CHAPTER - 20

LINEAR INEQUALITIES

LINEAR INEQUATIONS

Two real number or two algebraic expressions related by the symbol '<', '>', '≤' or '≥' form an inequality.

$5 < 7$, $10 > 8$ are examples of numerical inequalities, while $x < 5$, $3 \le x \le 5$, $2x + y \le 7$ are the examples of literal inequalities.

Results

1. The statement obtained by connecting two algebraic expressions by an inequality sign is called an inequality, if it is true for all values of the variable(s) involved.

   eg. $x^2 + \dfrac{1}{x^2} \ge 2$

2. The statement obtained by connecting two algebraic expressions by an inequality sign is called an inequation, if it is not true for all values of the variable(s) involved.

   eg. $x > 5$, $x + 2y \ge 1$

3. The values of the unknown variable for which an inequation holds good are called solution of the inequation.

4. An inequation is linear, if each term of the algebraic expression(s) in the inequation is of degree at most one.

5. The line $ax + by + c = 0$ divides the xy plane into two regions called half planes.

Properties of inequalities

1. If $a > b$, $b > c$ then $a > c$.

2. $a > b \Rightarrow a + c > b + c$ and $a - c > b - c$ for all $c \in R$.

3. $a > b \Rightarrow ac > bc$ and $\dfrac{a}{c} > \dfrac{b}{c}$ for all $c > 0$, $c \in R$.

4. If a > b > 0, then $\frac{1}{a} < \frac{1}{b}$.

5. If $a_1 > b_1$, $a_2 > b_2$ ---------, $a_n > b_n$

then $a_1 + a_2 + --- + a_n > b_1 + b_2 + ---- + b_n$

and $a_1 a_2 ....a_n > b_1 b_2 ....b_n$, provided all $a_i > 0$ and all $b_i > 0$.

6. If a, b are positive real numbers such that a > b and if 'n' is any positive rational number, then

$(i) a^n > b^n \quad (ii) a^{-n} < b^{-n} \quad (iii) a^{1/n} > b^{1/n}$

7. If $0 < a < 1$ and 'n' is any positive rational number, then $0 < a^n < 1$ and $a^{-n} > 1$.

8. If $0 < a < 1$, and m, n are positive rational numbers, then

(i) $m > n \Rightarrow a^m < a^n$ and $m < n \Rightarrow a^m > a^n$.

9. If a > 1, then $a^m > a^n$ for m > n and $a^m < a^n$ for m < n, where m and n are positive rational numbers.

10. If a is a positive real number, then $a + \frac{1}{a} \geq 2$

If a is a negative real number $a + \frac{1}{a} \leq -2$

11. $|a_1 + a_2 + --- + a_n| \leq |a_1| + |a_2| + --- + |a_n|$

12. $(a+b)^2 \geq 2ab$, $a, b \in R$

13. $(a+b+c)^2 \geq 3(ab+bc+ac)$, $a, b \in R$

PART I - (JEEMAIN)

SECTION - I - Straight objective type questions

1. If $|x+3| \geq 10$, then $x \in$

1) $[-13, 7]$ 2) $(-13, 7)$ 3) $(-\infty, 13) \cup (7, \infty)$ 4) $(-\infty, -13] \cup [7, \infty)$

2. If the area and perimeter of a rectangle are A and P respectively, then P and A satisfy the inequality

1) $P + A > PA$ 2) $P^2 \leq A$ 3) $A - P < 2$ 4) $P^2 \geq 16A$

3. Statement I : The solution set of $\frac{2x-1}{3} \geq \frac{3x-2}{4} - \frac{2-x}{5}$ is $(-\infty, 2]$

Statement II : The solution set of $-3 \leq 4 - \frac{7x}{2} \leq 18$ is $[-2, 4]$

1) Only Statement I is true
2) Only Statement II is true
3) Statement I and Statement II true
4) Statement I and Statement II false

4. The solution set of $\frac{x+3}{x-2} \leq 2$ is

1) R
2) $(-\infty, 2) \cup [7, \infty)$
3) $(\infty, 2] \cup [7, \infty)$
4) $(7, \infty)$

5. The solution of $2(2x+3) - 10 < 6(x-2)$ and $\frac{2x-3}{4} + 6 \geq 2 + \frac{4x}{3}$ is

1) $x \leq 4$
2) $x \geq 2$
3) $x \leq -4$
4) No solution

6. The set of all real x satisfying the inequality $\frac{3-|x|}{4-|x|} \geq 0$ is

1) $[-3, 3] \cup (-\infty, -4) \cup (4, \infty)$
2) $(-\infty, -4) \cup (4, \infty)$
3) $(-\infty, -3) \cup (4, \infty)$
4) $(-\infty, -3) \cup (3, \infty)$

