CHAPTER - 21
STATISTICS

JEE MAIN - SECTION I

1. 1 $\sum_{i=1}^{25} x_i = 1000$

$\frac{940+x}{25} = 39$

$\therefore x = 35$

2. 4 odd divisors are 1,3,5,9,15,45

$\therefore$ Required mean = 10

3. 3 Median = 170 $\Rightarrow$ 125, a, b, 170, 190, 210, 230

Mean deviation about

Median =

$$\frac{0+45+60+20+40+170-a+170-b}{7} = \frac{205}{7}$$

$\Rightarrow a + b = 300$

Mean = $\frac{170+125+230+190+210+a+b}{7} = 175$

Mean deviation

About mean =

$$\frac{50+175-a+175-b+5+15+35+55}{7} = 30$$

4. 1 $\sum f_i = 584 \Rightarrow \alpha + \beta = 390$

Median $= 45 \Rightarrow \alpha = 113$

$\therefore \beta = 277$

$|\alpha - \beta| = 164$

5. 1 Given $MSD_{(-2)} = 18 \Rightarrow \frac{\sum x_i^2}{n} + 4\bar{x} = 14....(1)$

Given $MSD_{(+2)} = 10 \Rightarrow \frac{\sum x_i^2}{n} - 4\bar{x} = 6 - (2)$

$(1)\&(2) \Rightarrow \frac{\sum x_i^2}{n} = 10 \ \& \bar{x} = 1$

$\therefore SD = 3$

6. 2 $\bar{x} = 14, \sum_{i=1}^{10} x_i^2 = 2000$

$\therefore \sigma = 2$

7. 3 $Am = 6 \Rightarrow a + b = 7$

$\sigma^2 = 6.8 \Rightarrow a^2 + b^2 = 25$

$\therefore ab = 12$

8. 3

9. 4 Outcomes are

$$\underbrace{\frac{1}{2}-d, \frac{1}{2}-d, ...\frac{1}{2}-d}_{10}, \underbrace{\frac{1}{2}, \frac{1}{2}, ...\frac{1}{2}}_{10}, \underbrace{\frac{1}{2}+d, \frac{1}{2}+d.....\frac{1}{2}+d}_{10}$$

$\therefore \bar{x} = \frac{1}{2} \ \& \ \sigma^2 = \frac{2}{3}d^2$

$\therefore \frac{2}{3}d^2 = \frac{4}{3} \Rightarrow d = \sqrt{2}$

10. 1

11. 3 $\sum x_i = 10n$

$\therefore n = 20$

$\sigma^2 = 3.96$

12. 2 $a_1 + a_3 = 10 \Rightarrow a_1 + d = 5$

$a_1 = 2 \& d = 3$

$\therefore \sigma^2 = \frac{105}{4}$

$8\sigma^2 = 210$

13. 2 New mean $= p\bar{x} - q$

$\Rightarrow 10 = 20p - q \quad -(1)$

New variance $(\sigma')^2 = p^2(\sigma)^2$

$\Rightarrow 1 = p^2 \times 4$

$\Rightarrow p = \pm\frac{1}{2}, \; q = 0, -20$

$\therefore |q| = 20$

14. 1 $\Sigma[x_i - 5] = 10 \Rightarrow \bar{x} = 6$

$\Sigma[x_i - 5]^2 = 40 \Rightarrow \Sigma x_i^2 = 390$

$\therefore \sigma^2 = 3$

By property new mean = 3

New variance = 3

15. 1

16. 1 $n_1 = 5, n_2 = 5, \sigma_1^2 = 4, \sigma_2^2 = 5, \bar{x}_1 = 2, \bar{x}_2 = 4$

$\therefore d_1 = -1, d_2 = 1 \; \& \; \bar{x} = 3$

$$\sigma^2 = \frac{n_1\left[\sigma_1^2 + d_1^2\right] + n_2\left[\sigma_2^2 + d_2^2\right]}{n_1 + n_2}$$

17. 3

$n_1 + n_2 = 250 \Rightarrow n_2 = 150$

$\bar{x} = 15.6 \Rightarrow \bar{x}_2 = 16$

$d_1 = 0.6, d_2 = 0.4$

Given $\sigma^2 = 13.44$

$\Rightarrow 150\sigma_2^2 = 2400$

$\Rightarrow \sigma_2 = 4$

18. 3

Given $\sum_{i=1}^{10}[xi - 50] = 0 \Rightarrow \bar{x} = 50$

$$SD = \sqrt{\frac{\sum_{i=1}^{n}\left[x_i - \bar{x}\right]^2}{n}} = 5$$

$\therefore CV = 10$

19. 2

$\sum_{i=1}^{n} x_i^2 + n + 2\sum_{i=1}^{n} x_i = 9n$ — ①

$\sum_{i=1}^{n} x_i^2 + n - 2\sum_{i=1}^{n} x_i = 5n$ — ②

Solving ① & ② we get, $\sum_{i=1}^{n} x_i^2 = 6n$ & $\sum_{i=1}^{n} x_i = n$

$\therefore S.D = \sqrt{5}$

20. 1

$$\frac{1+3+5+a+b}{5} = 5$$

$a + b = 16$ .....(1)

$$\sigma^2 = \frac{\sum x_i^2}{5} - \left(\frac{\sum x}{5}\right)^2$$

$$8 = \frac{1^2 + 3^2 + 5^2 + a^2 + b^2}{5} - 25$$

$a^2 + b^2 = 130$ .....(2)

by (1), (2)

$a = 7, b = 9$

SECTION II (NUMERICAL)

