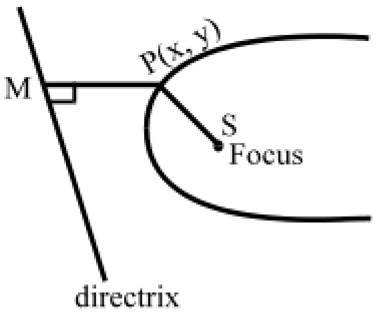
R

CHAPTER - 9

CONIC SECTIONS

1. Conic : The set of points in a plane whose distances from a fixed point (focus) and a fixed straight line (directrix) are in a constant ratio 'e' is called conic. The constant ratio 'e' is called the eccentricity of the conic. The conic is known as parabola, ellipse, hyperbola according as the value of e is equal to 1, less than 1, greater than respectively



$\frac{SP}{PM} = e$ (focus directrix property)

2. The general equation of a conic is of the form $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, it represent a parabola if $h^2 - ab = 0 \text{ and } \Delta \neq 0$

3. Axis of a conic : The line which is perpendicular to directrix and passing through the focus is called the axis

4. Vertices : The points of intersection of the conic and its axis are called the vertices of the conic

5. Centre : A point C is called the centre of the conic, if every chord of the conic, passing through c is bisected at c

6. Focal Chord : Any chord of the conic through the focus is called a focal chord

7. Double ordinate : A cord of a conic perpendicular to axis

8. Latus rectum : A focal chord of a conic perpendicular to its axis is called the latus rectum

9. Focal Distance : The distance from focus to any point on a conic is called focal distance

10. For any conic, if semi latus rectum is l and the perpendicular from the focus to directrix is d then $\frac{\ell}{d} = e$

PARABOLA

11. A parabola is the locus of a point which moves such that its distance from a fixed point is always equal to its distance from a fixed straight line

12. Equations of parabolas :

a) The equation $(y-k)^2 = 4a(x-h)$ represents a parabola with axis y = k, vertex (h, k), focus (h + a, k), directrix x = h – a and length of latus rectum 4a

b) The equation $(x-h)^2 = 4a(y-k)$ represents a parabola with axis x = h, vertex (h, k), focus (h, k + a), directrix y = k – a and length of latus rectum 4a

c) The equation $(y-k)^2 = -4a(x-h)$ represents a parabola with axis y = k, vertex (h, k), focus (h – a, k), directrix x = h + a and length of latus rectum 4a

d) The equation $(x-h)^2 = -4a(y-k)$ represents a parabola with axis x = h, vertex (h, k), focus (h, k – a), directrix y = k + a and length of latus rectum 4a

13. If the axis is parallel to x-axis then equation of the parabola is of the form $Ay^2+By + Cx + D = 0$

14. Length of its latus rectum = $\left|\frac{C}{A}\right|$

15. If the axis is parallel to y-axis then then the equation of the parabola is of the form $Ax^2 + Bx + Cy + D = 0$, length of its latus rectum = $\left|\frac{C}{A}\right|$

16. Equation of tangent at (x_1, y_1) to the parabola $y^2 = 4ax$ is $yy_1 = 2a(x + x_1)$

17. Equation of normal at (x_1, y_1) to $y^2 = 4ax$ is $y - y_1 = \frac{-y_1}{2a}(x - x_1)$

18. The condition for y = mx + c to touch $y^2 = 4ax$ is $c = \frac{a}{m}$ and the point of contact is $\left(\frac{a}{m^2} \frac{2a}{m}\right)$

19. The equations $x = at^2$, $y = 2at$ are called parametric equations of the parabola $y^2 = 4ax$. The point $(at^2, 2at)$ lies on $y^2 = 4ax, \forall t \in R$

20. Equation of the chord joining $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ on $y^2 = 4ax$ is $y(t_1 + t_2) = 2x + 2at_1t_2$

