MOTION ALONG A STRAIGHT LINE

Kinematics

The branch of mechanics that deals with the motion of objects without reference to the forces which cause the motion. i.e. in kinematics we don’t consider the cause and effects of motion

Position

The location of a point in space at an insant of time is known as position. A particle’s position is characterised by 2 factors

1. Its distance from the observer
2. Its direction with respect to observer

Rest and motion

If a body does not change its position as time passes, with respect to frame of reference, it is said to be at rest

If the body changes its position w.r.t frame of reference as time passes, it is said to be in motion

One dimensional motion

The motion of a body is said to be one dimensional if its motion is confined along a straight line.

In this type of motion, the position and motion can be represented with the help of just one coordinate

Eg. Motion of a freely falling body

Motion of a car on a straight road

Two dimensional motion

The motion of a body is known as two dimensional motion, if it is confined to move along a plane.

In this case, the position of the body can be represented using just two coordinates

Eg. Motion of a ship on calm water

Motion of an ant on a wall

Three dimensional motion

The motion is known as three dimensional motion if the particle can move in any direction in space.

In this case, we need three coordinates to represent its position

Eg. Motion of a flying insect.

Motion of a dust particle in air

Particle / Point mass

If the size of a body is negligible in comparison to its range motion then that body is termed as a particle. If all the points of a body have same velocity and displacement, then that body can be treated as a particle / point mass

Distance and Displacement

Distance : It is the actual length of the path covered by a moving particle in a given interval of time

Distance is a scalar quantity.

Dimensional formula $M^0L^1T^0$

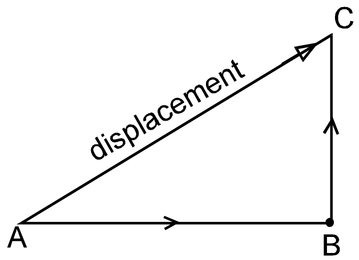
Unit : metre (m)

Displacement : Displacement is change in position. It is the vector joining initial and final position. Its magnitude is shortest distance between initial and final position.

Its direction is from initial position to final position

Dimensional formula $M^0L^1T^0$

Unit : metre (m)



If a body moves from A to B to C

distance = AB + BC

displacement = $\overrightarrow{AC}$

Comparison

→ distance $\geq |\text{displacement}|$

distance = $|\text{displacement}|$, when a particle moves along a straight line without reversing its direction

→ distance can only have positive value

Displacement can be positive, negative or zero. Sign of displacement gives its direction

→ For a moving particle, displacement can be zero, but distance can never be zero

→ If $\vec{r}_i$ and $\vec{r}_f$ are the initial and final position vectors of a particle, displacement $\vec{S} = \vec{r}_f - \vec{r}_i$

Speed and Velocity

Speed : The distance covered by a particle per unit time is known as speed. Like distance, speed is a scalar quantity. Dimension LT^{-1}, unit : ms^{-1}

→ **Uniform speed** : When a particle covers equal distances in equal intervals of time, it is said to be moving with uniform speed. Here the value of speed is same irrespective of time interval chosen.

Non-uniform speed : Speed is non uniform if the particle covers different distances in same intervals of time.

→ **Average speed**

Average speed of a particle in an interval of time is the ratio of total distance travelled by that particle to the total time taken.

Average speed = $\frac{\text{distance travelled}}{\text{time taken}} = \frac{\Delta d}{\Delta t}$

Different expressions for average speed

→ When a particle moves with speeds v_1, v_2, v_3 etc., average speed is

$$V_{av} = \frac{d_1 + d_2 + ...d_n}{t_1 + t_2 + ...t_n} = \frac{v_1 t_1 + v_2 t_2 ....v_n t_n}{t_1 + t_2 + ...t_n} = \sum \frac{v_i t_i}{\sum t_i}$$

When a particle travels with speed v_1, v_2, v_3 etc for equal intervals of time

$$V_{av} = \frac{v_1 + v_2 + ...v_n}{n}$$

→ When particle travels distances $d_1, d_2, d_3...$ etc. with speeds $v_1, v_2....$ etc. average speed is

$$V_{av} = \frac{d_1 + d_2 + ...d_n}{\frac{d_1}{v_1} + \frac{d_2}{v_2} + ...\frac{d_n}{v_n}}$$

