CHAPTER - 00
MOTION IN A STRAIGHT LINE

RECTILINEAR MOTION / MOTION IN A STRAIGHT LINE

Mechanics is a study of rest and motion

- Mechanics
  - Statics (Study of rest)
  - Dynamics (Study of motion)
    - Kinematics
      [Study of motion without considering its cause]
      [Kinematical variables of motion-distance,displacement, speed,velocity,acceleration,time]
    - Kinetics
      [Study of motion by considering the cause]
      [Force, Torque]

REFERENCE FRAME OR FRAME OF REFERENCE

A coordinate system used to take a measurement is called frame of reference.

The reference point is the origin of the coordinate system and observer is considered to be at origin.

If a body does not change its position as time passes with respect to frame of reference, it is said to be at rest.

And if a body changes its position as time passes with respect to frame of reference, it is said to be in motion.

Frame of Reference : It is a system to which a set of coordinates are attached and with reference to which observer describes any event.

A passenger standing on platform observes that tree on a platform is at rest. But when the same passenger is passing away in a train through station, observes that tree is in motion. In both conditions observer is rest . But observations are different because in first situation observer stands on a platform, which is reference frame at rest and in second situation observer moving in train, which is reference frame in motion.

So rest and motion are relative terms. It depends upon the frame of reference

Types of Motion

One dimensional	Two dimensional	Three dimensional
Motion of a body in a straight line is called one dimensional motion.	Motion of body in a plane is called two dimensional motion.	Motion of body in a space is called three dimensional motion.
When only one coordinate of the position of a body changes with time then it is said to be moving one dimensionally.	When two coordinates of the position of a body changes with time then it is said to be moving two dimensionally.	When all three coordinates of the position of a body changes with time then it is said to be moving three dimensionally.
e.g.. Motion of car on a straight road. Motion of freely falling body.	*e.g.* Motion of car on a circular turn. Motion of billiards ball.	*e.g.*. Motion of flying kite. Motion of flying insect.

Distance

It is the actual path length covered by the object

Properties : It is scalar

For a moving object distance always increases from O. Therefore, it can be zero or positive but never be negative.

Dimensions of distance is [L]

Distance doesn't give any idea about the final position

Dimensions

length $\rightarrow$ [L]

Mass $\rightarrow$ [M]

Time $\rightarrow$ [T]

Displacement

It is the shortest separation between initial and final position.

Properties : It is a vector [Vector quantities have both magnitude and direction and must obey laws of vector addition]

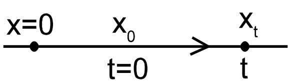
It is a vector from initial position to final position

Displacement may increase or decrease with time. It can be 0, +ve or –ve

Unit of displacement is m

Dimension is [L]

Displacement = Final position – initial position



$$\boxed{\Delta x = x_t - x_0}$$

x_0 = position at t = 0

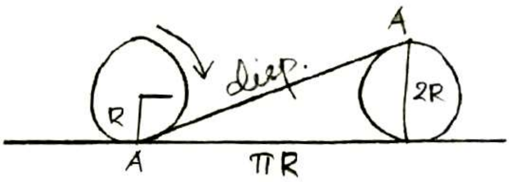
x_t = position at a time t

$|\text{Displacement}|$ = distance only when the object travels in a fixed direction

Generally, $\boxed{|\text{Displacement}| \le \text{distance}}$

i.e. $\dfrac{|\text{Displacement}|}{\text{distance}} \le 1$

Q. A circular ring of radius R is rolling along a horizontal surface. Find out the displacement of particle A on the surface of the ring when the ring completes half of the rotation



Displacement of A = $\sqrt{(2R)^2 + (\pi R)^2} = R\sqrt{4+\pi^3}$

Speed

Distance travelled in unit time

Constant Speed, $v = \dfrac{\text{Total distance}}{\text{Total time}} \Rightarrow v = \dfrac{s}{t}$

Properties : It is a scalar quantity

It can be 0 or positive but never be –ve

It may increase, decrease or may remain constant with time

Unit is m/s

Dimensions = $\left[LT^{-1}\right]$

When speed is non-uniform, then it can be measure by its average value and instantaneous value.

For any type of motion, average speed

$$V_{av} = \frac{\text{Total distance}}{\text{Total time}} \Rightarrow V_{av} = \frac{s}{t}$$

S = $V_{av}t$

Instantaneous Speed

Speed at any instant / at any position is called instantaneous speed

Velocity

Displacement in unit time / Rate of change of position w.r.t time

Uniform velocity, $\vec{v} = \frac{\text{total displacement}}{\text{total time}}$

$v = \frac{x_f - x_i}{t}$ where x_f is the initial position and x_i is the initial position

Properties
* It is vector
* Can be 0, +ve or –ve
* Dimensions = $[LT^{-1}]$

* It may increase, decrease or may remain constant with time
* Velocity constant only when its magnitude and direction remains unchanged

For any type of motion average velocity

$$\vec{V}_{av} = \frac{\text{total displacement}}{\text{total time}}$$

OR

$$V_{av} = \frac{x_f - x_i}{t} = \frac{\Delta x}{\Delta t}$$

OR

Displacement = $x_f - x_i = V_{av} \times t$

Instantaneous Velocity

Velocity at any instant or at any position

If Δx is the displacement in a small time interval Δt, then the instantaneous velocity

$$\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

$$\boxed{V_{inst} = \frac{dx}{dt} = \frac{d(x)}{dt}}$$

Instantaneous values are measured in small time interval. Therefore magnitude of instantaneous displacement = instantaneous distance

Therefore, magnitude of instantaneous velocity = instantaneous speed

$|\text{average velocity}|$ = average speed, only

when the object travels in a fixed direction

Generally,

$$|\text{average velocity}| \le \text{average speed}$$

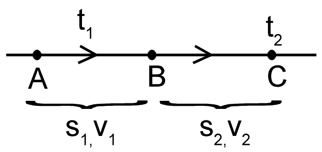
Point Object

size of the object is very small compared to the distance travelled by the object.

A body (Group of particles) to be known as a particle depends upon types of motion. For example in a planetary motion around the sun the different planets can be asumed to be the particles.

In above consideration when we treat body as particles, all parts of the body undergo same displacement and have same velocity and acceleration

AVERAGE VELOCITY



Object covers S_1 displacement with velocity V_1 in a time t_1 and the displacement S_2 with velocity V_2 in a time t_2.

$$V_{av} = \frac{\text{total displacement}}{\text{total time}}$$

$$V_{av} = \frac{s_1 + s_2}{t_1 + t_2} = \frac{v_1 t_1 + v_2 t_2}{t_1 + t_2}$$

$$V_{av} = \frac{s_1 + s_2}{\frac{s_1}{v_1} + \frac{s_2}{v_2}}$$

If two equal time intervals travelled with different velocities. i.e., $t_1 = t_2 = t$, the average velocity

$$V_{av} = \frac{v_1 t + v_2 t}{2t} \Rightarrow 1$$

$$\boxed{V_{av} = \frac{v_1 + v_2}{2}}$$

When n equal time intervals are travelled with different velocities, then average velocity

$$\boxed{V_{av} = \frac{v_1 + v_2 + ...v_n}{n}}$$

The above equation can be used to find out the average speed.

When two equal displacements travelled with different velocities v_1 and v_2, then average velocity

$$V_{av} = \frac{s_1 + s_2}{\frac{s_1}{v_1} + \frac{s_2}{v_2}} \qquad \text{but } s_1 = s_2 = s$$

$$= \frac{2s}{\frac{s}{v_1}+\frac{s}{v_2}} = \frac{2s}{\left[\frac{1}{v_1}+\frac{1}{v_2}\right]} \Rightarrow$$

$$\frac{2}{v_{av}} = \frac{1}{v_1}+\frac{1}{v_2}$$

$$\boxed{v_{av} = \frac{2v_1v_2}{v_1+v_2}}$$

For n equal displacement with different velocities

$$\boxed{\frac{n}{v_{av}} = \frac{1}{v_1}+\frac{1}{v_2}+\frac{1}{v_3}}$$

$$\boxed{v_{av} = \frac{3v_1v_2v_3}{v_1v_3+v_2v_3+v_1v_2}}$$

V_{av} = harmonic mean of v_1, v_2, ...v_n

Uniform speed : When a particle covers equal distances in equal intervals of time, (no matter how small the intervals are) then it is said to be moving with uniform speed. In given illustration motorcyclist travels equal distance (= $5m$) in each second. So we can say that particle is moving with uniform speed of 5 m/s.

Non-uniform (variable) speed : In non-uniform speed particle covers unequal distances in equal intervals of time. In the given illustration motorcyclist travels $5m$ in 1^{st} second, $8m$ in 2^{nd} second, $10m$ in 3^{rd} second, $4m$ in 4^{th} second *etc.*

Therefore its speed is different for every time interval of one second. This means particle is moving with variable speed.

Uniform velocity : A particle is said to have uniform velocity, if magnitudes as well as direction of its velocity remains same and this is possible only when the particles moves in same straight line without reversing its direction.

