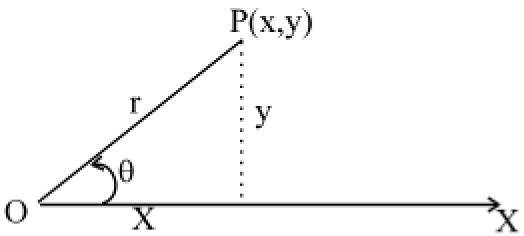
CHAPTER - 2
TRIGONOMETRIC FUNCTIONS

1. Some useful results



$\frac{x}{r} = \cos\theta; \frac{y}{r} = \sin\theta; \frac{y}{x} = \tan\theta$

2. $\cos^2\theta + \sin^2\theta = 1 \qquad \cos^2\theta = 1 - \sin^2\theta$

3. $1 + \tan^2\theta = \sec^2\theta \qquad \tan^2\theta = \sec^2\theta - 1$

4. $1 + \cot^2\theta = \operatorname{cosec}^2\theta \qquad \cot^2\theta = \operatorname{cosec}^2\theta - 1$

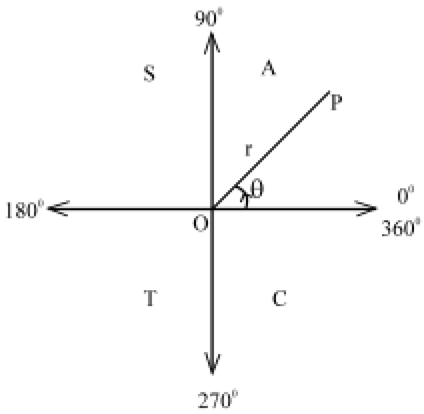
5. $\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta} \qquad \tan\theta = \frac{1}{\cot\theta}; \ \operatorname{cosec}\theta = \frac{1}{\sin\theta} \quad \sec\theta = \frac{1}{\cos\theta}$

6. $\pi \text{ radians} = 180° \qquad \frac{\pi}{2} = 90° \quad \frac{\pi}{3} = 60° \quad \frac{\pi}{4} = 45°; \ \frac{\pi}{6} = 38° \qquad \frac{\pi}{5} = 36° \quad \frac{\pi}{10} = 18°$

7.

θ	0	30°	45°	60°	90°	180°	270°	360°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	–1	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	–1	0	1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	$-\infty$	0

8.



9. $\sin(-\theta) = -\sin\theta, \cos(-\theta) = \cos\theta, \tan(-\theta) = -\tan\theta$

$\cot(-\theta) = -\cot\theta, \sec(-\theta) = \sec\theta \operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

10. sin (A+B) = sinA cos B + cos A sin B

sin (A-B) = sin A cos B - cos A sin B

cos (A+B) = cos A cos B - sin A sin B

cos (A-B) = cos A cos B + sin A sin B

$$\tan|A+B| = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan|A+B| = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

11. $\sin(A+B+C) = \cos A \cos B \cos C[\tan A + \tan B + \tan C - \tan A \tan B \tan C]$

12. $\cos(A+B+C) = \cos A \cos B \cos C[1 - \tan A \tan B - \tan B \tan C - \tan C \tan A]$

13. $\tan(A+B+C) = \dfrac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$

14. If A + B + C = π then

i) tan A + tan B + tan C = tan A tan B tan C

ii) cot A cot B + cot B cot C + cot C. cot A = 1

iii) sin 2A + sin 2B + sin 2C = 4 sin A sin B sin C

iv) cos 2A + cos 2B + cos 2C = –1 – 4cos A cos B cos C

v) $\cot\dfrac{A}{2} + \cot\dfrac{B}{2} + \cot\dfrac{C}{2} = \cot\dfrac{A}{2}.\cot\dfrac{B}{2}.\cot\dfrac{C}{2}$

vi) $\tan\frac{A}{2}.\tan\frac{B}{2}+\tan\frac{B}{2}\tan\frac{C}{2}+\tan\frac{C}{2}.\tan\frac{A}{2}=1$

vii) $\cos^2 A+\cos^2 B+\cos^2 C=1-2\cos A\cos B\cos C$

viii) $\sin^2 A+\sin^2 B+\sin^2 C=2+2\cos A\cos B\cos C$

15. If A + B + C = $\pi/_2$, tan A tan B + tan B tan C + tan C tanA = 1,

cot A + cot B + cot C = cot A. cot B. cotC and $\sin^2A + \sin^2B + \sin^2C = 1- 2 \sin A \sin B \sin C$,