→ When particle covers half distance with v_1 and rest half with v_2, then

$$V_{av} = \frac{2 v_1 v_2}{v_1 + v_2}$$

Instantaneous Speed

It is the speed at a particular instant of time. it is the ratio of distance covered to an infinitesimally small interval of time i.e. $\Delta t \to 0$

if s is distance

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

Velocity

The rate of change of position is known as velocity. It is the displacement per unit time. Its a vector quantity. Unit is ms^{-1}. Dimensional formula is $M^0 L T^{-1}$

→ Uniform velocity : A particle is said to have uniform velocity if the magnitude and direction of its velocity doesn't change with time. In this case, the particle moves along a straight line with constant speed.

→ Non-uniform velocity

A particle is said to have non-uniform velocity, if either the magnitude or direction of velocity changes with time, (or both magnitude and direction changes)

→ Average velocity

It is the ratio of total displacement to time taken.

Average velocity = $\dfrac{\text{displacement}}{\text{time}}, \overrightarrow{V_{av}} = \dfrac{\overrightarrow{\Delta r}}{\Delta t}$

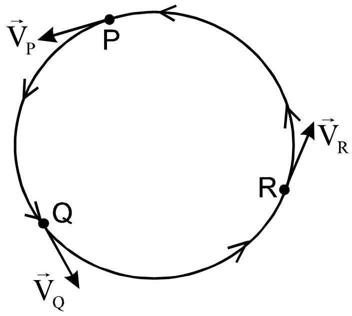
Instantaneous Velocity

It is the rate at which position is changing at an instant of time. Here the time interval considered is infinitesimally small.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

→ Instantaneous velocity is always tangential to the path followed by the particle

Eg.



→ The magnitude of instantaneous velocity is equal to instantaneous speed.

→ If a body is moving with uniform velocity, instantaneous velocity is equal to average velocity

→ First derivative of displacement is velocity

Acceleration

The rate of change of velocity is known as acceleration

Its a vector quantity, its direction is that of change in velocity

In one dimensional motion, the direction of velocity and acceleration can be parallel or anti-parallel

→ If velocity and acceleration are parallel, speed increases and if velocity and acceleration are anti-parallel, the speed decreases. i.e. the body slows down

Unit : $\dfrac{\text{metre}}{(\text{second})^2} = \dfrac{m}{s^2}$

Dimensional formula : $M^0L^1T^{-2}$

Uniform acceleration

A body is said to have uniform acceleration if the magnitude and direction of acceleration doesn't change with time.

Eg. Motion of a freely falling body is uniformly accelerated

Non-uniform acceleration : Acceleration is non-uniform if direction or magnitude (or both) of acceleration changes during motion

Average acceleration

$$\vec{a}_{av} = \frac{\text{Total change in velocity}}{\text{time}}$$

$$\vec{a}_{av} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t}$$

Instantaneous acceleration

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}_i}{\Delta t} = \frac{d\vec{v}}{dt}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\vec{x}}{dt}\right) = \frac{d^2\vec{x}}{dt^2}$$

first derivative of position is velocity and second derivative of position is acceleration

Equations of kinematics

There are equations relating u, v, a, t and s for a moving particles where

u = initial velocity of the particle

v = final velocity

a = acceleration

t = time

s = distance travelled by particle

s_n = distance travelled by the particle in n^{th} second

CASE-1

Uniform motion (a = 0)

→ Direction and speed doesn't change

→ Distance = $|\text{displacement}|$

→ v = u

→ s = ut

CASE-2

(Uniformly accelerated motion)

→ Here the magnitude and direction of acceleration remains constant

→ Uniformly accelerated motion is one-dimensional if velocity and acceleration are along same direction or along opposite direction

Equations for motion for uniformly accelerated motion

→ v = u + at

→ $s = ut + \frac{1}{2}at^2$

→ $v^2 = u^2 + 2as$

→ $S_n = u + \frac{a}{2}(2n-1)$

→ For uniformly accelerated motion, instantaneous acceleration = average acceleration

