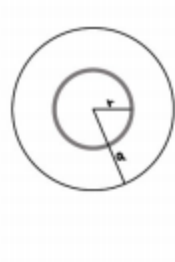


CHAPTER - 06

RIGID BODY DYNAMICS

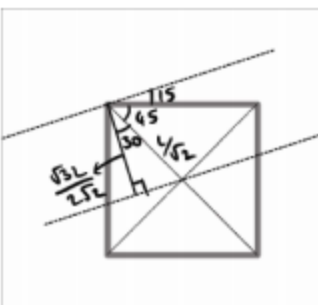
LEVEL - I

1. A



$$\begin{aligned}
 & \int_0^a [r \cdot 2\pi r dr] \cdot r^2 \\
 &= \int_0^a [A + B r] 2\pi r dr \cdot r^2 \\
 &= 2\pi A \frac{a^4}{4} + 2\pi B \frac{a^5}{5} \\
 &= 2\pi a^4 \left[\frac{A}{4} + \frac{B a}{5} \right] \rightarrow \boxed{A} \checkmark
 \end{aligned}$$

2. B



$$\begin{aligned}
 & \frac{m L^2}{12} + m \left(\frac{\sqrt{3} L}{2\sqrt{2}} \right)^2 \\
 &= m L^2 \left[\frac{1}{12} + \frac{3}{8} \right] \\
 &= m L^2 \frac{2+9}{24} = \frac{11}{24} m L^2 \rightarrow \boxed{B} \checkmark
 \end{aligned}$$

3. B

$$\begin{aligned}
 & \left\{ \frac{m R^2}{2} + 6 \left[\frac{m R^2}{2} + m \cdot (2R)^2 \right] \right\} + 7m \cdot (3R)^2 \\
 &= m R^2 \left[\frac{1}{2} + 6 \cdot \frac{9}{2} + 63 \right] \\
 &= \frac{181}{2} m R^2 \rightarrow \boxed{B} \checkmark
 \end{aligned}$$

4. A

$$\left. \begin{aligned} \tau_p &= 5 \times 20 = 100 \\ \tau_o &= 0 \end{aligned} \right\} \rightarrow \boxed{A} \checkmark$$

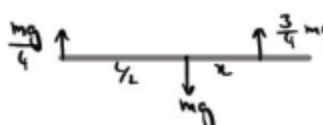
5. C

$$\frac{l}{2} \cdot mg + l \cdot mg = \left[\frac{ml^2}{3} + ml^2 \right] \kappa$$

$$\frac{3}{2} g = l \kappa \cdot \frac{4}{3}$$

$$\kappa = \frac{9}{8} \frac{g}{l} \rightarrow \boxed{C} \checkmark$$

6. A

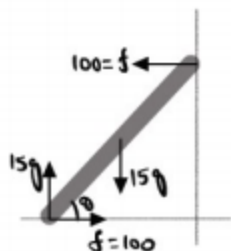


$$\frac{l}{2} \cdot \frac{mg}{4} = x \cdot \frac{3}{4} mg$$

$$\therefore x = \frac{l}{6}$$

$$\therefore \frac{l}{2} + \frac{l}{6} = \frac{4l}{6} = \frac{2l}{3} \rightarrow \boxed{A} \checkmark$$

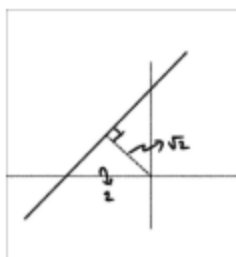
7. C



$$\frac{1}{2} \cdot 15g = l \cdot f$$

$$\therefore f = \frac{15g}{2} \cdot \frac{4}{3} = 100$$

8. B



$$\sqrt{2} \cdot \left(\frac{1}{2} \right) = 2\sqrt{2} \rightarrow \boxed{B} \checkmark$$

