CHAPTER - 09
HEAT AND THERMODYNAMICS

PART I - (JEEMAIN LEVEL)

SECTION - I

1. 1 Since, we have seen that $\frac{\Delta T_C}{100} = \frac{\Delta T_F}{180} = \frac{\Delta T}{100}$

$\Rightarrow \quad \Delta T_F = \frac{9}{5}\Delta T_C = \frac{9}{5}(90-30) = 108\ °F$

Further, rise in temperature for both the celcius and the kelvin scale is the same, so $\Delta T = \Delta T_C = 60$ K

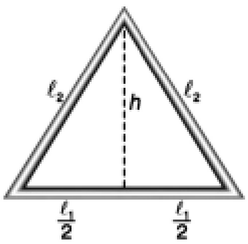
2. 2 **SOLUTION**

According to the problem, we have h = constant

$\Rightarrow \quad h^2 = l_2^2 - \frac{l_1^2}{4} = \text{constant}$

$\Rightarrow \quad \Delta(l_2^2) - \Delta\left(\frac{l_1^2}{4}\right) = 0$

$\Rightarrow \quad 2l_2\Delta l_2 - \frac{1}{4}2l_1\Delta l_1 = 0$



Since, by definition, $\Delta l = l\alpha\Delta T$

$\Rightarrow \quad \Delta l_2 = l_2\alpha_2\Delta T$ and $\Delta l_1 = l_1\alpha_1\Delta T$

$\Rightarrow \quad 2l_2(l_2\alpha_2\Delta T) = \frac{1}{4}(2l_1)(l_1\alpha_1\Delta T)$

$\Rightarrow \quad l_2^2\alpha_2 = \frac{1}{4}l_1^2\alpha_1$

$\Rightarrow \quad \frac{l_1^2}{l_2^2} = 4\left(\frac{\alpha_2}{\alpha_1}\right)$

$\Rightarrow \quad \frac{l_1}{l_2} = 2\sqrt{\frac{\alpha_2}{\alpha_1}}$

3. 2 Due to volume expansion of both liquid and vessel, the change in volume of liquid relative to container is given by $\Delta V = V_0[\gamma_L - \gamma_g]\Delta\theta$

Given V_0 = 1000 cc, $\alpha_g = 0.1\times10^{-4}$/°C

$\therefore \quad \gamma_g = 3\alpha_g = 3 \times 0.1 \times 10^{-4}$/°C $= 0.3\times10^{-4}$/°C

$\therefore \quad \Delta V = 1000\,[1.82 \times 10^{-4} - 0.3 \times 10^{-4}] \times 100$

$= 15.2$

4. 2 $m_1s_1(32-20) = m_2s_2(40-32)$

$$\frac{s_1}{s_2} = \frac{40-32}{32-20}$$

$$\frac{s_1}{s_2} = \frac{8}{12}$$

$$\frac{s_1}{s_2} = \frac{2}{3}$$

5. 2 Heat required for vapourisation = Rate × time

$$= 63 \times (30-20) = mL = 5 \times L$$

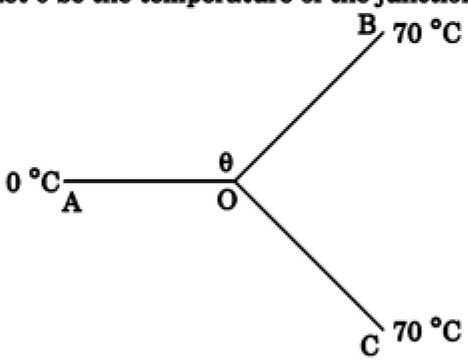
$$\Rightarrow L = 126 \text{ kJ/kg}$$

6. 1 10 kg ice at $-10°C$ + 40 kg of water at 45°C

$$10^4 \times \frac{1}{2} \times 10 + 10^4 \times 80 + 10^4(T-0)$$

$$= 40 \times 10^3 \times 1\,(45 - T) \Rightarrow T = 19°C$$

7. 2 **Let θ be the temperature of the junction**



If R is the thermal resistance of each rod
Then,
Heat current in BO + Heat current in CO
= Heat current in OA

$$\therefore \quad \frac{70-\theta}{R} + \frac{70-\theta}{R} = \frac{\theta - 0}{R}$$

$$\Rightarrow \quad 140 - 2\theta = \theta$$

$$\text{or} \quad \theta = \frac{140}{3} \approx 47\ °C$$

8. 3

As both rods are connected in series, the rate of flow of heat is same.

