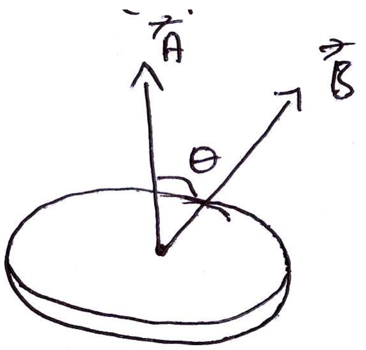
ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

Magnetic flux : Flux is a latin word which means 'flow'. Here flow means flow of a vector field through an area which is inside that field and it is numerically equal to the number of lines of forces passing through given area which is perpendicular to that area

$\phi = \vec{B}.\vec{A} = BA\cos\theta$



If the coil has N number of turns then $\phi = NBA\cos\theta$. Magnetic lines of force are imaginary, magnetic flux is a real scalar physical quantity with dimensions

$$\phi = B \times \text{area} = \frac{F}{IL} \times L^2 = \frac{MLT^{-2}L^2}{AL}$$

= $ML^2T^{-2}A^{-1}$

So unit of flux is Joule / Ampere

= $\frac{\text{Joule} \times \text{second}}{\text{coulomb}} = \text{Wb or } Tm^2$

C.g.s unit of magnetic flux is Maxwell (Mx)

1 Wb = 10^8 Mx

If $\theta = 0^o$ then $\phi_{max} = BA$, if $\theta = 90^o$ then $\phi_{min} = 0$

If $0 \le \theta < 90^o$ flux ϕ is +ve and

If $\theta = 90^o$ flux ϕ is zero

If $90^o < \theta \le 180^o$ flux ϕ is –ve

Electromagnetic Induction : The phenomenon of producing induced emf and hence current in a closed circuit due to the change in manetic flux associated with it.

Faraday's Laws of Electromagnetic Induction

First Law

Whenever the amount of magnetic flux linked with a closed circuit changes, an emf is induced in the circuit. The induced emf lasts so long as the change in magnetic flux continues.

Second Law

The magnitude of emf induced in a closed circuit is directly proportional to rate of change of magnetic flux linked with the circuit. i.e. $e \,\alpha\, \frac{d\phi}{dt}$. Second law does not give polarity of induced emf.

Lenz's Law : This law gives the direction of induced emf / induced current. According to this law the direction of induced emf or current in a circuit is such as to oppose the cause that produces it. This law is based on the law of conservation of energy. i.e. $e = -\frac{d\phi}{dt}$

Induced current = $-\frac{d\phi}{dt \times R}$. where R is the resistance of the coil.

Induced change = $-\frac{d\phi}{R}$

Note

1) Induced emf does not depends on nature of the coil and its resistance

2) Magnitude of induced emf is directly proportional to the relative speed of coil magnet system $(e \,\alpha\, v)$

3) Induced current is also depends on resistance of coil

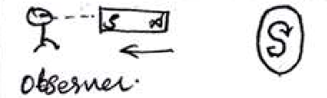
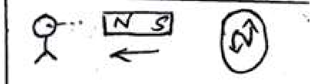
4) Induced emf does not depends on resistance of circuit, its exist in open circuit also

5) In all M.I. phenomenon induced emf is nonzero induced parameter

6) Induced charge in any coil or circuit does not depends on time in which change in flux occurs

7) Induced charge depends on change in flux through the coil and nature of the coil

The various positions of relative motion between the magnet and the coil.

Position magnet				
Direction of induced current	Anticlockwise direction	clockwise direction	Clockwise direction	Anticlockwise direction.
Behaviour of face of the coil	As a north pole	As a South pole	As a South pole	As a north pole
Type of magnetic force developed	Repulsive force	Attractive force	Repulsive force	Attractive force
magnetic field linked with the coil and its progress as viewed from left.	Cross (X), Increases	Cross(x) Decreases	Dot(·) Increases	Dot(·) Decreases.

Induced electric field

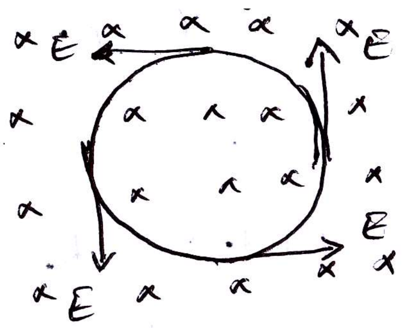
When the magnetic field changes with time there is an induced electric field in the conductor caused by the changing magnetic flux.

Important properties of induced electric field

1. It is non conservative in nature. The line integral of $\vec{E}$ around a closed path is not zero.

   Hence $\oint \vec{E}.\overrightarrow{d\ell} = e = -\frac{d\phi}{dt}$

2. Due to symmetry, the electric field E has the same magnitude at every point on the circle and it is tangential at each point.
3. Being nonconservative field, the concept of potential has no meaning for such a field

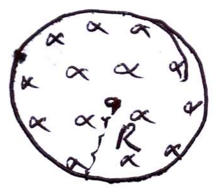


4. The relation F = qE is still valid for this field
5. This field can very with time. For symmetrical situations $E\ell = \left|\frac{d\phi}{dt}\right| = A\frac{dB}{dt}$.

   $\ell \Rightarrow$ the length of the closed loop in which electric field is to be calculated

   $A \Rightarrow$ Area in which magnetic field is changing. Direction of induced electric field is same as the direction of induced current.

Ex. The magnetic field at all points within the cylindrical region whose cross-section is indicated in the figure start increasing at a constant rate $kT/\sec ond$. Find the magnitude of electric as a function of r, the distance from the geometric centre of the region

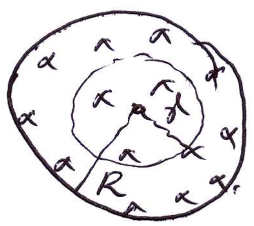


Solution for $r \leq R$

$$E\ell = A\left|\frac{dB}{dt}\right|$$

$$E \times 2\pi r = \pi r^2 \times k$$

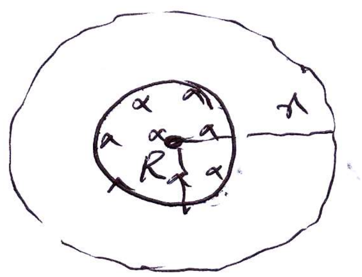
$$\therefore E = \frac{rk}{2} \quad \therefore E \, \alpha \, r$$



E - r - graph is a straight line passing through origin

If r = R, $E = \frac{Rk}{2}$

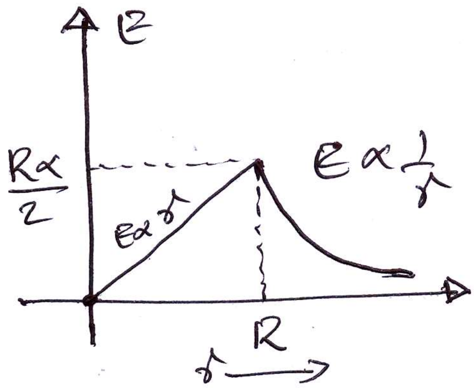
For $r \geq R$



$$E\ell = A\frac{dB}{dt}$$

$$E \times 2\pi r = \pi R^2 \times k$$

$$E = \frac{kR^2}{2r} \text{ or } E\alpha\frac{1}{r}$$



Methods of producing induced emf

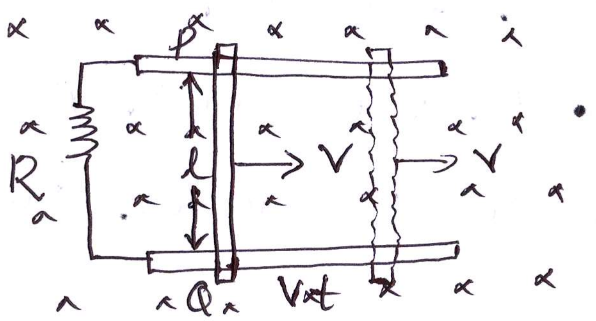
1) Changing the magnetic field B (static EMI)

2) Changing the area A of the coil (dynamic EMI)

3) Changing the relative orientation θ of $\vec{B}$ and $\vec{A}$ (periodic EMI)

Motional EMI in loop by Generated Area

If conducting rod moves on two parallel conducting rails as shown in following figure the phenomenon of induced emf can also be understood by the concept of generated area



Induced emf $|e| = \frac{d\phi}{dt} = B\ell v$

Induced current i = $\frac{e}{R} = \frac{B\ell v}{R}$

Magnetic force $F_m = BI\ell = B \times \left(\frac{B\ell v}{R}\right)\ell = \frac{B^2\ell^2 v}{R}$

Power dissipated in moving the conductor

$$P_{mech} = P_{ext.} = \frac{dw}{dt} = F_{ext.}v = \frac{B^2\ell^2 v}{R} \times v = \frac{B^2\ell^2 v^2}{R}$$

4) **Electrical power**

$$P_{thermal} = \frac{H}{t} = I^2R = \left(\frac{B\ell v}{R}\right)^2 R = \frac{B^2\ell^2 v^2}{R}$$

It is clear that $P_{mech} = P_{thermal}$ which is consistent with the principle of conservatory of energy

Motional emf from Lorentz Force

A conductor rod PQ is placed in a uniform magnetic field B, directed normal to the plane of paper outwards. PQ is moved with a velocity v, the free electrons of PQ also move with the same velocity.

