CHAPTER -
UNITS AND MEASUREMENTS

Measurement of a physical quantity involves its comparison with a standard value of the same kind is called the unit of that quantity. The process of measurement of a physical quantity involves,

1) selection of unit (u)

2) to find out the no. of times that unit is contained in the given physical quantity it is called the numerical value OR magnitude of the quantity (n)

$\therefore$ Any measurement (X) can be represented as the product of numerical value and unit

$$\boxed{X = nu}$$

Fundamental and Derived units

The physical units which can neither be derived from one another, nor they can be further resolved in to more simpler units are called fundamental units

eg. metre, kg, sec

All other physical units which can be expressed in terms of fundamental units are called derived units.

eg. $ms^{-1}, kg\ ms^{-2}\left(N\right)$

System of Units

A complete set of units which is used to measure all kinds of fundamental and derived quantities are called system of units

1) CGS system - Centimetre, gram, sec

2) FPS system - Foot, pound, sec
   1 foot = 0.3048 m
   1 pound = 0.4536 kg

3) MKS system - metre, kg, sec

4) SI system - (International system of units)

Basic SI units

Length - metre (m)

Mass - kilogram (kg)

Time - second (s)

Temperature - kelvin (K)

Electric current - Ampere (A)

Supplementary SI units

Plane angle - radian (rad)

Solid angle - Steradian (Sr)

Luminous intensity - Candela (Cd)

Amount of substance - mole (mol)

SI prefixes for powers of ten

10^1 - deca (da)	10^{-1} - deci (d)
10^2 - hecto (h)	10^{-2} - centi (c)
10^3 - kilo (k)	10^{-3} - milli (m)
10^6 - mega (M)	10^{-6} - micro (μ)
10^9 - giga (G)	10^{-9} - nano (n)
10^{12} - tera (T)	10^{-12} - pico (p)
10^{15} - peta (p)	10^{-15} - femto (f)
10^{18} - exa (E)	10^{-18} - atto (a)

Some common practical Units

Large distances

1) Light year (ly)

It is the distance travelled by light through vacuum in one year

$$\boxed{1\ \ell y = 9.46\times10^{15}\,\text{m}}$$

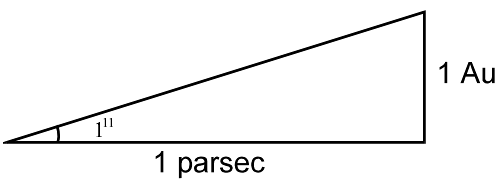
2) Astronomical Unit (Au)

It is the average distance between centre of earth and centre of Sun

$$\boxed{1\ \text{Au} = 1.496\times10^{11}\,\text{m}}$$

3) Par sec (parallactic sec)

It is the distance at which an arc of length one astronomical unit subtends an angle of 1 second of arc



$$\boxed{1\ \text{par sec} = 3.08\times10^{16}\,\text{m}}$$

$$\boxed{1\ \text{par sec} = 3.26\ \ell y}$$

Large Masses

1) tonne or metric ton = 1000 kg

2) quintal = 100 kg

3) slug = 14.57 kg

4) Chandra Shekhar Limit (CSL) = 1.4 times the mass of sun

Small masses

Atomic mass unit (amu) = It is defined as $\frac{1}{12}^{th}$ of the mass of one ${}_{6}^{12}C$ - atom

$\boxed{1\text{ amu} = 1.66\times10^{-27}\text{ kg}}$

Time

1) Solar day - One day (24 hour)

2) Solar year - 365.25 days

3) Lunar month - It is the time taken by the moon to complete one revolution around the earth in its orbit

$\boxed{1\text{ lunar month} = 27.3\text{ days}}$

4) Shake - It is the smallest practical unit of time

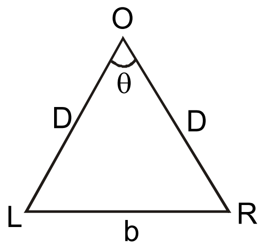
$\boxed{1\text{ shake} = 10^{-8}\text{ sec}}$

Small Areas

$\boxed{\text{Barn} = 10^{-28}\text{ m}^2}$

Parallax

It is the apparent shift in the position of an object with respect to another when we shift our eye sidewise. The distance between the two points of observation is called basis (b)



