OSCILLATION

Periodic Motion OR Harmonic Motion

Any motion that repeats itself at regular intervals of time is known as periodic motion

Eg. Revolution of earth around the sun

Oscillatory Motion

A body is said to be in oscillatory motion if it undergoes a to and fro motion about an equilibrium position or mean position

Eg. Motion of a simple pendulum

* Every oscillatory motion is periodic, but every periodic motion need not be oscillatory

  $\Rightarrow$ Example, circular motion is periodic, but not oscillatory

* **Time period (T)**

  * Time for one oscillation

  * SI unit is second(s)

* **Frequency** (ν)

  No. of oscillations per second

  $$\boxed{\nu = \frac{1}{T}}$$

  $\Rightarrow$ SI unit (Hz) or s^{-1}

  1Hz = $1s^{-1}$

There is no significant difference between oscillations & vibrations. If frequency is high, we call it as vibration. If frequency is low, it is oscillation

Displacement

In oscillatory motion, the term displacement means any physical quantity that changes with time.

Eg. In the case of oscillation of heart, displacement means volume

In oscillatory motion, displacement can be represented by using periodic functions. <u>A function f(t) is said to be periodic if and only if</u>

$$\boxed{f(t)=f(T+t)}$$

$\Rightarrow$ where T is the period of function

eg: $\boxed{\begin{array}{l}\sin\theta=\sin(\theta+2\pi)\\ \& \\ \cos\theta=\cos(\theta+2\pi)\end{array}}$

Simple Harmonic Motion [SHM]

A motion is said to be simple harmonic, if its acceleration at any instant is directly proportional to displacement from mean position, and acceleration is always directed towards the mean position.

i.e., a α - displacement

[Displacement is always away from mean position & it is always measured from mean position]

a α - y

a = $-\omega^2 y$

$a+\omega^2 y=0$

i.e., $\boxed{\frac{d^2y}{dt^2}+\omega^2 y=0}$

This is the differential equation of a SHM. Solution of this equation gives displacement of SHM, which is given by

$$\boxed{\begin{array}{l}y=A\sin(\omega t+\phi)\\ \text{OR}\\ y=A\cos(\omega t+\phi)\end{array}}$$

where $A\rightarrow$ Amplitude : Maximum displacement

$\omega\rightarrow$ Angular frequency

$$\boxed{\omega=\frac{2\pi}{T}=2\pi\nu}$$

$\Rightarrow$ SI unit of ω is rad/s

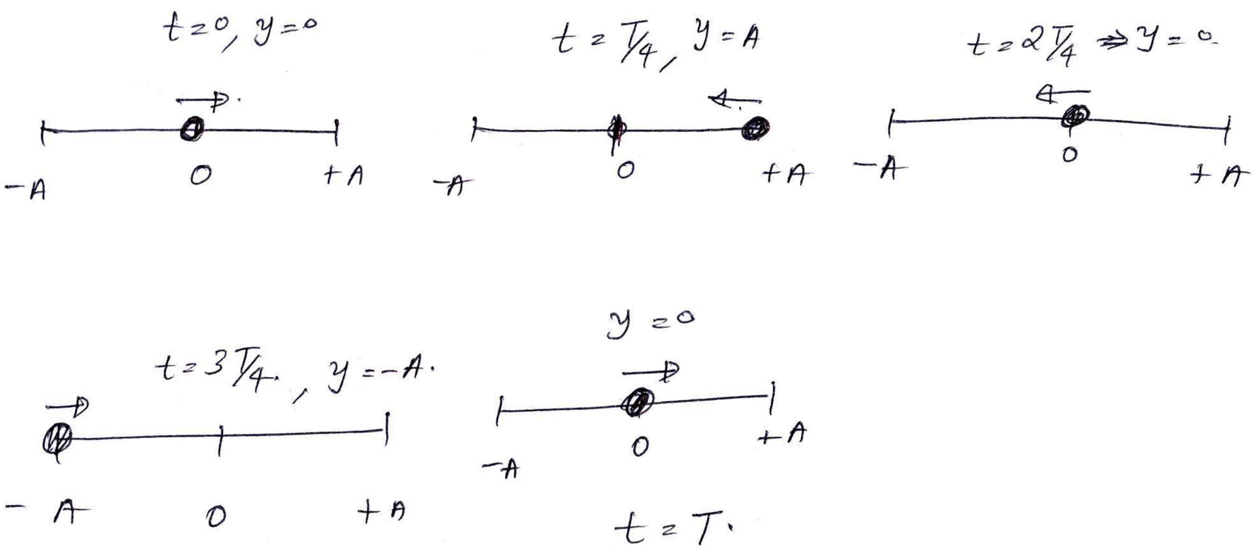
$(\omega t+\phi)\Rightarrow$ phase

$\phi\Rightarrow$ initial phase OR phase constant OR epoch

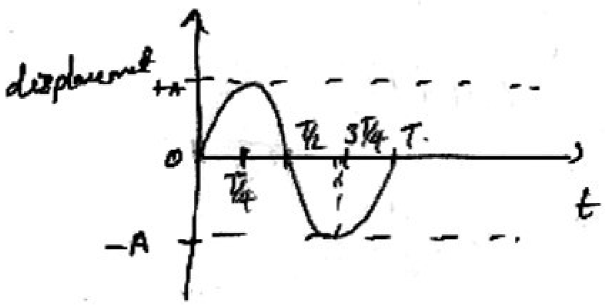
If $\phi=0$

$y = A\sin\omega t$	$y = A\cos\omega t$
when $t = 0$	when $t = 0$
$y = 0$	$y = A$
$\therefore \boxed{y = A\sin\omega t}$, represents oscillation starting from mean position	$\therefore \boxed{y = A\cos\omega t}$, represents, oscillation starting from extreme position
$\Rightarrow y = A\sin(\omega t + \phi)$ represents particle initially shifted from mean position by an angle ϕ	$\Rightarrow y = A\cos(\omega t + \phi)$. represents particle initially shifted from extreme position by an angle ϕ

Oscillation of a particle along a straight line



displacement - time graph of above motion



$y = A\sin\omega t \Rightarrow$ starting from mean position.

