# CHAPTER - 04
# COMPLEX NUMBERS

**JEE MAIN - SECTION I**

1. 1 Taking modulus on both sides , we get the result !

2. 4 Put z = x+iy, $(x+iy)+|x+iy| = 8+12i$

   Solving $x = -5$, $y = 12$

3. 4 $a = e^{i\alpha}, b = e^{i\beta}, c = e^{i\gamma} \Rightarrow$

4. 2 $|Z_1| = |Z_2| = |Z_1 - Z_2|$ and $Z_1^2 + Z_2^2 = Z_1 Z_2$ ; $Z_1^2, Z_1^2 + Z_2^2, Z_2^2$ are be vertices of an isosceles triangle

5. 4 put $x = 1, \omega, \omega^2$ and add

6. 4 $z_1 = 1 + i$, $z_2 = -1 + \sqrt{3}i \Rightarrow \arg z_1 = \frac{\pi}{4}$. $\arg z_2 = 2\pi/3$ by

   verification $\arg z_1$ and $\arg z_2$ lies between $\frac{\pi}{4} \& \frac{2\pi}{3}$

7. 1 $(3 + 2\sqrt{-54}) = 3 + 2 \times 3 \times \sqrt{6}i = (3 + \sqrt{6}i)^2$

   $(3 - 2\sqrt{54}) = (3 - \sqrt{6}i)^2$

   $(3 + 2\sqrt{-54})^{1/2} + (3 - 2\sqrt{-54})^{1/2}$

   $= \pm(3 + \sqrt{6}i) \pm (3 - \sqrt{6}i) = 6, -6, 2\sqrt{6}i, -2\sqrt{6}i$ .

8. 2

Given $z = \dfrac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely imaginary

So, real part becomes zero

$$z = \left(\frac{3+2i\sin\theta}{1-2i\sin\theta}\right)\times\left(\frac{1+2i\sin\theta}{1+2i\sin\theta}\right)$$

$$z = \frac{(3-4\sin^2\theta)+i(8\sin\theta)}{i+4\sin^2\theta}$$

Now, Re(z) = 0

$$\frac{3-4\sin^2\theta}{1+4\sin^2\theta}=0 \Rightarrow \sin^2\theta = \frac{3}{4}$$

$$\Rightarrow \sin\theta = \pm\frac{\sqrt{3}}{2} \Rightarrow \theta = -\frac{\pi}{3},\frac{\pi}{3},\frac{2\pi}{3} \quad \left(\because \theta\in\left(-\frac{\pi}{2},\pi\right)\right)$$

Then sum of the elements in A is $-\dfrac{\pi}{3}+\dfrac{\pi}{3}+\dfrac{2\pi}{3}=\dfrac{2\pi}{3}$

9. 2

The value of $\left(\dfrac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right) = \left(\dfrac{1+\sin\left(\frac{\pi}{2}-\frac{5\pi}{18}\right)+i\cos\left(\frac{\pi}{2}-\frac{5\pi}{18}\right)}{1+\sin\left(\frac{\pi}{2}-\frac{5\pi}{18}\right)-i\cos\left(\frac{\pi}{2}-\frac{5\pi}{18}\right)}\right)^3$

$$= \left(\frac{1+\cos\frac{5\pi}{18}+i\sin\frac{5\pi}{18}}{1+\cos\frac{5\pi}{18}-i\sin\frac{5\pi}{18}}\right)^3 = \left(\frac{2\cos^2\frac{5\pi}{36}+2i\sin\frac{5\pi}{36}\cos\frac{5\pi}{36}}{2\cos^2\frac{5\pi}{36}-2i\sin\frac{5\pi}{36}\cos\frac{5\pi}{36}}\right)^3$$

$$= \left(\frac{\cos\frac{5\pi}{36}+i\sin\frac{5\pi}{36}}{\cos\frac{5\pi}{36}-i\sin\frac{5\pi}{36}}\right)^3 = \left(\frac{e^{i\frac{5\pi}{36}}}{e^{-i\frac{5\pi}{36}}}\right)^3 = \left(e^{i\frac{5\pi}{18}}\right)^3$$

