CHAPTER - 5

PERMUTATION, COMBINATION & BINOMIAL THEOREM

FACTORIAL NOTATION

The notation n! represents the product of first n natural numbers, i.e., the product $1\times 2\times 3\times .....\times (n-1)\times n$ is denoted as n!. We read this symbol as 'n' factorial'. Thus, $1\times 2\times 3\times 4....\times (n-1)\times n = n!$

1 = 1!

$1\times 2 = 2!$

$1\times 2\times 3 = 3!$

$1\times 2\times 3\times 4 = 4!$ and so on. We define 0! = 1

MULTIPLICATION PRINCIPLE

If an operation can be performed in 'm' different ways and then a second operation can be performed in 'n' different ways, then the two operations taken together can be performed in m x n ways.

This can be extended to any finite number of operations.

Example 1: A hall has 6 gates. In how many ways can a man enter the hall through one gate and come out through a different gate?

Sol: Since there are 6 ways entering into the hall. Therefore, for coming out the hall through a different gate, number of ways = 5

Hence by the fundamental principle of multiplication, the total number of ways = 6 x 5 = 30 ways

Example: 35 buses are running between Kottayam and Ernakulam. In how many ways can a person go from Kottayam to Ernakulam and return by a different bus:

Ans: Required number of ways = 1190

ADDITION PRINCIPLE

If an operation can be performed in 'm' different ways and another operation, which is independent of the first operation, can be performed in n different ways. Then, either of the two operations can be performed in (m+n) ways.

This can be extended to any finite number of mutually exclusive events.

Ex: There are 30 students in a class in which there are 20 boys and 10 girls. The class teacher selects either a boy or a girl for monitor of the class. In how many ways the class teacher can make this selection

Sol: Clearly, there are 20 ways to select a boy and 10 ways to select a girl.

$\therefore$ by the fundamental principle of addition, reqd. number of ways = 20 + 10 = 30 ways

PERMUTATION

Each of the different arrangements which can be made by taking some or all of a number of things is called a Permutation

Eg : Arrangements of objects taking 2 at a time from given 3 objects (a, b, c) are ab, bc, ca , cb, ac , ba then total number of arrangements is 6 each of which is known as permutation

CIRCULATION PERMUTATIONS

(a) Number of circular permutations of n different things taken all at a time is $(n-1)!$ if clockwise and anticlockwise orders are taken as different

Example 1: 15 persons were invited to a party. In how many ways can they and the host be seated at a circular table?

Sol. Total number of persons on the circular table = 15 guests + 1 host = 16

They can be seated in $(16-1)! = (15)!$ ways

Eg 2 : Seven women and seven men are to sit around a circular table such that there is a man on either side of every woman. Find the total number of seating arrangements

Required number of ways = $6! \times 7!$

(b) Arrangement of beads or flowers (all different) around a circular necklace or garland.

In this case clockwise and anticlockwise order is not different

$\therefore$ reqd. number of ways = $\frac{1}{2}(n-1)!$

Example 2 . Find the number of ways in which 10 different beads can be arranged to form a necklace

Reqd. number of ways $= \frac{1}{2}(10-1)! = \frac{1}{2}(9!)$

COMBINATION

Each of the different groups or selections which can be made by some or all of a number of given things without reference to the order of the things in each group, is called a **combination.**

Eg: The groups made by taking 2 objects at a time from three objects (a, b, c) are ab, bc, ca. Then the number of group is 3 each of which is known as combination.

IMPORTANT RESULTS

1. No. of permutations of n things all different, taking r at a time, in a straight line is denoted by

$P(n, r)$ or $nP_r = n(n-1)(n-2) \text{--------} (n-r+1) = \dfrac{n!}{(n-r)!}$

Eg: The number of 4 letter words, with or without meaning, which can be formed out of the letters of the word ROSE, where the repetition of the letters is not allowed

Ans: Required number of words = ${}^4P_4 = 4! = 24$.

