CHAPTER - 05

PERMUTATION, COMBINATION & BINOMIAL THEOREM

JEE MAIN - SECTION I

1. 3

2. 4 Any two non-adjacent pillars are joined beams

$\therefore$ Number of beams = Number of diagonals

$={}^{20}C_2-20$

$=190-20=170$

3. 2 'EQUATION' contains 8 different letters, with consonants Q, T, N. 2 end places can be filled by Q, T, N in (${}^3P_2$) ways. Remaining 6 places can be filled by remaining 6 letters in ${}^6P_6$ i.e., 6! ways

$=({}^3P_2)\times 6!=4320$

4. 1 We have III, TT, D, S, R, B, U, O, N

Number of words with selection (a, a, a, b)

$={}^8C_1\times\frac{4!}{3!}=32$

Number of words with selection (a, a, b, b)

$=\frac{4!}{2!2!}=6$

Number of words with selection (a, a, b, c)

$={}^2C_1\times{}^8C_2\times\frac{4!}{2!}=672$

Number of words with selection (a, b, c, d)

$={}^9C_4\times 4!=3024$

$\therefore$ total $=3024+672+6+32$

$=3734$

5. 1 Selection of four number is ${}^{10}C_4$ and derangement of 6 numbers is D_6

6. 3 No. of ways of arranging 2 women in 4 chairs marked 1 to 4 = $({}^4P_2)$ or $({}^4C_2 \times \angle 2)$

Then no. of ways 3 men can be arranged in remaining 6 chairs = $({}^6P_3)$

$\therefore$ Total no. of ways of arranging women and men = $({}^4P_2 \times {}^6P_3)$

7. 2 Required no. of ways = (No. of surjections from A to B)

Where n(A) = 6, n(B) = 2 = $(2^6 - 2) = 62$

8. 1 $a, b, c \in N$ are in AP and $a + b + c = 21 \Rightarrow b = 7$

Then $a + c = 14$ for which the no. of solutions =

9. 1 $xyz = 24 = (2^3 \times 3^1) \Rightarrow (x_1 x_2 .....x_r) = (P_1^{n_1}, P_2^{n_2})$

$\Rightarrow$ Where $r = 3$, $n_1 = 3$, $n_2 = 1$

The number of positive integral solutions

$= ({}^{n_1+r-1}C_{r-1})({}^{n_2+r-1}C_{r-2}) = ({}^5C_2 \times {}^3C_2) = 30$

10. 4 001, 002, 003, ….., 999 are the numbers, where digit 3 occurs in each of unit, tens and 100's place for (10×10) times $\therefore$ Total number of times, the digit 3 will be written = 100 + 100 + 100 = 300.

11. 3 To count no. of cases for which $f(0) \le f(1) \le f(2)$

Then $f(0) = f(1) = f(2)$ or $f(0) = f(1) < f(2)$ or $f(0) < f(1) = f(2)$ or $f(0) < f$

No. of cases = $({}^8C_1 + 2X\,{}^8C_2 + {}^8C_3)$ $[\because n(B) = 8]$ = $({}^9C_2 + {}^9C_3) = {}^{10}C_3$

No. of non-decreasing function from A to B

$= ({}^{n+m-1}C_m)$ where $m = n(A)$, $n = n(B)$ $= ({}^{8+3-1}C_3) = ({}^{10}C_3)$

12. 1 If $f(a_i) \neq b_i$ for $i = 1, 2, \ldots\ldots, 5$ then the number of one-one functions is equal to the no. of derangements of 5 elements = $D_5 = 44$.

Then $f(a_1) = (b_2$ or b_3 or b_4 or $b_5) \Rightarrow f(a_1) = b_2$ s one of the 4 possible cases.

No. of mapping such that $(f(a_1) = b_2) = \frac{1}{4}(D_4) = 11$.

13. 2 No. of ways of choosing 10 questions out of 13, so that the selection contains

(i) 4 questions out of first 5 = $({}^5C_4 \times {}^8C_6) = 140$

(ii) 5 questions out of first 5 = $({}^5C_5 \times {}^8C_5) = 56$

$\therefore$ Total no. of ways = 140 + 56 = 196.