θ - is called parallax angle OR parallactic angle

$$\theta = \frac{b}{D}$$

$$\boxed{D = \frac{b}{\theta}}$$

Dimensional Analysis

The dimensions of a physical quantity are the powers to which the units of base quantities are raised to represent a derived unit of that quantity. It is denoted with square brackets [ ]

Eg. Force, F = ma = $\left[M^1L^1T^{-2}\right]$

• The physical quantities can be added or substracted which have the same dimensions

• Special functions such as trigonometric functions, logarithmic functions, and exponential functions must be dimensionless

• A pure number, ratio of similar physical quantities has no dimension. (Eg. Angle, refractive index, π,...etc)

Different quantities having same dimension

$$\left.\begin{array}{l}\text{Work}\\\text{Energy}\\\text{Heat}\\\text{Torque}\\\text{Moment of force}\end{array}\right\}\left[ML^{2}T^{-2}\right] \qquad \left.\begin{array}{l}\text{Linear momentum}\\\text{Impulse}\end{array}\right\}\left[MLT^{-1}\right] \qquad \left.\begin{array}{l}\text{Surface tension}\\\text{Surface Energy}\\\text{Spring constant}\end{array}\right\}\left[ML^{0}T^{-2}\right]$$

Dimensional constants :	Speed of light (C)
	Gravitational constant (G)
	Planks constant (h)
Dimensional variables :	Area, volume, force,....
Dimensionless constants:	Numbers, π, ...
Dimensionless variables :	Angle, strain, specific gravity, ...

A dimensionally correct equation need not be actually a correct equation, but dimensionally wrong equation must be wrong

Applications of Dimensional Analysis

1. Conversion of one system of units to another

This is based on the fact that magnitude of a physical quantity remains the same whatever be the system of units.

Q = nu = constant

$$\boxed{n_1u_1 = n_2u_2}$$

$$u_1 = M_1^aL_1^bT_1^c \qquad u_2 = M_2^aL_2^bT_2^c$$

$$n_2 = \frac{n_1u_1}{u_2}$$

$$\boxed{n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c}$$

Eg. Convert 1 N to dyne (CGS system)

$\left[F\right]=\left[M^1L^1T^{-2}\right]$. Here a = 1, b = 1, c = –2

In SI system M_1 = kg, L_1 = m, T_1 = sec

In CGS system M_2 = g, L_2 = cm, T_2 = sec

$n_1 = 1$ $n_2 = ?$

$$n_2 = 1\left[\frac{kg}{g}\right]^1\left[\frac{m}{cm}\right]^1\left[\frac{s}{s}\right]^{-2}$$

$$= \left[\frac{10^3 g}{g}\right]\left[\frac{10^2 cm}{cm}\right]$$

$\boxed{n_2 = 10^5}$ $\therefore 1N = 10^5 \text{dyne}$

2. <u>Checking the correctness of an equation</u>

   <u>(Principle of homogenecity of dimensions)</u>

   According to this principle, when a relation is dimensionally correct, then the dimensions of all the terms in that relation are equal

   Eg. $S = ut + \frac{1}{2}at^2$

   $$\left[s\right]=\left[ut\right]=\left[\frac{1}{2}at^2\right]$$

3. To derive the relationship among various physical quantities

   Using the principle of homogeneity of dimension we can derive the formula of a physical quantity.

   Eg. Derive an expression for the time period (T) of a simple pendulum depends mass (m), length (ℓ) and acceleration due to gravity (g)

   Let $T \,\alpha\, m^a \ell^b g^c$

   $T = K\, m^a \ell^b g^c$

   $$\left[M^0L^0T^1\right]=M^aL^b\left(LT^{-2}\right)^c$$

   $$M^0L^0T^1 = M^aL^{b+c}T^{-2c}$$

   Applying the homogeneity of dimension

   a = 0, b + c = 0, –2c = 1

   a = 0, $c = -\frac{1}{2}$, $b = \frac{1}{2}$

$$\therefore T = KM^0 \ell^{1/2} g^{-1/2}$$

$$T = K\sqrt{\frac{\ell}{g}} \quad k = 2\pi$$

$$\boxed{T = 2\pi\sqrt{\frac{\ell}{g}}}$$

Limitations

1. If a quantity depends on more than three factors having dimensions, the formula cannot be derived
2. The method of dimensions cannot be used to derive an exact form of relation, when it consists of more than one part on any side

   eg. $S = ut + \frac{1}{2}at^2$

3. It gives us no information about the dimensionless constants in the relation eg. π,1,2...
4. We cannot derive the formula containing trigonometrical function, exponential function, logarithmic function, etc. which are dimensionless

Significant Figures

In all instrumental values, last digit remains uncertain and the rest of the digits are certain or reliable digits. The total number of certain digits along with last uncertain digit gives the number of significant digits.

