CHAPTER - 04

WORK ENERGY POWER & CIRCULAR MOTION

SYNOPSIS

Work is said to be done when a force F displaces its point of application. Workdone is measured as

$W = \vec{F}.\vec{S} = FS\cos\theta$.

Unit of work:- Joule (J) in SI and erg in CGS system. Gravitational unit of work is kg.m. 1kgm = 9.8 J

1 J = 10^7 ergs. Dimensions ML^2T^{-2}

If $\theta = 0$ $\vec{F}$ and $\vec{S}$ are along the same direction. Work done W = FS is maximum.

If $\theta = 90^0$, $\vec{F}$ and $\vec{S}$ are perpendicular to each other. Work done by the force is zero. ie,

$W = FS\cos\theta = 0$

If $\theta < 90^0$, $W = FS\cos\theta$ work done is positive: work is said to be done by the force

If $\theta > 90^0$, $w = FS\cos\theta$ work done is negative: work is said to be done against the force

Retarding force performs negative work. Accelerating force performs positive work.

Calculation of work:

(i) For constant force, $W = \vec{F}.\vec{S}$

(ii) For uniformly varying force, W = Average force $\times$ Displacement

(iii) For non-uniformly varying force and in general, $W = \int \vec{F}.d\vec{s}$

The area under force - displacement graph gives the work done.

Energy of a body is the capacity for doing work. It is measured in terms of work. So the units and dimensions of energy are the same as those of work.

Mechanical energy exists in two forms:

(1) ***Kinetic energy*** - possessed by virtue of motion

$KE = \frac{1}{2}mv^2 = \frac{P^2}{2m}$ $\quad [P = mv]$

Work - Energy Principle: $W = \Delta KE$ Work done by all forces = Δ KE

(2) Potential energy is the energy possessed by virtue of position, or state of strain.

Gravitational PE = mgh. (by virtue of position)

Elastic PE (of a stretched spring or body) = $\frac{1}{2}kx^2$ - the energy possessed due to the state of strain

where k is the force constant (or, spring constant), x is the stretching length or strain produced.

$$k = \frac{F}{x} = \frac{mg}{x}$$

For an elastic spring the stretching force F = +kx

Work done in stretching = $\frac{1}{2}kx^2$. This amount of work will be stored in the spring as its elastic PE.

Conservation of Mechanical Energy

Total energy is conserved in a mechanical system if the force acting is conservative. ie; KE + PE = a constant.

Power is the rate of doing work. $P = \frac{W}{t} = \frac{\vec{F}.\vec{s}}{t} = \vec{F}.\vec{v}$.

If F is varying $P_{inst} = \frac{dW}{dt}$

Unit: - Watt (W) = Joule/s. Dimension ML^2T^{-3}

Kilowatt (kW) = 10^3 W

Another commonly used unit of power is horse power (HP)

1 HP = 746 W

Kilo watt - hour (kWh) is the commonly used unit of energy. It is the energy consumed at the rate of 1 kJ/s in one hour.

$\therefore$ 1 kWh = 1000 x 60 x 60 J = 3.6 x 10^6 J

Energy in kWh = Power in kW x time in hour.

COLLISION

Elastic and inelastic collisions.

Elastic collision 1. Total linear momentum is conserved

2. Total KE is conserved

3. TE is conserved

4. Forces involved during collision are conservative

In the case of a linear elastic collision

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

Solving ,
$$v_1 = \frac{2m_2u_2}{m_1+m_2} + \frac{(m_1-m_2)u_1}{(m_1+m_2)} \text{................}(1)$$

$$v_2 = \frac{2m_1u_1}{m_1+m_2} + \frac{(m_2-m_1)u_2}{(m_1+m_2)} \text{..............}(2)$$

$v_2 - v_1 = u_1 - u_2$ ....(3)

Relative velocity of separation = Relative velocity of approach

Special cases

(1) If $m_1 = m_2$ $v_1 = u_2$, $v_2 = u_1$ velocities interchange after collision

(2) If $m_1 >> m_2$ and $u_2 = 0$, then $v_1 = u_1$ and $v_2 = 2u_1$

(3) If $m_1 << m_2$ and $u_2 = 0$, then $v_1 = -u_1$ and $v_2 = 0$

Inelastic collision

(1) Total Linear momentum is conserved

(2) Total energy is conserved

(3) There is a loss of KE

(4) Some or all the forces involved during collision are non-conservative.

