CHAPTER - 02

TRIGONOMETRIC FUNCTIONS

1. 2

$$\sec\theta + \tan\theta = p \Rightarrow \sec\theta - \tan\theta = \frac{1}{p}$$

Subtracting second from first, we get $2\tan\theta = p - \frac{1}{p}$

$$\Rightarrow \tan\theta = \frac{p^2 - 1}{2p}.$$

2. 4

The expression is equal to $\sin(x-y) + \cos(x-y) = \sqrt{2}\left\{\sin\left(\frac{\pi}{4} + x - y\right)\right\}$,

which is zero, if $\sin\left(\frac{\pi}{4} + x - y\right) = 0$ i.e., $\frac{\pi}{4} + x - y = n\pi (n \in I) \Rightarrow x = n\pi - \frac{\pi}{4} + y$

3. 4

We have $\sin A = \frac{4}{5}$ and $\cos B = -\frac{12}{13}$

Now, $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$= \sqrt{1 - \frac{16}{25}}\left(-\frac{12}{13}\right) - \frac{4}{5}\sqrt{1 - \frac{144}{169}}$$

$$= -\frac{3}{5} \times \frac{12}{13} - \frac{4}{5}\left(-\frac{5}{13}\right) = -\frac{16}{65}$$ (Since A lies in first quadrant and B lies in third quadrant).

4. 2

The given expression can be written as

$$\frac{(\cos 6x + \cos 4x) + 5(\cos 4x + \cos 2x) + 10(\cos 2x + 1)}{\cos 5x + 5\cos 3x + 10\cos x}$$

After solving, we get the required result i.e. 2 cos x.

5. 2

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \; \tan\theta = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{1 - 2\sin^2\frac{\theta}{2}} = \frac{2\tan\frac{\theta}{2}}{1 - \tan^2\frac{\theta}{2}} \left[\begin{array}{l} \text{Using } \sin\frac{\theta}{2} = \sqrt{\frac{x-1}{2x}} \\ \therefore \; \cos\frac{\theta}{2} = \sqrt{1 - \sin^2\frac{\theta}{2}} = \sqrt{\frac{x+1}{2x}} \text{ and } \tan\frac{\theta}{2} = \frac{\sqrt{x-1}}{\sqrt{x+1}} \end{array}\right]$$

$\therefore \tan\theta = \sqrt{x^2 - 1}$.

6. 2

We have $A = \sin^2\theta + \cos^4\theta = \sin^2\theta + \cos^2\theta\cos^2\theta \le \sin^2\theta + \cos^2\theta$ (since $\cos^2\theta \le 1$)

$\Rightarrow \sin^2\theta + \cos^4\theta \le 1 \Rightarrow A \le 1$

Again, $\sin^2\theta + \cos^4\theta = 1 - \cos^2\theta + \cos^4\theta$

$= \cos^4\theta - \cos^2\theta + 1 = \left(\cos^2\theta - \frac{1}{2}\right)^2 + \frac{3}{4} \ge \frac{3}{4}$

Hence, $\frac{3}{4} \le A \le 1$.

7. 1

We have $\alpha + \beta - \gamma = \pi$.

Now $\sin^2\alpha + \sin^2\beta - \sin^2\gamma = \sin^2\alpha + \sin(\beta - \gamma)\sin(\beta + \gamma)$

$= \sin^2\alpha + \sin(\pi - \alpha)\sin(\beta + \gamma)$ $(\because \alpha + \beta - \gamma = \pi)$

$= \sin^2\alpha + \sin\alpha\sin(\beta + \gamma) = \sin\alpha\{\sin\alpha + \sin(\beta + \gamma)\}$

$= \sin\alpha\{\sin(\pi - \overline{\beta + \gamma}) + \sin(\beta + \gamma)\} = \sin\alpha\{-\sin(\gamma - \beta) + \sin(\gamma + \beta)\}$

$= \sin\alpha\{2\sin\beta\cos\gamma\} = 2\sin\alpha\sin\beta\cos\gamma$.

