UNITS & MEASUREMENTS

Measurement of a physical quantity is the comparison with a standard value of the same kind is called the unit of that quantity. The process of measurement of a physical quantity involves,

1) selection of unit (u)

2) to find out the no. of times that unit is contained in the given physical quantity. It is called the numerical value OR magnitude of the quantity (n)

$\therefore$ Any measurement (X) can be represented as the product of numerical value and unit

$\boxed{X = nu}$

Fundamental and Derived units

The physical units which can neither be derived from one another, nor they can be further resolved in to more simpler units are called fundamental units

eg. metre, kg, second

All other physical units which can be expressed in terms of fundamental units are called derived units.

eg. $ms^{-1}, kg\ ms^{-2}\ (N)$

System of Units

A complete set of units which is used to measure all kinds of fundamental and derived quantities are called system of units

1) CGS system -	centimetre, gram, second
2) FPS system -	foot, pound, second
	1 foot = 0.3048 m
	1 pound = 0.4536 kg
3) MKS system -	metre, kg, second
4) SI system -	(International system of units)

Basic SI units	Supplementary SI units
Length - metre (m)	Plane angle - radian (rad)
Mass - kilogram (kg)	Solid angle - steradian (sr)
Time - second (s)	

Temperature - kelvin (K)

Electric current - ampere (A)

Luminous intensity - candela (Cd)

Amount of substance - mole (mol)

SI prefixes for powers of ten

10^{1} - deca (da)	10^{-1} - deci (d)
10^{2} - hecto (h)	10^{-2} - centi (c)
10^{3} - kilo (k)	10^{-3} - milli (m)
10^{6} - mega (M)	10^{-6} - micro (μ)
10^{9} - giga (G)	10^{-9} - nano (n)
10^{12} - tera (T)	10^{-12} - pico (p)
10^{15} - peta (p)	10^{-15} - femto (f)
10^{18} - exa (E)	10^{-18} - atto (a)

Some common practical Units

Large distances

1) Light year (ly)

It is the distance travelled by light through vacuum in one year

$$\boxed{1\ \ell y = 9.46\times10^{15}\,\text{m}}$$

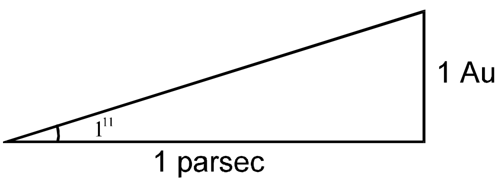
2) Astronomical Unit (Au)

It is the average distance between centre of earth and centre of Sun

$$\boxed{1\ \text{Au} = 1.496\times10^{11}\,\text{m}}$$

3) Par sec (parallactic sec)

It is the distance at which an arc of length one astronomical unit subtends an angle of 1 second of arc



$$\boxed{1\ \text{par sec} = 3.08\times10^{16}\,\text{m}}$$

$$\boxed{1\ \text{par sec} = 3.26\ \ell y}$$

Large Masses

1) tonne or metric ton = 1000 kg

2) quintal = 100 kg

3) Chandra Shekhar Limit (CSL) = 1.4 times the mass of sun

Small masses

Atomic mass unit (amu) = It is defined as $\frac{1}{12}^{th}$ of the mass of one ${}^{12}_{6}C$ - atom

$\boxed{1\,\text{amu} = 1.66\times10^{-27}\,\text{kg}}$

Time

1) Solar day - One day (24 hour)

2) Solar year - 365.25 days

3) Lunar month - It is the time taken by the moon to complete one revolution around the earth in its orbit

4) Shake - It is the smallest practical unit of time

$\boxed{1\,\text{shake} = 10^{-8}\,\text{sec}}$

Small Areas

$\boxed{\text{Barn} = 10^{-28}\,\text{m}^2}$

Order of Magnitude

The order of magnitude of a quantity means its value (in suitable power of 10) nearest to the actual value of that quantity. Consider a no. as $a\times10^{b}$ where a is in between 1 & 10, then a is replaced with 10^{0} OR 1 if $a \le 5$ and with 10^{1} if $5 < a \le 10$. The resulting power of 10 at which the number is reduced is called its order of magnitude.