9. C

$$Mr^2 \cdot \omega = [Mr^2 + 2mr^2] \Omega$$

$$\therefore \Omega = \frac{M\omega}{M+2m} \rightarrow \boxed{C} \checkmark$$

10. A

$$\frac{R}{2} \cdot \frac{M}{2} v = \left[\frac{MR^2}{2} + \frac{M}{2} R^2 \right] \omega$$

$$\frac{R}{4} Mv = MR^2 \omega$$

$$\therefore \omega = \frac{v}{4R} \rightarrow \boxed{A} \checkmark$$

11. C

$$\frac{\sqrt{3}}{2} a \cdot F = \frac{a}{2} \cdot mg$$

$$\therefore F = \frac{mg}{\sqrt{3}}$$

$$\therefore \mu N \geq \frac{mg}{\sqrt{3}}$$

$$\therefore \mu \geq \frac{1}{\sqrt{3}} \rightarrow \boxed{C} \checkmark$$

12. A

$$mg \frac{l}{4} = \frac{1}{2} \left[\frac{ml^2}{12} + m \frac{l^2}{16} \right] \omega^2$$

$$mg \frac{l}{4} = \frac{ml^2}{2} \cdot \frac{7}{48} \omega^2$$

$$\omega^2 l = \frac{24}{7} g \rightarrow \boxed{A} \checkmark$$

13. A

$$mg h = \frac{1}{2} \left(\frac{m R^2}{2} + m R^2 \right) \omega^2$$

$$mg h = \frac{1}{2} \cdot \frac{3}{2} m v^2$$

$$mg \cdot v = \frac{3}{4} m \cdot 2 v a$$

$$\therefore a = \frac{2g}{3} \rightarrow \boxed{A} \checkmark$$

14. B

$$mg \frac{l}{2} = \frac{1}{2} \frac{m l^2}{3} \omega^2$$

$$\omega = \sqrt{\frac{3g}{l}}$$

$$l\omega = \sqrt{3gl} \rightarrow \boxed{B} \checkmark$$

15. A

$$\frac{\frac{1}{2} m v^2}{\frac{1}{2} \left[1 + \frac{2}{5} \right] m v^2} = \frac{5}{7} \rightarrow \boxed{A} \checkmark$$

16. D

$$\frac{1}{2} (1+\beta) mv^2 = mg \cdot \frac{3v^2}{4g}$$

$$1+\beta = \frac{3}{2}$$

$$\beta = \frac{1}{2} \rightarrow \boxed{D} \checkmark$$

17. A

$$a = \frac{g\beta}{1+\beta} = \frac{g\beta}{2}$$

$$\frac{h}{\beta} = 0 + \frac{1}{2} \cdot \frac{g\beta}{2} \cdot t^2 \Rightarrow t = \frac{1}{\beta} \sqrt{\frac{4h}{g}} \rightarrow \boxed{A} \checkmark$$

18. 12

$$\left. \begin{aligned} \omega_0^2 - \left(\frac{\omega_0}{2}\right)^2 &= 2\kappa \cdot [36 \cdot 2\pi] \\ \left(\frac{\omega_0}{2}\right)^2 - 0^2 &= 2\kappa [n \cdot 2\pi] \end{aligned} \right\} \Rightarrow n = 12 \rightarrow \boxed{12} \checkmark$$

19. 20

$$10 + \frac{1}{2}\omega = 20 \Rightarrow \omega = 20 \rightarrow \boxed{20} \checkmark$$

20. 16

$$R \cdot 40 = MR^2 \kappa$$

$$\kappa = \frac{40}{MR} = \frac{40}{5 \times \frac{1}{2}} = 16 \rightarrow \boxed{16} \checkmark$$

LEVEL - II

21. D

$$mg \frac{l}{2} (1 - \phi) = \frac{1}{2} \cdot \frac{ml^2}{3} \cdot \omega^2 \Rightarrow \omega^2 l = 3g(1 - \phi)$$

$$\frac{l}{2} \cdot mg \phi = \frac{ml^2}{3} \kappa \Rightarrow \kappa = \frac{3}{2} g \phi$$

$$\begin{aligned} f = m \left[\frac{l}{2} \kappa \phi - \omega^2 \frac{l}{2} \phi \right] &= \frac{3}{4} mg \phi - \frac{3mg}{2} (1 - \phi) \phi \\ &= \frac{3mg}{2} \left[\frac{\phi}{2} - \phi + \phi^2 \right] \\ &= \frac{3mg}{2} \left[\frac{3}{2} \phi - \phi \right] \rightarrow \text{D} \checkmark \end{aligned}$$

