } 2[a^2 + b^2 + c^2 - bc - ca - ab] = (b-c)^2 + (c-a)^2 + (a-b)^2 \text{ which is always positive}$$

iii) Every equation of  $n^{\text{th}}$  degree ( $n \geq 1$ ) has less than or equal to  $n$  real roots & exactly  $n$  complex roots and if the equation has more than  $n$  roots, it is an identity

iv) If  $\alpha$  is a root of the equation  $f(x) = 0$  then the polynomial  $f(x)$  is exactly divisible by  $(x - \alpha)$  or  $(x - \alpha)$  is a factor of  $f(x)$

iv) Quadratic Equations containing modulus sign are solved considering both positive and negative values of the quantity containing modulus sign. Finally the roots of the given equation will be those values among the values of the variable so obtained which satisfy the given equation

**PART I - (JEEMAIN)**

**SECTION 1- Straight objective type questions**

1. If  $\alpha$  and  $\beta$  are the roots of  $ax^2 + bx + c = 0$  then the equation with roots  $\frac{1}{a\alpha + b}$  and  $\frac{1}{a\beta + b}$  is

$$1) ax^2 - bx + 1 = 0 \qquad 2) ax^2 + bx - 1 = 0$$

$$3) ax^2 - bx - 1 = 0 \qquad 4) bx^2 - ax + c = 0$$

2. If  $\sec \theta$  and  $\tan \theta$  are the roots of  $ax^2 + bx + c = 0$ , then  $\sec \theta - \tan \theta =$

$$1) \frac{c}{a} \qquad 2) \frac{-c}{a} \qquad 3) \frac{-b}{a} \qquad 4) \frac{-a}{b}$$

3. If  $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$  has equal roots, then
- 1)  $\frac{1}{a} + \frac{1}{c} = \frac{2}{b}$       2)  $a+c = \frac{2}{b}$       3)  $\frac{1}{a} + c = \frac{2}{b}$       4)  $a + \frac{1}{c} = \frac{2}{b}$
4. If  $\alpha, \beta, \gamma$  are the roots of the equation  $x^3 + 4x + 1 = 0$ , then  $(\alpha + \beta)^{-1} + (\beta + \gamma)^{-1} + (\gamma + \alpha)^{-1}$
- 1) 1      2) -4      3) 4      4)  $\frac{1}{2}$
5. If  $\alpha_1, \alpha_2, \alpha_3, \alpha_4$  are the roots of the equation  $x^4 + (2 - \sqrt{3})x^2 + 2 + \sqrt{3} = 0$  then the value of  $(1 - \alpha_1)(1 - \alpha_2)(1 - \alpha_3)(1 - \alpha_4) =$
- 1A) 1      2) 4      3)  $2 + \sqrt{3}$       4) 5
6. If  $x$  is real number, then  $\frac{x}{x^2 - 5x + 9}$  must lie between
- 1)  $\frac{1}{11}$  and 1      2) -1 and  $\frac{1}{11}$       3) -11 and 1      4)  $-\frac{1}{11}$  and 1
7. Let  $\alpha$  and  $\beta$  be the roots of equation  $x^2 - 6x - 2 = 0$ . If  $a_n = \alpha^n - \beta^n$ , for  $n \geq 1$  then the value of  $\frac{a_{10} - 2a_8}{2a_9}$  is equal to
- 1) 6      2) -6      3) 3      4) -3
8. The solutions of  $(2 + \sqrt{3})^{x^2 - 2x + 1} + (2 - \sqrt{3})^{x^2 - 2x - 1} = \frac{4}{(2 - \sqrt{3})}$  are
- 1)  $(1 \pm \sqrt{3}), 1$       2)  $(1 \pm \sqrt{2}), 1$       3)  $(1 \pm \sqrt{3}), 2$       4)  $(1 \pm \sqrt{2}), 2$
9. If  $(1 + i)$  is a root of the equation  $x^2 - x + (1 - i) = 0$ , then the other root is
- 1)  $1 - i$       2)  $i$       3)  $-i$       4)  $-2i$
10. The sum of the solutions of the equation  $|\sqrt{x} - 2| + \sqrt{x}(\sqrt{x} - 4) + 2 = 0, (x > 0)$  is equal to
- 1) 4      2) 9      3) 10      4) 12
11. If  $\alpha$  and  $\beta$  are roots of the equation,  $x^2 - 4\sqrt{2}kx + 2e^{4 \tan k} - 1 = 0$  for some  $k$ , and  $\alpha^2 + \beta^2 = 66$ , then  $\alpha^3 + \beta^3$  is equal to
- 1)  $-32\sqrt{2}$       2)  $280\sqrt{2}$       3)  $248\sqrt{2}$       4)  $-280\sqrt{2}$