* **Velocity in SHM**

Let $y = A\sin(\omega t + \phi)$

$$v = \frac{dy}{dt} = A\omega\cos(\omega t + \phi)$$

$$v = A\omega\sqrt{1 - \sin^2(\omega t + \phi)}$$

$$v = \omega\sqrt{A^2 - A^2\sin^2(\omega t + \phi)}$$

$$\boxed{v = \omega\sqrt{A^2 - y^2}}$$

$\Rightarrow$ Relation between velocity & displacement in SHM

$\Rightarrow$ At mean position (y = 0)

$\therefore \boxed{v_{max} = \omega A}$

$\Rightarrow$ At extreme position (y = A)

$$\boxed{v = 0}$$

* **Acceleration in SHM**

We have $y = A\sin(\omega t + \phi)$

$$v = \omega A\cos(\omega t + \phi)$$

$$a = -\omega^2 A\sin(\omega t + \phi)$$

i.e., $\boxed{a = -\omega^2 y}$

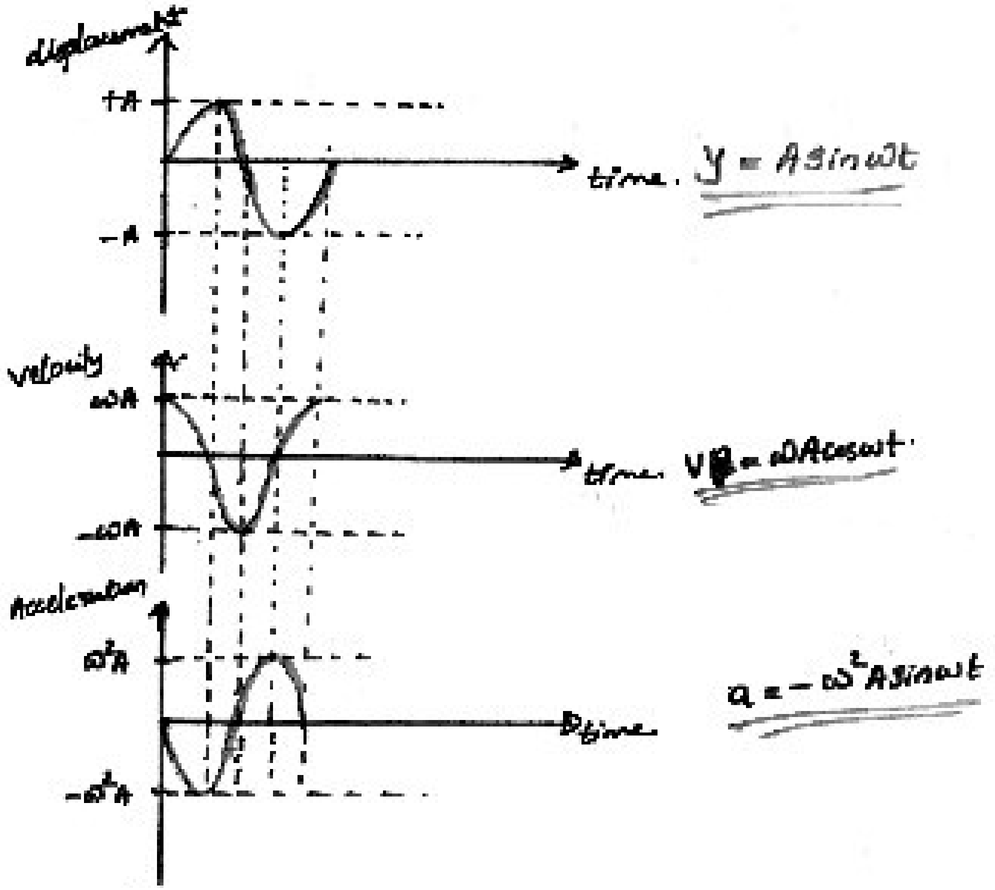
At mean position (y = 0)

$$\boxed{a = 0}$$

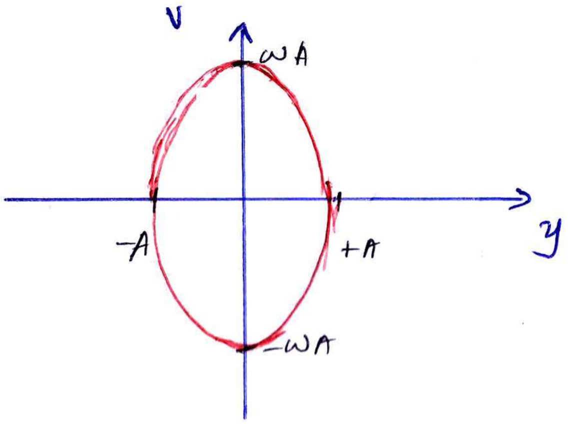
At extreme position (y = A)

$$\boxed{a_{max} = \omega^2 A}$$

Comparison of displacement-time graph, velocity-time graph, acceleration-time graph