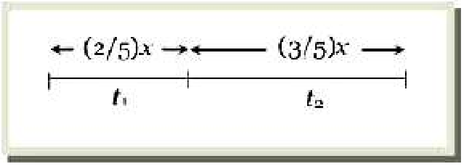
Non-uniform velocity : A particle is said to have non-uniform velocity, if either of magnitude or direction of velocity changes (or both changes).

Problem If a car covers $2/5^{th}$ of the total distance with v_1 speed and $3/5^{th}$ distance with v_2 then average speed is

(a) $\frac{1}{2}\sqrt{v_1v_2}$ (b) $\frac{v_1+v_2}{2}$ (c) $\frac{2v_1v_2}{v_1+v_2}$ (d) $\frac{5v_1v_2}{3v_1+2v_2}$

Solution : (d) Average speed = $\dfrac{\text{Total distance travelled}}{\text{Total time taken}} = \dfrac{x}{t_1 + t_2}$

$$= \frac{x}{\dfrac{(2/5)x}{v_1} + \dfrac{(3/5)x}{v_2}} = \frac{5v_1v_2}{2v_2 + 3v_1}$$



Acceleration

Change in velocity in unit time / Rate of change of velocity w.r.t time

$$\text{Uniform acceleration} = \frac{\text{Change in velocity}}{\text{time}}$$

$$\boxed{a = \frac{v - u}{t}}$$

* Acceleration is a vector in the direction of change in velocity

* Unit is ms^{-2}

* Dimensions = $\left[LT^{-2}\right]$

* It can be 0, +ve, or –ve

* It may increase, decrease or may remain constant with time

Average acceleration

Average acceleration = $\dfrac{\text{change in velocity}}{\text{Total time}}$

$$\boxed{a_{av} = \frac{v - u}{t_f - t_i} = \frac{\Delta u}{\Delta t}}$$

Instantaneous acceleration

Acceleration at any instant or at any position

$$\boxed{a_{inst} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}} \Rightarrow \boxed{a_{inst} = \frac{dv}{dt}}$$

If acceleration is positive, velocity increases. In negative acceleration, velocity decreases and it is called deceleration.

$\xrightarrow{a}$

motion

$\longrightarrow$ + x

$v \Rightarrow +ve,\ v > 0$

$a \Rightarrow +ve,\ a > 0$

Velocity $\uparrow$ $\therefore 0$

Speed $\uparrow$

$\xleftarrow{a}$

motion

$\longrightarrow$ +

$\text{velocity} \Rightarrow +ve,\ v > 0$

$a \Rightarrow -ve,\ a < 0$

Velocity $\downarrow$ $\therefore$ deceleration

Speed $\downarrow$ $\therefore$ retardation

$\xrightarrow{a}$

motion

-x $\longleftarrow$

$\text{velocity} \Rightarrow -ve,\ v < 0$

$a = +ve$

Velocity $\uparrow$ $\therefore$ acceleration

Speed $\downarrow$

$\xleftarrow{a}$

motion

$\longleftarrow$

$v \Rightarrow -ve,\ v < 0$

$a \Rightarrow -ve, a < 0$

Velocity $\downarrow$ $\therefore$ deceleration

Speed $\downarrow$ $\therefore$ retardation

If velocity and acceleration are opposite in sign, or opposite in directions, then speed of the particle will decrease.

If velocity and acceleration are in the same direction or same sign, then speed will increase

In motion along a straight line, angle between velocity and acceleration can be 0° and 180°

Note: A particle in rectilinear motion is speeding up when its instantaneous speed is increasing and is slowing down when its instantaneous speed is decreasing. An object that is speeding up is said to be "accelerating" and an object that is slowing down is said to be "decelerating", thus, one might expect that a particle in rectilinear motion will be speeding up when its instantaneous acceleration is positive and slowing down when it is negative. This is true for a particle moving in the positive direction and it is not true for a particle moving in the negative direction-a particle with negative velocity is speeding up when its acceleration is negative and slowing down when its acceleration is positive.

This is because a positive acceleration implies an increasing velocity and increasing a negative velocity decreases its absolute value. similarly, a negative acceleration implies a decreasing velocity and decreasing a negative velocity increases its absolute value.

Interpreting the sign of acceleration:

particle in rectilinear motion is speeding up when its velocity and acceleration have the same sign and slowing down when they have opposite signs.

Note: If a particle is moving with uniform acceleration, this does not necessarily imply that particle is moving in straight line. e.g. Projectile motion.

The direction of average acceleration vector is in the direction of the change in velocity vector, not in the direction of velocity vector.

Note : If velocity is uniform, then the displacement covered by an object in a time 't' is displacement

= velocity $\times$ time

S = vt

For any type of motion, total displacement

S= average velocity $\times$ time

$S = V_{av}\, t$

For any type of motion, average acceleration

$$a_{av} = \frac{\text{total change in velocity}}{\text{total time}}$$

$$a_{av} = \frac{v-u}{t}$$

Equations of motion of uniformly accelerated Motion from Graph

Consider a particle starts moving with an initial velocity 'u' under the influence of a uniform acceleration 'a'. v is the velocity of the particle at a time 't'. The velocity time graph of the motion is shown below

In the graph

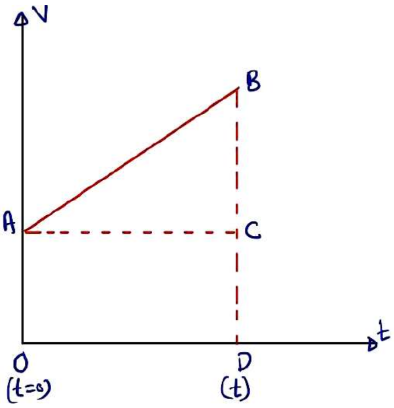
OA = u

DB = v

OD = t

1) Velocity in terms of time

Slope of the velocity time graph gives acceleration



$\therefore$ a = slope of the graph = $\frac{CB}{AC}$

$$a = \frac{BD - DC}{AC} = \frac{v-u}{t}$$

$$\therefore \boxed{v = u + at}$$

2) **Displacement in terms of time**

Displacement s = area under v – t graph

S = area OACD + area ACB

$$= (OA)(OD) + \frac{1}{2}(BC)(AC)$$

$$= ut + \frac{1}{2}(v-u)t$$ but v – u = at

$$\therefore \boxed{s = ut + \frac{at^2}{2}} \boxed{x(t) - x(0) = ut + \frac{at^2}{2}}$$

x(t) = position at a time ‘t’

x(0) = initial position, at t = 0

3) Displacement S = area under v - t graph

$$S = \frac{1}{2}(OA + DB)(OD)$$

$$= \frac{1}{2}(u+v)t$$

$\boxed{S=\left(\frac{u+v}{2}\right)t}$ but $\boxed{S=v_{av}t}$

$\therefore$ average velocity $\boxed{V_{av}=\frac{u+v}{2}}$

but $t=\frac{v-u}{a}$ $\quad \therefore S=\left(\frac{u+v}{2}\right)\left(\frac{v-u}{a}\right)$

$S=\frac{v^2-u^2}{2a}$ or $\boxed{\begin{array}{l} v^2=u^2+2as \\ v^2=u^2+2a\left[x(t)-x(0)\right] \end{array}}$

4) **Displacement in n[th] second**

Total displacement in a time 'n' is

$$S_{(n)}=un+\frac{an^2}{2}$$

Displacement in '(n – 1)' seconds is

$$S_{(n-1)}=u(n-1)+\frac{a(n-1)^2}{2}$$

$\therefore$ Displacement in n[th] second

$$S_n=S_{(n)}-S_{(n-1)}$$

$$= un+\frac{an^2}{2}-\left[u(n-1)+\frac{a(n-1)^2}{2}\right]$$

$$= un+\frac{an^2}{2}-\left[un-u+\frac{a}{2}\left[n^2-2n+1\right]\right]$$

$$= un+\frac{an^2}{2}-\left[un-u+\frac{an^2-an}{2}+\frac{a}{2}\right]$$

$$= un+\frac{an^2}{2}-un+u-\frac{an^2}{2}+an-\frac{a}{2}$$

$$\boxed{S_n=u+\frac{a}{2}[2n-1]}$$

When object starts from rest, u = 0 m/s, a = constant

then, v = at

$$S_n = \frac{a}{2}[2n-1]$$

$$v^2 = 2as$$

$$v_{av} = \frac{v}{2}$$

$$S = \frac{v}{2}t$$

Eg. Object starts from rest and moves with constant acceleration. The ratio of the displacements in 1s, in 2s, in 3s, ... is 1:4:9:....

$$s = \frac{at^2}{2}$$

$$s \, \alpha \, t^2$$

$$s_{(1)} = 1^2$$

$$s_{(2)} = 2^2$$

$$s_{(3)} = 3^2$$

$$s_{(1)} : s_{(2)} : s_{(3)} .... = 1:4:9...$$

Eg. Object starts from rest and moves with uniform acceleration. Then the ratio of velocities at 1^{st} second, 2^{nd}, 3^{rd} second etc. is 1:2:3....

$$v = at$$

$$c \, \alpha \, t$$

$$1 \, \alpha \, 1$$

$$2 \, \alpha \, 2$$

$$3 \, \alpha \, 1$$

$$v_1 : v_2 : v_3 .... = 1:2:3$$

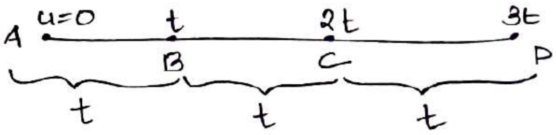
In the above situation, ratio of the displacement in 1^{st} second, 2^{nd} second, 3^{rd} second ... is 1:3:5:....

$$\because s_n = \frac{a}{2}[2n-1]$$

$$s_n \alpha \left(2n-1\right)$$

$$\left(2n-1\right) \Rightarrow \text{add values}$$

In the above situation, ratio of the displacements in successive equal intervals of time is 1:3:5 ....



