CHAPTER - 17
WAVE OPTICS

SYNOPSIS

1. | **Wave front** | **Intensity** | **Amplitude** |
|---|---|---|
| a. Spherical | $I \alpha \frac{1}{r^2}$ | $A \alpha \frac{1}{r}$ |
| b. Cylindrical | $I \alpha \frac{1}{r}$ | $A \alpha \frac{1}{\sqrt{r}}$ |
| c. Plane | $I \alpha\, r^0$ | $A\, \alpha\, r^0$ |

2. Relation between **phase difference and path difference** $\phi = \frac{2\pi}{\lambda}\Delta L$

3. **Amplitude of the resultant wave**

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\phi}$$

$$I\, \alpha\, A^2$$

ie $I \alpha \left(A_1^2 + A_2^2 + 2A_1A_2 \cos\phi\right)$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\phi$$

4. $$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2} = \frac{\left(A_1 + A_2\right)^2}{\left(A_1 - A_2\right)^2}$$

5. $I = I_{max} \cos^2 \phi / 2$

6. $$\frac{W_1}{W_2} = \frac{I_1}{I_2} = \frac{A_1^2}{A_2^2} \left(W \rightarrow \text{slit width}\right)$$

7. For constructive interference $\phi = 2n\pi\ (n = 0,1,2,3...)$

   $\Delta L = n\lambda\ (n = 0,1,2,3...)$

8. For destructive interference $\phi = (2n-1)\pi\ (n = 1,2,3...)$

   $\Delta L = (2n-1)\lambda/2\ \ (n = 1,2,3...)$

9. In YDSE for constructive interference $d\sin\theta = n\lambda$ (n = 0, 1, 2, 3...)

   and for destructive interference $d\sin\theta = (2n-1)\ \lambda/2\ (n = 1,2,3...)$

   Distance to the nth bright fringe from central maximum

   $y_{nb} = \frac{nD\lambda}{d}\ (n = 0,1,2,3...)$ and to the n^{th} dark fringe $y_{nd} = (2n-1)\ \frac{D\lambda}{2d}\ (n = 1,2,3...)$

   Fringe width, $\beta = \frac{D\lambda}{d}$; Angular fringe width $\theta = \frac{\beta}{D} = \frac{\lambda}{d}$

10. Fringe visibility $V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2}$

11. $n\lambda = a$ constant $n_1\lambda_1 = n_2\lambda_2$

12. $\frac{\Delta\beta}{\Delta D} = \frac{\lambda}{d}$

13. If YDSE is performed in a medium $\beta^1 = \beta/\mu$

14. If transparent sheet is introduced in the path of one of the two waves

    $y_0 = \frac{D}{d}(\mu-1)t;\quad y_0 = \frac{\beta}{\lambda}(\mu-1)t$

15. **Diffraction**

    Position of the secondary minimum $a\sin\theta = n\lambda\ (n = 1,2,3...)$

    Position of the secondary maximum

    $a\sin\theta = (2n+1)\frac{\lambda}{2}\ (n = 1,2,3..)$ First sec. min $\theta = \frac{\lambda}{a} = \frac{x}{D}$ ie $x = \frac{D\lambda}{a}$

    Angular width of the central maximum $2\theta = \frac{2\lambda}{a}$

    Linear width of the central maximum $2x = \frac{2D\lambda}{a} = (2\theta)D$

    The first minimum for the diffraction pattern of circular aperture of diameter d is located by $\sin\theta = \frac{1.22\lambda}{d}$

16. **Doppler effect of light**

$$\Delta\lambda = \lambda\frac{v}{c}$$

17. **Polarization**

Malu's law (Cosine squared law) $I_2 = I_1 \cos^2\theta$ $I_1 = \frac{I_0}{2}$

if θ = 90 (ie polaroids are crossed) $I_2 = 0$ if θ = 0 $I_2 = I_1$

Brewsters law

$$\frac{n_d}{n_r} = \tan\theta_B;\ \tan\theta_B = \frac{1}{\sin C}$$

Teaching Poins

Wave concept of light

Huygens's principle - Idea of wave fronts

Interference of Light

Condition of sustained interference

Coherent and incoherent sources

Youngs Double slit experiment

Experiment setup - Band width (Equation)

