CHAPTER - 06
SEQUENCES AND SERIES

JEE MAIN - SECTION I

1. 4

$x_1 + x_2 + x_3 + \dots\dots + x_{11} + x_{12} + x_{13} + \dots + x_{21} + x_{22} + x_{23}$

$x_{11} + x_{12} + x_{13} = 141 \Rightarrow 3x_1 + 33d = 141 \longrightarrow (1)$

$x_{21} + x_{22} + x_{23} = 261 \Rightarrow 3x_1 + 63d = 261 \longrightarrow (2)$

$(2) - (1) \Rightarrow 30d = 120 \Rightarrow d = 4$

$(1) \Rightarrow 3x_1 + 33 \times 4 = 141$

$\Rightarrow 3x_1 = 9 \Rightarrow x_1 = \underline{3}$

2. 1

$x_1 + x_4 + x_9 + x_{11} + x_{20} + x_{22} + x_{27} + x_{30} = 272$

$(x_1 + x_{30}) + (x_4 + x_{27}) + (x_9 + x_{22}) + (x_{11} + x_{20}) = 272$

$\Rightarrow 4[x_1 + x_{30}] = 272 \Rightarrow x_1 + x_{30} = 68$

$x_1 + x_2 + \dots + x_{30} = \frac{30}{2}[x_1 + x_{30}] \Rightarrow = 15 \times 68 = \underline{\underline{1020}}$

3. 2

$a_1 + a_2 + a_3 + \dots + a_{21} = 693$

$S_{21} = 693 \Rightarrow \frac{21}{2}[a_1 + a_{21}] = 693 \Rightarrow a_1 + a_{21} = 66$

$\sum_{r=0}^{10} a_{2r+1} = a_1 + a_3 + a_5 + \dots + a_{21} = \frac{11}{2}[a_1 + a_{21}] = 363$

4. 2

$S_{10} = \frac{1}{2}(S_{20} - S_{10})$

$2 \cdot S_{10} = S_{20} - S_{10} \Rightarrow 3 \cdot S_{10} = S_{20}$

$3\left[\frac{10}{2}[2a + 9d]\right] = \frac{20}{2}[2a + 19d]$

$6a + 27d = 4a + 38d \Rightarrow 2a = 11d \Rightarrow a = \frac{11d}{2}$

$T_2 = a + d = 13 \Rightarrow d = \underline{2}$

5. 2

$\sqrt{3} + 5\sqrt{3} + 9\sqrt{3} + 13\sqrt{3} + \dots \text{ n terms} = 435\sqrt{3}$

$S_n = \frac{n}{2}[2\sqrt{3} + (n-1)4\sqrt{3}] = 435\sqrt{3}$

$\Rightarrow n[1 + 2n - 2] = 435 \Rightarrow n(2n-1) = 435$

$\Rightarrow 15 \times 29 = 435 \Rightarrow n = \underline{15}$

6. 4

Let $a-d,\ a,\ a+d$ be the roots

Sum $= (a-d) + a + (a+d) = 3a = 12 \Rightarrow a = 4$

Product $= (a-d)a(a+d) = a(a^2-d^2) = 28$

$\Rightarrow 4(16-d^2) = 28 \Rightarrow d^2 = 9 \Rightarrow d = \pm\underline{3}$

7. 1

$a_1 = a_2 = \dots = a_{10} = 150$

currency counted in first 10 minute $= 1500$

Remaining currency $= 3000$

$a_{11} + a_{12} + a_{13} + \dots \text{ n terms} = 3000$

$148 + 146 + 144 + \dots \text{ n terms} = 3000$

$\frac{n}{2}[2 \times 148 + (n-1)(-2)] = 3000$

$\Rightarrow n^2 - 149n + 3000 = 0 \Rightarrow (n-125)(n-24) = 0$

$n = 125, 24$

n cannot be 125, $a_{125} < 0$

Remaining 3000 currency should be counted in 24 minutes $\therefore$ Total time $= 24 + 10 = \underline{34}$

8. 3

$\frac{a_3}{a_1} = 25 \Rightarrow r^2 = 25 = 5^2$

$\frac{a_9}{a_5} = \frac{ar^8}{ar^4} = r^4 = (r^2)^2 = \underline{5^4}$

9. 1

$\frac{a_9}{a_5} = \frac{ar^8}{ar^4} = r^4 = (r^2)^2 = \underline{5^4}$

$x = 1 + a + a^2 + \dots \infty \Rightarrow x = \frac{1}{1-a} \Rightarrow 1-a = \frac{1}{x} \Rightarrow a = \frac{x-1}{x}$

