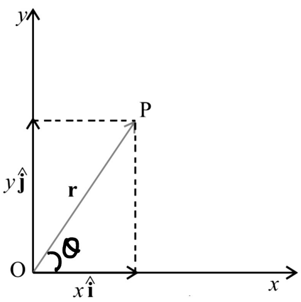
MOTION IN A PLANE

POSITION VECTOR



The vector which represent the position of a particle.

If $\vec{r}$ represents the position vector of a particle as shown, then

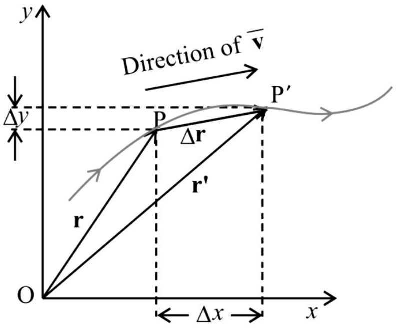
$\vec{r} = r_x\,\hat{i} + r_y\,\hat{j}$

$\vec{r} = x\,\hat{i} + y\,\hat{j}$

when x and y are the coordinates of the particle.

Magnitude $r = \sqrt{x^2 + y^2}$ and $\tan\theta = \dfrac{y}{x}$

DISPLACEMENT VECTOR



Let the particle moves from P to P' the displacement vector

$\overrightarrow{\Delta r} = \vec{r}' - \vec{r}$

$\overrightarrow{\Delta r} = (x'\hat{i} + y'\hat{j}) - (x\hat{i} + y\hat{j})$

$\overrightarrow{\Delta r} = (x' - x)\hat{i} + (y' - y)\hat{j}$

$\overrightarrow{\Delta r} = \Delta x\,\hat{i} + \Delta y\,\hat{j}$

$\Delta x =$ displacement along x direction

$\Delta y =$ displacement along y direction

$\Delta r = \sqrt{(\Delta x)^2 + (\Delta y)^2}$

VELOCITY VECTOR

Let $\overrightarrow{\Delta r}$ be the displacement vector of a particle in a time interval Δt, then the average velocity vector

$$\overrightarrow{V_{av}} = \frac{\overrightarrow{\Delta r}}{\Delta t} = \frac{\Delta x\hat{i} + \Delta y\hat{j}}{\Delta t}$$

$$\overrightarrow{V_{av}} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} = V_x\hat{i} + V_y\hat{j}$$

where V_x is the x-component of velocity.

V_y is the y-component of velocity.

Direction of average velocity is in the direction of displacement vector $\overrightarrow{\Delta r}$.

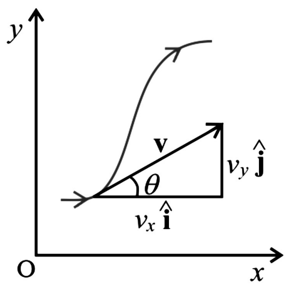
Magnitude of velocity $V_{av} = \sqrt{V_x^2 + V_y^2}$

Instantaneous velocity

$$\vec{V} = \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

$$\vec{V} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = V_x\hat{i} + V_y\hat{j}$$

The direction of velocity at any instant or at any point on the path of an object is tangential to the path at that point and is in the direction of motion



$$\tan\theta = \frac{V_y}{V_x}$$

or $\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right)$

$$V = \sqrt{V_x^2 + V_y^2}$$

ACCELERATION VECTOR

Let $\overrightarrow{\Delta V}$ be the change in velocity in a time interval Δt, then the average acceleration,

$$\overrightarrow{a_{av}} = \frac{\overrightarrow{\Delta V}}{\Delta t} = \frac{\vec{V} - \vec{u}}{\Delta t} = \frac{(V_x\hat{i} + V_y\hat{j}) - (u_x\hat{i} + u_y\hat{j})}{\Delta t}$$

$$= \left(\frac{V_x - u_x}{\Delta t}\right)\hat{i} + \left(\frac{V_y - u_y}{\Delta t}\right)\hat{j}$$

$$\overrightarrow{a_{av}} = \frac{\Delta V_x}{\Delta t}\hat{i} + \frac{\Delta V_y}{\Delta t}\hat{j} = a_x\hat{i} + a_y\hat{j}$$

$a_x = \frac{\Delta V_x}{\Delta t} =$ x-component of acceleration

$a_y = \frac{\Delta V_y}{\Delta t} =$ y-component of acceleration

Instantaneous acceleration

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\overrightarrow{\Delta V}}{\Delta t} = \frac{dV_x}{dt}\hat{i} + \frac{dV_y}{dt}\hat{j}$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

$$a_x = \frac{dV_x}{dt} = \frac{d^2x}{dt^2} \qquad a_y = \frac{dV_y}{dt} = \frac{d^2y}{dt^2}$$

In one dimensional motion, the velocity and acceleration are in the same straight line. However, for motion in two or three dimensions, velocity and acceleration vectors may have any angle between 0^0 and 180^0 between them.

