# MECHANICAL PROPERTIES OF SOLIDS AND FLUIDS

## ELASTICITY

* **Rigid Body**

  Shape of rigid body does not change under the action of external force. Hence rigid body is a hard solid object having a definite shape and size. But in reality, a body can be elongated, compfessed and bend that means no real body is perfectly rigid.

* **Elasticity**

  The property of a body by virtue of which it gends to regain its original size and shape after the removal of applied force is called elasticity.

  $\rightarrow$ Quarts is the nearest approach to a perfectly elastic body

* **Plasticity**

  The property of a body by virtue of which it does not tends to regain its original size and shape after the removal of applied force is called plasticity.

* **STRESS**

  When a body is subjected to deforming force a restoring force is developed in the body. This restoring force is equal and opposite to applied force. The restoring force per unit area is called stress.

  If F is the force applied and A is the area of cross section of the body.

  $$\boxed{\text{Stress} = \frac{F}{A}}$$

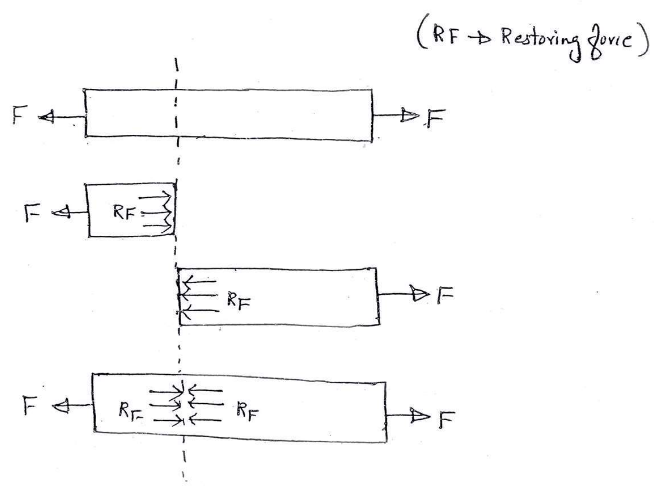
* SI unit of stress is $N/m^2$ or Pascal
* Stress = $ML^{-1}T^{-2}$
* Longitudinal / Normal stress

  When the elastic force developed is perpendicular to the surface, the stress is called longitudinal or normal stress

  Longitudinal stress → Tensile stress<br>
  Longitudinal stress → Compressive stress

* **Tensile stress**

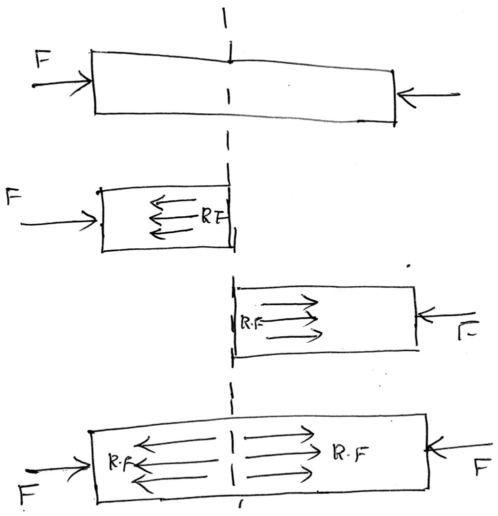
A body is stretched by equal and opposite forces applied normal to its cross sectional area. The restoring force per unit area is called tensile stress.




Tensile stress = $\frac{F}{A}$

* **Compressive stress**

If a body is compressed under the action of applied force, the restoring force per unit area is called compressive stress




Compressive stress = $\frac{F}{A}$

* **STRAIN**

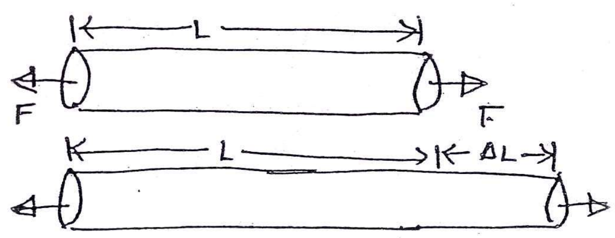
When a body is subjected to some external force, there is some change in dimension of the body. The ratio of change in dimension of the body to the original dimension is known as strain.

$$\text{Strain} = \frac{\text{change in dimension}}{\text{original dimension}}$$

$\rightarrow$ Strain is dimensionless

* Longitudinal strain = $\frac{\text{change in length}}{\text{original length}} = \frac{\Delta L}{L}$

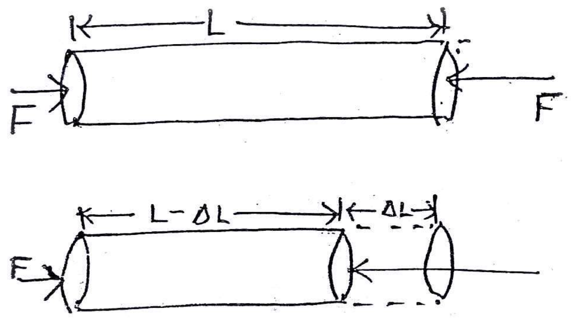
* **Tensile strain**




$$\text{Tensile strain} = \frac{\text{increase in length}}{\text{actual length}}$$

$$\text{Tensile strain} = \frac{\Delta L}{L}$$

* **Compressive strain**



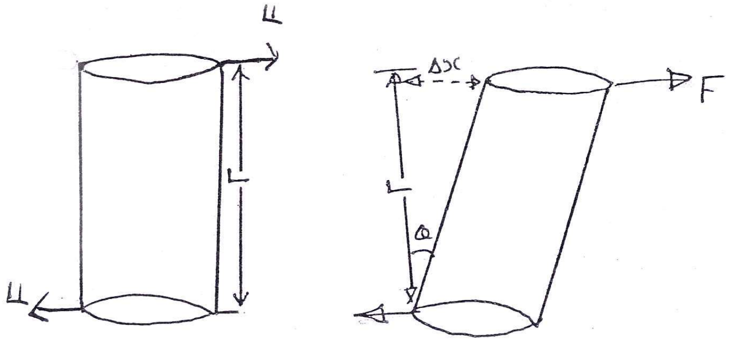

$$\text{Compressive strain} = \frac{\text{decrease in length}}{\text{actual length}}$$

$$\text{Compressive strain} = \frac{\Delta L}{L}$$

* **Shearing stress or tangential stress**

When elastic force developed is parallel (tangential) to the surface, the stress is called shearing stress or tangential stress

If two equal and opposite deforming forces are applied parallel to the cross sectional area, there is a relative displacement between the oppoiste faces. The restoring force per unit area developed due to the applied tangential force is called tangential or shearing stress.




shear stress = $\frac{F}{A}$

* **Shearing strain**

It is defined as the ratio of the relative displacement between two opposite faces to the length of the body.

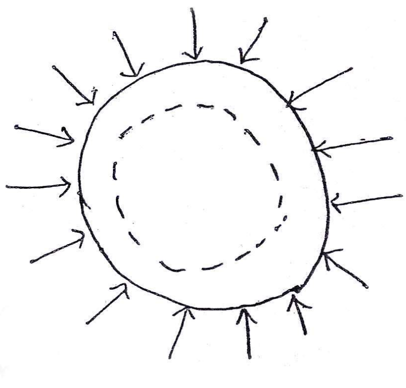
Shearing strain = $\frac{\Delta x}{L} = \tan\theta \simeq \theta$

* **Hydraulic stress [Volume stress]**

Consider a solid sphere immersed in a fluid at high pressure, the sphere is compressed by the fluid from all sides. The hydrostatic force acting at each point on the sphere is constant in magnitude and perpendicular to that point (along radial direction). Hence volume of the sphere is reduced without any change in shape. The body develops an internal restoring force that are equal and opposite to the forces applied by the fluid (The body restores its original shape and size when taken out from the fluid). The internal restoring force per unit area is called hydraulic stress.

The magnitude of hydraulic stress is equal to hydraulic pressure

* **Volume strain**



The strain produced by a hydraulic pressure is called volume strain and is defined as the ratio of change in volume to the original volume

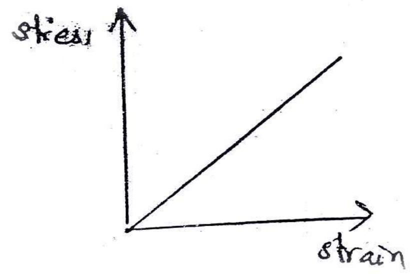
Volume strain = $\frac{\Delta V}{V}$

* **Elastic Limit**

No real body is perfectly elastic. But a body behaves like a perfectly elastic body and completely regains its original size and shape after the removal of deforming force if deforming force does not exceed a particular limit called elastic limit.

* **Hooke's Law**

For small deformation [with in proportional limit] stress is directly proportional to strain




stress $\alpha$ strain

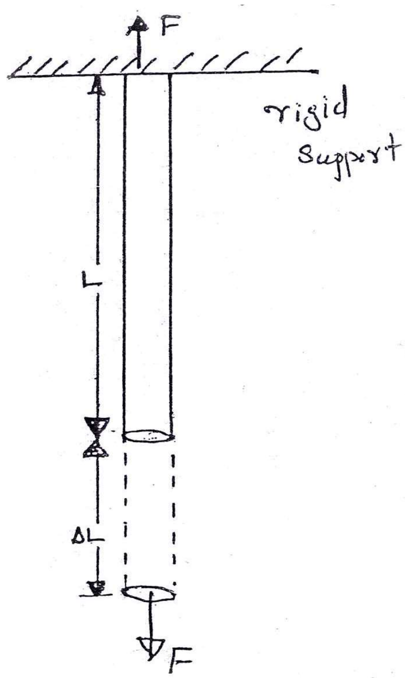
$$\frac{\text{stress}}{\text{strain}} = K$$

K is called modulus of elasticity

$\rightarrow$ Slope of stress - strain graph = K

* **Young's Modulus (Y)**

The ratio of the longitudinal stress (tensile or compressive) to the longitudinal strain is called Young's Modulus (Y)




$L \rightarrow$ Original length of the wire

$\Delta L \rightarrow$ Change in length

$A \rightarrow$ Area of cross section of the wire

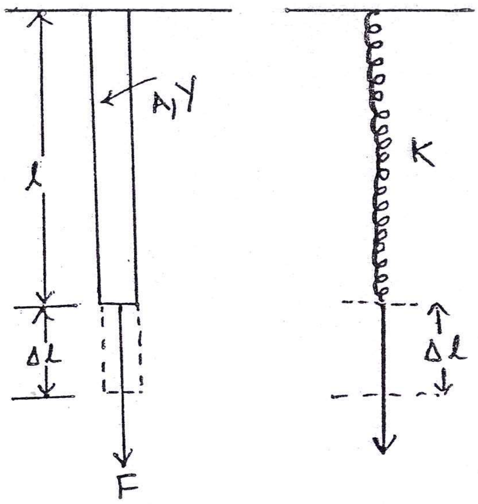
Longitudinal stress = $Y \times$ longitudinal strain

$$\boxed{\frac{F}{A} = Y\frac{\Delta L}{L}}$$

$$Y = \frac{FL}{A\Delta L}$$

* **Analogy of wire as a spring**

A thin wire can be imagined as a parallel combination of arrays of molecular springs. When we pull a wire, we really pull the spring. Let us taken a elastic wire of length $\ell$, area of cross section A and Youngs modulus Y and apply a force F. if the wire elongated under the action of force $\Delta\ell$ then