Displacement - Velocity graph



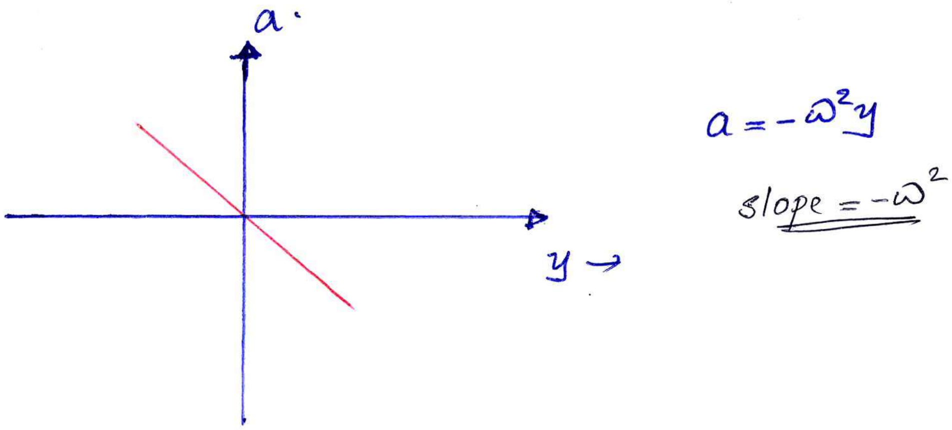
$$v = \omega\sqrt{A^2 - y^2}$$

$$\frac{v^2}{\omega^2} = A^2 - y^2$$

$$y^2 + \frac{v^2}{\omega^2} = A^2$$

$$\frac{y^2}{A^2} + \frac{v^2}{\omega^2 A^2} = 1 \quad \text{Equation of ellipse}$$

Acceleration - displacement graph



Force in SHM

Let 'm' be the mass of a particle in SHM. Then force acting on it is

F = ma

where $a = -\omega^2 y$

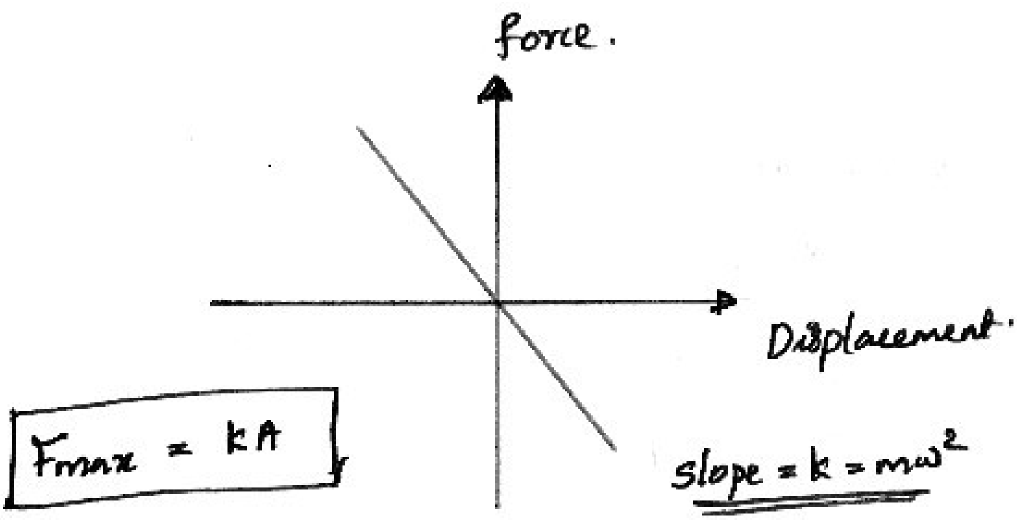
$\boxed{F = -m\omega^2 y}$ where $\boxed{m\omega^2 = k}$ force constant

$\boxed{F = -ky}$

–ve sign indicates that force is restoring

Here also, force is maximum at extreme position

$\boxed{F_{max} = KA}$



Energy in SHM

1) Kinetic energy

$$KE = \frac{1}{2}mv^2$$

where $v = \omega\sqrt{A^2 - y^2}$

$$KE = \frac{1}{2}m\omega^2\left[A^2 - y^2\right]$$

$$\boxed{KE = \frac{1}{2}k\left[A^2 - y^2\right]}$$

At mean position (y = 0)

$$kE_{max} = \frac{1}{2}kA^2$$

At extreme position (y = A)

KE = 0

2) Potential Energy

Potential energy of an oscillating particle at a point is defined as work done by an external agent to bring the body from mean position to that point

Let ‘y’ be the displacement from mean position, then potential energy is given by

$$\boxed{U = \frac{1}{2}ky^2}$$

At mean position (y = 0)

$\therefore$ PE = 0

At extreme position (y = A)