$y = 1 + b + b^2 + \dots \infty \Rightarrow y = \frac{1}{1-b} \Rightarrow 1-b = \frac{1}{y} \Rightarrow b = \frac{y-1}{y}$

$1 + ab + a^2b^2 + \dots \infty = \frac{1}{1-ab} = \frac{1}{1-\left(\frac{x-1}{x}\right)\left(\frac{y-1}{y}\right)} = \underline{\frac{xy}{x+y-1}}$

10. 4 Given $\frac{a+2}{2} = \sqrt{2a} + 1 \Rightarrow \frac{a+2}{2} - 1 = \sqrt{2a} \Rightarrow a = 2\sqrt{2a}$

$\Rightarrow a^2 = 8a \Rightarrow a = 8$

11. 3 $a_1, a_2, a_3, \ldots \ldots \ldots a_{50} \rightarrow G.P$

$$\frac{a_1 - a_3 + a_5 - \cdots + a_{49}}{a_2 - a_4 + a_6 - \cdots + a_{50}} = \frac{a - ar^2 + ar^4 - \cdots + ar^{48}}{ar - ar^3 + ar^5 - \cdots + ar^{48}}$$

$$= \frac{a(1 - r^2 + r^4 - \cdots + r^{48})}{ar(1 - r^2 + r^4 - \cdots + r^{48})}$$

$$= \frac{1}{r} = \frac{a_1}{a_2}$$

12. 4 $\frac{a+b}{2} = 5\sqrt{ab} \Rightarrow a+b = 10\sqrt{ab}$

$(a+b)^2 = 100\,ab \longrightarrow (1)$

$(a+b)^2 - 4ab = 96\,ab$

$(a-b)^2 = 96\,ab \longrightarrow (2)$

$\frac{(a+b)^2}{(a-b)^2} = \frac{100}{96} = \frac{25}{24} \quad \therefore \frac{a+b}{a-b} = \frac{5}{2\sqrt{6}} = \frac{5\sqrt{6}}{12}$

13. 2

$a-8 \quad a-6 \quad a-4 \quad a-2 \quad a \quad \frac{a}{2} \quad \frac{a}{4} \quad \frac{a}{8} \quad \frac{a}{16}$

$\Rightarrow a-4 = \frac{a}{4} \Rightarrow 4a - 16 = a \Rightarrow 3a = 16 \Rightarrow a = \frac{16}{3} \quad \therefore \frac{a}{4} = \frac{4}{3}$

14. 2 We have $a_{2n+1} = a_{2n} \times r$

$$\sum_{n=1}^{100} a_{2n+1} = \sum_{n=1}^{100} a_{2n} \times r$$

$\Rightarrow \beta = \alpha \times r \Rightarrow r = \frac{\beta}{\alpha}$

15. 3

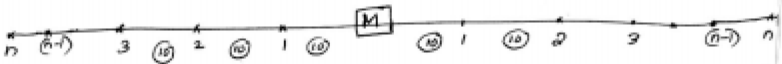
$$\text{Total distance} = 48 + \overset{\text{up}}{48\times\frac{2}{3}} + \overset{\text{down}}{48\times\frac{2}{3}} + 48\times\left(\frac{2}{3}\right)^2 + 48\times\left(\frac{2}{3}\right)^2 + 48\times\left(\frac{2}{3}\right)^3 + 48\left(\frac{2}{3}\right)^3 + \cdots$$

$$= 48 + 2\times 48\left[\frac{2}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \cdots\right]$$

$$= 48 + 96\left[\frac{\frac{2}{3}}{1-\frac{2}{3}}\right] = \underline{\underline{240}}$$

