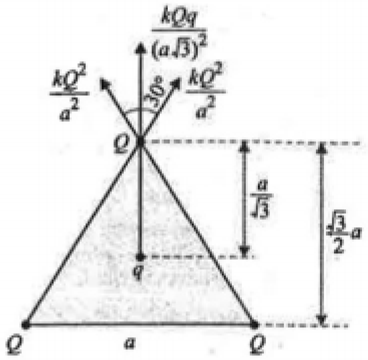
CHAPTER - 12
ELECTROSTATICS

1. 1 Force on charge q towards right is

$$F = \frac{kQq}{(l/2)^2} + \frac{k(4q)q}{l^2} = 0 \quad \therefore \quad Q = -q$$

2. 3

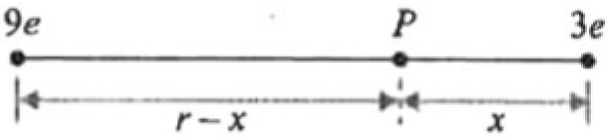


Charge q is in equilibrium, irrespective of value. As charge Q is also in equilibrium, net force acting on it is zero.

$$\Rightarrow \quad \frac{2kQ^2}{a^2}\cos 30° + \frac{kQq}{(a/\sqrt{3})^2} = 0$$

$$\Rightarrow \quad \sqrt{3}Q + 3q = 0 \qquad \therefore \quad q = -Q$$

3. 1

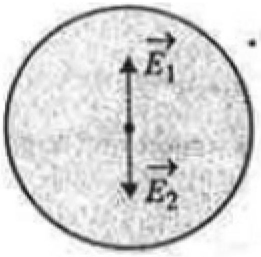


Let the field intensity be zero at point P.

Then, $\frac{k(3e)}{x^2} = \frac{k(9e)}{(r-x)^2} \quad \Rightarrow \left(\frac{r-x}{x}\right)^2 = 3$

$$\Rightarrow \quad r - x = \sqrt{3}x \qquad \therefore \quad x = \frac{r}{1+\sqrt{3}}$$

4. 1



Let $\vec{E_2}$ be the field due to the removed portion and $\vec{E_1}$ be the field due to balance ring. By superposition principle, the sum of these two is zero, *i.e.*, $\vec{E_1}+\vec{E_2}=0$.

Now, $\vec{E_2}=\frac{1}{4\pi\varepsilon_0}\left(\frac{d}{2\pi R}\right)\frac{Q}{R^2}$. It is away from gap and inversely proportional to R^3.

$\therefore\ \vec{E_1}=-\vec{E_2}$ is towards the gap and inverse proportional to R^3.

5. 2 $E_A=\frac{k(3Q+Q)}{(2R+x)^2}$ and $E_B=\frac{kQ}{(2R-x)^2}$

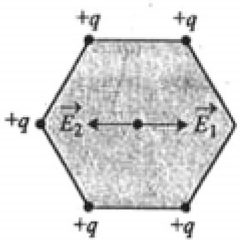
Since, $E_A=E_B$, we have

$\frac{4kQ}{(2R+x)^2}=\frac{kQ}{(2R-x)^2} \Rightarrow \frac{2R}{2R}$

$\therefore\ x=2R/3$

6. 4

7. 1



Let $\vec{E_1}$ be the electric field due to the five charges shown in fig. 13.223 and let $\vec{E_2}$ be the field due to the same charge $+q$ if placed at the sixth unoccupied corner. Then, due to all six charges (if present), the net electric field is zero, *i.e.*, $\vec{E_1}+\vec{E_2}=0$

$\Rightarrow\ \vec{E_1}=-\vec{E_2} \quad \therefore\ |\vec{E_1}|=|\vec{E_2}|=\frac{q}{4\pi\varepsilon_0 a^2}$

8. 4

The net force on the sphere is

$$F = \sqrt{(mg)^2 + (qE)^2}$$

The effective gravity is

$$g' = \frac{F}{m} = \sqrt{g^2 + (qE/m)^2}$$

$$\therefore \; T = 2\pi\sqrt{\frac{L}{g'}} = 2\pi\sqrt{\frac{L}{\sqrt{g^2 + (qE/m)^2}}}$$

9. 2

The electric field at point P

due to charge σ is $\sigma/2\,\varepsilon_0$ downwards

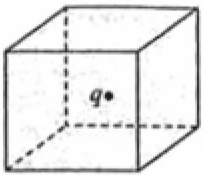
due to charge $-2\,\sigma$ is $2\,\sigma/2\,\varepsilon_0$ downwards

due to charge $-\,\sigma$ is $\sigma/2\,\varepsilon_0$ downwards

The net field is $\frac{\sigma}{2\varepsilon_0} + \frac{2\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}$ downwards.

$$\therefore \; E = -\frac{2\sigma}{\varepsilon_0}\hat{k}$$

10. 4



On completing the cube with the bottom s… as one of the face, we find that the c… $q = +6\,\mu C$ is symmetrically placed at the c… of cube.

