Applications Of Derivative

GEOMETRICAL SIGNIFICANCE OF DERIVATIVE

The derivative $\left[f'(x) \quad \text{or} \quad \frac{dy}{dx}\right]$ of the function y = f(x) at the point P(x, y) (when exists) is equal to the slope (or gradient) of the tangent line to the curve y = f(x) at P(x, y).
Slope of tangent to the curve y = f(x) at the point (x, y) is
$m = \tan\theta = \left[\frac{dy}{dx}\right]_{(x,\ y)}$

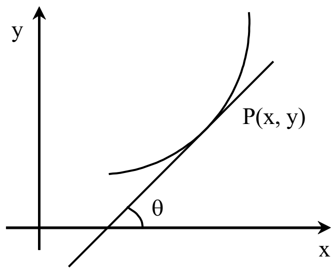


Fig. 1

Notes:

- If $\frac{dy}{dx}$ = 0 then the tangent to curve y = f(x) at the point (x, y) is parallel to the x-axis.
- If $\frac{dy}{dx} \to \infty$, or $\frac{dx}{dy}$ = 0, then the tangent to the curve y = f(x) at the point (x, y) is parallel to the y-axis.
- If $\frac{dy}{dx}$ = tanθ > 0, then the tangent to the curve y = f(x) at the point (x, y) makes an acute angle with positive x-axis and vice versa.

Tangent and Normal

1) Show that tangent at any point on $y = x^5 + x$ makes acuts angle with x-axis

2) Point on $3y - y^3 = 3x^2$ tangent is vertical

3) Equation of NRML to $y^2 = 8x$ at $(2,4)$

4. Show that tangent drawn at any point to $f(x) = x^7 + 3x^5 + 5x + 8$ make acute angle with x – axis.

5. The point on $y^3 + 3x^2 = 12y$ where tangent are vertical is $\left(\pm\frac{4}{\sqrt{3}}, 2\right)$.

$\Rightarrow m = \frac{6x}{12 - 3y^2} \Rightarrow y = \pm 2$.

6. Equation of Normal to $y = \left|x^2 - |x|\right|$ at $x = -2$ is $3y = x + 8$

7. If the line joining A(0,3) and B(5,-2) is a tangent to y (x+1)=c then c = 4
[⇒AB is x + y=3, eliminate y , quadratic in x.]

8. Equation of normal to $x^2 + 2xy - 3y^2 = 0$ at (1, 1) is $x + y = 2$

9. $f(x) = x^3 + x + 1, g(x)$ be its inverse then tgt to $g(x)$ at x = 3 is $x - 4y + 1 = 0$

Angle between Two Curves

The angle between two curves (or the angle of intersection of two curves) is defined as the angle between the two tangents at their point of intersection.
As the figure shows, ϕ, the angle between the two curves, is given by $\phi = \psi_1 - \psi_2$

$\Rightarrow \tan\phi = \tan(\psi_1 - \psi_2) = \frac{\tan\psi_1 - \tan\psi_2}{1 + \tan\psi_1 \tan\psi_2}$,

where $\tan\psi_1 = f'(x_1)$ and $\tan\psi_2 = g'(x_1)$.

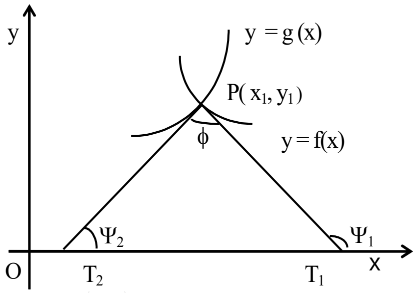


Fig. 3

1. Angle between $\frac{x^2}{4} + \frac{y^2}{1} = 1$ and $x^2 + y^2 = 2$ is θ then $\tan\theta = \frac{1}{\sqrt{2}}$

Point of intersection is $\left(\frac{2}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}\right)$ $m_1 = \frac{dy}{dx}, m_2 = \frac{dy}{dx}$ of both curves

2. Angle between $y^2 = 4x$ and $y = e^{-x/2}$ is $\frac{\pi}{2}$

3. Angle of intersection of $x^2 - y^2 = a^2$ and $x^2 + y^2 = \sqrt{2}\, a^2$ is $\theta = \frac{\pi}{4}$

[$\Rightarrow \tan\theta = \left|\frac{2xy}{y^2 - x^2}\right| = \frac{a^2}{\left|-a^2\right|}$.]