$\cos^2 A+\cos^2 B+\cos^2 C=2+2\sin A\sin B\sin C$

16. If A + B = $\pi/4$, (1 + tan A) (1 + tan B) = 2, (cot A-1) (cot B-1) =2

If $A-B=\frac{\pi}{4},(1+\tan A)(1-\tan B)=2,\text{if } A+B=\frac{3\pi}{4}$ then, $(1+\cot A)(1+\cot B)=2$

17. $\tan\left(\frac{\pi}{4}+\theta\right)=\frac{1+\tan\theta}{1-\tan\theta}$ and $\tan\left(\frac{\pi}{4}-\theta\right)=\frac{1-\tan\theta}{1+\tan\theta}$

Also $\tan\left(\frac{\pi}{4}+\theta\right)=\frac{1+\sin 2\theta}{\cos 2\theta}$ and $\tan\left(\frac{\pi}{4}-\theta\right)=\frac{1-\sin 2\theta}{\cos 2\theta}$

$$\cos\theta+\sin\theta=\begin{cases}\sqrt{2}\cos\left(\frac{\pi}{4}-\theta\right)\\ \sqrt{2}\sin\left(\frac{\pi}{4}+\theta\right)\end{cases};\qquad \cos\theta-\sin\theta=\begin{cases}\sqrt{2}\cos\left(\frac{\pi}{4}+\theta\right)\\ \text{or}\\ \sqrt{2}\sin\left(\frac{\pi}{4}-\theta\right)\end{cases}$$

18. $\dfrac{\tan\left(\frac{\pi}{4}+\theta\right)+\tan\left(\frac{\pi}{4}-\theta\right)}{\tan\left(\frac{\pi}{4}+\theta\right)-\tan\left(\frac{\pi}{4}-\theta\right)}=\operatorname{cosec}2\theta$

19. sin 2A = 2 sinA cos A => sin A = 2sin (A/2).cos (A/2)

Also $\sin A\cos A=\frac{\sin 2A}{2}$ and $\sin\frac{A}{2}\cos\frac{A}{2}=\frac{\sin A}{2}$

20. $\cos 2A = \cos^2A - \sin^2A = 2\cos^2A - 1 = 1 - 2\sin^2A$

$\Rightarrow \cos A=\cos^2\frac{A}{2}-\sin^2\frac{A}{2}=2\cos^2\frac{A}{2}-1=1-2\sin^2\frac{A}{2}$

21. $1 + \cos 2A = 2\cos^2A$ => $1 + \cos A = 2\cos^2(A/2)$

Also $\cos^2 A=\frac{1+\cos 2A}{2}$; $\cos^2\frac{A}{2}=\frac{1+\cos A}{2}$

$1- \cos 2A = 2\sin^2A$ => $1 - \cos A = 2\sin^2(A/2)$

Also $\sin^2 A = \dfrac{1-\cos 2A}{2}$; $\sin^2 \dfrac{A}{2} = \dfrac{1-\cos A}{2}$

$\tan^2 A = \dfrac{1-\cos 2A}{1+\cos 2A} \quad \Rightarrow \quad \tan^2 \dfrac{A}{2} = \dfrac{1-\cos A}{1+\cos A}$

22. cot A - tan A = 2 cot 2A

$\cot A + \tan A = \dfrac{1}{\sin A \cos A}$ = 2 cosec 2A

23. $\sin 2A = \dfrac{2\tan A}{1+\tan^2 A} \Rightarrow \sin A = \dfrac{2\tan|A/2|}{1+\tan^2|A/2|}$

$\cos 2A = \dfrac{1-\tan^2 A}{1+\tan^2 A} \Rightarrow \sin A = \dfrac{1-\tan^2|A/2|}{1+\tan^2|A/2|}$

24. $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A} \Rightarrow \tan A = \dfrac{2\tan|A/2|}{1-\tan^2|A/2|}$

25. $\sin 3A = 3\sin A - 4\sin^3 A \quad => \quad \sin^3 A = \dfrac{3\sin A - \sin 3A}{4}$

$\cos 3A = 4\cos^3 A - 3\cos A \quad => \quad \cos^3 A = \dfrac{3\cos A + \cos 3A}{4}$

26. $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

27. $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$

$\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$

$-\sin(A+B)\sin(A-B) = \cos^2 A - \cos^2 B$

$\tan(A+B)\tan(A-B) = \dfrac{\tan^2 A - \tan^2 B}{1-\tan^2 A . \tan^2 B}$ and $\cot|A+B|\cot|A-B| = \dfrac{\cot^2 A\cot^2 B - 1}{\cot^2 B - \cot^2 A}$

28. $\sin 75^0 = \cos 15^0 = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$, $\cos 75^0 = \sin 15^0 = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$

