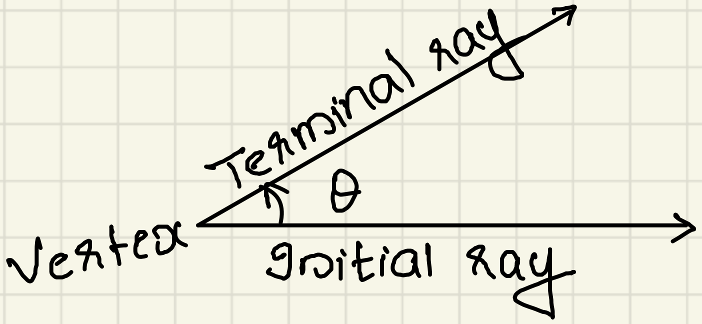
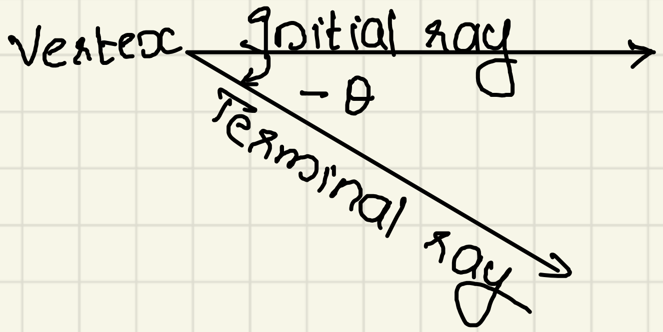
TRIGONOMETRY-1

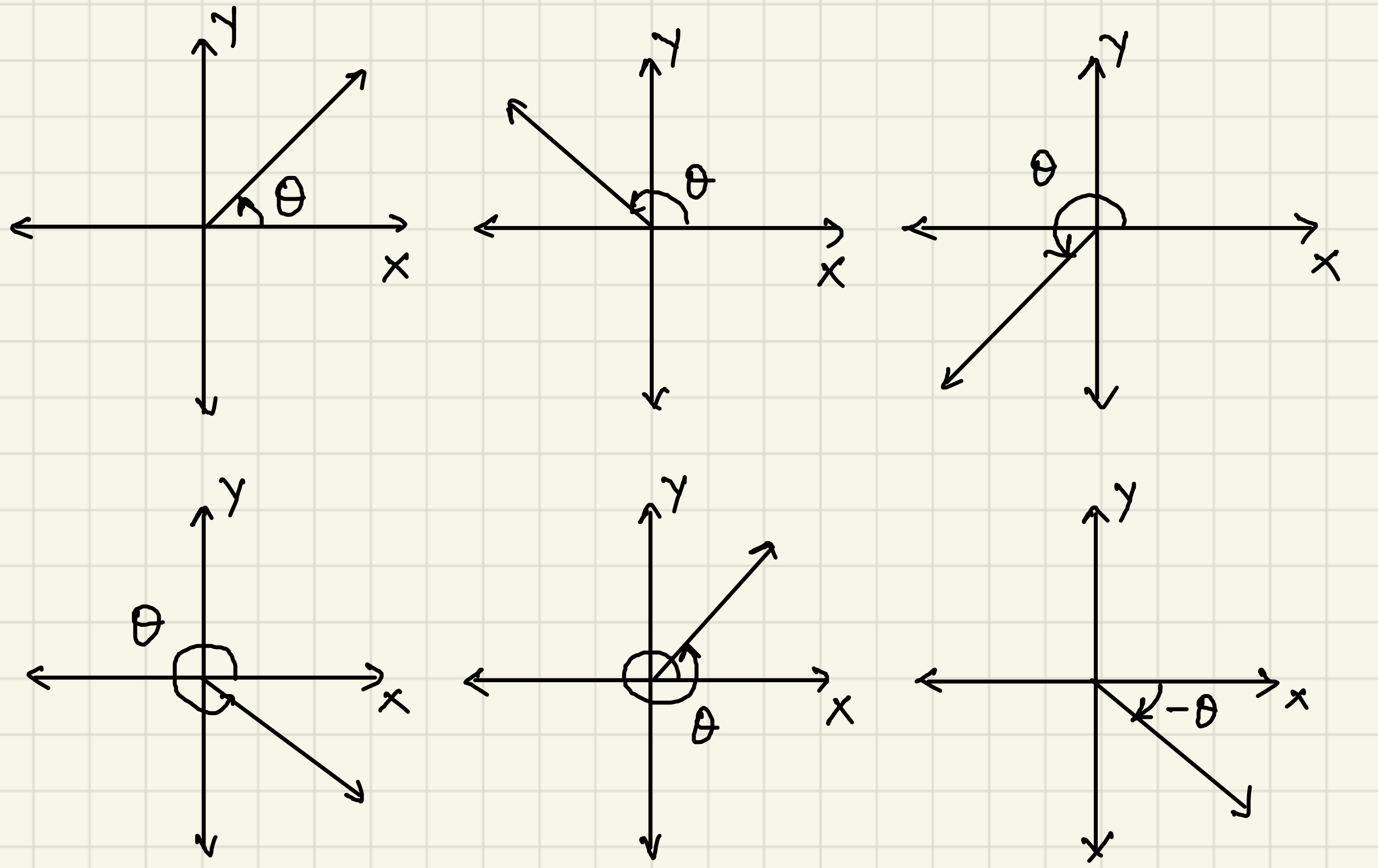
* Trigonometry- Ratios and identities

* Angle





Note: usually +ve x axis is taken as initial ray.



* Measurement of angles

1. Sexagesimal system - Degree
2. centesimal system - Grade
3. Circular system - Radian

1. Sexagesimal System - Degree

One right angle is divided into 90 equal parts, each part is termed as one degree (1°)

Each degree is again divided into 60 equal parts and each part is termed as one minute ($1'$)

Each minute again divided into 60 equal parts, each part is termed as one second ($1''$)

ie, $1^\circ = 60'$ $\quad 1' = 60''$

$$1' = \left(\frac{1}{60}\right)^\circ \quad 1'' = \left(\frac{1}{60}\right)'$$

2. Centesimal system - Grade

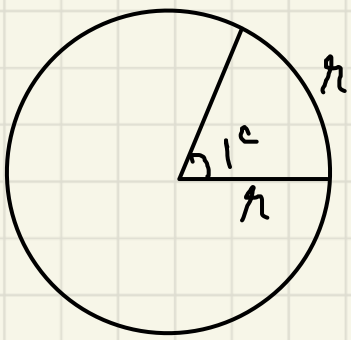
A right angle is divided into 100 equal parts, each part is termed as one grade

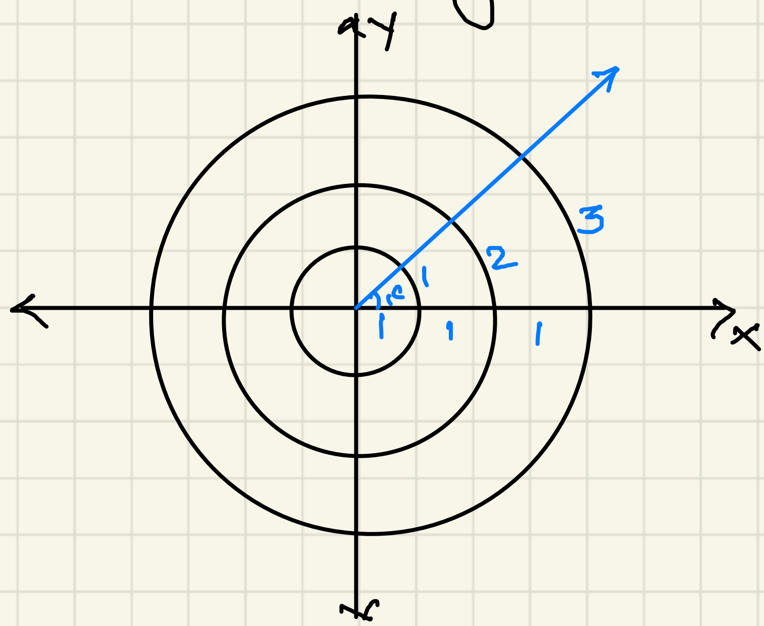
Each grade is again divided into 100 equal parts, each part is termed as one minute.

Each minute again divided into 100 equal parts, each part is termed as one second.

3. Circular system- Radian.

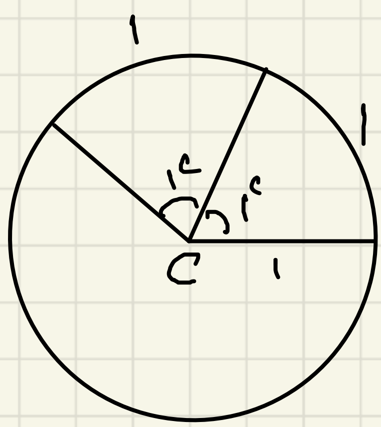
One radian is the angle subtended by an arc of length equal to the radius of the circle at its centre denoted by 1^c





* Relation between Degree and radian.

Consider a unit circle (r = 1)



each arc of length 1 unit subtend 1^c at centre

$\Rightarrow$ Total number of 1 unit arc = Circumference of circle

$= 2\pi r$

$= 2\pi \times 1$

$= 2\pi$

Total central angle $= 2\pi^c$

$\Rightarrow \quad 360° = 2\pi^c$

$180° = \pi^c$

$\Rightarrow 1° = \frac{\pi}{180}^c \qquad 1^c = \frac{180}{\pi}^{\circ}$

* Conversion:

Degree to radian $\times$ by $\frac{\pi}{180}$

Radian to degree $\times$ by $\frac{180}{\pi}$

* Some standard angles

$360^\circ = 2\pi^c$ $180^\circ = \pi^c$ $90^\circ = \frac{\pi}{2}^c$ $60^\circ = \frac{\pi}{3}^c$

$45^\circ = \frac{\pi}{4}^c$ $30^\circ = \frac{\pi}{6}^c$

* Arc length

l: arc length, r: radius, θ: central angle

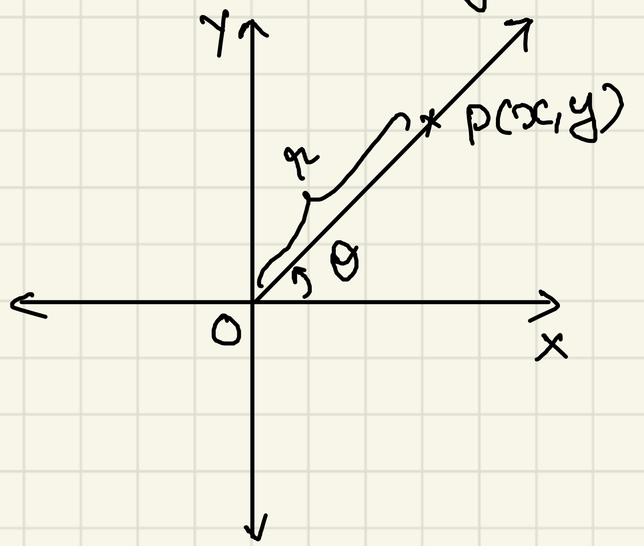
Arc length, $l = r\theta$, θ: in radians

arc length $= \frac{\theta}{360} \times 2\pi r$ θ: in degree

* Trigonometric ratios

There are three trigonometric ratios namely sine (sin), secant (sec), tangent (tan) and the corresponding complementary ratios namely cosine (cos), cosecant (cosec) and cotangent (cot).

* Trigonometry - In general.



Take any point $P(x, y)$ from terminal ray.

r: Distance Between P and vertex

$\sin\theta = \frac{y}{r}$

$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{r}{y}$

$\cos\theta = \frac{x}{r}$

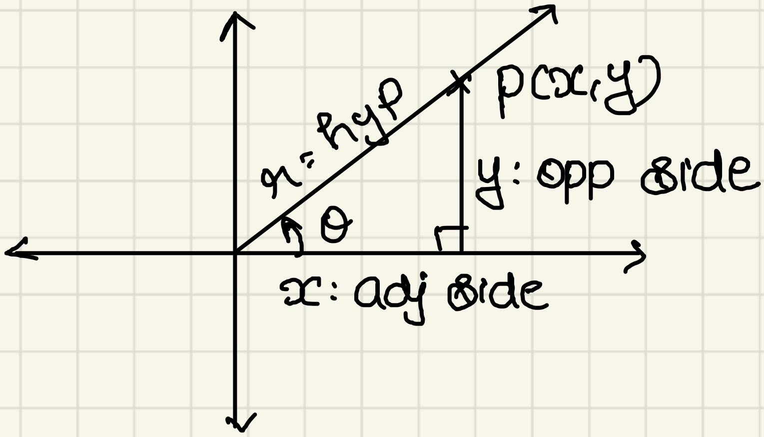
$\sec\theta = \frac{1}{\cos\theta} = \frac{r}{x}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{y}{x}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{x}{y}$

* Trigonometry in right triangle

Note: Terminal ray in 1st quadrant.