16. 2

16 Let the total number of stones be $2n+1$



$$\text{Total distance} = 10n + 2\times 10[(n-1)+(n-2)+\cdots+2+1] + 2\times 10[1+2+\cdots+n] = 3000$$

$$\Rightarrow 10n + 2\times 10\left[\frac{(n-1)n}{2}\right] + 2\times 10\left[\frac{n(n+1)}{2}\right] = 3000$$

$$= n + (n-1)n + n(n+1) = 300$$

$$\Rightarrow n(2n+1) = 300 \Rightarrow 12\times 25 = \underline{300}$$

$$\text{Total no. of stones} = 2n+1 = \underline{\underline{25}}$$

17. 2

$$\sum_{k=1}^{n} k(k^2-1) = \sum_{k=1}^{n}(k^3-k) = \sum_{k=1}^{n}k^3 - \sum_{k=1}^{n}k$$

$$= \frac{n^2(n+1)^2}{4} - \frac{n(n+1)}{2}$$

$$= \frac{n^2(n+1)^2}{4} - \frac{n^2}{2} - \frac{n}{2} = pn^4 + qn^3 + tn^2 + sn$$

$$s = \text{coeff. of } n = \underline{\underline{\frac{-1}{2}}}$$

18. 4

18. $S_n = 2+7+14+23+\cdots\cdots + T_{99} \rightarrow (1)$

$S_n = \quad 2+7+14+\cdots\cdots + T_{98} + T_{99} \rightarrow (2)$

(1)-(2) $0 = 2+5+7+9+\cdots\cdots$ upto 99 terms $- T_{99}$

$T_{99} = 2 + \{5+7+9+\cdots\cdots \text{upto 98 terms}\}$

$$= 2 + \frac{98}{2}[2\times 5 + 97\times 2] = \underline{9998}$$

19. 4

statement I is false, statement II is true

20. 1

$\frac{1}{5}, G_1, G_2, G_3 \ldots G_{49}, G_{50}, 5 \rightarrow G.P$

In a finite G.P. product of the terms equidistant from the beginning and end is same and is equal to the product of first and last terms.

SECTION II (NUMERICAL)

21. 20

$200 < S_9 < 220, \quad T_2 = a+d = 12$

$200 < \frac{9}{2}[2a+8d] < 220$

$200 < 9[a+4d] < 220$

$200 < 9[(a+d)+3d] < 220$

$200 < 9[12+3d] < 220$

$200 < 108+27d < 220$

$92 < 27d < 112$

$\frac{92}{27} < d < \frac{112}{27} \Rightarrow 3.. < d < 4 \ldots \Rightarrow d=4$

$T_4 = a+3d = (a+d)+2d = \underline{\underline{20}}$

22. 12100

$2^2 + 2(4)^2 + 3(6)^2 + \ldots$ 10 terms.

$2^2[1^3+2^3+3^3+\ldots+10^3] = \frac{2}{2}\left[\frac{10\times 11}{2}\right]^2 = 12100$

23. 7780

$\sum_{r=16}^{30}(r+2)(r-3) = \sum_{r=16}^{30}(r^2-r-6) = \sum_{r=16}^{30} r^2 - \sum_{r=16}^{30} r - 6\sum_{r=16}^{30} 1$

$= (16^2+17^2+\ldots+30^2)-(16+17+\ldots+30)-6\times 15$

$= \underline{\underline{7780}}$

24. 248

$$B = 1^2+2(2)^2+3^2+2(4)^2+5^2+2(6)^2+\dots+2(40)^2$$
$$B = S_{40} = (1^2+2^2+3^2+\dots+40^2)+(2^2+4^2+6^2+\dots+40^2)$$
$$= \frac{40\times41\times81}{6}+4(1^2+2^2+\dots+20^2) = \underline{33620}$$
$$A = S_{20} = (1^2+2^2+3^2+\dots+20^2)+(2^2+4^2+6^2+\dots+20^2)$$
$$= \frac{20\times21\times41}{6}+4\left(\frac{10\times11\times21}{6}\right) = 4410$$
$$B-2A = 33620-2\times4410 = 100T = 248 \Rightarrow T = \underline{248}$$

25. 101

$$S = \frac{8^2}{5^2}+\frac{12^2}{5^2}+\frac{16^2}{5^2}+\dots \text{ 10 terms.}$$
$$= \frac{4^2}{5^2}\left[2^2+3^2+4^2+\dots \text{ upto 10 terms}\right]$$
$$= \frac{16}{25}\left[1^2+2^2+3^2+4^2+\dots+10^2+11^2-1^2\right]$$
$$= \frac{16}{25}[505] \Rightarrow \frac{16}{25}[101] \Rightarrow m = \underline{\underline{101}}$$