2. $P(n, n) = P(n, n-1) = n!$

3. In a group of n things, p are exactly alike, q are exactly alike of a second type, r exactly alike of a third type and so on, the number of permutations of n things taken all at a time is $\dfrac{n!}{p!\,q!\,r!----}$

   Eg: How many words can be formed from the letters of the word 'COMMITTEE'

   Ans: Required number of words = $\dfrac{9!}{(2!)^3}$

4. $\dfrac{P(n, r)}{P(n, r-1)} = n - r + 1$

5. Number of permutations of n things all different, taking r at a time, if repetition allowed is n^r.

   Eg: The number of 4 letter words, with or without meaning, which can be formed out of the letters of the word ROSE, where the repetition of the letters is allowed

   Ans: Required number of words = $4^4 = 256$

6. No. of combinations of n things all different taking 'r' at a time, $nC_r = C_{(n,r)} = \dfrac{P_{(n,r)}}{r!}$

   or $C_{(n,r)} = \dfrac{n(n-1)(n-2)----(n-r+1)}{r!} = \dfrac{n!}{(n-r)!\,r!}$

7. $C_{(n, r)} = C_{(n, n-r)}$

8. $C_{(n, 0)} = C_{(n, n)} = 1$

9. $C_{(n, r)} + C_{(n, r-1)} = C_{(n+1, r)}$, $C_{(n, r)} + C_{(n, r+1)} = C_{(n+1, r+1)}$

10. $\dfrac{C_{(n,r)}}{C_{(n,r-1)}} = \dfrac{n-r+1}{r}$

11. If $C_{(n, r)} = C_{(n, s)}$, $\rightarrow r = s$ or $r + s = n$

12. If there are n points in a plane, and no three of them are collinear,

    (i) No. of straight lines that can be drawn by joining these points is nC_2

    (ii) No. of triangles, that can be drawn with these points as vertices is nC_3

13. Out of n points in a plane, if no 3 are collinear except m of them which are collinear

    (i) No. of lines that can be drawn is $nC_2 - mC_2 + 1$

    (ii) No. of triangles with these points as vertices is $nC_3 - mC_3$

Eg: The number of straight lines that can be formed by joining 12 points of which 4 are collinear is

Ans: $12C_2 - 4C_2 + 1 = 61$

14. Number of diagonals of a polygon of n sides is $C_{(n, 2)} - n = \frac{n(n-3)}{2}$

Eg: A polygon has 170 diagonals. How many sides will it have

$nC_2 - n = 170$; $\frac{n(n-1)}{2} - n = 170 \Rightarrow n = 20$

15. $nCo^2 + nC_1^2 + nC_2^2 + \ldots\ldots\ldots + nC_n^2 = 2nC_n$

SOME USEFUL RESULTS

1. Sum of the numbers formed by taking all the given n digits is

= (sum of all the n digits) $\times$ (n – 1) ! $\times$ (1 1 1.....1)n times

eg. Sum of the numbers formed by taking all the given digits 2, 3, 4, 5

= (2 + 3 + 4 + 5) $\times$ 3 ! $\times$ (1111) = 84 $\times$ 1111 = 93324.

2. If n items are arranged in row, then the number of ways in which they can be rearrnged so that no one of them occupies the place assigned to it is

$$n!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\ldots(-1)^n\frac{1}{n!}\right]$$

e.g, There are 5 lettres and 5 directed envelops. Find the number of ways in which all letters are put in the wrong envelopes.

Sol. The reqd. no. of ways

$$=5!\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right] = 60 - 20 + 5 - 1 = 44$$

3. The number of ways of selecting one or more items from a group of n distinct items is $2^n - 1$\

Eg. The number of different ways in which a man can invite one or more of his 6 friends to dinner is

Reqs no. of ways = $2^6 - 1 = 63$

4. The number of ways of selecting r items out of n identical items is 1

5. The total number of selections of some or all out of p + q + r items where p are alike of one kind, q are alike of second kind and rest are alike of third kind is [(p + 1) (q + 1) (r + 1)] – 1

Eg.The number of ways in which one one or more balls can be selected out of 10 white, 9 green and 7 blue balls is

Required number of ways = (10 + 1) (9 + 1) (7 + 1) – 1 = 879

6. The number of ways in which n identical things can be distributed into r different groups is

${}^{n+r-1}C_{r-1}$ or ${}^{n-1}C_{r-1}$ according as blanks groups are or are not adimissible.

Example : The number of ways in which 5 identical balls can be distributed into 3 different boxes so that no box remains empty

$= {}^{5-1}C_{3-1} = {}^{4}C_2 = 6$

Example : Four boys picked up 30 mangoes. In how many ways can they divide them if all mangoes are identical ?