4. The positive value of k for which $ke^x - x = 0$ has only one roots is $\frac{1}{e}$

INCREASING /DECREASING.

MONOTONOCITY

Let y = f (x) be a given function with 'D' as it's domain. Let $D_1 \subseteq D$ then;

Increasing Function & Decreasing Function:

- f (x) is said to be increasing in D_1 if for every $x_1, x_2 \in D_1$, $x_1 > x_2 \Rightarrow f(x_1) > f(x_2)$. It means that the value of f (x) will keep on increasing with an increase in the value of x. Refer to fig.5

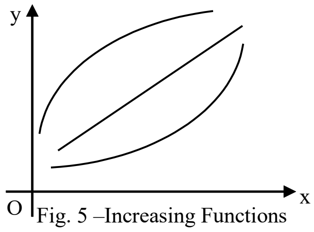


Fig. 5 –Increasing Functions

- f (x) is said to be decreasing in D_1 if for every $x_1, x_2 \in D_1$, $x_1 > x_2 \Rightarrow f(x_1) < f(x_2)$. It means that the value of f (x) would decrease with an increase in the value of x. Refer to fig.6.

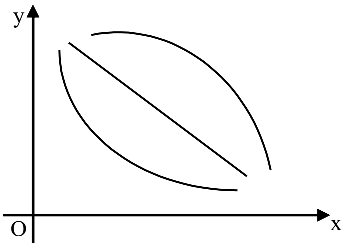


Fig. 6 –Decreasing Functions

Non-Decreasing Function & Non-Increasing Function:

- f (x) is said to be non-decreasing in D_1 if for every $x_1, x_2 \in D_1$, $x_1 > x_2 \Rightarrow f(x_1) \geq f(x_2)$. It means that the value of f (x) would never decrease with an increase in the value of x. Refer to fig.7.

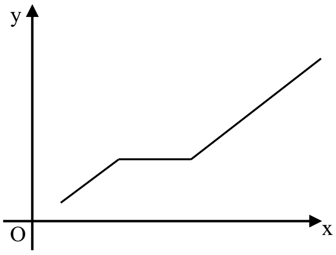


Fig. 7 –Non Decreasing Functions

- f (x) is said to be non-increasing In D_1 if for every $x_1, x_2 \in D_1$, $x_1 > x_2 \Rightarrow f(x_1) \leq f(x_2)$. It means that the value of f (x) would never increase with an increase in the value of x. Refer to fig. 8.

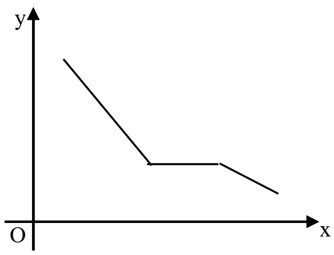


Fig. 8 –Non Increasing Functions

N : (1) Show that function f(x) = $4x^3-18x^2+27x-4$ is always increasing R

1) $f^1(x) = 12x^2 - 36x + 27$

$= 3(4x^2 - 12x + 9)$

$= 3(2x-3)^2$

≥ 0

$\Rightarrow f(x)$ is always $\uparrow$ in R

N (2) Find the interval in which $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$ is

Strictly increasing or strictly decreasing

$(1,2) \cup (3,\infty)$ or $(-\infty,1) \cup (2)$

Strictly increasing of $f(x) > 0$

$f^1(x) = 4x^3 - 8.3x^2 + 22.2x - 24$

$= 4(x^3 - 6x^2 + 11x - 6)$

$= 4(x-1)(x-2)(x-3) \qquad 1-6+11-6$

IIT. $f(x) = xe^{x(1-x)}$ is increasing in $\left[\frac{-1}{2}, 1\right]$

2. The interval in which f(x) is increasing if $f'(x) = (x-1)(x-2)^2(x-3)^3(x-4)^3(x-5)^5$

3. If $g(x) = 2f\left(\frac{x}{2}\right) + f(2-x)$ and $f''(x) < 0$ for all $x \in (0, 2)$. Then interval in which $g(x)$ increases in $\left(0, \frac{4}{3}\right)$.