21. If t_1 and t_2 are the extremities of a focal chord of a parabola, then $t_1t_2 = -1$

22. Equation of normal at 't' to the parabola $y^2 = 4ax$ is $y + xt = 2at + at^3$

23. The point of intersection of the tangent at t_1 and t_2 on the parabola $y^2 = 4ax$ is $\left(at_1t_2, a\left(t_1+t_2\right)\right)$

24. If the normal at $\left(at_1^2, 2at_1\right)$ cuts the parabola again at $\left(at_2^2, 2at_2\right)$, then $t_2 = -t_1 - \frac{2}{t_1}$

25. The tangents at the ends of a focal chord of the parabola meet on the directrix at right angles

26. If $\left(at^2, 2at\right)$ is one end of a focal chord of the parabola then the other end is $\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$

27. The length of the focal chord at t is a $\left(t+\frac{1}{t}\right)^2$

28. If PSQ is a focal chord of the parabola $y^2 = 4ax$, with focus S then $\frac{1}{SP}+\frac{1}{SQ}=\frac{1}{a}$

29. The least length of a focal chord of a parabola is its length of latus rectum

30. Length of focal chord of $y^2 = 4ax$ making an angle θ with its axis is $4a\,\mathrm{cosec}^2\theta$

31. If $\left(x_1, y_1\right)$ and $\left(x_2, y_2\right)$ are the extremities of a focal chord of the parabola $y^2 = 4ax$ then (i) $x_1x_2 = a^2$
(ii) $y_1y_2 = -4a^2$

32. Equation of chord of contact of $\left(x_1, y_1\right)$ with respect to $y^2 = 4ax$ is $yy_1 = 2a\left(x+x_1\right)$ ie. T = 0

33. If m_1, m_2 be the slopes of the two tangents drawn from (x_1, y_1) to $y^2 = 4ax$, then $m_1+m_2=\frac{y_1}{x_1}, m_1m_2=\frac{a}{x_1}$

34. Locus of point of intersection of perpendicular tangents to a parabola is its directrix

35.

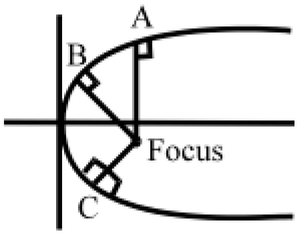
Equations of parabola	Parametric form t	Tangents at t
a) $y^2 = 4ax$	$\left(at^2, 2at\right)$	$yt = x + at^2$
b) $y^2 = -4ax$	$\left(-at^2, 2at\right)$	$yt = -x + at^2$
c) $x^2 = 4ay$	$\left(2at, at^2\right)$	$xt = y + at^2$
d) $x^2 = -4ay$	$\left(2at, -at^2\right)$	$xt = -y + at^2$

36. Normal at t

a) $y + xt = 2at + at^3$ b) $y - xt = 2at + at^3$

c) $x + yt = 2at + at^3$ d) $x - yt = 2at + at^3$

37. From a point which is inside the parabola, three normals can be drawn and their feet (points where they meet the parabola i.e. A,B,C) are called conormal points



ELLIPSE : (e < 1)

1. The general second degree equation $S = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents an ellipse if $h^2 - ab < 0$ and $\Delta \neq 0$

2. If S = 0 represents an ellipse then to find the centre of the ellipse, solve the equations $\frac{\partial s}{\partial x} = 0$ and $\frac{\partial s}{\partial y} = 0$

3. Let P be any point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and S, S¹ are foci then SP + S/P = 2a, where SP. S/P are called focal distances of P i.e., sum of the focal distances is equal to length of the major axis (2a)