$$\boxed{PE_{max} = \frac{1}{2}KA^2}$$

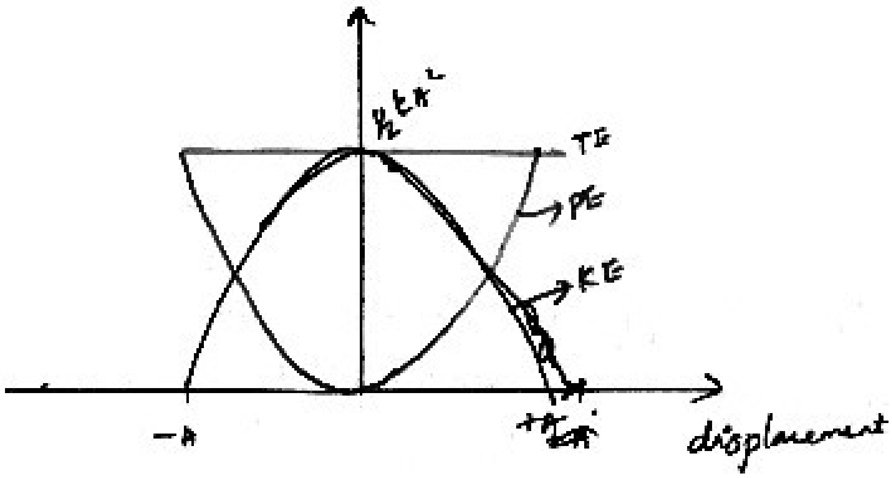
Total Energy

E = KE + PE

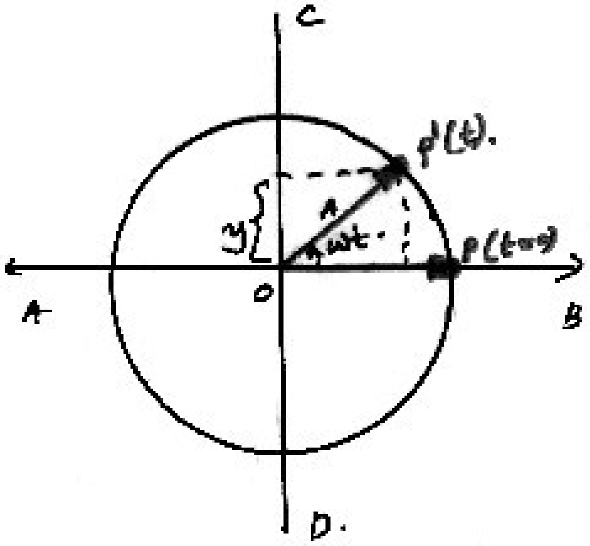
$$E = \frac{1}{2}m\omega^2\left[A^2 - y^2\right] + \frac{1}{2}m\omega^2A^2$$

$$E = \frac{1}{2}m\omega^2A^2$$

$$\boxed{E = \frac{1}{2}kA^2}$$ Total Energy of the oscillating particle always remains a constant



Relation between SHM & uniform circular motion



$\Rightarrow$ Consider a uniform circular motion

A $\rightarrow$ Radius of circular path

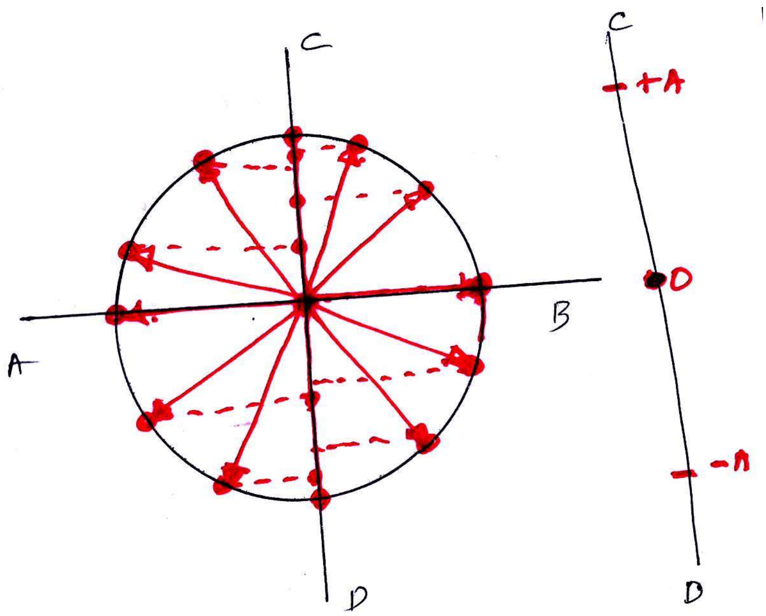
B $\rightarrow$ Angular velocity of circular motion

Let 'p' be the position of the particle at t = 0, and p' be the position at time t.

Projection of position vector op' on the diameter CD is

$$\boxed{y = A \sin \omega t}$$

This is displacement equation of SHM



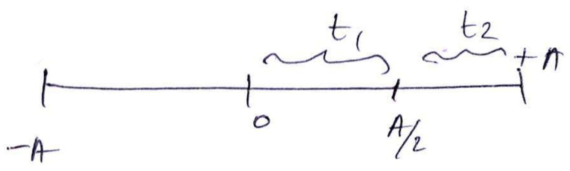
From the figure it is clear that particle only execute circular motion. But foot of the perpendicular drawn from the particle to the diameter CD will execute a SHM. Therefore SHM can also be defined as the projection of a uniform circular motion, on any of the diameter.

NOTE 1 :

1) Time taken to displace a particle from 0 to $A/2$ is $T/12$

2) Time taken to displace the particle from $A/2$ to A is $T/6$

Proof



$0 \rightarrow A/2$

$y = A \sin \omega t_1$

$\frac{A}{2} = A \sin \omega t_1 \qquad \Rightarrow \sin \omega t_1 = 1/2$

$\omega t_1 = \pi/6$

$$\boxed{t_1 = \frac{\pi}{6\omega} = \frac{\pi T}{6\times 2\pi} = \frac{T}{12}}$$

$$\frac{A}{2} \to A$$

$$\boxed{t_2 = \frac{T}{4} - t_1 = \frac{T}{4} - \frac{T}{12} = \frac{3T - T}{12} = \frac{T}{6}}$$

