VECTORS

Physical quantities can be classified into two types. Scalar quantities and vector quantities. Those quantities which have only magnitude are known as scalars.

Examples : Mass, distance, speed etc.

Those quantities which have both magnitude and direction but do not obey laws of vector addition are not vectors such as electric current.

A vector is represented by straight line with an arrow head. The length of the lines gives the magnitude of the vector and arrow head gives the direction of the vector.

The vectors are represented by boldface letters or with an arrow over simple letter. If A is a vector then it is represented by $\vec{A}$ and magnitude of that vector is represented by $|\vec{A}|$ or A

Types of Vectors

1. Equal vectors : Two vectors are equal if their magnitude and direction same
2. Negative vector : A vector is said to be negative vector if the magnitude is same but direction is opposite
3. Parallel vectors : Vectors in the same direction
4. Antiparallel vectors : Vectors in the opposite direction
5. Zero or null vector : A vector whose magnitude is zero and direction is in determinate
6. Unit vector : It is a vector of unit magnitude If A is a vector then its unit vector is denoted by $\vec{A}$

Then $\boxed{\vec{A} = \frac{\vec{A}}{|\vec{A}|}}$

$\therefore \vec{A} = |\vec{A}|\vec{A}$

7. Co-initial vectors : The vectors which have the same starting point are called coinitial vectors
8. Coplanar vectors : Three or more vectors are lying in the same plane or parallel to the same plane are known as coplanar vectors
9. Collinear vectors : Vectors which lie along the same line or parallel lines are known to be collinear vectors

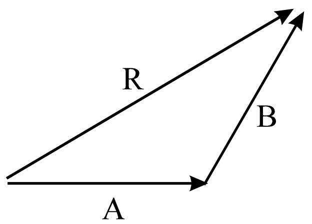
Multiplication of a vector by a scalar

When a vector $\vec{A}$ is multiplied by scalar K, then the resultant vector has magnitude KA, and direction same as that of $\vec{A}$

Addition of Vectors

1. Triangular Law of vector addition

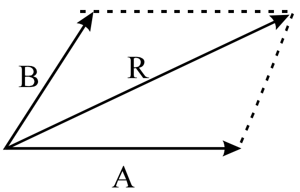
If two vectors are arranged as the adjacent sides of a triangle, then the third side taken in the opposite direction will represent their resultant



Then $\vec{R}$ is the resultant of $\vec{A}$ and $\vec{B}$

2. Parallelogram Law of vector addition

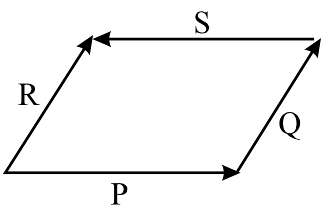
If two vectors are arranged as the adjacent sides of a parallelogram, then the diagonal passing through the point of intersection of these vectors represent their resultant



Here $\vec{R}$ is the resultant of $\vec{A}$ and $\vec{B}$

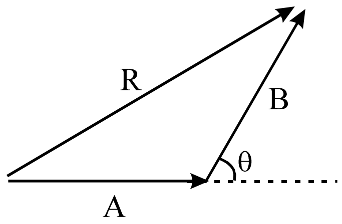
3. Polygon Law of vector addition

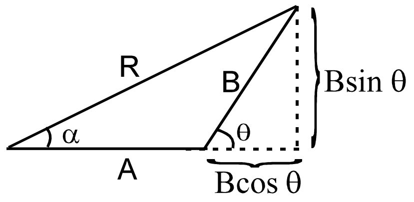
If we need to find resultant of $\vec{P}, \vec{Q}$ and $\vec{S}$. Then join the head of $\vec{P}$ to the tail of $\vec{Q}$, then head of $\vec{Q}$ to the tail of $\vec{S}$. Then join tail of $\vec{P}$ and head of $\vec{S}$ and it will represent their resultant.

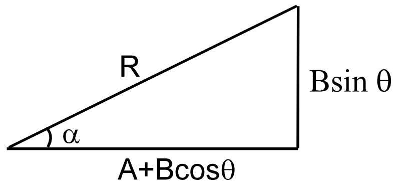


Here $\vec{R}$ represent the resultant of $\vec{P}, \vec{Q}$ and $\vec{S}$.