Let T be the temperature at the junction of rods.

$$A\frac{(T_1 - T)k_1}{l_1} = A\frac{(T - T_2)k_2}{l_2}$$

$$T = \frac{\boldsymbol{T_1 k_1 l_2 + T_2 k_2 l_1}}{\boldsymbol{k_1 l_2 + k_2 l_1}}$$

9. 2

From Newton's law of cooling

$$\frac{\theta_1 - \theta_2}{t} = -K\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right]$$

where θ_0 is the temperature of surrounding.

Initially, hot water cools from 60 °C to 50 °C in 10 minutes,

$$\frac{60 - 50}{10} = -K\left[\frac{60 + 50}{2} - \theta_0\right] \qquad ...(i)$$

Again, it cools from 50 °C to 42 °C in next 10 minutes.

$$\frac{50 - 42}{10} = -K\left[\frac{50 + 42}{10} - \theta_0\right] \qquad ...(ii)$$

Dividing equation (*i*) by (*ii*) we get

$$\frac{1}{0.8} = \frac{55 - \theta_0}{46 - \theta_0}$$

$$460 - 10\theta_0 = 440 - 8\theta_0$$

$$2\theta_0 = 20$$

$$\theta_0 = \mathbf{10\ °C}$$

10. 1

T is absolute temperature of body and λ_{max} is the wavelength of energy which is radiated maximum by the body. **B.**

So, $\lambda_{max_1} T_1 = \lambda_{max_2} T_2$

$$\Rightarrow \qquad \lambda_0 T = \frac{3\lambda_0}{4} T'$$

$$\Rightarrow \qquad T' = \frac{4}{3} T$$

Power radiated by the black body is proportional to the fourth power of the absolute temperature of the body. **B.**

$$\frac{P_2}{P_1} = \left(\frac{T'}{T}\right)^4 = \left(\frac{4}{3}\right)^4 = \frac{256}{81}$$

So, $$\frac{nP}{P} = \frac{256}{81}$$

$$n = \mathbf{\frac{256}{81}}$$

11. 4 $dQ = du + dW$ or $Q = (u_2 - u_1) + W$

$W = Q_{1b2} - (u_2 - u_1) = 36 - 30 = 6$ cal

12. 3 $\Delta U = nC_V\Delta T = n(5/2)R\Delta T$

$\Delta Q = nC_P\Delta T = n(7/2)R\Delta T$

$$W = \Delta Q - \Delta U = \frac{n7}{2}R\Delta T - \frac{n5}{2}R\Delta T = nR\Delta T$$

$$\frac{W}{\Delta U} = \frac{2}{5}$$

13. 3 Process AB is isochoric, $\therefore$ $W_{AB} = P\Delta V = 0$

Process BC is isothermal $\therefore$ $W_{BC} = RT_2 \cdot \ln\left(\frac{V_2}{V_1}\right)$

Process CA is isobaric

$$W_{CA} = nR\Delta T = R\left(T_1 - T_2\right)$$

14. 2 $$P_1V_1^{\gamma} = P_2V_2^{\gamma} \Rightarrow P_2 = \left(\frac{V_1}{V_2}\right)^{\gamma} \times P_1$$

$$P_2 = \left(\frac{V}{V/8}\right)^{4/3} \times P_0 = 16P_0$$

15. 2 $PV = \mu RT$

$$V = \frac{\mu RT}{P}$$

Slope = $\tan 53° = \dfrac{\mu R}{P \times 10^{-3}} = \dfrac{V}{T}$

$$\Rightarrow \frac{4}{3} = \frac{2 \times 25 \times 10^3}{3 \times P}$$

$$P = \frac{2 \times 25 \times 10^3}{3 \times \frac{4}{3}}$$

$P = 12500$

$P = 1.25 \times 10^4\ N/m^2$

SECTION - II

Numerical Type Questions

16. 20

17. 3

PART - II (JEE ADVANCED LEVEL)

SECTION - III (One correct answer)

18. C

Let T_{faulty} be the reading of faulty thermometer and T_C be the correct reading on Centigrade scale, then

$$\frac{T_{\text{faulty}}-10}{90-10}=\frac{T_C-0}{100-0}$$

Given that $T_{\text{faulty}}=T_C=T$ (say)

$$\Rightarrow \quad \frac{T-10}{90-10}=\frac{T-0}{100-0}$$

$$\Rightarrow \quad 100(T-10)=80T$$

$$\Rightarrow \quad 20T=1000$$

$$\Rightarrow \quad T=50\ ^\circ\text{C}$$

Hence, the correct answer is (C).