NOTE : 2

If two SHM are represented by

$y_1 = A_1 \sin(\omega_1 t + \phi_1)$ &

$y_2 = A_2 \sin(\omega_2 t + \phi_2)$

Then phase difference between them is

$$\boxed{\Delta\phi = (\omega_2 t + \phi_2) - (\omega_1 t + \phi_1)}$$

$\Rightarrow$ If $\Delta\phi$ is +ve, y_2 leads y_1 $\Rightarrow$ If $\Delta\phi$ –ve, y_1 leads y_2

$$\boxed{\begin{array}{l}\Rightarrow \text{If } \Delta\phi = 0, 2\pi, 4\pi, ... \\ \Rightarrow \text{Then they are said to be in} - \text{phase}\end{array}}$$

$$\left\{\begin{array}{l}\text{If } \Delta\phi = \pi, 3\pi, 5\pi, ... \\ \text{Then they are said to be out of phase}\end{array}\right\}$$

NOTE : 3

If a SHM is represented by

$y = A_1 \sin \omega t + A_2 \sin(\omega t + \phi)$

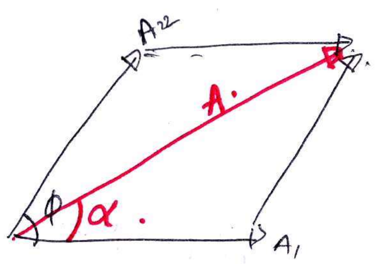
Then it can be represented as

$$\boxed{y = A \sin(\omega t + \alpha)}$$

where resulted amplitude

$$\boxed{A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\phi}}$$

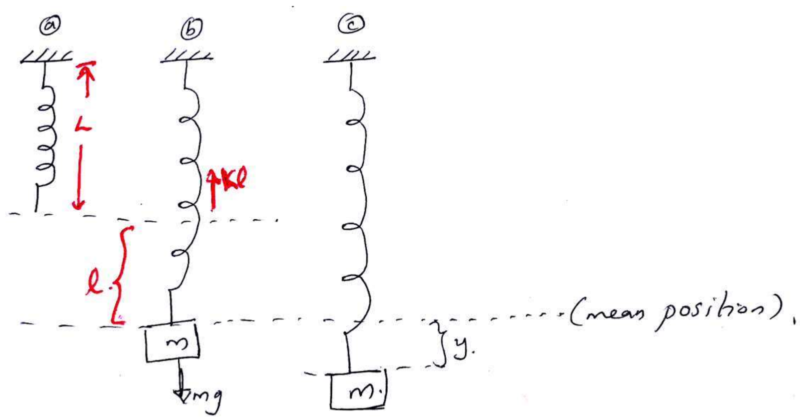
where $\boxed{\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}}$



Applications of SHM

1) Oscillations of a spring

Consider a light and elastic spring suspended vertically from a rigid support. Let a body of mass 'm' be attached to the lower end of the spring



$\ell \rightarrow$ elongation or extension

From figure (B), according to Hooke's Law, restoring force is directly proportional to elongation

i.e. $F \alpha \ell$ OR

$F = -k\ell$ | $k \rightarrow$ spring constant

–ve sign indicates that force is restoring

At equilibrium

$$mg = k\ell$$

$$\therefore \boxed{k = \frac{mg}{\ell}}$$

Now the body be pulled further down through a small distance 'y' and released, it starts vertical oscillations.

From fig. (c)

$$F_{net} = mg - k(\ell + y)$$

$ma = mg - k\ell - ky$

i.e. ma = –ky

$$\boxed{a = -\left(\frac{k}{m}\right)y}$$

This equation of the form $a = -\omega^2 y$. $\therefore$ Motion of spring is SHM

By comparing $\omega = \sqrt{\frac{k}{m}}$ & $k = m\omega^2$

$\therefore$ Time period of oscillation of a spring-block system is

$$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}}$$

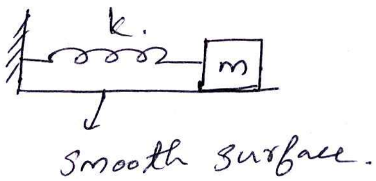
$\Rightarrow$ Spring constant 'k' is independent of acceleration due to gravity g

NOTE 1 :

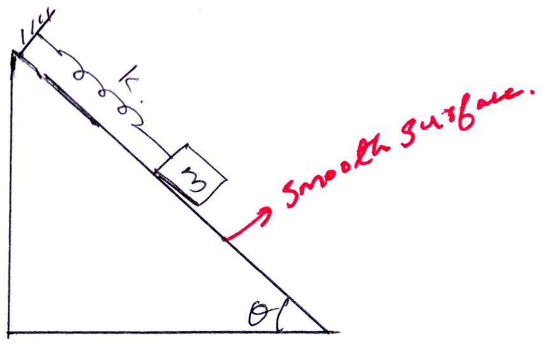
If the spring is not light, but has a mass m_s, then time period of oscillation of spring block system is

$$T = 2\pi\sqrt{\frac{m + \frac{m_s}{3}}{k}}$$

NOTE 2 :

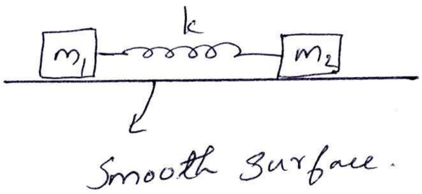


$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$



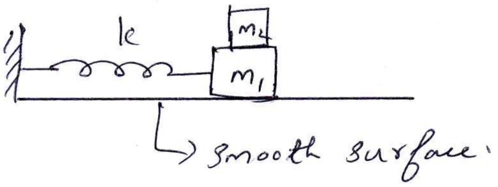
$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

NOTE : 3



$$T = 2\pi\sqrt{\frac{m_1 m_2}{(m_1 + m_2)k}}$$

NOTE : 4

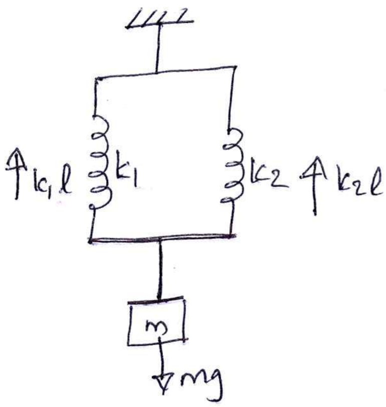


$$\boxed{T = 2\pi\sqrt{\frac{(m_1 + m_2)}{k}}}$$

⇒ Where restoring force is less than frictional force between m_1 & m_2

Combination of Springs

a) Parallel combination



⇒ For parallel combination both springs have same elongation.