8. 3 For $A = B = C = 60^0$ only option (3) satisfies the condition.

9. 2

$\frac{1}{2}(2\cos^2 10^0 - 2\cos 10^0\cos 50^0 + 2\cos^2 50^0)$

$\Rightarrow \frac{1}{2}(1 + \cos 20^0 - (\cos 60^0 + \cos 40^0) + 1 + \cos 100^0)$

$\Rightarrow \frac{1}{2}\left(\frac{3}{2} + \cos 20^0 + 2\sin 70^0\sin(-30^0)\right)$

$\Rightarrow \frac{1}{2}\left(\frac{3}{2} + \cos 20^0 - \sin 70^0\right) = \frac{3}{4}$

10. 2

On simplification, it reduces to $\cos 2\theta = \sin 2\theta$

$\Rightarrow \tan 2\theta = \tan\frac{\pi}{4} \Rightarrow 2\theta = n\pi + \frac{\pi}{4} \Rightarrow \theta = \frac{n\pi}{2} + \frac{\pi}{8}$.

11. 1

$\frac{1 - \tan^2\theta}{\sec^2\theta} = \frac{1}{2} \Rightarrow \cos^2\theta - \sin^2\theta = \frac{1}{2}$

$\Rightarrow \cos 2\theta = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$

$\Rightarrow 2\theta = 2n\pi \pm \frac{\pi}{3} \Rightarrow \theta = n\pi \pm \frac{\pi}{6}$.

12. 2 $4 + 2\sin^2 x = 5 \Rightarrow \sin^2 x = \frac{1}{2} = \sin^2\frac{\pi}{4} \Rightarrow x = n\pi \pm \frac{\pi}{4}$.

13. 1

$f(x) = \cos x - x + \frac{1}{2}$, $f(0) = \frac{3}{2} > 0$

$f\left(\frac{\pi}{2}\right) = 0 - \frac{\pi}{2} + \frac{1}{2} = \frac{1-\pi}{2} < 0$, $\left(\because \pi = \frac{22}{7} \text{ nearly}\right)$

$\therefore$ One root lies in the interval $\left[0, \frac{\pi}{2}\right]$.

14. 3

$$\frac{1}{\sin \pi/n} - \frac{1}{\sin \frac{3\pi}{n}} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)}$$

$$\frac{\sin \frac{3\pi}{n} - \sin \pi/n}{\sin \pi/n \sin \frac{3\pi}{n}} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)}$$

$$\frac{2\cos \frac{2\pi}{n} \sin \pi/n}{\sin \pi/n \sin 3\pi/n} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)}$$

$$2\sin \frac{2\pi}{n} \cos \frac{2\pi}{n} = \sin 3\pi/n$$

$$\sin \frac{4\pi}{n} = \sin 3\pi/n$$

$$\sin \frac{4\pi}{n} = \sin\left(\pi - \frac{3\pi}{n}\right)$$

15. 1

Since A.M. $\geq$ G.M. $\frac{1}{2}(2^{\sin x} + 2^{\cos x}) \geq \sqrt{2^{\sin x}.2^{\cos x}}$

$\Rightarrow 2^{\sin x} + 2^{\cos x} \geq 2.2^{\frac{\sin x + \cos x}{2}}$ $\Rightarrow 2^{\sin x} + 2^{\cos x} \geq 2^{1+\frac{\sin x + \cos x}{2}}$

and we know that $\sin x + \cos x \geq -\sqrt{2}$

$\therefore\ 2^{\sin x} + 2^{\cos x} > 2^{1-(1/\sqrt{2})}$, for $x = \frac{5\pi}{4}$.