* $Y = \dfrac{\text{stress}}{\text{strain}} \Rightarrow Y = \dfrac{F\ell}{A\Delta\ell}$
* $F = \dfrac{AY}{\ell}\Delta\ell$

$\left(\dfrac{AY}{\ell}\right) = K$ (constant) depends on type of material and geometry of wire

$F = K\Delta\ell$

$F \alpha \Delta\ell$

If we compare the relation with spring force ($F_{spring}$ = kx)

* $\boxed{K = \dfrac{\Delta Y}{\ell}} \Rightarrow$ equivalent spring constant and $x = \Delta\ell$

* **Elastic potential energy stored in a wire**

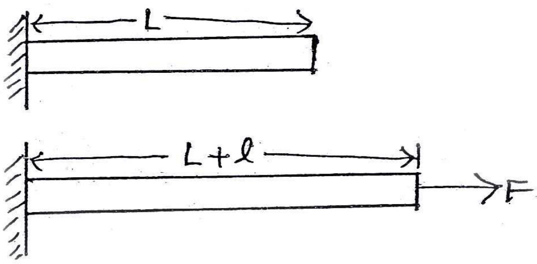
In order to deform a body, work has to be done on the body by an external agent. This work done or energy spent is stored in the body in the form of potential energy

→ The elastic potential energy stored per unit volume is called energy density

A wire of length 'L' is elongated by '$\ell$' under the action of a force 'F' as shown in fig. Let 'A' be the are of cross section and Y be the Young's modulus of material of the wire

$\dfrac{F}{A} = \dfrac{Y\ell}{L}$

$F = \dfrac{YA}{L}\ell$



* The work done by the determining force to stretch the wire through an additional amount 'd$\ell$' is given by

$$dW = F.d\ell$$

$$dW = \frac{YA\ell}{L}d\ell$$

The total work required to increase the elongation from 0 - $\ell$

$$W = \int_0^\ell dW$$

$$W = \frac{YA}{L}\int_0^x \ell d\ell$$

$$= \frac{YA}{L}\left[\frac{\ell^2}{2}\right]_0^\ell$$

$$W = \frac{YA\ell^2}{2L}$$

$$W = \frac{1}{2}\left(\frac{YA\ell}{L}\right)\ell \qquad \left\{\begin{array}{l} * \text{ Put } \frac{YA\ell}{L} = F \\ * \qquad \ell = \Delta L \end{array}\right.$$

$$W = \frac{1}{2}F(\Delta L)$$

This work done is stored as elasic potential energy

$$U = \frac{1}{2}F(\Delta L)$$

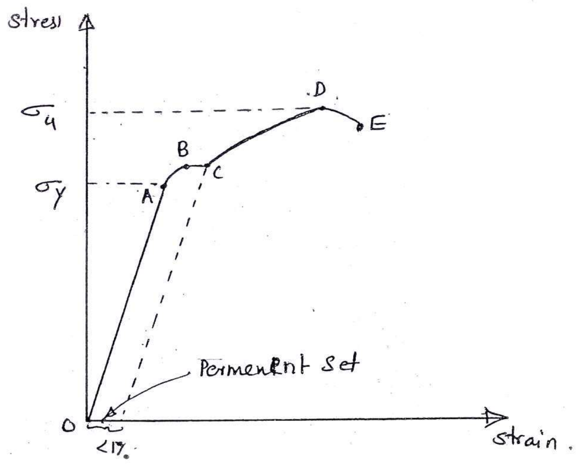
* Elastic energy density $\rightarrow$ Energy stored per unit volume

$$\frac{U}{V} = \frac{U}{AL} = \frac{1}{2}\left(\frac{F}{A}\right)\left(\frac{\Delta L}{L}\right)$$

$$\boxed{\text{Elastic energy density} = \frac{1}{2}(\text{stress})(\text{strain})}$$

$$\boxed{\text{Elastic potential energy} = \frac{1}{2}(\text{stress})(\text{strain})(\text{volume})}$$

* **Stress - Strain graph [Ductile material]**




⇒ In the region between 0 and A [Proportional limit (OA)]. The curve is linear, in this region Hookes law is obeyed. In this region the body regains its original dimension when the applied force is removed.

i.e., In this region stress is directly proportional to strain.

⇒ In the region from A to B stress and strain are not proportional. In this region the body still returns to its original dimension when the load is removed. The point B in the curve is known as yield point [also known as elastic limit] and corresponding stress is known as yield stress $(\sigma_Y)$

⇒ If the load is increased further the stress developed exceeds the yield strength and strain increases rapidly even for a small change in stress. When the load is removed at point C [lower yield point] the body does not regain its original dimension. In this case even when stress is zero, the strain is not zero. The metal is said to have permanent set. The deformation is said to be plastic deformation.

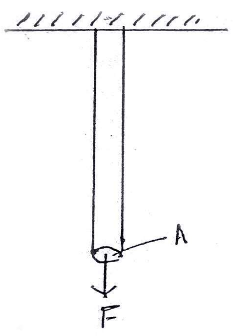
⇒ The point D on the graph is the ultimate stress point [ultimate tensile strength $(\sigma_U)$] of the material. It is the maximum strength point of the material that can handle maximum load. Beyond this point the breaking take place

⇒ The point E on the graph is the fracture or breaking point. In this point failure of the material takeplace

| |
|---|
| Note ⇒ In ductile material D and E are far apart<br>but in Brittle material D and E are very close |

* **Breaking of wire**

Breaking force depends up on the cross section of the wire



Breaking force $\alpha$ A

Breaking force = PA

P is a constant and known as Breaking stress

$\Rightarrow$ Breaking stress is a constant for given material and it does not depends upon the dimension of wire.

* **Elastic Fatigue**

The temporary loss of elastic properties because of the action of repeated alternating deforming forces is called elastic fatigue.

Eg.

1) Bridges are declared unsafe after a long time after their use

2) Spring balance show wrong readings after they have been used for a long time

3) We are able to break the wire by repeated bending

* **Shear modulus (Modulus of rigidity)** $[\eta \text{ or } G]$

The ratio of tangential stress to tangential strain is called rigidity modulus $(\eta)$

$$\boxed{\eta = \frac{F/A}{\Delta x/L} = \frac{\sigma_s}{\theta}} \quad \tau \text{ or } \sigma_s \rightarrow \text{shear stress}$$

* Solid oppose change in length, change in volume and change in shape. Thus solid posses all the three modulus of elasticity. But liquids and gases possess only volume elasticity. Gases are least elastic and solids are the most elastic while the elasticity of liquids is in between the two.

* **Young's modulus of steel are more than rubber**

If a rubber piece and steel piece having equal forces, then rubber will be elongated more than steel, that means Young's modulus of steel is more than rubber.

* $Y_s = \dfrac{FL}{A(\Delta L_s)}$     * $Y_r = \dfrac{FL}{A(\Delta Lr)}$

* **Bulk modulus (Volume elasticity)**

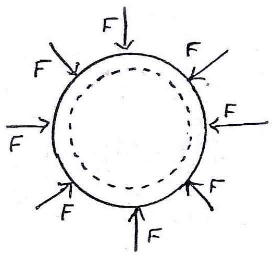
The ratio of volume stress to volume strain is called bulk modulus

Consider a spherical body which is being pressed from all sides by a uniform force F normal to its surface as shown in figure.

$V \rightarrow$ original volume of the body

$\Delta V \rightarrow$ small change in volume

$A \rightarrow$ surface area of the body



$$\frac{F}{A} = B\frac{\Delta V}{V}$$

Put $\frac{F}{A} = \Delta P \rightarrow$ change in pressure due to the action of force F

$$\boxed{\Delta P = -B\left(\frac{\Delta V}{V}\right)}$$

$$\boxed{B = \frac{-(\Delta P)V}{\Delta V}}$$

⇒ –ve sign shows that an increase in pressure causes a decrease in volume

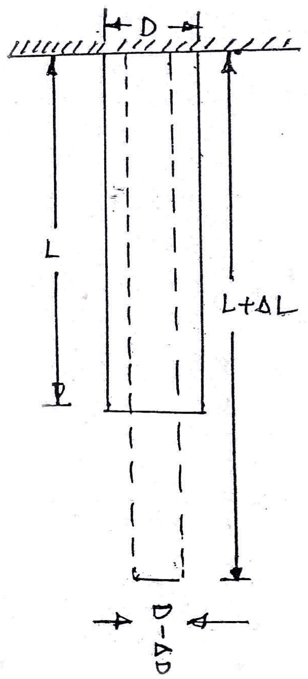
⇒ Bulk modulus of a material measures its tendency to recover its original volume, i.e. it is a measure of a compressibility of the body.

⇒ Compressibility = $\frac{1}{B} = -\frac{(\Delta V)/V}{\Delta P}$

⇒ Compressibility is the fractional decrease in volume per unit increase in pressure

* **Poisson's ratio**

If a wire is suspended from one end and loaded at the other end. Its length will increases and diameter will decreases



Longitudinal strain = $\frac{\text{change in length}}{\text{original length}} = \frac{\Delta L}{L}$

Lateral strain = $\frac{\text{change in diameter}}{\text{original diameter}} = \frac{-\Delta D}{D}$

Poissions ratio $[\sigma / \mu] = \frac{\text{lateral strain}}{\text{longitudinal strain}}$

$$\boxed{\sigma \text{ or } \mu = \frac{-\Delta D / D}{\Delta L / L} = \frac{-L(\Delta D)}{D(\Delta L)}}$$

* **Relations between Y, B, $\eta$ and $\sigma$**

* $\boxed{Y = 2\eta(1+\sigma)}$

* $\boxed{Y = 3B(1-2\sigma)}$

* $\boxed{Y = \frac{9\eta B}{3B+\eta}}$

* $\boxed{\sigma = \frac{3B-2\eta}{6B+2\eta}}$

**Application of Elasticity**

1. Metallic part of the machinery are never subjected to a stress beyond elastic limit, otherwise they will get permanently deformed
2. **Bridge and Buildings**

A bridge should be able to withstand

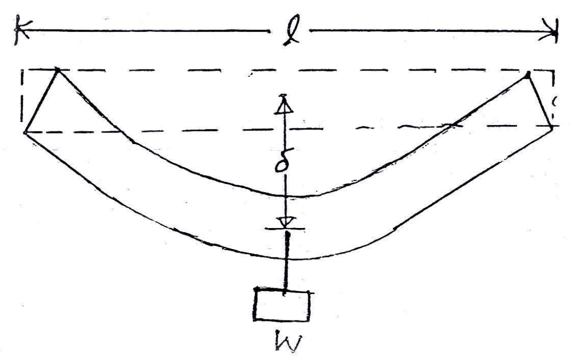
a) its own weight

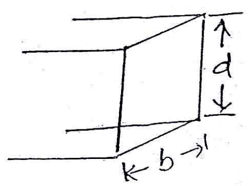
b) load of the heavy traffic

c) the force of the wind

In the design of the building beams and colums are commonly used. In all these cases the beam may bend and coloms may buckle under the load.