Coefficient of restitution

A ball moving with a velocity u_1 collides inelastically with another body moving with a velocity u_2 in the same direction and after collision if their velocities are v_1 and v_2 respectively, then,

$v_2 - v_1 = e(u_1 - u_2)$ where, e is the coefficient of restitution

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{\text{relative velocity of seperation}}{\text{relative velocity of approach}}$$ ....(4) The value of e lies between 0 and 1.

When a ball falls on a hard floor from a height h_1 with a velocity u and rebounds with a velocity v to a height h_2 then, it can be shown that $e = \frac{v}{u}$ ....(5) and also, $e = \sqrt{\frac{h_2}{h_1}}$...........(6)

Conservation of energy

Total energy is conserved

When a mass m is fully converted into energy, the energy produced $E = mc^2$

(PART 1 - JEE MAIN)

Section 1 - Only one option correct type

1. A bucket tied to a string is lowered at a constant acceleration of g/4. If the mass of the bucket is M and is lowered by a distance d, the work done by the string will be (assume the string to be massless)

(1) 1/4 Mg d (2) -3/4 Mgd (3) - 4/3Mgd (4) 4/3 Mgd

2. An object of mass 5 kg falls from rest through a vertical distance of 20 m and reaches a velocity of 10 m/s. How much work is done by the push of the air on the object? $\left(g = 10\ m/s^2\right)$

(1) – 375 J (2) – 750 J (3) – 1500 J (4) zero

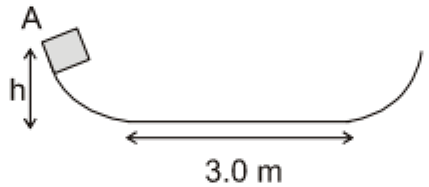
3. A force $\vec{F} = -k(y\hat{i} + x\hat{j})$, where k is a positive constant, acts on a particle moving in the xy – plane. Starting from the origin, the particle is taken along the positive x–axis to a point (a, 0) and then parallel to the positive y– axis to a point (a, a). Calculate the total work done by the force on the particle.

(1) ka^2 (2) $-ka^2$ (3) $ka^2/2$ (4) $-ka^2/2$

4. An object of mass 1 kg moves under the action of a force F in a straight line with its velocity V, changing with displacement as $v = 2\sqrt{x}$. Find the work done in a displacement from x = 0 to x = 2. in J is

(1) 1J (2) 2J (3) 3J (4) 4J

5. A small particle slides along a track with elevated ends and a flat central part, as shown in figure.The flat part has a length 3m.The curved portions of the track are frictionless, but for the flat part the coefficient of kinetic friction is =0.2.The particle is released at point A, which is at a height h=1.5m above the flat part of the track. Where does the particle finally come to rest?



(1) 1.5 m from left end of horizontal surface
(2) At point A again
(3) 1.0 m from left end of horizontal surface
(4) 2.0 m from left end of horizontal surface

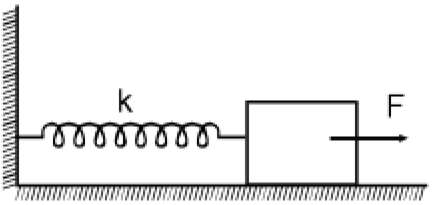
6. An elastic string of unstretched length L and force constant k is stretched by a small lengthx. It is further stretched by another small length y. The work done in the second stretching is :

(1) $\frac{1}{2}ky^2$ (2) $\frac{1}{2}k(x^2 + y^2)$ (3) $\frac{1}{2}k\ (x + y)^2$ (4) $\frac{1}{2}ky\ (2x + y)$

7. A particle of mass m = 1 kg lying on x-axis experiences a force given by law F = x (3x – 2) Newton, where x is the x-coordinate of the particle in meters. What is the minimum speed to be imparted to the particle placed at x = 4 meters such that it reaches the origin.