4. Equation of the ellipse of the type $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} (a > b)$

5. Centre (h, k)

6. Eccentricity $e = \sqrt{\frac{a^2 - b^2}{a^2}}, (ae)^2 = a^2 - b^2$

7. foci $(h \pm ae, k)$

8. Vertices $(h \pm a, k)$

9. Length of latus rectum $\frac{2b^2}{a}$,

10. Length of major axis is 2a
11. Length of minor axis is 2b

12. Equations of directrices $x = h \pm \frac{a}{e}$

13. Feet of the directrices $\left(h \pm \frac{a}{e}, k\right)$

14. Ends of minor axis $(h, k \pm b)$

15. Equation of the ellipse of the type $\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$

16. Centre (h, k)

17. Eccentricity $e=\sqrt{\frac{a^2-b^2}{a^2}}$, Distance between foci = 2ae

18. $\text{foci}(h, k \pm ae)$, Distance between Directrix = $\frac{2a}{e}$

19. Vertices $(h, k \pm a)$

20. Length of latus rectum = $\frac{2b^2}{a}$

21. Length of major axis = 2a

22. Length of minor axis = 2b

23. Equation of the directrices, $y=k \pm \frac{a}{e}$

24. Distance between foci = 2ae

25. Distance between directrices = $\frac{2a}{e}$

26. Equation of tangent at (x_1, y_1) to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$

27. If $y=mx+c$ is a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then the conditions is $c^2=a^2m^2+b^2$ and point of contact $\left(\frac{-a^2m}{\sqrt{a^2m^2+b^2}}, \frac{b^2}{\sqrt{a^2m^2+b^2}}\right)$

28. Equation of normal at (x_1, y_1) to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$

29. The locus of the point of intersection of perpendicular tangents to the ellipse is called director circle.

30. Equation of the director circle of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $x^2+y^2=a^2+b^2$

31. The parametric equations of the ellipse $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ are $x=h+a\cos\theta, y=k+b\sin\theta$

32. Equation of chord joining the points θ_1 and θ_2 on the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ is } \frac{x}{a}\cos\frac{(\theta_1+\theta_2)}{2}+\frac{y}{b}\sin\frac{(\theta_1+\theta_2)}{2}=\cos\left(\frac{\theta_1-\theta_2}{2}\right)$$

33. Equation of the tangent at θ to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$

34. Equation of the normal at θ to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2$

35. The product of the perpendicular drawn from the foci on any tangent of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is b^2.
Where a > b

36. The tangents at the ends of the focal chord meet on the directrix

37. The auxiliary circle of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ is
$x^2+y^2=a^2$

HYPERBOLA

1. If $S=ax^2+2hxy+by^2+2gx+2fy+c=0$ represents the hyperbola then $h^2-ab>0$ and $\Delta\neq 0$

2. Centre of S = 0 in obtained by solving $\frac{\partial S}{\partial x}=0$ and $\frac{\partial S}{\partial y}=0$

		Hyperbola	Conjugate Hyperbola
3.	Equation	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$ or $\frac{y^2}{b^2}-\frac{x^2}{a^2}=-1$
4.	Eccentricity	$e=\sqrt{\frac{a^2+b^2}{a^2}}$	$e=\sqrt{\frac{b^2+a^2}{b^2}}$
		$(ae)^2=a^2+b^2$	$(be)^2=b^2+a^2$
5.	foci	$(0\pm ae,0)$	$(0,0\pm be)$
6.	vertices	$(0\pm a,0)$	$(0,0\pm b)$
7.	Length of latus rectum	$\frac{2b^2}{a}$	$\frac{2a^2}{b}$
8.	Length of transverse axis	2a	2b
9.	Length of conjugate axis	2b	2a
10.	Equation of directrix	$x=0\pm a/e$	$y=0\pm b/e$
11.	Equation of director circle	$x^2+y^2=a^2-b^2$	$x^2+y^2=b^2-a^2$
12.	Equation of auxiliary circle	$x^2+y^2=a^2$	$x^2+y^2=b^2$
13.	$(sp-s'p)$	2a	2b

14. The line y = mx + c is a tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

15. If $c^2 = a^2m^2 - b^2$ and the point of contact is $\left(\frac{\pm a^2 m}{\sqrt{a^2m^2 - b^2}}, \frac{\pm b^2}{\sqrt{a^2m^2 - b^2}}\right)$