The depression of beam of length *l*, breadth b and thickness d, fixed at both ends and subjected to a load W hanging from its mid point.



$$\delta = \frac{W\ell^3}{4Ybd^3}$$

* For small depression

$\rightarrow$ Y should be large

$\rightarrow$ length should be small

$\rightarrow$ breadth should be large

$\rightarrow$ depth should be large

* Increasing the depth is more effective than increasing the breadth because

* $\delta \,\alpha\, \frac{1}{b}$

* $\delta \,\alpha\, \frac{1}{d^3}$

Note $\Rightarrow$ To avoid backling I sections are used for colums

3. **Designing of rope of cranes**

Cranes are used to shift heavy load from one place to another by lifting them using thick metalic rope.

Let us design a crane of maximum load 10000 Kg

The rope should be such that it does not get permanently deformed by the load.

Assuming that rop is made of steel whose elastic limit $3\times10^8\,M/m^2$.

We must ensure that the applied stress does not exceed the elastic limit of the material of the rope

Applied stress $\leq$ Elastic limit

$\frac{\text{Weight}}{\text{Area}} \leq$ Elastic limit

$\frac{Mg}{A} \leq$ Elastic limit

$$A = \frac{Mg}{\text{Elastic limit}}$$

$$A_{min} = \frac{10000\times10}{3\times10^8} = 3.3\times10^{-4}\,m^2$$

$$A_{min} = \pi r^2 = 3.3\times10^{-4}$$

$$r = \sqrt{\frac{3.3\times10^{-4}}{3.14}} \approx 10^{-2} = 1cm$$

In order to provide a safety factor of 10 the radius of the rope is kept about 3 cm $[10000\text{Kg}\times 10]$

A single wire of this radius is practically a rod which will not be flexible. Hence for flexibility the rope are always made of a large no. of thin wires braided together.

4. The maximum height of the mountain on the earth can be determined.

Let a mountain be of length h. At the bottom of the mountain the force per unit area due to the weight of the mountain will be $h\rho g$ . These shear component must be less than the elastic limit of the rock, lest the rock begins to flow.

Elastic limit of a typical rock is $3\times 10^8\,\text{N}/\text{m}^2$

density of the mountain is $3\times 10^3$

$h\rho g \le$ elastic limit

Maximum height of the mountain is

$h\rho g =$ Elastic limit

$h\times 3\times 10^3\times 10 = 3\times 10^8$

h = 10 Km

Which is more than the height of mount Everest.

## HYDROSTATICS AND HYDRODYNAMICS

* Density of a fluid = $\dfrac{\text{mass}}{\text{volume}}$ $\boxed{\rho = \dfrac{M}{V}}$

* Relative density = $\dfrac{\text{density of substance}}{\text{density of water at } 4^\circ\text{C}}$

* Relative density is also called specific gravity

Density of the mixture of liquids

* $\boxed{\text{Case I}}\Rightarrow$ Two liquids of densities $\rho_1$ and $\rho_2$ and masses $M_1$ and $M_2$ are mixed together

$$\rho_{mix} = \frac{\text{Total mass}}{\text{Total volume}} = \frac{m_1+m_2}{v_1+v_2} = \frac{m_1+m_2}{\dfrac{m_1}{\rho_1}+\dfrac{m_2}{\rho_2}}$$

If $m_1 = m_2 \Rightarrow \boxed{\rho_{mix} = \dfrac{2\rho_1\rho_2}{\rho_1+\rho_2}}$

* $\boxed{\text{Case II}}\Rightarrow$ Two liquids of densities $\rho_1$ and $\rho_2$ and volumes $v_1$ and $v_2$ are mixed together.

$$\rho_{mix} = \frac{m_1 + m_2}{v_1 + v_2} = \frac{\rho_1 v_1 + \rho_2 v_2}{v_1 + v_2}$$

If $v_1 = v_2 \Rightarrow \boxed{\rho_{max} = \frac{\rho_1 + \rho_2}{2}}$

**Effect of temperature on density**

$$dv = v\,\gamma\,d\theta$$

$$\frac{dv}{v} = \gamma d\theta$$

$$\int_{v}^{v^1} \frac{dv}{v} = \gamma \int_{\theta_1}^{\theta_2} d\theta$$

$$\log \frac{v^1}{v} = \gamma\left[\theta_2 - \theta_1\right]$$

$$\frac{v^1}{v} = e^{\gamma d\theta}$$

If $\gamma d\theta << 1$

$$e^{\gamma d\theta} = 1 + \gamma d\theta$$

$$\frac{v^1}{v} = 1 + \gamma d\theta$$

$$v^1 = v\left[1 + \gamma d\theta\right]$$

* $\rho = \frac{m}{v}$ $\qquad$ $\rho^1 = \frac{m}{v^1}$

$$\frac{m}{v^1} = \frac{m}{v\left[1+\gamma d\theta\right]} = \frac{\gamma}{\left[1+\gamma d\theta\right]}$$

$d\theta \rightarrow$ change in temperature

$$\rho^1 = \rho\left[1 + \gamma d\theta\right]^{-1}$$

if $\gamma d\theta << 1$

$$\boxed{\rho^1 = \rho\left[1 - \gamma d\theta\right]}$$

**Effect of pressure on density**

$$\boxed{\rho \alpha \frac{1}{v}}$$

* $dp = -B\left(\frac{dv}{v}\right)$

$$\frac{\rho^1}{\rho} = \frac{v}{v^1} = \frac{v}{v+dv} = \frac{v}{v-\left(\frac{dp}{B}\right)v}$$

$$dv = -\left(\frac{dp}{B}\right)V$$

$$\frac{\rho^1}{\rho} = \frac{1}{1-\frac{dp}{B}}$$

$$\boxed{\rho^1 = \frac{\rho}{1-\frac{dp}{B}}}$$

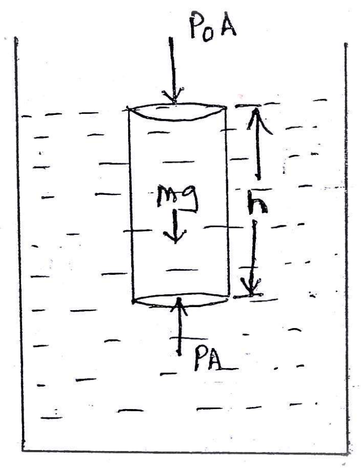
$$\rho^1 = \rho\left(1-\frac{dp}{B}\right)^{-1}$$

$$\rho^1 = \rho\left[1+\frac{dp}{B}\right]$$

**Pressure**

* SI unit of pressure is pascal [N/m$^2$] $\boxed{P = \frac{F}{A}}$
* 1 bar = $10^5$ Pa
* 1 atm = $1.013 \times 10^5 \text{Pa}$
* The excess pressure above atmspheric pressure is called gauge pressure and the total pressure is called absolute pressure
* Fluid pressure always act perpendicular to any surface in the fluid. It is a scalar

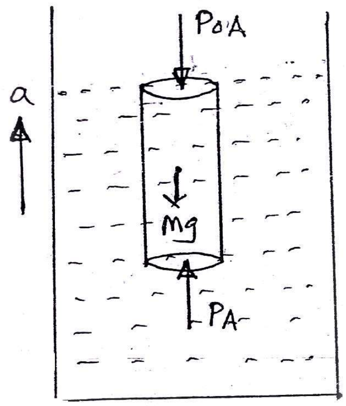
**Variation of pressure with depth in a non accelerating fluid**




$P_0 \rightarrow$ atmospheric pressure
$P \rightarrow$ Pressure at a depth h
Consider the equilibrium of liquid column of height h

$$P_0A + mg = PA$$
$$P_0A + hA\rho g = PA$$
$$P = P_0 + h\rho g$$
$$\boxed{P - P_0 = h\rho g}$$

**Pressure inside a vertically accelerating liquid**



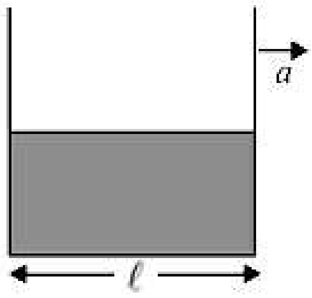

$$a = \frac{F}{m}$$
$$a = \frac{P_A - P_0A - mg}{m}$$
$$ma = P_A - P_0A - h\rho gA$$
$$P = P_0 + h\rho[g + a]$$
$$\boxed{P - P_0 = h\rho[g + a]}$$

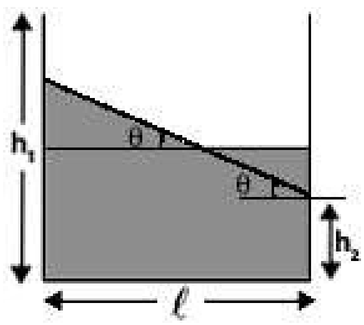
* If liquid moves down with an acceleration a then

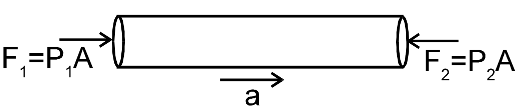
$P = P_0 + h\rho[g - a]$ $\boxed{P - P_0 = h\rho[g - a]}$

**Pressure inside a horizontally accelerating liquid**

* Free surface of a liquid accelerated in a beaker which is accelerating horizontally with an acceleration a then $\tan\theta = \frac{a}{g}$



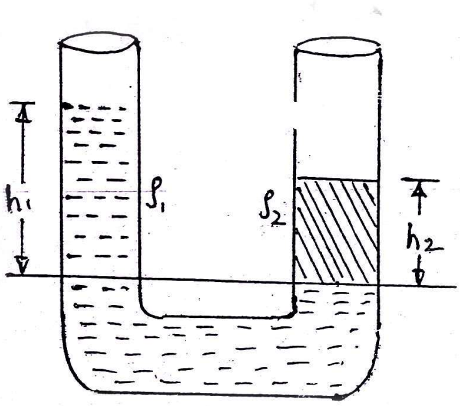




$$\tan\theta = \frac{h_1 - h_2}{\ell}$$

$$(P_1 - P_2)A = ma$$
$$(P_1 - P_2)A = A\ell\rho a$$
$$(P_1 - P_2) = \ell\rho a$$
$$P_1 = P_2 + \ell\rho a$$

$$P_1 - P_2 = \rho a\ell$$
$$P_1 - P_2 = (h_1 - h_2)\rho g$$
$$\rho a\ell = (h_1 - h_2)\rho g$$
$$\frac{h_1 - h_2}{\ell} = \frac{a}{g}$$
$$\boxed{\tan\theta = \frac{a}{g}}$$