In an instrumental value all nonzero digits, trapped zeros, and terminal zero's in a number with decimal point are measured as significant digits.

The insignificant digits are terminal zero's without a decimal point, the zero's on the right of decimal point (to the left of Ist non-zero digit in a number less than one), and the power of 10.

• Change of units does not change the no. of significant figures in a measurement

• The multiplying or dividing factors are exact values, they have infinite no. of significant figures as per the situation

ROUNDING OFF

1) If the digit to be dropped is smaller than 5, then the preceeding digit is left unchanged

2) If the digit to be dropped is greater than 5, then the preceeding digit is increased by 1

3) If the digit to be dropped is 5 followed by non-zero digits, then the preceeding digit is increased by 1

4) If the digit to be dropped is 5, then the preceeding digit is increased by 1 if it is odd, and left unchanged if it is even

Arithmetic Operations with Significant Figures

1. In addition and subtraction, the final result should retain the same number of decimal places as that of the original number with minimum number of decimal places.
2. In multiplication and division, the final result should retain the same number of significant figures as that of the original number with minimum number of significant figures.

Accuracy and Precision

The accuracy of a measurement means how close the measured, value to the true value

Precision gives the resolution or the limit to which the quantity is measured. The smaller the least count, greater is the precision

Errors in a measurement

Error in a measurement is equal to the difference between the true value and the measured value of the quantity

$$\boxed{\text{Error} = \text{True value} - \text{Measured value}}$$

Let $a_1, a_2, a_3, ... a_n$ are 'n' measured values, then the accepted true value is their average value

$$r_{mean} = \frac{a_1 + a_2 + ...a_n}{n} = \frac{1}{n}\sum_{i=1}^{n} a_i$$

1) Absolute error (Δ)

The magnitude of the difference between the true value and the individual measured value is called absolute error.

$$\Delta a_1 = |a_{mean} - a_1|$$

$$\Delta a_2 = |a_{mean} - a_2|$$

$$\Delta a_n = |a_{mean} - a_n|$$

$\therefore$ Mean absolute error $\boxed{\Delta a_{mean} = \frac{1}{n}\sum_{i=1}^{n} \Delta a_i}$

i.e. the final result of measurement may be written as $\boxed{a = a_{mean} \pm \Delta a_{mean}}$

2. Relative error (δ) / Fractional error

It is the ratio of mean absolute error to the mean value of the quantity measured

$$\boxed{\delta_a = \frac{\Delta a_{mean}}{a_{mean}}}$$

3. Percentage error (%)

The relative error is expressed in percent is called percentage error

$$\boxed{\% \, a = \frac{\Delta a_{mean}}{a_{mean}} \times 100\%}$$

Propagation of Errors

1. Error in sum and difference of two quantities

Z = A + B OR Z = A – B

$$\boxed{\Delta Z = \Delta A + \Delta B}$$

The maximum error in the result is equal to the sum of the absolute errors in the individual quantities

2. Error in product or quotient of two quantities

Z = AB OR $Z = A/B$

$$\boxed{\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}}$$

$$\boxed{\Delta Z = Z\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)}$$

$$\boxed{\%Z = \%A + \%B}$$

The maximum fractional error in the result is equal to the sum of their individual fractional errors.