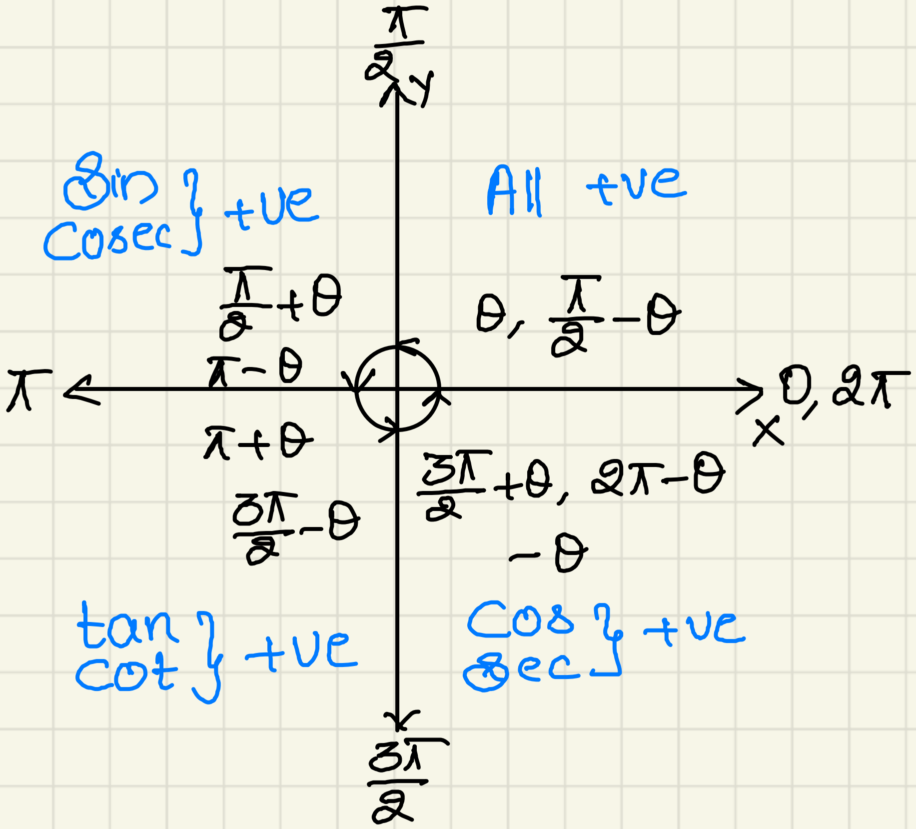
$$\sin\theta = \frac{\text{opp. side}}{\text{hyp}} \qquad \operatorname{cosec}\theta = \frac{\text{hyp}}{\text{opp. side}}$$

$$\cos\theta = \frac{\text{adj. side}}{\text{hyp.}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj side}}$$

$$\tan\theta = \frac{\text{opp. side}}{\text{adj side}} \qquad \cot\theta = \frac{\text{adj. side}}{\text{opp. side}}$$

* Quadrants

θ : acute angle



$0, \pi, 2\pi \dots$

multiple of π

$\Rightarrow$ No change

$\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \dots$

odd multiple of $\frac{\pi}{2}$

$\Rightarrow$ Change

Sin $\rightleftarrows$ Cos

Sec $\rightleftarrows$ Cosec

tan $\rightleftarrows$ Cot

1st quadrant

$$\frac{\pi}{2}-\theta$$

$$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$$

$$\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$$

$$\tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta$$

$$\operatorname{cosec}\left(\frac{\pi}{2}-\theta\right)=\sec\theta$$

$$\sec\left(\frac{\pi}{2}-\theta\right)=\operatorname{cosec}\theta$$

$$\cot\left(\frac{\pi}{2}-\theta\right)=\tan\theta$$

2nd quadrant

$$\frac{\pi}{2}+\theta$$

$$\sin\left(\frac{\pi}{2}+\theta\right)=\cos\theta$$

$$\cos\left(\frac{\pi}{2}+\theta\right)=-\sin\theta$$

$$\tan\left(\frac{\pi}{2}+\theta\right)=-\cot\theta$$

$$\operatorname{cosec}\left(\frac{\pi}{2}+\theta\right)=\sec\theta$$

$$\sec\left(\frac{\pi}{2}+\theta\right)=-\operatorname{cosec}\theta$$

$$\cot\left(\frac{\pi}{2}+\theta\right)=-\tan\theta$$

2nd quadrant

$\pi - \theta$

$\sin(\pi-\theta) = \sin\theta$

$\cos(\pi-\theta) = -\cos\theta$

$\tan(\pi-\theta) = -\tan\theta$

$\operatorname{cosec}(\pi-\theta) = \operatorname{cosec}\theta$

$\sec(\pi-\theta) = -\sec\theta$

$\cot(\pi-\theta) = -\cot\theta$

3rd quadrant

$\pi + \theta$

$\sin(\pi+\theta) = -\sin\theta$

$\cos(\pi+\theta) = -\cos\theta$

$\tan(\pi+\theta) = \tan\theta$

$\operatorname{cosec}(\pi+\theta) = -\operatorname{cosec}\theta$

$\sec(\pi+\theta) = -\sec\theta$

$\cot(\pi+\theta) = \cot\theta$

3^{rd} quadrant

$\frac{3\pi}{2} - \theta$

$\sin\left(\frac{3\pi}{2} - \theta\right) = -\cos\theta$

$\cos\left(\frac{3\pi}{2} - \theta\right) = -\sin\theta$

$\tan\left(\frac{3\pi}{2} - \theta\right) = \cot\theta$

$\operatorname{cosec}\left(\frac{3\pi}{2} - \theta\right) = -\sec\theta$

$\sec\left(\frac{3\pi}{2} - \theta\right) = -\operatorname{cosec}\theta$

$\cot\left(\frac{3\pi}{2} - \theta\right) = \tan\theta$

4^{th} quadrant

$\frac{3\pi}{2} + \theta$

$\sin\left(\frac{3\pi}{2} + \theta\right) = -\cos\theta$

$\cos\left(\frac{3\pi}{2} + \theta\right) = \sin\theta$

$\tan\left(\frac{3\pi}{2} + \theta\right) = -\cot\theta$

$\operatorname{cosec}\left(\frac{3\pi}{2} + \theta\right) = -\sec\theta$

$\sec\left(\frac{3\pi}{2} + \theta\right) = \operatorname{cosec}\theta$

$\cot\left(\frac{3\pi}{2} + \theta\right) = -\tan\theta$

4th quadrant

$2\pi - \theta$

$\sin(2\pi - \theta) = -\sin\theta$

$\cos(2\pi - \theta) = \cos\theta$

$\tan(2\pi - \theta) = -\tan\theta$

$\text{cosec}(2\pi - \theta) = -\text{cosec}\theta$

$\sec(2\pi - \theta) = \sec\theta$

$\cot(2\pi - \theta) = -\cot\theta$

1st quadrant

$2\pi + \theta$

$\sin(2\pi + \theta) = \sin\theta$

$\cos(2\pi + \theta) = \cos\theta$

$\tan(2\pi + \theta) = \tan\theta$

$\text{cosec}(2\pi + \theta) = \text{cosec}\theta$

$\sec(2\pi + \theta) = \sec\theta$

$\cot(2\pi + \theta) = \text{cosec}\theta$

4th quadrant

$-\theta : \quad 0-\theta$

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

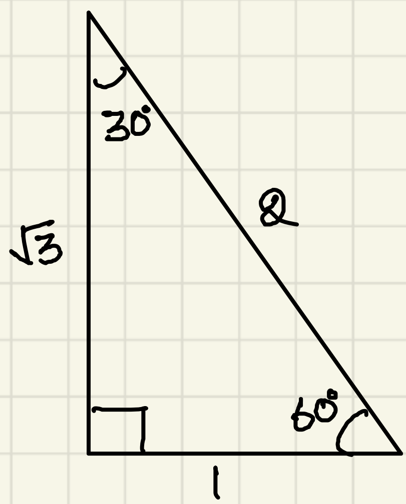
$\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

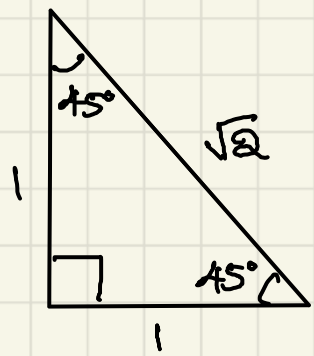
$\sec(-\theta) = \sec\theta$

$\cot(-\theta) = -\cot\theta$

* Trigonometric ratios of some standard angles

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	1	0
$30°, \frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$45°, \frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$60°, \frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$90°, \frac{\pi}{2}$	1	0	Not defined
$180°, \pi$	0	-1	0
$270°, \frac{3\pi}{2}$	-1	0	Not defined
$360°, 2\pi$	0	1	0





15, 18, 22½ 36, 54, 72, 75,

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
$15^\circ, \frac{\pi}{12}$	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$\frac{\sqrt{3}-1}{\sqrt{3}+1}$
$18^\circ, \frac{\pi}{10}$	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}$
$22\frac{1}{2}^\circ, \frac{\pi}{8}$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\frac{\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}$
$36^\circ, \frac{\pi}{5}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}+1}$
$54^\circ, \frac{3\pi}{10}$	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{5}+1}{\sqrt{10-2\sqrt{5}}}$
$72^\circ, \frac{4\pi}{10}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{\sqrt{5}-1}$
$75^\circ, \frac{5\pi}{12}$	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$\frac{\sqrt{3}+1}{\sqrt{3}-1}$

* Trigonometric Identities.

1. $\sin^2\theta + \cos^2\theta = 1$

   $\sec^2\theta - \tan^2\theta = 1$

   $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$$\sec\theta + \tan\theta = \frac{1}{\sec\theta + \tan\theta}$$

$$\operatorname{cosec}\theta + \cot\theta = \frac{1}{\operatorname{cosec}\theta - \cot\theta}$$

2. $\sin(A+B) = \sin A\cos B + \cos A\sin B$

   $\sin(A-B) = \sin A\cos B - \cos A\sin B$

   $\cos(A+B) = \cos A\cos B - \sin A\sin B$

   $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

3. C, D Formulae

$$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$$

$$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$$

4. $\sin 2A = 2\sin A\cos A$

$$= \frac{2\tan A}{1+\tan^2 A}$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 1 - 2\sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= \frac{1-\tan^2 A}{1+\tan^2 A}$$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

5. Half angle formula

$$\sin A = 2\sin\frac{A}{2}\cos\frac{A}{2}$$

$$= \frac{2\tan A/2}{1+\tan^2 A/2}$$

$$\cos A = \cos^2 A/2 - \sin^2 A/2$$

$$= 1 - 2\sin^2 A/2$$

$$= 2\cos^2 A/2 - 1$$

$$= \frac{1-\tan^2 A/2}{1+\tan^2 A/2}$$

$$\tan A = \frac{2\tan A/2}{1-\tan^2 A/2}$$

6. $\sin 3A = 3\sin A - 4\sin^3 A$

$\cos 3A = 4\cos^3 A - 3\cos A$

$$\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$$

7. $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$

$= \cos^2 B - \cos^2 A$

$\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$

$$\tan(A+B)\tan(A-B) = \frac{\tan^2 A - \tan^2 B}{1 - \tan^2 A \tan^2 B}$$

$$\cot(A+B)\cot(A-B) = \frac{\cot^2 A \cot^2 B - 1}{\cot^2 B - \cot^2 A}$$

8. $\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$\cos A \sin B = \frac{1}{2}[\sin(A+B) - \sin(A-B)]$

$\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$\sin A \sin B = \frac{-1}{2}[\cos(A+B) - \cos(A-B)]$

9. $\sin(A+B+C) = \cos A \cos B \cos C[\tan A + \tan B$

$+ \tan C - \tan A \tan B \tan C]$

$\cos(A+B+C) = \cos A \cos B \cos C[1 - \tan A \tan B$

$- \tan B \tan C - \tan A \tan C]$

$$\tan(A+B+C) = \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan A \tan C}$$