Reqd . no. of ways = ${}^{30+4-1}C_{4-1}$

$$= {}^{33}C_3 = \frac{33.32.31}{1.2.3} = 11 \times 16 \times 31 = 5456$$

BINOMIAL THEOREM

1. If n is a positive integer, for all values of a & b

$$(a+b)^n = nC_0\, a^n + nC_1\, a^{n-1}\, b + nC_2\, a^{n-2}\, b^2 + \text{-------}\ n\,C_r a^{n-r}\, b^r + \text{-----} + nC_n b^n = \sum_{r=0}^{n} nC_r\, a^{n-r}\, b^r$$

Properties of the expansion $(a+b)^n$

1. The expansion has (n + 1) terms

2. The first term is a^n and the last is b^n

3. The exponent of a decreases by one and that of b increases by one as we proceed from term to term, so that the sum of the exponents of a and b in any term is n.

4. Since C (n,r) = C (n,n – r), the coefficients of terms equidistant from either end of the expansion are equal

5. The (r +1) th term of expansion is $C(n,r)a^{n-r}b^r$. This term is generally referred to as the "general term".

2. $nC_0, nC_1, nC_2,$ -------- nC_n are the binomial coefficients

(i) $nC_{n/2}$ is the greatest if n is even

(ii) $nC_{\frac{n-1}{2}} = nC_{\frac{n+1}{2}}$ are the greatest if n is odd

these are the coefficient (coefficients) of the middle term (when n is even) & of middle terms if n is odd.

3. No. of terms in the expansion of $(a+b)^n$ is (n + 1) and number of terms in the expansion of $(a+b+c)^n$ is

$$\frac{(n+1)(n+2)}{2}$$

4. $C_0 + C_1 + C_2 + \text{------} + C_n = 2^n$

5. $C_0 - C_1 + C_2 - C_3 + \text{----} = 0$

6. $C_0 + C_2 + C_4 + C_6 + \text{----} = C_1 + C_3 + C_5 + \text{----} = 2^{n-1}$

7. For all rational values of n

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 +$$

$$----+\frac{n(n-1)(n-2)----(n-r+1)x^r}{r!}+---$$

Note: If n is not a positive integer; the expansion is an infinite series and valid only if $|x| < 1$

8. $(1+x)^{-n} = 1 - nx + \frac{n(n+1)}{1.2}x^2 - \frac{n(n+1)(n+2)}{1.2.3}x^3 + \ldots\ldots\ldots$

9. $(1-x)^{-n} = 1 + nx + \frac{n(n+1)}{2!}x^2 + \frac{n(n+1)(n+2)}{3!}$

$$x^3 + ---- + \frac{n(n+1)----(n+r-1)x^r}{r!} + ----$$

10. $(1-x)^n = 1 - nx + \frac{n(n-1)}{1.2}x^2 - \frac{n(n-1)(n-2)}{1.2.3}x^3 + \ldots\ldots\ldots$

11. $(1+x)^{-1} = 1 - x + x^2 - x^3 + \ldots\ldots\ldots\ldots \infty$

12. $(1 + x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \ldots\ldots \infty$

13. $(1+x)^{-3} = 1 - 3x + 6x^2 - 10x^3 + \ldots\ldots\ldots \infty$

14. $(1+x)^{-1/2} = 1 - \frac{x}{2} + \frac{3}{8}x^2 - \frac{5}{16}x^3 + \ldots\ldots\ldots\ldots\ldots \infty$

15. $(1 - x)^{-1} = 1 + x + x^2 + x^3 + \text{----} \infty$

16. $(1-x)^{-2} = 1 + 2x + 3x^2 + \text{------} \infty$

17. $(1-x)^{-3} = 1+\frac{2.3}{2}x+\frac{3.4}{2}x^2+\frac{4.5}{2}x^3+----$

$$(1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3+$$

PART I - (JEEMAIN)

SECTION - I - Straight objective type questions

1. The number of ways in which 6 boys and 6 girls can be arranged in a row so that boys and girls sit alternatively?

   1) (6!) 2) $(6!)^2$ 3) $2(6!)^2$ 4) $3(6!)^2$

2. Suppose that 20 pillars of the same height have been erected along the boundary of a circular stadium. If the top of each pillar has been connected by beams with the top of all non-adjacent pillars, then the total number of beams is

   1) 190 2) 180 3) 210 4) 170

3. The number of different words ending and beginning with a consonant, made out of the letters of the word EQUATION is

   (1) 5200 (2) 4320 (3) 1295 (4) 3000

4. The total number of words (with or without meaning) that can be formed out of the letters of the word 'DISTRIBUTION' taken four at a time, is equal to