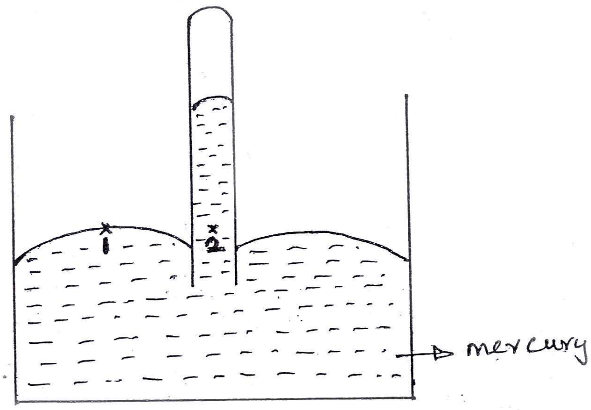
* Pressure force always push, it cannot pull
* In the same non accelerating liquid pressure will be same at all points at the same lvel




$$P_0 + h_1\rho_1 g = P_0 + h_2\rho_2 g$$
$$h_1\rho_1 = h_2\rho_2$$
$$\rho_2 > \rho_1 \qquad {*}h_1 > h_2$$
$$\boxed{h\ \alpha\ \frac{1}{\rho}}$$

**Mercury barometer**

Mercury barometer is used to measure atmospheric pressure



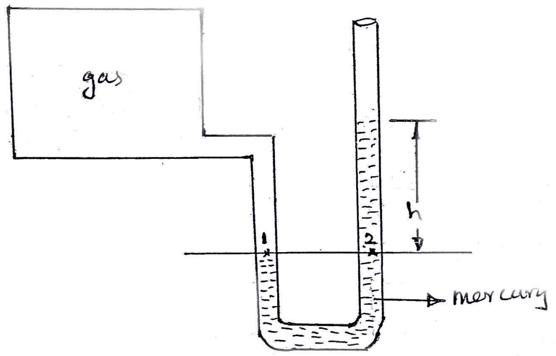

$$\rho_{Hg} = 13593\ kg/m^3$$
$$= 13.59\ g/cc$$

$P_1 = P_2$ $\quad$ $\rho \rightarrow$ density of Hg

$$\boxed{P_0 = h\rho g}$$

* **Manometer**

Manometer is used to measure the pressure of gas inside a chamber

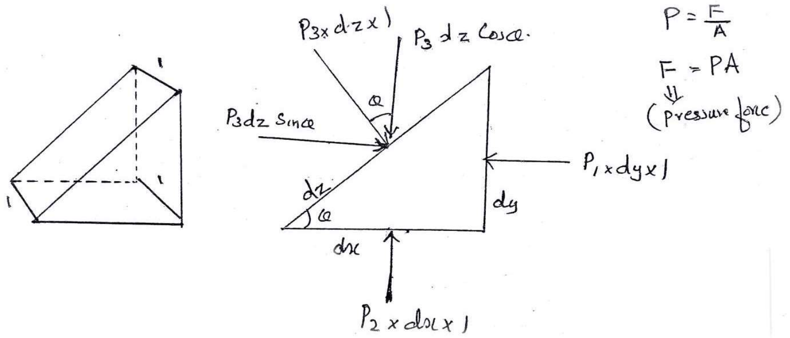



$P_1 = P_2$

$\boxed{P_{gas} = P_0 + h\rho g}$

**Pascals Law**

If state that the pressure or intensity of pressure at a point in a static fluid is equal in all direction. Consider an elementary small wedge shape fluid element at rest.




$$\boxed{\begin{aligned}\cos\theta &= \frac{dx}{dz}\\ \sin\theta &= \frac{dy}{dz}\end{aligned}}$$

* $P_3 dz\cos\theta = P_2 dx$

$P_3 dz\left(\frac{dx}{dz}\right) = P_2 dx$

$P_3 = P_2$

* $P_3 dz\sin\theta = P_1 dy$

$P_3 dz\left(\frac{dy}{dz}\right) = P_1 dy$

$P_3 = P_1$

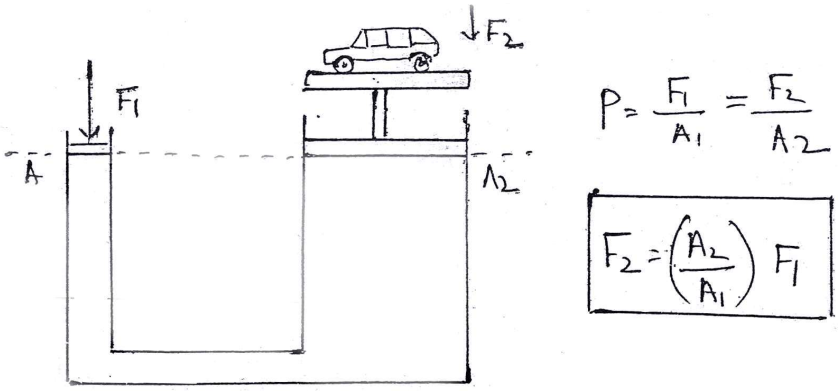
$P_1 = P_2 = P_3$

Pressure at any point is same in all direction

* **A change in pressure at any points in an enclosed incompressible fluid at rest is transmiteed its diminished to all points in the fluid**

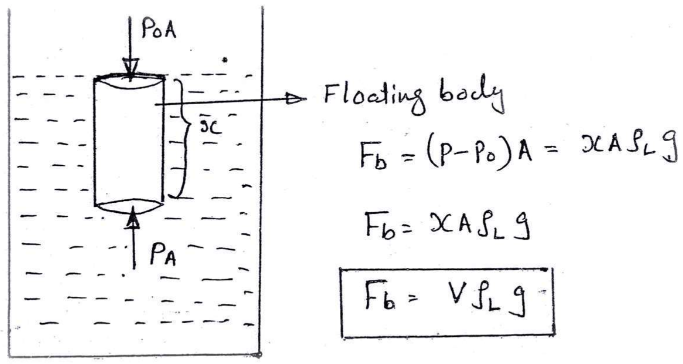
* **Hydraulic Lift**

  In hydraulic lift a heavy load can be lifted up by a small force




## ARCHIMEDES PRINCIPLE

**Buoyant Force or Upthrust:** The net hydrostatic force acting on a partially immersed or fully immersed body in a fluid is called Buoyant force. According to Archimedes this buoyant force is equal to the weight of the fluid displaced by the immersed part of the body.




$F_b \Rightarrow$ weight of liquid displaced

$\rho_L \Rightarrow$ density of liquid

$\rho \Rightarrow$ density of the material of the body

* Apparent weight of a body completely immersed in a liquid.

$$W_{app} = W_{actual} - \text{upthrust}$$

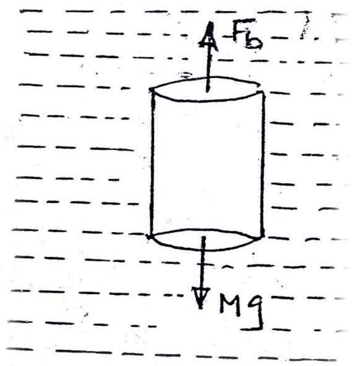
$Mg_{app} = Mg - V\rho_L g$

$Mg_{app} = Mg - \left(\frac{M}{\rho_S}\right)\rho_L g$

$$\boxed{g_{app} = g\left(1 - \frac{\rho_L}{\rho_S}\right)}$$

$\rho_L \rightarrow$ density of liquid

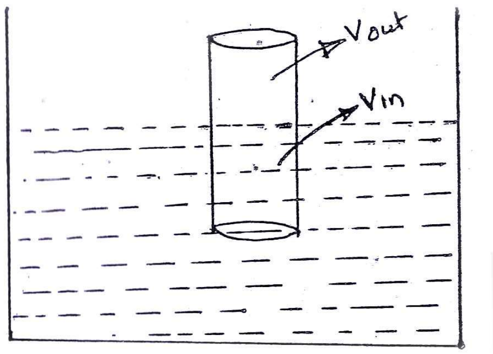
$\rho_S \rightarrow$ density of solid




- Volume of liquid displaced = Volume of body

[body is fully immersed]

* Fraction of volume of the flowing body inside and outside the liquid.




$V = V_{in} + V_{out} \Rightarrow$ Total Volume.

Mg = upthrust

$$V\rho g = V_{in}\rho_L g$$

$$\boxed{\frac{V_{in}}{V} = \frac{\rho}{\rho_L} = \frac{\text{density of body}}{\text{density liquid}}}$$

$$\frac{V - V_{out}}{V} = \frac{\rho}{\rho_L}$$

$$1 - \frac{V_{out}}{V} = \frac{\rho}{\rho_L}$$

$$\boxed{\frac{V_{out}}{V} = 1 - \frac{\rho}{\rho_L} = \frac{\rho_L - \rho}{\rho_L}}$$

* If $\rho < \rho_L$, then only fraction of body will be immersed in the liquid
* If $\rho = \rho_L$, then whole of the body will be immersed in the liquid
* If $\rho > \rho_L$, then the body will sink

**Case: 1**

* Ice floating in a liquid of density $\rho_L$
* Let V be the volume of liquid displaced by the floating ice of mass m

$$mg = V\rho_L g$$

$$\boxed{V = \frac{m}{\rho_L}}$$

¬ $\rho_L \rightarrow$ density of water

* If ice melts then $v^1$ volume of water having same mass of ice (m) will be formed

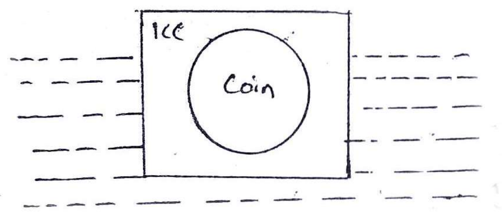
$$\boxed{v^1 = \frac{m}{\rho_w}}$$

* $\rho_w \rightarrow$ density of water

* If $\rho_L = \rho_W$, then $V = V'$ hence level of liquid will not change after the melting of ice.
* If $\rho_L > \rho_W$, then $V' > V$ hence level of liquid will increase after the melting of ice
* If $\rho_L < \rho_W$, then $V > V'$ hence level of liquid will fall after the melting of ice

**Case: 2**

* A piece of ice having a coin frozen in it is floating in water. Let V be the volume of water displaced by the floating ice, $m_1$ be the mass of ice and $m_2$ be the mass of coin. $\rho_W$ be the density of water.




$$(m_1 + m_2)g = V\rho_W g$$

$$V = \frac{m_1}{\rho_W} + \frac{m_2}{\rho_W}$$

* If ice melts then coin will sink. Volume of water displaced by the sinking coin = volume of coin $=\frac{m_2}{\rho_c}$

  $\rho_c$ is the density of coin.