The flux through the cube is $\phi = q/\varepsilon_0$

The flux through the bottom face is

$$\frac{\phi}{6} = \frac{q}{6\varepsilon_0} = \frac{6\times10^{-6}}{6\varepsilon_0} = \frac{1}{\varepsilon_0}\times10^{-6}$$

11. 4

Let's place eight cubes symmetrically wi… cubes at bottom and four cubes at the to… charge q exactly at the centre of new bi… formed. In all, there will be $3 \times 8 = 24$ fa… small cube through which the flux will be… The flux through face $ABCD$ will be th…

$\frac{q/\varepsilon_0}{24} = \frac{q}{24\varepsilon_0}$. The flux through the… $HGFE$ is zero since, no line crosses th…

12. 2 Net PE is zero.

$$\Rightarrow \frac{kq^2}{a}+\frac{kqQ}{a}+\frac{kQq}{\sqrt{2}a}=0$$

$$\therefore\ Q=-\frac{2q}{2+\sqrt{2}}$$

13. 4 $\because W=q\,(V_f-V_i)$

$$\therefore\ V_f=\frac{W}{q}+V_i=\frac{100}{-5}+0=-20\,\text{V}$$

14. 2 As the potential on a conductor is same at all points, we shall calculate the potential at its centre. Let q' be the net induced charge on the sphere which we know is zero. Potential at the centre of sphere is

$$V=\frac{1}{4\pi\varepsilon_0}\frac{q}{r}+\frac{1}{4\pi\varepsilon_0}\frac{q'}{R}=\frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

15. 1 If we add one more such hemisphere to the existing hemisphere, we now have a complete

sphere of charge $2Q$. The potential at the centre of sphere is

$$V=\frac{3}{2}\times\frac{1}{4\pi\varepsilon_0}\frac{2Q}{R}=\frac{1}{4\pi\varepsilon_0}\frac{3Q}{R}$$

The potential due to hemisphere at centre is

$$\frac{V}{2}=\frac{1}{4\pi\varepsilon_0}\frac{3Q}{2R}$$

16. 4 From the figure, we can see that the distance between two equipotential surfaces is

$$d=10\sin 30^\circ=5\text{ cm}$$

$$\therefore\ E=\frac{V}{d}=\frac{40-20}{5\times10^{-2}}=400\,\text{N/C}$$

17. 3 The field at point $(a, 0)$ due to the dipoles along X-axis and Y-axis are respectively

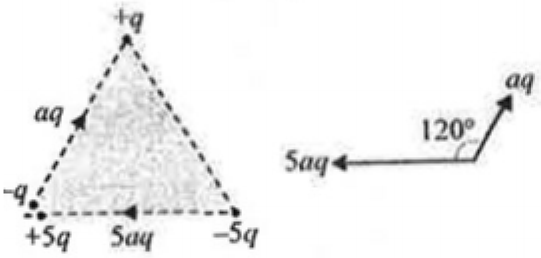
$$\vec{P_1}=\frac{k2p}{a^3}\hat{i}\quad\text{and}\quad\vec{P_2}=\frac{kp}{a^3}\hat{j}$$

The magnitude of net electric field is

$$\sqrt{\left(\frac{k2p}{a^3}\right)^2+\left(\frac{kp}{a^3}\right)^2}=\frac{k\sqrt{5}p}{a^3}$$

$$=\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{\sqrt{5}p}{a^3}$$

18. 1 The charges in given figure can be broken up as shown in fig. 13.235.



We now have two dipoles of dipole moments aq and 5 aq as shown with angle 120° between them.