Magnitude and direction of the resultant vector







$$R = \sqrt{(A + B\cos\theta)^2 + (B\sin\theta)^2}$$

$$= \sqrt{A^2 + 2AB\cos\theta + B^2\cos^2\theta + B^2\sin^2\theta}$$

$$= \sqrt{A^2 + 2AB\cos\theta + B^2\left(\sin^2\theta + \cos^2\theta\right)}$$

$$= \sqrt{A^2 + 2AB\cos\theta + B^2 \times 1}$$

α is the angle made by $\vec{R}$ with $\vec{A}$

then $\tan d = \dfrac{B\sin\theta}{A + B\cos\theta}$

$$\alpha = \tan^{-1}\left(\frac{B\sin\theta}{A + B\cos\theta}\right)$$

<u>Case (i)</u>

$\vec{A}$ and $\vec{B}$ in same direction

$\theta = 0$

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$= \sqrt{A^2 + B^2 + 2AB \times 1}$$

$$= \sqrt{A^2 + B^2 + 2AB}$$

$$= \sqrt{(A + B)^2} = A + B$$

Case (ii)

$\vec{A}$ and $\vec{B}$ in opposite direction

$\theta = 180^o$

$R = \sqrt{A^2 + B^2 + 2AB\cos 180}$

$= \sqrt{A^2 + B^2 + 2ABx - 1}$

$= \sqrt{A^2 + B^2 - 2AB}$

$= \sqrt{(A - B)^2}$

$= A - B$

Case (iii)

$\vec{A}$ and $\vec{B}$ are perpendicular

$\theta = 90^o$

$R = \sqrt{A^2 + B^2 + 2AB\cos 90}$

$= \sqrt{A^2 + B^2 + 2AB \times 0}$

$= \sqrt{A^2 + B^2}$

Note:

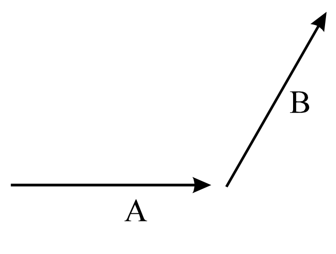
Max value of resultant is A + B and minimum value of resultant is A – B

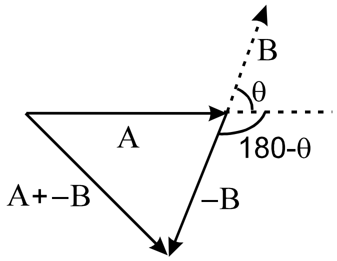
$\therefore A - B \le R \le A + B$

Subtraction of Vectors

$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

$\therefore$ Two subtract two vectors, we are adding one vector with negative vector of other vector





$$\left|\vec{A}-\vec{B}\right| = \sqrt{A^2+B^2+2AB\cos(180-\theta)}$$

$$= \sqrt{A^2+B^2+2ABx-\cos\theta}$$

$$= \sqrt{A^2+B^2-2AB\cos\theta}$$

Example 9

If $\left|\vec{A}+\vec{B}\right| = \left|\vec{A}-\vec{B}\right|$ find angle between $\vec{A}$ and $\vec{B}$

Ans.

$$\left|\vec{A}+\vec{B}\right| = \left|\vec{A}-\vec{B}\right|$$

$$\sqrt{A^2+B^2+2AB\cos\theta} = \sqrt{A^2+B^2-2AB\cos\theta}$$

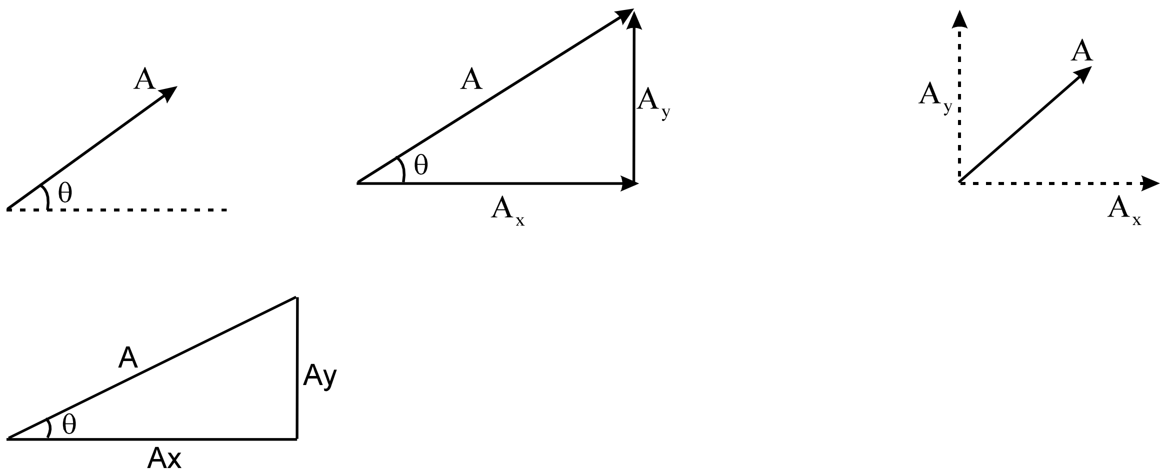
$$A^2+B^2+2AB\cos\theta = A^2+B^2-2AB\cos\theta$$