4. Find the range of x for which f(x) is increasing.
(i) $(x+1)e^x$
(ii) $x^3 - 15x^2 + 27x + 1$
(iii) $\sin^4 x + \cos^4 x$ in $[0,\pi]$

$$(2)\ f^1(x) = 4\sin^3 x \cos x - 4\cos^3 x \sin x$$

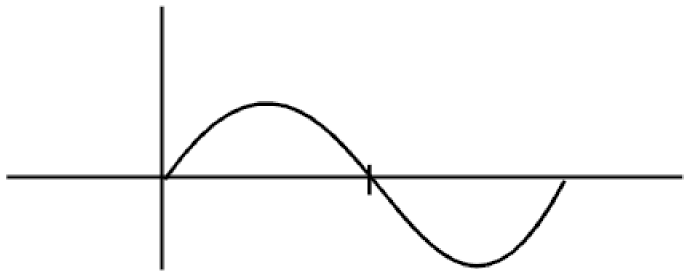
$$= 4\sin x \cos x\left(\sin^2 x - \cos^2 x\right)$$

$$= 2\sin 2x(-\cos 2x)$$

$$= -\sin 4x$$

$$f^1(x > 0) \quad \sin 4x < 0$$

$$0 < x < a/2 \quad \Rightarrow 0 < 4x < 2;\ 4x = \theta; y = \sin\theta$$



$$\pi < \theta < \quad 2\pi$$

$$\pi < 4x < \ 2\pi$$

$$\Rightarrow \left(\frac{\pi}{4} \quad \frac{\pi}{2}\right)$$

5. Find the range of a for which f(x) is increasing , f(x) = $x^3 + (a + 2)x^2 + 3ax + 5$

Inequality

1. Prove the following
   (i) $\sin x < x \ \ \forall\ x > 0$
   (ii) $\tan x > x \ \ \forall\ x > 0$
   (iii) $\ln(1 + x) < x ,\ \forall\ x > 0$
   (v) $\sin x/x$ is increasing
   (vi) $0 < x < \pi/2$, $\sin^2 x < x\sin(\sin x)$

Find the critical points of the following

1. $(x-2)^{2/3}(2x+1)$

2. max{sinx,cosx} in $(-2\pi, 2\pi)$.

Global maximum and Global minimum

Concept of Global Maximum / Minimum:

Let y = f (x) be a given function with domain D. Let [a, b] $\subseteq$ D. Global maximum/minimum of f (x) in [a, b] is basically the greatest/least value of f (x) in [a, b].
Global maximum and minimum in [a, b] would always occur at critical points of f(x) within [a, b] or at the end points of the interval.

Global Maximum / Minimum in [a, b]:

In order to find the global maximum and minimum of f(x) in [a, b], find out all the critical points of f(x) in (a, b). Let $c_1, c_2, \ldots, c_n$ be the different critical points. Find the value of the function at these critical points. Let $f(c_1), f(c_2), \ldots, f(c_n)$ be the values of the function at critical points.
Say, $M_1 = \max\{f(a), f(c_1), f(c_2), \ldots, f(c_n), f(b)\}$
and $M_2 = \min\{f(a), f(c_1), f(c_2), \ldots, f(c_n), f(b)\}$
Then M_1 is the greatest value of f (x) in [a, b] and M_2 is the least value of f (x) in [a, b].

f(x) be defined on [a , b] and [c , d] is the range then global max = d and global min = c .