14. 1 Unit digit of (6^p) is $6 \forall p \in \{1, 2, \ldots., 50\}$

Unit digit of (9^q) is 9 or 1 (each 25 cases) for $q \in \{1, 2, \ldots., 50\}$

$(6^p + 9^q)$ is divisible by 5 if sum of unit digits of $6^p, 9^q$ is 15 (i.e., 6 + 9)

Then number of pairs $(p, q) = 50 \times 25 = 1250$

15. 4 $a = {}^{19}C_{10}$, $b = {}^{20}C_{10}$ and $c = {}^{21}C_{10}$

$\Rightarrow a = {}^{19}C_9$, $b = 2({}^{19}C_9)$ and $c = \frac{21}{11}({}^{20}C_{10})$

$\Rightarrow b = 2a$ and $c = \frac{21}{11}, b = \frac{42a}{11}$

$\Rightarrow a : b : c = a : 2a : \frac{42a}{11}$ = 11 : 22 : 42

16. 1

10 Identical	21 Distinct	10 Object
0	10	${}^{21}C_{10}\times 1$
1	9	${}^{21}C_{9}\times 1$
⋮	⋮	⋮
⋮	⋮	⋮
⋮	⋮	⋮
10	0	${}^{21}C_{0}\times 1$

$${}^{21}C_0+...+{}^{21}C_{10}+{}^{21}C_1+.....+{}^{21}C_0=2^{21}\ (C_0+....+{}^{21}C_{10}=2^{20})$$

17. 4 Let $n = 10$ & $r = 3$

$\therefore$ No. of term $= n + r - 1C_{r-1} = 12C_2 = 66$

18. 4 In a chessboard, there are 9 horizontal lines and 9 vertical lines

$\therefore$ Number of rectangles of any size are ${}^{9}C_2\times{}^{9}C_2$.

Hence, Statement-I is false and statement-II is true.

19. 1

$$(1+x)(1-x^2)\left(1+\frac{3}{x}+\frac{3}{x^2}+\frac{1}{x^3}\right)^5$$

$$=(1+x)(1-x^2)\left(\left(1+\frac{1}{x}\right)^3\right)^5$$

$$=\frac{(1+x)^2(1-x)(1+x)^{15}}{x^{15}}$$

$$=\frac{(1+x)^{17}-x(1+x)^{17}}{x^{15}}$$

$=$ coeff (x^3) in the expansion $\approx$ coeff (x^{18}) in

$(1+x)^{17}-x(1+x)^{17}$

$= 0 - 1$

$= -1$

20. 4 $(27)^{999}=(28-1)^{999}$

$= 28N - 1 \quad = -7K - 1$

$\therefore$ the remainder will be 6

21. 3 It is $t_6 = {}^{10}C_5\left(\sin^{-1}\alpha\cos^{7}\alpha\right)^5$

$$= {}^{10}C_5\left[\frac{\pi}{2}\sin^{-1}\alpha - \left(\sin^{-1}\alpha\right)^2\right]$$

$$= {}^{10}C_5\left[\frac{\pi^2}{16} - \left(\sin^{-1}\alpha - \frac{\pi}{4}\right)^2\right] \Rightarrow \text{maximum}$$

22. 3 $(1+x)^{101}(1+x^2-x)^{100} \Rightarrow (1+x)[(1+x)(1+x^2-x)^{100}]$

$\Rightarrow (1+x)[1+x^3]^{100} = (1+x^3)^{100} + x(1+x^3)^{100}$

$\Rightarrow$ 101 terms $\Rightarrow$ 101 terms $\Rightarrow$ Total =202

23. 1 Coefficient of x^4 in $\left(1+2x+\frac{3}{x^2}\right)^6 \Rightarrow \frac{6!}{p!q!r!}1^p 2x^q\left(\frac{3}{x^2}\right)^r$

$\Rightarrow p+q+r=6 \Rightarrow p=2, q=4, r=0 \Rightarrow \frac{6!}{2!4!}2^4 = 240$

24. C

25. 7. 3 $\sum_{r=0}^{10} C_r \cdot \frac{2^{r+1}}{r+1} = \frac{1}{n+1}\sum_{r=0}^{10} {}^{n+1}C_{r+1}2^{r+1} = \frac{1}{n+1}[{}^{n+1}C_0 + {}^{n+1}C_1 + \ldots + {}^{n+1}C_n]$