22. B

$$R \cdot M v_0 + \frac{2}{5} M R^2 \cdot \frac{v_0}{2R} = \left(1 + \frac{2}{5} \right) R M v$$

$$\frac{6}{5} v_0 = \frac{7}{5} v$$

$$\therefore v = \frac{6}{7} v_0 \rightarrow \text{B} \checkmark$$

23. A

$$2J = M u_y$$

$$\therefore u_y = \frac{2J}{M}$$

$$\therefore T = \frac{2u_y}{g} = \frac{4J}{Mg}$$

$$2[L \cdot j] = \frac{M \cdot 4L^2}{12} \omega$$

$$\therefore \omega = \frac{6j}{ML}$$

$$\therefore \frac{6j}{ML} \cdot \frac{4J}{Mg} = \pi$$

$$jJ = M^2 \frac{\pi g L}{24}$$

$$\left[\sqrt{J^2 + j^2} \right]_{\min} = \sqrt{2} \cdot M \sqrt{\frac{\pi g L}{24}} = M \sqrt{\frac{\pi g L}{12}} = \frac{M}{2} \sqrt{\frac{\pi g L}{3}} \rightarrow \text{A} \checkmark$$

24. C

$$N = mg = 50g = 500 \Rightarrow f_{\max} = (0.3)500 = 150$$

$$500 - 150 = 50a \Rightarrow a = 7$$

$$1 \cdot 500 - 3 \cdot 150 = \frac{50 \cdot 3^2}{12} \kappa \Rightarrow \kappa = \frac{1}{3} \Rightarrow 3\kappa = 1$$

$$\therefore a - 3\kappa = 7 - 1 = 6 \rightarrow \boxed{C} \checkmark$$

25. B

$$\left. \begin{array}{l} N = Mg \phi \\ f = Mg \phi \end{array} \right\} \begin{array}{l} \mu N \geq Mg \phi \\ \therefore \mu \geq \tan \theta \end{array}$$

$$\therefore \mu \geq \frac{\kappa}{\sqrt{1-\kappa^2}} \rightarrow \boxed{B} \checkmark$$

26. A,B,D

$$R \cdot M v_0 - \frac{2}{5} M R^2 \cdot \frac{v_0}{2R} = \left(1 + \frac{2}{5}\right) R M v$$

$$\frac{4}{5} v_0 = \frac{7}{5} v$$

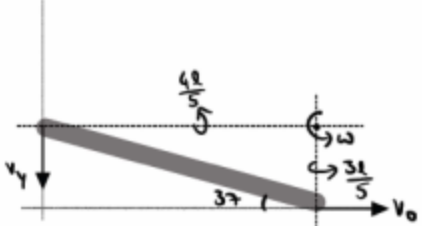
$$\therefore v = \frac{4}{7} v_0 \rightarrow \boxed{B} \checkmark$$

$\boxed{C} \times$

$$v_0 - \mu g t = \frac{4}{7} v_0 \Rightarrow t = \frac{3}{7} \frac{v_0}{\mu g} \rightarrow \boxed{A} \checkmark$$

$$\left. \begin{aligned} K_1 &= \frac{1}{2} M v_0^2 + \frac{1}{2} \cdot \frac{2}{5} M R^2 \left(\frac{v_0}{2R} \right)^2 = \frac{11}{20} M v_0^2 \\ K_2 &= \frac{1}{2} \left[1 + \frac{2}{5} \right] M \left(\frac{4}{7} v_0 \right)^2 = \frac{8}{35} M v_0^2 \end{aligned} \right\} K_1 - K_2 = \frac{9}{28} M v_0^2 \rightarrow \boxed{D} \checkmark$$