Special cases in YDSE

Bichromatic beam, white light, Electron beam

Introduction of transparent slab

Oblique incidence etc

Diffraction

Condition for diffraction

Diffraction at single slit

Linear and angular width of central maxima

Resolving power and limit of resolution

Fresnel distance - Validity of ray optics

Polarisation

Concept of polarisation

Polarised and unpolarised light, Malus law

Polarisation by reflection - Brewster's law

Doppler Effect in light

Red shift and Blue shift

PART I - (JEEMAIN)

SECTION - I - Straight objective type questions

1. The two light beams having intensities I and $9I$ interfere to produce a fringe pattern on a screen. The phase difference between the beams is $\frac{\pi}{2}$ at point P and π at point Q. Then the difference between the resultant intensities at P and Q will be:

   1) $2I$ 2) $6I$ 3) $5I$ 4) $7I$

2. The coherent light sources having intensity in the ratio 2x produce an interference pattern. The ratio $\frac{I_{max}-I_{min}}{I_{max}+I_{min}}$ will be

   1) $\frac{\sqrt{2x}}{2x+1}$ 2) $\frac{\sqrt{2x}}{x+1}$ 3) $\frac{2\sqrt{2x}}{x+1}$ 4) $\frac{2\sqrt{2x}}{2x+1}$

3. Two light waves having the same wavelength λ in vacuum are in phase initially. Then the first wave travels a path L, through a medium of refractive index n_1 while the second wave travels a path of length L_2 through a medium of refractive index n_2. After this the phase difference between the two waves is :

   1) $\frac{2\pi}{\lambda}\left(\frac{L_2}{n_1}-\frac{L_1}{n_2}\right)$ 2) $\frac{2\pi}{\lambda}\left(\frac{L_1}{n_1}-\frac{L_2}{n_2}\right)$ 3) $\frac{2\pi}{\lambda}(n_1L_1-n_2L_2)$ 4) $\frac{2\pi}{\lambda}(n_2L_1-n_1L_2)$

4. On a hot summer night, the refractive index of air is smallest near the ground and increases with height form the ground. When a light beam is directed horizontally the Huygens' principle leads us to conclude that as it travels, the light beam

   1) bends downwards 2) bends upwards
   3) becomes narrower 4) goes horizontally without any deflection

5. In Young's double slit experiment, the fringe width is 12 mm. If the entire arrangement is placed in water of refractive index $\frac{4}{3}$, then the fringe width becomes (in mm):

   1) 16 2) 9 3) 48 4) 12

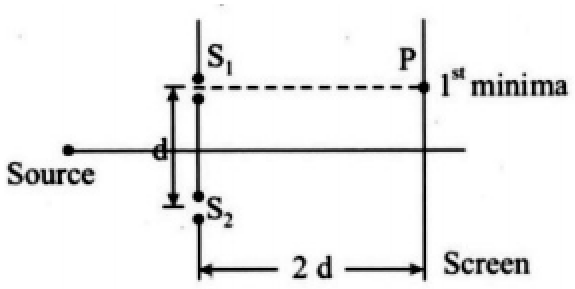
6. In a Young's double slit experiment, 16 fringes are observed in a certain segment of the screen when light of wavelength 700 nm is used. If the wavelength of light is changed to 400 nm, the number of fringes observed in the same segment of the screen would be:

   1) 24 2) 30 3) 18 4) 28

7. In a Young's double slit experiment, the slits are placed 0.320 mm apart. Light of wavelength $\lambda = 500\,nm$ is incident on the slits. The total number of bright fringes that are observed in the angular range $-30^0 \le \theta \le 30^0$ is

   1) 640 2) 320 3) 321 4) 641

8. Consider a Young's double slit experiment as shown in figure. What should be the slit separation d in terms of wavelength λ such that the first minima occurs directly in front of the slit (S_1)?