$$4AB\cos\theta = 0$$

$$\cos\theta = 0$$

$$\theta = 90^{o}$$

Resolution of Vectors

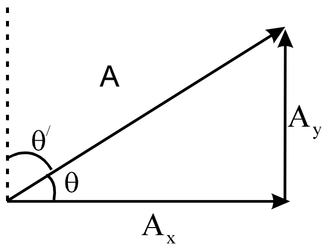


$$\cos\theta = \frac{Ax}{A} \qquad \sin\theta = \frac{Ay}{A}$$

$$\boxed{Ax = A\cos\theta} \qquad \boxed{Ay = A\sin\theta}$$

$$\vec{A} = \vec{A}x + \vec{A}y \qquad \boxed{\vec{A} = Ax\hat{i} + Ay\hat{j}}$$

$$\boxed{\vec{A} = A\cos\theta\hat{i} + A\sin\theta\hat{j}}$$



angle made by vector with horizontal = θ

$$\tan\theta^1 = \frac{Ax}{Ay}$$

$$\theta = \tan^{-1}\left(\frac{Ay}{Ax}\right)$$

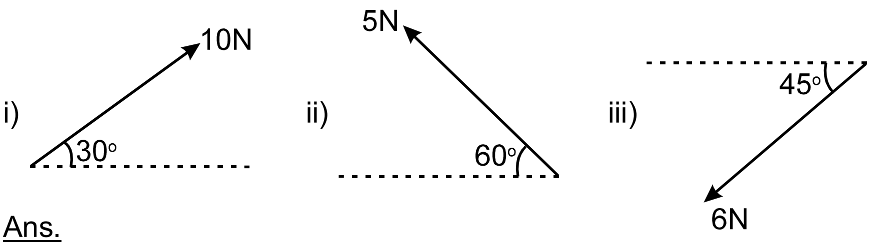
angle made by vector with vertical = θ^1

$$\tan\theta^1 = \frac{Ax}{Ay}$$

$$\boxed{\theta^1 = \tan^{-1}\left(\frac{Ax}{Ay}\right)}$$

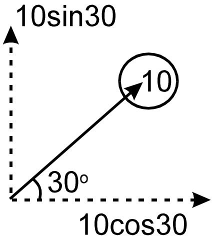
<u>Example 10</u>

Represent the following forces in vector form



<u>Ans.</u>

i)

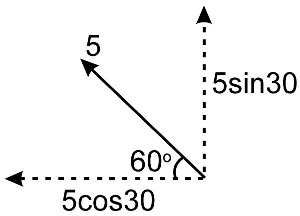


$$\vec{F} = 10\cos 30\hat{i} + 10\sin 30\hat{j}$$

$$= 10 \times \sqrt{3}/3\ \hat{i} + 10 \times \frac{1}{2}\hat{j}$$

$$= 5\sqrt{3}\hat{i} + 5\hat{j}$$

ii)

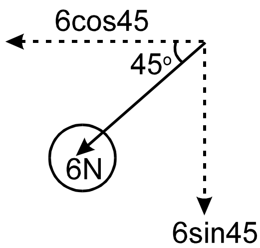


$$\vec{F} = 5\cos 60\left(-\hat{i}\right) + 5\sin 60\hat{j}$$

$$= 5 \times \frac{1}{2}\left(-\hat{i}\right) + 5\sqrt{3}/2\ \hat{j}$$

$$= -2.5\hat{i} + 2.5\sqrt{3}\hat{j}$$

iii)



$$\vec{F} = 6\cos 45\left(-\hat{i}\right) + 6\sin 45\left(-\hat{j}\right)$$

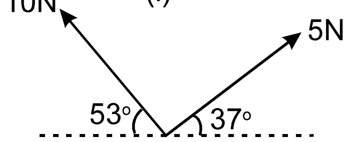
$$= -3\sqrt{2}\hat{i} - 3\sqrt{2}\hat{j}$$