16. 1

Since, $\cos\alpha + \cos\left(\alpha + \frac{2\pi}{3}\right) + \cos\left(\alpha + \frac{4\pi}{3}\right)$

$= \cos\alpha + 2\cos(\alpha+\pi)\cos\frac{\pi}{3} = \cos\alpha + (-2\cos\alpha)\left(\frac{1}{2}\right) = 0.$

$\therefore\ \cos^3\alpha + \cos^3\left(\alpha + \frac{2\pi}{3}\right) + \cos^3\left(\alpha + \frac{4\pi}{3}\right)$

$= 3\cos\alpha\cos\left(\alpha + \frac{2\pi}{3}\right)\cos\left(\alpha + \frac{4\pi}{3}\right).$

17. 4

Statement-I: $\cos(A-B) = \frac{4}{5}$

$\Rightarrow \frac{1-\tan^2\left(\frac{A-B}{2}\right)}{1+\tan^2\left(\frac{A-B}{2}\right)} = \frac{4}{5} \Rightarrow \frac{2\tan^2\left(\frac{A-B}{2}\right)}{2} = \frac{1}{9}.$

$\Rightarrow \tan\left(\frac{A-B}{2}\right) = \frac{1}{3}$ [as a > b $\Rightarrow$ A > B]

Using $\tan\left(\frac{A-B}{2}\right) = \left(\frac{a-b}{a+b}\right)\cot\frac{C}{2}$

We get, $\frac{1}{3} = \frac{6-3}{6+3}\cot\frac{C}{2} \Rightarrow \cot\frac{C}{2} = 1$

$\Rightarrow \angle C = 90^0 \Rightarrow$ Statement-I is false.

Statement-II: Using sine law in ΔABC, we get $\frac{a}{\sin A} = \frac{c}{\sin C}$

$\Rightarrow \frac{a}{\sin A} = \frac{\sqrt{a^2+b^2}}{\sin\frac{\pi}{2}} \Rightarrow \frac{6}{\sin A} = \sqrt{45} \Rightarrow \sin A = \frac{2}{\sqrt{5}}.$

$\therefore$ Statement-II is true

18. 1

$(b-c)\cot\frac{A}{2} = k(\sin B - \sin C)\cot\frac{A}{2}$

$= 2k\cos\frac{B+C}{2}\sin\frac{B-C}{2}\cot\frac{A}{2} = 2k\sin\frac{A}{2}.\sin\frac{B-C}{2}.\frac{\cos\frac{A}{2}}{\sin\frac{A}{2}}$

$= 2k\sin\left(\frac{B-C}{2}\right)\sin\left(\frac{B+C}{2}\right) = 2k\left(\sin^2\frac{B}{2} - \sin^2\frac{C}{2}\right)$

or we get L.H.S. $=\Sigma 2k\left(\sin^2\frac{B}{2}-\sin^2\frac{C}{2}\right)=0$.

19. 3 Let $a=\alpha-\beta, b=\alpha+\beta, c=\sqrt{3\alpha^2+\beta^2}$

$\therefore \cos C=\dfrac{a^2+b^2-c^2}{2ab}$

$\Rightarrow \cos C=\dfrac{\alpha^2+\beta^2-2\alpha\beta+\alpha^2+\beta^2+2\alpha\beta-3\alpha^2-\beta^2}{2(\alpha^2-\beta^2)}$

$\Rightarrow \cos C=-\dfrac{(\alpha^2-\beta^2)}{2(\alpha^2-\beta^2)}=\cos\left(\dfrac{2\pi}{3}\right)\Rightarrow \angle C=\dfrac{2\pi}{3}$, (largest angle).