16. The product of the perpendiculars from foci to any tangent to the hyperbola (ellipse) is b^2

17. The parametric equations of the hyperbola $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

18. are $x = h + a\sec\theta,\ y = k + b\tan\theta$

19. Equation of tangent at θ is $\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1$

20. The equation of the normal at θ is $\frac{ax}{\sec\theta} + \frac{by}{\tan\theta} = a^2 + b^2$

21. Equation of the chord joining θ_1 and θ_2 is

22. $\frac{x}{a}\cos\left(\frac{\theta_1 - \theta_2}{2}\right) - \frac{y}{b}\sin\left(\frac{\theta_1 + \theta_2}{2}\right) = \cos\left(\frac{\theta_1 + \theta_2}{2}\right)$

23. At most four normals can be drawn from any point to a hyperbola

24. The tangents at infinity to the hyperbola are called asymptotes of the hyperbola

25. The equation of the asymptotes of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

    are $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$ or $\frac{x}{a} + \frac{y}{b} = 0, \frac{x}{a} - \frac{y}{b} = 0$

26. If $e = \sqrt{2}$, then the hyperbola is called rectangular hyperbola (a = b)

27. Equation of RH is $x^2 - y^2 = a^2$ or $xy = c^2$

28. Parametric form of $xy = c^2$ are $x = ct,\ y = \frac{c}{t}$

PART I - (JEEMAIN)

SECTION - I - Straight objective type questions

PARABOLA

1. Vertex of the parabola $9x^2 - 6x + 36y + 9 = 0$ is

   (1) $(1/3, -2/9)$ (2) $(-1/3, -1/2)$ (3) $(-1/3, 1/2)$ (4) $(1/3, 1/2)$

2. The director circle of the parabola $(y-2)^2=16(x+7)$ touches the circle $(x-1)^2+(y+1)^2=r^2$, then r is equal to :

1) 10 2) 11 3) 12 4) 15

3. Consider a circle with its centre lying on the focus of the parabola $y^2=2px$ such that it touches the directrix of the parabola. Then, a point of intersection of the circle and the parabola is

(1) $\left(\frac{p}{2}, p\right)$ (2) $\left(\frac{-p}{2}, -p\right)$ (3) $\left(\frac{-p}{2}, p\right)$ (4) $\left(\frac{p}{2}, 0\right)$

4. If the line x - 1 = 0 is the directrix of the parabola $y^2-kx+8=0$, then one of the values of k is

(1) $\frac{1}{8}$ (2) 8 (3) 4 (4) $\frac{1}{4}$

5. If $L\equiv x+y-1=0$ is a line and $S\equiv y-x+x^2=0$ is a parabola, then which of the following is true

(1) L = 0 and S = 0 do not have common points

(2) L = 0 cuts S = 0 in two distinct points

(3) L = 0 touches the parabola S = 0

(4) L = 0 is the directrix of the parabola S = 0

6. The shortest distance between the line $y=x$ and the curve $y^2=x-2$ is:

(1) $\frac{7}{4\sqrt{2}}$ (2) $\frac{7}{8}$ (3) $\frac{11}{4\sqrt{2}}$ (4) 2

7. The tangent to the parabola $y^2=4x$ at the point where it intersects the circle $x^2+y^2=5$ in the first quadrant, passes through the point:

(1) $\left(-\frac{1}{3}, \frac{4}{3}\right)$ (2) $\left(-\frac{1}{4}, \frac{1}{2}\right)$ (3) $\left(\frac{3}{4}, \frac{7}{4}\right)$ (4) $\left(\frac{1}{4}, \frac{3}{4}\right)$

8. The angle between the tangents drawn from the point (1,4) to the parabola $y^2=4x$ is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