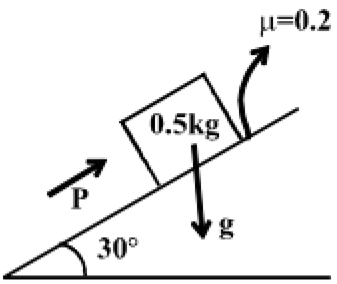
(1) $\sqrt{\frac{2600}{27}}$ m/s (2) $\sqrt{\frac{2300}{27}}$ m/s (3) $\sqrt{\frac{2500}{27}}$ (4) $\sqrt{\frac{2500}{23}}$ m/s

8. A block attached to a spring, pulled by a constant horizontal force, is kept on a smooth surface as shown in the figure. Initially, the spring is in the natural state. Then the maximum positive work that the applied force F can do is : [Given that spring does not break]



(1) $\frac{F^2}{K}$ (2) $\frac{2F^2}{K}$ (3) $\frac{F^2}{4K}$ (4) $\frac{F^2}{2K}$

9. A block of mass 0.5 kg is kept on a rough inclined plane making an angle of 30° with horizontal. What power will be required to move the block up the plane (along the plane) with a velocity of 5m/s? (Take μ = 0.2 between block and plane)



(1)16.825 N-m/s (2)16.822 N-m/s
(3)16.852 N-m/s (4)16.528 N-m/s

10. A stone of mass of 16 kg is attached to a string 144 m long and is whirled in a horizontal smooth surface. The maximum tension the string can withstand is 16 newton. The maximum speed of revolution of the stone without breaking it, will be :

(1) 20 ms^{-1} (2) 16 ms^{-1} (3) 14 ms^{-1} (4) 12 ms^{-1}

11. In a circus, stuntman rides a motorbike in a circular track of radius R in the vertical plane. The minimum speed at highest point of track should be :

(1) $\sqrt{2gR}$ (2) 2gR (3) $\sqrt{3gR}$ (4) $\sqrt{gR}$

12. A body is moved along a straight line by a machine delivering constant power. The distance moved by the body in time t is proportional to
(1) $t^{1/2}$ (2) $t^{3/4}$ (3) $t^{3/2}$ (4) t^2

13. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time as $a_c = k^2rt^2$, where k is a constant. The power delivered to the particle by the forces acting on it is –

(1) $2\pi mk^2r^2t$ (2) mk^2r^2t (3) $\frac{1}{3}mk^4r^2t^5$ (4) 0

Section 2 - Numerical type

14. Under the action of force, 2kg body moves such that its position x as a function of time t is given by $x = \frac{t^3}{3}$, x is in metre and t in second. Calculate the work done by the force in the first 2 second in J.

15. An automobile is moving at 100 kmph and is exerting attractive force of 3920 N. What horse power must the engine develop, if 20 % of the power developed is wasted (in KW) ? (Adjust the value to nearest integer)

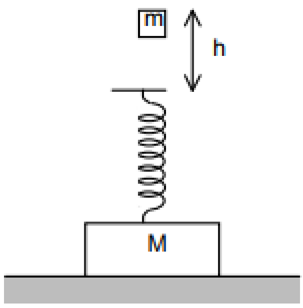
16. A rod of length 2 m and mass 0.5 kg is fixed at one end and allowed to hang vertically from a rigid support. Find the work done (in J) in raising the other end of the rod until it makes an angle of 60^0 with the vertical.($g = 10ms^{-2}$).

17. A coin placed on a rotating turntable just slips if it is placed at a distance of 4 cm from the centre. If the angular velocity of the turntable is doubled, it will just slip at a distance of (in cm)

PART 2 - JEE ADVANCED

Section 3 - One or more option correct type

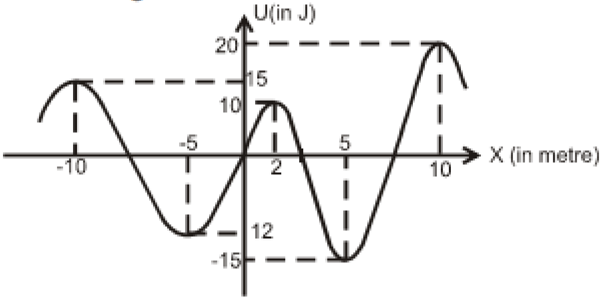
18. A block of mass m is dropped onto a spring of constant k from a height 'h'. The second end of the spring is attached to a second block of mass M as shown. Find the minimum value of h so that the block M bounces off the ground. if the block of mass m sticks to the spring immediately after it comes into contact with it.