$$= \cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}+\frac{i}{2}.$$

10. 1

$\alpha = \omega$, $a = (1+\omega)(1+\omega^2+\omega^4+....+\omega^{200})$

$$a = (1+\omega)\frac{\left(1-(\omega^2)^{101}\right)}{1-\omega^2} = 1$$

$$b = 1+\omega^3+\omega^6+....+\omega^{300} = 101$$

$$x^2 - 102x + 101 = 0$$

11. 3

The minimum value of $|Z-1+2i|+|4i-3-Z|$; $|Z_1+Z_2| \le |Z_1|+|Z_2|$

$$|Z-1+2i|+|4i-3-Z| \ge |Z-1+2i+4i-3-Z| \ge |-4+6i| \ge \sqrt{16+36} \ge 2\sqrt{13}$$

Minimum value = $2\sqrt{13}$

12. 2

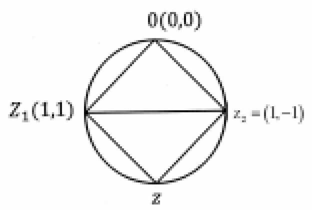
$$|Z_i| = \lambda, \Rightarrow \left|\frac{z_1^{-1}+z_2^{-1}+....+z_n^{-1}}{z_1+.....+z_n}\right| = \frac{1}{\lambda^2}$$

$$= \left|\frac{\frac{1}{\lambda}+\frac{1}{\lambda}+....+\frac{1}{\lambda}}{\lambda+\lambda+....+\lambda}\right|$$

$$= \left|\frac{n\frac{1}{\lambda}}{n\lambda}\right| = \frac{1}{\lambda^2}$$

13. 2

$Z_1 = (1,1)$, $Z_2 = (1,-1)$, $Z(0,0)$




Maximum distance 6 = OZ = diameter = 2

14. 4

$(x-1)^3 = -8 \Rightarrow x-1 = -2(1,\omega,\omega^2) \Rightarrow x = -1, 1-2\omega, 1-2\omega^2$

$\frac{\alpha-1}{\beta-1}+\frac{\beta-1}{\gamma-1}+\frac{\gamma-1}{\alpha-1} = \frac{-2}{-2\omega}+\frac{2\omega}{-2\omega^2}+\frac{-2\omega^2}{-2} = \frac{1}{\omega}+\frac{1}{\omega}+\omega^2 = 3\omega^2$

15. 1

$Z+\frac{1}{Z}=-1 \Rightarrow Z^2+Z+1=0 \Rightarrow Z=\omega$ or $\omega^2$

$\sum_{r=1}^{5}\left(Z^r+\frac{1}{Z^r}\right)^2 = \left(Z+\frac{1}{Z}\right)^2+\left(Z^2+\frac{1}{Z^2}\right)^2+\left(Z^3+\frac{1}{Z^3}\right)^2+\left(Z^4+\frac{1}{Z^4}\right)^2+\left(Z^5+\frac{1}{Z^5}\right)^2$

16. 3

$z = a+bw+bw^2 \Rightarrow \bar{z}\,|a+bw^2+cw| = |z|^2 = z\cdot\bar{z} = (a+bw+cw^2)(a+bw^2+cw)$

$= a^2+b^2+c^2-bc-ca-ab = \frac{1}{2}[(a-b)^2+(b-c)^2+(c-a)^2]$

$\Rightarrow |z| = \frac{1}{\sqrt{2}}\sqrt{(a-b)^2+(b-c)^2+(c-a)^2} \qquad a=1, b=2, c=3$

