CHAPTER - 10
OSCILLATIONS

1. 4
2. 4
3. 3
4. 1

5. 2

$k' = 5k$

$k_p = 3k' = 15k$

$\therefore T = 2\pi\sqrt{\frac{m}{k_p}} = 2\pi\sqrt{\frac{5}{15 \times 150}} = 2\pi\sqrt{\frac{1}{3 \times 150}}$

$= \frac{2\pi}{\sqrt{9 \times 5 \times 10}} = \frac{2}{3\sqrt{5}}$ second.

6. 3

Both the spring are in series

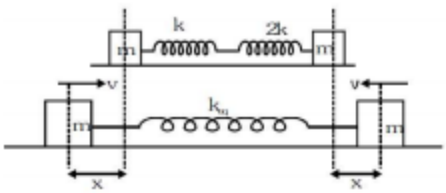
$\therefore K_{eq} = \frac{K(2K)}{K+2K} = \frac{2K}{3}$

Time period $T = 2\pi\sqrt{\frac{\mu}{K_{eq}}}$

where $\mu = \frac{m_1 m_2}{m_1 + m_2}$ Here $\mu = \frac{m}{2}$

$\therefore T = 2\pi\sqrt{\frac{m}{2}\cdot\frac{3}{2K}} = 2\pi\sqrt{\frac{3m}{4K}}$

OR



Total extension $= 2x$

By energy conservation

$E = \frac{1}{2}K_{eq}(2x)^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2$

$E = \frac{1}{2}\frac{2k}{3}4x^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = \frac{4}{3}kx^2 + mv^2$

$\frac{dE}{dt} = \frac{4}{3}k(2x)\frac{dx}{dt} + m(2v)\frac{dv}{dt}$

there is no loss of energy

$$\frac{dE}{dt} = 0 \Rightarrow \frac{8}{3}kxv + 2mva = 0 \Rightarrow \frac{8kxv}{3} = -2mva$$

$$a = -\frac{4kx}{3m} \Rightarrow -\omega^2 x = -\frac{4kx}{3m} \Rightarrow \omega = \sqrt{\frac{4k}{3m}}$$

$$T = \frac{2\pi}{\omega} \Rightarrow 2\pi\sqrt{\frac{3m}{4k}}$$

7. 1

8. 3

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$$

$$\frac{f_1}{f_2} = \sqrt{\frac{\ell_2}{\ell_1}} \Rightarrow \left(\frac{n}{n+1}\right)^2 = \frac{\ell_2}{\ell_1} \Rightarrow \frac{\ell_1}{\ell_2} = \left(\frac{n+1}{n}\right)^2$$

9. 4
10. 3
11. 1
12. 3
13. 3

$U(x) = ax^2 + bx^4$

$$F = -\frac{\partial U}{\partial x} = -2ax - 4bx^3 \approx -2ax \text{ for small } x$$

So $m\omega^2 = 2a \Rightarrow \omega = \sqrt{\frac{2a}{m}}$

14. 1

$$y = 5\sin\left[\omega t + \pi/6\right] + \sqrt{3}\sin\left[\omega t + \frac{2\pi}{3}\right]$$

here phase difference $\phi = 90^\circ$

$\therefore$ Resultant Amplitude

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

$$= \sqrt{25 + 3}$$

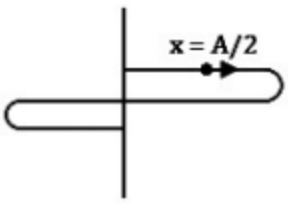
$$= \sqrt{28} = 2\sqrt{7}\ m$$

15. 6 Required time $= \frac{T}{4} - \frac{T}{12} = \frac{T}{6}$

16. 16

PART - II (JEE ADVANCED LEVEL)

17. A



Initial phase $\phi = \sin^{-1}\left(\frac{A/2}{A}\right) = \frac{\pi}{6}$

18. A w.r.t stationary block

$$T = 2\pi\sqrt{\frac{\mu}{k}} \text{ (for oscillation)}$$

So time of contact $T/2$

$$t = \frac{T}{2}$$

$$= \pi\sqrt{\frac{4\times4}{4+4}\over\pi^2}$$

$$= \sqrt{2} \text{ sec.}$$

19. C $$\frac{1}{k_{eq.}} = \frac{1}{k_1} + \frac{1}{k_2}$$

20. B $$T = 2\pi\sqrt{\frac{\frac{2m\ell^2}{3}}{2m\cdot g\cdot\frac{\ell}{2\sqrt{2}}}} = 2\pi\sqrt{\frac{2\sqrt{2}\ell}{3g}}$$

21. ABC (A) $K.E. = 0.64\,E$

$P.E. = 0.36\,E$

$$\frac{x^2}{A^2} = 0.36$$

$\therefore\ x = 6\ cm$

(B) $P.E. = 0.25\,E$

$K.E. = 0.75\,E$

(C) $E = KE_{max} = PE_{max}$

(D) $v = \frac{v_m}{2}$ then $A = \frac{\sqrt{3}}{2}A_m$

22. ACD (A) $$\vec{r} = (1+2\cos2\omega t)\hat{i} + \left(\frac{3}{2} - \frac{3}{2}\cos2\omega t\right)\hat{j} + (3)\hat{k}$$

$$\therefore\ m_P = \left(1, \frac{3}{2}, 3\right)$$

(B) It does not excute SHM about x-axis

(C) $$A = \sqrt{2^2 + \left(\frac{3}{2}\right)^2} = \frac{5}{2}$$

(D) Direction of SHM is $$\frac{\frac{2}{5}\hat{i} - \frac{3}{2}\hat{j}}{\frac{5}{2}} \Rightarrow \frac{4}{5}\hat{i} - \frac{3}{5}\hat{j}$$

23. BD

$x = A\sin\left(\frac{2\pi}{T}t + \frac{2\pi}{6}\right)$ (B)

$= A\sin\left(\frac{2\pi}{T}t + \frac{\pi}{3}\right)$ (D)

24. AB $T = 2\pi\sqrt{\frac{10}{360}} = \frac{\pi}{3}s$ (A)

$v_{max} = \frac{v}{m} = \frac{50}{10} = 5\ m/s$ (B)

$A = \frac{5}{6} = 0.83\ m$

25. C $F = \frac{-dU}{dx}$

$\Rightarrow$ At mean position net force is 0 and it is at $x = 2\ m$.

So, $x = -3m$ is it's left extreme position and amplitude is $(|-3| + 2) = 5m$

26. B Maximum x coordinate is $x = 7\ m$.

27. 19 $2\pi\sqrt{\frac{17L}{18g}}$

$d_{cm} = \frac{3\ell}{4}$

$I = \frac{m\ell^2}{3} + \frac{m\ell^2}{12} + m\ell^2 = \frac{17m\ell^2}{12}$

$2mg\frac{3\ell}{4} \times \theta = \frac{17m\ell^2}{12} \times \omega^2 \times \theta$

$\omega^2 = \frac{18g}{17\ell}$

$\therefore \quad T = 2\pi\sqrt{\frac{17\ell}{18g}} = \frac{2\pi}{3}\sqrt{\frac{17\ell}{2g}}$

28. 3

Speed just before collision

$$v = \omega\sqrt{a^2 - \left(\frac{a}{2}\right)^2}$$

$$= \omega\frac{\sqrt{3}a}{2}$$

Speed just after collision

$$v' = \frac{v}{2} = \frac{\omega\sqrt{3}a}{4} = \sqrt{\frac{k}{m}}\frac{\sqrt{3}a}{4}$$

$$= \sqrt{\frac{3k}{16m}}a$$