MOTION IN A PLANE WITH CONSTANT VELOCITY

In two dimensional two coordinators change during motion. When velocity is constant displacement

Vector $\overrightarrow{\Delta r} = \vec{V}\,t$

If motion is in x – y plane, then,

$$\overrightarrow{\Delta r} = (V_x\hat{i} + V_y\hat{j})t$$

$$\vec{r}' - \vec{r} = V_x t\,\hat{i} + V_y t\,\hat{j}$$

$$(x' - x)\hat{i} + (y' - y)\hat{j} = V_x t\,\hat{i} + V_y t\,\hat{j}$$

$\therefore x' - x = V_x t$ (1 D motion along x-direction)

$y' - y = V_y t$ (1 D motion along y-direction)

MOTION IN A PLANE WITH CONSTANT ACCELERATION

acceleration vector

$$\vec{a} = \frac{\vec{v} - \vec{u}}{t} \quad \text{or} \quad \vec{v} = \vec{u} + \vec{a}\,t$$

If the motion is in x-y plane

$$v_x\hat{i} + v_y\hat{j} = (u_x\hat{i} + u_y\hat{j}) + (a_x\hat{i} + a_y\hat{j})$$

$$= (u_x\hat{i} + a_x t)\hat{j} + (u_y\hat{i} + a_y t)\hat{j}$$

$\therefore$ $\boxed{v_x = u_x + a_x t}$ (1 D motion along x direction)

and $\boxed{v_y = u_y + a_y t}$ (1D motion along y-direction)

displacement vector $\vec{r}' - \vec{r} = \left(\dfrac{\vec{u} + \vec{v}}{2}\right) t$

$$\vec{r}' - \vec{r} = \left(\frac{\vec{u} + (\vec{u} + \overrightarrow{at}}{2}\right) t$$

$$\vec{r}' - \vec{r} = \vec{u}t + \frac{\vec{a}t^2}{2}$$

$$\boxed{\vec{r}' = \vec{r} + \vec{u}\,t + \frac{\vec{a}\,t^2}{2}}$$

If motion is in x-y plane, the above equation in component form is :

$$x' = x + u_x t + \frac{a_x t^2}{2}$$

$$y' = y + u_y t + \frac{a_y t^2}{2}$$

The above equations shows that, the motion in a plane can be treated as two separate simultaneous one-dimensional motions with constant acceleration along two perpendicular directions.

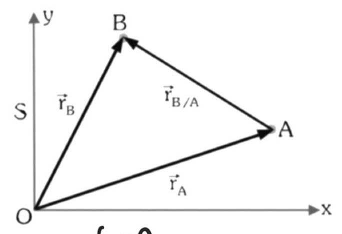
RELATIVE MOTION

Relative Motion

Motion of a body can only be observed, when it changes its position with respect to some other body. IN this sense, motion is a relative concept. To analyze motion of a body say A, therefore we have fix our reference **Relative position**, **Relative velocity** and **Relative Acceleration**.

Let two bodies represented by particles A and B at positions defined by position vectors $\vec{r_A}$ and $\vec{r_B}$, moving with velocities $\vec{v_A}$ and $\vec{v_B}$ and accelerations $\vec{a_A}$ and $\vec{a_B}$ with respect to a reference frame S. For analyzing motion off terrestrial bodies the reference frame S is fixed with the ground. The vectors $\vec{r}_{B/A}$ denotes position vector of B relative to A.

Following triangle law of vector addition, we have



(a)

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A} \qquad \text{...(i)}$$

First derivatives of $\vec{r_A}$ and $\vec{r_B}$ with respect to time equals to velocity of particle A and velocity of particle B relative to frame S and first derivative of $\vec{r}_{B/A}$ with respect to time defines velocity of B relative to A.

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} \qquad \text{...(ii)}$$

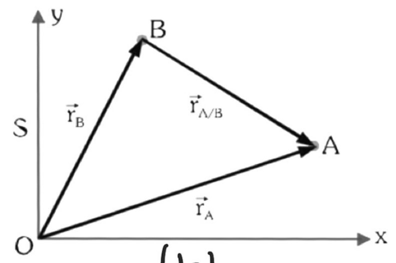
Second derivatives of $\vec{r_A}$ and $\vec{r_B}$ with respect to time equals to acceleration of particle A and acceleration of particle B relative to frame S and second derivative of $\vec{r}_{B/A}$ with respect to time defines acceleration of B relative to A.

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} \qquad \text{...(iii)}$$

In similar fashion motion of particle A relative to particle B can be analyzed with help of adjoining figure. You can observe in the figure that position vector of A relative to B is directed from B to A and therefore

$$\vec{r}_{B/A} = -\vec{r}_{A/B},\ \vec{v}_{B/A} = -\vec{v}_{A/B} \text{ and } \vec{a}_{A/B} = -\vec{a}_{A/B}$$

The above equations elucidate that how a body A appears moving to another body B is opposite to how body B appears moving to body A.