Displacement in first t seconds

$$AB = \frac{at^2}{2}$$

Displacement in 2[nd] t seconds

BC = displacement in 2t seconds – displacement in t seconds

$$= \frac{a}{2}(2t)^2 - \frac{at^2}{2}$$

$$= \frac{a}{2}3t^2$$

$$BC = 3(AB)$$

Displacement in 3[rd] t seconds

CD = displacement in 3t seconds – displacement in 2t seconds

$$= \frac{a}{2}(3t)^2 - \frac{a}{2}(2t^2)$$

$$= \frac{a}{2}\left[at^2 - 4t^2\right]$$

$$= \frac{5at^2}{2}$$

$$CD = 5(AB)$$

AB:BC:CD:.... = 1:3:5:......

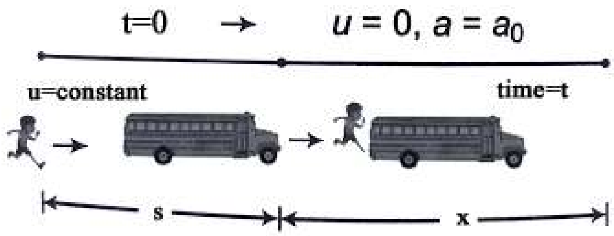
Q. A particle travels along a straight line with constant acceleration. If u and v are the initial and final velocities, then the velocity at the midpoint of the motion is

$$\frac{V_m^2 - u^2}{2a} = \frac{v^2 - v_m^2}{2a}$$

$$2v_m^2 = v^2 + u$$

$$\boxed{v_m = \sqrt{\frac{v^2 + u^2}{2}}}$$

Q. A student is standing at a distance 50m from the bus. As soon as bus begins its motion with an acceleration of 1 m/s^2, the student starts running towards the bus with a uniform velocity u. The minimum value of u so that the student is able to catch the bus is



Distance travelled by the student = initial separation + distance travelled by the bus

$$ut + s + \frac{at^2}{2}$$

$$\Rightarrow \frac{at^2}{2} - ut + s = 0$$

$$t = \frac{u \pm \sqrt{u^2 - 4 \times \frac{a}{2} \times s}}{2 \times \frac{a}{2}} \quad \left[\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right]$$

$$t = \frac{u \pm \sqrt{u^2 - 2as}}{a}$$

For real value of time,

$$u^2 - 2as \geq 0$$

$$u^2 \geq 2as$$

$$\boxed{\begin{array}{l} u \geq \sqrt{2as} \\ u_{min} = \sqrt{2as} \end{array}}$$

a = 1

s = 50 m

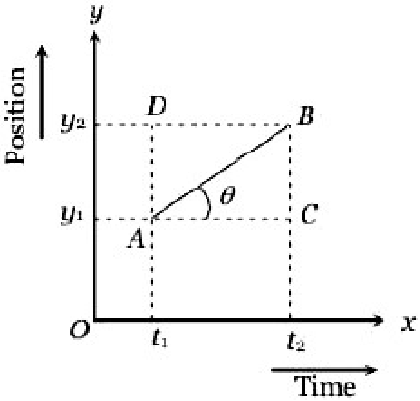
$$u_{min} = \sqrt{2 \times 1 \times 50} = 10 m/s$$

Position Time Graph

During motion of the particle its parameters of kinematical analysis (u, v, a, r) changes with time. This can be represented on the graph.

Position time graph is plotted by taking time t along x-axis and position of the particle on y-axis.

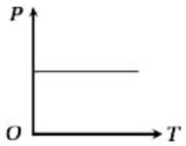
Let AB is a position-time graph for any moving particle



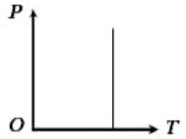
As Velocity = $\dfrac{\text{Change in position}}{\text{Time taken}} = \dfrac{y_2 - y_1}{t_2 - t_1}$ ...(i)

From triangle ABC $\tan\theta = \dfrac{BC}{AC} = \dfrac{AD}{AC} = \dfrac{y_2 - y_1}{t_2 - t_1}$ ....(ii)

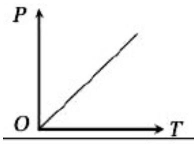
By comparing (i) and (ii) Velocity = $\tan\theta$



$\theta = 0^o$ so $v = 0$
i.e., line parallel to time axis represents that the particle is at rest.



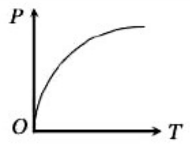
$\theta = 90^o$ so $v = \infty$
i.e., line perpendicular to time axis represents that particle is changing its position but time does not changes it means the particle possesses infinite velocity.
Practically this is not possible.



θ = constant so v = constant, $a = 0$
i.e., line with constant slope represents uniform velocity of the particle.

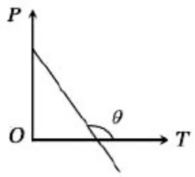
θ is increasing so v is increasing, a is positive.

i.e., line bending towards position axis represents increasing velocity of particle. It means the particle possesses acceleration.



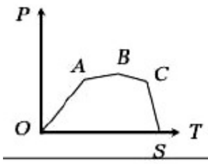
θ is decreasing so v is decreasing, a is negative

i.e., line bending towards time axis represents decreasing velocity of the particle. It means the particle possesses retardation.

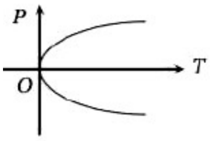


θ constant but > 90^o so v will be constant but negative

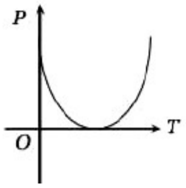
i.e., line with negative slope represent that particle returns towards the point of reference. (negative displacement).



Straight line segments of different slopes represent that velocity of the body changes after certain interval of time.



This graph shows that at one instant the particle has two positions. Which is not possible.



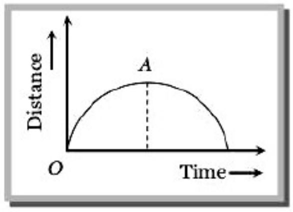
The graph shows that particle coming towards origin initially and after that it is moving away from origin.

$$v = \tan\theta$$

It is clear that slope of position-time graph represents the velocity of the particle.

Various position – time graphs and their interpretation

Note : ❑ If the graph is plotted between distance and time then it is always an increasing curve and it never comes back towards origin because distance never decrease with time. Hence such type of distance time graph is valid up to point A only, after point A it is not valid as shown in the figure.



❑ For two particles having displacement time graph with slopes θ_1 and θ_2 possesses velocities υ_1 and υ_2 respectively then $\frac{\upsilon_1}{\upsilon_2} = \frac{\tan\theta_1}{\tan\theta_2}$

<u>Velocity time Graph</u>

The graph is plotted by taking time t along x-axis and velocity of the particle on y-axis.

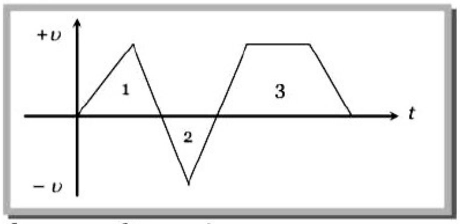
Distance and displacement : The area covered between the velocity time graph and time axis gives the displacement and distance travelled by the body for a given time interval.

Then Total distance $= |A_1| + |A_2| + |A_3|$

= Addition of modulus of different area. *i.e.* $s = \int |\upsilon|\, dt$

Total displacement $= A_1 + A_2 + A_3$

= Addition of different area considering their sign. *i.e.* $r = \int \upsilon\, dt$

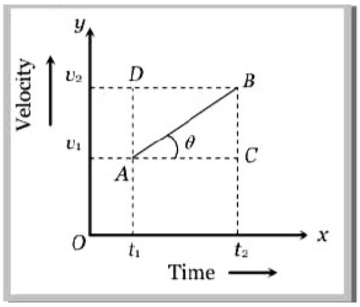


here A_1 and A_2 are area of triangle 1 and 2 respectively and A_3 is the area of trapezium .