10. If $A+B+C=\pi$ then

$$\tan A+\tan B+\tan C=\tan A\tan B\tan C$$

$$\cot A\cot B+\cot B\cot C+\cot A\cot C=1$$

$$\sin 2A+\sin 2B+\sin 2C=4\sin A\sin B\sin C$$

$$\cos 2A+\cos 2B+\cos 2C=-1-4\cos A\cos B\cos C$$

$$\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}=\cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}$$

$$\tan\frac{A}{2}\tan\frac{B}{2}+\tan\frac{B}{2}\tan\frac{C}{2}+\tan\frac{A}{2}\tan\frac{C}{2}=1$$

$$\cos^2 A+\cos^2 B+\cos^2 C=1-2\cos A\cos B\cos C$$

$$\sin^2 A+\sin^2 B+\sin^2 C=2+2\cos A\cos B\cos C$$

11. If $A+B+C=\frac{\pi}{2}$ then

$\tan A \tan B + \tan B \tan C + \tan A \tan C = 1$

$\cot A + \cot B + \cot C = \cot A \cot B \cot C$

$\sin^2 A + \sin^2 B + \sin^2 C = 1 - 2 \sin A \sin B \sin C$

$\cos^2 A + \cos^2 B + \cos^2 C = 2 + 2 \sin A \sin B \sin C$

12. If $A+B=\frac{\pi}{4}$ then

$$(1+\tan A)(1+\tan B)=2$$

$$(\cot A-1)(\cot B-1)=2$$

13. If $A-B=\frac{\pi}{4}$ then

$$(1+\tan A)(1-\tan B)=2$$

14. If $A+B=\frac{3\pi}{4}$ then

$$(1+\cot A)(1+\cot B)=2$$

15. $$\tan\left(\frac{\pi}{4}+\theta\right) = \frac{1+\tan\theta}{1-\tan\theta}$$

$$= \frac{1+\sin 2\theta}{\cos 2\theta}$$

$$\tan\left(\frac{\pi}{4}-\theta\right) = \frac{1-\tan\theta}{1+\tan\theta}$$

$$= \frac{1-\sin 2\theta}{\cos 2\theta}$$

16. $$\cos\theta+\sin\theta = \sqrt{2}\cos\left(\frac{\pi}{4}-\theta\right)$$

$$= \sqrt{2}\sin\left(\frac{\pi}{4}+\theta\right)$$

$$\cos\theta-\sin\theta = \sqrt{2}\cos\left(\frac{\pi}{4}+\theta\right)$$

$$= \sqrt{2}\sin\left(\frac{\pi}{4}-\theta\right)$$

17. $$\frac{\tan\left(\frac{\pi}{4}+\theta\right)+\tan\left(\frac{\pi}{4}-\theta\right)}{\tan\left(\frac{\pi}{4}+\theta\right)-\tan\left(\frac{\pi}{4}-\theta\right)} = \operatorname{cosec} 2\theta$$

18. Special results

$$\cos A \cos 2A \cos 4A \cdots \cos(2^{n-1}A) = \frac{1}{2^n \sin A} \times \sin(2^n A)$$

$$\cos\frac{\pi}{2n+1} \cos\frac{2\pi}{2n+1} \cos\frac{3\pi}{2n+1} \cdots \cos\frac{n\pi}{2n+1} = \frac{1}{2^n}$$

$$\cos A \cos(60-A) \cos(60+A) = \frac{\cos 3A}{4}$$

$$\sin A \sin(60-A) \sin(60+A) = \frac{\sin 3A}{4}$$

$$\tan A \tan(60-A) \tan(60+A) = \tan 3A$$

$$\cot A \cot(60-A) \cot(60+A) = \cot 3A$$

$$\sin^4 A + \cos^4 A = 1 - 2\sin^2 A \cos^2 A$$

$$\sin^6 A + \cos^6 A = 1 - 3\sin^2 A \cos^2 A$$

$$\sec^4 A + \tan^4 A = 1 + 2\sec^2 A \tan^2 A$$

$$\operatorname{cosec}^4 A + \cot^4 A = 1 + 2\operatorname{cosec}^2 A \cot^2 A$$

$$\operatorname{cosec}^6 A - \cot^6 A = 1 + 3\operatorname{cosec}^2 A \cot^2 A$$

$$\sec^6 A - \tan^6 A = 1 + 3\sec^2 A \tan^2 A$$

19. If angles are in AP

$$\cos A + \cos(A+B) + \cos(A+2B) + \cdots \text{ n terms}$$

$$= \frac{\sin \frac{nB}{2}}{\sin \frac{B}{2}} \times \cos\left[\frac{\text{first angle} + \text{last angle}}{2}\right]$$

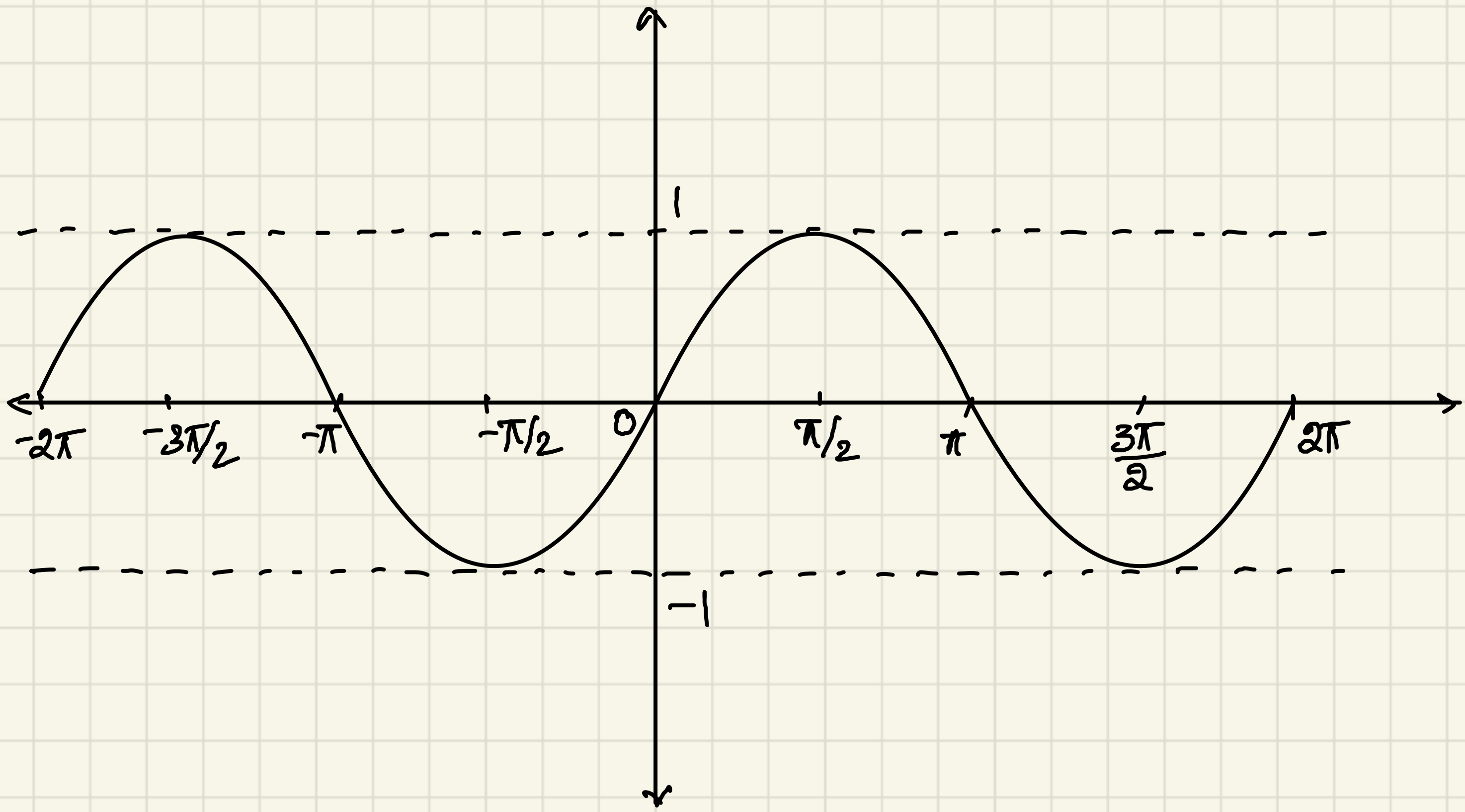
$$\sin A + \sin(A+B) + \sin(A+2B) + \cdots\cdots \text{ n terms}$$

$$= \frac{\sin \frac{nB}{2}}{\sin \frac{B}{2}} \times \sin\left[\frac{\text{first angle} + \text{last angle}}{2}\right]$$

Trigonometric functions

* $f(x) = \sin x$

Domain $\mathbb{R}$, Range : $[-1, 1]$



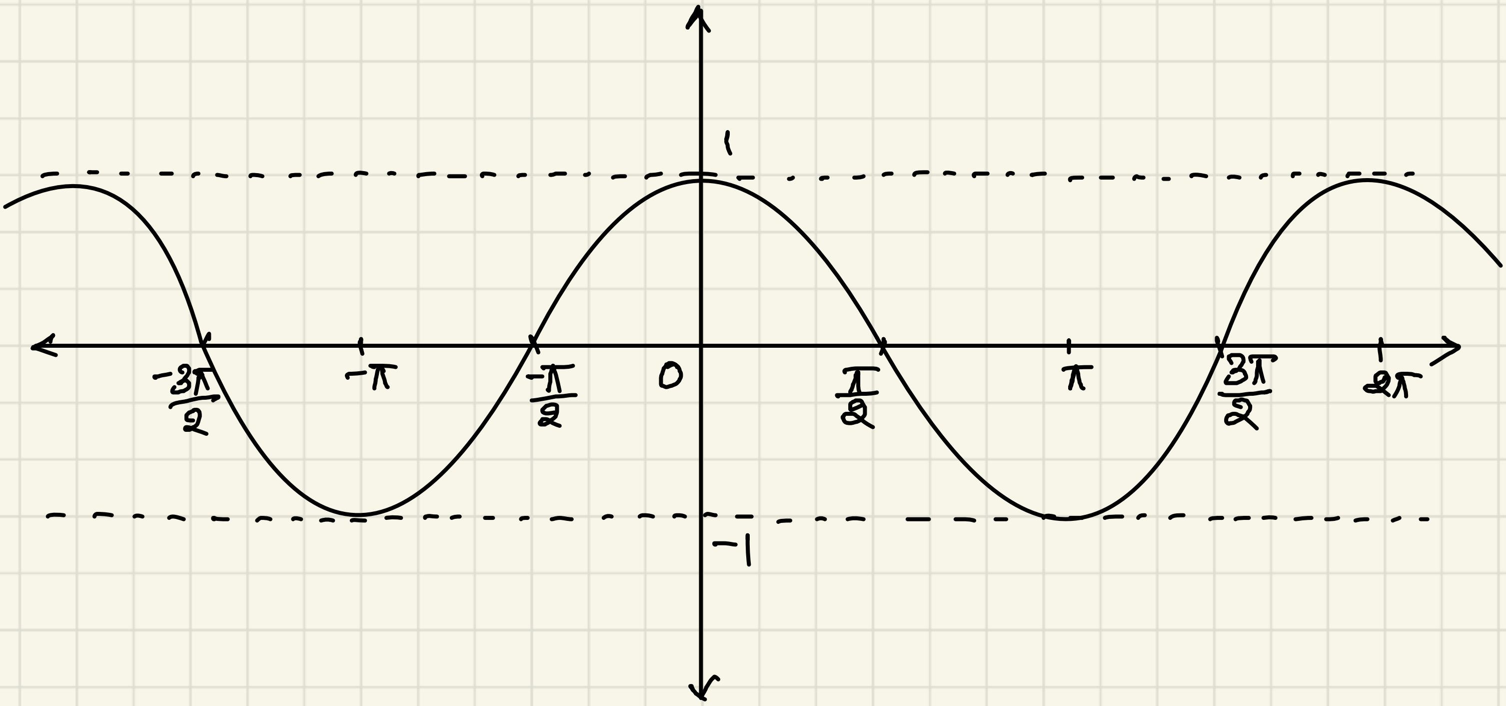
→ $\sin x$ is periodic with period 2π

→ $\sin x$ is an odd function.