* Volume of water formed due to the melting of ice $=\frac{m_1}{\rho_W}$

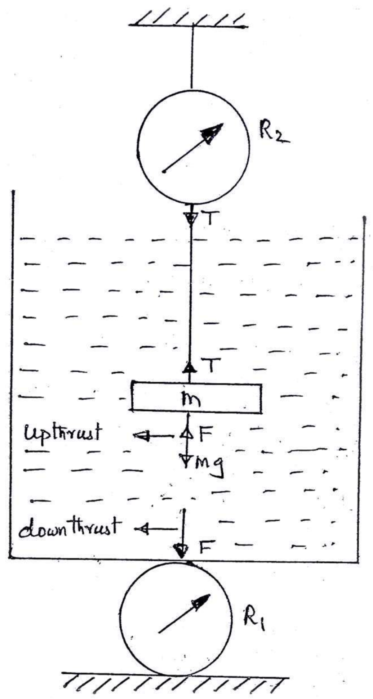
  Total volume $[V'] = \frac{m_1}{\rho_W} + \frac{m_2}{\rho_c}$

* $V' < V$ hence level will fall after the melting of ice.
* If an already floating body sinks, they level will fall.

**Case: 3**

* Instead of coin a cork is get embedded in the float ice then level of water will not change after the melting of ice because cork will again float after the melting of ice.

**Case: 4**




$m \rightarrow$ mass of hanging body

$M \rightarrow$ mass of (beaker + water)system

Consider the equilibrium of hanging body

T + F = Mg

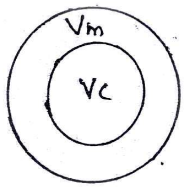
* $F \rightarrow$ weight of the water displaced by the hanging body

* Reading of hanging balance $(R_2) = T_2 = mg - F$

* Reading of platform balance $(R_1) = F + Mg$

$$\boxed{\text{upthrust acting on a body floating in a liquid} = \text{Loss of weight of body when floating in that liquid}}$$

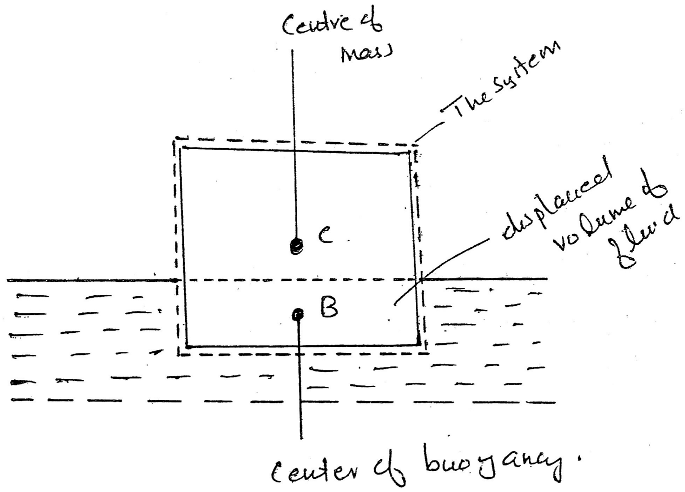
**Case: 5**




* measurement of volume of cavity inside an object
* $V_m \rightarrow$ Volume of material
* $V_c \rightarrow$ Volume of cavity
* $\rho \rightarrow$ density of material of body
* weight in air $= V_m \rho g$ ....(1)
* Loss of weight when body is completely immersed in water $= (V_m + V_c)\rho_W g$ ....(2)
* Solve (1) and (2) and find $V_c$

**Center of Buoyancy**

* In steady condition the weight of the system (floating body) acts at the centre of mass of the system. The position of centre of mass depends on the mass distribution of the system.
* The buoyant force acts at the position of the centre of mass of the fluid displaced, a point known as centre of buoyancy.

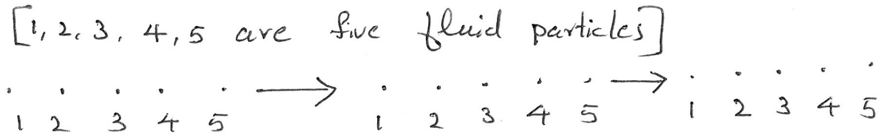



* When the system at rest is in equilibrium, the weight and buoyant force are collinear, ie, their line of action is same. Net torque in this system is zero, so no tilt occurs.

## Streamline flow and Turbulent flow

### Stream Line Flow

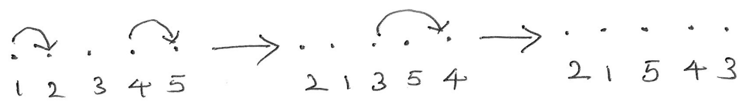
[1,2,3,4,5 are five fluid particles]




Order is same. Velocity is same.

each and every particle have same velocity then does not pressing the walls of the container.

### Turbulent flow




Overtaking take place in between particles. It is pressing the walls of container. [They are pushing the walls and pushing the ground] particle having different velocities and different order.

### Reyonld's no (Re)

According to Reynold, the critical velocity ($V_c$) of liquid flowing through a long narrow tube

$$V_C \alpha \eta$$

$$V_C \alpha \frac{1}{\rho}$$

$$V_C \alpha \frac{1}{D}$$

$\eta \rightarrow$ coefficient of viscosity

$\rho \rightarrow$ density of liquid

$D \rightarrow$ diameter of the tube.

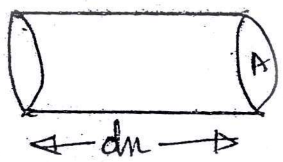
$$V_C \alpha \frac{\eta}{\rho D}; \qquad V_C \alpha \frac{R_e \eta}{\rho D}$$

$$\boxed{R_e = \frac{V_C \rho D}{\eta}}$$

* If the Reynolds number is less than 1000 the flow is laminar
* If the Reynold number is more than 2000 the flow is Turbulent
* If the Reynold number less between 1000 and 2000, the flow may be laminar or turbulent

## Equation of Continuity

At steady state rate of flow (volume of liquid flowing per sec) become constant.



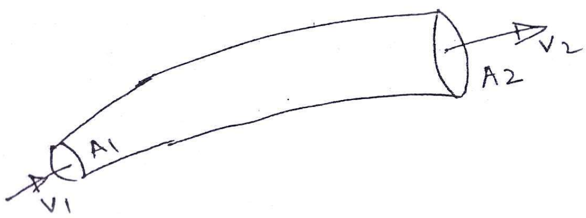

$$dV = A\,dx$$

$$\frac{dV}{dt} = \frac{A\,dx}{dt} = \text{const}$$

$$\frac{dx}{dt} = v \rightarrow \text{velocity}$$

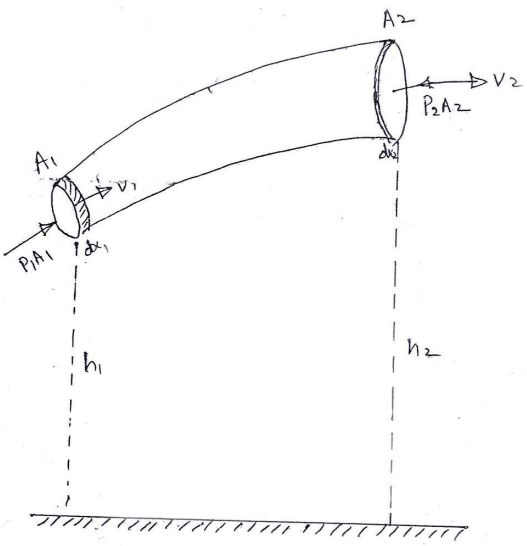
$$\boxed{Av = \text{const}}$$

* For an incompressible ideal flow.




$$\boxed{A_1V_1 = A_2V_2 = \text{constant}}$$

* **Bernoullis equation**



$$A_1\,dx_1 = A_2\,dx_2$$

$$dm_1 = dm_2 = dm = \rho A_1 dx_1 = \rho A_2 dx_2$$

W all forces = change in KE

$$\left[P_1A_1dx_1 - P_2A_2dx_2\right] - (dm)g\left(h_2 - h_1\right)\}$$

$$= \frac{1}{2}dm\left(V_2^2 - V_1^2\right)$$

work done by gravitational force (-ve)

$$\left(P_1A_1dx_1 - P_2A_1dx_1\right) - \rho A_1dx_1g\left(h_2 - h_1\right) = \frac{1}{2}\rho A_1dx_1\left(V_2^2 - V_1^2\right)$$

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

$\boxed{P + \frac{1}{2}\rho V^2 + \rho g h = \text{const}}$ $P \rightarrow$ pressure energy per unit volume

$\frac{1}{2}\rho V^2 \rightarrow \text{KE}$ per unit volume

$\boxed{\frac{P}{\rho g} + \frac{V_2}{2g} + h = \text{const}}$ $\rho g h \rightarrow \text{PE}$ per unit volume

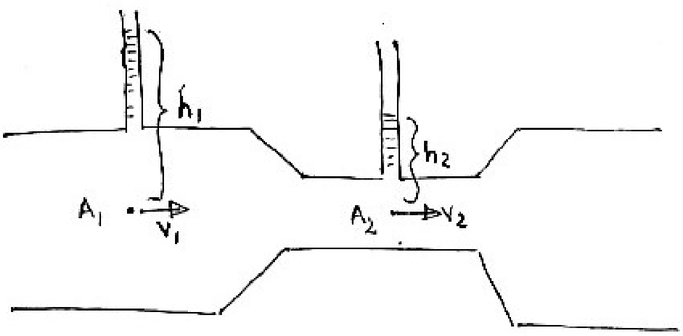
$\boxed{\frac{P}{\rho g} \rightarrow \text{Pressure head}}$

$\boxed{h \rightarrow \text{gravity head}}$

$\boxed{\frac{V^2}{2g} \rightarrow \text{velocity head}}$

* **Venturimeter**

It is used to measure flow speed in a pipe.