ELLIPSE

9. The equation $\frac{x^2}{2-r}+\frac{y^2}{r-5}+1=0$ represents an ellipse, if

(1) $r > 2$ (2) $2 < r < 5$ (3) $r > 5$ (4) $0 < r < 7$

10. Latus rectum of ellipse $4x^2+9y^2-8x-36y+4=0$ is

(1) 8/3 (2) 4/3 (3) $\frac{\sqrt{5}}{3}$ (4) 16/3

11. The area of the quadrilateral formed by the tangents at the end points of latus rectum to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$, is

(1) 27/4 sq. unit (2) 9 sq. unit

(3) 27/2 sq. unit (4) 27 sq. unit

12. If the distance between the foci of an ellipse is half the length of its latus rectum, then the eccentricity of the ellipse is

(1) $\frac{2\sqrt{2}-1}{2}$ (2) $\sqrt{2}-1$ (3) $\frac{1}{2}$ (4) $\frac{\sqrt{2}-1}{2}$

13. The locus of the foot of perpendicular drawn from the centre of the ellipse $x^2+3y^2=6$ on any tangent to it is

(1) $(x^2+y^2)^2=6x^2+2y^2$ (2) $(x^2+y^2)^2=6x^2-2y^2$

(3) $(x^2-y^2)^2=6x^2+2y^2$ (4) $(x^2-y^2)^2=6x^2-2y^2$

14. **Statement-I:** In an ellipse the distance between foci is always less than the sum of focal distances of any point on it

Statement-II: If 'e' be the eccentricity of the ellipse, then 0 < e < 1

(1) If both Statement-I and Statement-II are true and the reason is the correct explanation of the statement-I.

(2) If both Statement-I and Statement-II are true but reason is not the correct explanation of the statement-I.

(3) If Statement-I is true but Statement-II is false.

(4) If Statement-I is false but Statement-II is true.

15. The points (5,12) and (24,7) are the foci of an ellipse passing through the origin. Then the eccentricity of the ellipse is

(1) $\frac{\sqrt{356}}{38}$ (2) $\frac{\sqrt{286}}{38}$ (3) $\frac{\sqrt{286}}{28}$ (4) $\frac{\sqrt{386}}{35}$

HYPERBOLA

16. If e and e' are the eccentricities of the ellipse $5x^2+9y^2=45$ and the hyperbola $5x^2-4y^2=20$ respectively, then ee' =

(1) 9 (2) 4 (3) 5 (4) 6

17. The minimum length of intercept on any tangent to the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ cut by the circle $x^2+y^2=25$ is:

1) 8 2) 9 3) 2 4) 11

18. The locus of the midpoints of the chord of the circle, $x^2+y^2=25$ which is tangent to the hyperbola, $\frac{x^2}{9}-\frac{y^2}{16}=1$ is

(1) $(x^2+y^2)^2-16x^2+9y^2=0$ (2) $(x^2+y^2)^2-9x^2+144y^2=0$

(3) $(x^2+y^2)^2-9x^2-16y^2=0$ (4) $(x^2+y^2)^2-9x^2+16y^2=0$

19. Consider a hyperbola H : $x^2-2y^2=4$. Let the tangent at a point $P(4,\sqrt{6})$ meet the x-axis at Q and latus rectum at $R(x_1,y_1)$, $x_1>0$. If F is a focus of H which is nearer to the point P, then the area of ΔQFR is equal to

(1) $4\sqrt{6}$ (2) $\sqrt{6}-1$ (3) $\frac{7}{\sqrt{6}}-2$ (4) $4\sqrt{6}-1$

20. The locus of the point of intersection of two lines $\sqrt{3}x-y-4\sqrt{3}k=0$ and $k\sqrt{3}x+ky-4\sqrt{3}=0$ is a hyperbola whose eccentricity is

(1) 2 (2) $\sqrt{2}$ (3) $\frac{1}{2}$ (4) $\frac{3}{4}$

21. The equation of the sides AB and AC of the triangle ABC are $(\lambda+1)x+\lambda y=4$ and $\lambda x+(1-\lambda)y+\lambda=0$ respectively. Its Vertex A is on the y-axis and its orthocentre is (1,2). The length of the tangent from the point C to the part of the parabola $y^2=6x$ in the first quatrant is