→ $\sin n\pi = 0$ where $n \in \mathbb{Z}$

* $f(x) = \cos x$

Domain: $\mathbb{R}$ Range: $[-1, 1]$



→ $\cos x$ is periodic with period 2π

→ $\cos x$ is an even function, $\cos(-x) = \cos x$

→ $\cos(2n+1)\frac{\pi}{2} = 0$, $n \in \mathbb{Z}$,

$(2n+1)\frac{\pi}{2}$: odd multiple of $\frac{\pi}{2}$

→ $\cos 2n\pi = 1$, $n \in \mathbb{Z}$, $2n\pi$: even multiple of π

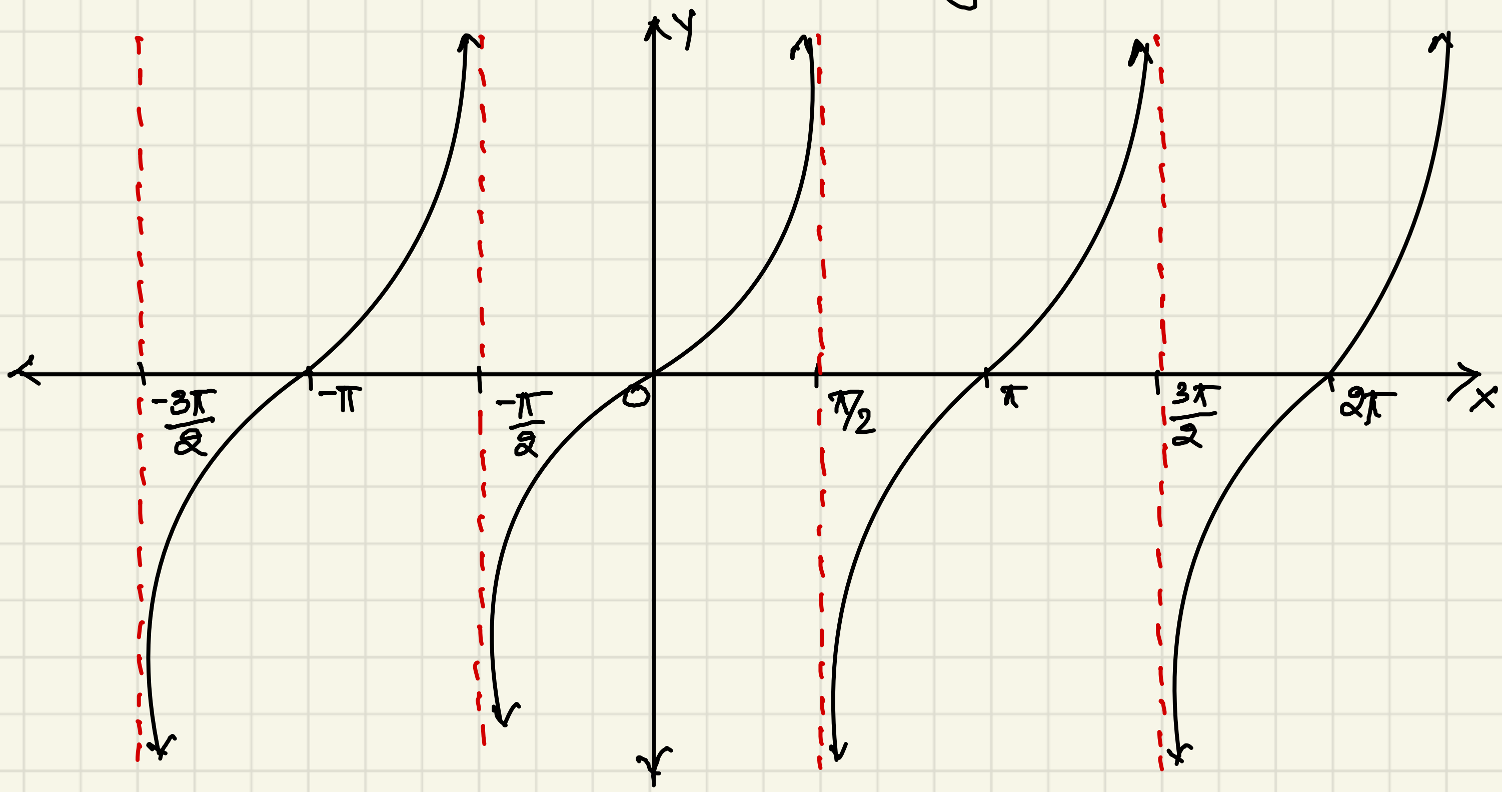
→ $\cos(2n+1)\pi = -1$, $n \in \mathbb{Z}$, $(2n+1)\pi$: odd multiple of π

$f(x) = \tan x$

$\tan x = \dfrac{\sin x}{\cos x}$, $\tan x$ is not defined when $\cos x = 0$

ie when $x = (2n+1)\frac{\pi}{2}$

Dom: $\mathbb{R} - \{(2n+1)\frac{\pi}{2}, n \in \mathbb{Z}\}$ Range : $\mathbb{R}$



→ $\tan x$ is periodic with period π

→ $\tan x$ is an odd function, $\tan(-x) = -\tan x$

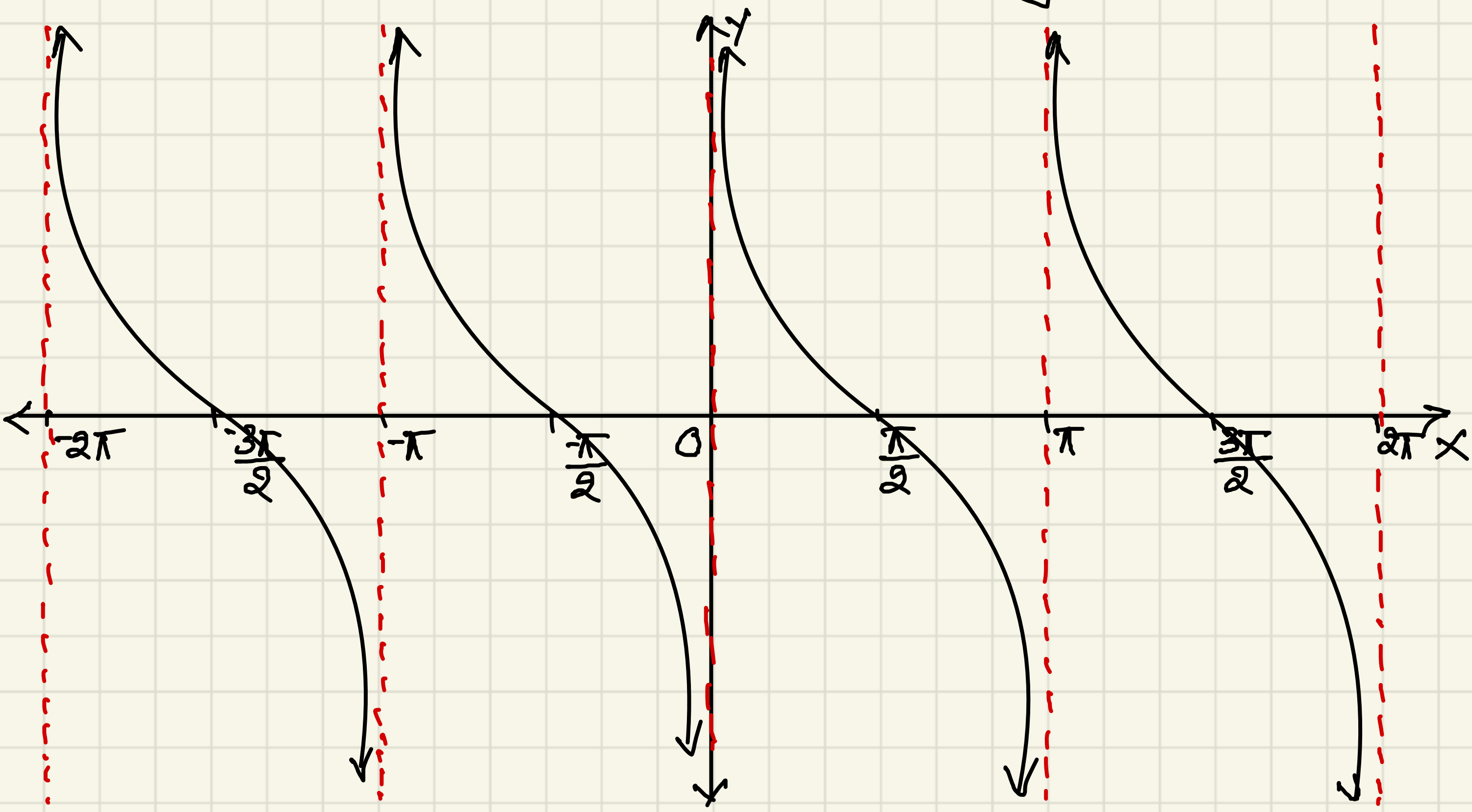
→ $\tan n\pi = 0$, $n \in \mathbb{Z}$

* $f(x) = \cot x$

$\cot x = \frac{\cos x}{\sin x}$, Not defined when $\sin x = 0$

ie when $x = n\pi$, $n \in \mathbb{Z}$

Domain : $\mathbb{R} - \{n\pi, n \in \mathbb{Z}\}$, Range : $\mathbb{R}$



→ $\cot x$ is periodic with period π

→ $\cot x$ is an odd function, $\cot(-x) = -\cot x$

→ $\cot(2n+1)\frac{\pi}{2} = 0$, $n \in \mathbb{Z}$

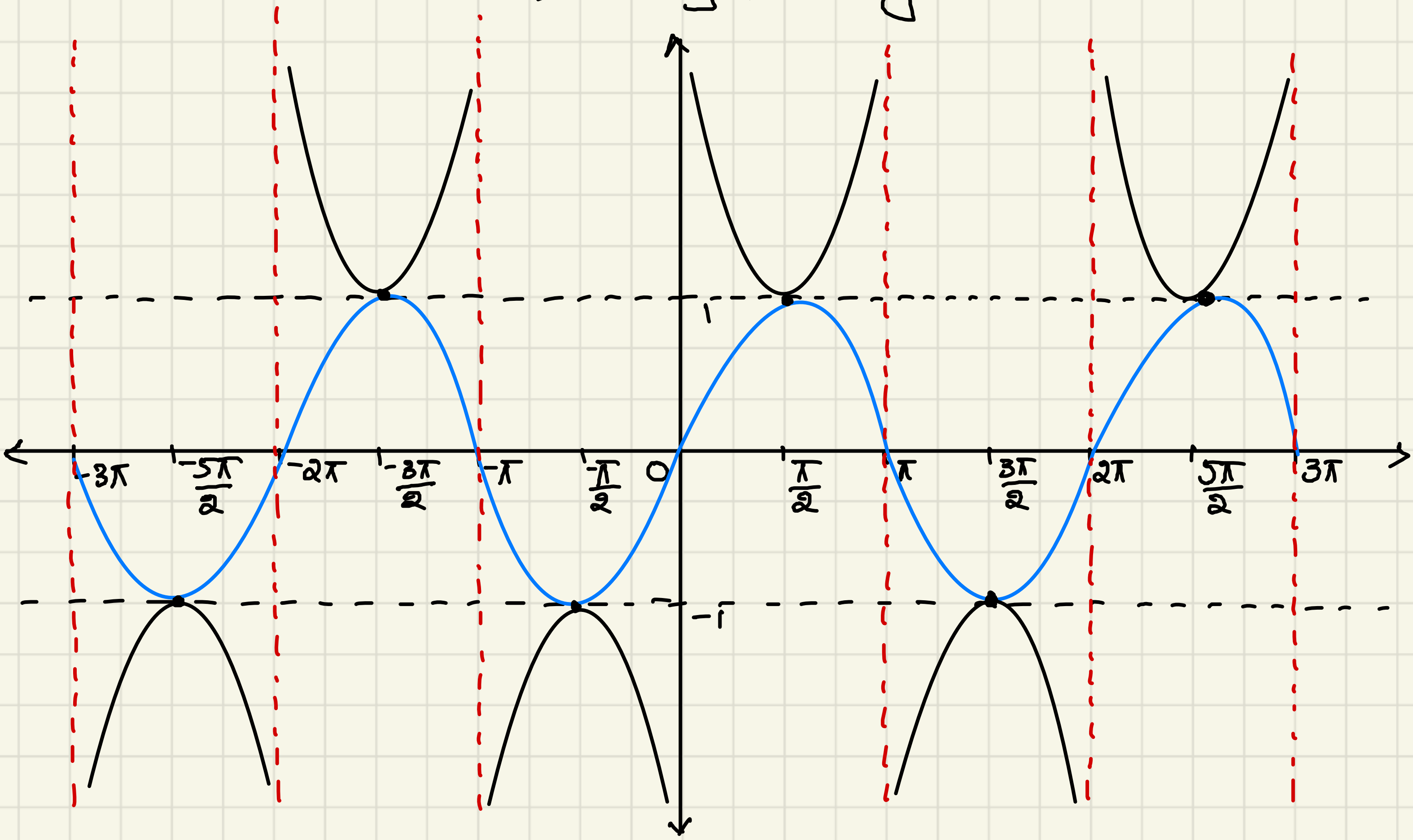
→ $\cot n\pi$, $n \in \mathbb{Z}$ is not defined.