3. Error of a quantity raised to a power

$Z = A^m B^n$ OR $Z = \frac{A^m}{B^n}$

$$\boxed{\frac{\Delta Z}{Z} = \frac{m\Delta A}{A} + \frac{nB}{B}}$$

$$\boxed{\%Z = m\%A + n\%B}$$

NOTE

If a value alone is given (eg. ℓ = 7.6 cm) without specifying error then the least count of the measuring device gives the value its absolute error

If ℓ = 7.6 cm then $\Delta\ell = 0.1\,\text{cm}$

If M = 12.28 kg then $\Delta m = 0.01\,\text{kg}$

ADDITIONAL NOTE

Measuring Devices

Vernier callipers

Principle

It is a device invented by a french mathematician, Pierre Vernier and is called Vernier after his name. It consists of a main scale along which another scale slides, known as vernier scale. If n vernier scale divisions (V.S.D) coincide with (n – 1) main scale divisions (M.S.D.), then

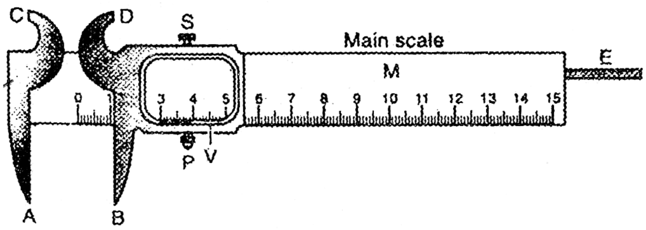
$n \text{ V.S.D} = (n-1)\text{M.S.D.}$ or $1 \text{ V.S.D} = \left(\frac{n-1}{n}\right)\text{M.S.D.}$

and 1 M.S.D. – 1 V.S.D = 1 M.S.D. – $\left(\frac{n-1}{n}\right)$ M.S.D

$$= \left(\frac{1}{n}\right)\text{M.S.D} = \frac{\text{smallest division of main scale}}{\text{No.of division on vernier scale}} = \frac{1}{10} = 0.1\,\text{mm}$$

LC = 1 M.S.D. – 1 V.S.D = 1 mm – 0.9 mm = 0.1 mm = 0.01 cm

The difference between the values of one main scale division and one vernier scale division is known as vernier constant (V.C.). This is the smallest distance that can be accurately measured with the vernier scale, also known as least count (L.C.) of the vernier scale.



Zero error

If the zero of the vernier scale does not coincide with the zero of main scale when the jaw B touches A and the straight edge of D touches the straight edge of C, then the instrument has an error called as zero error. It can be positive or negative depending upon whether the zero of vernier scale lies to the right or to the left of the zero of the main scale. Positive zero error is subtracted from the observed reading while negative zero error is added in observed reading. e.g. If zero error is +ve then to find zero error read the main scale reading (M.S.R). N on left zero of vernier scale (V.S) and also the vernier division x coinciding with any M.S. division, then

Zero error = x × L.C.

Total reading of Vernier callipers

If with the body between the jaws, the zero of vernier scale lies ahead of Nth division of main scale then main scale reading (M.S.R) = N.

If nth division of vernier scale coincides with any division of main scale, then vernier scale reading (V.S.R) = n × (L.C.).

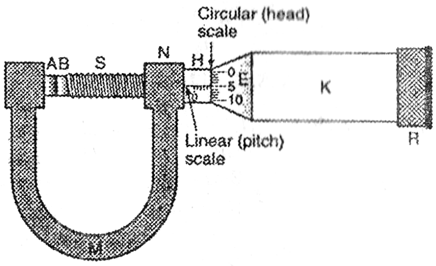
Total reading = M.S.R. + V.S.R = N+ (n × L. C)

Screw gauge

Principle

This instrument is based on the principle of micrometer screw. If an accurately cut single threaded screw is rotated in corresponding not having evenly spaced threads then in addition to the circular motion of the screw there is a linear motion of the screw head in the forward or backward direction along the axis of the screw. The linear distance moved by the screw, when it is given one complete rotation is equal to the distance between two consecutive threads, along the axis of the screw. This distance is called the pitch of the screw. A circular cap is fixed on one end of the screw and the circumference of the cap is normally divided into equal parts, typically 100 equal parts. If it is divided into 100 equal parts, then the screw moves forward or backward by 1/100 of the pitch for the rotation of circular scale by one circular scale division. It is the minimum distance which can be accurately measured and it is called as least count.

$$\therefore \text{ Least count} = \frac{\text{Pitch}}{\text{No.of division on the circular scale}} = \frac{1}{100} = 0.01\text{mm} = 0.001\text{cm}$$



Zero error and zero correction

Normally the zero on circular scale coincides with the zero of the pitch scale. Instruments possess zero error and zero correction, if the zero on circular scale does not coincide with the zero of the pitch scale. If the zero of the circular scale advances beyond the reference line, the zero error is negative and zero correction is positive. If it is left behind the reference line, the zero error is positive and zero correction is negative.