Let 'ℓ' be the elongation, then

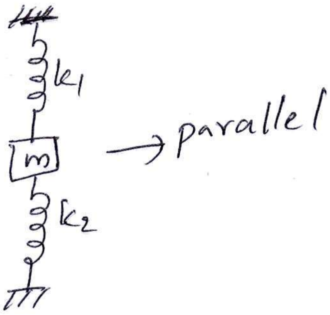
$mg = k_1\ell + k_2\ell$

= $(k_1 + k_2)\ell$

i.e., $mg = (k_p)\ell$

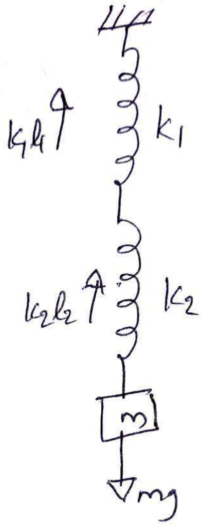
where $\boxed{k_p = k_1 + k_2}$ ⇒ Effective spring constant in parallel combination

$\therefore \boxed{T = 2\pi\sqrt{\dfrac{m}{k_p}}}$



b) Series combination

In series combination force is same in both springs, but elongation is different



For first spring $k_1\ell_1 = F$.

$$\ell_1 = \frac{F}{k_1}$$

For second spring $k_2\ell_2 = F$

$$\ell_2 = \frac{F}{k_2}$$

$\Rightarrow$ Let 'ℓ' be the total elongation of springs, then $\ell = \ell_1 + \ell_2$

$$\frac{F}{k_s} = \frac{F}{k_1} + \frac{F}{k_2}$$

i.e., $\boxed{\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2}}$

where $k_s \rightarrow$ effective spring constant in series combination

i.e., $\boxed{k_s = \frac{k_1 k_2}{k_1 + k_2}}$

NOTE : 1

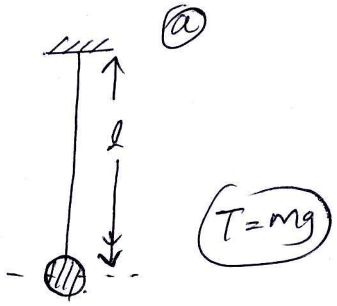
$\therefore \boxed{\text{Spring constant } \alpha \frac{1}{\text{length of spring}}}$

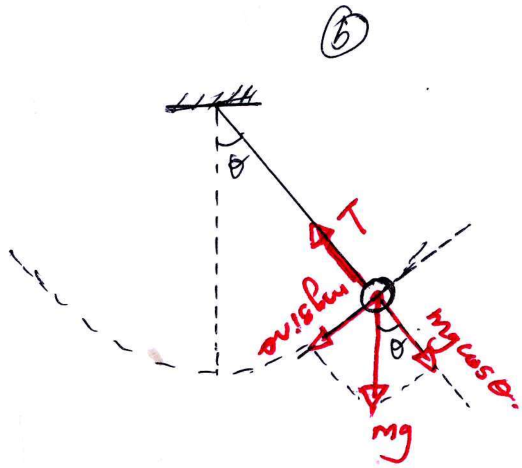
NOTE : 2

If a spring of spring constant 'k' is cut into 'n' equal parts, then spring constant of each part becomes nk.

2) Oscillations of a simple pendulum

Simple pendulum consist of a light & inextensible spring connected to a small mass or bob





m $\rightarrow$ mass of bob

$\ell \rightarrow$ length of pendulum

(Distance between point of suspension & centre of mass of bob)

From figure (B)

$T - mg\cos\theta = F_C \rightarrow$ centripetal force

$$\boxed{T = mg\cos\theta + \frac{mv^2}{\ell}}$$

Restoring torque is provided by $mg\sin\theta$

i.e., restoring torque $\tau = -\ell(mg\sin\theta)\sin(90)$ | – ve sign indicates restoring torque

i.e., $I\alpha = -\ell mg\sin\theta$

for small oscillations $\sin\theta \approx \theta$

$\therefore I\alpha = -\ell mg\theta$

$\boxed{\alpha = -\left(\frac{\ell mg}{I}\right)\theta}$ | $I \rightarrow$ moment of inertia

This is the form of $a = -\omega^2 y$

$\therefore$ It is a simple harmonic motion.