$\sin 18^0 = \sin \pi/10 = \dfrac{\sqrt{5}-1}{4}$, $\cos 18^0 = \cos(\pi/10) = \dfrac{\sqrt{10+2\sqrt{5}}}{4}$

$\sin 36^0 = \sin \pi/5 = \frac{\sqrt{10-2\sqrt{5}}}{4}$ $\cos 36^0 = \cos(\pi/5) = \frac{\sqrt{5}+1}{4}$

$\sin 54^0 = \sin(90^0-36^0) = \cos 36^0$, $\cos 54^0 = \sin 36^0$

$\sin 72^0 = \sin(90^0-18^0) = \cos 18^0$, $\cos 72^0 = \sin 18^0$

$\sin 22^1/_2{}^0 = \sin \pi/8 = \frac{\sqrt{2-\sqrt{2}}}{2}$, $\cos 22^1/_2{}^0 = \cos \pi/8 = \frac{\sqrt{2+\sqrt{2}}}{2}$ $\tan 22^1/_2{}^0 = \tan \pi/8 = \sqrt{2}-1$

29. $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

$\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$

30. $\sin A \cos B = {}^1/_2[\sin(A+B) + \sin(A-B)]$

$\cos A \sin B = {}^1/_2[\sin(A+B) - \sin(A-B)]$

$\cos A \cos B = {}^1/_2[\cos(A+B) + \cos(A-B)]$

$\sin A \sin B = -{}^1/_2[\cos(A+B) - \cos(A-B)]$

31. Special Results

(1) $\cos A . \cos 2A . \cos 4A . \cos 8A \ldots\ldots\ldots \cos(2^{n-1}A) = \frac{1}{2^n \sin A}.\text{Sin}(2^n A)$

(2) $\cos\frac{\pi}{2n+1}.\cos\frac{2\pi}{2n+1}.\cos\frac{3\pi}{2n+1} - - - - - \cos\frac{n\pi}{2n+1} = \frac{1}{2^n}$

$\cos\theta.\cos 2\theta.\cos 3\theta \ldots\ldots \cos n\theta = \frac{1}{2^n}$ if $\theta = \frac{\pi}{2n+1}$

(3) $\cos A . \cos(60^0-A) . \cos(60^0+A) = \frac{\cos 3A}{4}$

(4) $\sin A \sin(60^0-A)\sin(60^0+A) = \frac{\sin 3A}{4}$

(5) $\tan A.\tan\left|60^0 - A\right|.\tan\left|60^0 + A\right| = \tan 3A$

(6) cot A cot (60^0 - A) . cot (60^0 + A) = cot3A

(7) cos A + cos (A+B) + cos (A+2B) + ... n terms $= \dfrac{\sin nB/2}{\sin B/2} \times \cos\left|\dfrac{\text{first angle + last angle}}{2}\right|$

(8) sin A + sin (A+B) + sin (A+2B) + ... n terms $= \dfrac{\sin nB/2}{\sin B/2} \times \sin\left|\dfrac{\text{first angle + last angle}}{2}\right|$

(9) $\sin^4 A + \cos^4 A = 1 - 2\sin^2 A\cos^2 A$

(10) $\sin^6 A + \cos^6 A = 1 - 3\sin^2 A\cos^2 A$

(11) $\sec^4 A + \tan^4 A = 1 + 2\sec^2 A.\tan^2 A$

$\operatorname{cosec}^4 A + \cot^4 A = 1 + 2\operatorname{cosec}^2 A.\cot^2 A$

(12) $\operatorname{cosec}^6 A - \cot^6 A = 1 + 3\operatorname{cosec}^2 A.\cot^2 A$

$\sec^6 A - \tan^6 A = 1 + 3\sec^2 A\tan^2 A$

32. Periodic function

f(x) is periodic with period t, if t is the least positive number such that f(x+t) = f(x)

sin x is periodic with period 2π.

cos x is periodic with period 2π.

tan x is periodic with period π.

cot x is periodic with period π

sec x is periodic with period 2π

cosec x is periodic with period 2π

|sin x| is periodic with period π

|cos x| is periodic with period π

Note 1: If f(x) is periodic with period t, then f(ax+b) where a > 0, b any number, is also periodic with period t/a.

eg: f(x) = sin 3x is periodic with period $2\pi/3$,

$f(x) = \sin(-3x)$ is periodic with period $\dfrac{2\pi}{|-3|}$; ie, $\dfrac{2\pi}{3}$

Note 2 : If $f_1(x)$ and $f_2(x)$ are periodic functions with periods T_1 and T_2 then $f_1(x) + f_2(x)$ is periodic with period T where T is the LCM of T_1 and T_2 provided there is no positive c such that

$f_1(c + x) = f_2(x)$ and $f_2(c + x) = f_1(x)$

Note 3: If $f(x)=\frac{af_1(x)+bf_2(x)}{cf_3(x)+df_4(x)}$ where $f_1(x), f_2(x), f_3(x), f_4(x)$ are periodic then period of f(x) is the l.c.m of the different periods.