The magnitude of net dipole moment is

$$p=\sqrt{(aq)^2+(5aq)^2+2(aq)(5aq)\cos 120^\circ}$$

$$=\sqrt{21}\,aq\,.$$

19. 2 $\vec{r}_{AB}=\vec{r}_B-\vec{r}_A=2\hat{i}+3\hat{j}+4\hat{k}$

$\therefore\ V_A-V_B=\vec{E}\cdot\vec{r}_{AB}$

$$=(\hat{i}+2\hat{j}+\hat{k})\cdot(2\hat{i}+3\hat{j}+4\hat{k})=12\,V$$

20. 8 For the potential energy to be minimum, the larger charges must be placed at maximum distanceas shown in figure.

PE of system is $U=k\left(\frac{2q^2}{x}+\frac{8q^2}{24-x}+\frac{16q^2}{24}\right)$

For U to be minimum, dU/dx=0

$$\Rightarrow kq^2\left(-\frac{2}{x^2}+\frac{8}{(24-x)^2}\right)=0$$

$\Rightarrow 24-x=2x\ ;\ \therefore x=8\,cm$

21. 6

Consider an element of the rod as shown in fig. 13.274.

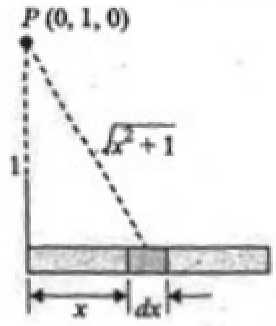


FIGURE 13.274

Its charge is $\lambda\, dx = ax\, dx$

The potential at P due to this element is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{ax\,dx}{\sqrt{x^2+1}}$$

$$\Rightarrow \quad V = \frac{a}{4\pi\varepsilon_0}\int_0^{\sqrt{0.44}} \frac{a\,dx}{\sqrt{x^2+1}}$$

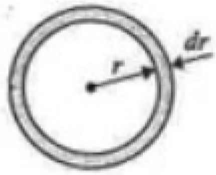
$$= \frac{a}{4\pi\varepsilon_0}\sqrt{x^2+1}\Big|_0^{\sqrt{0.44}}$$

$$= 9\times10^9\times\frac{20}{9}\times10^{-6}\times(1.2-1) = 4\times10^3\ \text{V}$$

Work done to bring charge q from infinity is

$W = qV = 1.5\times10^{-3}\times4\times10^3 = 6$ J

22. 2



The charge in a shell of radius r and thickness dr is

$dq = \rho\,(4\,\pi r^2\,dr) = kr^a\,(4\,\pi r^2\,dr)$

The charge in a sphere of radius r is

$$q = 4\pi k\int_0^r r^{a+2}\,dr = \frac{4\pi k}{a+3}r^{a+3}$$

The electric field E at distance r from the centre of sphere by Gauss law is given by

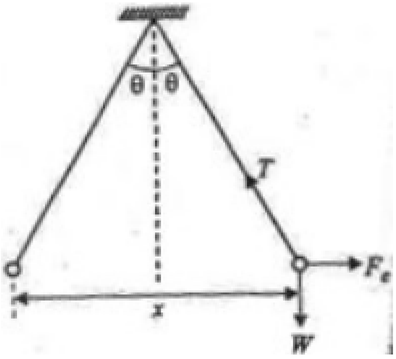
$$E(4\pi r^2) = \frac{q}{\varepsilon_0} = \frac{4\pi k r^{a+3}}{\varepsilon_0(a+3)} \quad \Rightarrow E = \frac{kr^{a+1}}{\varepsilon_0(a+3)}$$

Since, $E(R/2) = \frac{1}{8} E(R)$, we have

$$\frac{k(R/2)^{a+1}}{\varepsilon_0 (a+3)} = \frac{1}{8}\frac{kR^{a+1}}{\varepsilon_0 (a+3)}$$

$\Rightarrow 2^{a+1} = 8 \qquad \therefore a = 2$

23. 4



In equilibrium,

$T\cos\theta = W$ and $T\sin\theta = F_e$

$\Rightarrow F_e = W\tan\theta$

When suspended in air,

$$\frac{1}{4\pi\varepsilon_0}\frac{q^2}{x^2} = mg\tan\theta$$

When suspended in liquid, the app

is

$$W' = \rho V g - \rho_l V g = \rho V g\left(1 - \frac{\rho}{\rho}\right)$$

$$= mg\left(1 - \frac{900}{1200}\right) = \frac{mg}{4}$$

and electrostatic force is $F_e' = \frac{\quad}{4\pi}$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0 K}\frac{q^2}{x^2} = \frac{mg}{4}\tan\theta$$

