CHAPTER - 19
THEORY OF PROBABILITY

Probability

Random experiment : An experiment whose outcome can not be predicted with certainty

Sample space (S) : Set of all possible outcomes of a random experiment.

Event : Any finite sub set of sample space

Equaly likely event : Events are equally likely if they have the same chance to occer.

Mutually exclusive Events : Events are mutually exclusive if they can not occur at the same time. If A and B are Mutually exclusive $A \cap B = \phi$.

Exhaustive Events : Events are exhaustive if they include all the possible outcomes of a random experiment. If A and B are exhaustive $A \cup B = S$

Example : Consider the Tossing of a fair/unbaised coin. Then the events [H] and [T] are equally likely, Mutally exclusive and exhaustive.

Probability : Let A be an event. Probability of A is denoted by P(A) and is a numerical measure of the chance of occurance of A.

Classical Probability: In classical definition probability of the event A is $P(A) = \frac{m}{n}$

Where n = n (S) and m = n(A). Classical probability is also known as mathematical probability

Statistical/Empirical probability

In statistical definition probability of even A is defined as P(A) = $\underset{n\to\infty}{\mathrm{Lt}}\left(\frac{r}{n}\right)$ where 'r' is the frequency of the event A out of a total of n repetitions of the experiment. (r/n) is called the frequency ratio.

Axiomatic probability

Given a sample space of random experiment, the probability of the occurance of any event A is defined as a set function satisfying the following axioms

i) $P(A) \geq 0$

ii) $P(S) = 1$

iii) If A_1 and A_2 are mutually exclusive events $P(A_1 \cup A_2) = P(A_1) + P(A_2)$

Note : a) $0 \leq P(A) \leq 1$

b) sample space is called the sure event

c) Null set (ϕ) is called the impossible event

Addition Theorm

Let A and B be any two events. Then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

but $A \cup B$ = A or B; $A \cap B$ = A and B

Hence addition theorm can also be written as

P (A or B) = P (A) + P (B) – P (A and B)

Conditional Probability

Let A be an event and B be a condition. The probability of the occurrence of event A, given that B has already occurred is called the conditional probability. It is denoted by P (A/B) and is given by

$$P(A/B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0$$

Similarly P (B/A) = $\dfrac{P(A \cap B)}{P(A)}, P(A) \neq 0$

Multiplication (compound prob) Theorm

Let A and B be two events having non – zero probabilities. Then

$$P(A \cap B) \begin{cases} = & P(A/B) \quad P(B) \\ & \text{or} \\ = & P(B/A) \quad P(A) \end{cases}$$

Independent Events

Events A and B are independent if

P (A/B) = P(A) and P (B/A) = P (B)

Notes

1. When A and B are independent P (A $\cap$ B) = P (A) P (B) (multiplication theorm for independent events)

2. When A and B are independents P (A $\cup$ B) = 1 – P (A$'$) P (B$'$) where P (A$'$) = 1 – P (A), P (B$'$) = 1 – P (B) (Addition Theorm for independent events)

3. When A and B are independent

i) A$'$ and B$'$ are independent

ii) A and B$'$ are independent

iii) A$'$ and B are independent

Random Variable (R.V)

A variable which takes values based on the outcomes of a sample space is called a Random Variable and it is denoted by ‘x’. If a R.V takes only finite or countably infinite number of values, it is called discrete R.V. On the other hand a R.V taking any value in an interval is called a contnuous R.V

Probability Mass function (pmf)

The probability that a discrete Random variable ‘X’ takes a particular value x_1 is denoted by P (xi) and is called the pmf.

Probability Distribution of a R.V

Let 'X' be a discrete Random variable taking values x_i with pmf P (xi) where i = 1, 2, 3 ....n. Then the table consiting of the R.V and the pmf is called the probability distribution. It is given below

X	x_1	x_2	x_3	------	------	x_n
P(X)	$P(x_1)$	$P(x_2)$	$P(x_3)$	------	------	$P(x_n)$

Arithmetic Mean (Expeitation) of a R.V

The Arithmetic mean of the probability distribution is called the expected value of the R.V. If is denoted by E(x). Since $\sum P(x_i)=1$, the A.M = E (X) = $\sum x_i P(x_i)$

The varience of the R.V. is $V(x) = E(x^2) - [E(x)]^2$ where $E(x^2)$ = A.M of $x^2 = \sum x_i^2 P(x_i)$

Example : A random variable X has the following probability distribution.