   1) 3734 2) 3473 3)3347 4) 3437

5. The Number of Derangements of the integers 1 to10 inclusive of the Number line such that Exactly 4 integers are in natural positions where D_i represents the number of derrangements of i items

   1) $10_{C_4}.D_6$ 2) D_6

   3) $D_6+D_7+D_8+D_9+D_{10}$ 4) D_{10}

6. Eight chairs are numbered 1 to 8. Two women and three men wish to occupy one chair each. First the women choose the chairs from the chairs marked 1 to 4 and then the men select the chairs from the remaining. The number of possible arrangements is

(1) ${}^6C_3 \times {}^4C_2$ (2) ${}^4C_2 \times {}^6P_3$ (3) ${}^4P_2 \times {}^6P_3$ (4) ${}^4C_2 \times {}^6C_3$

7. The number of ways in which 6 different balls can be put in two boxes of different sixes so that no box is empty is

(1) 64 (2) 62 (3) 60 (4) 30

8. If a,b,c are three natural numbers in A.P., such that $a+b+c=21$, then the possible number of values of a,b,c is

(1) 13 (2) 14 (3) 15 (4) 16

9. The total number of positive integral solution for x, y, z such that $xyz = 24$, is

(1) 30 (2) 60 (3) 90 (4) 120

10. The total number of times the digit '3' will be written, when the integers having less than 4 digits are listed is equal to

(1) 302 (2) 206 (3) 306 (4) 300

11. The number of functions f from the set A = {0, 1, 2} into the set A = {0, 1, 2, 3, 4, 5, 6, 7} such that $f(i) \leq f(j)$ for $i < j$ and $i, j \in A$ is

(1) ${}^8C_3$ (2) ${}^8C_3 + 2({}^8C_2)$ (3) ${}^{10}C_3$ (4) ${}^8C_3 + {}^{10}C_3$

12. $A = \{a_1, a_2, a_3, a_4, a_5\}$, $B = \{b_1, b_2, b_3, b_4, b_5\}$. The number of one-one function from A to B such that

$f(a_i) \neq b_i$ for $i = 1, 2, 3, 4, 5$ and $f(a_1) = b_2$

(1) 11 (2) 14 (3) 24 (4) 18

13. A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five questions. The number of choice available to him is

(1) 140 (2) 196 (3) 280 (4) 346

14. The number of ordered pairs $(p,q) \in \{1,2,.....,50\}$ such that $6^p + 9^q$ is a multiple of 5 is

(1) 1250 (2) 2520 (3) 1520 (4) 250

15. If a, b and c are the greatest values of ${}^{19}C_p$, ${}^{20}C_q$ and ${}^{21}C_r$ respectively, then

(1) $\frac{a}{11}=\frac{b}{22}=\frac{c}{21}$ (2) $\frac{a}{10}=\frac{b}{11}=\frac{c}{21}$ (3) $\frac{a}{10}=\frac{b}{11}=\frac{c}{42}$ (4) $\frac{a}{11}=\frac{b}{22}=\frac{c}{42}$

16. The number of ways of choosing 10 objects out of 31 objects of which 10 are identical and the remaining 21 are distinct, is:

(1) 2^{20} (2) $2^{20}-1$ (3) $2^{20}+1$ (4) 2^{21}

17. Total number of terms in the expansion of $(x+y+z)^{10}$ is

1) 32 2) 55 3) 44 40 66

Assertion & Reasoning

(a) If both Statement-I and Statement-II are true and the reason is the correct explanation of the statement-I.
(b) If both Statement-I and Statement-II are true but reason is not the correct explanation of the statement-I.
(c) If Statement-I is true but Statement-II is false.
(d) If Statement-I is false but Statement-II is true.

18. **Statement-I:** Number of rectangles on a chess-board is ${}^8C_2 \times {}^8C_2$

Statement-II: To form a rectangle, we have to select any two of the horizontal lines and any two of the vertical lines

19. In the expansion of $(1+x)(1-x^2)\left(1+\frac{3}{x}+\frac{3}{x^2}+\frac{1}{x^3}\right)^5, x \neq 0$, the sum of the coefficient of x^3 and x^{-13} is equal to

1) 118 2) 119 3) 117 4) 115

20. If $(27)^{999}$ is divided by 7, then the remainder is

1) 1 2) 2 3) 3 4) 6

21. If $\alpha \in [-1,1]$ and $I(\alpha)$ denotes the term independent of x in the expansion of $\left(x\sin^{-1}\alpha+\frac{1}{x}\cos^{-1}\alpha\right)^{10}$ then maximum value of $|I(\alpha)|$ is