The electron experiences a magnetic force $F_m = e(v \times B)$. An electric field 'E' is set up in the conductor from P to Q

$F_e = eE$

$F_m + F_e = 0 \qquad E = -(v \times B)$

Potential difference between the ends P and Q is $V = E.\ell = (V \times B).\ell$

$\in = B\ell v\left(\text{for } \overline{B} \perp \overline{v} \perp \overline{\ell}\right)$

This this emf is produced due to the motion of the conductor, so it is called a motional emf.

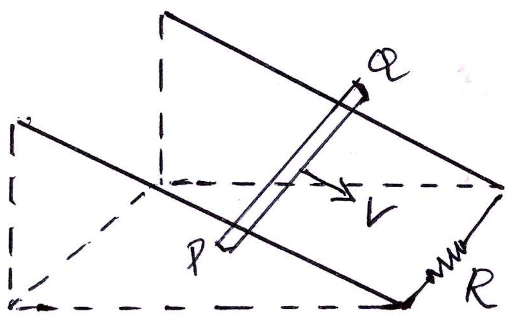
Direction of the induced current or Higher Potential (H.P.) end of the rod can be found out with the help of Fleming right hand rule.

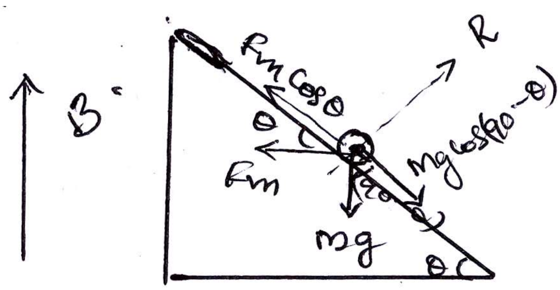
Fore finger $\Rightarrow$ In external field direction

Thumb $\Rightarrow$ In the direction of motion $(\overline{v})$ of conductor

Middle finger $\Rightarrow$ it indicates H.P. end of conductor or direction of induced current in the conductor

Motion of a conducting rod in an inclined plane





Induced emf across the ends of the conductor, $e = B\ell v\sin(90-\theta) = B\ell v\cos\theta$

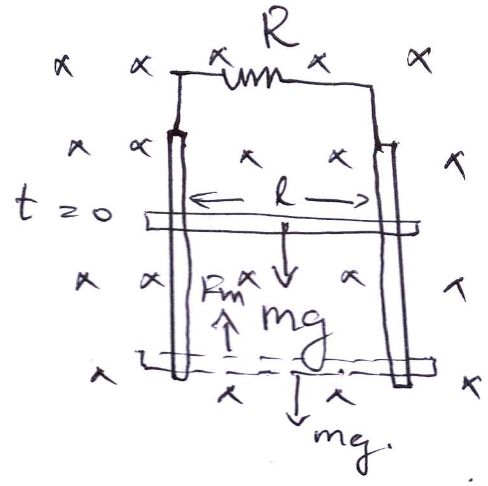
Induced current $i = \dfrac{B\ell v\cos\theta}{R}$

The rod will move down with constant velocity only if $F_m \cos\theta = Mg\cos(90-\theta) = mg\sin\theta$

$$Bi\ell\cos\theta = mg\sin\theta \Rightarrow \frac{B(BV_T\ell\cos\theta)}{R}\ell\cos\theta = Mg\sin\theta$$

$$V_T = {MgR\sin\theta}\Big/{B^2\ell^2\cos^2\theta}$$

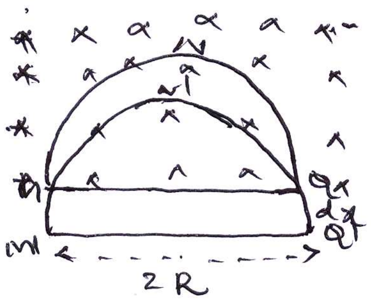
Motion of a conductor rod in a vertical plane



As the speed increases induced emf$_{(e)}$, inducced current (i), magnetic force (F_m) increases but its weight remains constant. Rod will attain a constant maximum (terminal) velocity V_T if F_m = mg

$$\frac{B^2V_T\ell^2}{R} = mg \Rightarrow V_T = \frac{mgR}{B^2\ell^2}$$

Ex. A thin semicircular conducting ring of radius R is falling with its plane vertical in a horizontal magnetic induction B(fig.). At the position MNQ, the speed of the ring is V. What is the p.d. developed across the ring at position MNQ.



$d\phi = B.dA = B - 2Rdx = -2RBdx$

Potential difference across the ring $e = -\frac{d\phi}{dt} = -\left(-2BR\frac{dx}{dt}\right)$

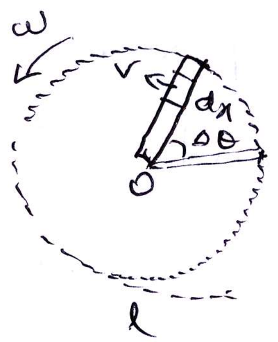
= 2BRV

Induced emf due to Rotation of a conductor rod in a uniform magnetic field

Induced emf in a small element dx is

$de = Bvdx = B\omega xdx$

So net induced emf = $\int_0^{\ell} B\omega xdx$



$$\boxed{e = \frac{1}{2}B\omega\ell^2}$$

Self Induction

Whenever the electric current passing through a coil or circuit changes, the magnetic flux linked with it also change. As a result of this, in accordance with Faraday's laws of electromagnetic induction, an emf is induced in the coil or circuit. Which opposes the change that causes it according to Lenz Law. This phenomenon is called self induction and the emf induced is called back emf. Current so produced in the coil is called self induced current.

Coefficient of self-induction

Flux linked with the coil is proportional to the current.

i.e. $\phi \,\alpha\, I$ or $\phi = LI$. Hence $L = \frac{\phi}{I}$. Coefficient of self-induction or self inductance

The coefficient of self induction of a coil is equal to the flux linked with the coil when the current in it is 1 ampere.

By Faraday's second law, induced emf $e = -\frac{d\phi}{dt}$ ie. $e = -L\frac{dI}{dt}$

Hence coefficient of self induction is numerically equal to the induced emf produced in the coil when the current varying at the rate of 1 ampere / second

Units and dimensional formula of 'L'

If S.I. unit $\Rightarrow \frac{\text{Weber}}{\text{amp}} = \frac{\text{tesla} \times \text{m}^2}{\text{amp}} = \frac{\text{N} \times \text{m}}{(\text{amp})^2}$

= $\frac{\text{Joule}}{(\text{amp})^2} = \frac{\text{coulomb} \times \text{volt}}{(\text{amp})^2} = \frac{\text{volt} \times \text{sec}}{\text{amp}} = \text{ohm} \times \text{sec}$

But practical unit is Henry (H)

Its dimensional formula is $\left[ML^2T^{-2}A^{-2}\right]$

Dependence of self inductance (L)

'L' does not depend on current flowing or change in current flowing but it depends upon number of turns (N), Area of cross section (A) and permeability of medium (μ).

'L' does not play any role till there is a constant current flowing in the circuit. 'L' comes to the picture only when there is a change in current.