(1) $\sqrt{6}$ (2) $2\sqrt{2}$ (3) 2 (4) 4

22. If the points of intersection of two distinct conics $x^2+y^2=4b$ and $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ lie on the curve $y^2=3x^2$, then $3\sqrt{3}$ times the area of the rectangle formed by the intersection points is equal to

(1) 432 (2) 400 (3) 632 (4) 423

23. Let e_1 be the eccentricity of the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ and e_2 be the eccentricity of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b,$ which passes through the foci of the hyperbola. If $e_1e_2=1,$ then the length of the chord of the ellipse parallel to the x-axis and passing through (0,2) is

(1) $4\sqrt{5}$ (2) $\frac{8\sqrt{5}}{3}$ (3) $\frac{10\sqrt{5}}{3}$ (4) $3\sqrt{5}$

SECTION - II

Numerical type Questions

24. The line x = 8 is the directrix of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with the corresponding focus (2,0) if the tangent to the ellipse at the point P in the first quadrant passes through the point $\left(0,4\sqrt{3}\right)$ and intersects the x-axis at Q, then $(3PQ)^2$ is equal to

25. Let H be the hyperbola, whose foci are $\left(1\pm\sqrt{2},0\right)$ and eccentricity is $\sqrt{2}$. Then the length of the latus rectum is equal to

26. If one end of a focal chord of the parabola, $y^2=16x$ is at (1,4), then the length of this focal chord is

27. The length of the latus rectum of the parabola whose focus is (3, 3) and directrix is $3x-4y-2=0$ is

28. If area of the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ inscribed in a square of side length $5\sqrt{2}$ is A, then $\frac{A}{\pi}$ equals to:

29. If the latus rectum of the hyperbola $9x^2-16y^2+72x-32y-16=0$ is $\frac{m}{n}$,

then mn equal to

30. The straight line $x+y=\sqrt{2}p$ will touch the hyperbola $4x^2-9y^2=36$, if $2p^2=k$. Find k?

PART - II (JEE ADVANCED LEVEL)

SECTION - III (Only one option corrrect type)

31. The axis of a parabola is along the line $y=x$ and the distances of its vertex and focus from origin are $\sqrt{2}$ and $2\sqrt{2}$ respectively. If vertex and focus both lie in the first quadrant, then the equation of the parabola is

A) $(x-y)^2=8(x+y-2)$ B) $(x+y)^2=2(x+y-2)$

C) $(x-y)^2=4(x+y-2)$ D) $(x+y)^2=2(x-y+2)$

32. Let $3x-y-8=0$ be the equation of tangent to a parabola at the point (7,13). If the focus of the parabola is at (–1,–1). Its directrix is

A) $x-8y+19=0$ B) $8x+y+19=0$ C) $8x-y+19=0$ D) $x+8y+19=0$

33. The locus of the point of intersection of tangents drawn at the extremities of a normal chord to the parabola $y^2=4ax$ is the curve

A) $y^2(x+2a)-4a^3=0$ B) $y^2(x+2a)+4a^3=0$

C) $y^2(x-2a)+4a^3=0$ D) $y^2(x-2a)-4a^3=0$

34. The locus of the point of intersection of the tangents to the parabola $y^2=4ax$ which include an angle α is

A) $(x+a)^2\tan^2\alpha=y^2-4ax$

B) $(x+a)^2\sec^2\alpha=y^2+(x+a)-4ax$

C) $(x+a)^2\cot^2\alpha=y^2-4ax$

D) $(x+a)^2\operatorname{cosec}^2\alpha=y^2+(x+a)-4ax$

35. Consider an ellipse with foci at (5,15) and (21,15). If the x-axis touches the ellipse, then the length of the major axis is