$\Rightarrow \frac{2^{11}-1}{11}$

26. 1 $(5+x)^{500} + x(5+x)^{499} + x^2(5+x)^{498} + \ldots + x^{500}$

$$= \frac{(5+x)^{501} - x^{501}}{(5+x) - x} = \frac{(5+x)^{501} - x^{501}}{5}$$

$\Rightarrow$ coefficient x^{101} in given expression

$$= \frac{{}^{501}C_{101}5^{400}}{5} = {}^{501}C_{101}5^{399}$$

27. 10 $T_6 = 8C_5\left(\frac{1}{x^{8/3}}\right)^3\left(x^2\log_{10}x\right)^5 = 5600$

$$\Rightarrow \frac{8\times7\times6}{3\times2\times1}\times\frac{1}{x^8}.x^{10}.(\log_{10}x)^5 = 5600$$

$$x^2(\log_{10}x)^5 = 100;\Rightarrow x^2(\log_{10}x)^5 = 10^2(\log_{10}10)^5 \Rightarrow x = 10$$

28. 4

$$200 = {}^6C_3\left(x^{\frac{1}{x+\log_{10}x}}\right)^{3/2}\times x^{1/4}$$

$$\Rightarrow 10 = x^{\frac{3}{2(1+\log_{10}x)}+\frac{1}{4}}$$

$$\Rightarrow 1 = \left(\frac{3}{2(1+t)}+\frac{1}{4}\right)t \text{, where } t = \log_{10}x$$

$$\Rightarrow t^2+3t-4 = 0 \Rightarrow t = 1,-4$$

$$\Rightarrow x = 10, 10^{-4} \Rightarrow x = 10 \text{ (as } x>1)$$

SECTION II (NUMERICAL)

29. 1120

n(B) = 10

n(a) = 5

The number of ways of forming a group of 3 girls of 3 boys.

$= {}^{10}C_3 \times {}^5C_3$

$= \frac{10\times9\times8}{3\times2}\times\frac{5\times4}{2} = 1200$

The number of ways when two particular boys B_1 of B_2 be the member of group together

$= {}^8C_1 \times {}^5C_3 = 8 \times 10 = 80$

Number of ways when boys B_1 of B_2 hot in the same group together

$= 1200 \times 80 = 1120$

30. 327

First arrange in alphabetical order

i.e. ADMNOY

A _ _ _ _ _ = 5!

D _ _ _ _ _ = 5!

$\boxed{M}$ A _ _ _ _ = 4!

$\boxed{M}$ D _ _ _ _ = 4!

$\boxed{M}$ N _ _ _ _ = 4!

$\boxed{M}$ $\boxed{O}$ A _ _ _ = 3!

$\boxed{M}$ $\boxed{O}$ D _ _ _ = 3!

$\boxed{M}$ $\boxed{O}$ $\boxed{N}$ A _ _ = 2!

$\boxed{M}$ $\boxed{O}$ $\boxed{N}$ $\boxed{D}$ $\boxed{A}$ $\boxed{Y}$ = 1

= 327

31. 1 Each of 10!, 11!, 12!, ….., 49! is a multiple of 100 and hence tens digit of each of them is zero.
$\therefore$ required = tens digit of $(1!+2!+3!+...+49!)$
= tens digit of (1+ 2+ 6 + 24 + 120 + 720 + 5040 + 40320 + 362880)

32. 6 r^{th} term from the cad is $(n-r+27)^{th}$ term from the begining

$T_7 : T_{n-5} = 1:6 \Rightarrow n = 9$

33. 13 $T_{r+1} = {}^{22}C_r (x^m)^{22-r} \left(\frac{1}{x^2}\right)^2 = {}^{22}C_r x^{22m-mr-2r}$ = ${}^{22}C_r x$

$\because {}^{22}C_3 = {}^{22}C_{19} = 1540$

$\therefore\ r = 3$ or 19

$22m - mr - 2r = 1$

$m = \frac{2r+1}{22-5},\ r = 3,\ m = \frac{7}{19} \notin N.$

$r = 19, m = \frac{38+1}{22-19} = \frac{39}{3} = 13$, m = 13.