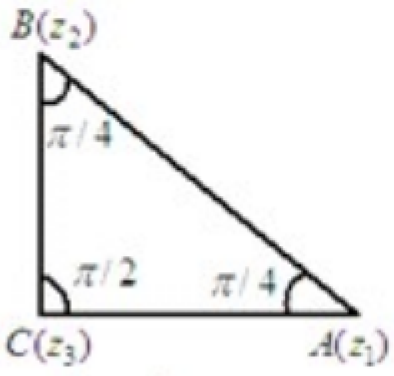
$= \frac{1}{\sqrt{2}}\sqrt{6} = \sqrt{3} \Rightarrow |a+bw+cw^2|+|a+bw^2+cw| = |z|+|z| = 2|z| = 2\sqrt{3}$

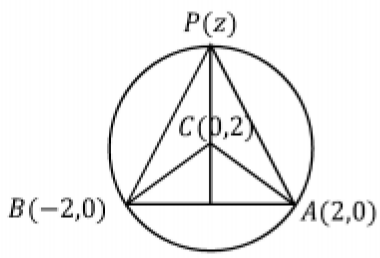
17. 2

Slope of CA × Slope of CB = −1

$\angle BCA = 90^0$, $\angle BPA = \pi/4$

$\arg\left(\frac{Z-2}{Z+2}\right) = \frac{\pi}{4}$.

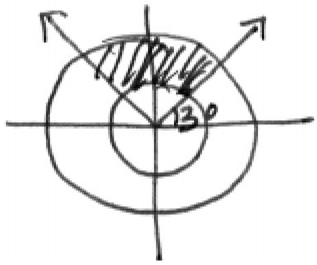


18. 4 $\frac{\pi}{6} < \arg Z < \frac{2\pi}{3}$ ; $3 < |z| < 5$

Area : $\frac{1}{2}\times 5^2 \times \frac{\pi}{2} - \frac{1}{2}\times 3^2 \times \frac{\pi}{2}$

$= \frac{\pi}{4}(25-9) = 4\pi$



19. 2 Statement-I

$\because\ |z_1 + z_2| = |z_1| + |z_2|$ .... (1)

If $\text{amp}(z_1) = \theta_1$ and $\text{amp}(z_2) = \theta_2$, then

$|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2|z_1||z_2|\cos(\theta_1 - \theta_2)$.

$\Rightarrow (|z_1| + |z_2|)^2 = |z_1|^2 + |z_2|^2 + 2|z_1||z_2|\cos(\theta_1 - \theta_2)$ [From eq. (1)]

$\Rightarrow |z_1|^2 + |z_2|^2 + 2|z_1||z_2| = |z_1|^2 + |z_2|^2 + 2|z_1||z_2|\cos(\theta_1 - \theta_2)$

$\therefore \cos(\theta_1 - \theta_2) = 1 \Rightarrow \theta_1 - \theta_2 = 0$ or $\text{amp}(z_1) - \text{amp}(z_2) = 0$

$\therefore$ Statement-I is true.

Statement-II: Since $z_1, z_2$ and O (origin) are collinear, then

$\text{amp}\left(\frac{O - z_1}{O - z_2}\right) = 0$ or $\pi \Rightarrow \text{amp}\left(\frac{z_1}{z_2}\right) = 0$

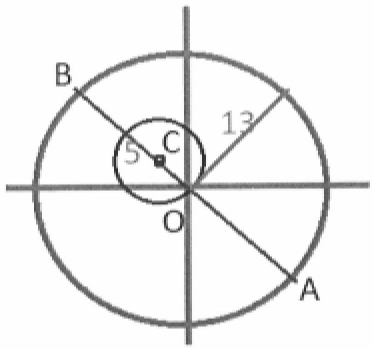
$\Rightarrow \text{amp}(z_1) - \text{amp}(z_2) = 0$.

$\therefore$ Statement-II is true, which is not a correct explanation of statement-I

20. 1 Distance between the centres = $3\sqrt{2}$ = sum of radii

**SECTION II (NUMERICAL)**

21. 8




$|z_1| \le 13$

$|z_2 - (-3+4i)| \le 5$ , OA = 13, OC =5

From diagram maximum value of $|z_1 - z_2| = 23$

Thus ,p=8.