Note 4: $f(x)=\cos\sqrt{x}$ is not periodic, $f(x)=\sin(x^2)$ is not periodic

33. $\sin 75^0 = \cos 15^0 = \frac{\sqrt{3}+1}{2\sqrt{2}}$, $\cos 75^0 = \sin 15^0 = \frac{\sqrt{3}-1}{2\sqrt{2}}$

$\sin 18^0 = \sin \pi/10 = \frac{\sqrt{5}-1}{4}$, $\cos 18^0 = \cos(\pi/10) = \frac{\sqrt{10+2\sqrt{5}}}{4}$

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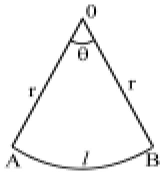
$\sin 22^1/_2{}^0 = \sin \pi/8 = \frac{\sqrt{2-\sqrt{2}}}{2}$, $\cos 22^1/_2{}^0 = \cos\pi/8 = \frac{\sqrt{2+\sqrt{2}}}{2}$ $\quad \tan 22^1/_2{}^0 = \tan \pi/8 = \sqrt{2}-1$

34. $l = r\theta$, θ is measured in radians, $\quad$ Area of the sector = $\frac{1}{2}r^2\theta$



35. Maximum value of $a\cos\theta + b\sin\theta$ is $\sqrt{a^2+b^2}$ and minimum value of $a\cos\theta + b\sin\theta$ is $-\sqrt{a^2+b^2}$.

If a and b are positive numbers such that $a > b$ then the minimum value of $a\sec\theta - b\tan\theta$ where $0<\theta<\pi/2$ is $\sqrt{a^2-b^2}$

36. If P(x, y) is any point in the cartesian plane then $\sin\theta = y/r$, $\cos\theta = x/r$, $\tan\theta = y/x$; $x^2 + y^2 = r^2$

37. Trigonometric equations

1. xxxxIf $\sin\theta = \sin\alpha$, $\theta = n\pi + (-1)^n\alpha$, $\alpha \in [-\pi/2, \pi/2]$, $n \in I$

If $\cos\theta = \cos\alpha$, $\theta = 2n\pi \pm \alpha$, $\alpha \in [0, \pi]$, $n \in I$

If $\tan\theta = \tan\alpha$, $\theta = n\pi + \alpha$, $\alpha \in (-\pi/2, \pi/2)$, $n \in I$

2. If $\sin^2\theta = \sin^2\alpha$, $\cos^2\theta = \cos^2\alpha$, $\tan^2\theta = \tan^2\alpha$ $\quad \theta = n\pi \pm \alpha$

3. If $\sin\theta = 0$, $\theta = n\pi$

   If $\sin\theta = 1$; $\theta = 2n\pi + \pi/2$; If $\sin\theta = -1$, $\theta = 2n\pi - \pi/2$

4. If $\cos\theta = 0$, then $\theta = (2n+1)\frac{\pi}{2}$

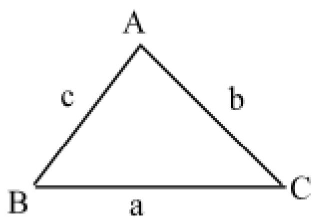
   If $\cos\theta = 1$, $\theta = 2n\pi$

   If $\cos\theta = -1$, $\theta = (2n+1)\pi$

5. If $\sin\theta = \sin\alpha$, $\cos\theta = \cos\alpha$ then $\theta = 2n\pi + \alpha$ where α is the common value satisfying the given equations and lying between 0 and 2π