12. The product of the roots of the equation  $x|x| - 5x - 6 = 0$  is equal to  
 1) 36                      2) -36                      3) -18                      4) 6
13. The quadratic equation  $p(x) = 0$  with real coefficients has purely imaginary roots. Then the equation  $p(p(x)) = 0$  has  
 1) only purely imaginary root                      2) all real roots  
 3) two real and two purely imaginary root                      4) neither real nor purely imaginary root

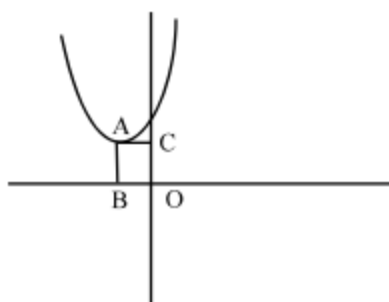
**Reasoning Type**

- A) Both statement I and statement II are true and statement II is correct explanation of statement I  
 B) Statement I and statement II are true but statement II is not the correct explanation of statement I  
 C) Statement I is true but statement II is false  
 D) Statement I is false but statement II is true
14. Statement I : The roots of equation  $p(q-r)x^2 + q(r-p)x + r(p-q) = 0$  are 1 and  $\frac{r(p-q)}{p(q-r)}$   
 Statement II: If  $a + b + c = 0$  then roots of  $ax^2 + bx + c = 0$  are 1 and  $\frac{c}{a}$   
 1) A                      2) B                      3) C                      4) D
15. Statement I : If equations  $ax^2 + bx + c = 0, a, b, c \in \mathbb{R}$  and  $2x^2 + 3x + 4 = 0$  have a common root then  $a : b : c = 2 : 3 : 4$   
 Statement II: Roots of  $2x^2 + 3x + 4 = 0$  are imaginary  
 1) A                      2) B                      3) C                      4) D
16. If  $\alpha$  and  $\beta$  are the roots of  $ax^2 + bx + c = 0$  then the eqn.  $ax^2 - bx(x-1) + c(x-1)^2 = 0$  has roots  
 1)  $\frac{\alpha}{1-\alpha}, \frac{\beta}{1-\beta}$       2)  $\frac{1-\alpha}{\alpha}, \frac{1-\beta}{\beta}$       3)  $\frac{\alpha}{\alpha+1}, \frac{\beta}{\beta+1}$       4)  $\frac{\alpha+1}{\alpha}, \frac{\beta+1}{\beta}$
17. Let  $\alpha(a)$  and  $\beta(a)$  be the roots of the equation  $(\sqrt[3]{1+a} - 1)x^2 + (\sqrt{1+a} - 1)x + (\sqrt[6]{1+a} - 1) = 0$  where  $a > -1$ . Then  $\lim_{a \rightarrow 0^+} \alpha(a)$  and  $\lim_{a \rightarrow 0^+} \beta(a)$  are  
 1)  $-\frac{5}{2}$  and 1      2)  $-\frac{1}{2}$  and -1      3)  $-\frac{7}{2}$  and 2      4)  $-\frac{9}{2}$  and 3

18. If  $\left(\sqrt{(49+20\sqrt{6})}\right)^{\sqrt{a\sqrt{a\sqrt{a\ldots\infty}}}} + (5-2\sqrt{6})^{x^2+x-3-\sqrt{x\sqrt{x\sqrt{x\ldots\infty}}}} = 10$  where  $a = x^2 - 3$ , then  $x$  is