JEE ADVANCED LEVEL

SECTION III

26. A The odd numbers of four digits which are divisible by 9 are 1017, 1035,.....9999

Thesea re in A.P with C.D = 18

$$t_n = a+(n-1)d$$

$$a = 1017 \qquad d = 18 \qquad t_n = 9999$$

$$9999 = 1017+(n-1)18 \Rightarrow n = 500$$

$$S_n = \frac{n}{2}(a_1+a_2) = \frac{500}{2}(1017+9999) = 275400$$

27. A

$$\text{Sn} = 5+7+13+31+\dots\dots\text{Tn}$$
$$\text{Sn} = \quad 5+7+13+\dots\dots+\text{Tn}$$
$$\text{O} = 5+(2+6+18+\dots\dots)-\text{Tn}$$

$$\text{Tn} = 5+\frac{2(3^{n-1}-1)}{2} = 4+3^{n-1}; \text{Sn} = \sum_1^n \text{Tn} = \frac{1}{2}(3^n+8n-1)$$

28. C Let the two numbers a and b; given $a+b=\frac{13}{6}$

and A.M.'s are $A_1, A_2, ...A_{2n}$ inserted between a and b.

Here $a, A_1, A_2, ..., A_{2n}, b$ are in A.P. then given condition

$A_1 + A_2 + ... + A_{2n} = 2n+1$

or $(a+A_1+A_2+....+A_{2n}+b)-(a+b)=2n+1$

or $\frac{(2n+2)}{2}(a+b)-(a+b)=2n+1$

or $n=(a+b)=2n+1$ or $13n=12n+6$

Hence number of means are 12

29. D $\sum_{n=1}^{\infty}\frac{K}{2^{n+K}}=\frac{K}{2^K}\sum_{n=1}^{\infty}\frac{1}{2^n}$; $=\frac{K}{2^K}\left(\frac{1}{2}+\frac{1}{2^2}+....\text{ to } \alpha\right)$; $=\frac{K}{2^K}\left(\frac{1/2}{1-1/2}\right)=\frac{K}{2^K}$

$\sum_{K=1}^{\infty}\sum_{n=1}^{\infty}\frac{K}{2^{n+K}}=\sum_{K=1}^{\infty}\frac{K}{2^K}$; $=\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+ .... \text{ to } \infty = 2$

30. C $S=2+\frac{6}{7}+\frac{12}{7^2}+\frac{20}{7^3}+\frac{30}{7^4}+.....$

Considering infinite sequence, $S=2+\frac{6}{7}+\frac{12}{7^2}+\frac{20}{7^3}+\frac{30}{7^4}+.....$

$\frac{S}{7}=\frac{2}{7}+\frac{6}{7^2}+\frac{12}{7^3}+\frac{20}{7^4}+........$

$\Rightarrow \frac{6S}{7}=2+\frac{4}{7}+\frac{6}{7^2}+\frac{8}{7^3}+\frac{10}{7^4}+.......$

$\Rightarrow \frac{6S}{7^2}=\frac{2}{7}+\frac{4}{7^2}+\frac{6}{7^3}+\frac{8}{7^4}+.....$

$\Rightarrow \frac{6S}{7}\left(1-\frac{1}{7}\right)=2+\frac{2}{7}+\frac{2}{7^2}+\frac{2}{7^3}+......$

$$\Rightarrow \frac{6^2 S}{7^2} = \frac{2}{1-\frac{1}{7}} = \frac{2}{6}\times 7 \;;\Rightarrow S = \frac{2\times 7^3}{6^3} \Rightarrow 4S = \frac{7^3}{3^3} = \left(\frac{7}{3}\right)^3$$

SECTION IV (More than one correct)

31. B,D We have $1072 < 10(2a + 19d) < 1162$ and $a + 5d = 32$ $\Rightarrow 1072 < 640 + 90d < 1162$

$\frac{432}{90} < d < \frac{522}{90}$ and d is natural number, so $d = 5 \Rightarrow a = 7$

32. A,C $S_n = \sum_{r=1}^{n} \frac{8r}{4r^4+1}$

$$= \sum_{r=1}^{n} \frac{8r}{(2r^2-2r+1)(2r^2+2r+1)}$$

$$= 2\sum_{r=1}^{n}\left(\frac{1}{2r^2-2r+1} - \frac{1}{2r^2+2r+1}\right)$$

$$= 2\left[\frac{1}{1} - \frac{1}{5} + \frac{1}{5} - \frac{1}{13} + \frac{1}{13} ..... + \frac{1}{(2n^2-2n+1)} - \frac{1}{(2n^2+2n+1)}\right]$$

33. B,C,D $a, b, \frac{1}{18}$ are in G.P ;

$\frac{a}{18} = b^2$............(1)