27. A, B, C, D



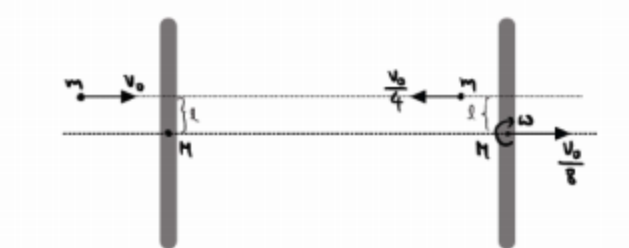
$$\omega = \frac{v_0}{\frac{3l}{5}} = \frac{v_y}{\frac{4l}{5}}$$

\downarrow \boxed{B} \downarrow \boxed{A} \checkmark

$$v_c = \frac{r}{2} \omega = \frac{5}{6} v_0 \rightarrow \boxed{C} \checkmark$$

$$\frac{1}{2} \cdot \frac{m l^2}{3} \cdot \omega^2 = \frac{1}{2} \cdot \frac{m}{3} \cdot \frac{25}{9} v_0^2 = \frac{25}{54} m v_0^2 \rightarrow \boxed{D} \checkmark$$

28. A, B, C, D



$$m v_0 = M \frac{v_0}{8} - m \frac{v_0}{4} \Rightarrow \frac{5m}{4} = \frac{M}{8} \Rightarrow \frac{M}{m} = 10 \rightarrow \boxed{A} \checkmark$$

$$0 = l M \frac{v_0}{8} - \frac{M l^2}{12} \omega \Rightarrow \omega = \frac{3}{128} \frac{v_0}{l} \rightarrow \boxed{B} \checkmark$$

$$e = \frac{\frac{v_0}{8} + l \omega + \frac{v_0}{4}}{v_0} = \frac{\frac{3v_0}{8} + \frac{3}{128} v_0}{v_0} = \frac{51}{128} \rightarrow \boxed{C} \checkmark$$

$$\left. \begin{aligned} \frac{v_0}{8} + 4l\omega &= \frac{v_0}{8} + \frac{3}{32} v_0 = \frac{7}{32} v_0 \\ \frac{v_0}{8} - 4l\omega &= \frac{v_0}{8} - \frac{3}{32} v_0 = \frac{1}{32} v_0 \end{aligned} \right\} \rightarrow \boxed{D} \checkmark$$

29. A,B,C,D

Diagram showing a vertical rod of length L and mass M pivoted at the bottom. A horizontal force F is applied at the top. A sphere of radius $R = \frac{L}{3}$ and mass M is in contact with the rod at a height of $\frac{L}{3}$ from the pivot. The sphere is on a horizontal surface. Forces on the rod: Mg at center, N at pivot, F at top. Forces on the sphere: Mg at center, N at contact point, normal force at bottom, friction force at bottom.

Equations for the rod:

$$LF - \frac{L}{3}N = \frac{ML^2}{3}\alpha$$

$$N = MA$$

$$\frac{L}{3}\alpha = A$$

Equations for the sphere:

$$F + X - N = M\left(\frac{L}{2}\alpha\right)$$

$$F + X - \frac{3F}{4} = M\frac{9F}{8M}$$

$$\therefore X = \frac{7F}{8} \rightarrow \text{D} \checkmark$$

For the rod, solving for N and A :

$$\therefore \frac{F}{M} - \frac{N}{3M} = \frac{N}{M}$$

$$\Rightarrow N = \frac{3F}{4} \rightarrow \text{B} \checkmark$$

$$\Rightarrow A = \frac{3F}{4M} \rightarrow \text{A} \checkmark$$

$$\Rightarrow L\alpha = \frac{9F}{4M} \rightarrow \text{C} \checkmark$$

30. A,B,C,D

$$\frac{55}{2} \cdot 2x = \frac{1}{2} \cdot 1 \cdot [2v]^2 + \frac{1}{2} \left[1 + \frac{1}{4}\right] 1 \cdot v^2$$