A)) 17 B) 34 C) 13 D) $\sqrt{416}$

36. If $\dfrac{x^2}{f(4a)}+\dfrac{y^2}{f(a^2-5)}=1$ represents an ellipse with major axis as y-axis and f is a decreasing function positive for all 'a' then a belongs to

A) $(0,6)$ B) $(-1,-1)$ C) $(-1,5)$ D) $(5,\infty)$

37. The locus of the point of intersection of the tangents at the extremities of the chord of the ellipse $x^2+2y^2=6$ which touches the ellipse $x^2+4y^2=4$ is

A) $x^2+2y^2=4$ B) $x^2+2y^2=6$ C) $x^2+y^2=9$ D) $x^2+y^2=3$

38. The chord of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, whose equation is $x\cos\alpha+y\sin\alpha=P$, subtends a right angle at it centre and it always touches a circle of radius

A) $\frac{ab}{\sqrt{b^2-a^2}}$ B) $\frac{a^2b^2}{\sqrt{b^2-a^2}}$ C) $\frac{a^2b^2}{\sqrt{b-a}}$ D) $\frac{ab}{\sqrt{b-a}}$

39. A hyperbola having transverse axis of length $2\sin\theta$ is confocal with the ellipse $3x^2+4y^2=12$. Then its equation is ....

A) $x^2\operatorname{cosec}^2\theta-y^2\sec^2\theta=1$ B) $x^2\sec^2\theta-y^2\operatorname{cosec}^2\theta=1$

C) $x^2\sin^2\theta-y^2\cos^2\theta=1$ D) $x^2\cos^2\theta-y^2\sin^2\theta=1$

40. If the normal to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at any point $P(a\sec\theta, b\tan\theta)$ meets the transverse axis in G, F is the foot of the perpendicualr drawn from centre C to the normal at P then the geometric mean of PF and PG

A) a B) b C) 2a D) 2b

SECTION - IV (More than one correct answer)

41. The equation of tangent to the parabola $y^2=8x$ which makes an angle 45^0 with the line $y=3x+5$ is

A) $2x+y+1=0$ B) $y=2x+1$

C) $x-2y+8=0$ D) $x+2y-8=0$

42. Let A and B be two distanct points on the parabola $y^2=4x$. If the axis of the parabola touches a circle of radius r and AB is its diameter, then the slope of the line AB is

A) $\frac{-1}{r}$ B) $\frac{1}{r}$ C) $\frac{2}{r}$ D) $-\frac{2}{r}$

43. The tangent PT and the normal PN to the parabola $y^2=4ax$ at a point P on it meet its axis at points T and N, respectively. The locus of the centroid of the triangle PTN is a parabola whose :

A) vertex is $\left(\frac{2a}{3},0\right)$ B) directrix is $x=0$

C) latus rectum is $\frac{2a}{3}$ D) focus is $(a,0)$

44. Two ellipse $\frac{x^2}{\cos^2\alpha}+\frac{y^2}{\sin^2\alpha}=1$ and $\frac{x^2}{\sin^2\alpha}+\frac{y^2}{\cos^2\alpha}=1\left(0<\alpha<\frac{\pi}{4}\right)$ intersect at four points P, Q, R, S then which of the following statement(s) is/are true?

A) PQRS is a square with length of the side $\sin 2\alpha$

B) PQRS lie on a circle whose centre is origin and with radius $\frac{\sin 2\alpha}{\sqrt{2}}$

C) Eccentricity of the two given ellipse are same

D) there are two points on $\frac{x^2}{\sin^2\alpha}+\frac{y^2}{\cos^2\alpha}=1$ whose reflection in $y=x$ lie on the same ellipse

45. If the equation of an ellipse is $2x^2+3y^2-8x+6y+5=0$, then which of the following is/are true?

A) Equation of auxiliary circle is $x^2+y^2-4x+2y+2=0$

B) Equation of director circle is $x^2+y^2-4x+2y=0$

C) The director circle will pass through $(4,-2)$

D) Director circle will pass through $(4,4)$

46. A tangent to the ellipse $4x^2+9y^2=36$ is cut by the tangent at the extremities of the major axis at T and T^1, the circle on TT^1 as diameter passes through the point