6. To solve $a\cos\theta + b\sin\theta = c$ where $c \leq \sqrt{(a^2+b^2)}$, divide by $\sqrt{(a^2+b^2)}$

The following symbols in relation to ΔABC are universally adopted



$m\angle BAC = A$

$m\angle ABC = B$

$m\angle BCA = C$

$A + B + C = \pi$

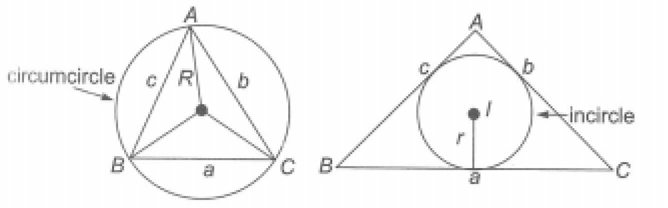
$AB = c, BC = a, CA = b$

* Semi-perimeter of the triangle, $s = \frac{a+b+c}{2}$ so, $a+b+c = 2s$

* The radius of the circumcircle of the triangle, i.e., circumradius = R

* The radius of the incircle of the triangle, i.e., intadius = r

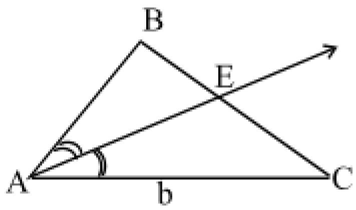
* Area of the triangle $= \Delta$



* If H is the orthocentre of the ΔABC then orthocentre of ΔAHB is C and that of ΔAHC in B.etc

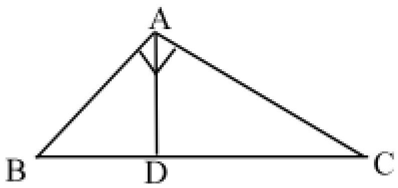
* Image (reflexion) of orthocentre on any side of ΔABC lies on the circumcircle of the triangle

* Angle bisector of ΔABC divides the oposite side in the ratio of other two sides



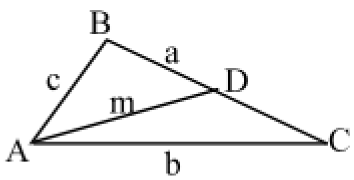
$BE : EC = c : b$

* If $AD \perp BC$ in a right triangle ABC with BC is the hypotenuse then $AD^2 = BD.DC$



* In a parallelogram ABCD, $AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + DA^2 = 2\left(AB^2 + BC^2\right)$

* In $\Delta ABC, AD$ is the median, the $4AD^2 = 2\left(AB^2 + AC^2\right) - BC^2$



$4m^2 = 2\left(b^2 + c^2\right) - a^2$, m length of the median

In any polygon of 'n' sides, the sum of the internal angles is $(n-2)\pi$ and sum of external angle is 2π

For regular polygon of n side and 'a' is length of the side, then radius of incribed circle of the polygon $r=\frac{a}{2}\cot\left(\frac{\pi}{n}\right)$ and radius of circumcribed circle $R=\frac{a}{2}\operatorname{cosec}\left(\frac{\pi}{n}\right)$

Some basic formula

(1) Sine rule : In any triangle ABC

$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=2R$; where R is the circumradius of ΔABC

(2) Cosine Rule, In any ΔABC $\cos A=\frac{b^2+c^2-a^2}{2bc}$; $\cos \mathbf{B}=\frac{c^2+a^2-b^2}{2ca}$; $\cos C=\frac{a^2+b^2-c^2}{2ab}$

(3) Tangent Rule (Napiers Analogy)

(1) $\tan\left(\frac{B-C}{2}\right)=\frac{b-c}{b+c}\operatorname{Cot}\frac{A}{2}$ (2) $\tan\left(\frac{C-A}{2}\right)=\frac{c-a}{c+a}\cot\frac{B}{2}$ (3) $\tan\left(\frac{A-B}{2}\right)=\frac{a-b}{a+b}\cot\frac{C}{2}$

(4) Half Angle formulae

In any triangle ABC, If a+b+c = 2s, then

a) (i) $\sin\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{bc}}$ b)(i) $\cos\frac{A}{2}=\sqrt{\frac{s(s-a)}{bc}}$ c) (i) $\tan\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$

(ii) $\sin\frac{B}{2}=\sqrt{\frac{(s-c)(s-a)}{ca}}$ (ii) $\cos\frac{B}{2}=\sqrt{\frac{s(s-b)}{ca}}$ (ii) $\tan\frac{B}{2}=\sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$

(iii) $\sin\frac{C}{2}=\sqrt{\frac{(s-a)(s-b)}{ab}}$ (iii) $\cos\frac{C}{2}=\sqrt{\frac{s(s-c)}{ab}}$ (iii) $\tan\frac{C}{2}=\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$

(5) Projection formulae -In any ΔABC

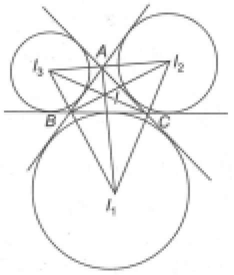
$a=b\cos C+c\cos B$; $b=c\cos A+a\cos C$; $c=a\cos B+b\cos A$

(6) Area of $\Delta ABC=\frac{1}{2}bc\sin A=\frac{1}{2}ca\sin B=\frac{1}{2}ab\sin c$