19. C

As two arms of U-tube are maintained at different temperatures, densities in two arms will be different but pressure at the bottom is same. So, we have

$$h_1\left(\frac{\rho_0}{1+\gamma T_1}\right)g=h_2\left(\frac{\rho_0}{1+\gamma T_2}\right)g$$

$$\Rightarrow \quad 49\left(\frac{\rho_0}{1+50\gamma}\right)=50\left(\frac{\rho_0}{1+60\gamma}\right)$$

$$\Rightarrow \quad 49+49(60\gamma)=50+50(50\gamma)$$

$$\Rightarrow \quad \gamma(2940-2500)=1$$

$$\Rightarrow \quad \gamma=\frac{1}{440}\approx 2.3\times10^{-3}\ ^\circ\text{C}^{-1}$$

Hence, the correct answer is (C).

20. C Heat released when 5 kg of water cools from 20 °C to 0 °C is

$$Q_1 = m_{\text{water}} c_{\text{water}} \Delta T = (5)(1)(20) = 100 \text{ kcal}$$

Heat required to raise temperature of 2 kg of ice from −20 °C to 0 °C is

$$Q_2 = m_{\text{ice}} c_{\text{ice}} \Delta T = (2)(0.5)(20) = 20 \text{ kcal}$$

Heat required to melt 2 kg of ice at 0 °C is

$$Q_3 = m_{\text{ice}} L_{\text{ice}} = (2)(80) = 160 \text{ kcal}$$

Since $Q_1 > Q_2$, so temperature of ice will reach 0 °C. However we observe that $Q_1 < Q_2 + Q_3$, therefore the complete ice will not melt and final mixture will have both ice and water. The amount of ice melted m is

$$m = \frac{\text{Available Heat}}{\text{Latent Heat}} = \frac{(100-20) \text{ kcal}}{80 \text{ calg}^{-1}} = 1 \text{ kg}$$

So in equilibrium, the vessel will have water given by

$$m'_{\text{water}} = 5 + 1 = 6 \text{ kg}$$

Hence, the correct answer is (C).

21. C

22. B Total work done in this cycle

$$\begin{aligned}\Delta W &= \Delta W_{AB} + \Delta W_{BC} + \Delta W_{CD} + \Delta W_{DA} \\ &= \Delta W_{AB} + 0 + \Delta W_{CD} + 0 \\ &= 2P_0(V_0) + P_0(V_0 - 2V_0) \\ &= 2P_0V_0 - P_0V_0 \\ &= P_0V_0\end{aligned}$$

$$\eta = \frac{\Delta W}{\Delta Q}$$

$$= \frac{P_0V_0}{\frac{13}{2}P_0V_0}$$

$$= \frac{2}{13}$$

Percentage efficiency $= \frac{2}{13} \times 100$

$$= \mathbf{15.38\% \approx 15.4\%}$$

23. C (*c*): Equation of line BC, point B(V_0, $3P_0$) and C($2V_0$, P_0)

$$P = P_0 - \frac{2P_0}{V_0}(V - 2V_0)$$

Using $PV = (1)RT$, we can eliminate P as follows:

$$T = \frac{P_0V - \frac{2P_0V^2}{V_0} + 4P_0V}{1 \times R} \qquad ...(i)$$

$$T = \frac{P_0}{R}\left[5V - \frac{2V^2}{V_0}\right]$$

For maxima or minima $\frac{dT}{dV} = 0$

$$\Rightarrow \qquad 5 - \frac{4V}{V_0} = 0, \Rightarrow V = \frac{5}{4}V_0$$

From equation (*i*)

$$T = \frac{P_0}{R}\left[\left(5 \times \frac{5V_0}{4}\right) - \left(\frac{2}{V_0} \times \frac{25}{16}V_0^2\right)\right]$$

$$T = \frac{25}{8}\frac{P_0V_0}{R}$$

24. ABCD
25. ABCD

Sol. 11 (All) At a given temperature the average kinetic energy per molecule is given by $(f/2)kT$ which is same for all diatomic gases hence option (A) is correct. *RMS* velocity of gas molecules at same temperature is inversely proportional to the square root of molar mass of gas hence option (B) is correct. For a gaseous mixture the pressure exerted by a gas is proportional to m/M hence option (C) is correct. From gas law $PV = NkT$ we can see that option (D) is also correct.

26. AD

Sol. 15 (A, D) The rate of collisions of the molecules with per square meter of the wall is $(1/6)n_0 v$ where n_0 is the molecular density and v is *RMS* speed of molecules and pressure exerted by the gas on wall is given by $(1/6)n_0 v \times 2m'v$ where m' is the mass of each molecule.