P1 Minimum value of $ax + by$, where $xy = c^2$ is $2\sqrt{ab}\ c$

1. If xy = 10. Find the minimum value of $12x^2 + 13y^2$

2. let f(x) = $\frac{(x+1)(x+2)}{x}$ $\forall$ x > 0. Find the minimum value of f(x)

3. let f(x) = $\frac{(x-1)(x-2)}{(x-3)}$ $\forall$ x > 3. Find the minimum value of f(x)

4. If $4x^2 + y^2 = 1$ maximum value of $12x^2 - 3y^2 + 16xy$ is 5.

5. Minimum value of $(x_2 - x_1)^2 + \left(\sqrt{1-x_1^2} - \sqrt{4-x_2^2}\right)^2$ is 1

6. Maximum value of $\left(\sqrt{-3+4x-x^2}+4\right)^2 + (x-5)^2, 1 \le x \le 3$ is 36

7. Min value of 3x + 4y , x > 0 , x> 0 y > 0 and $x^2y^3 = 6$ is 10
$\Rightarrow \frac{3x}{2} + \frac{3x}{2} + \frac{4y}{3} + \frac{4y}{4} + \frac{4y}{4} \ge 5(\text{product})^{\frac{1}{5}}$

8. Max. value of f(x) = $x^3(2-x)^4 \Rightarrow \frac{x}{3} + \frac{y}{3} + \frac{y}{3} + \frac{2-x}{4} + \frac{2-x}{4} + \frac{2-x}{4} \ge 7(\text{product})^{\frac{1}{7}}$.

9. The maximum value of $\sin x_1 \cos x_2 + \sin x_2 \cos x_3 + \ldots\ldots\ldots\ldots + \sin x_{n-1} \cos x_n + \sin x_n \cos x_1$ is $\frac{n}{2}$

$\Rightarrow \sin x_1 \cos x_2 \le \frac{\sin^2 x_1 + \cos^2 x_2}{2}$.

Length of Tangent, Normal, Sub-Tangent and Sub-Normal:

Since PT = PP_1 cosec $\theta \Rightarrow$ |PT| = |y| $\sqrt{1+\cot^2\theta}$

$\Rightarrow$ length of the tangent PT = $|y|\sqrt{1+\left(\frac{dx}{dy}\right)^2}$

Similarly, Length of the normal PN = $|y|\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

Sub-tangent, $TP_1 = \left|y.\frac{dx}{dy}\right|$. Sub-normal, $NP_1 = \left|y\frac{dy}{dx}\right|$

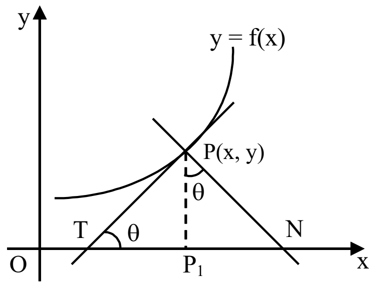


Fig. 4

1) Length of sub tangent to $y = e^{x/3}$

2) Length of the light to $y^2 = 8x$ at $(2,4)$

3) Length of sub normal to $x = t^2 + 3t - 8$ and $y = 2t^2 - 2t - 5$ at $(2,-1)$ is $\frac{6}{7}$

Rate of Change

limit of ratio of change in y write x

$$= \frac{dy}{dx}$$

Rate of change of y wrt to time t = $\frac{dy}{dt}$

Rate of change of displacement $= \frac{ds}{dt}$

1. If radius of a circle increasing at the rate of 2cm/s. If radius =20 cm then ratio of change of area of $80\pi cm^2/s$

1) $A = \frac{\sqrt{3}}{4} a^2$

$$\frac{dA}{dt} = \frac{\sqrt{3}}{4}.\ 2a = \frac{da}{dt} = \frac{\sqrt{3}}{4}\ 2 \times 10 \times 2 = 10\sqrt{3} cm^2 / s$$

2. Find the curve y = 5x -2x^3 . If x increases at the rate of 2 unit/s find the rate of change of slope of the curve when x = 3 An : 72 unit/s

3. Use differentiation approximate value of

$\sqrt{101}$ $\quad$ Ans : 10.05

4. Approximate value of $(25)^{1/3}$ is $3 - \frac{2}{7}$

1. The side of an equilateral triangle are increasing at the rate of 2cm/s Find a rate at which its area increases when side 10 cm log.