1) $\frac{\lambda}{2\left(\sqrt{5}-2\right)}$ 2) $\frac{\lambda}{\left(\sqrt{5}-2\right)}$ 3) $\frac{\lambda}{2\left(5-\sqrt{2}\right)}$ 4) $\frac{\lambda}{\left(5-\sqrt{2}\right)}$

9. In a Young's double slit experiment, the distance between the two identical slits is 6.1 times larger than the slit width. Then the number of intensity maxima observed within the central maximum of the single slit diffraction pattern is:
1) 3 2) 6 3) 12 4) 24

10. This question has statement -I and Statement-2. Of the four choices given after the Statements, choose the one that best describes the two Statements.
Statement I: In Young's double slit experiment, the number of fringes observed in the field of view is small with longer wavelength of light and is large with shorter wavelength of light.
Statement II : In the double slit experiment the fringe width depends directly on the wavelength of light.
1) Statement -I is true, Statement-2 is true and the Statement-2 is correct explanation of the | Statement-I
2) Statement -I is false and the Statement -2 is true
3) Statemennt-I is true Statement-2 is true and the Statement -2 is not correct explanation of the Statement -I
4) Statement -I is true and the Statement -2 is false

11. In Young's double slit experiment, one of the slit is wider than other, so that amplitude of the light from one slit is double of that other slit. If I_m be the maximum intensity the resultant intensity I when they interfere at phase difference ϕ is given by :

1) $\frac{I_m}{9}(4+5\cos\phi)$ 2) $\frac{I_m}{3}\left(1+2\cos^2\frac{\phi}{2}\right)$ 3) $\frac{I_m}{5}\left(1-4\cos^2\frac{\phi}{2}\right)$ 4) $\frac{I_m}{9}\left(1+8\cos^2\frac{\phi}{2}\right)$

12. In a Young's double slit experiment the intensity at a point where the path difference is $\frac{\lambda}{6}$ (λ being the wavelength of light used) is I. If I_0 denotes the maximum intensity, $\frac{I}{I_0}$ is equal to

1) $\frac{3}{4}$ 2) $\frac{1}{\sqrt{2}}$ 3) $\frac{\sqrt{3}}{2}$ 4) $\frac{1}{2}$

13. The maximum number of possible interference maxima for slit-separation equal to twice the wavelength in Young's double-slit experiment is
1) three 2) five 3) infinite 4) zero

14. A single-slit of a width *a* is illuminated by a monochromatic light of wavelength 600nm. The value of 'a' for which first minimum appears at $\theta = 30^0$ on the screen will be
1) $0.6\mu m$ 2) $1.2\mu m$ 3) $1.8\mu m$ 4) $3\mu m$

15. In a double-slit experiment, green light $\left(5303\overset{0}{A}\right)$ falls on a double slit having a separation of $19.44\mu m$ and a width of $4.05\mu m$. The number of bright fringes between the first and the second diffraction minima is:
1) 10 2) 05 3) 04 4) 09

16. Two polaroids A and B are placed in such a way that the pass-axis of polaroids are perpendicular to each other. Now, another polaroid C is placed between A and B bisecting angle between them. If intensity of unpolarised light is I_0 then intensity of transmitted light after passing through polaroid B will be:
1) $\frac{I_0}{4}$ 2) $\frac{I_0}{2}$ 3) $\frac{I_0}{8}$ 4) Zero

17. 'n' polarizing sheets are arranged such that each makes an angle 45^0 with the preceding sheet. An unpolarized light of intensity I is incident into this arrangement. The output intensity is found to be I/64. The value of n will be :
1) 6 2) 3 3) 5 4) 4

18. A beam of plane polarised light of large cross-sectional area and uniform intensity of 3.3 Wm^{-2} falls normally on a polariser (cross sectional are $3\times10^{-4}m^2$) which rotates about its axis with an angular speed of 31.4 rad/s. The energy of light passing through the polariser per revolution, is close to:
1) $1.0\times10^{-5}J$ 2) $1.0\times10^{-4}J$ 3) $1.5\times10^{-4}J$ 4) $5.0\times10^{-4}J$

19. Unpolarized light of intensity I is incident on a system of two polarizers. A followed by B. The intensity of emergent light is 1/2. If a third polarizer C is placed between A and B, the intensity oif emergent light is reduced to 1/3. The angle between the polarizers A and C is θ. Then
1) $\cos\theta = \left(\frac{2}{3}\right)^{1/4}$ 2) $\cos\theta = \left(\frac{1}{3}\right)^{1/4}$ 3) $\cos\theta = \left(\frac{1}{3}\right)^{1/2}$ 4) $\cos\theta = \left(\frac{2}{3}\right)^{1/2}$

20. The angle of incidence at which reflected light is totally polarized for reflection from air to glass (refractive index n), is
1) $\tan^{-1}(1/n)$ 2) $\sin^{-1}(1/n)$ 3) $\sin^{-1}(n)$ 4) $\tan^{-1}(n)$

SECTION - II

Numerical Type Questions

21. A young's double-slit experiment is performed using monochromatic light of wavelength λ. The intensity of light at a point on the screen, where the path difference is λ, is K units. The intensity of light at a point where the path difference is $\frac{\lambda}{6}$ is given by $\frac{nK}{12}$, where n is an integer. The value of *n* is ____________.