Eg. 0.005289 $\Rightarrow 5.289\times10^{-3}$

5.289 is replaced with 10

$\Rightarrow 10\times10^{-3} \Rightarrow 10^{-2}$ then its order of magnitude is –2

Dimensional Analysis

The dimensions of a physical quantity are the powers to which the units of base quantities are raised to represent a derived unit of that quantity. It is denoted with square brackets [ ]

Eg. Force, F = ma = $\left[M^{1}L^{1}T^{-2}\right]$

• The physical quantities can be added or substracted which have the same dimensions

• Special functions such as trigonometric functions, logarithmic functions, and exponential functions must be dimensionless

• A pure number, ratio of similar physical quantities has no dimension. (Eg. Angle, refractive index, π,...etc)

Different quantities having same dimension

$$\left.\begin{array}{l}\text{Work}\\ \text{Energy}\\ \text{Heat}\\ \text{Torque}\\ \text{Moment of force}\end{array}\right\}\left[ML^2T^{-2}\right] \qquad \left.\begin{array}{l}\text{Linear momentum}\\ \text{Impulse}\end{array}\right\}\left[MLT^{-1}\right]$$

$$\left.\begin{array}{l}\text{Surface tension}\\ \text{Surface Energy}\\ \text{Spring constant}\end{array}\right\}\left[ML^0T^{-2}\right]$$

Dimensional constants :	Speed of light (C)
	Gravitational constant (G)
	Planks constant (h)
Dimensional variables :	Area, volume, force,....
Dimensionless constants:	Numbers, π, ...
Dimensionless variables :	Angle, strain, specific gravity, ...

A dimensionally correct equation need not be actually a correct equation, but dimensionally wrong equation must be wrong

Applications of Dimensional Analysis

1. Conversion of one system of units to another

This is based on the fact that magnitude of a physical quantity remains the same whatever be the system of units.

Q = nu = constant

$$\boxed{n_1u_1 = n_2u_2}$$

$$u_1 = M_1^aL_1^bT_1^c \qquad u_2 = M_2^aL_2^bT_2^c$$

$$n_2 = \frac{n_1u_1}{u_2}$$

$$\boxed{n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c}$$

Eg. Convert 1 N to dyne (CGS system)

$[F]=\left[M^1L^1T^{-2}\right]$. Here a = 1, b = 1, c = –2

In SI system M_1 = kg, L_1 = m, T_1 = sec

In CGS system M_2 = g, L_2 = cm, T_2 = sec

n_1 = 1 n_2 = ?

$$n_2 = 1\left[\frac{kg}{g}\right]^1\left[\frac{m}{cm}\right]^1\left[\frac{s}{s}\right]^{-2}$$

$$= \left[\frac{10^3 g}{g}\right]\left[\frac{10^2 cm}{cm}\right]$$

$$\boxed{n_2 = 10^5} \qquad \therefore 1N = 10^5 \text{dyne}$$

2. Checking the correctness of an equation

   (Principle of homogeneity dimensions)

   According to this principle, when a relation is dimensionally correct, then the dimensions of all the terms in that relation are equal

   Eg. $S = ut + \frac{1}{2}at^2$

   $$[s] = [ut] = \left[\frac{1}{2}at^2\right]$$

3. To derive the relationship among various physical quantities

   Using the principle of homogeneity of dimension we can derive the formula of a physical quantity.

   Eg. Derive an expression for the time period (T) of a simple pendulum depends mass (m), length (ℓ) and acceleration due to gravity (g)

   Let $T \,\alpha\, m^a \ell^b g^c$

   $$T = K\, m^a \ell^b g^c$$

   $$\left[M^0L^0T^1\right] = M^aL^b\left(LT^{-2}\right)^c$$

   $$M^0L^0T^1 = M^aL^{b+c}T^{-2c}$$

   Applying the homogeneity of dimension

   a = 0, b + c = 0, –2c = 1

   a = 0, $c = -\frac{1}{2}$, $b = \frac{1}{2}$

   $$\therefore T = KM^0\ell^{1/2}g^{-1/2}$$

   $$T = K\sqrt{\frac{\ell}{g}} \qquad k = 2\pi$$

   $$\boxed{T = 2\pi\sqrt{\frac{\ell}{g}}}$$

Limitations

1. If a quantity depends on more than three factors having dimensions, the formula cannot be derived
2. The method of dimensions cannot be used to derive an exact form of relation, when it consists of more than one part on any side

   eg. $S = ut + \frac{1}{2}at^2$

3. It gives us no information about the dimensionless constants in the relation eg. π,1,2...
4. We cannot derive the formula containing trigonometrical function, exponential function, logarithmic function, etc. which are dimensionless

Significant Figures

In all instrumental values, last digit remains uncertain and the rest of the digits are certain or reliable digits. The total number of certain digits along with first uncertain digit gives the number of significant digits.