(1) ${}^{10}C_5\frac{\pi^{10}}{2^{20}}$ (2) ${}^{10}C_5\frac{\pi^{5}}{2^{10}}$ (3) ${}^{10}C_5\frac{\pi^{10}}{2^{5}}$ (4) ${}^{10}C_5\left(\frac{1}{2}\right)^{10}$

22. The number of terms in the expansion of $(1+x)^{101} \cdot (1+x^2-x)^{100}$ is

(1) 10100 (2) 50×101 (3) 202 (4) 102

23. The coefficient of x^4 in the expansion of $\left(1+2x+\frac{3}{x^2}\right)^6$ is

(1) 240 (2) 250 (3) 260 (4) 230

24. The sum of coefficients of integral powers of x in the expansion of $\left(1-2\sqrt{x}\right)^{50}$

1) 1 2) $\frac{3^{50}-1}{2}$ 3) $\frac{3^{50}+1}{2}$ 4) –1

25. $\sum_{r=0}^{10} C_r \cdot \frac{2^{r+1}}{r+1}$ = (where $C_r = {}^{10}C_r$)

(1) $\frac{3^{11}}{11}$ (2) $\frac{2^{11}}{11}$ (3) $\frac{3^{11}-1}{11}$ (4) $\frac{2^{11}-1}{11}$

26. The coefficient of x^{101} in the expression $(5+x)^{500} + x(5+x)^{499} + x^2(5+x)^{498} + \ldots. x^{500}, x>0$ is

1) ${}^{501}C_{101}(5)^{399}$ 2) ${}^{501}C_{101}(5)^{400}$

3) ${}^{501}C_{100}(5)^{400}$ 4) ${}^{500}C_{101}(5)^{399}$

27. If the 6th term in the expansion of $\left(\frac{1}{x^{8/3}} + x^2 \log_{10} x\right)^8$ is 5600, then x equals

1) 0 2) $\log_e 10$ 3) 10 4) x does not exist

28. If the fourth term in the binomial expansion of $\left(\sqrt{\frac{1}{x^{1+\log_{10} x}}} + x^{1/12}\right)^6$ is equal to 200, and $x>1$, then the value of x is

(1) 10^3 (2) 100 (3) 10^4 (4) 10

SECTION - II

Numerical Type Questions

29. There are ten boys $B_1, B_2, ...,B_{10}$ and five girls $G_1, G_2, ...,G_5$ in a class. Then the number of ways of forming a group consisting of three boys and three girls, if both B_1 and B_2 together should not be the members of a group, is

30. All words, with or without meaning, are made using all the letters of the word MONDAY. These words are written as in a dictionary with serial numbers. The serial number of the word MONDAY is

31. The tens digit of $1! + 2! + 3! + ... + 49!$ is

32. If in the expansion of $\left(2^{1/3} + 3^{-1/3}\right)^n$ the ratio of 7^{th} term from begining to the 7^{th} term from end is 1:6, then n is

33. The natural number m, for which the coefficient of 'x' in the binomial expansion of $\left(x^m + \frac{1}{x^2}\right)^{22}$ is 1540, is _______

34. The number of integral terms in the expansion of $\left(\sqrt{3} + \sqrt[8]{5}\right)^{256}$ is

35. If the term independent of 'x' in the expansion of $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$ is k, then 18k is _________

PART - II (JEE ADVANCED)

SECTION - III (Only one optipn correct type)

36. Let $S = \{1, 2, 3, ...9\}$. For K=1,2, ...5 let N_K be the number of subsets of S, each containing five elements out of which exactly K are odd. Then $N_1 + N_2 + N_3 + N_4 + N_5 =$

A) 210 B) 252 C) 125 D) 126

37. A candidate is required to answer 7 out of 15 questions which are divided into three groups A, B, C each containing 4, 5, 6 questions respectively. He is required to select at least 2 questions from each group. He can make up his choice in................ways

A) 1200 B) 2700 C) 2000 D) 1800

38. The number of three digit numbers of the form xyz such that $x < y$ and $z \le y$ is

A) 276 B) 285 C) 240 D) 244

39. 3 ladies have each brought their one child for admission toi a school. The principal wants to interview the six persons one byone subject to the condition that no mother interviewed before her child. The number of ways in which interviews can be arranged is 10λ ways then λ is

A) 8 B) 9 C) 11 D) 10

40. Number of ways in which 5 boys and 4 girls can be arranged on a circular table such that no two girls sit together and two particular boys are always together.