Acceleration : Let AB is a velocity-time graph for any moving particle

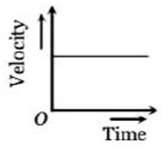
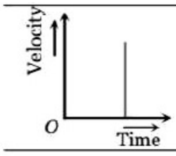
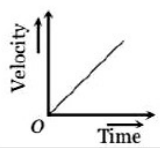
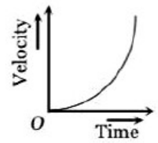
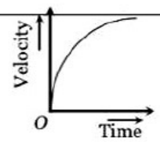
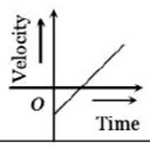
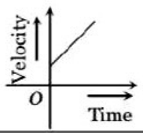
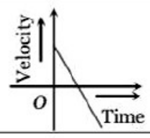
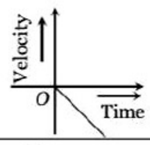
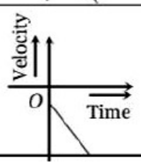
As Acceleration $= \frac{\text{Change in velocity}}{\text{Time taken}} = \frac{v_2 - v_1}{t_2 - t_1}$ ...(i)

From triangle ABC, $\tan\theta = \frac{BC}{AC} = \frac{AD}{AC} = \frac{v_2 - v_1}{t_2 - t_1}$ ....(ii)



It is clear that slope of velocity-time graph represents the acceleration of the particle.

Various velocity – time graphs and their interpretation

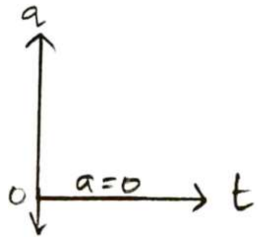
	$\theta = 0$, $a = 0$, v = constant *i.e.*, line parallel to time axis represents that the particle is moving with constant velocity.
	$\theta = 90^o$, $a = \infty$, v = increasing *i.e.*, line perpendicular to time axis represents that the particle is increasing its velocity, but time does not change. It means the particle possesses infinite acceleration. Practically it is not possible.
	θ=constant, so a = constant and v is increasing uniformly with time *i.e.*, line with constant slope represents uniform acceleration of the particle.
	θ increasing so acceleration increasing *i.e.*, line bending towards velocity axis represent the increasing acceleration in the body.
	θ decreasing so acceleration decreasing *i.e.* line bending towards time axis represents the decreasing acceleration in the body
	Positive constant acceleration because θ is constant and < 90^o but initial velocity of the particle is negative.
	Positive constant acceleration because θ is constant and < 90^o but initial velocity of particle is positive.
	Negative constant acceleration because θ is constant and > 90^o but initial velocity of the particle is positive.
	Negative constant acceleration because θ is constant and > 90^o but initial velocity of the particle is zero.
	Negative constant acceleration because θ is constant and > 90^o but initial velocity of the particle is negative.

Acceleration-time Graph

a) Uniform motion

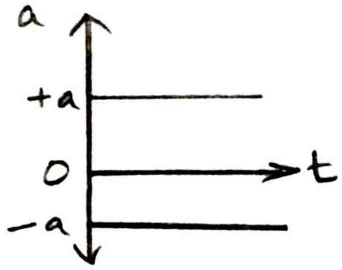
$\text{velocity} = \text{constant}$

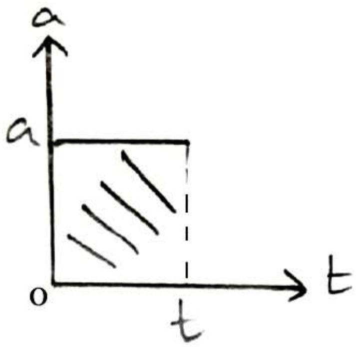
$a = 0$



b) Uniformly accelerated motion

a = constant





Area under a – t graph

A = at

but at = v – u

Area = change in velocity

Area under acceleration - time graph given change in velocity

<u>**For acceleration**</u>

v = u + at

v = 0 + αt_1

$t_1 = \dfrac{v}{\alpha}$

Deceleration

$$t_2 = \frac{v-u}{a} = \frac{0-v}{-\beta}$$

$$t_2 = \frac{\alpha}{\beta}$$

$$\boxed{\frac{t_2}{t_1} = \frac{\alpha}{\beta}}$$

Displacement in deceleration

$$= s_2 = \frac{v^2-u^2}{2a} = \frac{0-v^2}{-2\beta} = \frac{v^2}{2\beta}$$

$$\Rightarrow s_2 = \frac{v^2}{2\beta}$$

$$\boxed{\frac{s_2}{s_1} = \frac{\alpha}{\beta} = \frac{t_2}{t_1}}$$

$$t = t_1 + t_2$$

$$t_1 = \frac{v}{\alpha}$$

$$t_2 = \frac{v}{\beta}$$

$$t = \frac{v}{\alpha} + \frac{v}{\beta} = v\left(\frac{\alpha+\beta}{\alpha\beta}\right)$$

Motion under Gravity

In motion under gravity, object is moving with constant acceleration which is equal to acceleration due to gravity. In the following situations all the resistive forces are neglected.

FREE FALL

Here object is released from a height.

$\therefore$, initial velocity is 0. Starting point is taken as origin and downward direction is represented by –ve y-axis.

Thus all the vectors in the downward direction are negative and all the vectors in the upward direction are positive.

Acceleration due to gravity is downward.

$\therefore$, acceleration a is –ve

$a = -g = -9.8\ ms^{-2}$

EQUATIONS OF MOTION OF FREE FALL

$u = 0$

$a = -g$

$v = -gt$

$$y = s = \frac{-1}{2}gt^2$$

$$v^2 = -2g(s) = 2gs = 2gy$$

$$s_n = \frac{-g}{2}[2n-1]$$

$$v_{av} = \frac{u+v}{2}[\text{Av.speed}]$$

$$\Rightarrow V_{av} = \frac{v}{2}[\because u = 0]$$

Time taken to reach ground

When the object is dropped from a height h, its displacement when it reaches ground is

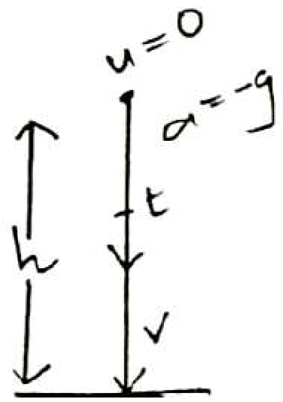
$$y = -h$$

$$y = \frac{-gt^2}{2}$$

$$-h = \frac{-gt^2}{2}$$

$$h = \frac{gt^2}{2}$$

$$\boxed{t = \sqrt{\frac{2h}{g}}}$$



The speed with which the object hits ground

$$v^2 = 2gy$$

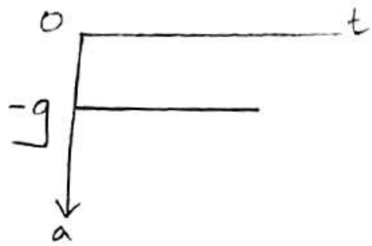
$$v^2 = 2gh$$

$$\boxed{v = \sqrt{2gh}}(\text{speed})$$

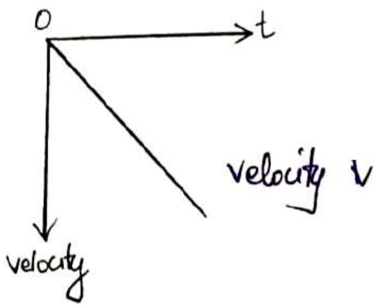
Velocity $\boxed{v = -\sqrt{2gh}}$

Time of motion and final speed are independent of mass, size and shape of the object

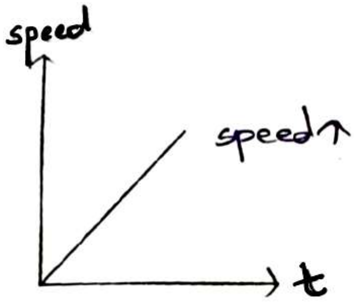
Acceleration time graph



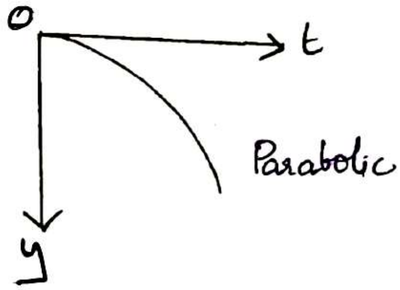
Velocity - time graph



Speed - time graph



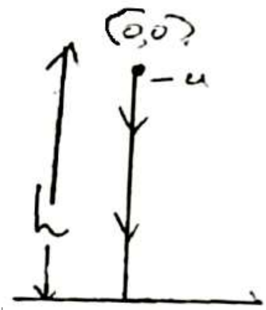
Displacement-time graph



Projectile motion

Motion of an object with an initial velocity and moving with uniform acceleration is called projectile motion.

Eg.1 : Object is thrown vertically downward.