<u>A vector addition in component method</u>

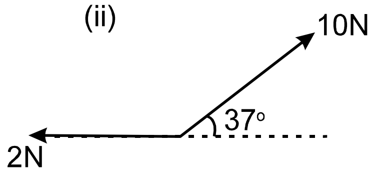
<u>Example 11</u>

Find resultant of following forces

(i)



(ii)



(i) $\vec{F}_1 = 5\cos 37\hat{i} + 5\sin 37\hat{j} = 5\times \frac{4}{5}\hat{i} + 5\times \frac{3}{5}\hat{j} = 4\hat{i} + 3\hat{j}$

$\vec{F}_2 = 10\cos 53\left(-\hat{i}\right) + 10\sin 53\left(\hat{j}\right)$

$= 10\times \frac{3}{5}\left(-\hat{i}\right) + 10\times \frac{4}{5}\hat{j}$

$= -6\hat{i} + 8\hat{j}$

$\vec{F} = \vec{F}_1 + \vec{F}_2 = \left(4\hat{i} + 3\hat{j}\right) + \left(-6\hat{i} + 8\hat{j}\right)$

$= -2\hat{i} + 11\hat{j}$

$F = \left|\vec{F}\right| = \sqrt{2^2 + 11^2} = \sqrt{125}$

ii) $\vec{F}_1 = 10\cos 37\hat{i} + 10\sin 37\hat{j} = 10\times \frac{4}{5}\hat{i} + 10\times \frac{3}{5}\hat{j} = 8\hat{i} + 6\hat{j}$

$\vec{F}_2 = -2\hat{i}$

$\vec{F} = \vec{F}_1 + \vec{F}_2 = 8\hat{i} + 6\hat{j} + -2\hat{i} = 6\hat{i} + 8\hat{j}$

$\left|\vec{F}\right| = \sqrt{6^2 + 6^2} = 6\sqrt{2}$

Scalar Product of Vectors

The dot product of two vector is scalar and it is given by

$\vec{A}.\vec{B} = \left|\vec{A}\right|\left|\vec{B}\right|\cos\theta$

Where θ is the angle between $\vec{A}$ and $\vec{B}$

$\vec{B}\,.\,\vec{A} = \left|\vec{B}\right|\left|\vec{A}\right|\cos\theta$

*) The scalar product is scalar. it can be +ve, –ve or zero

If $0 < \theta < 90$ scalar product is positive

If $90 < \theta < 180$ scalar product is negative

If $\theta = 90^\circ$, scalar product is zero

*) $\vec{A}.\vec{A} = \left|\vec{A}\right|\left|\vec{A}\right|\cos\theta = \left|\vec{A}\right|^2 = A^2$

*) Angle between two vectors is given by

$\vec{A}\,.\,\vec{B} = \left|\vec{A}\right|\left|\vec{B}\right|\cos\theta$

$$\cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}$$

$$\boxed{\theta = \cos^{-1}\left(\frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}\right)}$$

*) If two vectors are perpendicular, then their dot product will be zero

*) $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

*) $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$

If $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

then $\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$

*) Component of $\vec{A}$ along $\vec{B}$ is given by

$$A\cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{B}|}$$

<u>Vector or Cross Product</u>

The product of two vectors are given by

$$\vec{A}\times\vec{B} = |\vec{A}||\vec{B}|\sin\theta\hat{n}$$

Where $\hat{n}$ is a unit vector perpendicular to both $\vec{A}$ and $\vec{B}$

*) $\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$

*) $\sin\theta = \dfrac{|\vec{A}\times\vec{B}|}{|\vec{A}||\vec{B}|}$

*) $\vec{A}\times\vec{A} = 0$

*) $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$

*) $\hat{i}\times\hat{j} = \hat{k} = -\left(\hat{j}\times\hat{k}\right)$

*) $\hat{j}\times\hat{k} = \hat{i} = -\left(\hat{k}\times\hat{j}\right)$

*) $\hat{k}\times\hat{i} = \hat{j} = -\left(\hat{i}\times\hat{k}\right)$