Self inductance of a plane coil

Total magnetic flux linked with N turns

$$\phi = NBA = N\left(\frac{\mu_0 NI}{2r}\right)A = \frac{\mu_0 N^2 I}{2r}A = \frac{\mu_0 N^2 I}{2r} \times \pi r^2$$

$$\phi = LI = \frac{\mu_0 N^2 I \pi r}{2} \quad \therefore L = \frac{\mu_0 N^2 \pi r}{2}$$

Self inductance of a solenoid

$$\phi = NBA = N\left(\frac{\mu_0 NI}{\ell}\right)A = \frac{\mu_0 N^2 A}{\ell}I$$

$L = \frac{\mu_0 N^2 A}{\ell}$ with medium $L_m = \frac{\mu_0 \mu_r N^2 A}{\ell}$

Mutual Induction

Whenever the current passing through a coil or circuit changes, the magnetic flux linked with a neighbouring coil or circuit will also change. Hence an emf will be induced in the neighbouring coil or circuit. This phenomenon is called mutual induction.

Coefficient of mutual induction

Total flux linked with the secondary due to current in the primary is $N_2\phi_2$ and $N_2\phi_2 \,\alpha\, i_1 \quad N_2\phi_2 = MI_1$

Where $M = \frac{N_2 \phi_2}{I_1}$ called coefficient of mutual induction

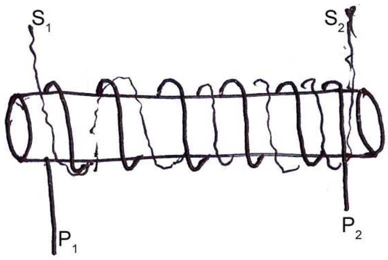
It is numerically equal to the flux linked with the secondary coil when a current of 1A flows through the primary. According to Faraday's second Law emf induced in the secondary $e_2 = -M\frac{dI}{dt}$. Hence coefficient of mutual inductctions is equal to the emf induced in the secondary when current varying at the rate of 1A/s in the primary coil.

M depends on :

- Number of turns (N_1 and N_2)
- Area of cross section (A)
- Distance between two coils (As $d \downarrow \Rightarrow M \uparrow$)
- Coupling factor between two coils
- Coefficient of self inductance (L_1 and L_2)
- Magnetic permeability of medium (μr)
- Orientation between two coils

Diffrent coefficient of mutual inductance

- a) Two-co-axial solenoids (Ms_1s_2)



$$Ms_1s_2 = \frac{N_2 B_1 A}{I_1} = \frac{N_2}{I_1}\left(\frac{\mu_0 N_1 I_1}{\ell}\right)A$$

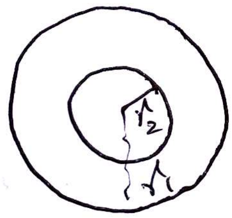
$$Ms_1s_2 = \frac{\mu_0 N_1 N_2 A}{\ell}$$

$A \rightarrow$ Smaller area

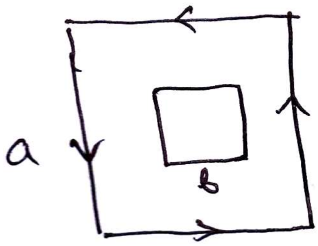
b) Two plane concentric coils (Mc_1c_2)

Two concentric loop

$$M\,\alpha\,\frac{r_2^2}{r_1}$$

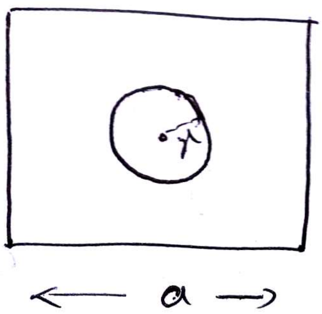


Two concentric square loop



$$M\,\alpha\,\frac{b^2}{a}$$

A square and a circular loop



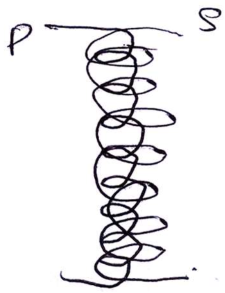
$$M \,\alpha\, \frac{r^2}{a}$$

In terms of L_1 and L_2

$M = K\sqrt{L_1 L_2}$ Here K is coupling factor between two coils.

$0 \geq K \leq 1$

- Different fashion of coupling

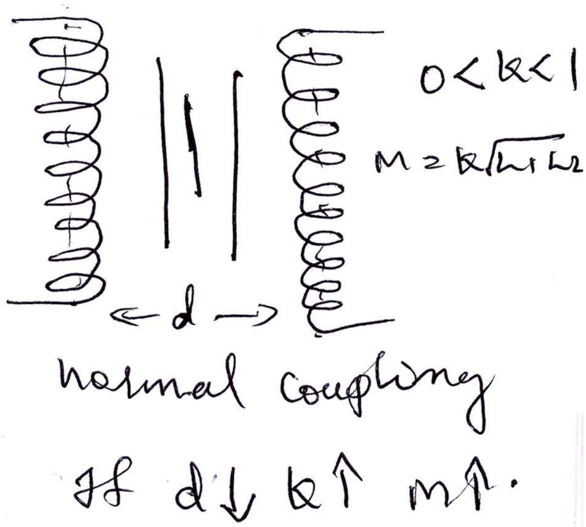


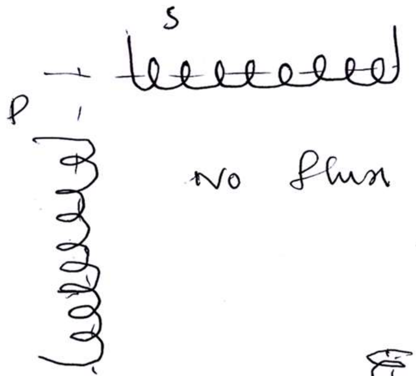
$K = 1$

$M = \sqrt{L_1 L_2}$

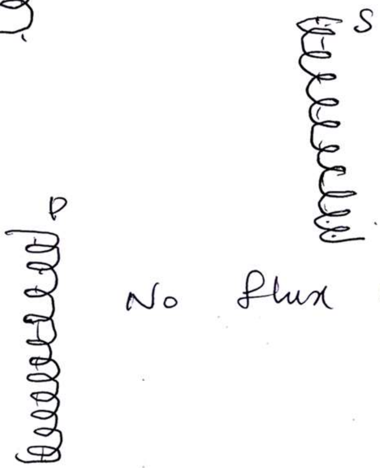
ideal coupling

coaxial fashion





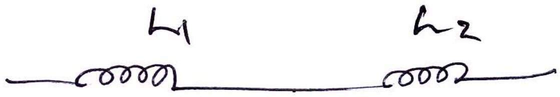
No flux coupling, K = 0, M = 0.

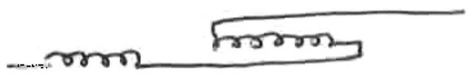


No flux coupling, K = 0, M = 0

K is also defined as, $K = \frac{\phi_s}{\phi_p}$

Inductance in series



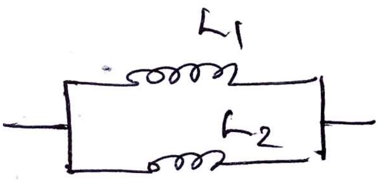


$L = L_1 + L_2 - 2M$

If m = 0, $L = L_1 + L_2$ If $M \neq 0$

$L = L_1 + L_2 + 2M$

Two coils are connected in parallel



If M = 0, $\frac{1}{L}=\frac{1}{L_1}+\frac{1}{L_2}$

If $M \neq 0$, then $\frac{1}{L}=\frac{1}{(L_1+M)}+\frac{1}{(L_2+M)}$

Magnetic potential energy of Inductor

In building a steady current in the circuit, the source of emf has to do work against self inductance of coil and whatever energy consumed for this work stored in magnetic field of coil. This energy called as magnetic potential energy.

$$U=\int_0^i LI\,di=\frac{1}{2}LI^2$$

$$U=\frac{1}{2}\frac{LI\times I}{2}=\frac{\phi I}{2}$$

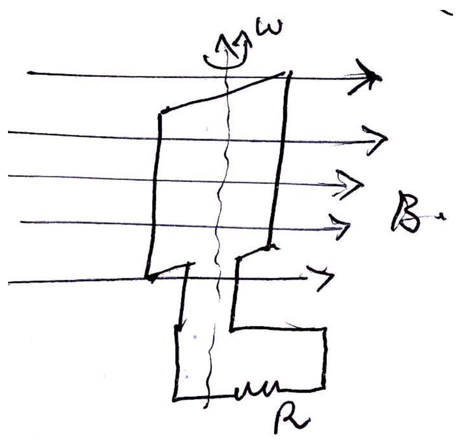
Magnetic energy density

$$u=\frac{U}{v}=\frac{1}{2}\mu_0 n^2 i^2=\frac{1}{2}\frac{(\mu_0 nI)^2}{\mu_0}=\frac{B^2}{2\mu_0}$$

Periodic E.M.I

Suppose a rectangular coil having N turns is placed initially in a magnetic field such that magnetic field is perpendicular to its plane.