(b)

Relative velocity

Relative velocity of a particle A with respect to B defined as the velocity with which A appears to move if B is considered to be at rest.

$\vec{V}_A \Rightarrow$ Velocity of A with respect to ground

$\vec{V}_B \Rightarrow$ Velocity of B with respect to ground

Then velocity of A with respect to B is given by

$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

Similarly velocity of B with respect to A is given by

$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$

Example

find v_{AB} and v_{BA}

$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

$= 10\hat{i} - 2\hat{i}$

$= 8\hat{i}$

$|\vec{v}_{AB}| = 8\, m/s$

$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$

$= (2\hat{i}) - (10\hat{i})$

$= -8\hat{i}$

$|\vec{v}_{BA}| = 8\, m/s$

Note :

If two particles are moving in the same direction, then the magnitude of their relative velocity is the difference of individual velocities

$$\boxed{v_{AB} = v_A - v_B}$$

Example

find v_{AB} and v_{BA}

$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

$= (10\hat{i}) - (-2\hat{i})$

$= 12\hat{i}$

$|\vec{v}_{AB}| = 12\ m/s$

$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$

$= (-2\hat{i} - 10\hat{i})$

$= -12\hat{i}$

$|\vec{v}_{BA}| = 12\ m/s$

Note :

If two particles are moving in the opposite direction, then magnitude of their relative velocity is the sum of their of individual velocities

$$\boxed{v_{AB} = v_A + v_B}$$

Relative acceleration

It is the rate at which relative velocity is changing Acceleration of A with respect to B is given by

$$\vec{a}_{AB} = \frac{d\left(\vec{v}_{AB}\right)}{dt} = \frac{d}{dt}\left(\vec{v}_A - \vec{v}_B\right)$$

$$= \frac{d}{dv}\vec{v}_A - \frac{d}{dv}\vec{v}_B$$

$$= \vec{a}_A - \vec{a}_B$$

$$\boxed{\vec{a}_{AB} = \vec{a}_A - \vec{a}_B}$$

Short cut to find relative velocity

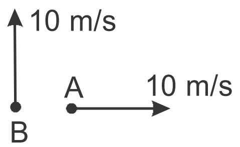
How we need to find the relative velocity

i.e. velocity of A with respect to B

For that keep ‘B’ at rest and give its velocity to A in the opposite direction

So relative velocity is 8 m/s

Example



Find v_{AB} and v_{BA}

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B = \left(10\,\hat{i}\right) - \left(10\,\hat{j}\right)$$

$$\left|\vec{v}_{BA}\right| = \sqrt{10^2 + (10)^2} = 10\sqrt{2}\ m/s$$

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = \left(10\,\hat{j}\right) - \left(10\,\hat{i}\right)$$

$$\left|\vec{v}_{BA}\right| = \sqrt{10^2 + (-10)^2} = 10\sqrt{2}\ m/s$$

Example

Velocity of A is $\left(\hat{i} + \hat{j}\right) m/s$. Velocity of A w.r.t. B is $2\hat{i} + 3\hat{j}$. Then the actual velocity of B is

$\vec{v}_A = \hat{i} + \hat{j}$ $\quad \vec{v}_B = ?$ $\quad \vec{v}_{AB} = 2\,\hat{i} + 3\,\hat{j}$

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

$$\vec{v}_B = \vec{v}_A - \vec{v}_{AB}$$

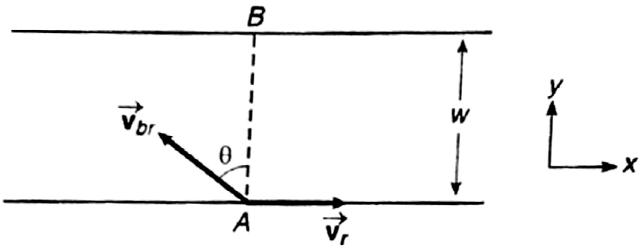
$$= \left(\hat{i} + \hat{j}\right) - \left(2\hat{i} + 3\hat{j}\right) = -\,\hat{i} - 2\hat{j}$$

River-Boat Problems

In river-boat problems we come across the following three terms:

drift of the boat $x = v_x t$

$$\boxed{x = \frac{v_r w}{v_b r}}$$



$\vec{v}_r$ = absolute velocity of river.

$\vec{v}_{br}$ = velocity of boatman with respect to river or velocity of boatman in still water.

and $\vec{v}_b$ = absolute velocity of boatman.

Here, it is important to note that $\vec{v}_{br}$ is the velocity of boatman with which he steers and $\vec{v}_b$ is the actual velocity of boatman relative to ground.

Further, $\vec{v}_b = \vec{v}_{br} + \vec{v}_r$

Now, let us derive some standard results and their special cases.