$f(x) = \operatorname{cosec} x$

$\operatorname{cosec} x = \dfrac{1}{\sin x}$, Not defined when $\sin x = 0$

ie when $x = n\pi$, $n \in \mathbb{Z}$

Domain: $\mathbb{R} - \{n\pi, n \in \mathbb{Z}\}$, Range $= \mathbb{R} - (-1, 1)$



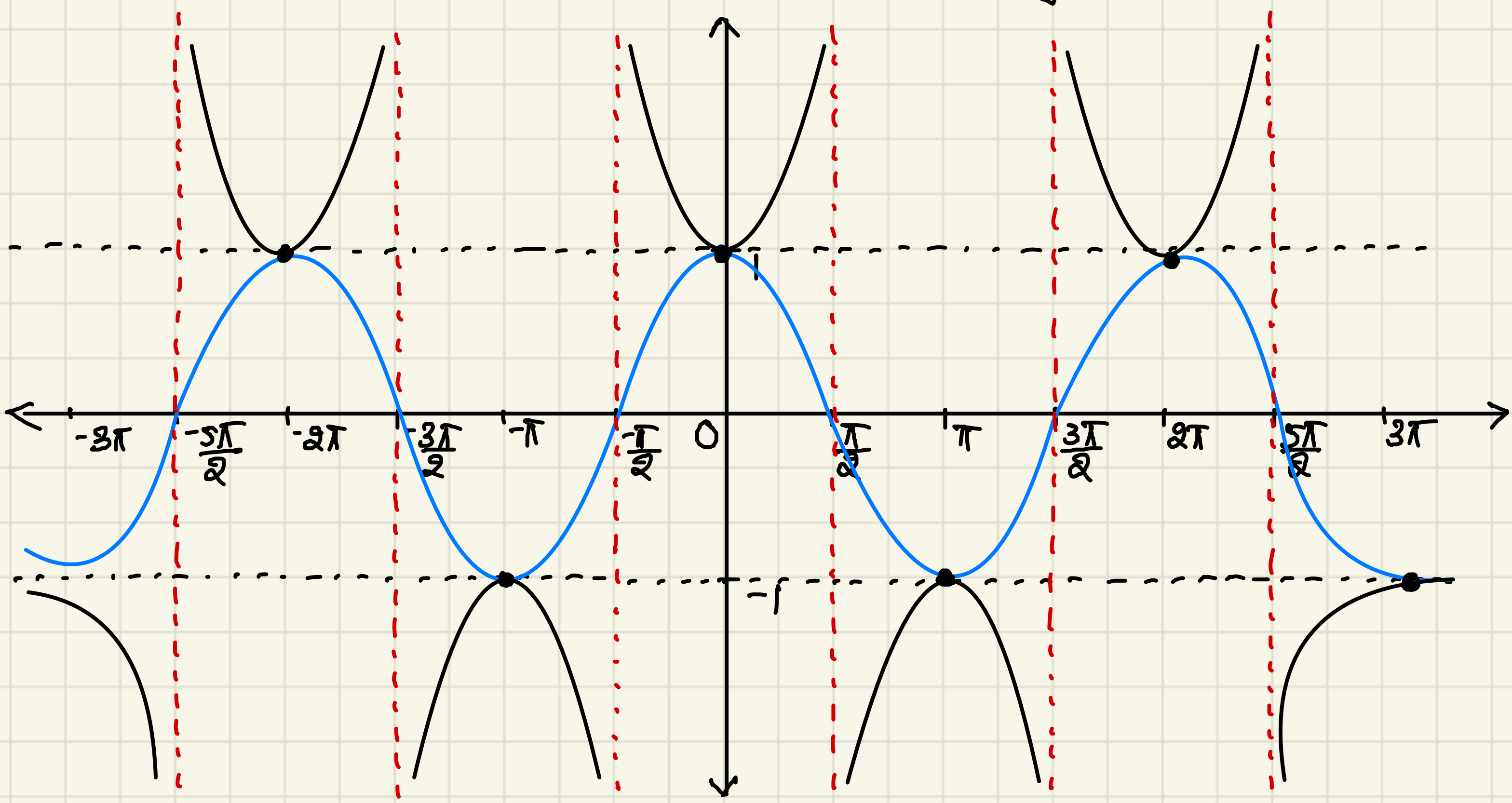
* Periodic with period 2π
* $\operatorname{cosec} x$ is an odd function.

$f(x) = \sec x$

$\sec x = \dfrac{1}{\cos x}$, Not defined when $\cos x = 0$

ie when $x = (2n+1)\frac{\pi}{2}$, $n \in \mathbb{Z}$

Domain: $\mathbb{R} - \{(2n+1)\frac{\pi}{2}, n \in \mathbb{Z}\}$ Range: $\mathbb{R} - (-1, 1)$



* Periodic with period 2π
* $\sec x$ is an even function
* $\sec x = 1$ if $x = 2n\pi$, $n \in \mathbb{Z}$
* $\sec x = -1$ if $x = (2n+1)\pi$, $n \in \mathbb{Z}$

* Graph transformations

→ x replaced by $x+k$

⇒ graph shifts k units to left

→ x replaced by $x-k$

⇒ graph shifts k units to right

→ y replaced by $y+k$

⇒ graph shifts k units downwords

→ y replaced by $y-k$

⇒ graph shifts k units upwards

→ x replaced by kx $(k>1)$

⇒ graph shrinks $(\frac{1}{k}$th$)$

→ x replaced by $\frac{x}{k}$ $(k>1)$

⇒ graph expands

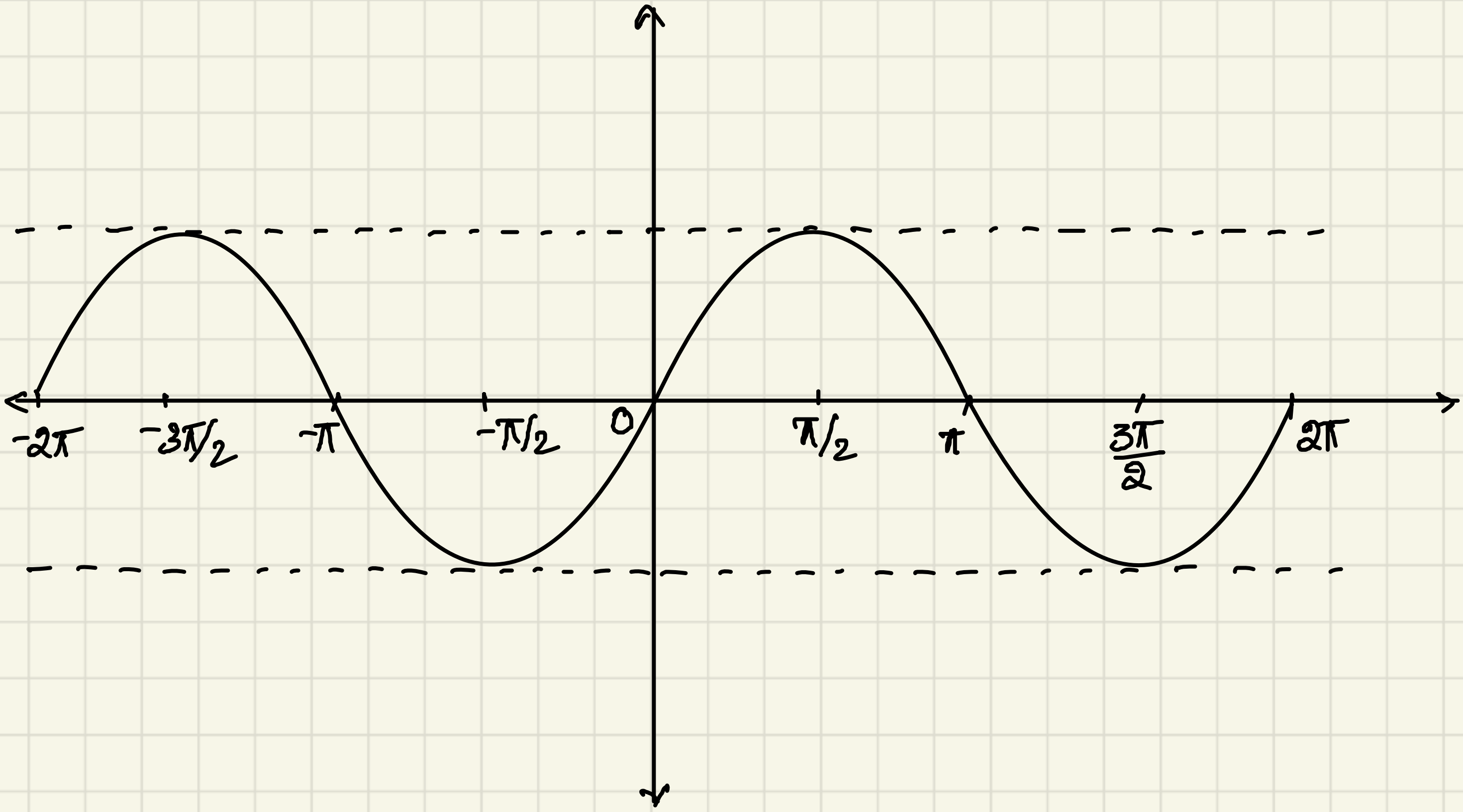
→ y replaced by ky $(k>1)$

⇒ graph compresses

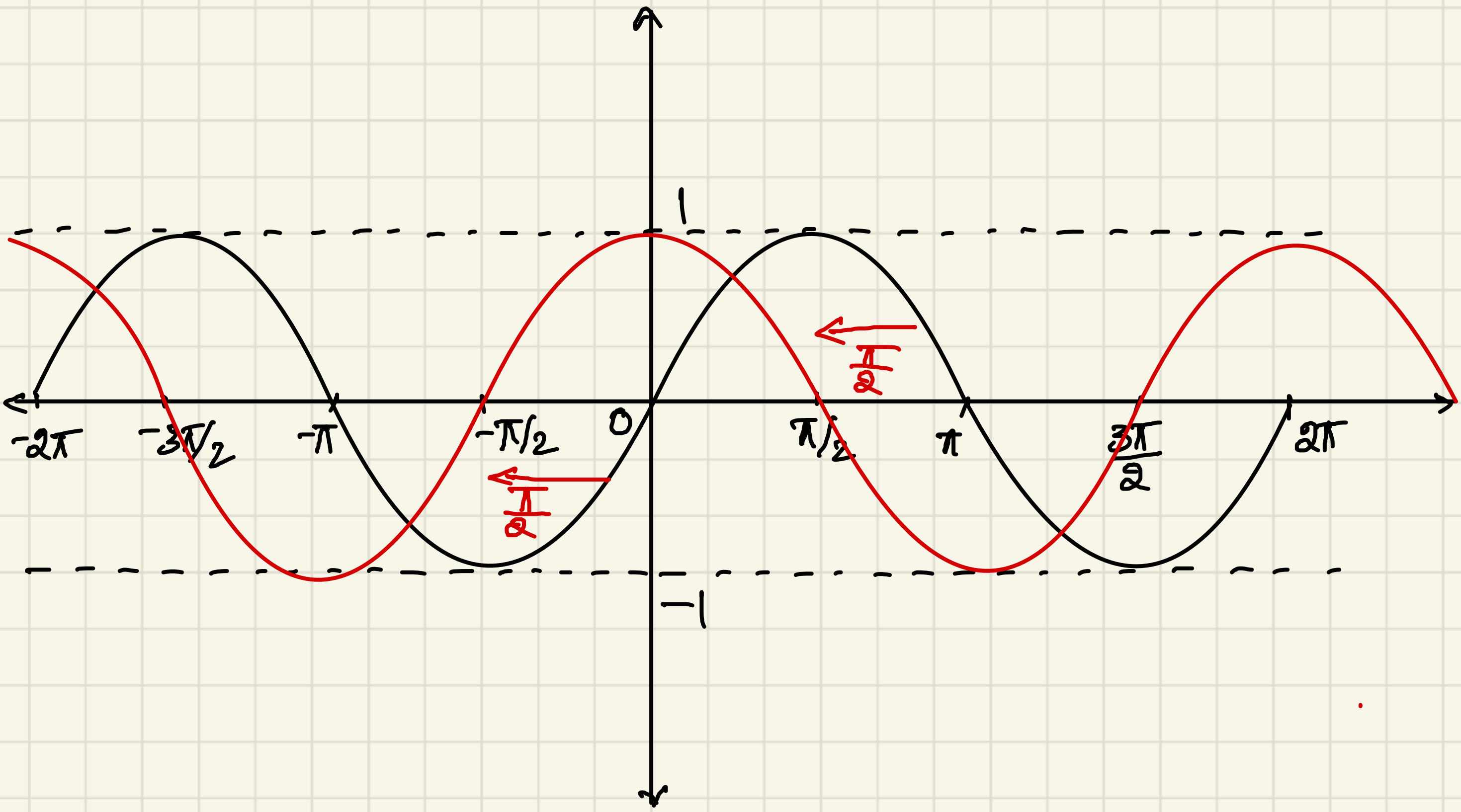
→ y replaced by $\frac{y}{k}$,

⇒ graph expands vertically.