In an instrumental value all nonzero digits, trapped zeros, and terminal zero's in a number with decimal point are measured as significant digits.

The insignificant digits are terminal zero's without a decimal point, the zero's on the right of decimal point (to the left of Ist non-zero digit in a number less than one), and the power of 10.

• Change of units does not change the no. of significant figures in a measurement

• The multiplying or dividing factors are exact values, they have infinite no. of significant figures as per the situation

ROUNDING OFF

1) If the digit to be dropped is smaller than 5, then the preceeding digit is left unchanged

2) If the digit to be dropped is greater than 5, then the preceeding digit is increased by 1

3) If the digit to be dropped is 5, then the preceeding digit is increased by 1 if it is odd, and is left unchanged if it is even

Arithmetic Operations with Significant Figures

1. In addition and subtraction, the final result should retain the same number of decimal places as that of the original measurement with minimum number of decimal places.
2. In multiplication and division, the final result should retain the same number of significant figures as that of the original measurement with minimum number of significant figures.

Errors in a measurement

Error in a measurement is equal to the difference between the true value and the measured value of the quantity

$$\boxed{\text{Error} = \text{True value} - \text{Measured value}}$$

Let $a_1, a_2, a_3, ... a_n$ are 'n' measured values, then the accepted true value is their average value

$$r_{mean} = \frac{a_1 + a_2 + ...a_n}{n} = \frac{1}{n}\sum_{i=1}^{n} a_i$$

1) Absolute error (Δ)

The magnitude of the difference between the true value and the individual measured value is called absolute error.

$$\Delta a_1 = |a_{mean} - a_1|$$

$$\Delta a_2 = |a_{mean} - a_2|$$

$$\Delta a_n = |a_{mean} - a_n|$$

$\therefore$ Mean absolute error $\boxed{\Delta a_{mean} = \frac{1}{n}\sum_{i=1}^{n} \Delta a_i}$

i.e. the final result of measurement may be written as $\boxed{a = a_{mean} \pm \Delta a_{mean}}$

2. Relative error (δ) / Fractional error

   It is the ratio of mean absolute error to the mean value of the quantity measured

$$\boxed{\delta_a = \frac{\Delta a_{mean}}{a_{mean}}}$$

3. Percentage error (%)

   The relative error is expressed in percent is called percentage error

$$\boxed{\%\, a = \frac{\Delta a_{mean}}{a_{mean}} \times 100\%}$$

Propagation of Errors

1. Error in sum and difference of two quantities

   Z = A + B OR Z = A – B

$$\boxed{\Delta Z = \Delta A + \Delta B}$$

   The maximum error in the result is equal to the sum of the absolute errors in the individual quantities

2. Error in product or quotient of two quantities

   Z = AB OR $Z = A/B$

$$\boxed{\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}}$$

$$\boxed{\Delta Z = Z\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)}$$

$$\boxed{\%Z = \%A + \%B}$$

The maximum fractional error in the result is equal to the sum of their individual fractional errors.

3. Error of a quantity raised to a power

$Z = A^m B^n$ OR $Z = \dfrac{A^m}{B^n}$

$$\boxed{\frac{\Delta Z}{Z} = \frac{m\Delta A}{A} + \frac{nB}{B}}$$

$$\boxed{\%Z = m\%A + n\%B}$$

NOTE

If a value alone is given (eg. ℓ = 7.6 cm) without specifying error then the least count of the measuring device gives the value its absolute error

If ℓ = 7.6 cm then $\Delta\ell = 0.1\,\text{cm}$

If M = 12.28 kg then $\Delta m = 0.01\,\text{kg}$