X	0	1	2	3	4	5	6	7	8
P(x)	k	3k	5k	7k	8k	11k	0	k	3k

1) Find k

2) Evaluate P (x < 4), P (x $\geq$7), P (2 < x< 5)

In a probability distribution sum of probability = 1

$k + 3k + 5k + 7k + 8k + 11k + 0 + k + 3k = 1 \Rightarrow k = \frac{1}{39}$

$P(x < 4) = P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3) = \frac{16}{39}$

$P(x \geq 7) = P(x = 7) + P(x = 8) = \frac{4}{39}$

$P(2 < x < 5) = P(x = 3) + P(x = 4) = \frac{15}{39}$

Binomial distribution: Random experiments having only two possible outcomes namely success (S) and failure (F) are called Bernoulli trial or Binomial experiments. Trials of random experiment are called by Bernoulli trials if they satisfy the following conditions.

1. There should be a finate no of trials
2. These trials are independent
3. Each trial has exactly two outcomes; success or failure
4. The probability of an outcome remains the same in each trial

Let P (S) = P and P (F) = q so that p + q = 1. The Binomial distribution, describes the probability of getting 'x' successes when a Binomial experiment is independently repeated 'n' times. This probability is given by

$P(X) = nC_x P^x q^{n-x}$

where x = 0, 1, 2 ------n

for a Binomial Distribution Mean =nP, variance = npq, S.D = $\sqrt{npq}$

Note

When a fair coin is tossed 'n' times, the probability of getting x heads (Tails) is given by

$$P(x) = nC_x \left(\frac{1}{2}\right)^n$$

Example : 10 coins are tossed. Find the probability of getting

a) exactly 6 head   b) 0 head   c) at least 1 head   d) at most 1 head

n = 10, p = P (H) = 1/2, q = 1–p = 1/2

$P(x) = nC_x\, p^x q^{n-x} = 10C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{10-x} = 10C_x \left(\frac{1}{2}\right)^{10}, x = 0,1,2,3 - - - - - - - 10$

a) P (6 head) = P(6) = $10C_6 \left(\frac{1}{2}\right)^{10} = \frac{105}{512}$ b) P (0 head) = P(0) = $10C_0 \left(\frac{1}{2}\right)^{10} = \frac{1}{1024}$

Total probability = 1

P(0) + P(1) + P(2) + P(3) + - - - - - - - - + P(10) = 1

c) P (at least one head) = $1 - P(0) = 1 - \frac{1}{1024} = \frac{1023}{1024}$

d) P(at most one head) = P(0) + P(1) = $\frac{11}{1024}$

Odds in favour and against

Let n(A) = number of cases in favour of event A

n(A′) = number of cases against event A

1. Odds in favour of event $A = \frac{n(A)}{n(A')}$ 2. Odds against event $A = \frac{n(A^1)}{n(A)}$

3. Odds is favour of A are m:n means $P(A) = \frac{m}{m+n}$

4. Odds against A are m:n means $P(A) = \frac{n}{m+n}$

5. P(A) = P $\Rightarrow$ odds in favour of $A = \frac{P}{1-P}$

6. P (A) = P $\Rightarrow$ odds against $A = \frac{1-P}{P}$

Packet of cards

Sl No	Spade (Black)	Club (Black)	Hearts (Red)	Diamond (Red)	Total
1	KING	KING	KING	KING	4
2	QUEEN	QUEEN	QUEEN	QUEEN	4
3	JACK	JAKC	JACK	JACK	4
4	ACE	ACE	ACE	ACE	4
5	2	2	2	2	4
6	3	3	3	3	4
7	4	4	4	4	4
8	5	5	5	5	4
9	6	6	6	6	4
10	7	7	7	7	4
11	8	8	8	8	4
12	9	9	9	9	4
13	10	10	10	10	4
TOTAL	**13**	**13**	**13**	**13**	**52**