2) If X changes from 4 to 4.01 find the appoximate change in In x

$$y = \text{In X} ; \frac{dy}{dx} = \frac{1}{X} \quad dy = \frac{1}{x} dx ; x = 4 \quad \text{x changes t red } x = 4 + 0.01$$

$$dy = \frac{1}{x}.dx = \frac{1}{4}.0.01 = 0.25 \times 0.01 = 0.025$$

$$\text{change in y ; y dey} = \text{In } 4 + 0.0025$$

Local maximum and local minimum

Concept of Local Maximum and Local Minimum

Let y = f (x) be a function defined at x = a and also in the vicinity of the point x = a. Then, f (x) is said to have a local maximum at x = a, if the value of the function at x = a is greater than the value of the function at the neighbouring points of x = a. Mathematically, f (a) > f (a – h) and f (a) > f (a + h) where h > 0.

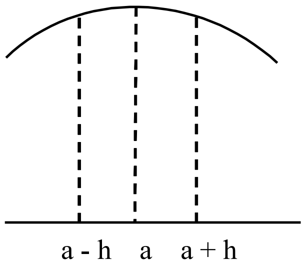


Fig. 9- Local Maxima

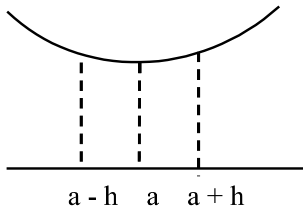


Fig. 10- Local Minima

Similarly, f (x) is said to have a local minimum at x = a, if the value of the function at x = a is less than the value of the function at the neighbouring points of x = a. Mathematically, f (a) < f (a – h) and f (a) < f (a + h) where h > 0.
A local maximum or a local minimum is also called a local extremum.

Test for Local Maximum / Minimum:

If f (x) is differentiable at x = a and if it is a critical point of the function (i.e. $f'(a)$ = 0) then we have the following three tests to decide whether f(x) has a local maximum or local minimum or neither at x = a,

1. First Derivative Test

If $f'(a)$ = 0 and $f'(x)$ changes it's sign while passing through the point x = a , then

(i) f(x) would have a local maximum at x = a if $f'(a-0) > 0$ and $f'(a+0) < 0$. It means that $f'(x)$ should change it's sign from positive to negative
e.g. f (x) = $-x^2$ has local maxima at x = 0.

(ii) f(x) would have local minimum at x = a if $f'(a-0) < 0$ and $f'(a+0) > 0$. It means that $f'(x)$ should change it's sign from negative to positive.
e.g. f (x) = x^2 has local minima at x = 0.

(iii) If f(x) doesn't change it's sign while passing through x = a, then f (x) would have neither a maximum nor minimum at x = a.
e.g. f (x) = x^3 doesn't have any local maxima or minima at x = 0.

1. Find the extreme value of
(i) $1 - x^6$ (ii) $-|x|$

1. If $f'(x) = \pi_{n=1}^{10} (x-n)^n$ then the points at which f(x) having maximum is $1,5,9 \Rightarrow$ use wavy course

6. Find the point of max.min of $\int_0^{x^2} \frac{t^2 - 5t + 4}{2 + e^t}$

7. The roots of $2x^3 - 3x^2 - 12x + a = 0$ are real and distinct then the range of 'a' is $(-7, 20)$

1. f(x) = $2x^3 - 9ax^2 + 12a^2x + 1$ is local max at x_1 and local min at x_2 and $x_2 = x_1^2$ then a = 2 .

2. F(X) = $\begin{cases} 4x - x^3 + \ln(b^2 - 3b + 3), 2 \le x < 3 \\ x - 18 \qquad\qquad\qquad\qquad x \ge 3 \end{cases}$ has Local min at x = 3 then range of $b \in (-\infty, 1] \cup [2, \infty)$.