A boatman starts from point A on one bank of a river with velocity $\vec{v}_{br}$ in the direction shown in Fig.

River is flowing along positive x-direction with velocity $\vec{v}_r$. Width of the river is w, then

$$\vec{v}_b = \vec{v}_r + \vec{v}_{br}$$

Therefore, $v_{bx} = v_{rx} + v_{brx} = v_r - v_{br}\sin\theta$

and $v_{by} = v_{ry} + v_{bry} = 0 + v_{br}\cos\theta = v_{br}\cos\theta$

Now, time taken by the boatman to cross the river is:

$$t = \frac{w}{v_{by}} = \frac{w}{v_{br}\cos\theta}$$

or $$\boxed{t = \frac{w}{v_{br}\cos\theta}}$$

Further, displacement along x-axis when he reaches on the other bank (also called drift) is:

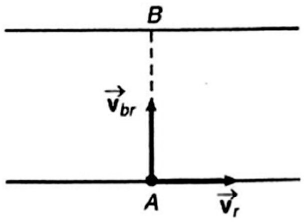
$$x = v_{bx} t = \left(v_r - v_{br} \sin\theta\right) \frac{w}{v_{br} \cos\theta}$$

or $$\boxed{x = \left(v_r - v_{br} \sin\theta\right) \frac{w}{v_{br} \cos\theta}}$$

Three special cases are:

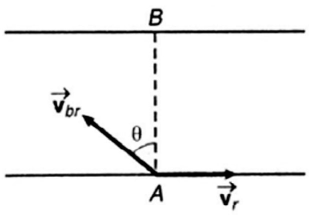
(i) Condition when the boatman crosses the river in shortest interval of time.

From Eq. (i) we can see that time (t) will be minimum when $\theta = 0°$, i.e., the boatman should steer his boat perpendicular to the river current.



Also, $\boxed{t_{min} = \frac{w}{v_{br}}}$ as $\cos\theta = 1$

(ii) Condition when the boatman wants to reach point B, i.e., at a point just opposite from where be started (shortest path).



$$\vec{v}_b = \vec{v}_r + \vec{v}_{br}$$

$$v_b = \sqrt{v_{br}^2 - v_r^2}$$

$$t = \frac{w}{v_b} = \frac{w}{\sqrt{v_{br}^2 - v_r^2}}$$

In this case, the drift (x) should be zero.

$\therefore \quad x = 0$

or $\left(v_r - v_{br} \sin\theta\right) \frac{w}{v_{br} \cos\theta} = 0$

or $v_r = v_{br} \sin\theta$

or $\boxed{\sin\theta = \frac{v_r}{v_{br}}}$ or $\theta = \sin^{-1}\left(\frac{v_r}{v_{br}}\right)$

Hence, to reach point B the boatman should row at an angle $\theta = \sin^{-1}\left(\frac{v_r}{v_{br}}\right)$ upstream from AB.

Further, since $\sin\theta \not> 1$

So, if $v_r \geq v_{br}$, the boatman can never reach at point B. Because if $v_r = v_{br}$, $\sin\theta = 1$ or $\theta = 90^\circ$ and it is just impossible to reach at B if $\theta = 90^\circ$. Moreover it can be seen that $v_b = 0$ if $v_r = v_{br}$ and $\theta = 90^\circ$. Similarly, if $v_r > v_{br}$, $\sin\theta > 1$, i.e., no such angle exists. Practically it can be realized in this manner that it is not possible to reach at B if river velocity (v_r) is too high.

(iii) Shortest path

Path length travelled by the boatman when he reaches the opposite shore is

$$s = \sqrt{w^2 + x^2}$$

Here, w = width of river is constant. So for s to be minimum modulus of x (drift) should be minimum. Now two cases are possible.

When $v_r < v_{br}$; In this case x = 0, when $\theta = \sin^{-1}\left(\frac{v_r}{v_{br}}\right)$

or $\boxed{s_{min} = w}$ at $\boxed{\theta = \sin^{-1}\left(\frac{v_r}{v_{br}}\right)}$

When $v_r > v_{br}$: In this case x is minimum, where $\frac{dx}{d\theta} = 0$

or $\frac{d}{d\theta}\left\{\frac{w}{v_{br}\cos\theta}\left(v_r - v_{br}\sin\theta\right)\right\} = 0$

or $-v_{br}\cos^2\theta - \left(v_r - v_{br}\sin\theta\right)\left(-\sin\theta\right) = 0$

or $-v_{br} + v_r\sin\theta = 0$

or $\theta = \sin^{-1}\left(\frac{v_{br}}{v_r}\right)$

or $\boxed{\sin\theta = \frac{v_{br}}{v_r}}$

Aircraft Wind Problems

This is similar to river boat problem. The only difference is that $\vec{v}_{br}$ is replaced by $\vec{v}_{aw}$ (velocity of aircraft with respect to wind or velocity of aircraft in still air), $\vec{v}_r$ is replaced by $\vec{v}_w$ (velocity of wind) and $\vec{v}_b$ is replaced by $\vec{v}_a$ (absolute velocity of aircraft). Further, $\vec{v}_a = \vec{v}_{aw} + \vec{v}_w$. The following example will illustrate the theory.