ω – angular speed
ν – frequency of rotation
R – resistance of coil
ϕ – NBA cos θ
ϕ – NBA cos ωt



Induced emf in coil : Induced emf also changes in periodic manner that is why this phenomenon called periodic E.M.I

$e = -\frac{d\phi}{dt} = NBA\omega \sin \omega t$ i.e. $e = e_0 \sin \omega t$

$i_0 = \frac{R_0}{R} = \frac{NBA\omega}{R} = \frac{\phi_0 \omega}{R}$

Eddy Currents : are basically the induced currents set up inside the body of conductor whenever the magnetic flux linked with it changes. Eddy currents tend to follow the path of least resistance inside the conductor. So they form irregularly shaped loops. However their directions are not random, but guided by Lenz's law.

Applications of eddy currents

1) Induction furnace

2) Electromagnetic damping

3) Electric brakes

4) Speedometers

5) Induction motor

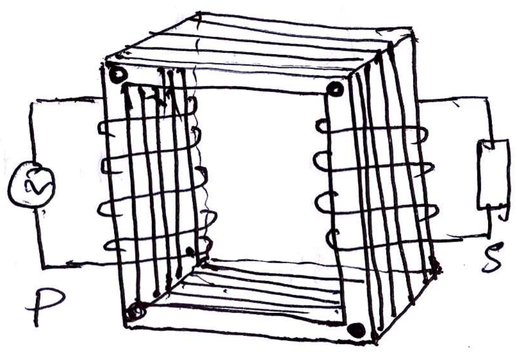
6) Electromagnetic shielding

7) Inductothermy

8) Eergy meters

Transformer

Working principle : Mutual induction transformer has basic two sections



Shell : Consists of primary and secondary coil of copper

Core : Which is between two coil and magnetically coupled two coils. Two coils of transformer are wound on same core.

Work : It regulate A.C. voltage and transfer electrical power without change in frequency of input supply.

Special points :

It can't work with D.C

It can't called amplifier as it has no power gain like transistor

It has no moving part, hence there are no mechanical losses in transformer

If no flux leakage $\phi_s = \phi_p = \frac{d\phi_s}{dt} = \frac{d\phi_p}{dt}$

$e_s = e_p = e$ induced emf/turn of each coil is same.

Total induced emf in secondary $E_s = N_s e$

Total induced emf in primary $E_p = N_p e$

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

No power loss

$P_{out} = P_{in}$ i.e. $V_s I_s = V_p \times I_p$ $\therefore \frac{V_s}{V_p} = \frac{I_p}{I_s}$

Efficiency of transformer

$$\eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{V_s I_s}{V_p I_p} \times 100$$

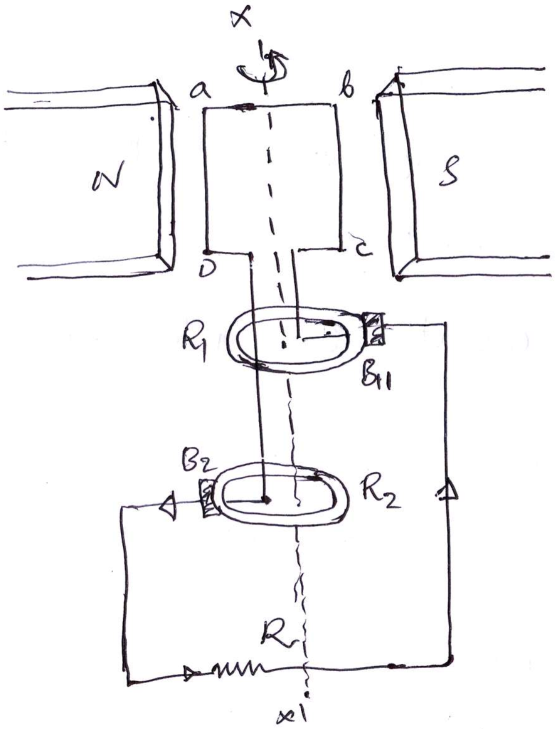
Energy loss in the transformer

- Copper or joule heating losses
- Flux leakage losses
- Iron losses
- Hysteresis losses

Generator or dynamo : It is a device which converts mechanical energy into electrical energy

A.C. Generator

It is based upon the principle of electromagnetic induction.



R_1, R_2 slip rings, B_1 and B_2 Brushes and abcd armature, field magnet.(NS)

Theory : Flux linked with the coil at any instant t.

$$\phi = NBA\cos\omega t$$

$$\frac{d\phi}{dt} = -NBA\omega\sin\omega t$$

emf $E = NBA\omega\sin\omega t$

or $E = E_0\sin\omega t$ where $E_0 = NBA\omega$

ALTERNATING CURRENT AND VOLTAGE

Voltage or current is said to be alternating if it changes continuously in magnitude and peridically in direction with time. It can be represented by a sine curve or cosine curve

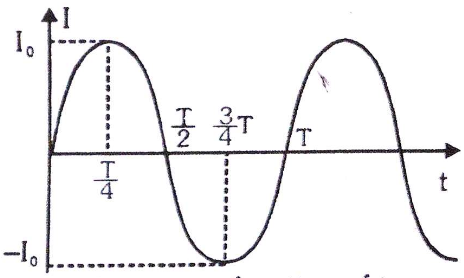
$I= I_0 \sin \omega t$ or $I=I_0 \cos \omega t$

where I= Instantaneous value of current at time t,

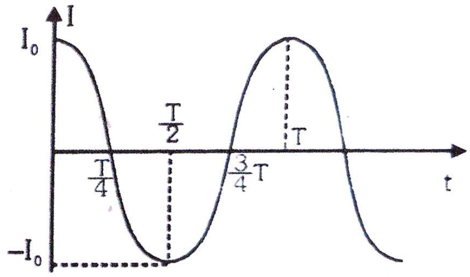
I_0= Amplitude or peak value

ω= Angular frequency, $\omega=\frac{2\pi}{T} = 2\pi f$

T= time period, f= frequency



I as a sine function of t



I as a cosine function of t

- **Amplitude of AC** (I_0)

  The maximum value of current in either direction is called peak value or amplitude of current. It is represented by I_0. Peak to peak value = $2I_0$

- **Periodic time** (T)

  The time taken by alternating current to complete one cycle of variation is called periodic time or time period of the current.

- **Frequency** $(f \text{ or } \nu)$

  The number of cycle completed by an alternating current in one second is called the frequency of the current.

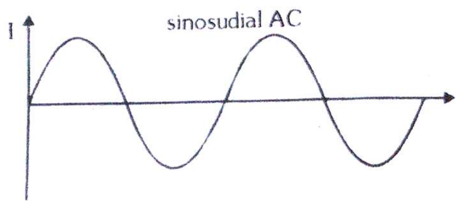
  UNIT: cycle/s; (Hz)
  In India : f= 50Hz, supply voltage= 220 volt (rms)
  In USA : f=60 Hz, supply voltage= 110 volt (rms)

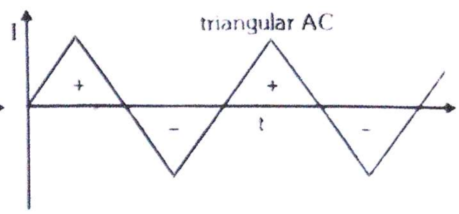
- Condition required for current / voltage to be alternating

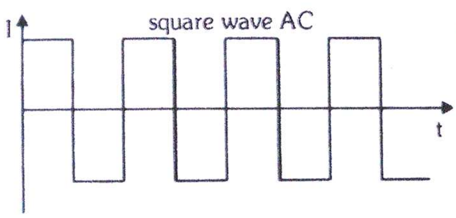
  Amplitude is constant

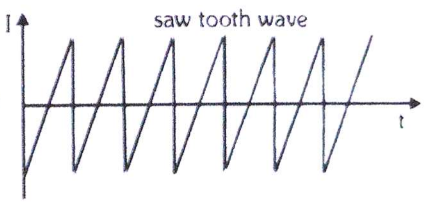
  Alternate half cycle is positive and half negative

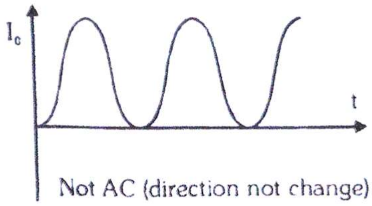
  The alternating current continuously varies in magnitude and periodically reverses its direction.