Total reading of screw gauge

If for a object placed between stud and screw, the edge of the cap (circular scale) lies ahead of Nth division of linear scale and nth division of circular scale lies over reference line, then linear scale reading (L.S.R.) = N and circular scale reading (C.S.R.) = n × (least count)

Total reading = L.S.R. + C.S.R.

<u>Length Measuring Instruments</u>

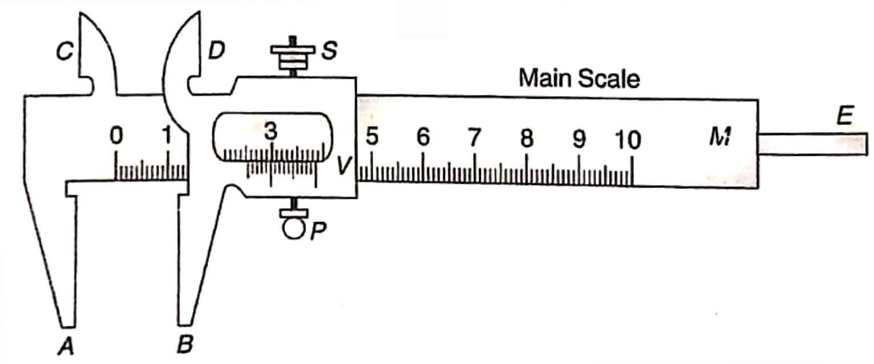
Length is an elementary physical quantity. The device generally used in everyday life for measurement of the length is a metre scale. It can be used for measurement of length with an accuracy of 1 mm. So, the least count of a metre scale is 1 mm. To measure length accurately upto $(1/10)^{th}$ of $\left(\frac{1}{100}\right)^{th}$ of a millimetre, the following instruments are used.

1) Vernier callipers 2) Micrometer 3) Screw gauge

1. **Vernier Callipers**

It has three parts.

i) Main scale: It consists of a steel metallic strip M, graduated in cm and mm at one edge. It carries two fixed jaws A and C as shown in figure.



ii) Vernier Scale: Vernier scale V slides on metallic strip M. It can be fixed in any position by screw S. The side of the vernier scale which slide over the mm sides has ten divisions over a length of 9 mm. B and D two movable jaws are fixed with it. When vernier scale is pushed towards A and C, then B touches A and B straight side of C will touch straight side of D. In this position, if the instrument is free from error, zeros of vernier scale will coincide with zeros of main scales. To measure the external diameter of an object it is held between the jaws A and B, while the straight edges of C and D are used for measuring the internal diameter of a hollow object.

iii) Metallic strip: There is a thin metallic strip E attached to the back side of M and connected with vernier scale. When jaws A and B touch each other, the edge of E touches the edge of M. When the jaws A and B are separated E moves outwards. This strip E is used for measuring the depth of a vessel.

Principle (Theory)

In the common form, the divisions on the vernier scale V are smaller in size than the smallest division on the main scale M, but in some special cases the size of the vernier division may be larger than the main scale division.

Let n vernier scale divisions (V.S.D.) coincide with (n - 1) main scale divisions (M.S.D.). Then,

$$n \text{ V.S.D.} = (n - 1) \text{ M.S.D.}$$

or
$$1 \text{ V.S.D.} = \left(\frac{n-1}{n}\right)\text{M.S.D.}$$

$$1\text{M.S.D.} - 1\text{V.S.D.} = 1\text{M.S.D.} - \left(\frac{n-1}{n}\right)\text{M.S.D.} = \frac{1}{n}\text{M.S.D.}$$

The difference between the values of one main scale division and one vernier scale division is known as Vernier constant (V.C.) or the Least count (L.C.). This is the smallest distance that can be accurately measured with the vernier scale. Thus,