$$A_1V_1 = A_2V_1 \qquad\qquad P_1 = P_0 + h_1\rho g$$

$$V_2 = \frac{A_1}{A_2}V_1 \qquad\qquad P_2 = P_0 + h_2\rho g$$

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$P_0 + h_1\rho g + \frac{1}{2}\rho V_1^2 = P_0 + h_2\rho g + \frac{1}{2}\rho V_2^2$$

$$h_1 g + \frac{1}{2}V_1^2 = h_2\rho + \frac{1}{2}\left(\frac{A_1}{A_2}\right)^2 V_1^2$$

$$2h_1 g + V_1^2 = 2h_2 g + \left(\frac{A_1}{A_2}\right)^2 V_1^2$$

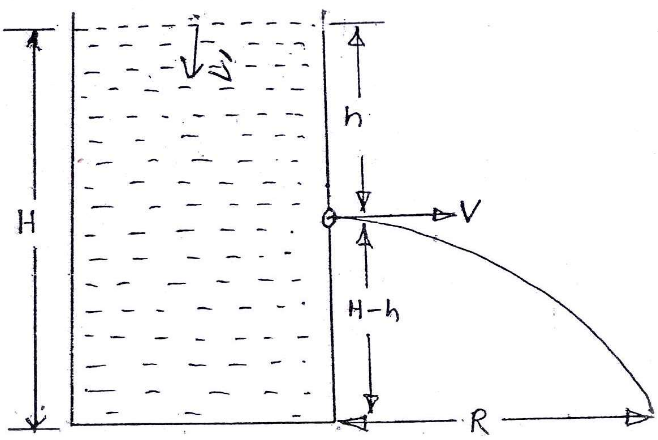
$$2g\left(h_1 - h_2\right) = \left(\frac{A_1}{A_2}\right)^2 V_1^2 - V_1^2$$

$$2g\left(h_1 - h_2\right) = \left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]V_1^2$$

$$V_1^2 = \frac{2g\left(h_1 - h_2\right)}{\left(\frac{A_1}{A_2}\right)^2 - 1}$$

$$V_1 = \sqrt{\frac{2g\left(h_1 - h_2\right)}{\left(A_1/A_2\right)^2 - 1}}$$

**Velocity of efflux (Torricellis theorem)**



$A_{base}\ V' = A_{hole}\ V$

$$P_0 + H\rho g + \frac{1}{2}\rho\left(V'\right)^2 = P_0 + \left(H - h\right)\rho g + \frac{1}{2}\rho V^2$$

$$h\rho g + \frac{1}{2}\rho\left(V'\right)^2 = \frac{1}{2}\rho V^2$$

$2h\rho g + \rho\left(V'\right)^2 = \rho V^2$ Put $V' = \dfrac{A_{hole}V}{A_{base}}$

$$2hg + \left(\frac{A_{hole}V}{A_{base}}\right)^2 = V^2$$

$$2hg = V^2 - \left(\frac{A_{hole}V}{A_{base}}\right)^2$$

$$2gh = V^2\left[1 - \left(\frac{A_{hole}}{A_{base}}\right)^2\right]$$

$$V^2 = \frac{2gh}{1 - \left(\frac{A_{hole}}{A_{bse}}\right)^2}$$

$$V = \sqrt{\frac{2gh}{1 - \left(\frac{A_{hole}}{A_{base}}\right)^2}}$$

If $A_{hole} <<< A_{base}$

$$\boxed{V = \sqrt{2gh}}$$

Time taken by the water to reach the ground $t = \sqrt{\dfrac{2\left(H - h\right)}{g}}$

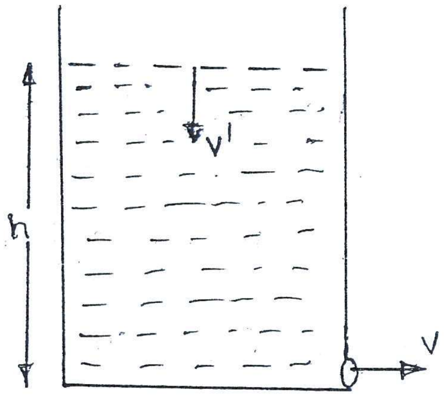
Range $= V \times t$

$$= \sqrt{2gh} \times \sqrt{\frac{2\left(H - h\right)}{g}}$$

Range $= 2\sqrt{h(H-h)}$

* For maximum range $h = H - h = \frac{H}{2}$

* **Time taken to empty a tank**




$*A_{base} = A \qquad * A_{hole} = a$

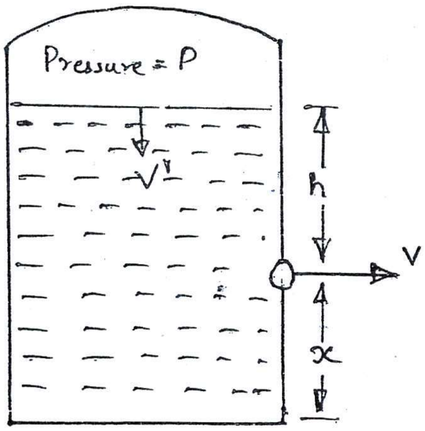
$AV' = av$

$-A\frac{dh}{dt} = a\sqrt{2gh}$

$\int_0^t dt = \frac{-A}{a}\int_H^0 \frac{dh}{\sqrt{2gh}}$

$\boxed{t = \frac{A}{a}\sqrt{\frac{2H}{g}}}$

* **Velocity of efflux from a container whose Tap is closed**




$A_{hole} \times v = A_{base} \times v' \quad * v' = \frac{A_{hole} \times v}{A_{base}}$

$A_{hole} <<< A_{base} \times v' = 0$

$P + \rho g(h + x) = P_0 + \rho g x + \frac{1}{2}\rho v^2$

$(P - P_0) + \rho g h = \frac{1}{2}\rho v^2$

$\boxed{v = \sqrt{\frac{2(P - P_0)}{\rho} + 2gh}}$

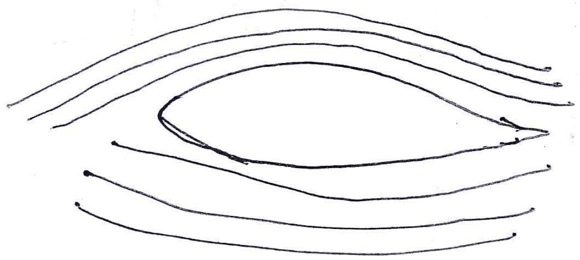
When top of the tank is open to atmosphere then $P = P_0$

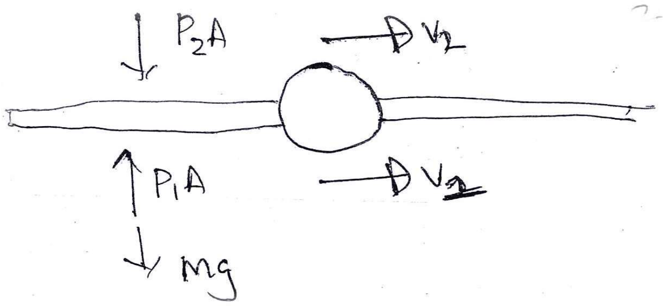
$v = \sqrt{2gh}$

If $\frac{2(P-P_0)}{\rho} >>> 2gh$ then $\boxed{v = \sqrt{\frac{2(P-P_0)}{\rho}}}$

## Lifting force on an aeroplane wing

The upper surfaced aeroplane wing is more curved them lower surface and its head is more thicker than tail. When aeroplane moves forward the air blown in the form of streamlines over the wings as shown in figure. The velocity of ai`r flow near the upper surface is more than that near the lower surface because upper surface is more curved than lower surface. Hence air pressure below the wings is more than air pressure above it hence aeroplane experiences a net force in upward direction.

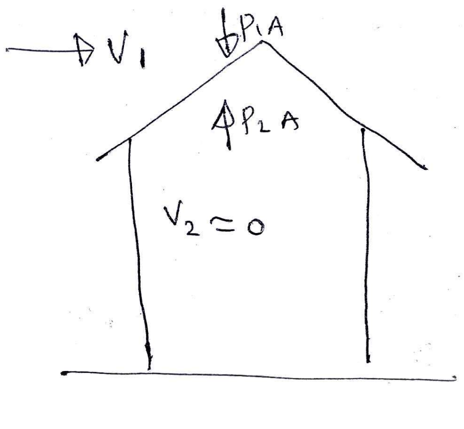





$$V_2 > V_1 \; ; \; \frac{1}{2}\rho V_1^2 + P_1 = \frac{1}{2}\rho V_2^2 + P_2 \; ; \quad P_1 > P_2$$

## Blowing off the roofs during a storm

During a high wind the roofs of the huts are generally blown off with out causing any damage to the walls of the hut. Wind flows with high speed near the top of the roof hence pressure below the roof much more than pressure above it. Hence roof experiences of heat force on upward direction.



$$\frac{1}{2}\rho V_1^2 + P_1 = \frac{1}{2}\rho V_2^2 + P_2$$

$$V_1 >>> V_2$$

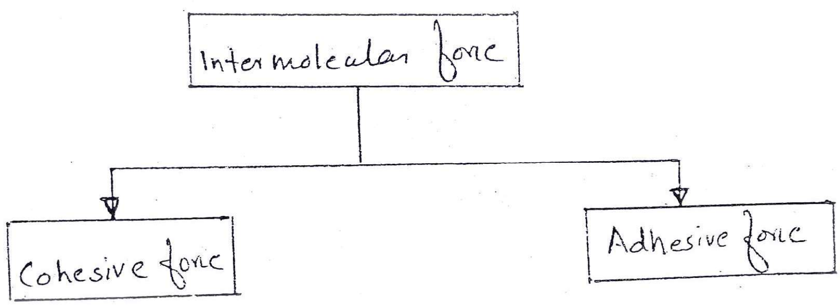
$$P_2 >>> P_1$$

## SURFACE TENSION & VISCOSITY

### Intermolecular Force

The force of attraction or repulsion acting between the molecules are known as intermolecular force. The nature of intermolecular force is electromagnetic.

The intermolecular forces of attraction may be classified into two types




### Cohesive force

The force of attraction between molecules of same substance is called the force of cohesion. This force is lesser in liquids and least in gas.

**Examples**

i) Two drop of a liquid coalesce into one when brought in mutual contact.

ii) It is difficult to separate two sticky plates of glass welded with water.

iii) It is difficult to break a drop of mercury into small droplets because of large cohesive force between the mercury molecules.

### Adhesive force

The force of attraction between the molecules of the different substances is called the force of adhesion.

**Examples**

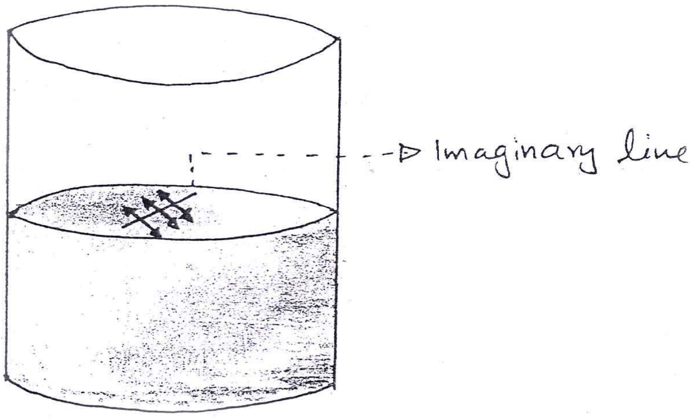
i) Adhesive force enables us to write on the blackboard will chalk

ii) A piece of paper sticks to another due to large force adhesion between the paper and gum molecules.