Initial velocity = –u

a = –g

Equations of motion

v = –u – gt

$$y = -ut - \frac{gt^2}{2}$$

$$v^2 = u^2 + 2(-g)(-y)$$

$$\text{i.e, } v^2 = u^2 + 2gy$$

$$y_n = -u - \frac{g}{2}[2n-1]$$

Speed with which object hits ground

$$v^2 = u^2 + 2gh$$

$$\Rightarrow \boxed{v = \sqrt{u^2 + 2gh}}\ (\text{speed})$$

Velocity $\boxed{v = -\sqrt{u^2 + 2gh}}$

Time with which object hits the ground

v = –u – gt

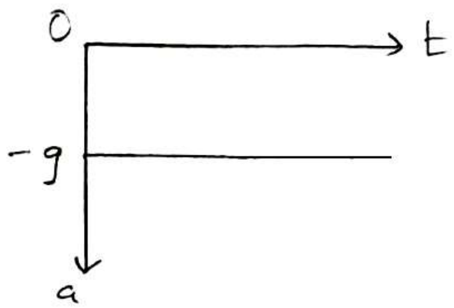
$$t = \frac{(v+u)}{-g} = \frac{v+u}{g} = \frac{-\sqrt{u^2 + 2gh} - (-u)}{-g}$$

$$t = \frac{-u + \sqrt{u^2 + 2gh}}{g}$$

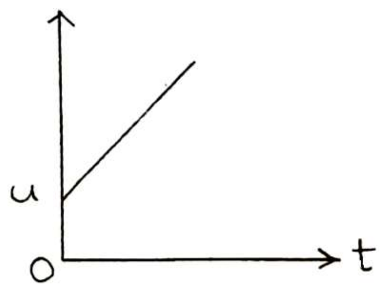
$$t = \frac{-u + \sqrt{u^2\left[1 + \frac{2gh}{u^2}\right]}}{g}$$

$$\boxed{t = \frac{u}{g}\left[-1 + \sqrt{1 + \frac{2gh}{u^2}}\right]}$$

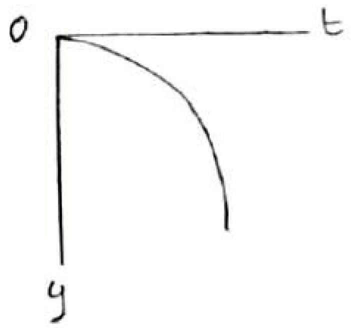
ACCELERATION-TIME GRAPH



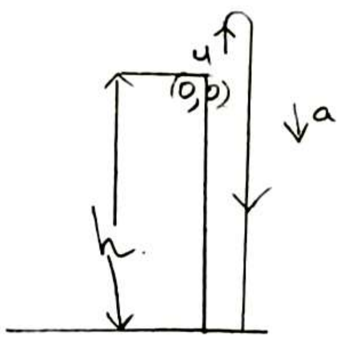
SPEED-TIME GRAPH



POSITION-TIME GRAPH



OBJECT IS THROWN VERTICALLY UPWARD FROM A HEIGHT



Upward direction is taken as +ve and downward direction is taken as –ve. Therefore, initial velocity = +u [upward].

Acceleration due to gravity is downward. $\therefore$, In both upward and downward motion, acceleration due to gravity is –ve.

Thus, equations of motion becomes

$$v = u - gt$$

$$y = ut - \frac{gt^2}{2}$$

$$v^2 = u^2 - 2gh$$

$$y_n = u - \frac{g}{2}[2n - 1]$$

SPEED WITH WHICH THE OBJECT HITS GROUND

h is the height from which the object is thrown u is the speed with

When the object reaches ground, displacement

y = –h

$\therefore$ Final speed $v = \sqrt{u^2 + 2gh}$

Final velocity $v = -\sqrt{u^2 + 2gh}$

TIME OF FLIGHT (T)

The time in which the object moves through air or time taken to reach the ground is called the time of flight.

$$t = \frac{v - u}{a}$$

$$= \frac{-\sqrt{u^2 + 2gh - u}}{-g}$$

$$= \frac{u + \sqrt{u^2 + 2gh}}{g}$$

$$= \frac{u + \sqrt{u^2\left[1 + \frac{2gh}{u^2}\right]}}{g}$$

$$\boxed{\frac{u}{g}\left[1 + \sqrt{1 + \frac{2gh}{u^2}}\right]}$$

$$y = \frac{v^2 - u^2}{2a}$$

If H is the max height from the projection point, then at y = H, v = 0

$$H = \frac{0 - u^2}{-2g}$$

$$\boxed{H = \frac{u^2}{2g}}$$

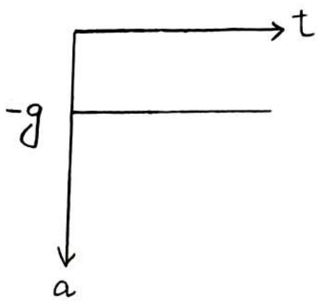
Total distance travelled by the object when it reaches ground

$$= 2H + h$$

$$= 2\frac{u^2}{2g} + h$$

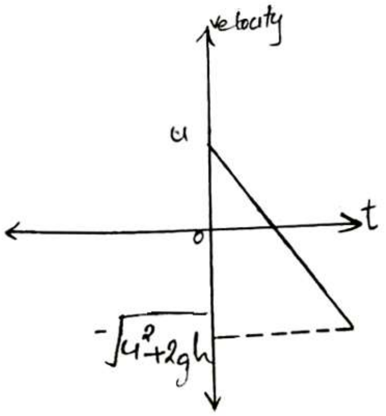
$$= \boxed{\frac{u^2}{g} + h}$$

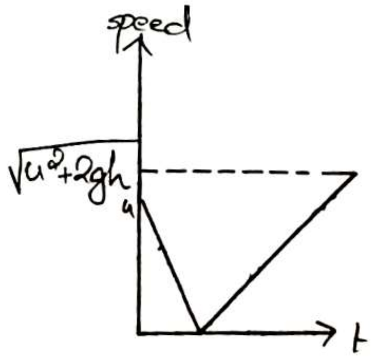
ACCELERATION-TIME GRAPH



VELOCITY-TIME GRAPH

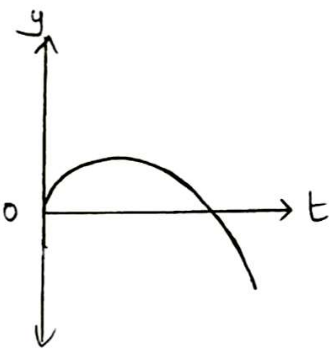
SPEED-TIME GRAPH

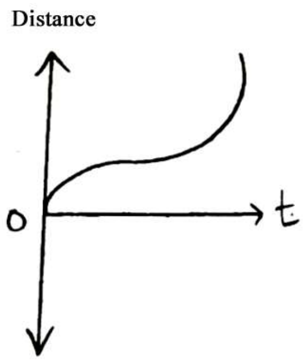




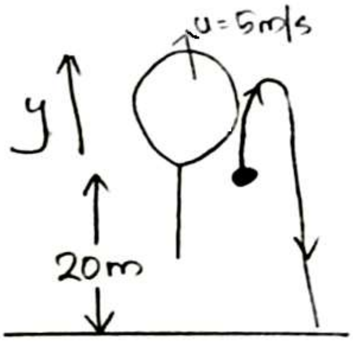
SPEED INITIALLY DECREASES, reduces to 0 at max. height [0 min] and then increases. But velocity continuously decreases and it is a uniformly decelerated motion

POSITION-TIME GRAPH **DISTANCE-TIME GRAPH**





Q. Eg. A balloon is ascending with a constant speed 5 m/s. A stone is dropped from it when it is at a height 20m from the ground. Find out the height of the balloon from the ground when the stone hits the ground.



Initial velocity of an object when it is released from a moving system = velocity of the moving system. Thus, initial velocity of the stone = velocity of the balloon which is vertically upward from a height

Time taken to reach the ground,

$$t = \frac{u + \sqrt{u^2 + 2gh}}{g}$$

$$= \frac{5\sqrt{5^2 + 2 \times 10 \times 20}}{10}$$

$$t = 2.5s$$

Distance travelled by the balloon

$$y = ut$$
$$= 5 \times 2.5$$
$$= 12.5m$$

Height from ground = 20 + 12.5 = 32.5m

OBJECT IS THROWN VERTICALLY UPWARD FROM GROUND

Initial velocity is upward. So it is +ve. Acceleration due to gravity is downward. So throughout motion, acceleration

a = –g

$$y = ut - gt^2$$
$$v^2 = u^2 - 2gy$$
$$y_n = u - \frac{g}{2}[2n - 1]$$

Maximum height

At max height, velocity is zero and acceleration, $a = -g \neq 0$. Thus object reverses its direction of motion.

$$v^2 = u^2 - 2gy$$
$$\text{at } y = h, v = 0$$
$$0 - u^2 - 2gh$$
$$\Rightarrow 2gh = u^2$$
$$\Rightarrow \boxed{h = \frac{u^2}{2g}} ...(1)$$