22. In a single slit diffraction experiment light of wavelength 600 nm is used and the first minimum is observed at an angle of 30^0. The width of the slit is (in μm)

23. A source of light is placed in front of a screen. Intensity of light on the screen is I. Two polaroids P_1 and P_2 are so placed in between the source of light and screen that the intensity of light on screen is $I/2$. P_2 should be rotated by an angle of ....(degrees) so that the intensity of light on the screen becomes $\frac{3I}{8}$.

24. Orange light of wavelength $6000\times10^{-10}\,m$ illuminates a single slit of width $0.6\times10^{-4}\,m$. The maximum possible number of diffraction minima produced on both sides of the central maximum is ______________

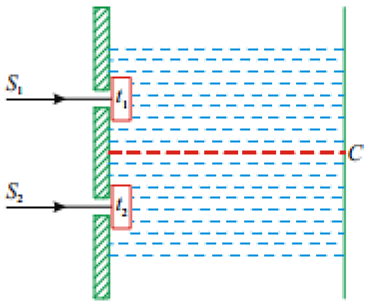
PART - II (JEE ADVANCED)

SECTION - III (Only one option correct type)

25. In Young's double slit experiment intensity at a point is $\left(\frac{1}{4}\right)$ of the maximum intensity. Angular position of this point is:

A) $\sin^{-1}\left(\frac{\lambda}{d}\right)$ B) $\sin^{-1}\left(\frac{\lambda}{2d}\right)$ C) $\sin^{-1}\left(\frac{\lambda}{3d}\right)$ D) $\sin^{-1}\left(\frac{\lambda}{4d}\right)$

26. A screen is at a distance D=80cm from a diaphragm having two narrow slits S_1 and S_2 which are $d=2mm$ apart. Slit S_1 is covered by a transparent sheet of thickness $t_1=2.5\mu m$ and slit S_2 is covered by another sheet of thickness $t_2=1.25\mu m$ as shown in figure. Both sheets are made of same material having refractive index $\mu=1.4$. Water is filled in the space between the diaphragm and screen. A monochromatic light beam of wavelength $\lambda=5000\text{Å}$ is incident normally on the diaphragm. Assuming intensity of beam to be uniform calcualte ratio of intensity at C to the maximum intensity of interference patern obtained on the screen? $\left[\mu_\omega = 4/3\right]$



A) $\frac{3}{4}$ B) $\frac{2}{3}$ C) $\frac{8}{9}$ D) $\frac{5}{7}$

27. In the ideal double-slit experiment, when a glass-plate (refractive index 1.5) of thickness t is introduced in the path of one of the interfering beams (wavelength λ), the intensity at the position where the central maximum occurred previously remains unchanged. The minimum thickness of the glass-plate is :

A) 2λ B) $\frac{2\lambda}{3}$ C) $\frac{\lambda}{3}$ D) λ

28. In Young's double slit experiment, 5th dark fringe is obtained at a point. If a thin transparent film is placed in the path of one of waves, then 7th bright fringe is obtained at the same point. The thickness of the film in terms of wavelength λ and refractive index μ will be

A) $\frac{1.5\lambda}{(\mu-1)}$ B) $1.5(\mu-1)\lambda$ C) $2.5(\mu-1)\lambda$ D) $\frac{2.5\lambda}{(\mu-1)}$

29. A small aperture is illuminated with a parallel beam of $\lambda = 628nm$. The emergent beam has an angular divergence of 2^0. The size of the aperture is

A) $9\mu m$ B) $18\mu m$ C) $27\mu m$ D) $36\mu m$

30. A person wants to see two pillars distant 11 km, separately. The distance between the pillars must be approximately

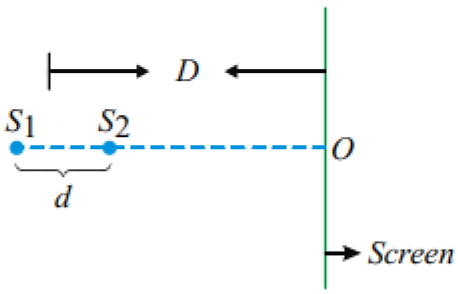
A) 3m B) 1m C) 0.25 m D) 0.5 m

31. Unpolarised light of intensity $32Wm^{-2}$ passes through three polarisers such that the transmission axis of the last polariser is crossed with first. If the intensity of the emerging light is $3Wm^{-2}$, the angle between the axes of the first two polarisers is