$\Delta=\sqrt{s(s-a)(s-b)(s-c)}$; $R=\frac{abc}{4\Delta}$, $\Delta=\frac{abc}{4R}$, $\Delta=rS$; $r=\frac{\Delta}{S}$

$\Delta=2R^2\sin A\sin B\sin C$

(7) In ΔABC I incircle r inradius I_1, I_2, I_3 are excircles and r_1, r_2, r_3 are exradii



$$\tan\frac{A}{2}=\frac{\Delta}{S-a};\tan\frac{B}{2}=\frac{\Delta}{S-b};\tan\frac{C}{2}=\frac{\Delta}{S-c}$$

$$r_1=\frac{\Delta}{S-a};r_2=\frac{\Delta}{S-b};\ r_3=\frac{\Delta}{S-c}$$

$$\tan\frac{A}{2}=\frac{r_1}{S};\ \tan\frac{B}{2}=\frac{r_2}{S};\tan\frac{C}{2}=\frac{r_3}{S}$$

$$\frac{1}{r}=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}$$

PART I - (JEEMAIN)

SECTION - I - Straight objective type questions

1. If $\sec\theta+\tan\theta=p$, then $\tan\theta$ is equal to

(1) $\frac{2p}{p^2-1}$ (2) $\frac{p^2-1}{2p}$ (3) $\frac{p^2+1}{2p}$ (4) $\frac{2p}{p^2+1}$

2. The value of $\cos y\cos\left(\frac{\pi}{2}-x\right)-\cos\left(\frac{\pi}{2}-y\right)\cos x+\sin y\cos\left(\frac{\pi}{2}-x\right)+\cos x\sin\left(\frac{\pi}{2}-y\right)$ is zero, if

(1) x = 0 (2) y = 0 (3) x = y (4) $x=n\pi-\frac{\pi}{4}+y,(n\in I)$

3. If $\sin A=\frac{4}{5}$ and $\cos B=-\frac{12}{13}$, where A and B lie in first and third quadrant respectively, then $\cos(A+B)=$

(1) $\frac{56}{65}$ (2) $-\frac{56}{65}$ (3) $\frac{16}{65}$ (4) $-\frac{16}{65}$

4. The expression $\frac{\cos 6x + 6\cos 4x + 15\cos 2x + 10}{\cos 5x + 5\cos 3x + 10\cos x}$ is equal to

(1) cos 2x (2) 2cos x (3) $\cos^2 x$ (4) 1 + cos x

5. If θ is an acute angle and $\sin\frac{\theta}{2} = \sqrt{\frac{x-1}{2x}}$, then $\tan\theta$ is equal to

(1) $x^2 - 1$ (2) $\sqrt{x^2 - 1}$ (3) $\sqrt{x^2 + 1}$ (4) $x^2 + 1$

6. If $A = \sin^2\theta + \cos^4\theta$, then for all real values of θ

(1) $1 \le A \le 2$ (2) $\frac{3}{4} \le A \le 1$ (3) $\frac{13}{16} \le A \le 1$ (4) $\frac{3}{4} \le A \le \frac{13}{16}$

7. If $\alpha + \beta - \gamma = \pi$, then $\sin^2\alpha + \sin^2\beta - \sin^2\gamma =$

(1) $2\sin\alpha\sin\beta\cos\gamma$ (2) $2\cos\alpha\cos\beta\cos\gamma$

(3) $2\sin\alpha\sin\beta\sin\gamma$ (4) None of these

8. In any triangle ABC , $\sin^2\frac{A}{2} + \sin^2\frac{B}{2} + \sin^2\frac{C}{2}$ is equal to

(1) $1 - 2\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$ (2) $1 - 2\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

(3) $1 - 2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$ (4) $1 - 2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

9. The value of $\cos^2 10^0 - \cos 10^0 \cos 50^0 + \cos^2 50^0$ is

(1) $\frac{3}{2}(1 + \cos 20^0)$ (2) $\frac{3}{4}$ (3) $\frac{3}{4} + \cos 20^0$ (4) $\frac{3}{2}$

10. General solution of the equation $\cot\theta - \tan\theta = 2$ is

(1) $n\pi + \frac{\pi}{4}$ (2) $\frac{n\pi}{2} + \frac{\pi}{8}$ (3) $\frac{n\pi}{2} \pm \frac{\pi}{8}$ (4) $\frac{n\pi}{4} + \frac{\pi}{16}$

11. If $\frac{1 - \tan^2\theta}{\sec^2\theta} = \frac{1}{2}$, then the general value of θ is

(1) $n\pi \pm \frac{\pi}{6}$ (2) $n\pi + \frac{\pi}{6}$ (3) $2n\pi \pm \frac{\pi}{6}$ (4) $n\pi + \frac{\pi}{4}$