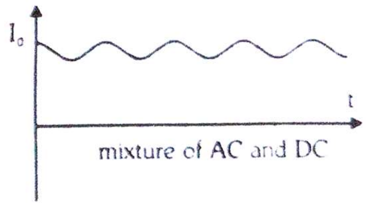












- The mean value of A.C over any half cycle (either positive or negative) is that value of DC which would send same amount of charge through a circuit as is sent by the AC through same circuit in the same time.

$$\text{average value of current for half cycle } \langle I \rangle = \frac{\int_0^{T/2} I dt}{\int_0^{T/2} dt}$$

Average value of $I = I_0 \sin \omega t$ over the positive half cycle :

$$I_{av} = \frac{\int_0^{\frac{T}{2}} I_0 \sin \omega t \, dt}{\int_0^{\frac{T}{2}} dt} = \frac{2I_0}{\omega T}[-\cos \omega t]_0^{\frac{T}{2}} = \frac{2I_0}{\pi}$$

$\langle \sin\theta \rangle = \langle \sin 2\theta \rangle = 0$
$\langle \cos\theta \rangle = \langle \cos 2\theta \rangle = 0$
$\langle \sin\theta \cos\theta \rangle = 0$
$\langle \sin^2\theta \rangle = \langle \cos^2\theta \rangle = \frac{1}{2}$

For symmetric AC, average value over full cycle = 0,
Average value of sinusoidal AC

Full cycle	(+ve) half cycle	(–ve) half cycle
0	$\frac{2I_0}{\pi}$	$\frac{-2I_0}{\pi}$

As the average value of AC over a complete cycle is zero, it is always defined over a half cycle which must be either positive or negative

- **Root mean square (rms) value**

  It is the value of DC which would produce same heat in given resistance in given time as is done by the alternating current when passed through the same resistance for the same time.

$$I_{rms} = \sqrt{\frac{\int_0^T I^2 dt}{\int_0^T dt}}$$ rms value = Virtual value = Apparent value

rms value of $I = I_0 \sin \omega t$:

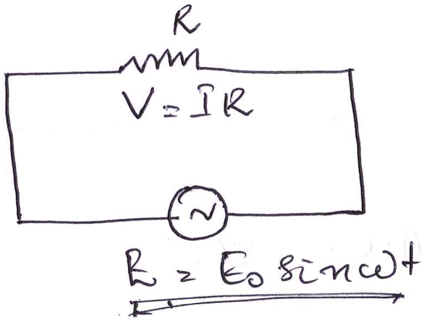
$$I_{rms} = \sqrt{\frac{\int_0^T (I_0 \sin \omega t)^2 dt}{\int_0^T dt}} = \sqrt{\frac{I_0^2}{T}\int_0^T \sin^2 \omega t \, dt} = I_0\sqrt{\frac{1}{T}\int_0^T \left[\frac{1-\cos 2\omega t}{2}\right] dt} = I_0\sqrt{\frac{1}{T}\left[\frac{t}{2} - \frac{\sin 2\omega t}{2\times 2\omega}\right]_0^T} = \frac{I_0}{\sqrt{2}}$$

If nothing is mentioned then values printed in a.c circuit on electrical appliances, any given or unknown values, reading of AC meters are assumed to be RMS.

Current	Average	Peak	RMS	Angular fequency
$I_1 = I_0 \sin \omega t$	0	I_0	$\frac{I_0}{\sqrt{2}}$	ω
$I_2 = I_0 \sin \omega t \cos \omega t = \frac{I_0}{2} \sin 2\omega t$	0	$\frac{I_0}{2}$	$\frac{I_0}{2\sqrt{2}}$	2ω
$I_3 = I_0 \sin \omega t + I_0 \cos \omega t$	0	$\sqrt{2} I_0$	I_0	ω

For above varieties of current rms $= \frac{\text{Peak value}}{\sqrt{2}}$

- **Different types of A.C circuits**
- **AC circuit containing pure resistance**



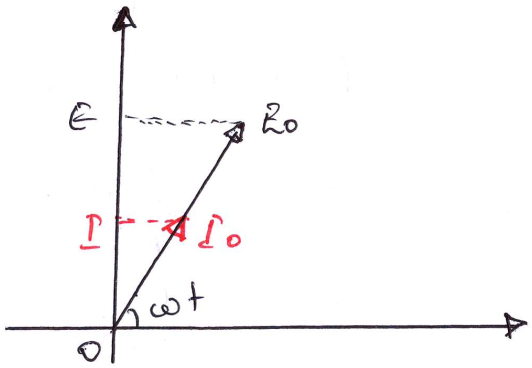
With the help of Kirchoff's circuital law E – IR = 0

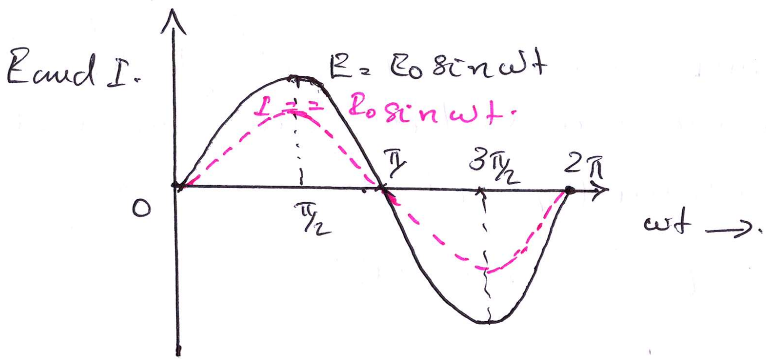
$$E_0 \sin \omega t = IR$$

$$I = \frac{E_0}{R} \sin \omega t$$

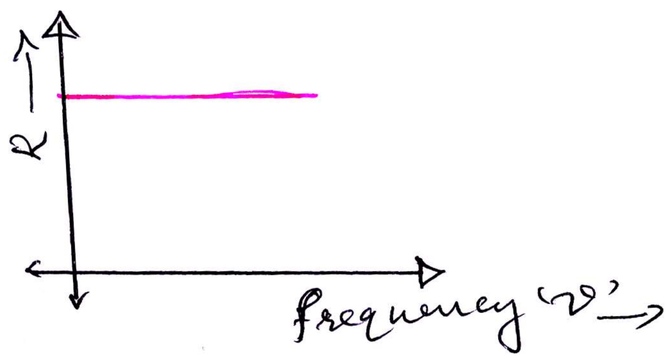
$I = I_0 \sin \omega t$ and $E = E_0 \sin \omega t$

So in a resistor, current and emf are in the same phase. The phaser diagram of emf and current are as shown in the figure



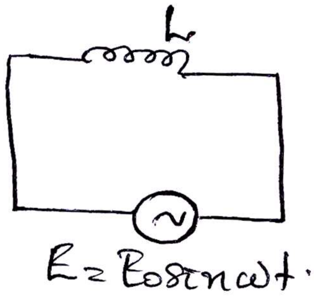


Variation resistance 'R' with frequency ν



A.C. circuit containing pure inductance

Let emf E = $E_0 \sin \omega t$



When a.c flows through the circuit, emf induced across inductance

= $-L\frac{dI}{dt}$

$E + -L\frac{dI}{dt} = 0$ or $E = L\frac{dI}{dt}$

$dI = \frac{E_0 \sin \omega t \, dt}{L}$

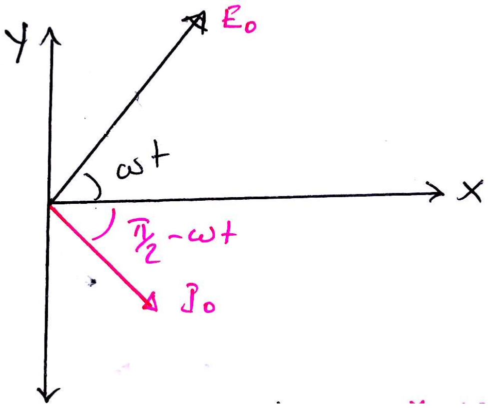
$I = \frac{E_0}{L\omega} \times \sin\left[\omega t - \pi/2\right]$ So current lags due emf with a phase difference of $\pi/2$