22. 3 $\quad GV = \frac{2+\sqrt{2^2+4(3)}}{2} = 3$

23. 91 $\quad \left(-2-\frac{i}{3}\right)^3 = -\frac{(6+i)^3}{27} = \frac{-198-107i}{27} = \frac{x+iy}{27}$

Hence, $y-x = 198-107 = 91$

24. 0

$$\frac{a}{|z_1-z_2|} = \frac{b}{|z_3-z_2|} = \frac{c}{|z_3-z_1|} = k$$

$$a^2 = k^2(z_1-z_2)(\overline{z}_1-\overline{z}_2)$$

$$b^2 = k^2(z_2-z_3)(\overline{z}_2-\overline{z}_3)$$

$$c^2 = k^2(z_3-z_1)(\overline{z}_3-\overline{z}_1)$$

$$\frac{a^2}{z_1-z_2} + \frac{b^2}{z_2-z_3} + \frac{c^2}{z_3-z_1} = k^2[\overline{z}_1-\overline{z}_2-\overline{z}_3+\overline{z}_2+\overline{z}_3-\overline{z}_1] = k^2(0) = 0$$

25. 0

If the equation $z^2+(a+ib)z+(c+id)=0$ has real roots, say $\alpha$

$\alpha^2+(a+ib)\alpha+(c+id)=0 \Rightarrow \alpha^2+a\alpha+c=0,\ b\alpha^2+d=0 \Rightarrow \alpha = -\frac{d}{b}$

$\therefore \frac{d^2}{b^2} - \frac{ad}{b} + c = 0 \Rightarrow d^2 - abd + cb^2 = 0$

**JEE ADVANCED LEVEL**

**SECTION III**

26. B $a^2+b^2=1-c^2 \Rightarrow a-ib=\frac{(1-c)(1+c)}{a+ib}$ ....(i)

Now $z=\frac{b+ic}{1+a} \Rightarrow \frac{iz}{1}=\frac{-c+ib}{1+a}, \frac{1+iz}{1-iz}=\frac{1-c+a+ib}{1+c+a-ib}$

$=\frac{1-c+a+ib}{1+c+(1-c)(1+c)/(a+ib)}=\frac{a+ib}{1+c}$

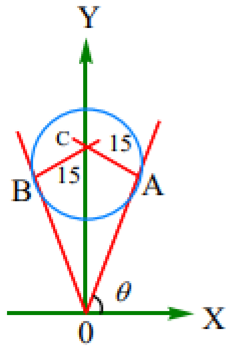
27. B $|z_1+z_2+......+z_n| \le |z_1|+|z_2|+.........+|z_n|$

$\therefore |z_1+z_2+(5+12i) \le |z_1|+|z_2|+|5+12i||$

$=2+3+13=18$

28. B The given equation is $(z^2+z+1)(z^2+1)=0 \Rightarrow z=\pm i, \omega, \omega^2$, $\omega$ being an imaginary cube root of unity. Thus $|z|=1$

29. B If $|z-25i| \le 15$, then z lies either in the interior or on the boundary of the circle with centre (0,25) and radius is 15, thus $\Delta OAC$




$\cos\left(\frac{\pi}{2}-\theta\right)=\frac{20}{25}=\frac{4}{5} \Rightarrow \frac{\pi}{2}-\theta=\cos^{-1}\left(\frac{4}{5}\right)$

$|\max.\arg(z)-\min.\arg(z)|=|\arg(B)-\arg(A)|$

$=\underline{|BOA}=\underline{|BOX}-\underline{|AOX}$

$=\frac{\pi}{2}+\left(\frac{\pi}{2}-\theta\right)-\theta=\pi-2\theta=2COS^{-1}\left(\frac{4}{5}\right)$

30. C $\bar{z}+i\bar{w}=0 \Rightarrow z-iw=0 \Rightarrow z=iw----(1)$

arg zw = $\pi$ $\Rightarrow \arg z+\arg w=\pi$, $\arg z-\arg w=\frac{\pi}{2} \Rightarrow \arg z=\frac{3\pi}{4}$