From eqns. (1) and (2), we get

24. 0 Since, $V_A = V_C$, we have

$$k\left[\frac{\sigma(4\pi a^2)}{a} - \frac{\sigma(4\pi b^2)}{b} + \frac{\sigma(4\pi c^2)}{c}\right]$$

$$= k\left[\frac{\sigma(4\pi a^2)}{c} - \frac{\sigma(4\pi b^2)}{c} + \frac{\sigma(4\pi c^2)}{c}\right]$$

$$\Rightarrow a-b=\frac{a^2-b^2}{c}\Rightarrow c=a+b$$

$$\therefore c-(a+b)=0$$

PART - II (JEE ADVANCED LEVEL)

SECTION - I (One correct answer type including passage)

25. B As the solid sphere is grounded, its potential is zero. Let q_1 be the charge that appears on it. Its potential is

V_1 = (Pot. due to q_1) + (Pot. due to outer sphere) + (Pot. due to q)

$$\Rightarrow \quad 0=\frac{kq_1}{a}+0+\frac{kq}{4a} \qquad \therefore \quad q_1=-\frac{q}{4}$$

26. C The inside region of the outer sphere will be shielded from charge q. The induced charge on inner sphere and inner surface of outer shell will be uniform whereas the induced charge on outer surface of outer shell will be non-uniform.

27. D As the net charge inside the outer shell is zero and inner shell acquires charge q_1, the inner surface of outer shell will acquire charge $-q_1$ and the outer surface of outer shell will acquire charge q_1.

28. C The net effective gravity on the particle in downward direction is

$$g'=\frac{mg+q_0E}{m}=g+\frac{\sigma q_0}{\varepsilon_0 m}$$

The minimum horizontal velocity required is

$$v=\sqrt{5g'r}=\sqrt{5l\left(g+\frac{\sigma q_0}{\varepsilon_0 m}\right)}$$

29. D For the bullet to penetrate through the sphere, it must be able to reach the centre of sphere.

The potential at the centre of sphere is

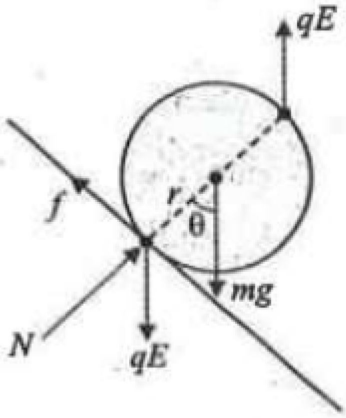
$$V = \frac{3}{2} \times \frac{1}{4\pi\varepsilon_0} \frac{q}{R}$$

For the minimum speed u of the bullet, its speed at the centre is zero. Applying cons. of mechanical energy, we have

$$\frac{1}{2}mu^2 + 0 = 0 + q \times \left(\frac{3}{2} \times \frac{1}{4\pi\varepsilon_0} \frac{q}{R}\right)$$

$$\therefore \quad u = \frac{\sqrt{3}\,q}{\sqrt{4\pi\varepsilon_0 mR}}$$

30. B

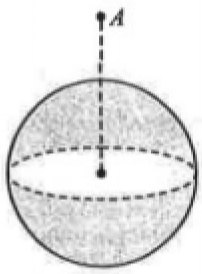


For the wheel to be in equilibrium, the net force and the net torque on it must be zero. Taking the torque about point of contact to be zero, we have

$mgr \sin\theta - qE\,(2\,r\sin\theta) = 0$

$\therefore\ E = mg/2\,q$

31. B



Consider a complete sphere consisting of two hemispheres as shown in . If E and E' are the fields at A due to lower hemisphere and upper hemisphere respectively, then the total field at A is $E_T = E + E'$

$$\therefore\ E' = E_T - E = \frac{1}{4\pi\varepsilon_0} \times \left(\frac{4}{3}\pi R^3\right)\rho \times \frac{1}{(2R)^2} - E$$

$$= \frac{\rho R}{12\varepsilon_0} - E$$

This shall also be the field at B due to lower hemisphere.

32. ABC $V(x) = 4 + 5x^2$

$\Rightarrow\ V(1) = 9\text{ V}$ and $V(-2) = 24\text{ V}$

$\Rightarrow\ |V(1) - V(-2)| = 15\text{ V}$ $\therefore$ (*a*) is correct.