Total no. of cards = $13 \times 4 = 52$
number of Red cards = 26
number of Black cards = 26
number of Kings/Queens/Jack/Ace cards = 4
number of spade / clubs/Hearts/Diamonds = 13

The Law of Total Probability

Let S be the sample space and let $E_1, E_2 ...... E_n$ be n mutually exclusive and exhaustive events associated with a random experiment. If A is any event which occurs with E_1 or E_2 or ...... or E_n, then

$$P(A) = P(E_1) . P(A/E_1) + P(E_2) P(A/E_2) + ...+ P(E_n) P(A/E_n) = \sum_{i=1}^{n} P(E_i) P(A/E_i)$$

Baye's theorem

Let S be the sample space and let $E_1, E_2 .... E_n$ be n mutually exclusive and exhaustive events associated with a random experimnet. If A is any event which occurs with E_1 or E_2 or...or E_n, then

$$P(E_i/A) = \frac{P(E_i)P(A/E_i)}{\sum_{i=1}^{n} P(E_i)P(A/E_i)}, i = 1, 2...., n$$

Example : A bag contains 2 white and 3 red balls. Another bag contains 4 white and 1 red ball. A bag is selected at random and ball is taken. It is found to be red. What is the probability that it was taken from the second bag.

$B_1 \rightarrow 1^{st}$ bag, $B_2 \rightarrow 2^{nd}$ bag, $R \rightarrow$ Red ball

$P(B_1) = 1/2$, $P(B_2) = 1/2$ $\quad P(A/B_1) = 3/5 \quad P(A/B_2) = 1/5$

$$P(B_2/A) = \frac{P(B_2)P(A/B_2)}{P(B_1).P(A/B_1) + P(B_2).(A/B_2)} = \frac{1}{4}$$

PART I - (JEEMAIN)

QUESTIONS

SECTION - I - Straight objective type questions

1. A seven digit number is formed using digits 3, 3, 4, 4, 4, 5, 5. The probability, that number so formed is divisible by 2, is

(1) $\frac{6}{7}$ (2) $\frac{1}{7}$ (3) $\frac{3}{7}$ (4) $\frac{4}{7}$

2. From a group of 10 men and 5 women, four member committee is to be formed which should have at least one women. The probability that the committee to have more women than men

1) $\frac{21}{220}$ 2) $\frac{3}{11}$ 3) $\frac{1}{11}$ 4) $\frac{2}{23}$

3. If two events A and B are such that $P(\overline{A}) = 0.3$, P(B)=0.4 and $P(A \cap \overline{B})$=0.5, then $P\left(\frac{B}{A \cup \overline{B}}\right)$ =

1) 1/4 2) 1/5 3) 3/5 4) 2/5

4. An integer is choosen at random from the interval $-10 \le q \le 10$. What is the probability that $x^2 + qx + \frac{3q}{4} + 1 = 0$ has real roots

1) $\frac{16}{21}$ 2) $\frac{15}{21}$ 3) $\frac{14}{21}$ 4) $\frac{17}{21}$

5. A 2n digit number starts with 2 and all its digits are prime. Then the probability that the sum of all two consecutive digits of the number is prime

1) 4×2^{-2n} 2) 4×2^{-3n} 3) 2^{3n} 4) 2^{-3n}

Assertion & Reasoning

(1) If both Statement-I and Statement-II are true and the reason is the correct explanation of the statement-I.

(2) If both Statement-I and Statement-II are true but reason is not the correct explanation of the statement-I.

(3) If Statement-I is true but Statement-II is false.

(4) If Statement-I is false but Statement-II is true.

6. **Statement-I:** Out of 21 tickets with number 1 to 21, 3 tickets are drawn at random, the chance that the numbers on them are in AP is $\frac{10}{133}$.

Statement-II: Out of $(2n+1)$ tickets consecutively numbered three are drawn at random, the chance that the numbers on them are in AP is $\frac{(4n-10)}{(4n^2-1)}$.