7. The values of x which satisfy both the inequations $x^2 - 1 \leq 0$ and $x^2 - x - 2 \geq 0$ lie in

1) $(-1, 1)$
2) $(-1, 2)$
3) $(1, 2)$
4) $\{-1\}$

8. The solution of the inequality $\frac{|x+2| - |x|}{\sqrt{8 - x^3}} \geq 0$ is

1) $[-1, 1]$
2) $[-1, 2)$
3) $[1, 2]$
4) $(-1, 2]$

9. In the first 4 papers each of 100 marks, Rishi got 95,72,73,83 Marks. If he wants an average greater than or equal to 75 marks and less than 80 marks, then the range of marks he should score in the fifth paper is

1) $52 \leq x \leq 75$
2) $52 < x \leq 77$
3) $52 \leq x < 77$
4) $52 < x < 76$

10. Which of the following is not the solution of the inequality $\frac{2x+3}{x^2+x-12}<\frac{1}{2}$

1) $(-\infty,-4)$ 2) $(-3,3)$ 3) $(9,\infty)$ 4) $(3,6)$

SECTION - II

Numerical Type Questions

11. If $A=\{x\in R:|x-2|>1\}$

$B=\{x\in R:\sqrt{x^2-3}>1\}$

$C=\{x\in R:|x-4|\geq 2\}$

and Z is the set of all integers then the number of elements in the set $Z\cap(A\cap B\cap C)^c$ is

12. The number of positive integers for which the inequality $\frac{(x+3)(x-1)}{x^2(x-2)^3}\leq 0$ hold is

13. If the solution set of the inequality $|a+3x|\leq 6$ is $\left[\frac{-8}{3},\frac{4}{3}\right]$ then the value of a is equal to

14. The smallest prime number satisfying the inequality $\frac{2x-3}{3}\geq\frac{x-1}{6}+1$ is

15. Sum of the least positive integer and greatest negative integer which satisfy the inequality $\frac{(x+2)(x-1)(x-3)(x+4)}{x(x-4)}\geq 0$ is

PART - II (JEE ADVANCED)

SECTION - III (Only one option correct type)

16. If r is a real number such that $|r|<1$ and if $a=5(1-r)$, then

A) $0<a<5$ B) $-5<a<5$ C) $0<a<10$ D) $0\leq a<10$

17. A manufacturer has 600 litres of a 12% solution of acid. If x litres of a 30% acid solution must be added to it so that acid content in the resulting mixture will be more than 15% but less than 18%, then which one of the following is true

A) $120<x<200$ B) $120<x<250$ C) $120<x<300$ D) $120<x<350$

18. The set of all x satisfying the inequality $\frac{4x-1}{3x+1} \geq 1$ is

A) $\left(-\infty,-\frac{1}{3}\right) \cup \left[\frac{1}{4},\infty\right)$
B) $\left(-\infty,-\frac{2}{3}\right) \cup \left[\frac{5}{4},\infty\right)$

C) $\left(-\infty,-\frac{1}{3}\right) \cup [2,\infty)$
D) $\left(-\infty,-\frac{2}{3}\right) \cup [4,\infty)$

19. If $|2x-3| < |x+5|$, then x lies in the interval

A) $(-3,5)$
B) $(5,9)$
C) $\left(\frac{-2}{3},8\right)$
D) $\left(-8,\frac{2}{3}\right)$

20. If $a^2+b^2+c^2=1$, then $ab+bc+ca$ lies in the interval

A) $\left[\frac{1}{2},2\right]$
B) $[-1,2]$
C) $\left[-\frac{1}{2},1\right]$
D) $\left[-1,\frac{1}{2}\right]$

21. If a,b,c, d are positive real numbers such that a+b+c+d=2 then $m=(a+b)(c+d)$ satisfies the relation

A) $0<m\leq 1$
B) $1\leq m\leq 2$
C) $2\leq m\leq 3$
D) $3\leq m\leq 4$

22. If $\alpha \in \left(0,\frac{\pi}{2}\right)$ then $\sqrt{x^2+x}+\frac{\tan^2\alpha}{\sqrt{x^2+x}}$ is always greater than or equal to

A) 2 tan α
B) 1
C) 2
D) $\sec^2\alpha$

23. If $a>0,b>0,c>0$ then the minimum value of $(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ is

A) 3
B) 6
C) 9
D) 27

24. Let $S_1=\left\{x\in R-\{1,2\}:\frac{(x+2)(x^2+3x+5)}{-2+3x-x^2}\geq 0\right\}$ and $S_2=\left\{x\in R:3^{2x}-3^{x+1}-3^{x+2}+27\leq 0\right\}$. Then

$S_1 \cup S_2$ is equal to

A) $(-\infty,-2]\cup(1,2)$
B) $(-\infty,-2]\cup[1,2]$
C) $(-2,1]\cup(2,\infty)$
D) $(-8,2]$

25. Consider the two sets

$A=\{m\in R:\text{both the roots of } x^2-(m+1)x+m+4=0 \text{ are real}\}$ and $B=[-3,5)$, which of the following is not true?

A) $A-B=(-\infty,-3)\cup[5,\infty)$

B) $A\cap B=\{-3\}$

C) $B-A=(-3,5)$

D) $A\cup B=R$