→ for uniformly accelerated motion

Average velocity $V_{av} = \frac{u+v}{2}$

Hence $S = v_{av}t = \left(\frac{u+v}{2}\right)t$

→ for uniformly accelerated motion starting from rest

$s \propto t^2$

→ If a body starting from rest has uniform acceleration, distance travelled in n^{th} second is proportional to (2n - 1)

$S_n \propto (2n-1)$

$S_1 : S_2 : S_3 = 1:3:5:......$

→ If a body travels from A to B with uniform acceleration such that v_1 = velocity at A, v_2 = velocity at B then velocity at midpoint of AB

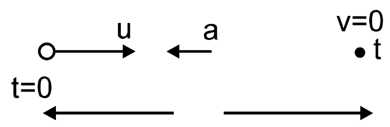
$$v = \sqrt{\frac{v_1^2 + v_2^2}{2}}$$

→ **Retardation / Deceleration**

When the velocity and acceleration of a body are along opposite direction, the speed of the body decreases. In this case, the body is said to be decelerated.

i.e. If the deceleration of a body is a, it means that the speed of the body is decreasing at the rate a. Then acceleration is –a

Stopping time and stopping distance



Let the speed of a body be u at time t = 0. Its speed decreases at a rate a. It will come to rest in time t, after covering a distance s

Stopping time $t = \frac{u}{a}$	$o = u + (-a)t$ $t = \frac{u}{a}$
Stopping distance $S = \frac{u^2}{2a}$	$o = u^2 + 2(-a)s$ $s = \frac{u^2}{2a}$

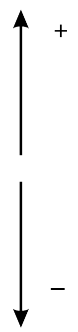
MOTION UNDER GRAVITY

Motion under the effect of only gravity near the surface of earth is an example of uniformly accelerated motion

If a body is moving under the effect of only gravity near the surface, it will experience an acceleration of 9.8 ms^{-2} downwards. This acceleration is known as acceleration due to gravity.

Two approaches commonly used in motion under gravity

Here use take the upward direction as positive and downward direction as negative direction



Here a = –g

Then equations of motion can be represented as

$$v = u - gt$$

$$s = ut - \frac{1}{2}gt^2$$

$$v^2 = u^2 - 2gs$$

FREE FALL FROM REST

Here, the body is dropped from rest from a height

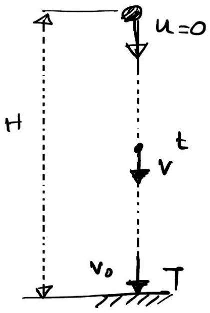
Hence a = -g

u=0

velocity after time t, v= gt (v negative)

Displacement after time t

$$y=\frac{1}{2}gt^2$$



$$S_n=\frac{g}{2}(2n-1)$$

$$S_1=\frac{g}{2},S_2=\frac{3g}{2},S_3=\frac{5g}{2}$$

$$S_1:S_2:S_3=1:3:5...$$

$\rightarrow$Time taken to reach the ground

When the body reaches the ground

S=H, t= T

We know $S=\frac{1}{2}gt^2$

$$H=\frac{1}{2}gT^2$$
$$T=\sqrt{\frac{2H}{g}}$$

Speed with which body hits the ground

$v^2=2gS,$ when body reaches ground

$$V_0^2=2gH$$
$$V_0=\sqrt{2gH}$$

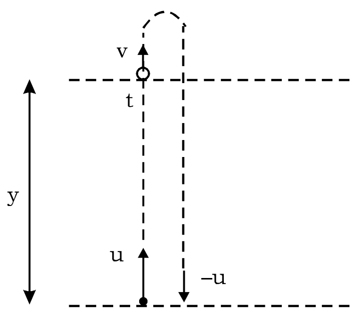
Vertical Projection

Here a body is projected from the ground with a speed. Here are consider the upward direction as positive and downward direction as negative direction

Then a= –g

‘t’ seconds after projection, velocity of the body is ‘v’ and the height above the ground is ‘y’.