Time of ascent

$$t_{up} = \frac{v-u}{-g} = \frac{0-u}{-g} = \frac{u}{g}$$

From (1),

$$u = \sqrt{2gh}$$

$$t_{up} = \frac{\sqrt{2gh}}{g} = \sqrt{\frac{2gh}{g^2}}$$

$$\boxed{t_{up} = \sqrt{\frac{2h}{g}}}$$

Time of flight (T)

at $t = T, y = 0$

$$y = ut - \frac{gt^2}{2}$$

$$0 = uT - \frac{gT^2}{2}$$

$$\Rightarrow \frac{gT}{2} = ut$$

$$\Rightarrow \boxed{T = \frac{2u}{g} = 2\sqrt{\frac{2h}{g}}}$$

$$t_{descend} = T_{ascend}$$

$$= \frac{2u}{g} - \frac{u}{g}$$

$$t_{descend} = \frac{u}{g} = \sqrt{\frac{2h}{g}}$$

$$t_{ascend} = t_{descend}$$

Speed with which the object hits ground

$v^2 = u^2 - 2gy$

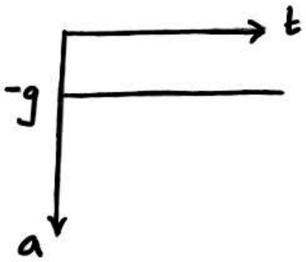
At ground, $y = 0$

$v^2 = u^2$

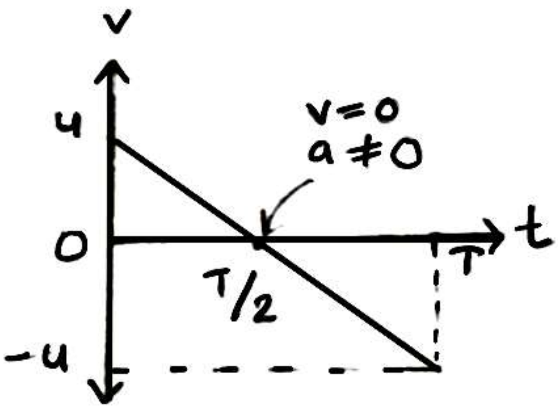
$\boxed{v = u}$ speed

$\boxed{v = -u}$ Velocity

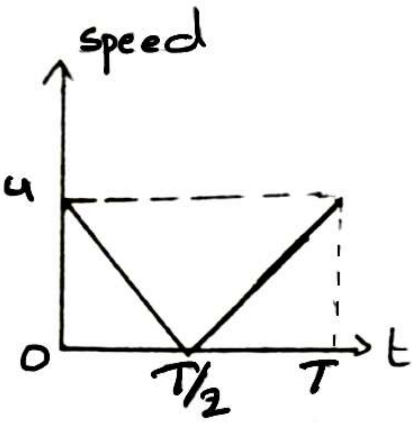
ACCELERATION-TIME GRAPH



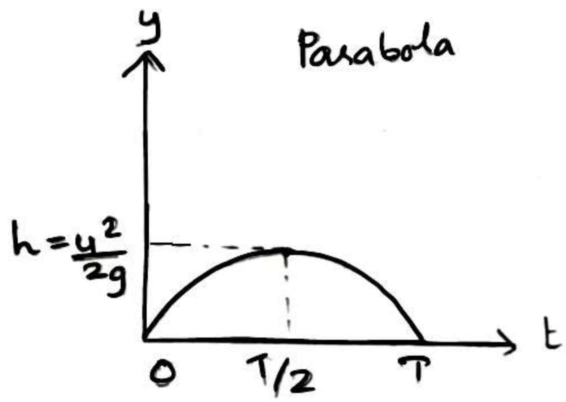
VELOCITY-TIME GRAPH



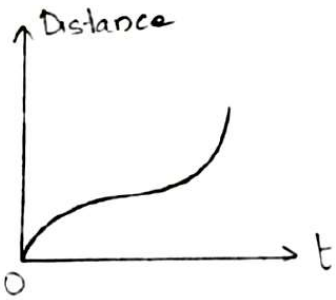
SPEED-TIME GRAPH



DISPLACEMENT-TIME GRAPH



DISTANCE-TIME GRAPH



Speed initially decreases, minimum 0 at max. height and then increases. But velocity continuously decreases and it is uniformly decelerated motion.

$$K.E = \frac{mv^2}{2}$$

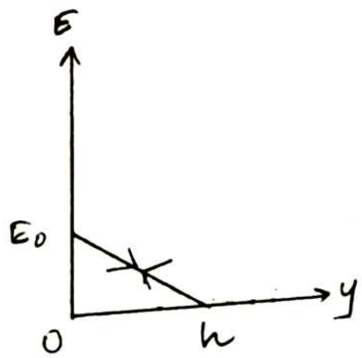
$$K.E. \frac{m}{2}\left[u^2 - 2gy\right]$$

$$E = \frac{mu^2}{2} - mgy$$

$$E = E_0 - mgy$$

$$E_0 = \frac{mv^2}{2} = \text{initial K.E}$$

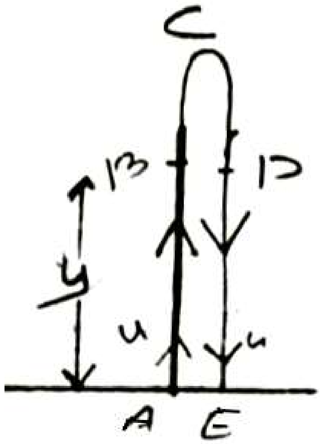
E – y graph is a straight line



If t_1 and t_2 are the times at which the object passes through a height y during ascend and descend, then

$$y = ut - \frac{1}{2}gt^2$$

$$\boxed{\frac{gt^2}{2} - ut + y = 0}$$



$$t_1 = t_{AB}$$

$$t_2 = t_{AD}\left(\text{ABCD path}\right)$$

Solution of this equation gives t_1 and t_2

$$t = \frac{u \pm \sqrt{u^2 - 4^2 \times \frac{g}{2} \times y}}{2 \times \frac{gt^2}{2}}$$

$$= \ t = \frac{u \pm \sqrt{u^2 - 2gy}}{g}$$

$$\boxed{t_1 = \frac{u - \sqrt{u^2 - 2gy}}{g}, \ t_2 = \frac{u + \sqrt{u^2 - 2gy}}{g}}$$

$$t_1 + t_2 = \frac{2u}{g} = T$$

$$u = \frac{g\left(t_1 + t_2\right)}{2}$$

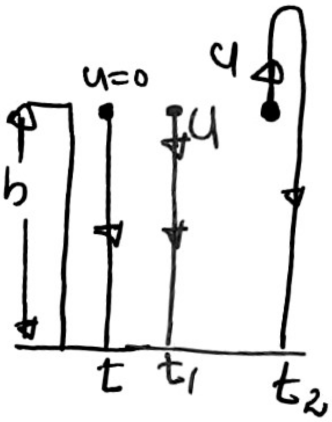
$$\boxed{\begin{array}{l} t_1 t_2 = \dfrac{2y}{g} \\ y = g\dfrac{t_1 t_2}{2} \end{array}}$$

If v is the velocity at a height y, then $t_1 - t_1 = \frac{2v}{g}$ or $v = \frac{\left(t_2 - t_1\right)}{2}$

Problem :

A stone dropped from a building of height h and it reaches after t seconds on earth. From the same building if two stones are thrown (one upwards and other downwards) with the same velocity u and they reach the earth surface after t_1 and t_2 seconds respectively, then

(a) $t = t_1 - t_2$ (b) $t = \frac{t_1 + t_2}{2}$ (c) $t = \sqrt{t_1 t_2}$ (d) $t = t_1^2\, t_2^2$



For dropped object $h = \frac{gt^2}{2}$ ....(1)

for object thrown vertically downward $-h = -ut_1 - \frac{gt_1^2}{2}$

$h = ut_1 + \frac{gt_1^2}{2}$ ....(2)

for the object thrown vertically upward

$-h = ut_2 - \frac{gt_2^2}{2}$ $h = -ut_2 - \frac{gt_2^2}{2}$ ......(3)

$t_2 \times (1) \Rightarrow ht_2 = ut_1 t_2 + \frac{gt_1^2 t_2}{2}$ ....(4)

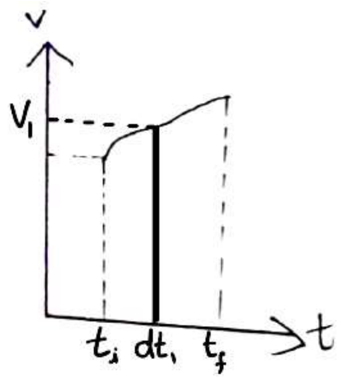
$t_1 \times (3) \Rightarrow ht_1 = -ut_1 t_2 + \frac{gt_2^2 t_1}{2}$ ....(5)

(4) + (5) $\Rightarrow h[t_1 + t_2] = 0 + \frac{gt_1 t_2}{2}[t_1 + t_2]$

$$h = \frac{gt_1t_2}{2}$$

From (1) $\frac{gt^2}{2} = \frac{gt_1t_2}{2}$ $\boxed{t = \sqrt{t_1t_2}}$

Relation between Velocity and Acceleration



If v_1 is the velocity in a small time interval dt_1, then displacement in this small time interval is

$$dx_1 = v_1 dt_1$$

If v_2 is the velocity in a small time interval dt_2 then displacement $dx_2 = v_2dt_2$