12. The solution of the equation $4\cos^2 x + 6\sin^2 x = 5$ is

(1) $x = n\pi \pm \frac{\pi}{2}$ (2) $x = n\pi \pm \frac{\pi}{4}$ (3) $x = n\pi \pm \frac{3\pi}{2}$ (4) $x = n\pi \pm \frac{3\pi}{4}$

13. One root of the equation $\cos x - x + \frac{1}{2} = 0$ lies in the interval

(1) $\left[0, \frac{\pi}{2}\right]$ (2) $\left[-\frac{\pi}{2}, 0\right]$ (3) $\left[\frac{\pi}{2}, \pi\right]$ (4) $\left[\pi, \frac{3\pi}{2}\right]$

14. The +ve integer value of n>3 satisfying the equation $\frac{1}{\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)}$ is

1) 5 2) 6 3) 7 4) 8

15. The only value of x for which $2^{\sin x} + 2^{\cos x} > 2^{1-(1/\sqrt{2})}$ holds, is

(1) $\frac{5\pi}{4}$ (2) $\frac{3\pi}{4}$ (3) $\frac{\pi}{2}$ (4) All values of x

16. **Statement-I:** $\cos^3\alpha + \cos^3\left(\alpha + \frac{2\pi}{3}\right) + \cos^3\left(\alpha + \frac{4\pi}{3}\right) = 3\cos\theta\cos\left(\alpha + \frac{2\pi}{3}\right)\cos\left(\alpha + \frac{4\pi}{3}\right)$

Statement-II: If $a + b + c = 0 \Leftrightarrow a^3 + b^3 + c^3 = 3abc$.

(1) If both Statement-I and Statement-II are true and the reason is the correct explanation of the assertion.
(2) If both Statement-I and Statement-II are true but reason is not the correct explanation of the assertion.
(3) If Statement-I is true but Statement-II is false.
(4) If Statement-I is false but Statement-II is true.

17. In a ΔABC, let a = 6, b = 3 and $\cos(A - B) = \frac{4}{5}$

(All symbols used have usual meaning in a triangle)

Statement-I: $\angle B = \frac{\pi}{2}$

Statement-II: $\sin A = \frac{2}{\sqrt{5}}$

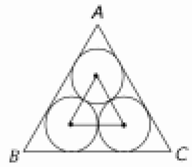
18. In ΔABC, $(b - c)\cot\frac{A}{2} + (c - a)\cot\frac{B}{2} + (a - b)\cot\frac{C}{2}$ is equal to

(1) 0 (2) 1 (3) ± 1 (4) 2

19. The lengths of the sides of a triangle are $\alpha-\beta, \alpha+\beta$ and $\sqrt{3\alpha^2+\beta^2}$, $(\alpha>\beta>0)$. Its largest angle is

(1) $\frac{3\pi}{4}$ (2) $\frac{\pi}{2}$ (3) $\frac{2\pi}{3}$ (4) $\frac{5\pi}{6}$

20. The area of the equilateral triangle which containing three coins of unity radius is



(1) $6+4\sqrt{3}$ *sq.units* (2) $8+\sqrt{3}$ *sq. units*

(3) $4+\frac{7\sqrt{3}}{2}$ *sq. units* (4) $12+2\sqrt{3}$ *sq. units*

SECTION - II

Numerical type Questions

21. If $\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}}=\frac{1}{7}$ and $\sqrt{\frac{1-\cos 2\beta}{2}}=\frac{1}{\sqrt{10}}$, $\alpha,\beta\in\left(0,\frac{\pi}{2}\right)$, then $\tan(\alpha+2\beta)$ is equal to________

22. If $\tan^2\theta=2\tan^2\phi+1$, then $\cos 2\theta+\sin^2\phi$ equals ________

23. The number of ordered pairs (x, y) satisfying |x| + |y| = 2 and $\sin\left(\frac{\pi x^2}{3}\right)=1$ is

24. Number of integral solutions of the equation $\log_{\sin x}\sqrt{\sin^2 x}+\log_{\cos x}\sqrt{\cos^2 x}=2$, where $x\in[0,6\pi]$ is

25. $\frac{\cos 5A}{\cos A}+\frac{\sin 5A}{\sin A}=a+b\cos 4A$ then the value of a+b is

PART - II (JEE ADVANCED)

SECTION - III (Only one option option correct type)

26. $0\le a\le 3, 0\le b\le 3$ and the equation $x^2+4+3\cos(ax+b)=2x$ has at least one solution then the value of (a+b)