Then $v = u - gt$ $\qquad y = ut - \frac{1}{2}gt^2$



Then $v = u - gt$

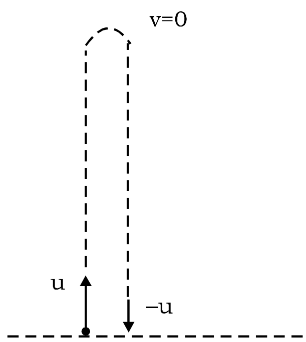
$$y = ut - \frac{1}{2}gt^2$$

At the topmost point, velocity is zero

If t_a is time of ascent

$$0 = u - gt_a$$

$$t_a = \frac{u}{g}$$



Time of flight

When $t = T, y = 0$

$$0 = uT - \frac{1}{2}gT^2$$

$$T = \frac{2u}{g}$$

$\therefore$ Time of descent $t_d = T - t_a = \frac{u}{g}$; Hence $t_a = t_d$

If air resistance is also taken into account, $t_a < t_d$

Maximum Height

At maximum height

y= H, v= 0

$$v^2 = u^2 - 2gy$$

$$o = u^2 - 2gH$$

$$H = \frac{u^2}{2g}$$

Motion with variable acceleration

Case-1 Acceleration is a function of position

$$a = f(t)$$

$$\frac{dv}{dt} = a$$

$$dv = adt$$

$\int_u^v dv = \int_0^t adt$; **v-u** $= \int_0^t adt$

$\boxed{v = u + \int_0^t adt}$; $\frac{dx}{dt} = u + \int_0^t adt$

displacement $S = ut + \int_0^t \left[\int_0^t adt\right] dt$

Case -2 : Acceleration is a function of position

a= f(x)

$$v\frac{dv}{dx} = a$$

$$v.dv = a \cdot dx$$

$$\int_u^v v \cdot dv = \int_{x_1}^{x_2} adx$$

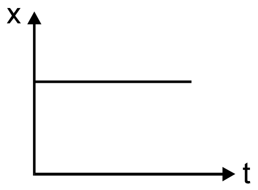
$\frac{v^2 - u^2}{2} = \int_{x_1}^{x_2} adx$; $\boxed{v^2 = u^2 + 2\int_{x_1}^{x_2} adx}$

GRAPHICAL ANALYSIS OF 1-D MOTION

→ POSITION TIME GRAPH

Position time graph is plotted by taking time t along horizontal axis and position x on vertical axis

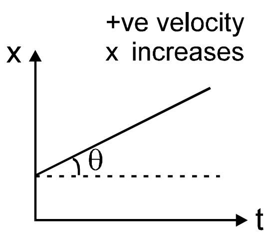
* Slope of position time graph at any instant of time gives instantaneous velocity

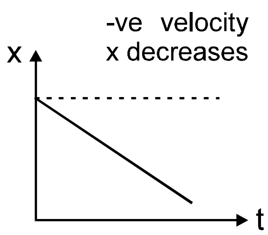


v = slope = 0

Uniform velocity

Graph : Straight line having non-zero slope

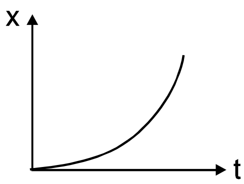




$$v = \frac{dx}{dt}$$

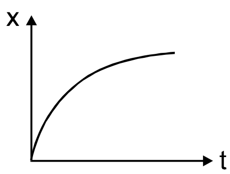
Uniformly accelerated motion

x – t graph will be parabolic



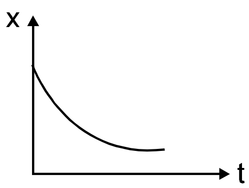
v increases with time

v is +ve, a is +ve



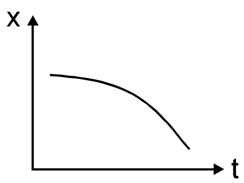
v decreases with time

v is +ve, a is -ve



Speed decreases with time

v is -ve,
a is +ve



Speed increases with time

v is -ve,
a is -ve

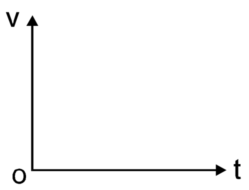
Velocity - time graph

v–t graph is plotted by taking time on horizontal axis and velocity on vertical axis