A) 45^0 B) 60^0 C) 30^0 D) zero

SECTION - IV (More than one correct answer)

32. Two point monochromatic and coherent sources of light of wavelength λ are each placed as shown in the figure below. The initial phase difference between the sources is zero.(Assume D>>>d). Select the correct statement(s).



A) If $d = \frac{7\lambda}{2}$, the point 'O' will be minima.

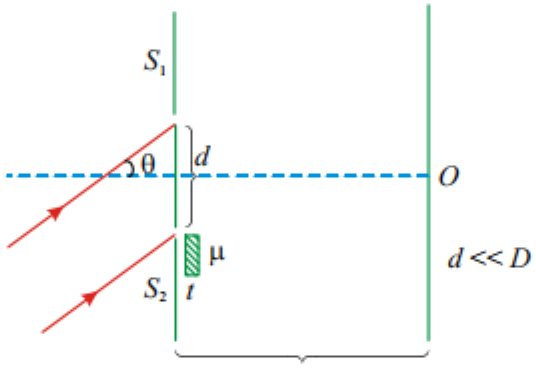
B) If $d = \lambda$, only one maxima canbe observed on screen

C) If $d = 4.8\lambda$,then a total 10 minimas would be there on screen.

D) If $d = \frac{5\lambda}{2}$, then Intensity at 'O' would be minimum.

Passage-I

A monochromatic beam of light falls on Young's double slit experiment apparatus as shown in figure. A thin sheet of glass is inserted in front of lower slit S_2 ($\lambda = 600nm$ is wavelength of source)



33. The central bright fringe can be obtained
A) At 'O'
B) At 'O' or below 'O'
C) at 'O' or above 'O'
D) Any where on the screen

34. If central bright fringe is obtained onscreen at 'O' then
A) $(\mu-1)t = d\sin\theta$
B) $(\mu-1)t = d\cos\theta$
C) $(\mu t) = d\theta$
D) $\dfrac{t}{(\mu-1)} = \dfrac{d}{\sin\theta}$

35. The phase difference between central maxima and 5th minima is
A) $\dfrac{\pi}{6}$
B) 9π
C) $\dfrac{3\pi}{2}$
D) $8\pi \pm \dfrac{\pi}{6}$

SECTION - V (Numerical Type - Upto two decimal place)

36. In Young's double slit experiment interference bands are produced on the screen placed at 1.5m from two slits 0.15 mm apart and illuminated by light of wavelength 6000Å. If the screen is now taken away from the slits by 50cm then find the change in fringe width (in mm)?

37. In YDSE, the intensity of light at a point on the screen is I for a path difference λ. The intensity of light at a point where the path difference becomes $\dfrac{\lambda}{3}$ is $\dfrac{I}{P}$. Find the value of P?

38. In YDSE, the wavelength of red light is 7.5×10^{-5} cm and that of blue light is $5\times10^{-5} cm$. Find the value if 'n' for which $(n+1)^{th}$ blue bright band coincides with n^{th} red bright band?

39. In YDSE, the slits have different widths. As a result, amplitude of waves from two slits are A and 2A respectively. If I_0 be the maximum intensity of the interference pattern then the intensity of the pattern at a point where the phase difference between waves is ϕ is given by $\dfrac{I_0}{P}(5+4\cos\phi)$. Where P is in in integer. Find the value of P?

SECTION - VI (Matrix Matching)

40. In Column-I the effect on fringe pattern in YDSE is mentioned when the changes mentioned in Column-II are made. Match the entries of Column-I with entries of Column-II.

Column-I	Column-II
A) Angular Fringe width changes	P) Screen is moved away from the plane of the slits
B) Fringe width [linear seperation	Q) Wavelength of light used is decreased.
	between two consecutive fringes]
C) Angular Fringe width remains same	R) The seperation between the slits is increased
D) The fringe pattern disappear	S) The width of the source slit is increased
	T) The source slit is moved closer to the double slit plane.