Similarly, divide the given time interval into small time intervals dt_1, dt_2, dt_3, ... and dx_1, dx_2, dx_3.... are the displacements in this time intervals, then the total displacement from time t_1 to t_f is

$$\Delta x = dx_1 + dx_2 + ...$$

$$x_f = x_i = v_1dt_1 + v_2dt_2 + v_3dt_3 + ...$$

$$x_f - x_i = \sum_{t_i}^{t_f} v_i dt_i$$

Velocity variation is continuous, therefore, summation can be replaced by integration

$$\boxed{x_f - x_i = \int_{t_i}^{t_f} vdt}$$

If v = constantly

$$x_f - x_i = v\int_{t_i}^{d_f} dt = v\left[t\right]_{t_i}^{t_f}$$

$\boxed{x_f - x_i = v\left[t_f - t_i\right]}$ Displacement = v $\times$ time

Relation between velocity and acceleration

$a = \frac{dv}{dt}$ $\therefore$, Change in velocity

$$\boxed{v - u = \int_{t_i}^{t_f} a\,dt}$$

EQUATIONS OF MOTION OF UNIFORMLY ACCELERATED MOTION USING METHOD OF CALCULUS

1) Velocity at any instant

We have instantaneous acceleration

$$a = \frac{dv}{dt}$$

$\Rightarrow$ a dt = dv

Integrating on both sides

$$\int_{t=0}^{t} a dt = \int_{u}^{v} dv$$

a = constant

$$a\int_{0}^{t} dt = \int_{u}^{v} dv$$

$$a[t]_0^t = [v]_u^v$$

$$a \times t = v - u$$

$$\Rightarrow v = u + at$$

2) Displacement at an instant

We have

$$x_f - x_i = \int_0^t v dt$$

$$x_f - x_i = \int_0^t (u + at) dt$$

$$= \int_0^t u dt + \int_0^t at\,dt$$

$$= u\int_0^t dt + a\int_0^t tdt$$

$$= u[t]_0^t + a\left[\frac{t^2}{2}\right]_0^t$$

$$= ut + a\left(\frac{t^2}{2}\right)$$

$$x_f - x_i = ut + \frac{at^2}{2}$$

Displacement $\boxed{S = ut + \frac{at^2}{2}}$ where

s is the total displacement in a time t starting from velocity u

3) Velocity at any position

We have instantaneous acceleration,

$$a = \frac{dv}{dt}$$

$$a = \frac{dv}{d}\frac{dx}{dt} \quad \text{chain rule}$$

$$\frac{dx}{dt} = v$$

$$\boxed{a = \frac{vdv}{dx}}$$

adx = vdv

Integrating on both sides

$$\int_{x_i}^{x_f} adx = \int_u^v vdv$$

but a = constant

$$a \times \int_{x_i}^{x_f} dx = \int_u^v vdv$$

$$\Rightarrow [x]_{x_i}^{x_f} = \left[\frac{v}{2}\right]_u^v$$

$$\Rightarrow a\left[x_f - x_i\right] = \frac{v^2}{2} - \frac{u^2}{2}$$

$$\Rightarrow 2a\left[x_f - x_i\right] = v^2 - u^2$$

$$\boxed{\begin{array}{l} \Rightarrow v^2 = u^2 + 2a\left[x_f - x_i\right] \\ \Rightarrow v^2 = u^2 + 2as \end{array}}$$

$$v^2 - u^2 = 2as$$
$$(v+u)(v-u) = 2as$$

but v-u = at

$$\Rightarrow (v+u)at = 2as$$

$$\Rightarrow \boxed{s = \left(\frac{v+u}{2}\right)t}$$

but s = $V_{av}t$

$$v_{av} = \frac{s}{t}$$

$$\boxed{v_{av} = \frac{u+v}{2}}$$

RELATIVE MOTION

Motion is a combined property of the object under study as well as the observer. It is always relative ; there is no such thing like absolute motion or absolute rest. Motion is always defined with respect to an observer or reference frame.

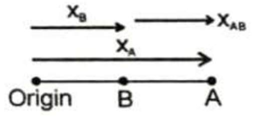
RELATIVE MOTION IN ONE DIMENSION :

Relative Position :

It is the position of a particle w.r.t. observer.

In general if position of A w.r.t. to origin is x_A and that of B w.r.t. origin is x_B then "Position of A w.r.t. B" x_{AB} is

$$\boxed{x_{AB} = x_A - x_B}$$



RELATIVE VELOCITY

Consider the motion of the car moving towards right and two observes Q_1 and Q_2 are coming from opposite directions as shown in figure

Observer O_1 finds that car is moving slower while observer O_2 finds that car is moving faster in comparison to when observer is at rest. The motion of same object looks different for two different observers. To understand such observations, there is a need of the concept of relative velocity.

Definition : Relative velocity of a particle A with respect to B is defined as the velocity with which A appears to move if B is considered to be at rest. In other words, it is the velocity with which A appears to move as seen by B considering itself to be at rest

Consider two objects A and B moving uniformly with average velocities V_A and V_B in one dimension, say along x-axis. Here V_A and V_B are with respect to ground.

If $x_A(0)$ and $x_B(0)$ are positions of objects A and B, respectively at time t = 0, their positions $x_A(t)$ and $x_B(t)$ at time t are given by

$$x_A(t) = x_A(0) + v_A t$$
$$x_B(t) = x_B(0) + v_B t$$

Then, the displacement of B w.r.t to A is

$$x_{BA}(t) = x_B(t) - x_A t$$

$$= x_B(0) = v_B t - [x_A(0) + v_A t]$$

$$x_{BA}(t) = x_B(0) - x_A(0) + (v_B - v_A)t$$

$x_B(0) - x_A(0)$ is the initial separation between A and B. Thus object B has a velocity $(V_B - V_A)$ because the displacement from A to B changes steadily by the amount $(V_B - V_A)$ in each unit of time.

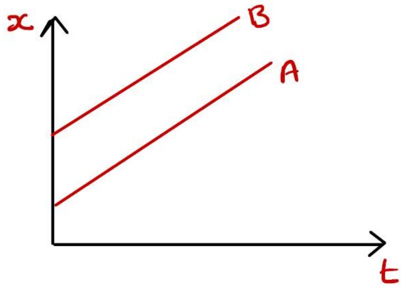
Thus velocity of object B relative to object A is : $\boxed{v_{AB} = v_B - v_A}$

Similarly, velocity of object A relative to object B is : $\boxed{v_{AB} = v_A - v_B}$

$$v_{AB} = -(v_B - v_A) \quad \therefore \boxed{v_{AB} = -v_{BA}}$$

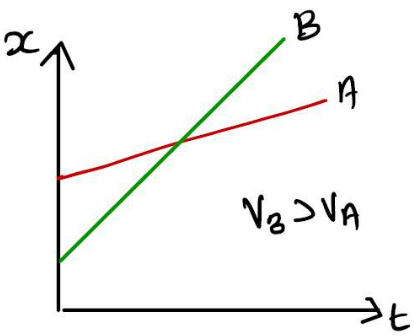
* If $v_B = v_A$, then $v_{BA} = v_B - v_A = 0$. Therefore, the two objects stay at a constant separation $x_B(0) - x_A(0)$ and their position time graph are straight lines parallel to each other.

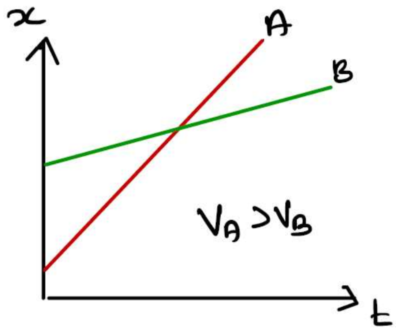
Position-time graph of two objects with equal velocities



* If $v_B > v_A$, then $v_{BA} = v_B - v_A$ is positive and v_{BA} is in the direction of v_B and v_A. One graph is steeper than the other and they meet at a common point.

* If $v_A > v_B$, then $v_{BA} = v_B - v_A$ is negative and v_{BA} is opposite to the direction of v_A and v_B.





* If two bodies *A* and *B* are moving in straight line same direction with velocity V_A and V_B, then relative velocity of *A* with respect to *B* is $v_{AB} = v_A - v_B$. Similarly $v_{BA} = v_B - v_A$

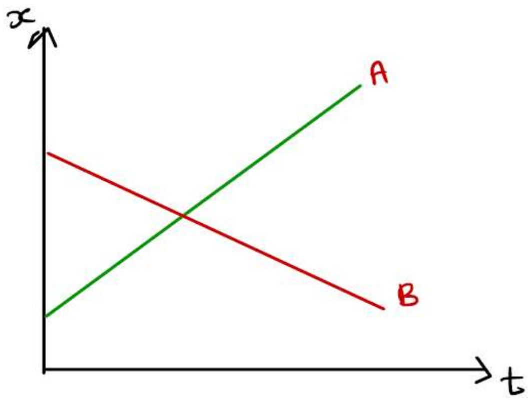
* If two bodies *A* and *B* are moving in straight line in opposite direction then

$v_{AB} = v_A + v_B$ (towards *B*)

$v_{BA} = v_B + v_A$ (towards *A*)

$v_{AB} = -v_{BA}$

Same concept is used for acceleration also.