Applications of Derivatives in polynomials

Basic : Quadratic $ax^2 + bx + c > 0 \quad D < 0$

ODD degree Equation have at least one

$\Rightarrow$ $f(x)$ be polynomial with L max at $x = x_1$ and L.min at $x = x_2$

Then $f'(x) = k(x - x_1)(x - x_2)$

Q.1) $f(x)$ be a polynomial with least degree has L.max at x=1 and L.Min at $x = 3$. If f(1) =6, f(3)=2 then $f^1(o) = 9$

Q.2) Number of Distinct roots of

$x^4 - 4x^3 + 12x^2 + x - 1 = 0$ is 2

$\Rightarrow$ α be Repeated root of n times

Then $f(\alpha) = f^1(\alpha) = f''(\alpha) = \ldots\ldots f^{(n-1)}(\alpha) = 0$

3) If 1 is Double repeated root of

$f(x) = px^3 + q(x^2 + x) + r = 0$; then $\frac{p}{q} = 1$

1. If $x^5 - 10a^3x^2 + b^4x + c^5 = 0$

has three equal roots then $\frac{b^4}{a^4} = 15$

2. Number of roots of

$x^2 - x\sin x - \cos x = 0$ is 2

3. $f(x) = x^3 - 3ax^2 + 3(a^2 - 1)x + 1$ has L max point greater than > -2 and L minimum point < 4 then range of $a \in (-1, 3)$

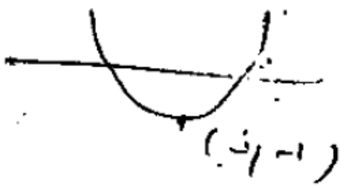
3) $p + 2q + r = 0;\ 3px^2 + q(2x + 1) = 0$

$$3p + 3q = 0 = \frac{p}{q} = 1;\ p - 2p + r \Rightarrow \frac{p}{r} = 1$$

1) $f''(x) = 0 \Rightarrow x = a;\ f'(a) = 0 \Rightarrow 5a^4 - 20a^4 + b^4 = 0$

2) $f'(x) = 2x - x\cos x; = x(2 - \cos x)$

$f^1(x) = 0$ only once $f(0) = -1$



3) $f'(x) = 3(x^2 - 2ax + a^2 - 1); = 3(x - (a+1))(x - (a-1))$

L More at $a - 1 > -2$

and L min at a+1 <4

$\Rightarrow 3$

$$f(a) \text{ is max} \Rightarrow f'(a) = 0 \text{ and } f''(a) < 0$$

1) Two positive numbers whose sum is 15 and sum of squares is minimum find the numbers $\left(\frac{15}{2},\frac{15}{2}\right)$

1) $x+y=15$

$$f(x)=x^2+y^2=x^2+(15-x)^2$$

$$f'(x)=2x+2(15-x)(-1)$$

$$f'(x)=0 \Rightarrow \quad x=15-x \Rightarrow x=\frac{15}{2}$$

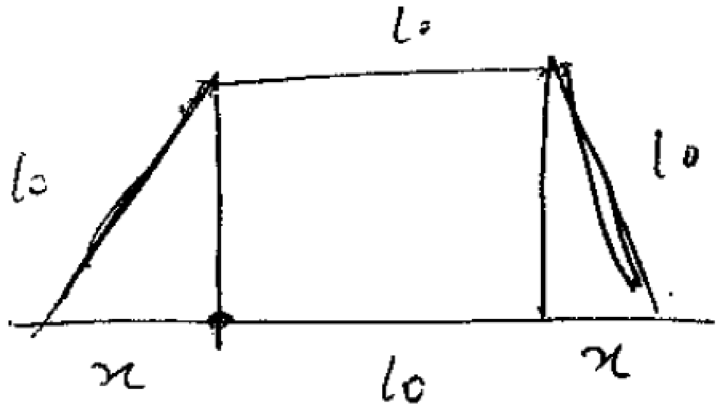
$$f''(x)=0 \quad \Rightarrow x=15-x \Rightarrow x=\frac{15}{2}$$

$$f''(x)=2+2=4>0$$

$$\text{At } \ x=\frac{15}{2} \quad f''(x)>0$$

$f\left(\frac{15}{2}\right)$ is minimum

2) If length of sides of a trapezium other than base is 10cm each. Find the area of trapezium when it is maximum $75\sqrt{3}$