7. The probabilities of three events A, B and C are given by P(A) = 0.6, P(B) = 0.4 and P(C) = 0.5. If $P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$, $P(A \cap B \cap C) = 0.2$, $P(B \cap C) = \beta$ and $P(A \cup B \cup C) = \alpha$, where $0.85 \le a \le 0.95$, then β lies in the interval:

(1) [0.36, 0.40] (2) [0.35, 0.36] (3) [0.25, 0.35] (4) [0.20, 0.25]

8. Let A,B and C be three events such that the probability that exactly one of A and B occurs is (1-k), the probability that exactly one of B and C occurs is (1-2k), the probability that exactly one of C and A occurs is 1-k and the probability of all A, B and C occurs simultaneously is k^2, where $0 < k < 1$. Then the probability that at least one of A,B and C occur is

1) greater than $\frac{1}{8}$ but less than $\frac{1}{4}$ 2) greater than $\frac{1}{2}$

3) greater than $\frac{1}{4}$ but less than $\frac{1}{2}$ 4) exactly equal to $\frac{1}{2}$

9. Two integers are selected at random from the set {1, 2, ….., 11}. Given that the sum of selected numbers is even, the conditional probability that both the numbers are even is:

(1) $\frac{2}{5}$ (2) $\frac{1}{2}$ (3) $\frac{3}{5}$ (4) $\frac{7}{10}$

10. Let $\overline{E}$ denote the complement of an event E. Let E_1, E_2 and E_3 be any pairwise independent events with $P(E_1) > 0$ and $P(E_1 \cap E_2 \cap E_3) = 0$. Then $P(\overline{E}_2 \cap \overline{E}_3 / E_1)$ is equal to

(1) $P(\overline{E}_3) - P(E_2)$ (2) $P(\overline{E}_2) + P(E_3)$ (3) $P(\overline{E}_3) - P(\overline{E}_2)$ (4) $P(E_3) - P(\overline{E}_2)$

11. There are 4 defective items in a lot of 15 items. The items are selected one by one at random without replacement till the last defective item is drawn. The probability that the 10^{th} item examined is the last defective item is

(1) $\frac{1}{65}$ (2) $\frac{2}{65}$ (3) $\frac{3}{65}$ (4) $\frac{4}{65}$

12. $A_1, A_2, ..., A_n$ are n independent events with $P(A_j) = \frac{1}{1+j}$ $(1 \le j \le n)$. The probability that none of $A_1, A_2, ..., A_n$ occur, is

(1) $\frac{n!}{(n+1)!}$ (2) $\frac{n}{n+1}$ (3) $\frac{1}{(n+1)!}$ (4) none of these

13. A box contains 5 bulbs of which two are defective. Test is carried out on bulbs one by one until the two defective bulbs are found out. The probability that the process stops after third test is

1) $\frac{1}{10}$ 2) $\frac{2}{10}$ 3) $\frac{3}{10}$ 4) $\frac{5}{7}$

14. In a random experiment, a fair die is rolled until two fours are obtained in succession. What is the probability that experiment will end in the 5^{th} throw

1) $\frac{175}{65}$ 2) $\frac{175}{6^5}$ 3) $\frac{175}{6^6}$ 4) $\frac{225}{6^5}$

15. From set A = $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$ two numbers are taken one by one with replacement. If x and y are the numbers taken, what is the probability that $x^2 - y^2$ is divisible by 3

1) $\frac{96}{169}$ 2) $\frac{97}{169}$ 3) $\frac{100}{169}$ 4) $\frac{42}{169}$

16. Let X be a random variable such that the probability function of a distribution is given by

$$P(X=0) = \frac{1}{2}, P(X=j) = \frac{1}{3^j}(j = 1, 2, 3...., \infty)$$

Then the mean of the distribution and P (X is positive and even) respectively are

1) $\frac{3}{8}$ and $\frac{1}{8}$ 2) $\frac{3}{4}$ and $\frac{1}{8}$ 3) $\frac{3}{4}$ and $\frac{1}{9}$ 4) $\frac{3}{4}$ and $\frac{1}{16}$

17. A fair coin is tossed a fixed number of times. If the probability of getting 7 heads is equal to probability of getting 9 heads, then the probability of getting 2 heads is

(1) $\frac{15}{2^{13}}$ (2) $\frac{15}{2^{12}}$ (3) $\frac{15}{2^8}$ (4) $\frac{15}{2^{14}}$

18. An urn contains 5 Red and 2 Green balls. A ball is drawn at random. If the ball is green a red ball is added to the urn. If the ball is red, a green ball is added to the urn. Original ball is not returned to the urn. Now a second ball is drawn. What is the probability that it is red.