### Surface Tension

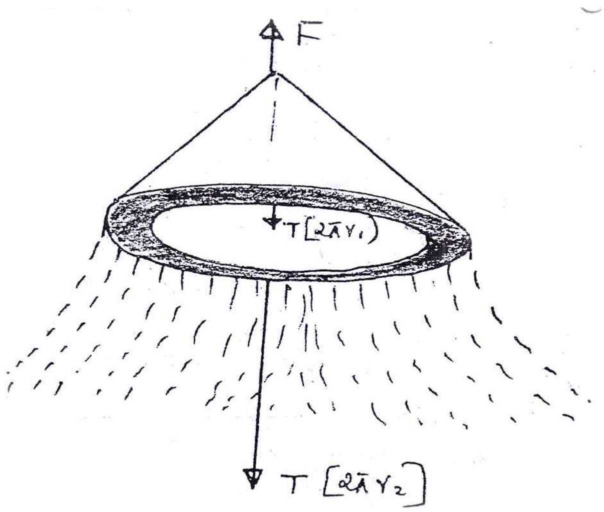
It is the properties of liquid at rest by virtue of which its free surface behaves like a stretched elastic membrane under tension and tries to occupied as small area as possible. If we consider an imagine line of length L on the free surface of the liquid, the liquid molecule on one side of the line will pull the liquid molecule on the other side. This pulling force per unit length of the line is called surface tension.

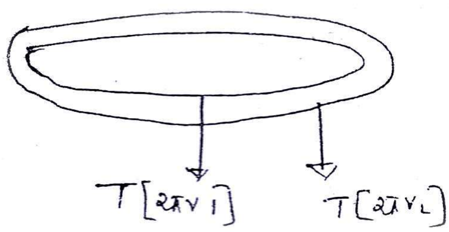
Since this imaginary line can be drawn any where on the free surface of the liquid, the surface tension has no intrinsic direction of its own. Hence surface tension is a scalar quantity.




$$\boxed{T = \frac{F}{L}}$$

* SI unit of surface tension is N/m (SI) and dyne/cm [CGS]
* SI unit of surface tension is same as that of spring constant. $\frac{\text{Force}(F)}{\text{Length}(L)} = \frac{MLT^{-2}}{L} = MT^{-2}$
* Dimension: ($MT^{-2}$)
* The minimum force required to take an annular disc of inner radii $r_1$ and outer radii $r_2$ from the surface of a liquid is



* $T[2\pi r_1] \rightarrow$ surface tension force on inner perimeter

* $T[2\pi r_2] \rightarrow$ surface tension force on outer perimeter

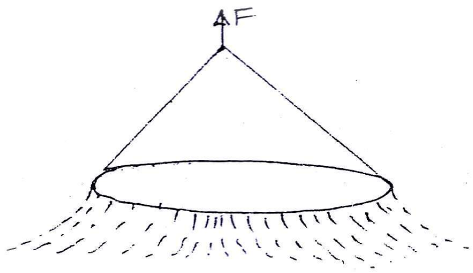
* $W = mg \rightarrow \omega t$ of the annular disc

$F = W + T[2\pi r_1 + 2\pi r_2]$

$F = W + T2\pi[r_1 + r_2]$

* **The minimum force required to pull it away from the water**

* $\dfrac{\textbf{Thin ring}}{(\text{radius r})}$

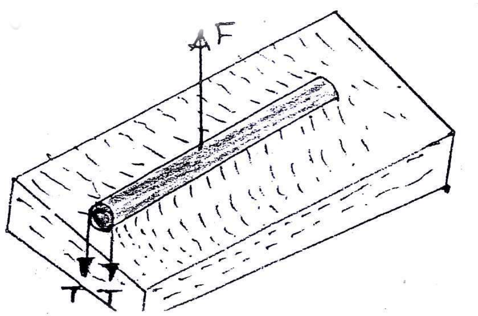



$F = W + (2\pi r + 2\pi r)T$

$F = W + 2\pi T[r + r]$

$F = W + 4\pi rT$

* **Needle of length** $\ell$




$F = 2\ell T + W$

* Surface tension arises due to the cohesive force between water molecules

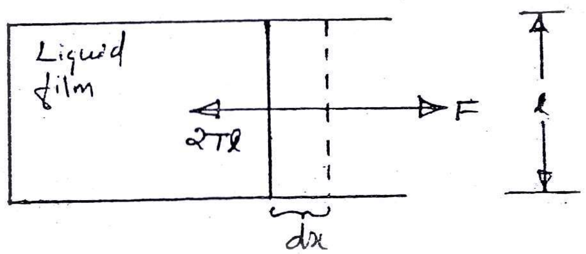
* The spiders and insects move and run along the free surface of water with out sinking because elastic membrane is formed on water due to surface tension.

* Hair of shaving brush eling together when it is removes from water due to surface tension.
* Small droplets of liquid are usually more spherical in shape than larger drops of the same liquid because force of surface tension predominates the force of gravity. [Due to surface tension free surface of liquid tries to occupy minimum surface area]
* Dancing of camphor piece over the surface of water is due to surface tension.
* Surface tension of a liquid at its boiling point become zero.
* **Surface Energy**

  To increase the area of the free surface of liquid the external agent has to perform work against the force due to surface tension. This work done is stored as in liquid molecules as surface energy.
* A liquid film is trapped between a wire frame and a movable wire of length $\ell$. The area of this film can be increased by pulling the movable wire. Since the film has two free surfaces. [both upper and lower surface of the liquid film is in contact with air]. The total surface tension force on the movable wire is $2T\ell \cdot$

  If external force F pulls the movable wire with constant speed through dx

  Then total change in area of the free surface is given by $dA = 2\ell dx$




Work required $[dW] = F\,dx = 2T\ell\,dx$

This work done is stored as surface energy.

$\boxed{dW = T\,dA}$

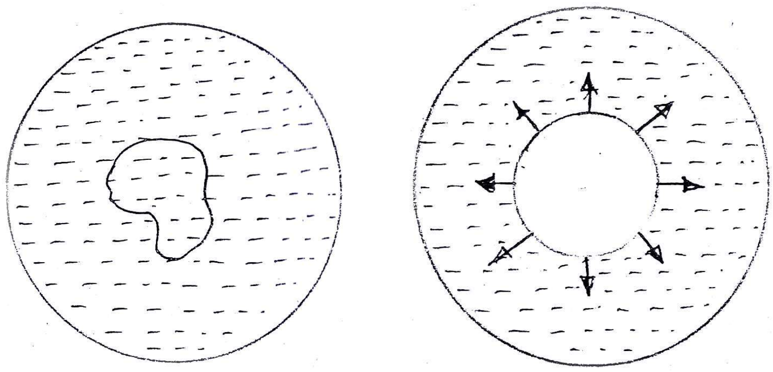
$\boxed{dV = T\,dA}$

Hence surface tension is equal to surface energy per unit surface area.

$$\boxed{T = \frac{dU}{dA}}$$

* Surface tension decreases with increase in temperature.
* Adding detergent into water surface tension decreases.
* For perfect washing water must pass through the tiny fibers of cloth. This requires increasing the surface area of the water. Which is difficult to do because of surface tension. Hence hot water and water mixed with detergent is better for washing.
* If salt is mixed with water then surface tension will increase.

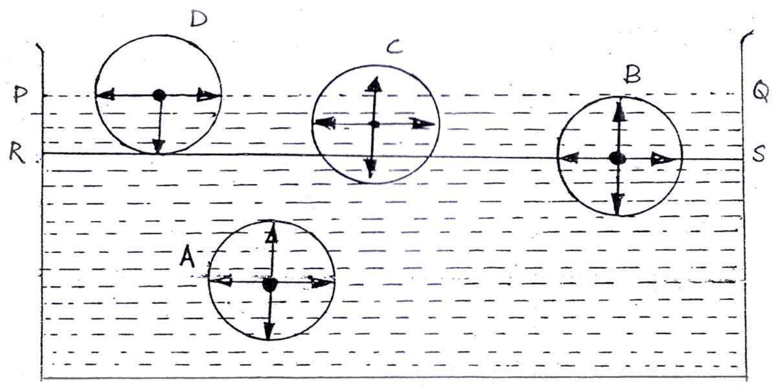
* A loop of thread is gently placed on a soap film trapping inside a ring. If a hole is pricked inside the loop, then the thread will be radially pulled by the film surface outside and it will take a circular shape.



* If phenol is mixed with water the surface tension will decrease.

**Molecular theory by surface tension**

The maximum distance upto which the force of attraction between two molecules is appreciable is called molecular range $\left(\approx 10^{-9}\,\mathrm{m}\right)$. A sphere with a molecule as centre and radius equal to molecular range is called the sphere of influence. The liquid enclosed between free surface (PQ) of the liquid and an imaginary plane (RS) at a distance r (equal to molecular range) from the free surface of the liquid form a liquid flow.




To understand the tension acting on the free surface of a liquid, let us consider four liquid molecules like A,B,C and D. Their sphere of influence are shown in figure.

1) Molecule A is well within the liquid, so it is attracted equally in all direction. Hence the net force on this molecule is zero and it moves freely inside the liquid.

2) Molecule B is little below the free surface of the liquid and it is also attracted equally in all directions. Hence the resultant on it is also zero.

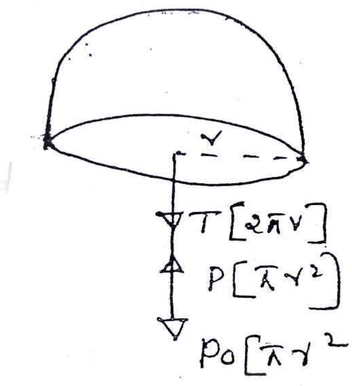
3) Molecule C is just below the upper surface of the liquid film and the part of its sphere of influence is outside the free liquid surface. So the number of molecules in the upper half (attracting the molecules upward) is less than the number of molecules in the lower half (attracting the molecules downward). Thus the molecule C experiences a net downward force.

4) Molecule D is just on the free surface of the liquid. The upper half of the sphere of influence has no liquid molecules. Hence the molecules D experiences a maximum downward force.

Thus all molecules lying in surface film experiences a net downward force. Therefore, free surface of the liquid behaves like a stretched membrane.