21. 400 $\sum x_i = 800$ & $\sum x_i^2 = 2.16^2.50$

$\therefore$ New $\bar{x} = 400$

22. 5 $d_1 = \dfrac{-n}{10+n}$ $d_2 = \dfrac{10}{10+n}$

Combined variance $= \dfrac{17}{9} \Rightarrow n = 5$

23. 37 $a = 7$ & $b = \dfrac{70}{6}$

$\therefore a + 3b - 5 = 37$

24. 500 $\sigma_1^2 + \sigma_2^2 = 500$

25. 2 $\sum x_1^2 = 90 - 18\beta^2 + 2\beta[36 + 18\alpha].....(1)$

Using equation (1) $\sigma^2 = 1$, we get $|\alpha - \beta| = 4$

JEE ADVANCED LEVEL

SECTION III

26. C $MD_{\bar{x}} = 6.2$

$\bar{x} - MD_{\bar{x}} = 3.8$ and $\bar{x} + MD_{\bar{x}} = 16.2$

$\therefore$ No of items $= 8$

27. C $\sum f_i = a + b + 26$

$\sum x_i f_i = 3a + 9b + 504$

$\bar{x} = \dfrac{309}{22} \Rightarrow 81a + 37b = 1018....(1)$

Median $= \ell + \dfrac{\left[\dfrac{n}{2} - c\right]h}{f}$

Median $= 14 \Rightarrow a + b = 18....(2)$

$(1)\&(2) \Rightarrow a = 8, b = 10$

$\therefore (a-b)^2 = 4$

28. C $\quad \bar{x} = 6 \Rightarrow 2\alpha + 3\beta = 16....(1)$

$\sigma^2 = 6.8 \Rightarrow 14\alpha - 11\beta = 48...(2)$

$(1)\&(2) \Rightarrow \alpha = 5 \& \beta = 2$

$\therefore$ New $\bar{x} = \dfrac{17}{3}$

29. B $\quad \displaystyle\sum_{k=1}^{10} a_k = 50$

$a_1 + a_2 + \ldots + a_{10} = 50 \quad ....(i)$

$\displaystyle\sum_{\forall k<j} a_k a_j = 1100 \quad ....(ii)$

If $a_1 + a_2 + \ldots + a_{10} = 50.$

$(a_1 + a_2 + \ldots + a_{10})^2 = 2500$

$\Rightarrow \displaystyle\sum_{i=1}^{10} a_i^2 + 2\sum_{k<j} a_k a_j = 2500$

$\Rightarrow \displaystyle\sum_{i=1}^{10} a_i^2 = 2500 - 2(1100)$

$\sum_{i=1}^{10} a_i^2 = 300$, Standard deviation '$\sigma$'

$= \sqrt{\dfrac{\sum a_i^2}{10} - \left(\dfrac{\sum a_i}{10}\right)^2} = \sqrt{\dfrac{300}{10} - \left(\dfrac{50}{10}\right)^2}$

$= \sqrt{30-25} = \sqrt{5}$

30. A

$x_1, x_2 \ldots\ldots x_{10}$

$$\sum_{i=1}^{10}(x_i-\alpha)=2 \Rightarrow \sum_{i=1}^{10}x_i-10\alpha=2$$

$$\text{Mean } \mu=\frac{6}{5}=\frac{\Sigma x_i}{10}$$

$$\therefore \ \Sigma x_i=12$$

$$10\alpha+2=12 \ \therefore \alpha=1$$

$$\text{Now } \sum_{i=1}^{10}(x_i-\beta)^2=40 \quad \text{Let } y_i=x_i-\beta$$

$$\therefore \sigma_y^2=\frac{1}{10}\sum y_i^2-(\bar{y})^2$$

$$\sigma_x^2=\frac{1}{10}\sum(x_i-\beta)^2-\left(\frac{\sum_{i=1}^{10}(x_i-\beta)}{10}\right)^2$$

$$\frac{84}{25}=4-\left(\frac{12-10\beta}{10}\right)^2$$

$$\therefore \left(\frac{6-5\beta}{5}\right)^2=4-\frac{84}{25}=\frac{16}{25}$$

$$6-5\beta=\pm 4 \Rightarrow \beta=\frac{2}{5} \text{ (not possible) or } \beta=2$$

Hence $\frac{\beta}{\alpha}=2$

31. A

$$\Sigma f_i=(p+q)^n=1$$

$$\Sigma f_i x_i=np(p+q)^{n-1}=np$$

$$\therefore \text{mean}=np$$

32. B

$$S.D = \sqrt{\frac{\Sigma(x_i-a)^2}{n} - \left(\frac{\Sigma(x_i-a)}{n}\right)^2}$$
$$= \sqrt{\frac{na}{n} - 1}$$
$$= \sqrt{a-1}$$

33. B

$$a+b=10 \quad \text{—(1)}$$
$$a^2+b^2=62 \quad \text{—(2)}$$
From (1) & (2) we get, $ab=19$
$\therefore$ eq. is $x^2-10x+19=0$

34. 3

$$X=\{1,2,\ldots 17\}$$
Let μ = mean $= \frac{a\sum_{i=1}^{17} x_i + \sum_{i=1}^{17} b_i}{n}$
$\therefore \quad 17 = 9a+b \quad \text{—(1)}$
Variance $= \frac{\sum_{i=1}^{17}(y_i-\mu)^2}{n}$
$\therefore 216 = \frac{\sum_{i=1}^{17}(a^2x_i^2 - 18a^2x_i + 81a^2)}{17}$
$= 105a^2 - 162a^2 + 81a^2$
$\Rightarrow a = \pm 3 \Rightarrow a=3, \; b=-10$

35. 1

$$\frac{3^n-1}{2^n} = \frac{728}{2^n}$$
$$\Rightarrow n=6$$