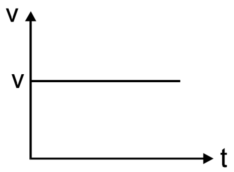
→ Slope of velocity time graph gives acceleration

→ Area under v – t graph gives displacement. If only the magnitude of area is considered, it gives distance

A body at rest

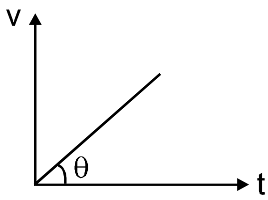


Uniform motion



acceleration = slope = 0

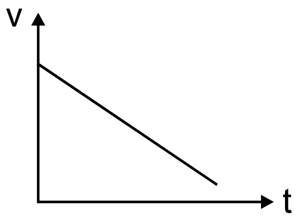
Uniformly accelerated motion



$a = \tan\theta$

u = 0

positive acceleration

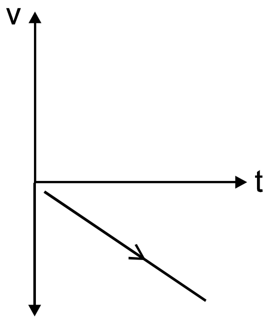


$u \neq 0$

Positive velocity

negative acceleration

speed decreases



negative velocity

negative acceleration

speed increases

ACCELERATION TIME GRAPH

This is plotted by taking acceleration along vertical axis and time along horizontal axis

Area under acceleration time graph gives “change” in velocity

RELATIVE MOTION IN ONE DIMENSIONS

If the position of two particles A and B are x_A and x_B, then the position of A relative to B

$$x_{AB} = x_A - x_B$$

$$x_{BA} = x_B - x_A$$

$$x_{AB} = -x_{BA}$$

Relative displacement

If S_A = displacement of A

S_B = displacement of B

then

$$S_{AB} = S_A - S_B$$

$$S_{BA} = S_B - S_A$$

$$S_{AB} = -S_{BA}$$

Relative Velocity

$V_A = \frac{dx_A}{dt}$ = velocity of A

$V_B = \frac{dx_B}{dt}$ = velocity of B

$$V_{AB} = \frac{dx_{AB}}{dt} = \frac{dx_A}{dt} - \frac{dx_B}{dt}$$

$$V_{AB} = V_A - V_B$$

$$V_{BA} = V_B - V_A$$

$$V_{AB} = -V_{BA}$$

Relative acceleration

a_A = acceleration of A

a_B = acceleration of B

$$a_{AB} = a_A - a_B$$

$$a_{BA} = a_B - a_A$$

$$a_{AB} = -a_{BA}$$

Kinematics equations for Relative Motion

$$V_{REL} = U_{REL} + a_{REL}t$$

$$S_{REL} = U_{REL}t + \frac{1}{2}a_{REL}t^2$$

$$V_{REL}^2 = U_{REL}^2 + 2a_{REL}S_{REL}$$

V_{REL} = final relative velocity

U_{REL} = initial relative velocity

a_{REL} = relative acceleration

S_{REL} = relative displacement

Note :

When two particles are moving in same direction with speeds v_1 and v_2, relative “speed” is

$$|V_{REL}| = |V_2 - V_1|$$

When two particles are moving in opposite direction with speeds v_1 and v_2, relative “speed” is

$$|V_{REL}| = v_1 - (-v_2) = v_1 + v_2$$

→ When two particles are moving along same line and the distance between them is decreasing with time, the their relative speed is known as speed of approach

The time in which they will meet is

$$t = \frac{\text{initial separation}}{\text{speed of approach}}$$

→ When two particles are moving simultaneously under gravity, their relative acceleration is zero. Hence, relative velocity will be constant

→ If a boat has a still water speed v and river is flowing with speed u, then actual speed of boat while moving upstream

$$V_{upstream} = v - u$$

Speed of boat while moving downstream

$$V_{downstream} = v + u$$