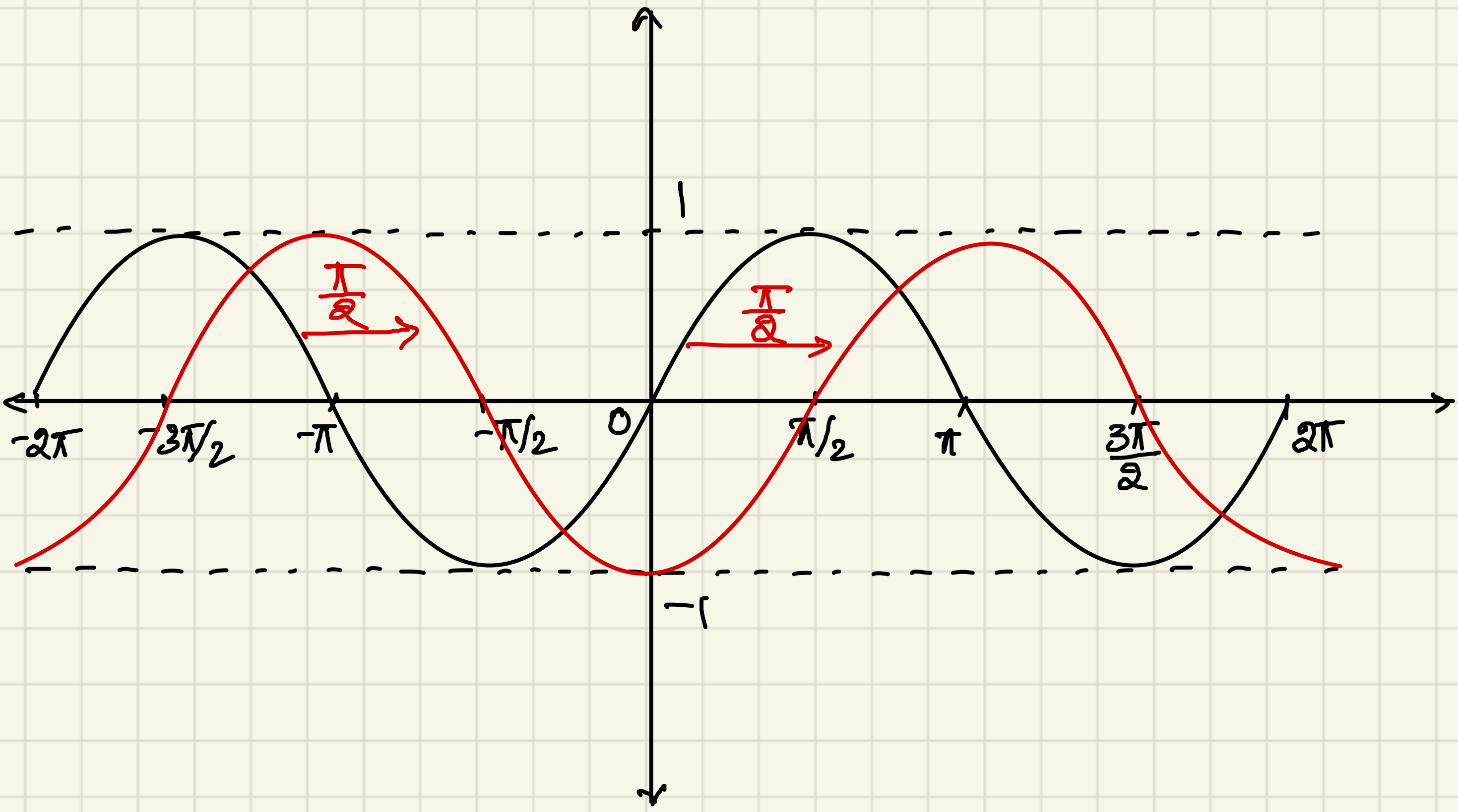
Eg: consider the graph of $y = \sin x$



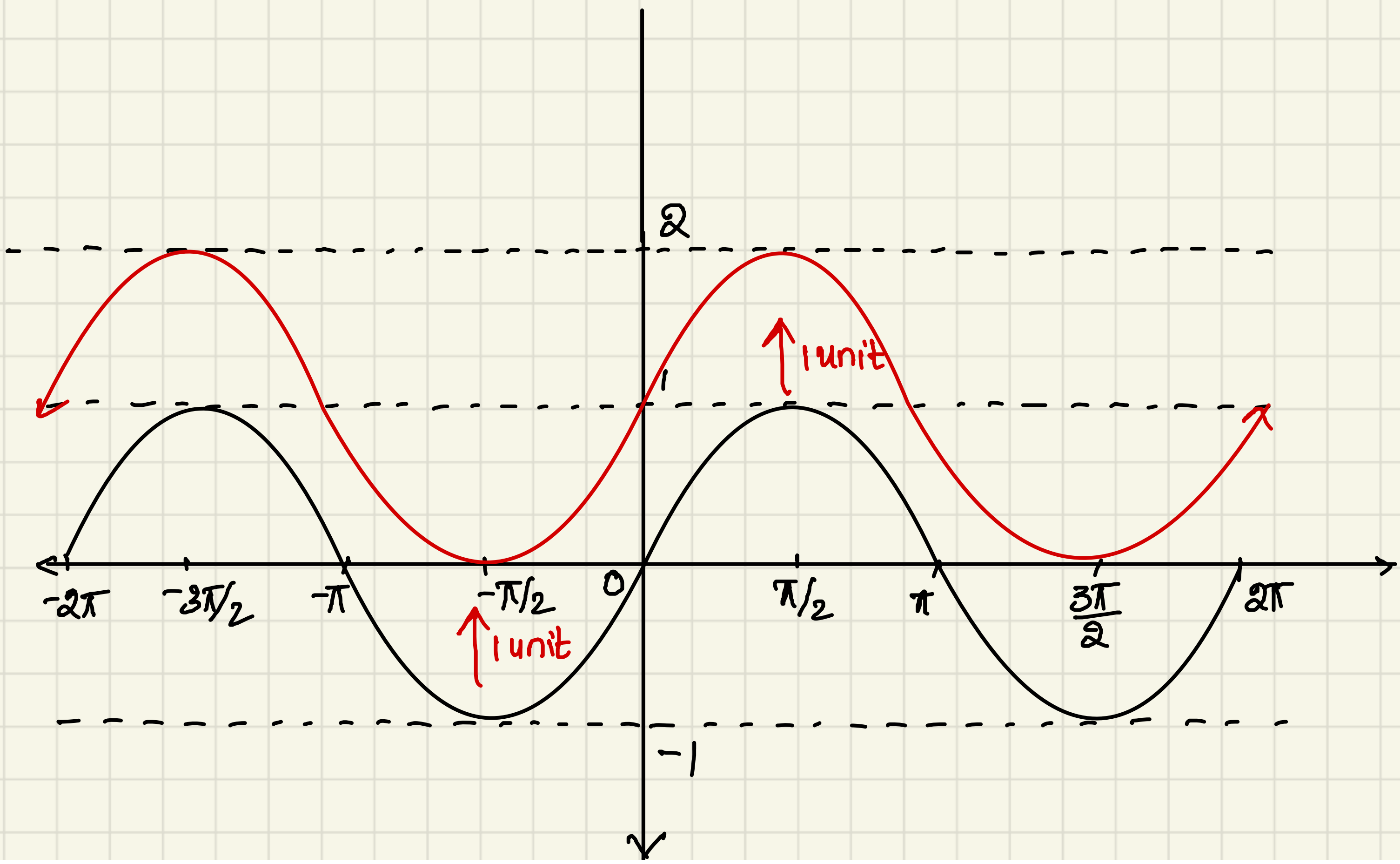
$y = \sin(x + \frac{\pi}{2}) \rightarrow$ shift $\frac{\pi}{2}$ units left



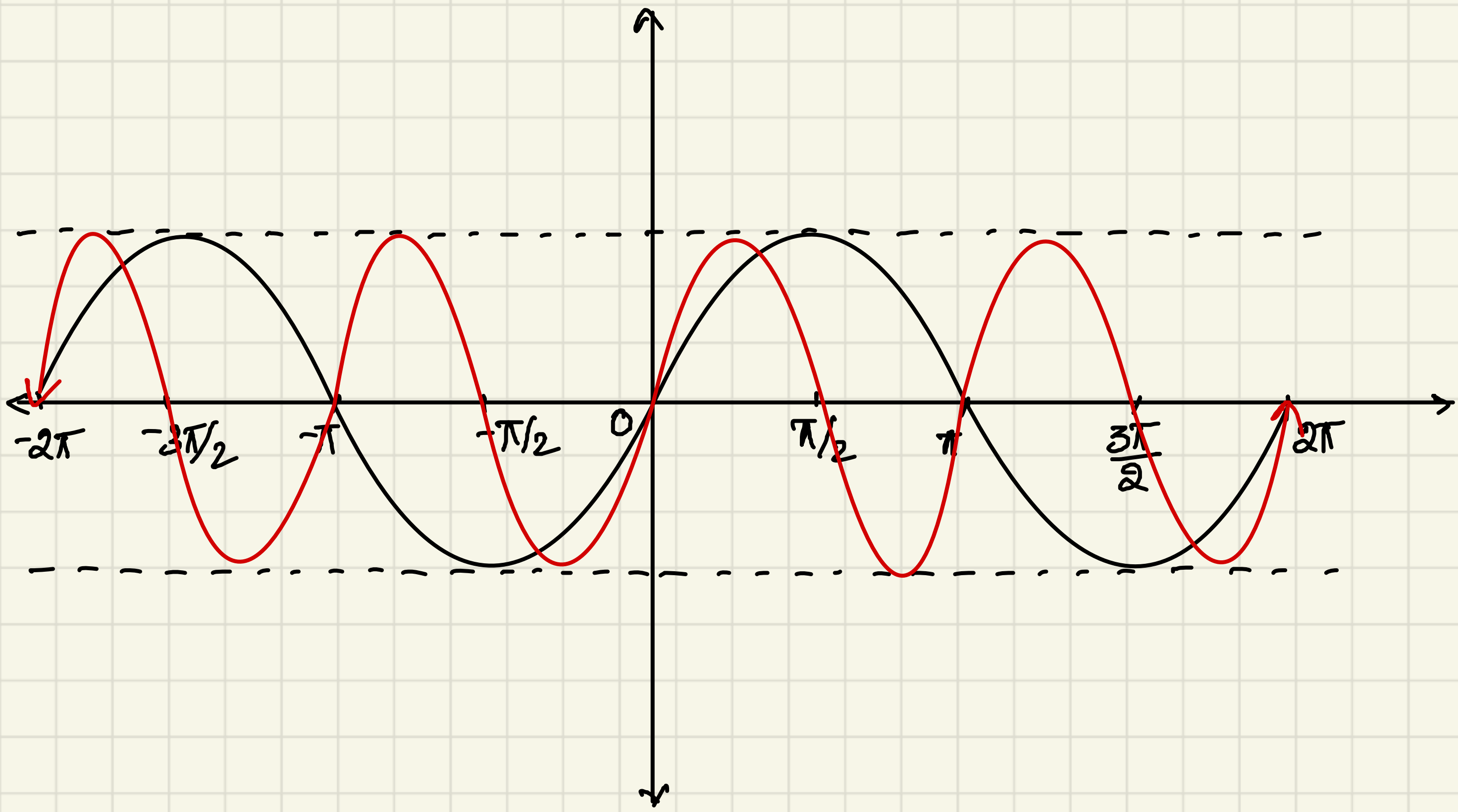
$y = \sin(x - \frac{\pi}{2}) \rightarrow$ shift $\frac{\pi}{2}$ units to right.



$y = \sin x - 1 \Rightarrow y + 1 = \sin x$ shift 1 unit upward



y = sin2x, graph shrinks

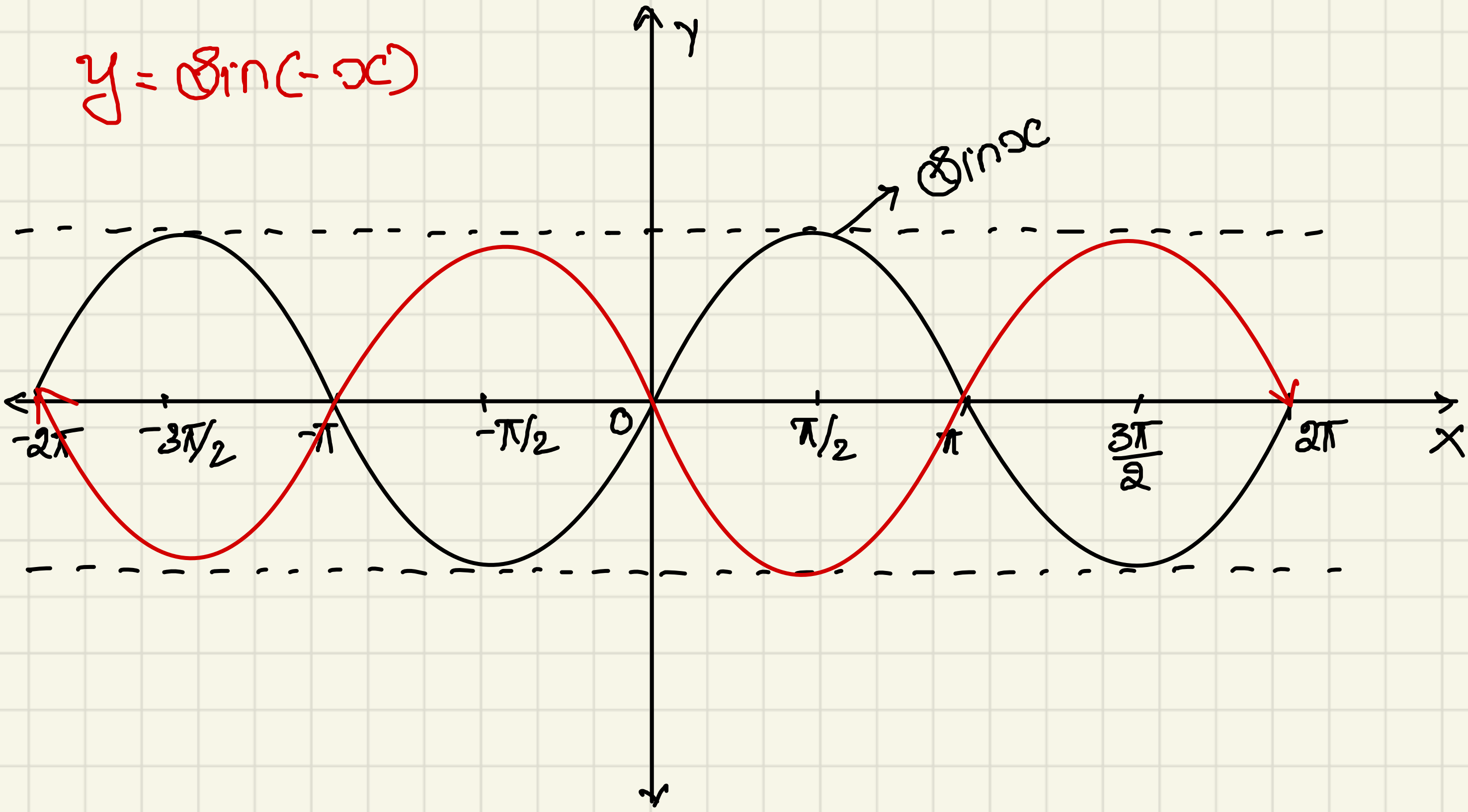


* x is replaced by $-x$

Draw the mirror image about Y axis

Fold the graph on the right side of Y axis to the left and graph on left side to right.