- 1)  $-\sqrt{2}$       2)  $\sqrt{2}$       3)  $-2$       4)  $2$

19. Consider the graph of  $f(x) = ax^2 + bx + c$  in the given figure such that (length  $\ell$ )  $\ell(AB) = 1$ ,  $\ell(AC) = 4$  and  $b^2 - 4ac = -4$ . The value of  $(a + b + c)$  is equal to



- 1) 1      2) 9      3) 10      4) 26

20. The number of (+)ve integral solution of  $\frac{x^2(3x-4)^3(x-2)^4}{(x-5)^5(2x-7)^6} \leq 0$  is

## SECTION - II

### Numerical Type Questions

21. Let  $a, b \in \{1, 2, 3, 4\}$ . The number of equations of the form  $ax^2 + bx + 1 = 0$  having real roots is....
22. If one root of the quadratic equation  $ax^2 - 3x + c = 0$  is 5<sup>th</sup> power of the other, then  $(a^5c)^{1/6} + (ac^5)^{1/6}$  is equal to
23. If  $\alpha, \beta, \gamma$  be such that  $\alpha + \beta + \gamma = 2, \alpha^2 + \beta^2 + \gamma^2 = 6, \alpha^3 + \beta^3 + \gamma^3 = 8$ , find value of  $\alpha^4 + \beta^4 + \gamma^4$
24. If  $\alpha, \beta$  be the roots of  $x^2 + px - q = 0$  and  $\gamma, \delta$  be the roots of  $x^2 + px + r = 0$ , find the value of  $\frac{(\alpha - \gamma)(\alpha - \delta)}{(\beta - \gamma)(\beta - \delta)}$
25. Sum of the roots of the equation  $|x - 3|^2 + |x - 3| - 2 = 0$  is equal to

**PART - II (JEE ADVANCED)**

**SECTION - III (Only one option correct type)**

26. Number of real solution of the equation  $x^2 + \left(\frac{x}{x-1}\right)^2 = 8$
- A) 1                      B) 2                      C) 3                      D) 4
27. The set of all values of the parameter  $a$  for which  $x^2 - 4x - 2|x - a| + a + 2 = 0$  has two roots.
- A)  $(-\infty, +\infty)$                       B)  $(-\infty, 1) \cup \left(\frac{7}{3}, \infty\right)$                       C)  $\left(-\infty, \frac{7}{3}\right)$                       D)  $(-\infty, 2) \cup \left(\frac{7}{3}, \infty\right)$
28. Find all possible values of ' $a$ ' for which the expression  $\frac{ax^2 - 7x + 5}{5x^2 - 7x + a}$  may be capable of all values  $x$  being any real quantity
- A)  $a \in [-12, 2]$                       B)  $a \in (-12, 2)$                       C)  $a \in (-12, 2]$                       D)  $a \in [-12, 2)$

**Passage**

Let  $p, q$  be integers and let  $\alpha, \beta$  be the roots of the equation,  $x^2 - x - 1 = 0$ , where  $\alpha \neq \beta$ . For  $n=0, 1, 2, 3, \dots$ , let  $a_n = p\alpha^n + q\beta^n$ . Fact: If  $a$  and  $b$  are rational numbers and  $a + b\sqrt{5} = 0$ , then  $a = 0 = b$

29.  $a_{12} =$
- A)  $a_{11} - a_{10}$                       B)  $a_{11} + a_{10}$                       C)  $2a_{11} + a_{10}$                       D)  $a_{11} + 2a_{10}$
30. If  $a_4 = 28$ , then  $p - 2q =$
- A) 21                      B) 14                      C) 7                      D) 12