[8]

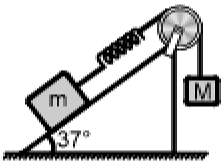
(A) $h = \frac{(M+2m)g}{2km}$ (B) $h = \frac{(M^2+2mM)g}{2km}$ (C) $h = \frac{(m^2+2M)g}{2m}$ (D) $h = \frac{K(M+2m)}{2M}g$

19. In the figure the variation of potential energy of a particle of mass m = 2kg is represented w.r.t. its x-coordinate. The particle moves under the effect of this conservative force along the x-axis.



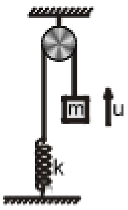
If the particle is released at the origin then :
(A) it will move towards positive x-axis.
(B) it will move towards negative x-axis.
(C) it will remain stationary at the origin.
(D) its subsequent motion cannot be decided due to lack of information.

20. A block of mass m is attached to a massless spring of force constant k. The block is placed over a fixed rough inclined surface for which the coefficient of friction is $\mu = \frac{3}{4}$. The block of mass m is initially at rest. The block of mass M is released from rest with spring in unstretched state. The minimum value of M required to move the block up the plane is (neglect mass of string and pulley and friction in pulley.)



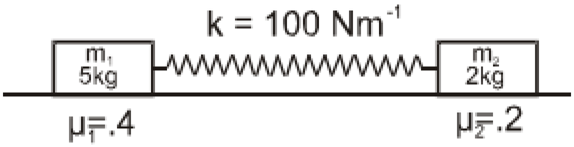
(A) $\frac{3}{5}$m (B) $\frac{4}{5}$m (C) $\frac{6}{5}$m (D) $\frac{3}{2}$m

21. A massless inelastic thread passes over a small frictionless fixed pulley. One end of the thread is fixed to a block of mass m and the other end is fixed to spring as shown in the figure. The lower end of the spring of spring constant k is rigidly fixed with ground. The system is in equilibrium.
the maximum speed u the block can be imparted from its equilibrium position upwards so that the string never gets slack is $n\sqrt{\frac{mg^2}{k}}$. Find the value of n.



A) 1 B) 2 C) 3 D) 4

22. Two blocks $m_1 = 5$kg and $m_2 = 2$kg are connected at the two ends of a spring of force constant $k = 100\ Nm^{-1}$. Friction coefficient between m_1 and ground is 0.4, and between m_2 and ground is 0.2. The minimum horizontal velocity v that must be imparted to the m_2 towards right in order to just move m_1 over surface is : (g = 10m/s^B)



(A) $\sqrt{1.4}$ m/s (B) $\sqrt{1.8}$ m/s (C) $\sqrt{2.2}$ m/s (D) $\sqrt{2.8}$ m/s

23. The kinetic energy k of a particle moving along a circle of radius R depends on the distance covered s as $k = as^2$ where a is a positive constant. The total force acting on the particle is :

(A) $2a\frac{s^2}{R}$ (B) $2as\left(1+\frac{s^2}{R^2}\right)^{1/2}$ (C) 2 as (D) $2a\frac{R^2}{s}$

24. A proton with mass m moves in one dimension. The potential-energy function is $U(x) = \frac{\alpha}{x^2} - \frac{\beta}{x}$, where a and b are positive constants. The proton is released from rest at $x_o = \frac{\alpha}{\beta}$. The correct option(s) is/are

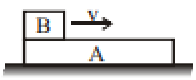
(A) speed of the particle is maximum at $x = 2x_o$

(B) maximum speed of the particle is $\frac{1}{x_o}\sqrt{\frac{\alpha}{2m}}$

(C) maximum speed of the particle is $\frac{1}{x_o}\sqrt{\frac{\beta}{2m}}$

(D) the particle will never come back and reach to the infinity with zero speed.