20. 1 In ΔBC_1M ; $BM=(C_1M)\cdot\cot 30^0\Rightarrow BM=\sqrt{3}$

$\Rightarrow$ Similarly, $CN=\sqrt{3}$ and $MN=C_1C_2=1+1=2$

Hence, side $BC=\sqrt{3}+\sqrt{3}+2=2(1+\sqrt{3})$

$\Rightarrow$ Area of equilateral triangle

$=\dfrac{\sqrt{3}}{4}[2(1+\sqrt{3})]^2=6+4\sqrt{3}$ *sq units.*

SECTION II (NUMERICAL)

21. 1 $\dfrac{\sqrt{2}\sin\alpha}{\sqrt{2}\cos\alpha}=\dfrac{1}{7}\Rightarrow \tan\alpha=\dfrac{1}{7}$

$\sin\beta=\dfrac{1}{\sqrt{10}}\Rightarrow \tan\beta=\dfrac{1}{3}\Rightarrow \tan 2\beta=\dfrac{3}{4}$

$\tan(\alpha+2\beta)=\dfrac{\tan\alpha+\tan 2\beta}{1-\tan\alpha\tan 2\beta}=1$

22. 4 $1+\tan^2\theta = 2(\tan^2\phi+1)$

$\sec^2\theta = 2\sec^2\phi$

$2\cos^2\theta = \cos^2\phi$

$\cos 2\theta+\sin^2\phi = 2\cos^2\theta-1+\sin^2\phi$

$= \sin^2\phi+\cos^2\phi-1$

$= \underline{0}$.

23. 4

$|x| + |y| = 2 \Rightarrow |x|, |y| \in [0,2]$

Also, $\sin\left(\frac{\pi x^2}{3}\right) = 1 \Rightarrow \frac{\pi x^2}{3} = (4n+1)\frac{\pi}{2} \Rightarrow x^2 = (4n+1)\frac{3}{2}$

$\because |x| \in [0,2]$, then only possible value of x^2 is $\frac{3}{2}$

$\therefore |x| = \sqrt{\frac{3}{2}}, |y| = 2 - \sqrt{\frac{3}{2}}$

Hence, total number of ordered pairs is 4

24. 4

$\log_{\sin x} \sqrt{\sin^2 x} + \log_{\cos x} \sqrt{\cos^2 x} = 2$.

$\therefore$ sin x > 0 and $\sin x \neq 1$

cos x > 0 and $\cos x \neq 1$

Domain $x \in \left(0, \frac{\pi}{2}\right) \cup \left(2\pi, \frac{5\pi}{2}\right) \cup \left(4\pi, \frac{9\pi}{2}\right)$

$\Rightarrow$ Number of integers = 1 + 1 + 2= 4.

25. 6

$$\text{L.H.S} = \frac{\cos 5A \sin A + \sin 5A \cos A}{\sin A \cos A} = \frac{2 \sin 6A}{\sin 2A}$$

$$= \frac{2\left[3\sin 2A - 4\sin^3 2A\right]}{\sin 2A}$$

$$= 6 - 8\sin^2 2A = 6 - 4(1 - \cos 2A)$$

$$= 2 + 4\cos 2A \Rightarrow a + b = 6$$

JEE ADVANCED LEVEL

SECTION III

26. D $x^2 - 2x + 4 = -3\cos(ax+b) \Rightarrow (x-1)^2 + 3 = -3\cos(ax+b)$

As $-1 \le \cos(ax+b) \le 1$ and $(x-1)^2 \ge 0$

$\therefore$ equation (i) is only possible if, $\cos(ax+b) = -1$ *and* $(x-1) = 0$

so $a+b = \pi, 3\pi, 5\pi$,......and $3\pi > 6 \Rightarrow a+b = \pi$ where $a+b \le 6$

27. D Taking first two members, then $2\cos(\alpha+\theta)\sin(\gamma+\alpha) = 2\cos(\beta+\theta)\sin(\beta+\gamma)$

$\Rightarrow \sin(2\alpha+\theta+\gamma) - \sin(\theta-\gamma) = \sin(2\beta+\theta+\gamma) - \sin(\theta-\gamma)$

$\Rightarrow \sin(2\alpha+\theta+\gamma) = \sin(2\beta+\theta+\gamma)$

$\Rightarrow 2\alpha+\theta+\gamma = n\pi + (-1)^n(2\beta+\theta+\gamma)$ for $n = 0,$ $2\alpha+\theta+\gamma = 2\beta+\theta+\gamma$