SECTION - V (Numerical Type - Upto two decimal place)

27. 30

Sol

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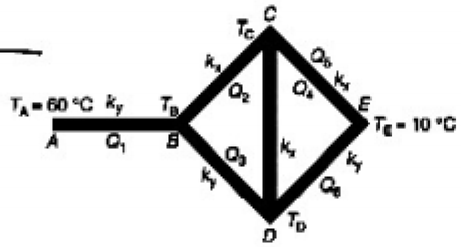
(5)

(6)



At junction B, we have

$$\frac{k_yA(60-T_B)}{l}=\frac{k_xA(T_B-T_C)}{l}+\frac{k_yA(T_B-T_D)}{l}$$

$$\Rightarrow \quad k_y(T_A-T_B)=k_x(T_B-T_C)+k_y(T_B-T_D)$$

$$\Rightarrow \quad 60-T_B=\frac{k_x}{k_y}(T_B-T_C)+k_y(T_B-T_D)$$

Given that $\frac{k_x}{k_y}=\frac{9.2\times10^{-2}}{4.6\times10^{-2}}=2$, so we get

$$(60-T_B)=2(T_B-T_C)+(T_B-T_D)$$

$$\Rightarrow \quad 4T_B-2T_C-T_D=60 \qquad \ldots(1)$$

At junction C, we have

$$\frac{k_xA(T_B-T_C)}{l}=\frac{k_xA(T_C-T_D)}{l}+\frac{k_xA(T_C-10)}{l}$$

$$\Rightarrow \quad -T_B+3T_C-T_D=10 \qquad \ldots(2)$$

At junction D, we have

$$\frac{k_yA(T_B-T_D)}{d}+\frac{k_xA(T_C-T_D)}{d}=\frac{k_yA(T_D-10)}{d}$$

$$\Rightarrow \quad (T_B - T_D) + \frac{k_x}{k_y}(T_C - T_D) + (T_D - 10)$$

as.
at
D.

Since $\frac{k_x}{k_y} = 2$, so we get

$$(T_B - T_D) + 2(T_C - T_D) = (T_D - 10)$$

$$\Rightarrow \quad T_B + 2T_C - 4T_D = -10 \qquad \ldots(3)$$

Solving equation (1), (2) and (3), we get

Y

28. 0.3 If emissivity of body is e, the power required equals the rate of emission of radiation.

$$P = e\sigma A\left(T^4 - T_5^4\right) \qquad \ldots(1)$$

For a black body

$$P_B = \sigma A\left(T^4 - T_5^4\right) \qquad \ldots(2)$$

Dividing, we get $\frac{P}{P_B} = e$

$$\Rightarrow \quad e = \frac{210}{700} = 0.3$$

29. Since, $T_i = 400$ K and $T_f = 2T_i = 800$ K

$$\Rightarrow \quad \Delta T = T_f - T_i = 400 \text{ K}$$

$$\Rightarrow \quad \Delta U = nC_V \Delta T$$

$$\Rightarrow \quad \Delta U = (1)\left(\frac{3}{2}R\right)(400) = 600R$$

The given process is V^2T = constant

Substituting $T = \frac{PV}{R}$, we get

$$PV^3 = \text{constant}$$

Comparing this equation with equation of a polytropic process i.e., PV^x = constant we observe that $x = 3$ and so molar heat capacity of polytropic process is

$$C = C_V + \frac{R}{1-x} = \frac{3R}{2} + \frac{R}{1-3} \qquad \left\{\because C_V = \frac{3R}{2}\right\}$$

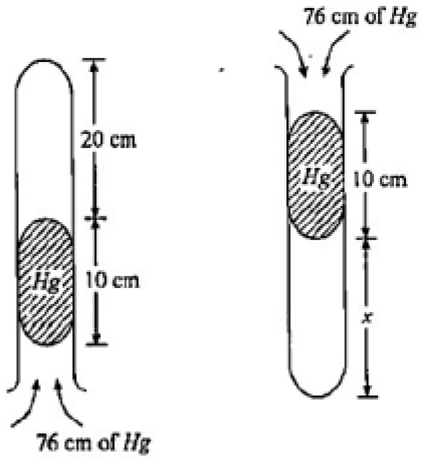
$$\Rightarrow \quad C = \frac{3}{2}R - \frac{R}{2} = R$$

Since, $Q = nC\Delta T = (1)(R)(400) = 400R$

Now from First Law, we have

$$W = Q - \Delta U = -200R$$

30. ***Solution***



(a) (b)

Figure-2.17(a) shows the initial state of the tube. Here pressure below the mercury pallet is the atmospheric pressure. 76 cm of Hg and due to opposite of pallet's weight the pressure of air column is $76 - 10 = 66$ cm of Hg.

When the tube is inverted with its open end upward, situation is shown in figure-2.17(b). If the length of air column is x and the pressure on this air column is atmospheric pressure planes the weight of mercury pallet which becomes $76 + 10 = 86$ cm of Hg, then according to Boyle's Law we have

$$P_1V_1 = P_2V_2$$

$$66 \times 20A = 86 \times xA$$

[If A is the cross sectional area of tube]

or $\qquad x = 15.35$ cm