1) $\frac{32}{49}$ 2) $\frac{29}{49}$ 3) $\frac{33}{49}$ 4) $\frac{32}{47}$

19. It is known that in a bag there are five balls of different colours, out of which one is red. A perso who speaks truth 3 times out of 4, draws a ball at random. The probability that he will say that b is red, is

(1) $\frac{3}{29}$ (2) $\frac{3}{4}$ (3) $\frac{2}{3}$ (4) $\frac{7}{20}$

20. Box I contains 30 cards numbered 1 to 30 and Box II contains 20 cards numbered 31 to 50. A box is selected at random and a card is drawn from it. The number on the card is found to be a non-prime number. The probability that the card was drawn from Box I is:

(1) $\frac{8}{17}$ (2) $\frac{2}{3}$ (3) $\frac{4}{17}$ (4) $\frac{2}{5}$

SECTION - II

Numerical Type Questions

21. A bag contains a white and b black balls. Two players A and B alternatively draw a ball from the bag, replacing the ball each time after the draw till one of them draws a white ball and wins the game. A begins the game if the probability of A winning the game is three time that of B, then a/b =

22. A quadratic equation is choosen from the set of all quadratic equations which are unchanged by squaring its roots. If P is the probability that the equation has equal roots. Then 2P =

23. One point is taken at random from the set of all extreme points of $f(x)=|x|+|x^2-1|$. The probability that it is a local maximum point is P. Then 5P is

24. Of the three independent events E_1, E_2 and E_3, the probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let the probability p that none of events E_1, E_2 and E_3 occurs satisfy the equations $(\alpha-2\beta)\ p=\alpha\beta$ and $(\beta-3\gamma)p=2\beta\gamma$. All the given probabilities are assumed to lie in the interval (0, 1)

Then $\frac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3}$ =

25. Suppose ‘a’ and ‘b’ are single digit natural numbers choosen with replacement at random. The probability that point (a,b) lies above the parabola $y=ax^2-bx$ is p. Then 9(9p-1)

PART - II (JEE ADVANCED)

SECTION - III (Only one option correct type)

26. Consider $f(x)=x^3+ax^2+bx+c$. Parameters a, b, c are chosen, respectively, by throwing a die three times. Then the probability that f(x) is an increasing function is

A) 5/36 B) 8/36 C) 4/9 D) 1/3

27. $\begin{vmatrix} e^{2iA} & e^{-iC} & e^{-iB} \\ e^{-iC} & e^{2iB} & e^{-iA} \\ e^{-iB} & e^{-iA} & e^{2iC} \end{vmatrix} = n$

Given A,B,C are angles of a triangle. Let a fair coin is tossed (- n) times. Then the probability of getting exactly 3 heads

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{1}{5}$

28. A student can solve 2 out of 4 problems of mathematics, 3 out of 5 problems of physics and 4 out of 5 problems of chemistry. There are equal number of books of maths, physics and chemistry in his shelf. He selects one book randomly and attempts 10 problems from it. If he solves the first problem, then the probability that he will be able to solve the second problem is

A) 2/3 B) 25/38 C) 13/21 D) 14/23

29. A bag contains 5 white and 5 black balls. Pairs of balls are drawn without replacement until the bag is empty. Then the probability that in which each pair consists of one black and one white ball is

A) $\frac{4}{63}$ B) $\frac{5}{63}$ C) $\frac{7}{63}$ D) $\frac{8}{63}$

30. A random variable takes values 0,1,2....with respective probabilities $p, qp, q^2p, q^3p.....$ If $p+q=1$, then the A.M of the random variable is

A) p B) q C) $\frac{p}{q}$ D) $\frac{q}{p}$

SECTION - IV (More than one correct answer)

31. If A and B are two events such that $P(A\cup B)\geq\frac{3}{4}$ and $\frac{1}{8}\leq P(A\cap B)\leq\frac{3}{8}$ then which of the following is/are necessarily true

A) $P(A)+P(B)\leq\frac{11}{8}$ B) $P(A).P(B)\leq\frac{3}{8}$ C) $P(A)+P(B)\geq\frac{7}{8}$ D) $P(A).P(B)\leq\frac{1}{8}$.