By comparing $\omega = \sqrt{\frac{\ell mg}{I}}$ where $\boxed{I = m\ell^2}$

$\therefore \boxed{\omega = \sqrt{\frac{\ell mg}{m\ell^2}} = \sqrt{\frac{g}{\ell}}}$

$\therefore$ Time period of oscillation

$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\ell}{g}}}$

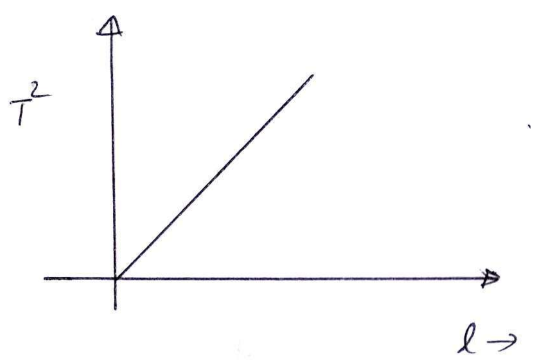
1) Time period is independent of mass of bob

2) $T \,\alpha\, \sqrt{\ell}$

3) $T \,\alpha\, \frac{1}{\sqrt{g}}$

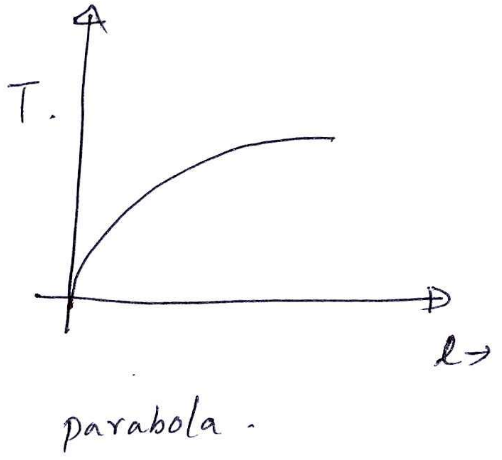
Graph

1) $\underline{\ell - T^2}$



$\boxed{T^2 = 4\pi^2 \frac{\ell}{g}}$ Slope $= \frac{4\pi^2}{g}$ $T^2 \,\alpha\, \ell$

2) $\ell - \mathrm{T}$ graph

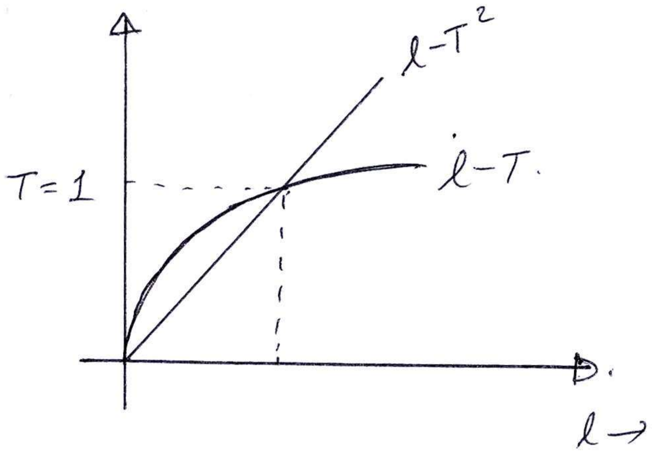


$$T^2 = \frac{4\pi^2}{g}\ell \rightarrow \text{Eqn. of parabola}$$

$$\uparrow \qquad \uparrow$$

$$y^2 = 4ax \rightarrow$$

$\ell - \mathrm{T}^2$ & $\ell - \mathrm{T}$ graph intersect at T = 1



NOTE

Second's pendulum : A pendulum whose time period is 2 second

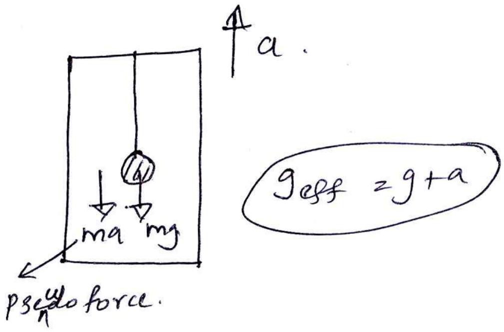
* Length of second's pendulum on the surface of earth is approximately 1m

Special Cases

* If a pendulum is suspended from the roof of a lift moving vertically upwards with an acceleration 'a', then effective acceleration on the bob becomes (g + a)

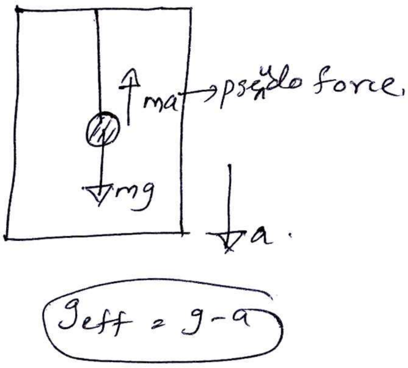
$\therefore$ Time period of oscillation

$$T = 2\pi\sqrt{\frac{\ell}{g+a}}$$



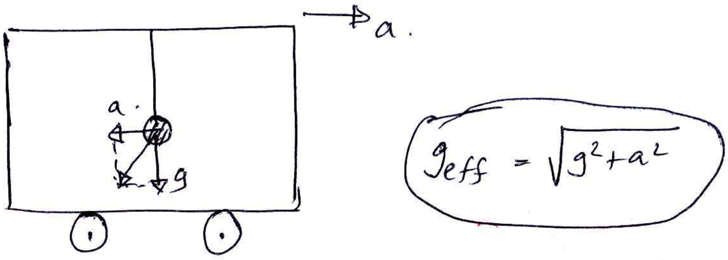
* If a pendulum is suspended from the roof of a lift moving vertically downwards with an acceleration 'a', then effective acceleration on the bob becomes (g – a)

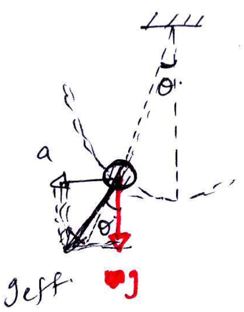
$$T = 2\pi\sqrt{\frac{\ell}{g-a}}$$



* If a pendulum is suspended on the roof of a car moving horizontally with an acceleration 'a', then effective acceleration on the bob becomes