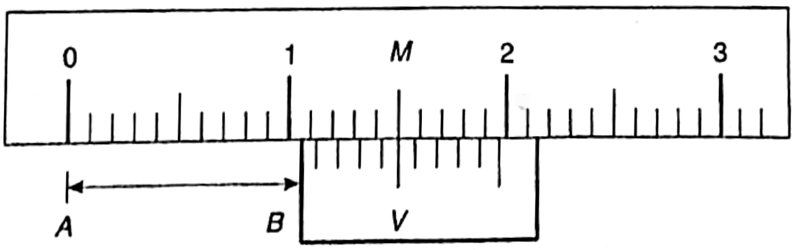
V.C. = L.C. = 1 M.S.D. - 1 V.S.D. $=\left(\frac{1}{n}\right)\text{M.S.D.}=\frac{\text{Smallest division on main scale}}{\text{Number of divisions on vernier scale}}$

In the ordinary vernier callipers one main scale division be 1 mm and 10

Reading a vernier Callipers

If we have to measure a length AB, the end A is coincided with the zero of main scale, suppose the end B lies between 1.0 cm and 1.1 cm on the main scale. Then,

1.0 cm < AB < 1.1 cm



Let 5th division of vernier scale coincides with 1.5 cm of main scale.

Then, $AB = 1.0 + 5 \times \text{V.C.} = (1.0 + 5 \times 0.01)\,\text{cm} = 1.05\,\text{cm}$

Thus, we can make the following formula, Total reading = N + n × V.C.

Here, N = main scale reading before on the left of the zero of the vernier scale.

n = Number of vernier division which just coincides with any of the main scale division.

Note: That the main scale reading with which the vernier scale division coincides has no connection with reading.

Zero error and zero correction

If the zero of the vernier scale does not coincide with the zero of main scale when jaw B touches A and the straight edge of D touches the straight edge of C, then the instrument has an error called zero error. Zero error is always algebraically subtracted from measured length.

Zero correction has a magnitude equal to zero error but its sign is opposite to that of the zero error. Zero correction is always algebraically added to measured length.

Zero error $\rightarrow$ algebraically subtracted

Zero correction $\rightarrow$ algebraically added

Positive and negative zero error

If zero of vernier scale lies to the right of the main scale the zero error is positive and if it lies to the left of the main scale the zero error is negative (when jaws A and B are in contact).

Positive zero error = (N + x × V.C.)

Here, N = main scale reading on the left of zero of vernier scale.

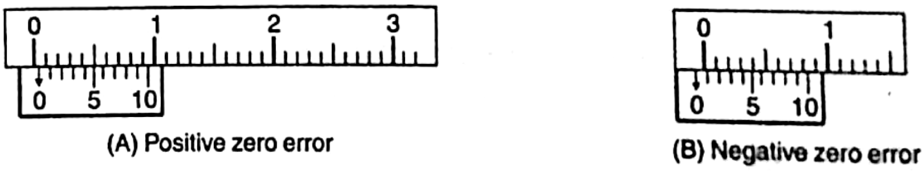
x = vernier scale division which coincides with any main scale division.

When the vernier zero lies before the main scale zero the error is said to be negative zero error. If 5^{th} vernier scale division coincides with the main scale division, then

Negative zero error = - [0.00 cm + 5 × V.C.]

= -[0.00 cm + 5 × 0.01 cm]

= -0.05 cm



Positive and negative zero error

Summary

1) V.C. $= \text{L.C.} = \dfrac{1\,\text{M.S.D.}}{n} = \dfrac{\text{Smallest division on main scale}}{\text{Number of divisions on vernier scale}}$

= 1 M.S.D. - 1 V.S.D.

2) In ordinary vernier callipers, 1 M.S.D. = 1 mm and n = 10

$\therefore \quad \text{V.C. or L.C.} = \dfrac{1}{10}\,\text{mm} = 0.01\,\text{cm}$

3) Total reading = (N + n × V.C)

4) Zero correction = -zero error

5) Zero error is algebraically subtracted while the zero correction is algebraically added.

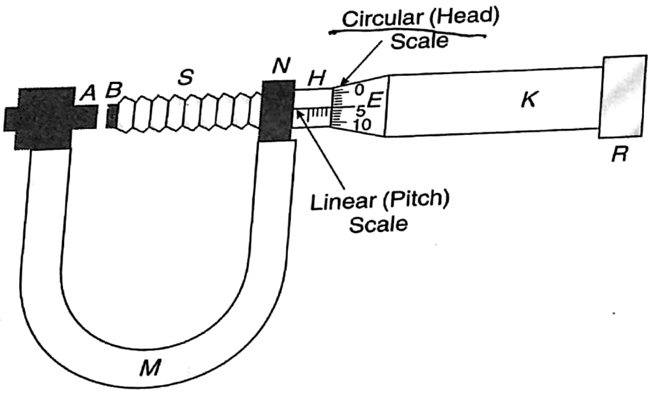
6) If zero of vernier scale lies to the right of zero of main scale the error is positive. The actual length in this case is less than observed length.