32. The probabilities that a student passes in Physics, Chemistry and Biology are p, c and b respectively. Of these subjects, the student has a 75% chance of passing in at least one, a 50% chance of passing is at least two, and 40% chance of passing in exactly two. Which of the following is/are true

A) p + c + b = 19/20

B) p + c + b = 27/20

C) pcb = 1/10

D) pcb = 1/4

33. Let X and Y be two events such that $P\left(\frac{X}{Y}\right)=\frac{1}{2}, P\left(\frac{Y}{X}\right)=\frac{1}{3}$ and $P(X\cap Y)=\frac{1}{6}$. Which of the following is (are) correct?

A) $P(X\cap Y)=\frac{2}{3}$

B) X and Y are independent

C) X and Y are not independent

D) $P(\overline{X}\cap Y)=\frac{1}{3}$

34. A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities $\frac{1}{2},\frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and let X_1, X_2 and X_3 denotes respectively the events that the engines E_1, E_2 and E_3 are functioning. Which of the following is (are) true ?

A) $P[X_1^c|X]=\frac{3}{16}$ $P\left(\frac{\overline{X}_1}{X}\right)$

B) P[exactly two engines of the ship are functioning $|X] = \frac{7}{8}$

C) $P[X|X_2]=\frac{5}{16}$ $P\left(\frac{X}{X_2}\right)$

D) $P[X|X_1]=\frac{7}{16}$ $P\left(\frac{X}{X_1}\right)$

35. There are n urns having (n+1) balls in each urn so that the i^{th} urn contains i white balls and (n+1-i) red balls. Let U_i be the event of taking the i^{th} urn and 'W' be the event of taking a white ball. Then which of the following is / are true

A) If $P(U_i)=\frac{1}{n}$ Then $\underset{n\to\infty}{Lt} P(w)=\frac{2}{3}$

B) If $P(U_i)=C$, a constant. Then $P(U_n/W)=\frac{2}{n+1}$

C) If 'n' is even and E denotes the event of choosing an even numbered urn with $P(U_i)=\frac{1}{n}$, then the value of $P(W/E)=\frac{n+2}{2(n+1)}$

D) If $P(U_i)$ is proportional to i then $\underset{n\to\infty}{Lt} P(W)=\frac{2}{3}$

SECTION - V (Numerical Type)

36. An urn contains two white and two black balls, A ball is drawn at random. If it is white, it is not replaced into urn otherwise it is replaced along with another ball of the same colour. The process is repeated. If p is the probability that the third ball drawn is black, then the value of $\frac{23}{5p}$ is

37. Suppose families always have one, two or three children with probabilities $\frac{1}{4},\frac{1}{2}$ and $\frac{1}{4}$ respectively. Assume everyone eventually gets married and has children, the probability of a couple having exactly four grand children is

38. The chance that a doctor will diagonse a certain disease correctly is 60%. The chance that a patient of the doctor will die by this treatment after correct diagonsis is 40% and the chances of death by wrong diagonsis is 70%. The chance that a patient of the doctor having the particular disease will survive is $\frac{2K}{25}$. Then K =

39. A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is $\frac{3}{4}$. If the signal received at station B is green, then the probability that the original signal was green is

SECTION VI - (Matrix match type)

40. p and q are randomly choosen from the set $\{1,2,3,4,5,6,7,8,9,10\}$ with replacement. The probability that the roots of $x^2+px+q=0$ are

i) imaginary	a) 0.38
ii) Real	b) 0.03
iii) Equal	c) 0.62
	d) 0.05

A) (i,a) (ii,b) (iii,c)

B) (i,a) (ii,c) (iii,b)

C) (i,c) (ii,b) (iii,a)

D) (i,b) (ii,c) (iii,d)