A) $\frac{\pi}{2}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{3}$ D) π

27. If α, β, γ do not differ by a multiple of π and if $\frac{\cos(\alpha+\theta)}{\sin(\beta+\gamma)} = \frac{\cos(\beta+\theta)}{\sin(\gamma+\alpha)} = \frac{\cos(\gamma+\theta)}{\sin(\alpha+\beta)} = k$. Then k equals

A) ± 2　　B) $\pm\frac{1}{2}$　　C) 0　　D) ± 1

28. $\sum_{r=1}^{10} \cos^3 \frac{r\pi}{3} =$

A) $-\frac{1}{8}$　　B) $-\frac{7}{8}$　　C) $-\frac{9}{8}$　　D) $\frac{1}{8}$

29. If $\frac{\tan(\alpha+\beta-\gamma)}{\tan(\alpha-\beta+\gamma)} = \frac{\tan\gamma}{\tan\beta}$ $(\beta \neq \gamma)$ then $\sin 2\alpha + \sin 2\beta + \sin 2\gamma =$

A) 0　　B) 1　　C) 2　　D) 1/2

30. If $\frac{x}{y} = \frac{\cos A}{\cos B}$ then $\frac{x\tan A + y\tan B}{x+y} =$

A) $\tan\frac{A+B}{2}$　　B) $\tan\frac{A-B}{2}$　　C) $\cot\frac{A+B}{2}$　　D) $\cot\frac{A-B}{2}$

31. The number of distinct real roots of the equation $\sqrt{\sin x} - \frac{1}{\sqrt{\sin x}} = \cos x$ is (where $0 \leq x \leq 2\pi$)

(A) 1　　(B) 2　　(C) 3　　(D) more than 3

SECTION - IV (More than one correct answer)

32. If $\cos x + \cos y + \cos z = 0 = \sin x + \sin y + \sin z$ then the possible value of $\cos\left(\frac{x-y}{2}\right) =$

A) $\frac{1}{2}$　　B) $-\frac{1}{2}$　　C) 1　　D) -1

33. If $\frac{\tan 3A}{\tan A} = k$ $(k \neq 1)$ then

A) $\frac{\cos A}{\cos 3A} = \frac{k^2-1}{2k}$　　B) $\frac{\sin 3A}{\sin A} = \frac{2k}{k-1}$　　C) $k < \frac{1}{3}$　　D) $k > 3$

34. For $0 < \phi < \pi/2$, if $x = \sum_{n=0}^{\infty} \cos^{2n}\phi$, $y = \sum_{n=0}^{\infty} \sin^{2n}\phi$ and $z = \sum_{n=0}^{\infty} \cos^{2n}\phi \sin^{2n}\phi$ then xyz =

A) xy + z　　B) xz + y　　C) x + y + z　　D) yz + x

35. The equation $\sin^2 x + \sin x - a = 0 (0 \le x < 2\pi)$

A) has solutions for every $a \ge -\frac{1}{4}$

B) has two solutions for $a = -\frac{1}{4}$

C) has four solutions for $-\frac{1}{4} < a < 0$

D) has two solutions for $-\frac{1}{4} < a < 0$

36. The number of distinct real roots of the equation $\tan^2 2x + 2\tan 2x \tan 3x - 1 = 0$ in the interval $\left[0, \frac{\pi}{2}\right]$ is

37. Number of solutions of the equation $\frac{\sin x}{\cos 3x} + \frac{\sin 3x}{\cos 9x} + \frac{\sin 9x}{\cos 27x} = 0$ in the interval $\left(0, \frac{\pi}{4}\right)$ is

Passage-I

SECTION - V (Numerical Type)

38. $16\left(\cos\theta - \cos\frac{\pi}{8}\right)\left(\cos\theta - \cos\frac{3\pi}{8}\right)\left(\cos\theta - \cos\frac{5\pi}{8}\right)\left(\cos\theta - \cos\frac{7\pi}{8}\right) = \lambda \cos 4\theta$

then the value of λ is

39. The maximum value of the expression $\frac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta}$ is

SECTION VI - (Matrix match type)

40.

	Column - I		Column - II
A	Number of distinct real roots of the equation $x = \left(\frac{5\pi}{2}\right)^{\cos x}$	P	0
B	If 'a' is irrational then the number of real roots of the equation $1 + \sin^2 ax = \cos x$	Q	1
C	The number of real roots of the equation 4sin2x+cos2x = 5 is	R	2
D	The number of distinct real roots of the equation $4\cos^2 x + 2\left(\sqrt{3}+1\right)\sin x - \sqrt{3} - 4 = 0$ in the interval $\left[0, \frac{\pi}{2}\right]$ in the interval is	S	3
		T	4