A) 276 B) 288 C) 296 D) 304

41. Let $x_1x_2x_3x_4x_5x_4x_3x_2x_1$ be a nine digit palindrome such that either the sequence (x_1,x_2,x_3,x_4,x_5) is a strictly ascending or strictly descending. Then the number of such palindromes is

A) $9 \times {}^9P_4$ B) $3 \times {}^9P_5$ C) $9 \times {}^9C_5$ D) $3 \times {}^9C_5$

42. The coefficient of x^{50} in the expansion of $(1+x)^{1000}+2x(1+x)^{999}+3x^2(1+x)^{998}+...1001x^{1000}$ is

A) ${}^{1000}C_{50}$ B) ${}^{1001}C_{50}$ C) ${}^{1002}C_{50}$ D) 2^{1001}

43. If $6^{83}+8^{83}$ is divided by 49, then the remainder is

A) 35 B) 5 C) 1 D) 0

44. The value of $\sum_{r=0}^{20} r(20-r)\left({}^{20}C_r\right)^2$ is equal to

A) $400\,{}^{39}C_{20}$ B) $400\,{}^{40}C_{29}$ C) $400\,{}^{39}C_{19}$ D) $400\,{}^{38}C_{20}$

SECTION - IV (More than one correct answer)

45. A number is called a palindrome if it reads the same backward or forward. For example 234432 is a palindrome. The number of 6 digit palindromes are divisible by 495 is λ, then λ divides

A) 495 B) 8976 C) 154 D) 288

46. If λ be the number of terms in the expansiion of $\left(3^{1/4}+4^{1/3}\right)^{99}$ which are irrational, λ is divisible by

A) 5 B) 7 C) 9 D) 13

47. In the expression of $(x+a)^n$ if the sum of odd terms be P and sum of even terms be Q, then

A) $P^2-Q^2=\left(x^2-a^2\right)^n$ B) $4PQ=(x+a)^{2n}-(x-a)^{2n}$

C) $2\left(P^2+Q^2\right)=(x+a)^{2n}+(x-a)^{2n}$ D) $4PQ=(x+a)^{2n}+(x-a)^{2n}$

48. If $\left(1+x+2x^2\right)^{20} = a_0 + a_1x + a_2x^2 + ... + a_{40}x^{40}$ then

A) $a_0 - a_2 + a_4 - ..... + a_{40} = -2^{10}$

B) $a_0 + a_4 + a_8 + ... + a_{40} = a_2 + a_6 + a_{10} + ... + a_{38}$

C) $a_1 + a_5 + a_9 + ... + a_{37} = a_3 + a_7 + a_{11} + ... + a_{39}$

D) $a_1 - a_3 + a_5 - a_7 + ... - a_{39} = 2^{10}$

49. A contest consists of ranking 10 songs of which 6 are Indian classic and 4 are western songs. Number of ways of ranking so that

A) There are exactly 3 Indian classic songs in top 5 is $(5!)^3$

B) Top rank goes to Indian classic song is 6(9!)

C) The ranks of all western songs are consecutive is 4!7!

D) The 6 Indian classic songs are in a specified order is ${}^{10}P_4$

SECTION - V (Numerical Type)

50. The term independent of x in expansion of $\left(\dfrac{x+1}{x^{2/3}-x^{1/3}+1} - \dfrac{x-1}{x-x^{1/2}}\right)^{10}$ is :

51. If all the six digit numbers $x_1\ x_2\ x_3\ x_4\ x_5\ x_6$ with $0 < x_1 < x_2 < x_3 < x_4 < x_5 < x_6$ are arranged in the increasing order, then the sum of the digits in the 72^{th} number is

52. Number of ways in which 9 different toys be distributed among 4 children belonging to different age groups in such a way what distribution among the 3 elder children is even and the youngest one is to receive one toy more, is $\dfrac{9P_5}{\lambda}$ then λ is

53. The number of 4-digit integers in the closed interval [2022,4482] formed by using the digits 0,2,3,4,6,7 is

54. If $\sum_{k=1}^{31}\left({}^{31}C_k\right)\left({}^{31}C_{k-1}\right) - \sum_{k=1}^{30}\left({}^{30}C_k\right)\left({}^{30}C_{k-1}\right) = \dfrac{\alpha(60!)}{(30!)(31!)}$ Where $\alpha \in R$, then the value of 16α is equal to