$\therefore\ \alpha = \beta$

Similarly taking last two members, we get $\beta = \gamma$

Hence, $\alpha = \beta = \gamma$

Also, take $n = 1, -1$

Then, we get $\alpha + \beta + \gamma + \theta = \frac{\pi}{2}, -\frac{\pi}{2}$

$$\therefore\ k = \frac{\cos(\alpha + \theta)}{\sin(\beta + \gamma)} = \frac{\cos\left(\frac{\pi}{2} - (\beta + \gamma)\right)}{\sin(\beta + \gamma)} = 1 \text{ and } k = \frac{\cos\left(-\frac{\pi}{2} - (\beta + \gamma)\right)}{\sin(\beta + \gamma)} = -1$$

Hence, $k = \pm 1$

28. C Conceptual

29. A Apply componendo and dividendo We get $\frac{\sin 2\alpha}{\sin 2(\beta - \gamma)} = \frac{\sin(\gamma + \beta)}{\sin(\gamma - \beta)}$

$$\Rightarrow \sin 2(\beta - \gamma)\sin(\beta + \gamma) + \sin 2\alpha \sin(\beta - \gamma) = 0$$

$$\Rightarrow \sin(\beta - \gamma)\left(2\cos(\beta - \gamma)\sin(\beta + \gamma) + \sin 2\alpha\right) = 0$$

$$\Rightarrow \sin(\beta - \gamma)\left(\sin 2\alpha + \sin 2\beta + \sin 2\gamma\right) = 0$$

30. A $\dfrac{\frac{x}{y}\tan A + \tan B}{\frac{x}{y} + 1} = \dfrac{\sin A + \sin B}{\cos A + \cos B} = \tan\dfrac{(A + B)}{2}$

31. A The equation clearly holds if $\sin x = 1$

SECTION IV (More than one correct)

32. A,B $\cos x + \cos y = -\cos z$ ....(1)

$\sin x + \sin y = -\sin z$ ....(2)

squaring and adding

$$\cos^2 x + \cos^2 y + 2\cos x \cos y = \cos^2 z$$

$$\sin^2 x + \sin^2 y + 2\sin x \sin y = \sin^2 z$$

$$1 + 1 + 2\cos(x - y) = 1$$

$$2\left(1 + \cos(x - y)\right) = 1$$

$$2 \times 2\cos^2\left(\frac{x - y}{2}\right) = 1$$

$$\cos\left(\frac{x - y}{2}\right) = \pm\frac{1}{2}$$

33. A,D $2x = \cos(\alpha - \beta - \gamma + \delta) - \cos(\alpha - \beta + \gamma - \delta)$

$2y = \cos(\beta - \gamma - \alpha + \delta) - \cos(\beta - \gamma + \alpha - \delta)$
and similarly for 2z
Adding $2x + 2y + 2z = 0$
$\Rightarrow x + y + z = 0$ then $x^3 + y^3 + z^3 = 3xyz$
Hence, a and d are correct answer

34. A,C $x = \sum_{n=0}^{\infty} \cos^{2n}\phi = \operatorname{cosec}^2\phi$; $y = \sum_{n=0}^{\infty} \sin^{2n}\phi = \sec^2\phi$

$$z = \sum_{n=0}^{\infty} \cos^{2n}\phi \sin^{2n}\phi = \frac{1}{1-\cos^2\phi\sin^2\phi} = \frac{1}{1-\frac{1}{xy}} = \frac{xy}{xy-1}$$

so xyz = xy + z or xyz = x + y + z as xy = x + y.

35. B,C $\sin x = \frac{1+\sqrt{1+4a}}{2}$

We must have $1 + 4a \geq 0$

and $1 \leq \frac{1+\sqrt{1+4a}}{2} \leq +1$

The above two conditions are equivalent to $-\frac{1}{4} \leq a < 0$.