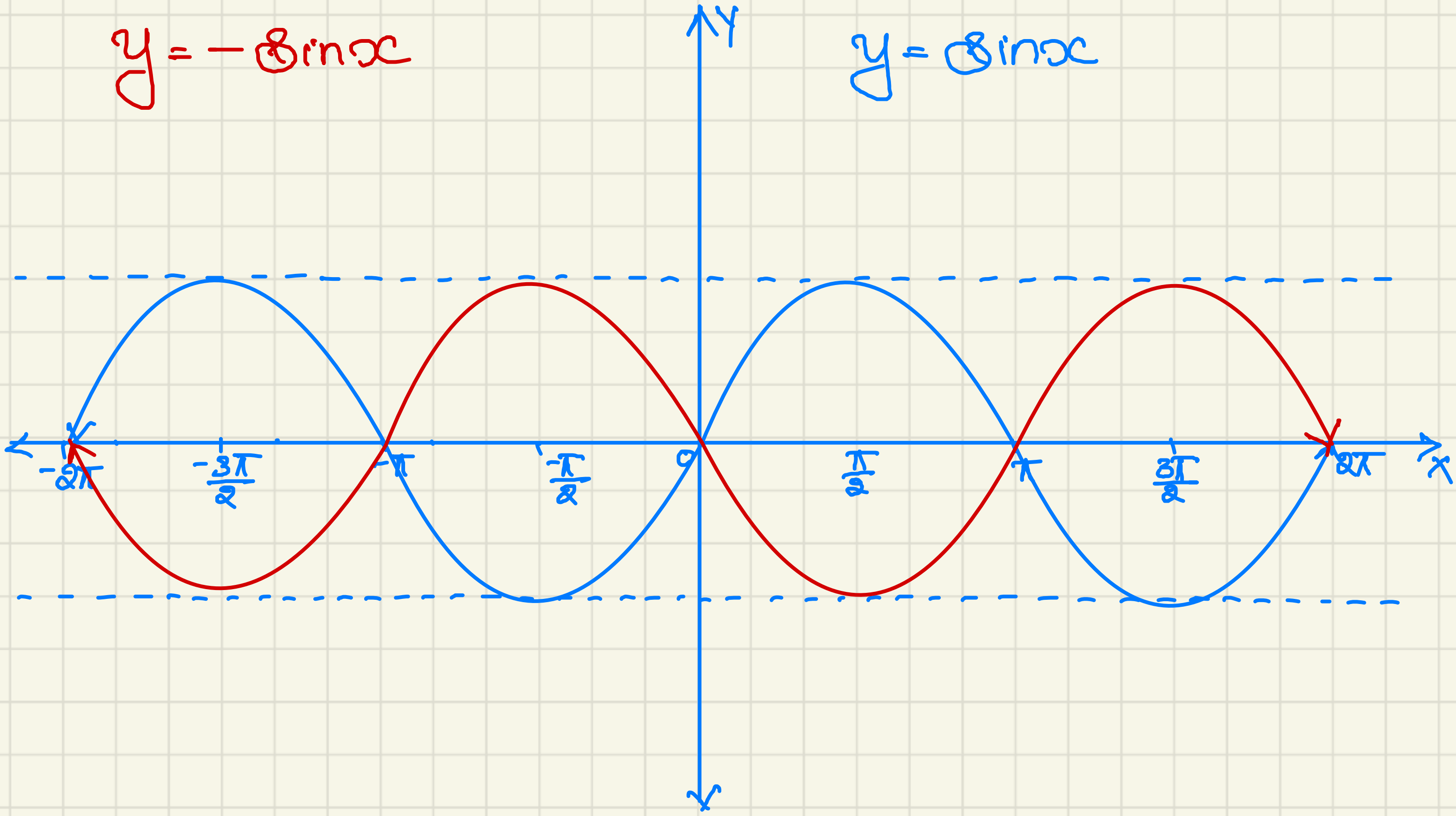
$y = \sin(-x)$



* The whole Function replaced by it negative

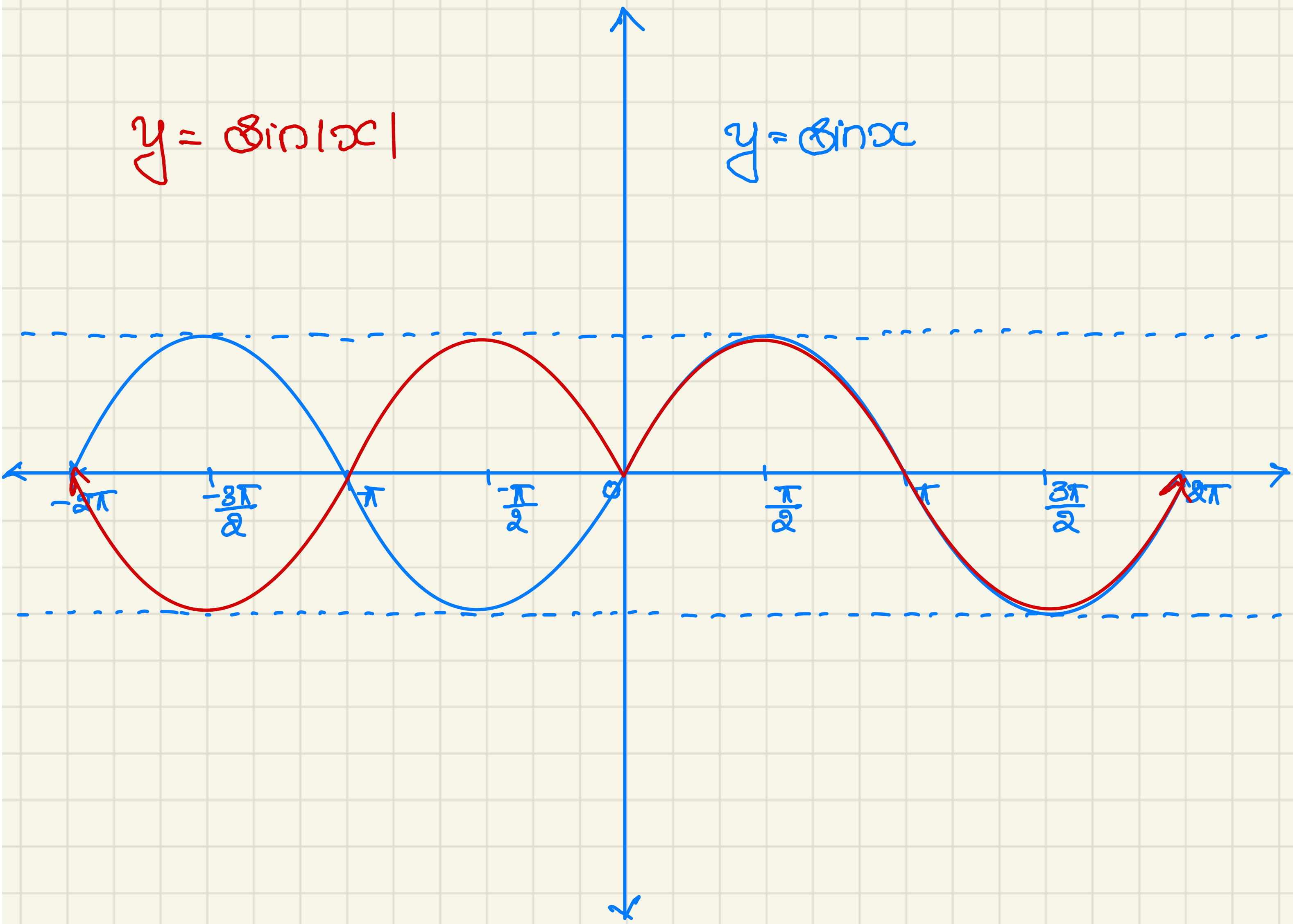
Fold the portions of the graph above the x axis downward

Fold the portions of graph below x axis upward.



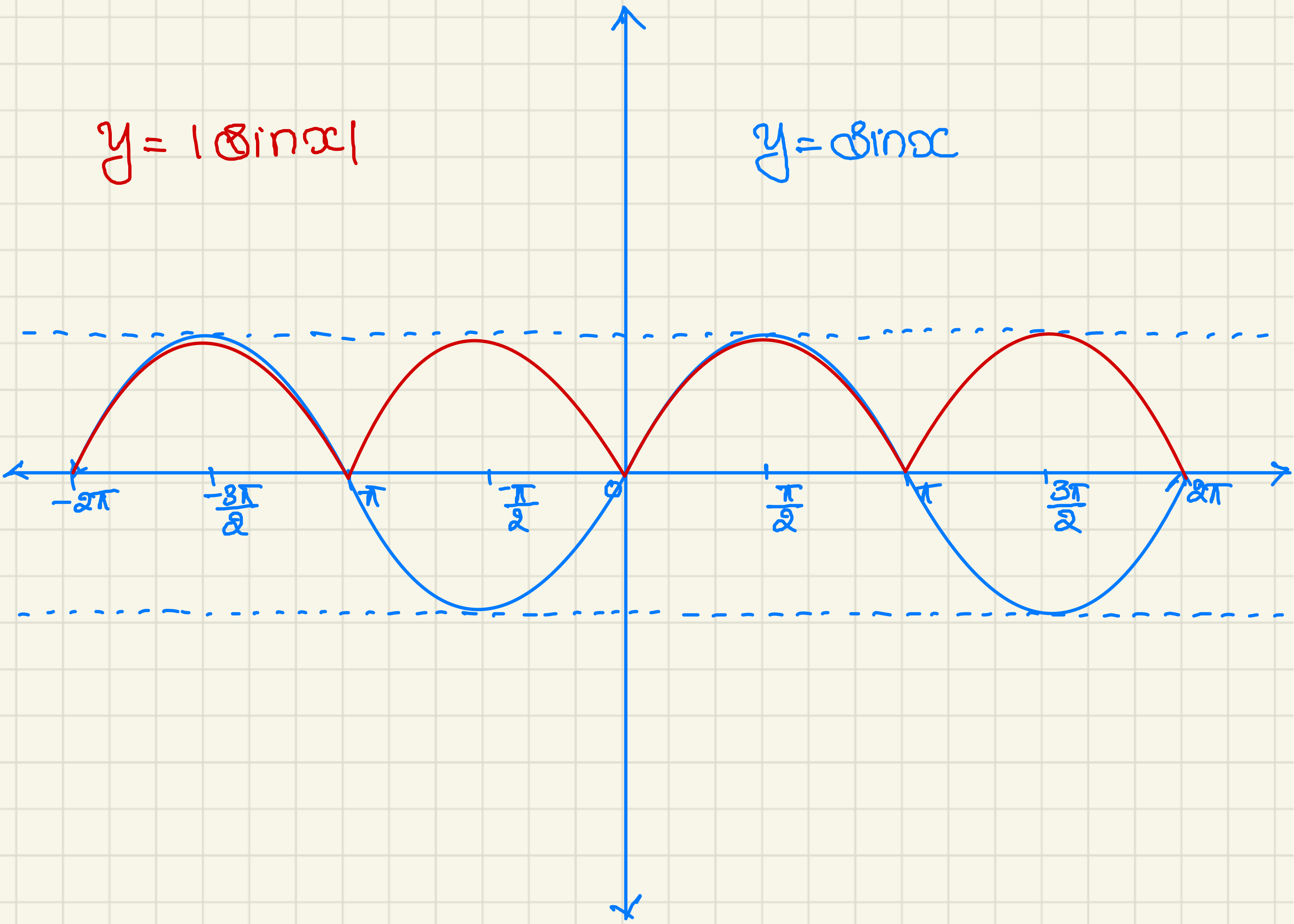
* Replacing x by $|x|$

Draw graph in the right side of y axis as it is and draw the mirror image of this to the left side.