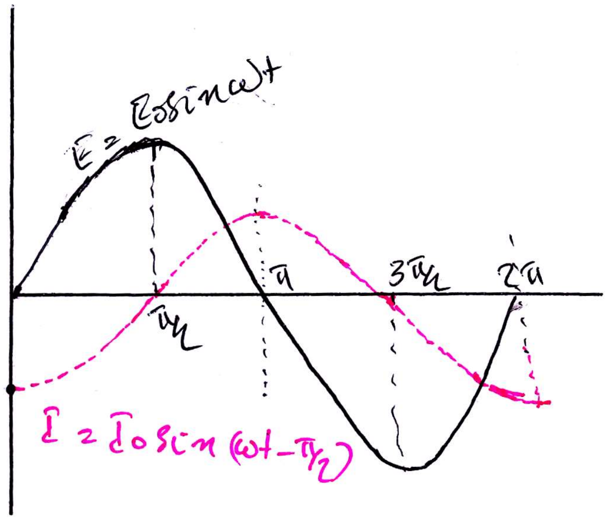
$I = I_0 \sin\left(\omega t - \pi/2\right)$ where $I_0 = \frac{E_0}{L\omega}$

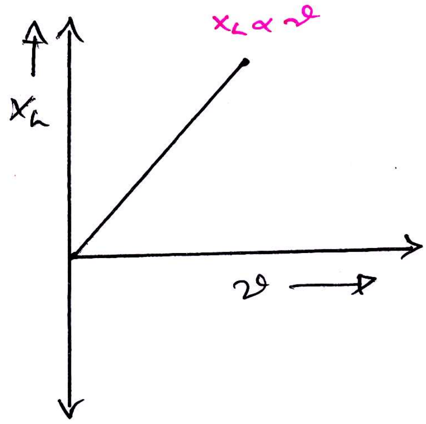
This non resistive opposition to the flow of A.C. in a circuit is called the inductive reactance (X_L) of the circuit.

$X_L = L\omega = L \times 2\pi\nu$. Its unit is 'Ω'

Its phasor diagram is as shown in the figure



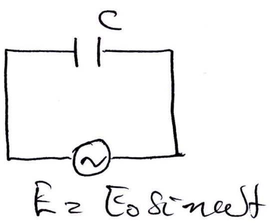




Foer d.c. $\nu = 0$, $X_L = L2\pi\nu = 0$

Hence inductor offers no opposition to the flow of d.c. whereas a resistive path to a.c

A.C. circuit containing pure capacitance



Instantaneous p.d. across the capacitor $E = q/c$

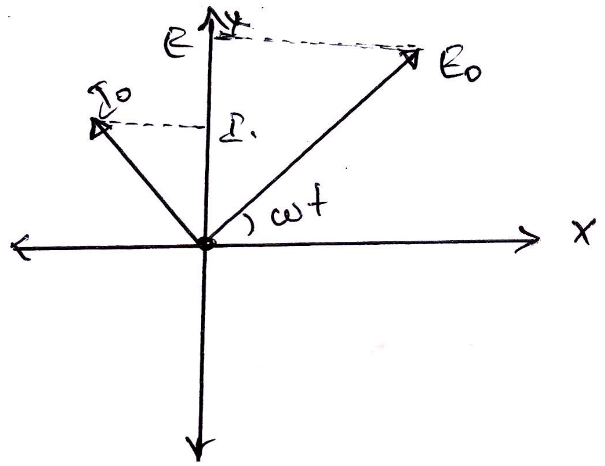
$$q/c = E_0 \sin \omega t$$

$$q = cE_0 \sin \omega t$$

$$\frac{dq}{dt} = cE_0 \omega \cos \omega t$$

$$I = \frac{E_0}{1/c\,\omega} \times \sin\left[\omega t + \pi/2\right]$$ i.e. $I = I_0 \sin\left[\omega t + \pi/2\right]$

$X_C = \dfrac{1}{c\omega}$ is called capacitive reactance. Its phasor diagram is as shown in the fig.

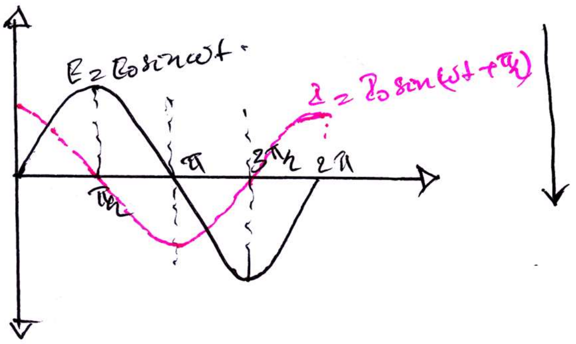


Here current loads the emf with a phase difference of $\pi/2$

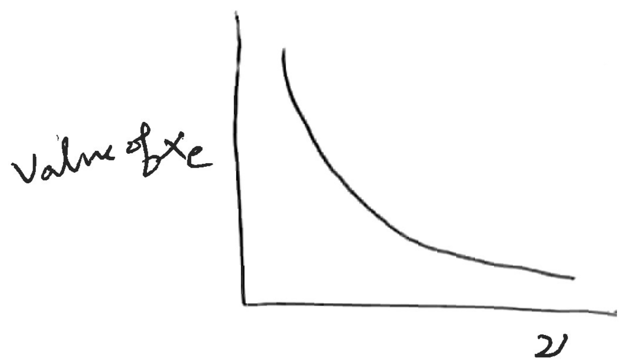
For d.c. $\nu = 0, X_C = \frac{1}{c\omega} = \frac{1}{c \times 2\pi\nu} = \infty$

So a capacitor applies infinite opposition to the d.c. i.e. It does'nt alow d.c. and an easy path for a.c.

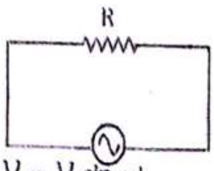
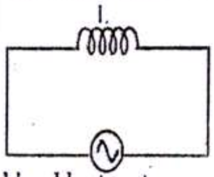
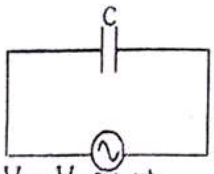
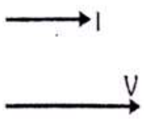
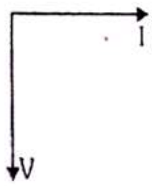
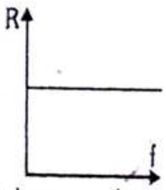
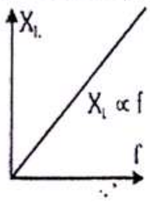
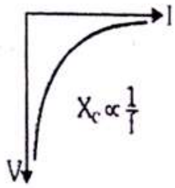
X_C, ν graph is as shown in the figure



unit of X_c is ohm

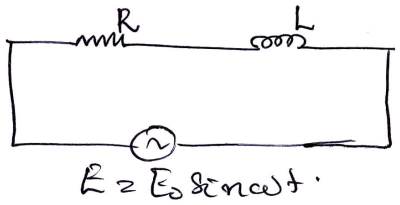


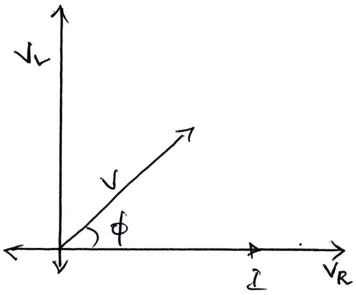
INDIVIDUAL COMPONENTS (R or L or C)