25. A long block A is at rest on a smooth horizontal surface. A small block B, whose mass is half of A, is placed on A at one end and projected along A with some velocity u. The coefficient of friction between the blocks is μ



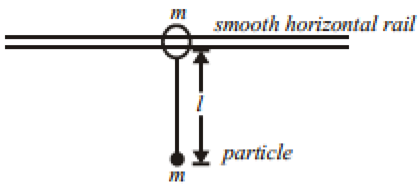
(A) the blocks will reach a final common velocity $\frac{u}{3}$

(B) the work done against friction is two-thirds of the initial kinetic energy of B

(C) before the blocks reach a common velocity, the acceleration of A relative to B is $\frac{2}{3}\mu g$

(D) before the blocks reach a common velocity, the acceleration of A relative to B is $\frac{3}{2}\mu g$

26. The ring shown in the figure is given a constant horizontal acceleration (a_0 = $g/\sqrt{3}$). Maximum deflection of the string from the vertical is θ_0, then



(A) $\theta_0 = 30°$
(B) $\theta_0 = 60°$
(C) at maximum deflection, tension in string is equal to mg
(D) at maximum deflection, tension in string is equal to $\frac{2mg}{\sqrt{3}}$

27. A sledge moving over a smooth horizontal surface of ice at a velocity v_0 drives out on a horizontal road and comes to a halt as shown. The sledge has a length *l*, mass *m* and friction between sledge and road is μ .

A) No work is done by the friction to switch the sledge ice to the road.

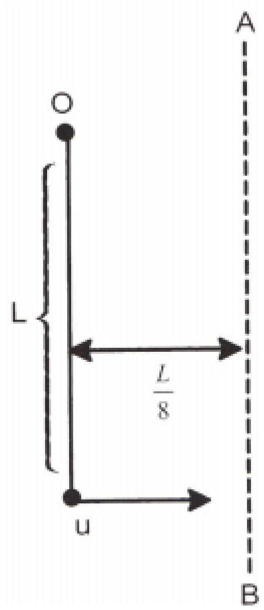
B) A work of $\frac{1}{2}\mu mgl$ is done against friction while sledge switches completely on to road.

C) The distance covered by the sledge on the road is $\left(\frac{v_0^2}{2\mu g} - \frac{l}{2}\right)$

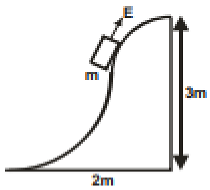
D) Total distance moved by the sledge before stopping is

Section 4 - Numerical type

28. A particle is suspended vertically from a point O by an inextensible massless string of length L. A vertical line AB is at a distance of L/8 from O as shown. The object is given a horizontal velocity u. At some point, its motion ceases to be circular and eventually the object passed through the line AB. At the instant of crossing AB, its velocity is horizontal. Then $u=\sqrt{\frac{1}{2}\left(x+3\sqrt{3}\right)gL}$, then the value of x is

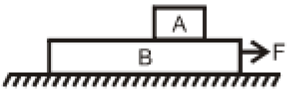


29. A body of mass 0.2 kg was slowly hauled up the hill as shown in the figure by a force E which at each point was directed along a tangent to the trajectory. Find the work performed by this force, if the height of the hill is 3m, and the length of the base is 2m and coefficient of kinetic friction $\mu = 0.5$. [given : g = 10 m/s^2]



Matrix Match type

30. A block A of mass m kg lies on block B of mass m kg. B in turn lies on smooth horizontal plane. The coefficient of friction between A and B is μ. Both the blocks are initially at rest. A horizontal force F is applied to lower block B at t = 0 such that there is relative motion between A and B. In the duration from t = 0 second till the lower block B undergoes a displacement of magnitude L, match the statements in column-I with results in column-II.



Column I	Column II
(A) Work done by friction force on block A is	(p) positive
(B) Work done by friction force on block B0 is	(q) negative
(C) Work done by friction on block plus work done by friction on block B is	(r) less than mmgL in magnitude A
(D) Work done by force F on block B is	(s) equal to mmgL in magnitude