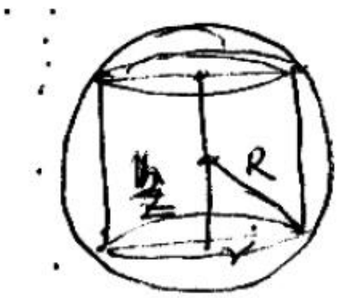


$$A = \frac{1}{2}(10+10+2x)\sqrt{100-x^2}$$

$$= (10+x)\sqrt{100-x^2}$$

A is max then A^2 is also max

4) Show that Height of a cylinder of max. volume inscribed in a sphere of rad = R is $\frac{2R}{\sqrt{3}}$



1. Show that all rectangles of gives area, square has smallest perimeter

1) $xy = k, \text{given}$

$$f(x) = \text{perimeter}$$

$$= 2(x+y); 2\left(x+\frac{k}{x}\right)$$

$$f'(x) = 0$$

$$2\left(1-\frac{k}{x^2}\right) = 0 \quad \Bigg| \quad x^2 = k; \ f''(x) = 2\left(\frac{2k}{x^3}\right) = \frac{4}{\sqrt{K}} = +ve$$

$$x = \sqrt{k} \ \Rightarrow y = \sqrt{k} \quad \text{rectangle square}$$

2. Sum of perimeter of a circle and a square is k, constant. Prove that area is Least when side is double the radius

2) $2\pi r + 4a = k$

$$A = \pi^2 + \left(\frac{k-2\pi}{4}\right)^2$$

$$f(r) = \pi r^2 + \frac{1}{16}(k-2\pi r)^2$$

$$f^1(r) = 2\pi r + \frac{1}{16}(2)\ (k-2\pi r)(-2\pi)$$

$$= 2\pi r - \frac{\pi}{4}(k-2\pi r)$$

$$f''(r) = 2\pi + \frac{\pi}{4}\ 2\pi\ > 0$$

$$f'(r) = 0 \quad 2\pi r = \frac{\pi}{4}(k-2\pi r);$$

$$8r = k - 2\pi r; 8r = 4a; a = 2r$$

3. A window is in the form a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10m. Find the dimensions of window of it admits

Maximum light

Large of rectangle = $\frac{20}{\pi+4}$

Breadth of rectangle = $\frac{10}{\pi+4}$

radius of the semi circle = $\frac{10}{\pi+4}$

Rolle's Theorem:

It is one of the most fundamental theorem of Differential calculus and has far reaching consequences. It states that if y = f (x) be a given function and satisfies,

- f (x) is continuous in [a , b]
- f (x) is differentiable in (a , b)
- f (a) = f (b). Then $f'(x) = 0$ at least once for some $x \in (a, b)$

Lagrange's Mean Value Theorem:

This theorem is in fact the general version of Rolle's theorem. It says that if y = f(x) be a given function which is;

- Continuous in [a , b]
- Differentiable in (a , b)

Then $f'(x) = \frac{f(b)-f(a)}{b-a}$, at least once for some $x \in (a, b)$. Let A $\equiv$ (a , f (a)) and B $\equiv$ (b , f (b))

Slope of Chord $AB = \frac{f(b)-f(a)}{b-a}$

1. Verify LMVt on $f(x) = |x|$ in $[-1, 1^{-}$

2. Prove that $\frac{b-a}{b} < \ln\frac{b}{a} < \frac{b-a}{a}$, where 0 < a < b.

3. If $a_0x^n + a_1x^{n-1} + \ldots\ldots\ldots\ldots + a_{n-1}x = 0$ has a real positive real root α , then prove that $na_0x^{n-1} + (n-1)a_1x^{n-2} + \ldots\ldots\ldots\ldots + a_{n-1} = 0$ has a positive real root less than α.