* **Excess pressure inside a liquid drop in air**

[Liquid drop has only one free surface only outer surface is in contact with air]




* Consider the equilibrium of upper hemispherical portion of liquid drop
* $T[2\pi r] \rightarrow$ surface tension force exerted by lower half on upper half.
* $P_0 \rightarrow$ atmospheric pressure
* $P \rightarrow$ pressure inside liquid drop

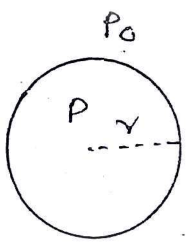
$$P\left[\pi r^2\right] = P_0\left[\pi r^2\right] + T[2\pi r]$$

$$\boxed{\text{Excess pressure}(\Delta P) = P - P_0 = \frac{2T}{r}}$$

**Excess pressure inside a soap bubble located in air**

Soap bubble in air has two free surfaces [Both inner and outer surfaces are in contact with air]

Hence surface tension force exerted by lower half on upper half is $T[4\pi r]$

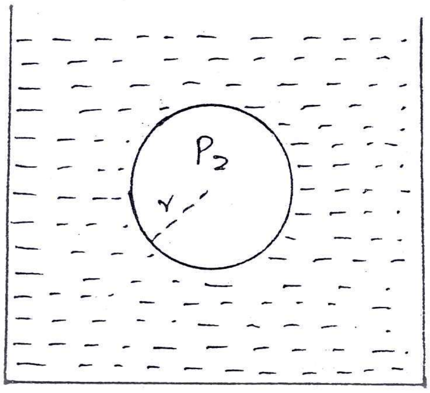



$$\boxed{\text{Excess pressure}(\Delta P) = P - P_0 = \frac{4T}{r}}$$

**Excess pressure inside an air bubble inside a liquid**

Soap bubble has one free surface [only inner portion are contact with air]

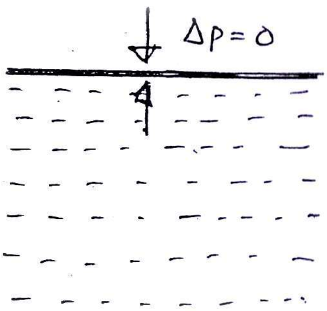
$$\boxed{\text{Excess pressure } (\Delta p) = P_2 - P_1 = \frac{2T}{r}}$$




* **Pressure on the concave side of a spherical liquid surface is always greater than the pressure on convex side.**

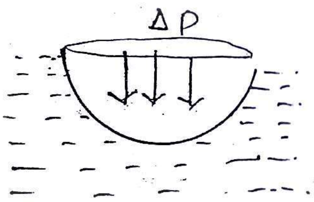
* **Excess pressure in different Cases**

Plane surface



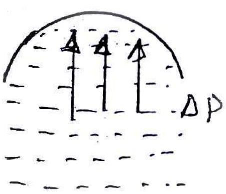

$$\boxed{\Delta p = 0}$$

Concave surface




$$\boxed{\Delta p = \frac{2T}{R}}$$

Convex surface

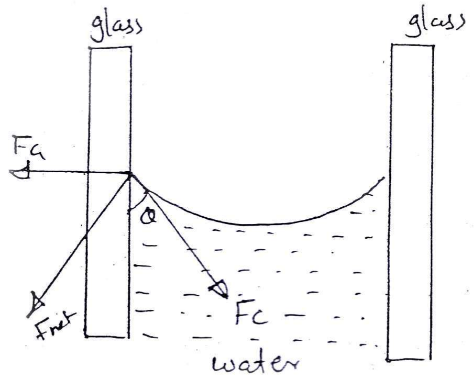



$$\boxed{\Delta p = \frac{2T}{R}}$$

**Shape of liquid Surface**

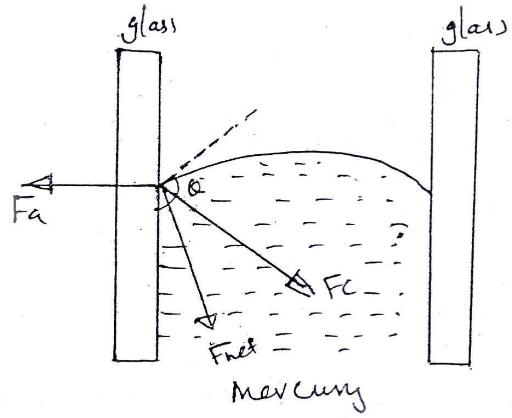
Liquid surface usually curves up or down when it meets the wall of the container. The angle at which liquid surface meets the solid surface is called angle of contact.

* When the adhesive force ($F_a$) between solid and liquid molecules is more than the cohesive for ($F_C$) betwee liquid molecules, shape of the meniscus is concave and the angle of contact is less than 90° [eg. water in a glass bottle]



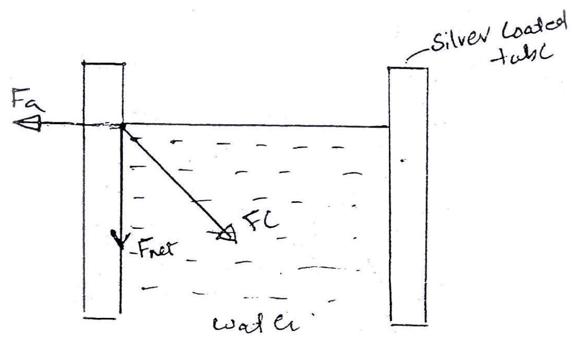

* When $F_a < F_c$ shape of meniscus is convex and angle of contact is more than 90°

Example : glass and mercury




* When shape of meniscus is plane [$F_a = F_c \cos 45$]

$F_a = \frac{F_c}{\sqrt{2}}$ and $\theta = 90$ in [silver coated tube and water] i.e. net force become vertically downwards

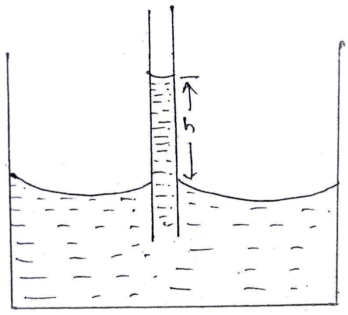



* Net force on any point on the free surface of the liquid must be perpendicular to the surface at that point

* **Capillarity**

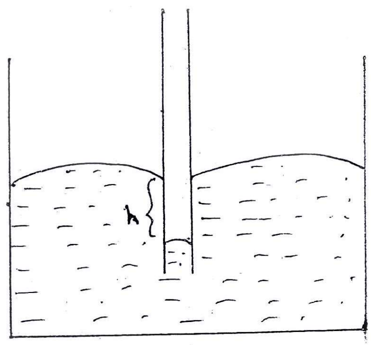
  If a narrow tube is dipped in a liquid then due to surface tension liquid in the tube will rise above [capillary rise] or fall below [capillary fall] the normal level

* **Capillary rise**



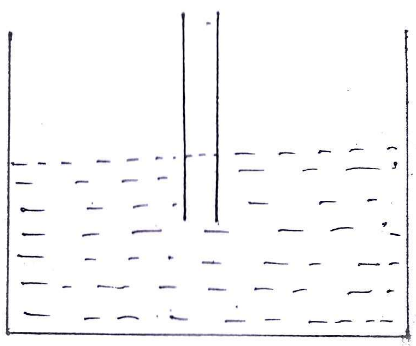

  * Water - glass tube
  * $F_a > F_C$
  * Concave meniscus

* **Capillary fall**

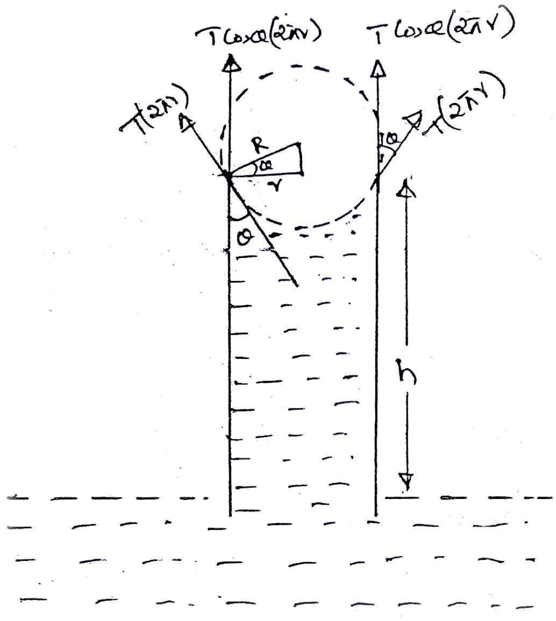


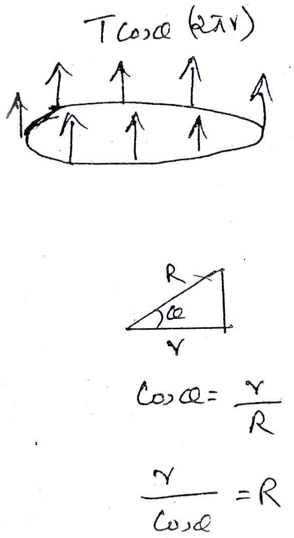

  * Mercury-glass tube
  * $F_a < F_c$
  * Convex meniscus

* In silver coated capillary tube and water system there is no capillary rise or fall



* Length of capillary rise




* $r \rightarrow$ radius of the tube
* $R \rightarrow$ radius of curvature of meniscus
* $\theta \rightarrow$ angle of contact
* Surface tension = weight of liquid
force column of height h
* $T\cos\theta(2\pi r) = (\pi r^2 h)\rho g$

$$\boxed{h = \frac{2T\cos\theta}{r\rho g}}$$

Put $\frac{r}{\cos\theta} = R$

$$\boxed{h = \frac{2T}{R\rho g}}$$

$$\boxed{hR = \frac{2T}{\rho g} = \text{cons tan t}}$$

* If length of the tube is less than $\frac{2T\cos\theta}{r\rho g}$ then liquid will rise to the top of the tube but will not flow out like fountain.

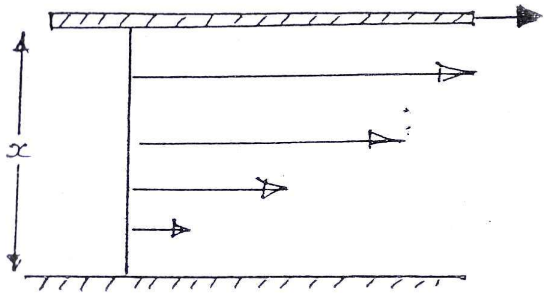
In this case free surface of the liquid at the top of the tube becomes fleet $[R \rightarrow \infty]$

$$\boxed{hR = \frac{2T}{\rho g} = \text{constant}}$$

**VISCOSITY**

It is the internal friction in a liquid, it opposes the relative motion between adjacent layers of liquid

$$\boxed{F = -\eta A \frac{dv}{dx}}$$




$\eta \rightarrow$ Coefficient of viscosity

$A \rightarrow$ Area of the layers in contact $\quad \frac{dv}{dx} \rightarrow$ Velocity of gradient

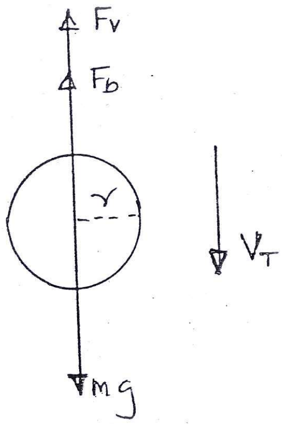
* **STOKES LAW**

Viscous force acting on a small spherical body of radius r falling through a viscous medium is given by

$\boxed{F = 6\pi\eta r V_T}$ $\quad V_T \rightarrow$ Terminal velocity

* Body falls with constant speed called terminal speed when its weight is balanced by viscous force and buoyant force. Let $\rho$ be the density of the material of the spherical drop and $\sigma$ be the density of the medium

$$mg = F_v + F_b$$
$$mg = 6\pi\eta r v_T + F_b$$
$$\frac{4}{3}\pi r^3 \rho g = 6\pi\eta r v_T + \frac{4}{3}\pi r^3 \sigma g$$
$$\frac{4}{3}\pi r^3 [\rho - \sigma] g = 6\pi\eta r v_T$$




$$\boxed{\text{Terminal velocity}\,(V_T) = \frac{2r^2(\rho - \sigma)g}{9\eta}}$$