TERM	R	L	C
Circuit			
Supply Voltage	$V = V_0 \sin \omega t$	$V = V_0 \sin \omega t$	$V = V_0 \sin \omega t$
Current	$I = I_0 \sin \omega t$	$I = I_0 \sin (\omega t - \frac{\pi}{2})$	$I = I_0 \sin (\omega t + \frac{\pi}{2})$
Peak Current	$I_0 = \frac{V_0}{R}$	$I_0 = \frac{V_0}{\omega L}$	$I_0 = \frac{V_0}{1/\omega C} = V_0 \omega C$
Impedance (Ω)	$\frac{V_0}{I_0} = R$	$\frac{V_0}{I_0} = \omega L = X_L$	$\frac{V_0}{I_0} = \frac{1}{\omega C} = X_C$
$Z = \frac{V_0}{I_0} = \frac{V_{rms}}{I_{rms}}$	R = Resistance	X_L=Inductive reactance.	X_C=Capacitive reactance.
Phase difference	zero (in same phase)	$+\frac{\pi}{2}$ (V leads I)	$-\frac{\pi}{2}$ (V lags I)
Phasor diagram			
Variation of Z with f	 R does not depend on f		
G, S_L, S_C (mho, seiman)	G=1/R = conductance.	$S_L = 1/X_L$ Inductive susceptance	$S_C = 1/X_C$ Capacitive susceptance
Behaviour of device in D.C. and A.C	Same in A C and D C	L passes DC easily (because $X_L = 0$) while gives a high impedance for the A.C. of high frequency ($X_L \propto f$)	C - blocks DC (because $X_C = \infty$) while provides an easy path for the A.C. of high frequency $\left[X_C \propto \frac{1}{f}\right]$
Ohm's law	$V_R = IR$	$V_L = IX_L$	$V_C = IX_C$

- The phase difference between capacitive and inductive reactance is π
- Inductor called low pass filter because it allow to pass low frequency signal
- Capacitor is called high pass filter because it allow to pass high frequency signal

Resistance and inductance in series (L-R circuit)





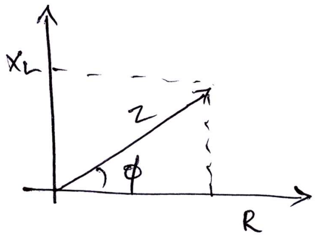
$$V = \sqrt{V_L^2 + V_R^2}$$

$I \times Z = \sqrt{(IR)^2 + (I \times L)^2}\ |Z \Rightarrow$ Impedance of the circuit

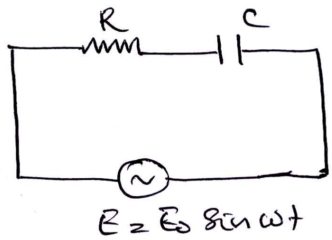
$$Z = \sqrt{R^2 + X_L^2} \qquad I = \frac{E}{\sqrt{R^2 + X_L^2}}$$

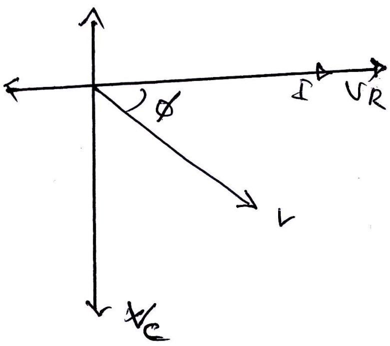
$$\tan\phi = \frac{V_L}{V_R} = \frac{I \times L}{IR} = \frac{X_L}{R} = \frac{L\omega}{R} \qquad I = I_0 \sin(\omega t - \phi)$$

Reciprocal of the impedance is called admittance



Resistance and capacitor are in series (R.C. circuit)





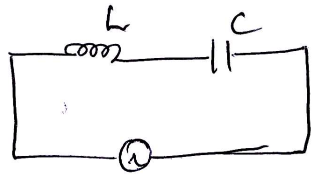
$$V = \sqrt{v_R^2 + V_C^2}$$

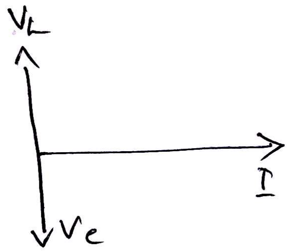
$$Z = \sqrt{R^2 + X_C^2}$$

Here emf lags behind the current with a phase difference of ϕ

$$\tan\phi = \frac{V_C}{V_R} = \frac{X_C}{R} = \frac{1/c\omega}{R} \qquad I = I_0 \sin(\omega t + \phi)$$

L.C. circuit



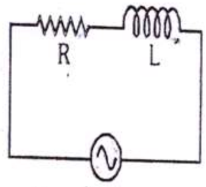
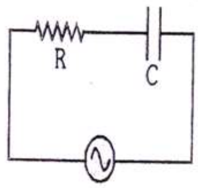
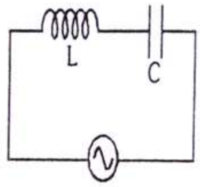
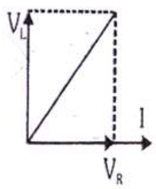
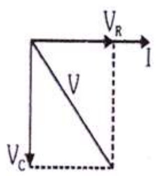
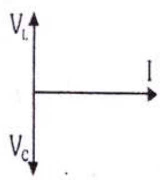
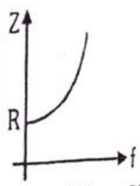
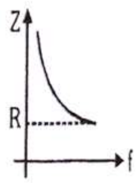
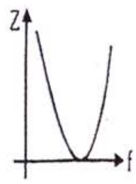


$V = V_L - V_c$

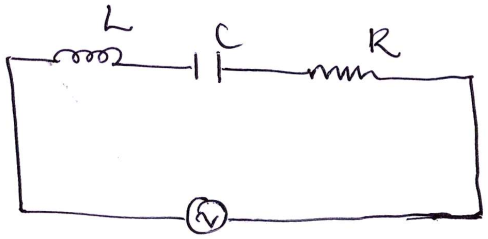
$Z = X_L - X_c$

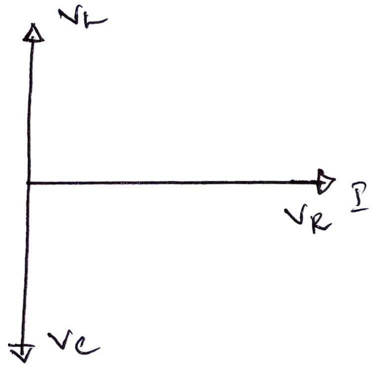
OR $(X_C - X_L)$

COMBINATION OF COMPONENTS (R-L or R-C or L-C)

TERM	R-L	R-C	L-C
Circuit			
	I is same in R & L	I is same in R & C	I is same in L & C
Phasor diagram			
	$V^2 = V_R^2 + V_L^2$	$V^2 = V_R^2 + V_C^2$	$V = V_L - V_C (V_L > V_C)$ $V = V_C - V_L (V_C > V_L)$
Phase difference in between V and I	V leads I ($\phi = 0$ to $\frac{\pi}{2}$)	V lags I ($\phi = -\frac{\pi}{2}$ to 0)	V lags I ($\phi = -\frac{\pi}{2}$, if $X_C > X_L$) V leads I ($\phi = +\frac{\pi}{2}$, if $X_L > X_C$)
Impedance	$Z = \sqrt{R^2 + X_L^2}$	$Z = \sqrt{R^2 + (X_C)^2}$	$Z = \lvert X_L - X_C \rvert$
Variation of Z with f	as f↑, Z↑ 	as f↑, Z↓ 	as f↑, Z first ↓ then↑ 
At very low f	$Z \simeq R (X_L \to 0)$	$Z \simeq X_C$	$Z \simeq X_C$
At very high f	$Z \simeq X_L$	$Z \simeq R (X_C \to 0)$	$Z \simeq X_L$