7) If zero of vernier scale lies to the left of zero of main scale the error in negative and the actual length is more than the observed length

8) Positive zero error = (N + x × V.C.)

Screw Gauge

Screw gauge works on the principle of micrometer screw. It consists of a U-shaped metal frame M. At one end of it is fixed a small metal piece A. It is called stud and it has a plane face. The other end N of M carries a cylindrical hub H. It is graduated in millimetres and half millimetre depending upon the pitch of the screw. This scale is called Linear scale or pitch scale.

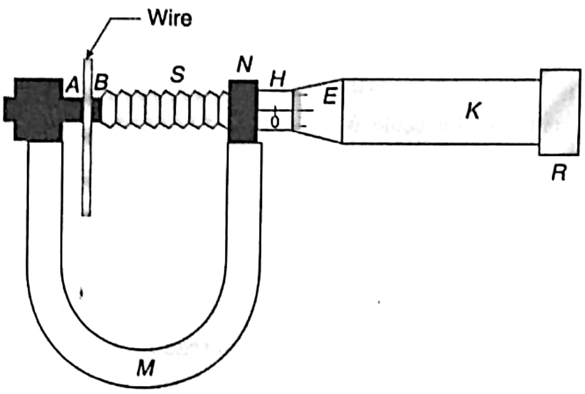


A nut is threaded through the hub and the frame N. Through the nut moves a screw S. The front face B of the screw, facing the plane face A is also plane. A hollow cylindrical cap K is capable of rotating over the hub when screw is rotated. As the cap is rotated the screw either moves in or out. The surface E of the cap K is divided into 50 or 100 equal parts. It is called the circular scale or head scale. In an accurately adjusted instrument when the faces A and B are just touching each other. Zero of circular scale should coincide with zero of linear scale.

To measure diameter of a given wire using a screw gauge

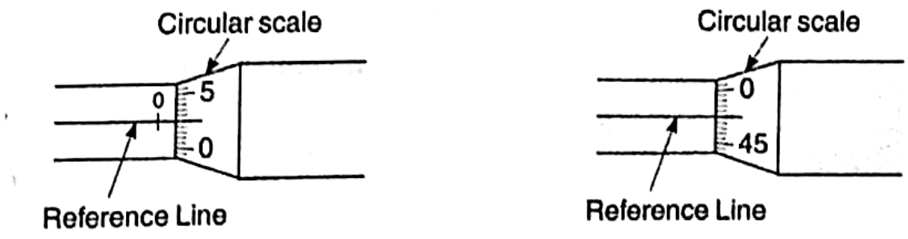
If with the wire between plane faces A and B, the edge of the cap lies ahead of N^{th} division of linear scale, and n^{th} division of circular scale lies over reference line.

Then, Total reading = N + n × L.C.



Zero error and zero correction

If zero mark of circular scale does not coincide with the zero of the pitch scale when the faces A and B are just touching each other, the instrument is said to possess zero error. If the zero of the circular scale advances beyond the reference line the zero error is negative and zero correction is positive. If it is left behind the reference line the zero is positive and zero correction is negative. For example if zero of circular scale advances beyond the reference line by 5 divisions, zero correction = + 5 × (L.C.) and if the zero of circular scale is left behind the reference line by 4 divisions, zero correction = -4 × (L.C.)



(A) Positive zero error (B) Negative zero error

Order of Magnitude

In scientific notation, a number can be expressed as N = a × 10^b

Where $1 \leq a < 10$

Order of magnitude is the power of 10 to which the number can be approximated.

i.e., if $a \leq 5$

order of magnitude is b

if a > 5

order of magnitude is b + 1

eg. order of magnitude of 54

54 = 5.4 × 10^1

a < 5

O.M. = b = 1

Order of magnitude of 9.1 × 10^{-31} kg

O.M. = -31 + 1 = -30