$E(x) = -dV(x)/dx = -10x$

$\Rightarrow\ F(x) = qE(x)$

$\Rightarrow\ F(-1) = 1 \times (10) = 10\text{ N}$

$\therefore$ (*b*) and (*c*) are correct.

Since, E depends upon x, E along x-axis constant.

$\therefore$ (*d*) is incorrect.

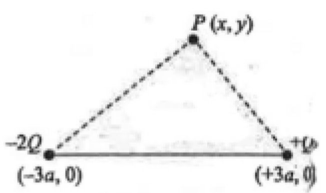
33. A,C When the particle P is displaced along Z-axis, a restoring attractive force acts on it by the ring. The motion is therefore, oscillatory and periodic for all values of z_0.

$\therefore$ (*a*) is correct.

If can be proved that if $z_0 << R$, the motion is approximately SHM.

$\therefore$ (*c*) is correct.

34. A,C



35. A,B,C,D

36. A,B

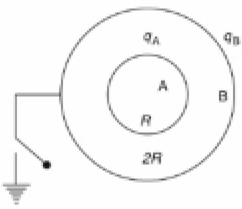
Before earthing:

$$V_A = 2V = k\left[\frac{q_A}{R} + \frac{q_B}{2R}\right] \qquad (1)$$

$$V_B = \frac{3}{20}V = k\left[\frac{q_A + q_B}{2R}\right] \qquad (2)$$

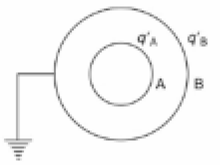
Dividing Eq. (1) by Eq. (2), we get

$$\frac{4}{3} = \frac{2q_A + q_B}{q_A + q_B} \Rightarrow q_A : q_B = \frac{1}{2}$$



After earthing,

$$V_B = 0 = k\left[\frac{q'_A + q'_B}{2R}\right] \Rightarrow q'_A = -q'_B$$



37. 1.73

38. 8 Since electrostatic energy stored is

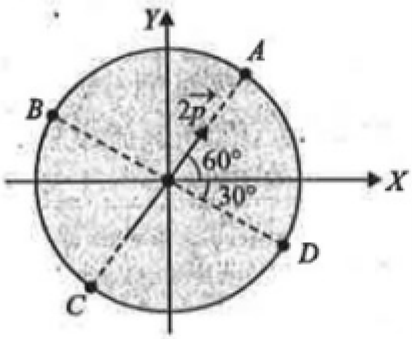
$$U = \int \frac{1}{2}\varepsilon_0 E^2 d\tau \text{ where } dr \text{ is small volume}$$

$$\Rightarrow \quad U = \frac{1}{2}\varepsilon_0 E^2 \int d\tau \text{, where } E = \frac{\sigma}{2\varepsilon_0} = \text{constant}$$

$$\Rightarrow \quad U = \frac{1}{2}\varepsilon_0\left(\frac{\sigma^2}{4\varepsilon_0^2}\right)a^3 = \frac{\sigma^2 a^3}{8\varepsilon_0}$$

39. $A \to p$, $B \to r,s$, $C \to p,q$, $D \to r,s$

The resultant of two dipoles $p\hat{i}$ and $\sqrt{3}\,p\hat{j}$ has a magnitude $\sqrt{1^2+(\sqrt{3})^2}\,p = 2p$ making an angle of 60° with X-axis as shown in the figure.



The potential is maximum at A, *i.e.*, at $\left(\frac{R}{2}, \frac{\sqrt{3}R}{2}\right)$

The potential is zero at B and D, *i.e.*, at $\left(-\frac{\sqrt{3}}{2}R, \frac{R}{2}\right)$ and $\left(\frac{\sqrt{3}}{2}R, -\frac{R}{2}\right)$

The magnitude of electric field is maximum at A and C,

i.e., at $\left(\frac{R}{2}, \frac{\sqrt{3}}{2}R\right)$ and $\left(-\frac{R}{2}, -\frac{\sqrt{3}}{2}R\right)$.

The magnitude of electric field is minimum at B and D,

i.e., at $\left(-\frac{\sqrt{3}}{2}R, \frac{R}{2}\right)$ and $\left(\frac{\sqrt{3}}{2}R, -\frac{R}{2}\right)$