Example

A man can row a boat with 4 km/h in still water. If he is crossing a river where the current is 2 km/h.

(a) In what direction will his boat be headed, if he wants to reach a point on the other bank directly opposite to starting point?

(b) If width of the river is 4 km, how long will the man take to cross the river, with the condition in part (a)?

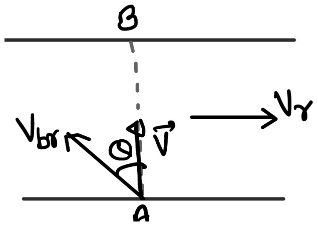
(c) In what direction should be head the boat if he wants to cross the river in shortest time and what is this minimum time?

(d) How long will it take him to row 2 km up the stream and then back to his starting point?

Solution:

(a) Given, that $v_{br} = 1 \text{km/h}$ and $v_r = 2 \text{km/h}$

$$\theta = \sin^{-1}\left(\frac{v_r}{v_{br}}\right) = \sin^{-1}\left(\frac{2}{4}\right) = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$$



Hence, to reach the point directly opposite to starting point he should head the boat at an angle 30° with AB or 90° + 30° = 120° with the river flow.

(b) Time taken by the boatman to cross river

w = width of river = 4 km

v_{br} = 4 km/h and $\theta = 30^\circ$

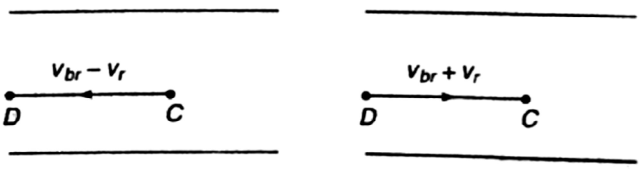
$$t = \frac{4}{4\cos 30^\circ} = \frac{2}{\sqrt{3}} h$$

(c) For shortest time $\theta = 0^\circ$

and $t_{min} = \dfrac{w}{v_{br}\cos 0^\circ} = \dfrac{4}{4} = 1h$

Hence, he should head his boat perpendicular to the river current for crossing the river in shortest time and this shortest time is 1 h.

(d) $t = t_{CD} + t_{DC}$



or $t = \frac{CD}{v_{br} - v_r} + \frac{DC}{v_{br} + v_r} = \frac{2}{4-2} + \frac{2}{4+2} = 1 + \frac{1}{3} = \frac{4}{3}h$

THE MAN AND THE RAIN PROBLEM

The aim is to determine the angle at which the man should hold the umbrella to prevent himself from wetting.

Here, v_r = velocity of rain w.r.t. ground

v_m = velocity of man w.r.t. ground

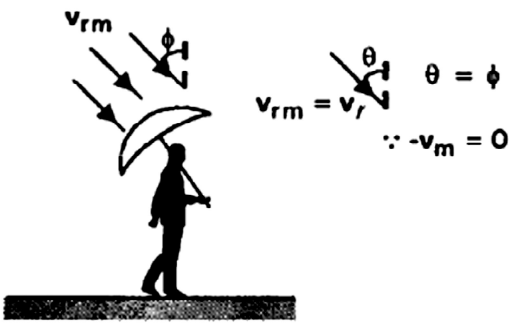
v_{rm} = velocity of rain w.r.t. man

$\vec{v}_{rm} = \vec{v}_r - \vec{v}_m = \vec{v}_r + (-\vec{v}_m)$

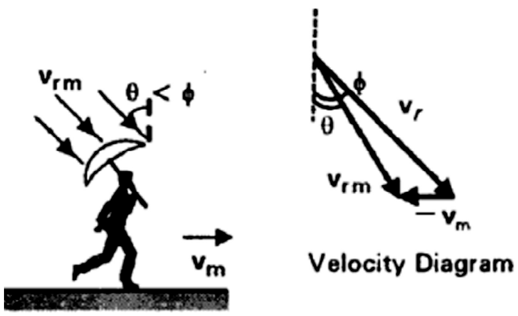
umbrella must be hold against the relative velocity v_{rm}

The answer to the problem is that he should hold the umbrella in the direction from where the rain appears to be falling.

1. The man is stationary and the rain is falling at his back at an angle ϕ with the vertical.

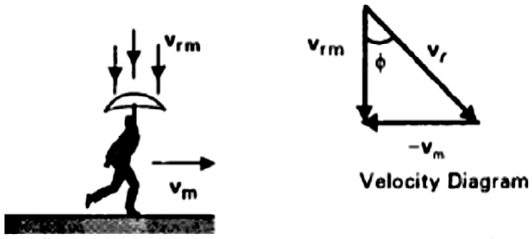


2. The man starts moving forward. The relative velocity of rain w.r.t. man shifts towards vertical direction (not away from it as a general misconception may be).