3. Prove that $\dfrac{\beta-\alpha}{1+\beta^2} < \tan^{-1}(B)-\tan^{-1}(\alpha)$

$< \dfrac{\beta-\alpha}{1+\alpha^2},\quad 0<\alpha<\beta\,;$

4. $f(x), g(x)$ are differentiable on [0,1] with

$f(0)=g(0)=0, f(1)=6\ \ g(1)=2\,,\ \dfrac{f'(c)}{g'(c)}=3$

5. The point on $y=\sqrt{x}$ tangent at which parallel to line joining (0,0) ad $(4,2)$ is $(1,1)$

6. Use LMVT, f(x) is differentiable on [2,5] with $f(2)=\frac{1}{5}, f(5)=\frac{1}{2}$

then $2<C<5$ then $f'(c)=\frac{1}{10}$

7. In $[1,2]$ tangent to $y=x^3-6ax^2+5x$ at $x=\frac{7}{4}$ is parallel to chord joining the curve at

$x=1, x=2$ then $a=\frac{35}{48}$

8. Verify LMVT for $f(x)=x^2+2x+3$ in $[4,6]$ $(C=5)$

6. $f'(c)=\dfrac{2}{4}\Rightarrow\dfrac{1}{2\sqrt{c}}\ \ \dfrac{1}{2}\Rightarrow c=1$

7. $f'(c)=\dfrac{f(5)-f(2)}{5-2}=10$

8. $f(1)=6-6a\quad f(2)=18-24a$

$f'(7/4)=\dfrac{227}{6}-21a\,;\Rightarrow a=\dfrac{35}{48}$

9. $f(x) = f(x)^3$

$$F^1(x) = 3\left(f(x)\right)^2 f'(x)$$

10. $f(x) = x^2 + 2x + 3$

f(x) is polynomial 1 degree 2

$\therefore$ f(x) is constant

$$f^1(x) = 2x + 2 \Rightarrow f(x) \text{ is differentiableis } (4,6)$$

$$f'(c) = \frac{f(6) - f(4)}{6-4} \Rightarrow c - 5 \leftarrow (4,6)$$

1) For local max /local minimum

$$f'(x) = 0$$

(i) $f''(x) > 0 \Rightarrow$ Local minimum

(ii) $f''(x) < 0 \Rightarrow$ Local minimum

$$f'(x) = 0 \Rightarrow \tan x = 7$$

$$\Rightarrow x = \pi - \frac{\pi}{4},\ 2\pi - \frac{\pi}{4}$$

$$f'(x) = \sin x + \cos x$$

$$f''(x) = \cos x - \sin x$$

$$\text{at } x = \frac{3\pi}{4} \quad f''(x) \quad \cos(\pi - \pi/4) - \sin(\pi - 12) = -\text{ve}$$

2) $f(x) = x^2 - x$

$f(-1) = 2$

$f(0) = 0, \quad f(1) = 0, f(2) = 2$

$g(x) = 2\left|x - \frac{1}{2}\right| - 1$

$g(-1) = 2, g(0) = 2, g(1) = 0$

$g(2) = 2$

At $x = -1, 0, 1, 2 \quad f(x) = g(x)$

f and q are equal

Rolle's Theorem

1. Find value of C in Rolle's theorem for $f(x) = x^3 - 3x \quad \text{in} \quad \left[-\sqrt{3}, 0\right]$

α is a root repeated n times then $f(\alpha) = f'(\alpha) = f''(\alpha) = \quad f^{n-1}(\alpha) = c$

1. If 1 is double repeated Root of

$f(x) = p + 3 + q\left(x^2 + x\right) + r = 0 \;\; \text{then} \frac{p}{r} = 1$

If $x^5 - 10a^3x^2 + b^4 + c^5 = 0$ has there equal roots $\frac{b^4}{a^4} = 15$