**SECTION - IV (More than one correct answer)**

31. Let  $S$  be the set of all non-zero real numbers  $\alpha$  such that the quadratic equation  $\alpha x^2 - x + \alpha = 0$  has two distinct real roots  $x_1$  and  $x_2$  satisfying the inequality  $|x_1 - x_2| < 1$ . Which of the following intervals is (are) a subset (s) of  $S$ ?
- A)  $\left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right)$                       B)  $\left(-\frac{1}{\sqrt{5}}, 0\right)$                       C)  $\left(0, \frac{1}{\sqrt{5}}\right)$                       D)  $\left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$
32. The set of all values of  $x$  for which  $[x^2 + x + 1] = 1$  satisfied
- A)  $\frac{-1-\sqrt{5}}{2} < x \leq -1$                       B)  $0 \leq x < \frac{\sqrt{5}-1}{2}$                       C)  $-\infty < x \leq \frac{-1-\sqrt{5}}{2}$                       D)  $\frac{\sqrt{5}-1}{2} < x < \infty$
33. A value of  $b$  for which the equations  $x^2 + bx - 1 = 0, x^2 + x + b = 0$  have one root is common is
- A)  $-\sqrt{2}$                       B)  $-i\sqrt{3}$                       C)  $i\sqrt{5}$                       D)  $i\sqrt{3}$

34. If every pair from among the equations  $x^2 + ax + bc = 0$ ,  $x^2 + bx + ca = 0$  and  $x^2 + cx + ab = 0$  has a common root, then
- A) the sum of the three common roots is  $-\frac{1}{2}(a + b + c)$
- B) the sum of the three common roots is  $2(a + b + c)$
- C) one of the values of the product of the three common roots is  $abc$
- D) the product of the three common roots is  $a^2b^2c^2$
35. If the equation  $ax^2 + 2bx + 4c = 16$  has no real roots and  $a + c > b + 4$  then integral value of  $c$  can be equal to
- A) 2                                      B) 6                                      C) 12                                      D) 20
36. The equation  $(\log_{10} x + 2)^3 + (\log_{10} x - 1)^3 = (2\log_{10} x + 1)^3$  has
- A) no natural solution                                      B) two rational solutions
- C) no prime solution                                      D) no irrational solution

### SECTION - V (Numerical Type)

37. If the sum of all real values of  $x$  satisfy the equation  $(x^2 - 5x + 5)^{x^2 + 4x - 60} = 1$  is  $k$ , then  $|110k|$  is equal to

38. If the value of  $5 + \frac{1}{4 + \frac{1}{5 + \frac{1}{4 + \frac{1}{5 + \dots \infty}}}}$  is  $\lambda$ , then the value of  $2\lambda - \sqrt{30}$  is equal to

39. If  $\alpha, \beta$  are the roots of the equation  $\lambda(x^2 - x) + x + 5 = 0$  and if  $\lambda_1$  and  $\lambda_2$  are two values of  $\lambda$  obtained from  $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{4}{5}$ , then  $\frac{1}{100} \left( \frac{\lambda_1}{\lambda_2^2} + \frac{\lambda_2}{\lambda_1^2} \right)$  is equal to

### SECTION VI - (Matrix match type)

40. Match the conditions on column I with the intervals in column II

Let  $f(x) = x^2 - 2px + p^2 - 1, p \in \mathbb{R}$  then

#### Column I

- A) Both the roots of  $f(x) = 0$  are less than 4, if  $p$  in
- B) Both the roots of  $f(x) = 0$  are greater than  $-2$  if  $p$  in
- C) Exactly one root of  $f(x) = 0$  lie in  $(0, 2)$ , if  $p$  in  $(0, 2)$
- D) 1 lies between the roots of  $f(x) = 0$ , if  $p$  in

#### Column II

- p)  $(-1, \infty)$
- q)  $(-\infty, 3)$
- s)  $(-3, -1) \cup (3, 5)$

A)  $A \rightarrow q, B \rightarrow p, C \rightarrow s, D \rightarrow r$

B)  $A \rightarrow q, B \rightarrow q, C \rightarrow s, D \rightarrow r$

C)  $A \rightarrow q, s, B \rightarrow p, C \rightarrow s, D \rightarrow r$

D)  $A \rightarrow q, B \rightarrow p, C \rightarrow s, D \rightarrow r, s$