For every a in this interval, the equation will give four values of x [Since over $(0, 2\pi)$ any value by sinx is attained for two values of x and the quadratic is giving two values of sin x]

SECTION V - (Numerical type)

36. 3 $\tan 2x(\tan 2x + \tan 3x) = 1 - \tan 2x \tan 3x$

$\Rightarrow \tan 2x \tan 5x = 1 \Rightarrow \cos 7x = 0$

37. 6 $\frac{\sin x}{\cos 3x} + \frac{\sin 3x}{\cos 9x} + \frac{\sin 9x}{\cos 27x} = 0$

$$\frac{1}{2}\left(\frac{\sin 2x}{\cos 3x \cos x} + \frac{\sin 6x}{\cos 3x \cos 9x} + \frac{\sin 18x}{\cos 9x \cos 27x}\right) = 0$$

$$\frac{1}{2}\left((\tan 3x - \tan x + \tan 9x - \tan 3x + \tan 27x - \tan 9x\right) = 0$$

$\tan 27x - \tan x = 0$
$\sin 27x.\cos x - \sin x \cos 27x = 0$
$\sin(27x - x) = 0$
$\sin 26x = 0$
$26x = n\pi$
$x = \frac{n\pi}{26}$

$n=1 \Rightarrow x=\frac{\pi}{26},\frac{2\pi}{26},\frac{3\pi}{26},\frac{4\pi}{26},\frac{5\pi}{26},\frac{6\pi}{26}$

Number of solutions : 6

38. 2 $\text{LHS}=16\left(\cos\theta-\cos\frac{\pi}{8}\right)\left(\cos\theta-\cos\frac{3\pi}{8}\right)\times\left(\cos\theta-\cos\frac{5\pi}{8}\right)\left(\cos\theta-\cos\frac{7\pi}{8}\right)$

$=16\left(\cos^2\theta-\cos^2\frac{\pi}{8}\right)\left(\cos^2\theta-\cos^2\frac{3\pi}{8}\right)=16\left(\cos^4\theta-\cos^2\theta+\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right)$

$=16\left(\cos^4\theta-\cos^2\theta+\frac{1}{8}\right)=16\left(-\cos^2\theta\sin^2\theta+\frac{1}{8}\right)=16\left(\frac{-\sin^2 2\theta}{4}+\frac{1}{8}\right)$

$16\left(\frac{1-2\sin^2 2\theta}{8}\right)=16\left(\frac{\cos^2 2\theta-\sin^2 2\theta}{8}\right)=\frac{16\cos 4\theta}{8}$

39. 2 $\frac{1}{4\cos^2\theta+1+\frac{3}{2}\sin 2\theta} \Rightarrow \frac{1}{2[1+\cos 2\theta]+1+\frac{3}{2}\sin 2\theta}$

lies between $\frac{1}{2}$ *to* $\frac{11}{2}$

$\therefore$ maximum value is 2.

Minimum value of 1+4 $\cos^2\theta+3\sin\theta\cos\theta$

$1+\frac{4(1+\cos 2\theta)}{2}+\frac{3}{2}\sin 2\theta=1+2+2\cos 2\theta+\frac{3}{2}\sin 2\theta$

$3+2\cos 2\theta+\frac{3}{2}\sin 2\theta \therefore =3-\sqrt{4+\frac{9}{4}}=3-\frac{5}{2}=\frac{1}{2}$

So maximum value of $\frac{1}{4\cos^2\theta+1+\frac{3}{2}\sin 2\theta}$ *is* 2

SECTION VI - (Matrix match type)

40. A-Q,B-Q,C-P,D-R

(i) Draw the graphs of $y=\cos x$ and $y=\log x$

(ii) If ‘a’ is irrational then x = 0 is the only real root of the given equation.

(iii) $|4\sin 2x+\cos 2x|\le\sqrt{17}$

(iv) The given equation is a quadratic $\sin x$ solving, $\sin x=\frac{\sqrt{3}}{2}$ or $\frac{1}{2}$