$|v_m| < |v_r \sin\phi|$ = horizontal component of rain velocity.

3. As the man further increases his speed, then at a particular value, the rain appears to be falling vertically.



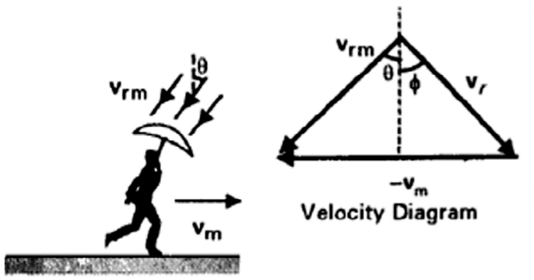
$|v_m| = |v_r \sin\phi|$

$\sin\phi = \frac{v_m}{v_r}$

$\tan\phi = \frac{v_m}{v_{rm}}$ $\quad\quad v_{rm} = \sqrt{v_r^2 - v_m^2}$

4. If the man increases his speed further more, then the rain appears to be falling from the forward direction.

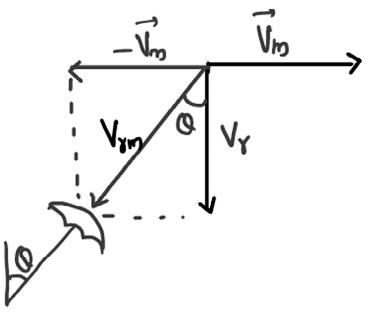
$|v_m| > |v_r \sin\phi|$



5. Rain is falling vertically downward.

$\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$; $\quad\quad \vec{v}_{rm} = \vec{v}_r + (-\vec{v}_m)$

$\tan\theta = \frac{v_m}{v_r}$; $\quad\quad \sin\theta = \frac{v_m}{v_{rm}}$



$v_{rm} = \sqrt{v_r^2 + v_m^2}$

VELOCITY OF APPROACH

Velocity of approach ($v_{app.}$) of particle A w.r.t. B is the component of the relative velocity of A w.r.t. B along the line joining A and B and is directed towards B.

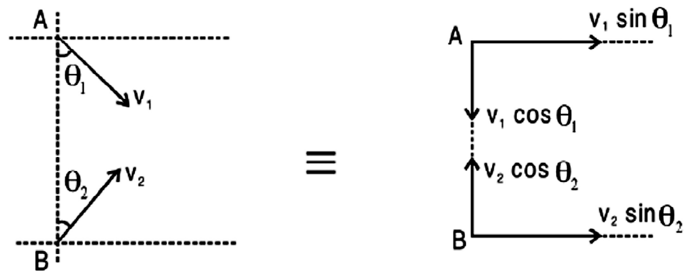
1. It's rate of decrease of separation if A is moving towards B.
2. If velocity of approach is constant then.

$$\text{time of collision} = \frac{\text{initial separation}}{\text{velocity of approach}}$$

3. If $v_{app.}$ is variable and t_0 = time taken for collision then

$$\text{initial separation } s_0 = \int_0^{t_0} v_{app.}\, dt$$

ILLUSTRATIVE CASES



$(v_{app} = v_1 \cos\theta_1 + v_2 \cos\theta_2)$

Example

A river has width 120 metre. Velocity of river is 10 m/s. Velocity of boat in still water is 12 m/s. The boat is rowed perpendicular to river flow.

Find a) actual velocity of boat

b) time taken to cross the river

c) drift

Here $\theta = 90^\circ$ θ is the angle between $\vec{v}_{mr}$ and $\vec{v}_r$ $v_r = 10\,m/s$ d = 120 m v_{MR} = 12 m/s

$$v_M = \sqrt{v_{MR}^2 + v_R^2 + 2v_{MR}v_R\cos\theta} = \sqrt{12^2 + 10^2 + 2\times12\times10\times\cos\theta} = \sqrt{244}\ m/s$$

$$t = \frac{d}{v_{MR}\sin 90} = \frac{120}{12\times1} = 10\,\text{sec}$$

$$x = (v_{MR}\cos\theta + v_R)\frac{d}{v_{MR}\sin\theta} = (12\times\cos 90 + 10)\frac{120}{12\times\sin 90} = 10\times\frac{120}{12} = 100\,\text{metre}$$

Example

A man can swim at the rate of 5 km/hr in still water a 1 km wide River flows at the rate of 3 km/hr. The man wishes to swim across the river directly opposite to the starting point.

a) Along what direction he must swim?

b) What is his resultant velocity?

c) Time taken to cross the river

v_{MR} = 5 km/hr

d = 1 km

v_R = 3 km/hr

$$t = \frac{d}{\sqrt{v_{MR}^2 - v_R^2}} = \frac{1}{\sqrt{5^2 - 3^2}} = \frac{1}{4}\text{hour} = 15 \text{ minutes}$$

$$v_M = \sqrt{v_{MR}^2 - v_R^2} = \sqrt{5^2 - 3^2} = 4\,\text{km / hr}$$

$$\theta = \cos^{-1}\left(\frac{-v_R}{v_{MR}}\right) = \cos^{-1}\left(\frac{-3}{5}\right) = 127^o$$

$\theta \Rightarrow$ angle made with the direction of river flow.