$\frac{1}{a} + \frac{1}{b} = 20$

$a + b = 20ab$ ....(2)

$18b^2 + b = 360b^3 ; 360b^2 - 18b - 1 = 0, b \neq 0 ; b = \frac{1}{12}$ $a = \frac{1}{8}$

34. A,C $5S_n = (1)(5)^2 + (2)(5^3) + ... + (n-1)5^n + (n)5^{n+1}$

Subtracting from S_n, we obtain

$$-4S_n = 5 + 5^2 + ... + 5^n - n(5^{n+1}) = \frac{5(5^n - 1)}{4} - n(5^{n+1})$$

$$\therefore S_n = \frac{1}{16}\left[(4n-1)5^{n+1} + 5\right]$$

SECTION V - (Numerical type)

35. 3 $\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ca}=\frac{1}{abc}$; $\frac{a+b+c}{3}\geq(abc)^{\frac{1}{3}}$

$(abc)^{\frac{1}{3}}\leq\frac{1}{3}\Rightarrow abc\leq\frac{1}{27}$

36. 1 $\frac{\frac{n}{2}\left[2a_1+(n-1)d_1\right]}{\frac{n}{2}\left[2a_2+(n-1)d_2\right]}=\frac{7n+1}{4n+17}$; $\frac{a_1+\frac{n-1}{2}d_1}{a_2+\frac{n-1}{32}d_2}=\frac{7n+1}{4n+17}$

Now $\frac{n-1}{2}=m-1\Rightarrow n=2m-1$

$\frac{a_1+(m-1)d_1}{a_1+(m-1)d_2}=\frac{7(2m-1)+1}{4(2m-1)+17}$

Replace m and n; $\frac{a_1+(n-1)d_1}{a_1+(n-1)d_2}=\frac{14n-6}{8n+13}$

$\Rightarrow\ \lambda=13$

37. 5 $\alpha=\frac{1}{14}+\frac{1}{24}+\frac{1}{34}+\frac{1}{44}+\text{.....to }\alpha$

$\beta=\frac{1}{14}+\frac{1}{34}+\frac{1}{54}+\frac{1}{74}+\text{.....to }\alpha$

$=\left(\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+....\right)-\left(\frac{1}{2^4}+\frac{1}{4^4}+\frac{1}{6^4}+....\right)$

$=\alpha-\frac{1}{2^4}\left(\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+....\right)=\alpha-\frac{\alpha}{16}$; $=\frac{15\alpha}{16}$; $\therefore\frac{16\beta}{\alpha}=15$; $\therefore\frac{16\beta}{15}-10=5$

38. 12 $\frac{a_6}{a_4}=\frac{4}{1}\Rightarrow\frac{ar^5}{ar^3}=4$

$\Rightarrow r^2=4$

Now $a_5+a_7=340$

$\Rightarrow ar^4+ar^6=340$

$\Rightarrow ar^4(1+r^2)=340 \Rightarrow a\times16(1+4)=340$

$\Rightarrow a=\dfrac{340}{16\times5}=3$

$a_3=ar^2;=3\times h=12$

39. 5 First A.P is

3,7,11,15,19,23,27,31,35,39,43,47,51,......

Second A.P is

2,9,16,23,30,37,44,51,58,.....

Common terms are

23,51,.....

d = 28 (product of common difference of the two A.P's = 4×7=28

If n is the numbers of common terms

$a_n \le 142$

$=123+(n-1)28\le142;\ (n-1)28\le119$

$n-1\le\dfrac{119}{28}\le4.25\Rightarrow n-1\le4; n\le5$

SECTION VI - (Matrix match type)

40. A A-PQRS; B-RS; C-PQ; D-RS

A. $\sum n=210\Rightarrow n(n+1)=420$

B. $G^2_{n+1}=4(2916)$

C. $\dfrac{1}{30}-\dfrac{1}{40}=\dfrac{1}{24}-\dfrac{1}{30}=\dfrac{1}{20}-\dfrac{1}{24}=\dfrac{1}{120}$ hence

$\dfrac{1}{40},\dfrac{1}{30},\dfrac{1}{24},\dfrac{1}{20}$ are in AP with common difference $\dfrac{1}{120}$

D. $s-\dfrac{1}{4}s=1+3\left(\dfrac{1}{5}+\dfrac{1}{5^2}+\dfrac{1}{5^3}+.........\infty\right)$