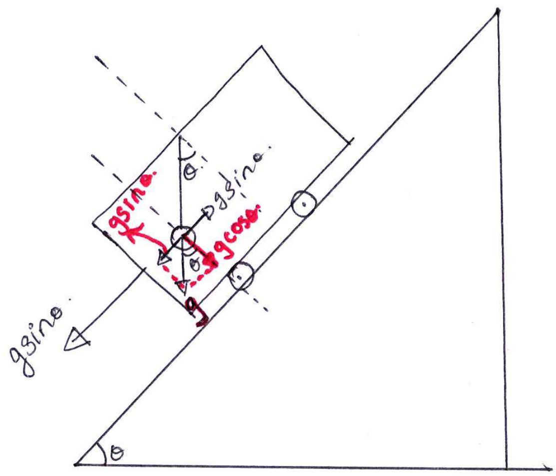
$$g_{eff} = \sqrt{g^2 + a^2}$$





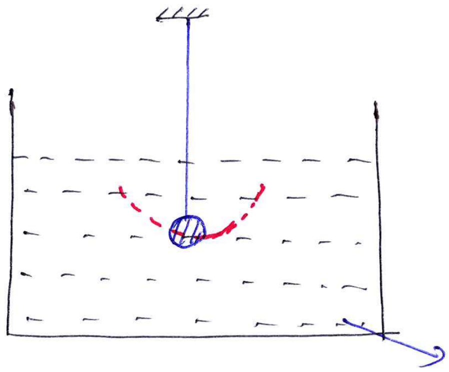
$\Rightarrow$ In such a case mean position of the pendulum is shifted backwards by an angle $\boxed{\tan\theta = \frac{a}{g}}$

* If a pendulum is on the roof of a car moving down an inclined plane of inclination 'θ', then effective acceleration on the bob becomes $g\cos\theta$



$\therefore \boxed{T = 2\pi\sqrt{\frac{\ell}{g\cos\theta}}}$

from figure $\boxed{g_{eff} = g\cos\theta}$

* 

$\rho \rightarrow$ density of bob.

$\sigma \rightarrow$ density of liquid.

$\boxed{\rho > \sigma}$

Non viscous liquid.

Net force on the bob is

F = mg – F_B

mg_{eff} = mg – weight of liquid displaced

$V \rightarrow$ volume of bob

$$(V\rho)g_{eff} = (V\rho)g - (V\times\sigma)g$$

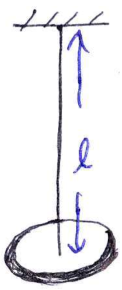
$$\rho g_{eff} = \rho g - \sigma g$$

$$g_{eff} = \left(\frac{\rho-\sigma}{\rho}\right)g \Rightarrow \boxed{g_{eff} = \left(1-\frac{\sigma}{\rho}\right)g}$$

$$\boxed{T = 2\pi\sqrt{\frac{\ell}{\left(1-\frac{\sigma}{\rho}\right)g}}}$$

* **Torsional Pendulum**

$\Rightarrow$ It consists of a horizontal circular disc suspended with the help of wire



$\Rightarrow$ If it is twisted by an angle θ & then released, it executes torsional oscillations

Restoring torque $\tau = -C\theta$ | $C \rightarrow$ restoring torque per unit twist

$$I\alpha = -C\theta$$

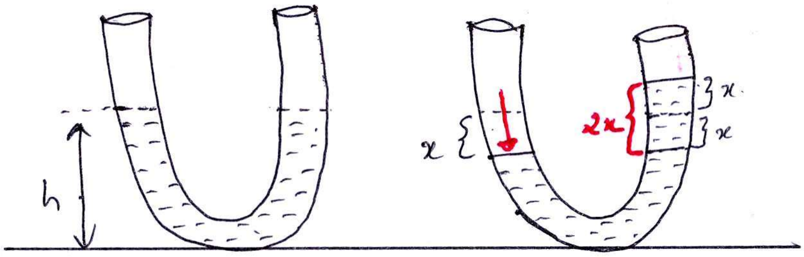
$$\boxed{\alpha = -\frac{C}{I}\theta} \rightarrow a = -\omega^2 y$$

SHM

By comparing $\omega = \sqrt{\frac{C}{I}}$ | $I \rightarrow$ moment of inertia of disc

$$\therefore \boxed{T = 2\pi\sqrt{\frac{I}{C}}}$$

U-Tube Oscillator



$A \rightarrow$ Area of cross section

$h \rightarrow$ height of liquid in the tube in the state of equilibrium

$d \rightarrow$ density of liquid

If we withdraw applied force, then liquid will oscillate

$\Rightarrow$ Here restoring force is provided by the weight of liquid in '2x' height.

i.e., F = –weight of liquid in 2x height

= $-A(2x)dg$

$M \rightarrow$ Total mass of liquid in the tube

$Ma = -2Adg\ x$

$$\boxed{a = -\left(\frac{2Adg}{M}\right)x} \Rightarrow a = -\omega^2 y$$

SHM

By comparing $\omega = \sqrt{\frac{2Adg}{M}}$

$$\therefore \boxed{T = 2\pi\sqrt{\frac{M}{2Adg}}}$$

But $M = (A2h)d$ $\qquad \therefore T = 2\pi\sqrt{\frac{A2h.d}{2Adg}}$

$$\boxed{T = 2\pi\sqrt{\frac{h}{g}}}$$