$\therefore 127^o$ downstream or 53^o upstream.

Sample

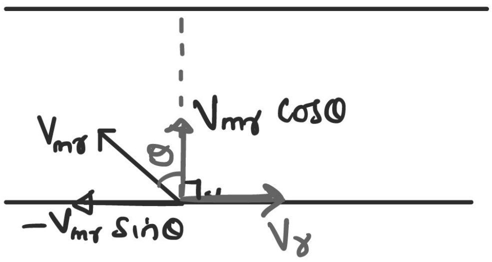
A man can swim at a speed 5 km/h in the still water. He wants to cross a 1600 m wide river flowing at 4 km/h. He keep himself at an angle 127° with river flow, while swimming. Find the time he takes to cross the river, also find the distance travelled by him along the flow.

v_{MR} = 5 km/h

d = 1600 m = 1.6 km

v_R = 4 km/h

$\theta = 127 - 90 = 37^0$



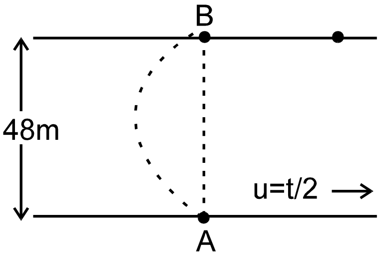
$$t = \frac{d}{v_{MR}\cos\theta} = \frac{1.6}{5 \times \sin 37} = \frac{1.6}{5 \times \frac{4}{5}} = 0.4\,\text{hour}$$

$$x = \left(v_R - v_{MR}\sin\theta\right)\frac{d}{v_{MR}\cos\theta} = \left(4 - 5 \times \cos 37\right) \times \frac{1.6}{5 \times \sin 37}$$

$$= \left(5 \times \frac{-3}{5} + 4\right) \times \frac{1.6}{5 \times \frac{4}{5}} = 1 \times 0.4 = 0.4\,\text{km} = 400\,\text{m}$$

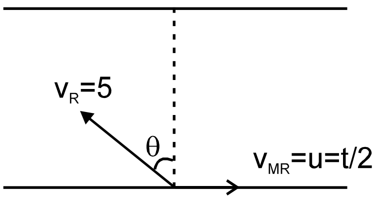
Example

A man starts swimming at time t = 0 from point A on the ground and he wants to reach the point B directly opposite the point A. His velocity in still water is 5 m/s and width of river is 48 m. River flow velocity u varies with time is given by $u = t/2$ m/s. He always tries to swim in particular fixed direction with river flow (given $\sin^{-1}\frac{24}{25} = 74^o$)



a) direction in which he should make stroke and the time taken by the man to cross the river

Trajectory of path



Horizontal motion

$vel = v_R - v_{MR}\sin\theta$

$v_x = t/2 - 5\sin\theta$

$\frac{dx}{dt} = t/2 - 5\sin\theta$

$dx = t/2\, dt = 5\sin\theta dt$

Integrate

$x = \frac{t^2}{4} - 5\sin\theta t$

drift t = 0

x = 0

$\frac{t^2}{4} = 5\sin\theta t$

Vertical motion

vel = $v_{MR}\cos\theta = 5\cos\theta$

disp = 48

$\boxed{t = \frac{48}{5\cos\theta}}$

$\boxed{t = 20\sin\theta}$

$$20\sin\theta = \frac{48}{5\cos\theta}$$

$$100\sin\theta\cos\theta = 48$$

$$50 \times \sin 2\theta = 48$$

$$\sin 2\theta = {}^{24}\!/_{25}$$

$$2\theta = 74^{\circ}$$

$$\theta = 37^{\circ}$$

Rain Umbrella problems

$\vec{v}_R$ - velocity of rain

$\vec{v}_M$ - velocity of man

$\vec{v}_{RM}$ - velocity of rain with respect to man

$\boxed{\vec{v}_{RM} = \vec{v}_R - \vec{v}_M}$

Example

The rain is falling vertically downward with velocity 6 m/s and man is moving horizontally with velocity 8 m/s. Find the velocity of rain with respect to man?