* The whole function inside modulus

Draw the graph above x axis as it is and fold the graph below x axis upwards



* Trigonometric equations.

• Theorems

1. $\sin x = \sin y \Rightarrow x = n\pi + (-1)^n y,\ n \in \mathbb{Z},\ y \in [-\frac{\pi}{2}, \frac{\pi}{2}]$
2. $\cos x = \cos y \Rightarrow x = 2n\pi \pm y,\ n \in \mathbb{Z}\quad y \in [0, \pi]$
3. $\tan x = \tan y \Rightarrow x = n\pi + y,\ n \in \mathbb{Z},\ y \in (-\pi/2, \pi/2)$

• Corollary

1. $\sin\theta = 0 \Rightarrow \theta = n\pi,\ n \in \mathbb{Z}$

   $\sin\theta = 1 \Rightarrow \theta = 2n\pi + \frac{\pi}{2},\ n \in \mathbb{Z}$

   $\sin\theta = -1 \Rightarrow \theta = 2n\pi - \frac{\pi}{2},\ n \in \mathbb{Z}$

2. $\cos\theta = 0 \Rightarrow \theta = (2n+1)\frac{\pi}{2},\ n \in \mathbb{Z}$

   $\cos\theta = 1 \Rightarrow \theta = 2n\pi,\ n \in \mathbb{Z}$

   $\cos\theta = -1 \Rightarrow \theta = (2n+1)\pi,\ n \in \mathbb{Z}$

3. $$\left.\begin{aligned} \sin^2 x &= \sin^2 y \\ \cos^2 x &= \cos^2 y \\ \tan^2 x &= \tan^2 y \end{aligned}\right\} \Rightarrow x = n\pi \pm y$$

* Maximum value of $a\sin\theta + b\cos\theta = \sqrt{a^2+b^2}$

minimum value of $a\cos\theta + b\sin\theta = -\sqrt{a^2+b^2}$

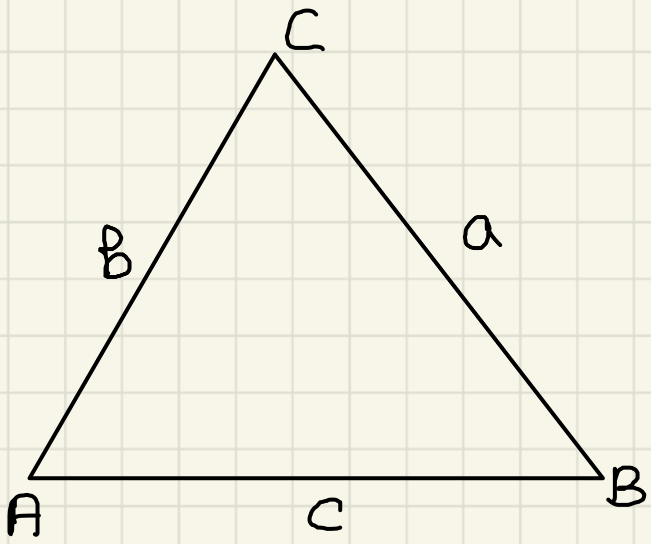
ie, $-\sqrt{a^2+b^2} \leq a\sin\theta + b\cos\theta \leq \sqrt{a^2+b^2}$

Note, to solve $a\sin\theta + b\cos\theta = k$,

divide LHS and RHS by $\sqrt{a^2+b^2}$

* Properties of triangles.

The following symbols are universally adopted.



s: Semiperimeter, $s = \dfrac{a+b+c}{2}$

Δ: area of triangle

$$\Delta = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} bc \sin A$$

$$= \frac{1}{2} ac \sin B$$

$$= \sqrt{s(s-a)(s-b)(s-c)}$$ { Heron's formula

1. Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$, R: circumradius of ΔABC

2. Cosine rule

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

3. tangent rule

$$\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c}\cot\frac{A}{2}$$

$$\tan\left(\frac{C-A}{2}\right) = \frac{c-a}{c+a}\cot\frac{B}{2}$$

$$\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b}\cot\frac{C}{2}$$

4. Half angle Formula

$$\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}} \qquad \cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$$

$$\sin\frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{ac}} \qquad \cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ac}}$$

$$\sin\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}} \qquad \cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$$

$$\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

$$\tan\frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$$

$$\tan\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

5. Projection Formula

$$a = b\cos C + c\cos B$$

$$b = c\cos A + a\cos C$$

$$c = a\cos B + b\cos A$$

Miscellaneous.

* Periodic functions:

A function $f(x)$ is said to be periodic with period t, if t is the least positive number such that $f(x+t) = f(x)$

Properties.

If $f(x)$ is periodic with period t, then

1. $f(x \pm a)$ is also periodic with period t where a is a constant.
2. $f(kx)$ is periodic with period $\frac{t}{|k|}$
3. If $f_1(x)$ and $f_2(x)$ are periodic with periods t_1 and t_2 respectively, then $a f_1(x) + b f_2(x)$, where a and b are constants, is periodic with period

LCM $\{t_1, t_2\}$

4. Period of $\dfrac{a f_1(x) + b f_2(x)}{c f_3(x) + d f_4(x)}$, where

f_1, f_2, f_3, f_4 are periodic functions and a, b, c, d are constants, is LCM of periods of f_1, f_2, f_3 and f_4.

Note: In a choice based question, check with options too.

Function	Period
$\sin x$	2π
$\cos x$	2π
$\operatorname{cosec} x$	2π
$\sec x$	2π
$\tan x$	π
$\cot x$	π
$\lvert\sin x\rvert$	π
$\lvert\cos x\rvert$	π

$\lvert\operatorname{cosec} x\rvert$	π
$\lvert\sec x\rvert$	π
$\lvert\tan x\rvert$	π
$\lvert\cot x\rvert$	π

* Arithmetic Progression and Geometric Progression.

<u>AP</u> : $a, a+d, a+2d, \dots$

1st term: a , common difference : d

nth term, $a_n = a+(n-1)d$

Sum of 1st n terms,

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$= \frac{n}{2}\left[\text{1st term} + \text{last term}\right]$$

number of terms,

$$n = \frac{\text{last term} - \text{1st term}}{d} + 1$$

<u>GP</u> $a, ar, ar^2, ar^3 \dots$

a: 1st term, r: common ratio

nth term, $a_n = ar^{n-1}$

Sum of 1st n terms

$$S_n = \frac{a(r^n-1)}{r-1}$$

$$\text{or} = \frac{a(1-r^n)}{1-r}$$

* Sum of infinite terms of a G.P.

$$a + ar + ar^2 + \cdots \infty = \frac{a}{1-r} \quad ; \quad |r|<1$$

* Inequalities

$$AM \geq GM \geq HM$$

AM : Arithmetic mean : $\dfrac{x_1 + x_2 + \cdots + x_n}{n}$

GM : Geometric mean : $(x_1 \times x_2 \times \cdots \times x_n)^{1/n}$

HM : Harmonic mean : $\dfrac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}}$

* Increasing/Decreasing functions

→ A function $f(x)$ is said to be increasing

if $x<y \Rightarrow f(x)<f(y)$

or $x>y \Rightarrow f(x)>f(y)$

→ A function f is said to be decreasing

if $x<y \Rightarrow f(x)>f(y)$

or $x>y \Rightarrow f(x)<f(y)$

* Odd / Even functions

→ f is odd if $f(-x) = -f(x)$

eg: $x, x^3, x^5, \sin x, \tan x, \dots$

→ f is even if $f(-x) = f(x)$

eg: $x^2, x^4, \cos x, \sec x \dots$

* Componendo dividendo rule

If $\frac{a}{b} = \frac{c}{d}$ then

→ Componendo $\frac{a+b}{b} = \frac{c+d}{d}$

→ Dividendo $\frac{a-b}{b} = \frac{c-d}{d}$

→ Componendo and dividendo $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

* Standard points with respect to a triangle

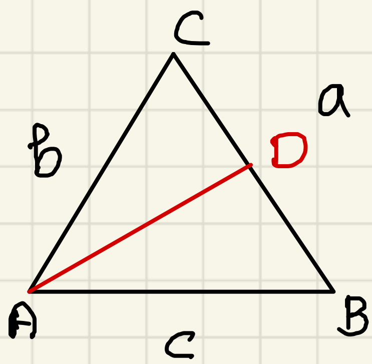
1. Centroid : Point of intersection of medians

→ centroid divides median in the ratio 2:1

→ Centroid of a triangle having vertices $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ is

$$G\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

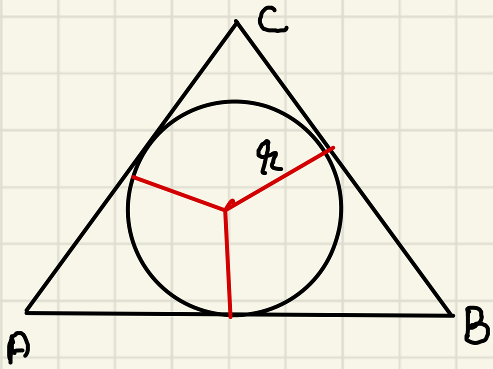
→ Length of median can be obtained by the formula below



$$4AD^2 = 2(AC^2 + AB^2) - BC^2$$

2. Incentre : Meeting point of internal angle bisectors.

→ Centre of incircle



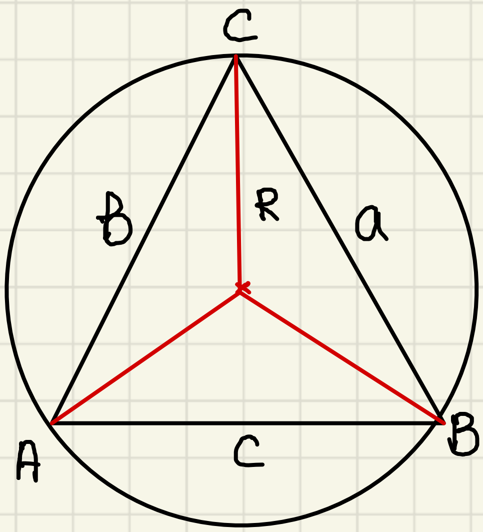
→ Radius of incircle r

$r = \frac{\Delta}{s}$, Δ: area of triangle

s: semiperimeter

3. Circumcentre : Meeting point of $\perp^{ar}$ bisectors of sides.

→ Centre of circumcircle

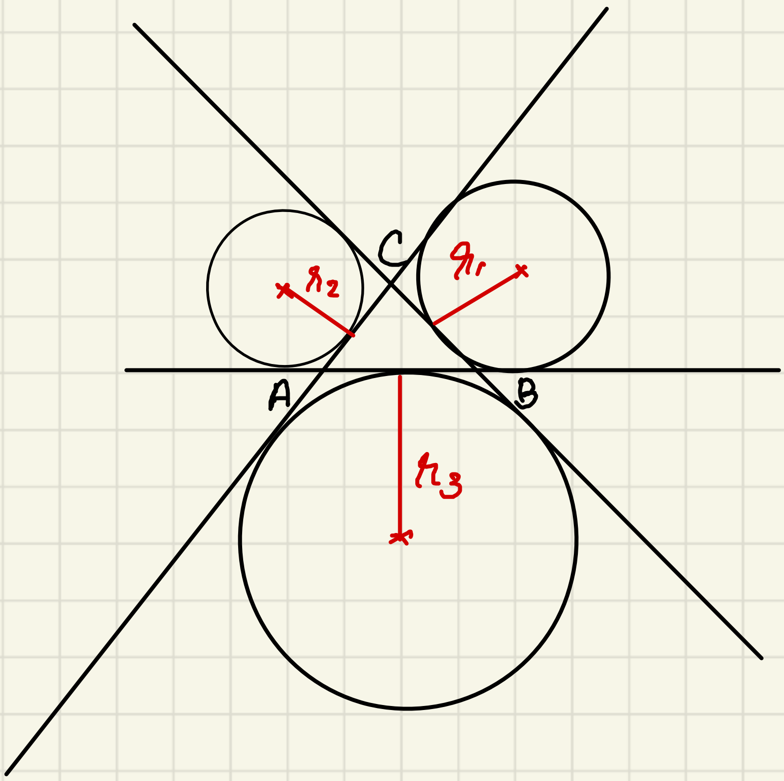


→ R: circumradius

$R = \frac{abc}{4\Delta}$

4. Orthocentre: Meeting point of altitudes.

5. Excentre: Meeting point of external angle bisectors



r_1, r_2, r_3 are exradii then,

$$r_1 = \frac{\Delta}{s-a}, \quad r_2 = \frac{\Delta}{s-b}, \quad r_3 = \frac{\Delta}{s-c}$$

and $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$

where r is the inradius of ΔABC