Position-time graphs of two objects with unequal velocities in opposite directions

Note : All velocities are relative and have no significance unless observer is specified. However, when we say ‘velocity of A’, what we means is, velocity of A w.r.t ground which assumed to be at rest.

Note : Velocity of an object w.r.t itself is always zero.

Relative Acceleration

Acceleration of A w.r.t B

$$a_{AB} = \frac{dv_{AB}}{dt}$$

When accelerations a_A and a_B are in the same direction,

$$a_{AB} = a_A - a_B$$

and $$a_{BA} = a_B - a_A$$

$$a_{BA} = -(a_A - a_B)$$

$$\therefore \boxed{a_{BA} = -a_{AB}}$$

When a_A and a_B are opposite in directions

$$a_{AB} = a_A + a_B$$

and $$a_{BA} = a_B + a_A$$

Note : By using relative velocity and relative acceleration two body motion problems can be reduced to one body motion problem. i.e. If velocity of A e.r.t B is calculated, then B is considered to be at rest and A is considered to be moving with relative velocity v_{AB}.

Equations of motion when relative acceleration is constant

u_{rel} = initial relative velocity

a_{rel} = relative acceleration

v_{rel} = relative velocity at a time t

s_{rel} = relative displacement

$$\boxed{\begin{aligned} v_{rel} &= u_{rel} + a_{rel}t \\ s_{rel} &= u_{rel}t + \frac{a_{rel}t^2}{2} \\ v_{rel}^2 &= u_{rel}^2 + 2a_{rel}s_{rel} \end{aligned}}$$

When velocities of the objects are constant values

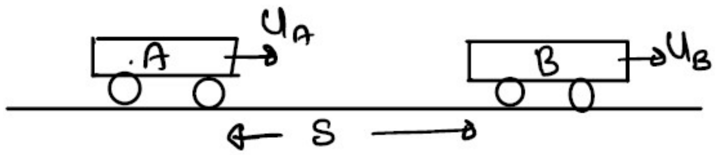
$$\boxed{s_{rel} = v_{rel}t}$$

Velocity Approach / Separation

If the separation is decreasing, we say it is velocity of approach and if separation is increasing, then we say it is velocity of separation.

In one dimension, since relative velocity is along the line joining A and B, hence velocity of approach / separation is simply equal to magnitude of relative velocity of A w.r.t. B.

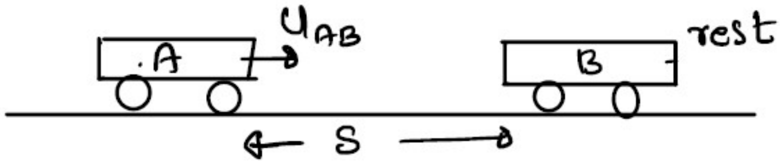
* Two cars A and B are moving in the same direction with velocities u_A and u_B $(u_A > u_B)$. Initial separation between A and B is s. Find out the deceleration of A to avoid collision.



Initial velocity of A w.r.t B

$u_{AB} = u_A - u_B$

Relative acceleration $a_{AB} = a_A - a_B = -a_A - 0$ [–ve sign shows that a_{AB} is opposite to u_A)



When B is considered to be at rest, the A is moving with a_{AB} and u_{AB}. Thus to avoid collision velocity of A w.r.t B reduces to zero before covering the initial separation s distance travelled, when relative velocity reduces to zero.

$$d = \frac{v_{AB}^2 - u_{AB}^2}{2a_{AB}} = \frac{0-(u_A - u_B)^2}{-2a_A} = \frac{(u_A - u_B)^2}{2a_A}$$

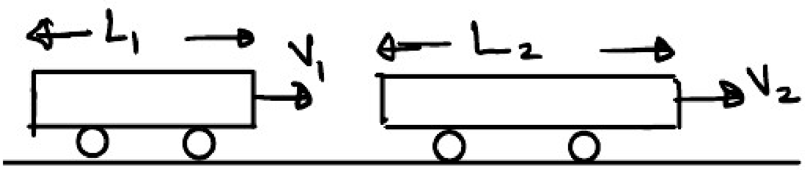
To avoid collision $d \le s$

$$\therefore \frac{(u_A - u_B)^2}{2a_A} \le s \qquad \boxed{a_A \ge \frac{(u_A - u_B)^2}{2s}}$$

Thus minimum acceleration required

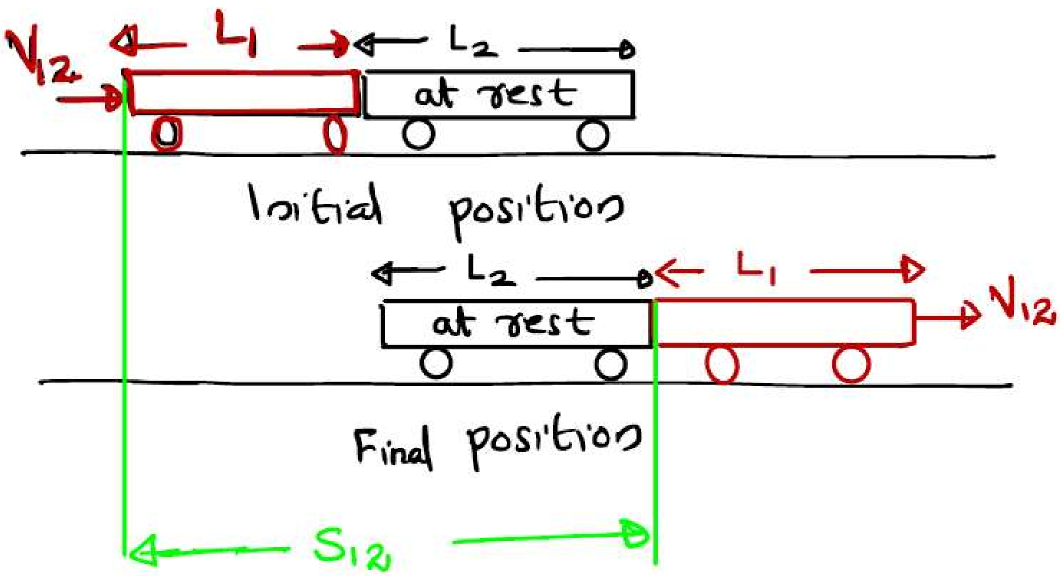
$$\boxed{(a_A)_{min} = \frac{(u_A - u_B)^2}{2s}}$$

Example : Two trains A and B of lengths L_1 and L_2 are travelling in the same direction with velocities v_1 and v_2. Find out the time taken by one of the trains to overtake the other train

Answer : 

Relative velocity $v_{12} = v_1 - v_2$

Relative displacement $s_{12} = L_1 + L_2$



Time taken to just overtake

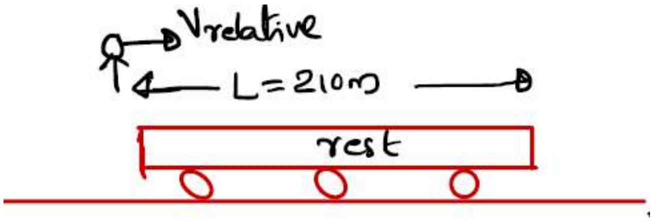
$t=\frac{s_{12}}{v_{12}}$ $\boxed{t=\frac{L_1+L_2}{v_1-v_2}}$

Example :A 120m long train is moving in a direction with speed 20 m/s. A train in a direction with speed 20 m/s. A train B moving with 30 m/s in the opposite direction and 130m long crosses the first train in a time

Answer : $t=\frac{L_1+L_2}{v_1+v_2}=\frac{120+130}{20+30}=\frac{250}{50}=5s$

Example : A 210m long train is moving due north at a speed of 25 m/s. A small bird is flying due south a little above the train with speed 5 m/s. The time taken by the bird to cross the train is

Answer :



Velocity of train and bird are in the opposite direction. $\therefore$ Relative velocity v_{rel} = 25 + 5 = 30 m/s

It train is considered to be at rest, then bird is flying with relative velocity

$$\therefore \text{time } t=\frac{\text{relative displacement}}{\text{relative velocity}}$$

$$t=\frac{210}{30}=7s$$

Example : Two cars, initially at a separation of 12m, start simultaneously. First car A, starting from rest, moves with an acceleration 2 m/s^2, whereas the car B, which is ahead moves with a constant velocity 1 m/s along the same direction. Find the time when car A overtakes car B.

Answer : Initial relative velocity, $v_{AB}=v_A-v_B=0-1$

$$v_{AB}=-1m/s$$

Relative acceleration $a_{AB}=a_A-a_B=2-0=2m/s$

$$s_{AB}=v_{AB}t+\frac{a_{AB}t^2}{2}$$

$$12=-1\times t+\frac{2}{2}t^2,\ t^2-t-12=0$$

$$(t+3)(t-4)=0 \quad \therefore t=4s$$