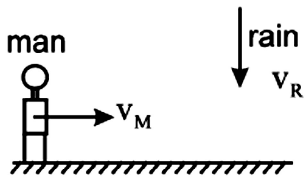
$\vec{v}_R = 6\left(-\hat{j}\right) = -6\hat{j}$

$\vec{v}_M = 8\hat{i}$

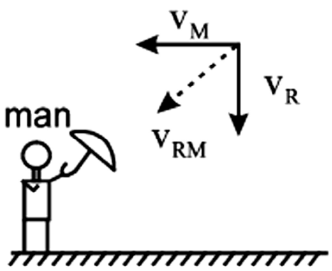
$\vec{v}_{RM} = \vec{v}_R - \vec{v}_M = -6\hat{j} - 8\hat{i}$

$|\vec{v}_{RM}| = v_{RM} = \sqrt{6^2 + 8^2} = 10 \text{ m/s}$

Note:



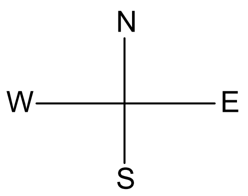
Take man at rest



Angle made by umbrella with vertically = angle made by $\vec{v}_{RM}$ with vertical.

Example

A man is walking due west with a velocity 3 km/h and rain appears to be falling vertically, with a velocity 4 km/h. Find the velocity of rain with respect to the ground.



$$\vec{v}_M = -3\hat{i}$$

$$\vec{v}_{RM} = -4\hat{i}$$

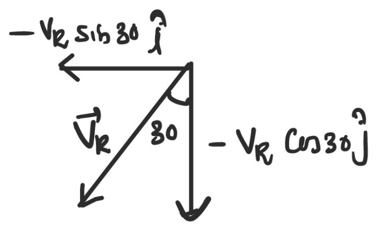
$$\vec{v}_{RM} = \vec{v}_R - \vec{v}_M$$

$$\vec{v}_R = \vec{v}_{RM} + \vec{v}_M = -4\hat{j} - 3\hat{i}$$

$$v_R = |\vec{v}_R| = \sqrt{4^2 + 3^2} = 5 \text{ km/h}$$

Example

A standing man observes rain falling with velocity 20 m/s at an angle 30° with the vertical. Find the velocity with which the man should move so that the rain appears to fall vertically to him.



$$\vec{v}_R = 20\sin 30\left(-\hat{i}\right) + 20\cos 30\left(-\hat{j}\right)$$

$$\vec{v}_R = 10\hat{i} - 10\sqrt{3}\hat{j}$$

$$\vec{v}_M = a\hat{i}$$

$$\vec{v}_{RM} = \vec{v}_R - \vec{v}_M = \left(-10\hat{i} - 10\sqrt{3}\hat{j}\right) - a\hat{i}$$

$$-(10 + a)\hat{i} - 10\sqrt{3}\,\hat{j}$$

v_{RM} is vertical so no horizontal component.

$\therefore$ horizontal comp = 0.

10 + a = 0

a = –10

$\vec{v}_M = a\hat{i} = -10\hat{i}$

$v_M = |\vec{v}_M| = 10\,m/s$

Aircraft wind Problems

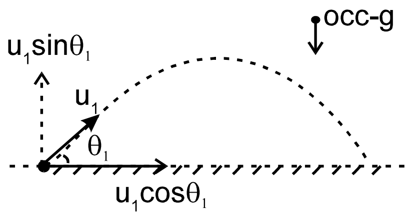
This is similar to river boat problem.

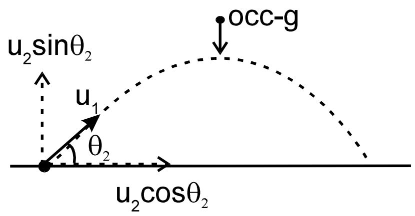
$\vec{v}_{MR}$ is replaced by $\vec{v}_{AW}$ (velocity of aircraft with respect to wind)

$\vec{v}_R$ is replaced by $\vec{v}_W$ (velocity of wind)

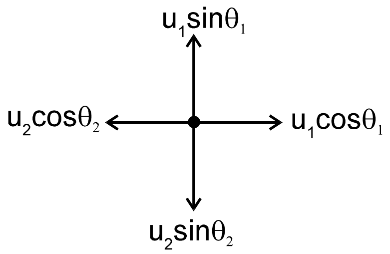
Relative Motion between two Projectiles

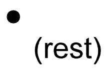
Consider two projectiles projected simultaneously.





1st projectile is seen from second projectile, for that second projectile makes at rest





Since acceleration is zero both horizontal and vertical component remains constant. Therefore it looks like a straight line

Note

If $u_1 \cos\theta_1 = u_2 \cos\theta_2$

then it will be a vertical straight line

If $u_1 \sin\theta_1 = u_2 \sin\theta_2$

then it will be a horizontal straight line and they will collide after some time

$\therefore$ Condition for collision is $u_1 \sin\theta_1 = u_2 \sin\theta_2$