A) $(\sqrt{5},1)$ B) $(-\sqrt{5},0)$ C) $(\sqrt{5},0)$ D) $(3,1)$

47. Let a hyperbola passes through the focus of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$. The transverse and conjugate axes of this hyperbola coincide with the major and minor axes of the given ellipse, also the product of eccentricities of given ellipse and hyperbola is 1, then

A) the equation of hyperbola is $\frac{x^2}{9}-\frac{y^2}{16}=1$ B) the equation of hyperbola is $\frac{x^2}{9}-\frac{y^2}{25}=1$

C) focus of hyperbola is (5, 0) D) vertex of hyperbola is $(5\sqrt{3},0)$

48. On the ellipse $4x^2+9y^2=1$, the points at which the tangents are parallel to the line $9y=8x$ are

A) $\left(\frac{2}{5},\frac{1}{5}\right)$ B) $\left(\frac{-2}{5},\frac{1}{5}\right)$ C) $\left(\frac{-2}{5},-\frac{1}{5}\right)$ D) $\left(\frac{2}{5},\frac{-1}{5}\right)$

SECTION - V (Numerical Type)

49. If the line $y=mx+c$ is normal to the parabola $y^2=8x$, then $c+pm+qm^3=0$. where $\frac{p+q}{10}$ is equal to

50. The radius of circle which passes through the focus of parabola $x^2 = 4y$ and touches it at point $(6,9)$ is $k\sqrt{10}$, then $k =$

51. An ellipse whose focii are (2,4) and (14,9) and touches x-axis then its eccentricity is $\frac{13}{\sqrt{k}}$. Sum of digits of $\frac{k}{5}$ is

52. The length of the chord of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$, whose midpoint is $\left(\frac{1}{2},\frac{2}{3}\right)$ is $\frac{7}{5}\sqrt{k}$. Sum of digits of $\frac{k}{2}$ is

53. The line $2x+y=1$ is tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If this line passes through the point of intersection of the nearest directrix and the x-axis, then the eccentricity of the hyperbola is

SECTION VI - (Matrix match type)

54.

	Column-I		Column-II
A	If a mid-point of a chord of the ellipse $\frac{x^2}{16}+\frac{y^2}{25}=1$ is (0,3), then length of the chord is $\frac{4k}{5}$, then k is	p	6
B	If the line $y=x+\lambda$ touches the ellipse $9x^2+16y^2=144$, then the sum of values of is	q	8
C	If the distance between a focus and corresponding directrix of an ellipse be 8 and the eccentricity be $\frac{1}{2}$, then length of the minor axis is $\frac{k}{\sqrt{3}}$ then k is	r	0
D	Sum of distances of point on the ellipse $\frac{x^2}{9}+\frac{y^2}{16}=1$ from the focii	s	16

A) $A \to q, B \to r, C \to S, D \to q$

B) $A \to q, B \to r, C \to q, D \to s$

C) $A \to q, B \to r, C \to r, D \to q$

D) $A \to q, B \to s, C \to r, D \to q$

55.

	Column I		Column II
A	The point from which perpendicular tangents can be drawn to the parabola $y^2 = 8x$ is	p	$(-2,1)$
B	The line $x+y+3=0$ touches the parabola $y^2 = 12x$ at the point	q	$(4,6)$
C	$4x+3y=34$ is normal to the parabola $y^2 = 9x$ at the point	r	$(3,-6)$
D	The line parallel to $4y-x+3=0$ touches the parabola $y^2 = 7x$ at the point	s	$(28,14)$
		t	$(-2,5)$

A) $A \to p, B \to r, C \to q, D \to s$

B) $A \to t, B \to r, C \to q, D \to s$

C) $A \to p, t, B \to r, C \to q, D \to s$

D) $A \to p, B \to r, C \to s, D \to q$