$$55x = 2v^2 + \frac{3}{4}v^2$$

$$55x = \frac{11}{4} \cdot 2 \cdot a$$

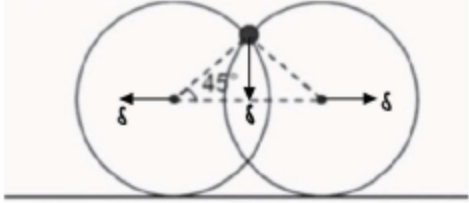
$$\therefore a = 10 \rightarrow \text{A} \checkmark$$

$$\text{D} \checkmark$$

$$\frac{55}{2} - f_1 = 1 \cdot 20 \Rightarrow f_1 = 7.5 \rightarrow \text{B} \checkmark$$

$$7.5 - f_2 = 1 \cdot 10 \Rightarrow f_2 = -2.5 \rightarrow \text{C} \times$$

31. A,B



$$mg\delta = \frac{1}{2}mv^2 + \frac{1}{2}(1+1)mv^2 + \frac{1}{2}(1+1)mv^2$$

$$mg\delta = \frac{5}{2}mv^2$$

$$mgv = \frac{5}{2}m \cdot 2va$$

$$\therefore a = \frac{g}{5} \rightarrow \boxed{B} \checkmark$$

$$\boxed{A} \times$$

$$mg - \left[\frac{N}{\sqrt{2}} + \frac{N}{\sqrt{2}} \right] = m a_{\frac{g}{5}}$$

$$\therefore N = \frac{2\sqrt{2}}{5}mg \rightarrow \boxed{B} \times$$

$$\boxed{D} \times$$

32. A,B,C,D

$$V_C + RW = 2V$$

$$RW - V_C = V$$

$$\therefore RW = \frac{3V}{2} \rightarrow \boxed{B} \checkmark$$

$$\therefore V_C = \frac{V}{2} \rightarrow \boxed{C} \checkmark$$

$$\pi\omega = \frac{V}{2} \Rightarrow \pi \cdot \frac{3V}{2R} = \frac{V}{2} \Rightarrow \pi = \frac{R}{3} \Rightarrow R - \frac{R}{3} = \frac{2R}{3} \rightarrow \boxed{A} \checkmark$$

$$\frac{1}{2}M\left(\frac{V}{2}\right)^2 + \frac{1}{2} \cdot \frac{MR^2}{2} \cdot \left(\frac{3V}{2R}\right)^2 = \frac{11}{6}MV^2 \rightarrow \boxed{D} \checkmark$$

33. 3

$$mR^2 \cdot 5 = 2mR^2 \cdot \omega$$

$$\therefore \omega = \frac{5}{2}$$

$$\frac{5}{2} = 5 - \frac{5}{6}t$$

$$\frac{5}{6}t = \frac{5}{2}$$

$$t = 3 \rightarrow \boxed{3} \checkmark$$

$$\int_0^R r \cdot \mu \cdot 2\pi r dr \cdot g = \frac{MR^2}{2} \alpha$$

$$\mu \cdot 2\pi g \frac{R^3}{3} = \frac{MR^2}{2} \alpha$$

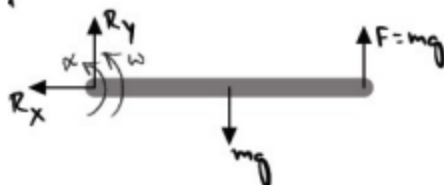
$$\mu \cdot \frac{M}{\pi R^2} \cdot 2\pi g \cdot \frac{R^3}{3} = \frac{MR^2}{2} \alpha$$

$$R\alpha = \frac{4Hg}{3}$$

$$\alpha = \frac{4}{3} \times \frac{\frac{1}{8} \times 10}{2}$$

$$\alpha = \frac{5}{6}$$

34. 15



$$lmg - \frac{l}{2}mg = \frac{ml^2}{3} \alpha$$

$$l\alpha = \frac{3g}{2}$$

$$R_y + mg - mg = m \left[\frac{l}{2} \alpha \right]$$

$$R_y = \frac{3mg}{4}$$

$$= \frac{3}{4} \times 2 \times g$$

$$= 15 \rightarrow \boxed{15} \checkmark$$