31. B The new complex number is $2(3+4i)e^{i\pi/4}=\sqrt{2}(-1+7i)$

32. D The image of z in the real axis is $\bar{z}$

$\therefore$ The image is given by $\arg\left(\frac{\bar{z}-3}{\bar{z}-i}\right)=\frac{\pi}{6}$. But $\arg\bar{z}=-\arg z$

$\therefore \arg\left(\frac{\bar{z}-3}{\bar{z}-i}\right)=\frac{\pi}{6} \Rightarrow \arg\left(\frac{z-3}{z+i}\right)=-\frac{\pi}{6} \Rightarrow \arg\left(\frac{z+i}{z-3}\right)=\frac{\pi}{6}$

33. B

**SECTION IV (More than one correct )**

34. A,C AC=|-i$z_1$+i$z_2$|=|$z_1$-$z_2$|=AB

Also, BC=|(1-i)($z_1$-$z_2$)|=$\sqrt{2}$ AB

Thus, A, B, C are vertices of an isosceles right triangle.

35. A,B Let z=x+iy; $\arg(z)=\frac{\pi}{6} \Rightarrow y=\tan\frac{\pi}{6} \Rightarrow y=\frac{1}{\sqrt{3}}x$, which is a straight line.

Also, $\left|z-2\sqrt{3}i\right|$=r, represents a circle with centre at $\left(0,2\sqrt{3}\right)$ and radius r. The straight line will intersect the circle if the perpendicular distance from the centre on the line $<r \Rightarrow \left|\frac{0-2\sqrt{3}.\sqrt{3}}{2}\right|<r \Rightarrow r>3$.

Therefore $[r]\geq 3$

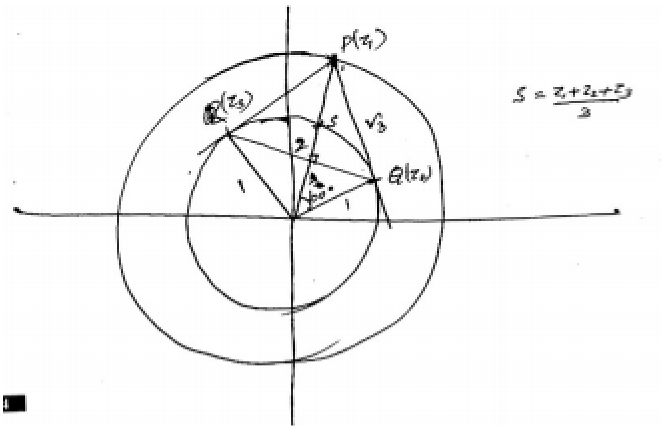
36. B,D $\frac{az+b}{z+1}=\frac{ax+b+aiy}{(x+1)+iy}=\frac{(ax+b+aiy)((x+1)-iy)}{(x+1)^2+y^2}$

$\therefore \operatorname{Im}\left(\frac{az+b}{z+1}\right)=\frac{-(ax+b)y+ay(x+1)}{(x+1)^2+y^2}$

$\Rightarrow \frac{(a-b)y}{(x+1)^2+y^2}=y$; $\because a-b=1$

$(x+1)^2+y^2=1$; $x=-1\pm\sqrt{1-y^2}$

37. A,B,C,D



## SECTION V - (Numerical type )

38. 9 $x+iy=3cis\theta+\frac{1}{3}cis(-\theta)\Rightarrow\frac{x^2}{100}+\frac{y^2}{64}=\frac{1}{9}$

39. 1 If implies that z lies on or inside the circle of radius 2 and centre (3, 2)

$$|2z-6+5i|_{\min}=2\left|z-3+\left(\frac{5}{2}\right)i\right|_{\min}=2\left(\frac{5}{2}\right)=5$$

## SECTION VI - (Matrix match type)

40. A A-R, B-R, C-R, D-P