34. 44 General term $= 256C_r \left(\sqrt{3}\right)^{256-r} \left(8\sqrt{5}\right)^r = 256\text{ Cr } 3^{\frac{256-r}{2}}\ 5^{r/8}$

$\therefore$ The terms are integral for values of $r = 0, 8, 16, 24, 32\ ......256$

$\therefore$ Total terms $= 33$

35. 7 $T_{r+1} = {}^9C_r \left(\frac{3}{2}x^2\right)^{9-r} \left(-\frac{1}{3x}\right)^r$

$T_{r+1} = {}^9C_r \left(\frac{3}{2}\right)^{9-r} \left(-\frac{1}{3}\right)^r x^{18-3r}$

For independent of 'x', $18 - 3r = 0, r = 6$

$\therefore\ T_7 = {}^9C_6 \left(\frac{3}{2}\right)^3 \left(-\frac{1}{3}\right)^6 = \frac{21}{54} = k$

$\therefore\ 18k = \frac{21}{54} \times 18 = 7.$

JEE ADVANCED LEVEL

SECTION III

36. D Let $S = \{1, 2, ...9\}$ and k is odd

$N_1 - 5C_1 \times 4C_4$

$N_2 = 5C_2 \times 4C_3$

$N_3 = 5C_3 \times 4C_2$

$N_4 = 5C_4 \times 4C_1$

$N_5 = 5 \Rightarrow N_1 + N_2 + N_3 + N_4 + N_5 = 126$

37. 2

From three groups A, B, C he can choose 3, 2, 2, or 2, 3, 2 or 2, 2, 3 questions respectively.

Total no. of ways of choosing = $({}^4C_3 \cdot {}^5C_2 \cdot {}^6C_2) + ({}^4C_2 \cdot {}^5C_3 \cdot {}^6C_2) + ({}^4C_2 \cdot {}^5C_3 \cdot {}^6C_3)$

$= 4 \times 10 \times 15 + 6 \times 10 \times 20 = 2700$

39. A

If zero is included it will be at $z \Rightarrow {}^9C_2$ no's

If zero is excluded

$$\begin{cases} x, y, z \text{ all diff.} & \Rightarrow {}^9C_3 \times 2! \\ x = z < y & \Rightarrow {}^9C_2 \\ x < y = z & \Rightarrow {}^9C_2 \text{ No's} \end{cases}$$

Total number of ways = 276

Alternative y can be from 2 to 9 so total number of ways = $\sum_{r=2}^{9}(r^2 - 1) = 276$

40. B

$10\lambda = \frac{6!}{2!2!2!}$

41. B

Total number of ways = (3!) (2!) (4!) = 288

42. D

Strictly descending $\rightarrow {}^{10}C_5$

Strictly ascending $\rightarrow {}^9C_5$ (because zero can't be at x_1)

${}^{10}C_5 + {}^9C_5 = 2.{}^9C_4 + {}^9C_4 = 3.{}^9C_4 = 3.{}^9C_5$

43. C

Let $S = (1+x)^{1000} + 2x(1+x)^{999} + 3x^2(1+x)^{998} + ... + 1001x^{1000}$

$$\frac{x}{1+x}S = x(1+x)^{999} + 2x^2(1+x)^{998} + ... + 1000x^{1000} + \frac{1001x^{1001}}{1+x}$$

Substract above equations

$$S = (1+x)^{1001} + x(1+x)^{1000} + x^2(1+x)^{999} + ... + \quad x^{1000}(1+x) - 1001x^{1001}$$

$$= (1+x)^{1001}\frac{\left[\left(\frac{x}{1+x}\right)^{1001} - 1\right]}{\frac{x}{1+x} - 1} - 1001x^{1001} \text{ (sum of G.P)}$$

$$= (1+x)^{1002} - x^{1002} - 1002x^{1001}$$

The coefficeint of x^{50} in $S = {}^{1002}C_{50}$

44. A $(1+7)^{83}+(7-1)^{83}=(1+7)^{83}-(1-7)^{83}$ $=2[{}^{83}C_1.7+{}^{83}C_3.7^3+.....+{}^{83}C_{83}.7^{83}]$

$= (2.7.83)+49I$ where I is an integer $14\text{x}83=1162$

$\frac{1162}{49}=23\frac{35}{49}$

remainder is 35.

45. D $\sum_{r=0}^{20} r(20-r)\left({}^{20}C_r\right)^2 = \sum_{r=0}^{20}(20-r)^{20}C_r \times r \times {}^{20}C_r = \sum_{r=0}^{19} 20\times{}^{19}C_r \times 20\times{}^{19}C_{r-1}$

$= 400\sum_{r=0}^{19}{}^{19}C_r \times {}^{19}C_{r-1} = 400\times{}^{38}C_{20}$

SECTION IV (More than one correct)