L.C.R. Series circuit

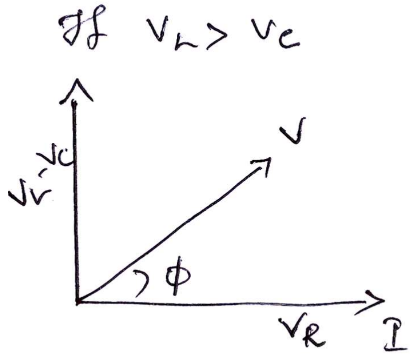




If $V_L > V_C$

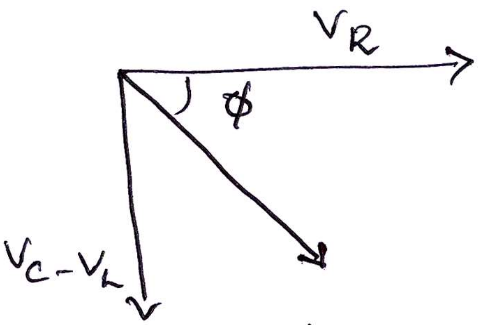
$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$Z = \sqrt{R^2 + (x_L - X_C)^2}$$



Here emf leads the current with a phase difference ϕ and $\tan\phi = \frac{V_L - V_C}{VR} = \frac{X_L - X_C}{R}$

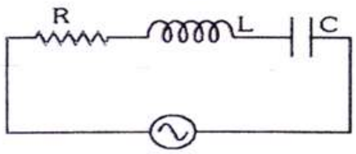
If $V_C > V_L$



Here emf lags behind the current with a phase dfference ϕ

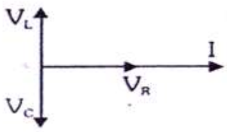
SERIES L-C-R CIRCUIT

1. Circuit diagram

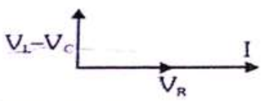


I same for R, L & C

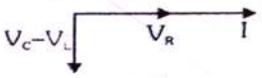
2. Phasor diagram



(i) If $V_L > V_C$ then



(ii) If $V_C > V_L$ then

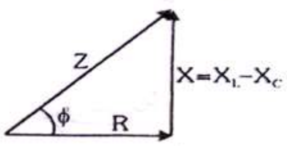


(iii) $V = \sqrt{V_R^2 + (V_L - V_C)^2}$

Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{V_L - V_C}{V_R}$

(iv) Impedance triangle



Resonance : A circuit is said to be resonant when the natural frequency of circuit is equal to frequency of the applied voltage. For resonance both L and C must be present in circuit.

Series Resonance

At resonance $X_L = X_C$, $V_L = V_C$ and $\phi = 0$ V and I in same phase, $Z_{min} = R$

$$I_{max} = \frac{V}{R}$$

Resonance Frequency

$$\because X_L = X_C,\ L\omega_r = \frac{1}{c\omega_r} \quad \text{or} \quad \omega_r^2 = \frac{1}{LC}$$

$$\omega_r = \frac{1}{\sqrt{LC}} \text{ and } \nu = \frac{1}{2\pi\sqrt{LC}}$$

Variation of I with ν

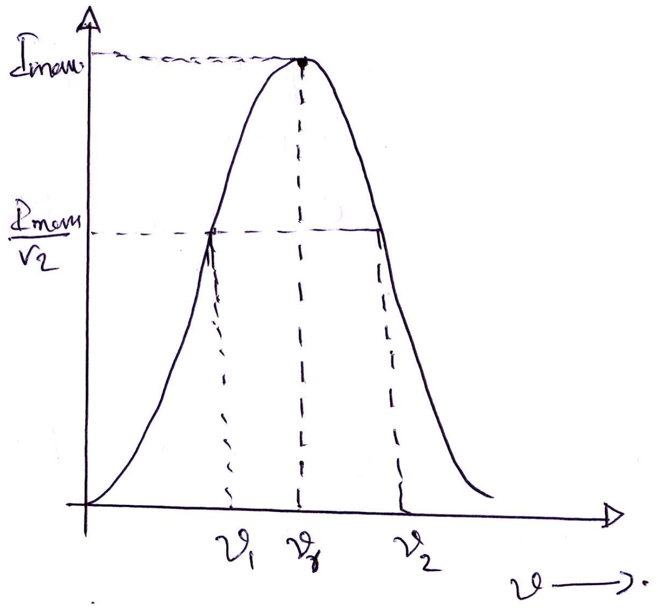
1) If $\nu < \nu_r$ then $X_L < X_C$ circuit nature capacitive, (ϕ negative)

2) At $\nu = \nu_r$ then $X_L = X_C$ circuit nature

Resistive $\phi = 0$

3) If $\nu > \nu_r$ then $X_L > X_C$ circuit nature is inductive, (ϕ positive)

Variation of I with frequency 'ν'



- At resonance impedance of the series resonant circuit is minimum it is called acceptor circuit as it most readily accepts that current out of many currents whose frequency is equal to its natural frequency.

Half Power Frequencies

The frequencies of which, power become half of its maximum value called half power frequencies

Band width $\Delta\nu = \nu_2 - \nu_1$

Quality factor or Q-factor of A.C. circuit basically gives an ideal about stored energy and lost energy.

$$Q = 2\pi \frac{\text{max. energy stored / cycle}}{\text{max. energy loss / cycle}}$$

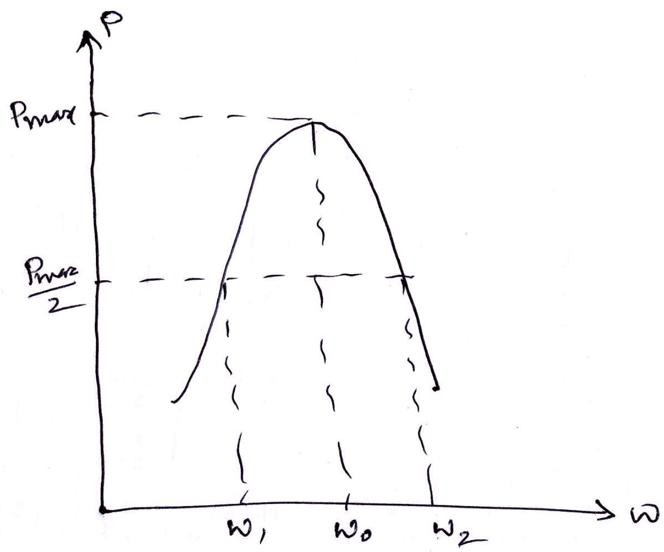
It represents sharpness of resonance. It is a unitless and dimensionless quantity

$$Q = \frac{(V_L)_r}{R} = \frac{(X_C)_r}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{\nu_r}{\Delta\nu}$$

Magnification factor = Q $\rightarrow$ factor

Sharpness α Q factor & magnification factor

Power in AC circuit



Power in an AC circuit

The average power dissipation in LCR AC circuit

Let $V = V_0 \sin\omega t$ and $I = I_0 \sin(\omega t - \phi)$

Instantaneous power $P = (V_0 \sin\omega t)(I_0 \sin(\omega t - \phi)) = V_0 I_0 \sin\omega t\,(\sin\omega t\cos\phi - \sin\phi\cos\omega t)$

Average power $\langle P\rangle = \frac{1}{T}\int_0^T (V_0 I_0 \sin^2\omega t\cos\phi - V_0 I_0 \sin\omega t\cos\omega t\sin\phi)dt$

$$= V_0 I_0\left[\frac{1}{T}\int_0^T \sin^2\omega t\cos\phi\, dt - \frac{1}{T}\int_0^T \sin\omega t\cos\omega t\sin\phi\, dt\right] = V_0 I_0\left[\frac{1}{2}\cos\phi - 0\times\sin\phi\right]$$

$$\Rightarrow \quad \langle P\rangle = \frac{V_0 I_0 \cos\phi}{2} = V_{rms}\, I_{rm,s} \cos\phi$$

Instantaneous power	Average power/actual power/ dissipated power/power loss	Virtual power/ apparent Power/rms Power	Peak power
$P = VI$	$P = V_{rms} I_{rms} \cos\phi$	$P = V_{rms} I_{rms}$	$P = V_0 I_0$

- $I_{rms}\cos\phi$ is known as active part of current or wattfull current, workfull current. It is in phase with voltage.
- $I_{rms}\sin\phi$ is known as inactive part of current, wattless current, workess current. It is quadrature (90°) with voltage.

Power factor :

Average power $\bar{P} = E_{rms} I_{rms} \cos\phi = \text{r m s power} \times \cos\phi$

Power factor $(\cos\phi) = \dfrac{\text{Average power}}{\text{r.m.s Power}}$ and $\cos\phi = \dfrac{R}{Z}$

Power factor : (i) is leading if I leads V (ii) is lagging if I lags V