46. A,B,C We know $495 = 5\times11\times9$

5	a	b	b	a	5

$a, b \in \{0,1,2....9\}$

$2(a+b)+10=9k \Rightarrow a+b=4 \text{ or } a+b=13$

$\Rightarrow 5\,\text{cases}+6\,\text{cases}=11\,\text{cases}$

47. B,D General term, $T_{r+1} = 99Cr\left(3^{114}\right)^{99-r}\left(4^{113}\right)^r$

$= 99Cr3^{\frac{99-r}{4}}2^{r/3}, r = 0, \text{i.e.} 99$

for $r = 99, 87, 75, 63, 51, 39, 27, 15, 3$

the terms becomes rational

no. of irrational terms = 100–9=91

48. A,B,C $P= T_1+T_3+T_5+....$

$Q = T_2+T_4+T_6+....$

$\therefore (x+a)^n = P+Q \rightarrow (1)$

$(x-a)^n = P-Q \rightarrow (2)$

49. A,C

50. A,B,C,D A) ${}^6C_3.{}^4C_2.5!.5! = (5!)^3$ B) ${}^6C_1.9!$ C) $(6+1)!4!$ D) ${}^{10}P_4$

SECTION V - (Numerical type)

51. 210 The given expansion is reduced to $\left(x^{1/3}-x^{-1/2}\right)^{10}$

Term independent of $x = 10C_4 = 210$

52. 32

$\boxed{1\ |\ 2\ |\ \ |\ \ |\ \ |\ \ } = {}^7C_4 = 35$

$\boxed{1\ |\ 3\ |\ \ |\ \ |\ \ |\ \ } = {}^6C_4 = 15$

$\boxed{1\ |\ 4\ |\ \ |\ \ |\ \ |\ \ } = {}^5C_4 = 5$

$\boxed{1\ |\ 5\ |\ \ |\ \ |\ \ |\ \ } = {}^4C_4 = 1$

$\boxed{2\ |\ 3\ |\ \ |\ \ |\ \ |\ \ } = {}^6C_4 = 15$

71 words

$2\ 4\ 5\ 6\ 7\ 8 \rightarrow 72^{\text{th}}$ word

$2+4+5+6+7+8=32$

53. 2 No of ways of distribution = $\dfrac{9!}{\left(2!\lambda^3 3!3!\right)} \times 3!$

54. 569

(1) $\boxed{2}\,\boxed{0}\,\boxed{2}\,\boxed{2,3,4,6,7} \longrightarrow 5$

(2) $\boxed{2}\,\boxed{0}\,\underset{4}{\boxed{3,4,6,7}}\,\underset{6}{\boxed{}} \longrightarrow 24$

(3) $\boxed{2}\,\underset{5}{\boxed{2,3,4,6,7}}\,\underset{6}{\boxed{}}\,\underset{6}{\boxed{}} \longrightarrow 180$

(1)

(4) $\boxed{3}\,\underset{6}{\boxed{}}\,\underset{6}{\boxed{}}\,\underset{6}{\boxed{}} \longrightarrow 216$

(5) $\boxed{4}\,\underset{4}{\boxed{0,2,3,4}}\,\underset{6}{\boxed{}}\,\underset{6}{\boxed{}} \longrightarrow 144$

Number of 4 digit integers in [2022,4482]

$= 5 + 24 + 180 + 216 + 144 = 569$

55. 141

$$\sum_{R=1}^{31} {}^{31}C_R \cdot {}^{31}C_{R-1}$$

$$= {}^{31}C_1 \cdot {}^{31}C_0 + {}^{31}C_2 \cdot {}^{31}C_1 + \ldots + {}^{31}C_{31} \cdot {}^{31}C_{30}$$

$$= {}^{31}C_0 \cdot {}^{31}C_{30} + {}^{31}C_1 \cdot {}^{31}C_{29} + \ldots + {}^{31}C_{30} \cdot {}^{31}C_0$$

$$= {}^{62}C_{30}.$$

Similarly

$$\sum_{R=1}^{30}\left({}^{30}C_R \cdot {}^{30}C_{R-1}\right) = {}^{60}C_{29}$$

$${}^{62}C_{30} - {}^{60}C_{29} = \frac{62!}{30!32!} - \frac{60!}{29!31!}$$

$$= \frac{60!}{29!\,31!}\left\{\frac{62\cdot 61}{30\cdot 32} - 1\right\}$$

$$= \frac{60!}{30!31!}\left(\frac{2822}{32}\right)$$

$$\therefore\ 16\alpha = 16 \times \frac{2822}{32} = 1411$$