CHAPTER - 20

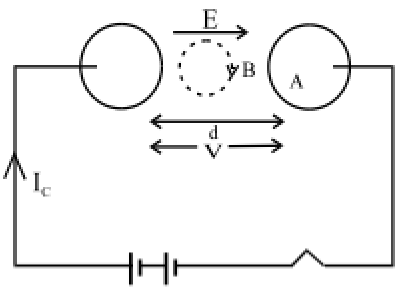
ELECTROMAGNETIC WAVES

SYNOPSIS

Displacement current (I_D)

Maxwell developed the theory of EM waves by introducing the idea of the displacement current.

Consider the working of a capacitor.



According to Faradays Law of electromagnetic induction, a changing magnetic field induces a current. Maxwell said that the converse of this, is also true. A changing electric field induces a changing magnetic field, which in turn induces an e.m.f and hence the displacement current.

$$I_D = \epsilon_0 \frac{d}{dt}(\varphi_E) \quad \text{(Given by Maxwell)}$$

$$= \epsilon_0 \frac{d}{dt}(EA)$$

$$= \epsilon_0 A \frac{d}{dt}(E)$$

$$= \epsilon_0 A \frac{d}{dt}\left(\frac{V}{d}\right)$$

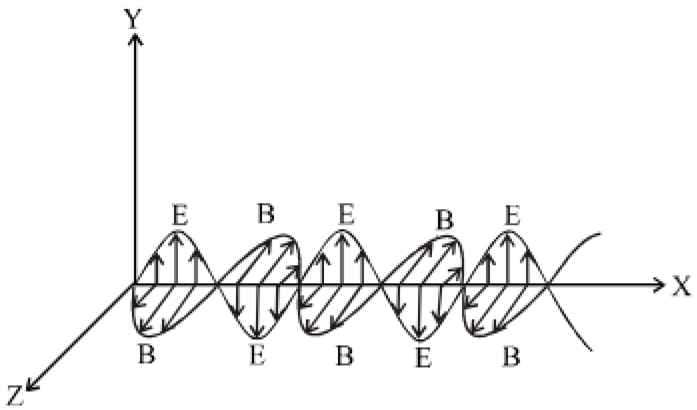
$$= \frac{\epsilon_0 A}{d}\frac{d}{dt}(V) = C\frac{dV}{dt}$$

The time varying electric field between the plates of a capacitor, is directed perpendicular to the plane of the capacitor plates and the time varying magnetic field (produced by the electric field), is along the perimeter of a circle, in a plane, parallel to the plates of the capacitor. Thus the time varying electric field and the magnetic field are perpendicular to each other. The two fields combine to constitute EM waves, which propagates in a direction perpendicular to the directions of both the fields. The value of I_D is numerically equal to the conduction current I_C. Inside the capacitor, $I_C = 0$ and $i = I_D$. Outside the capacitor $I_D = 0$ and $i = I_C$.

Hertz produced EM waves in the laboratory. J.C. Bose transmitted EM waves through short distances. Marconi transmitted EM waves through long distances.

Propagation of EM waves

EM waves can be produced in the laboratory, due to accelerated charges. EM waves propagate through space as coupled electric and magnetic fields, oscillating perpendicular to each other and to the direction of propagation of the waves.



EM waves are transverse progressive waves and the equation may be written as

$$E_x = E_0 \sin(kz - \omega t)$$

$$B_y = B_0 \sin(kz - \omega t)$$

where $k = \dfrac{2\pi}{\lambda}$

The direction of EM wave is along $\vec{E} \times \vec{B}$. The direction may be along any axis x, y, or z. EM waves do not require a medium for the propagation. The variation of E and B have the same frequency and in phase. At any instant, the ratio of the amplitudes of electric and magnetic fields, is a constant and is equal to the velocity of EM waves. $\left(\dfrac{E_0}{B_0} = c\right)$

EM waves are governed by Maxwell's Equations.

1) $\oint_s \overline{E}.\overline{ds} = \dfrac{1}{\epsilon_0} q$ (Gauss' Law in Electrostatics)

2) $\oint_s \overline{B}.\overline{ds} = 0$ (Gauss' Law in Magnetism)

3) $\oint_c \bar{E}.\overline{dl} = \frac{d}{dt}(\phi_B)$ (Faraday's Law of electromagnetic induction)

4) $\oint_c \bar{B}.\overline{dl} = \mu_0[I + I_D] = \mu_0\left[I + \epsilon_0 \frac{d}{dt}(\phi_E)\right]$ (Ampere-Maxwell Law)

Magnetic field (B) on a circular loop of wire of radius r, kept between the capacitor plates of radius R, is given by

$$B = \frac{\mu_0 I_D r}{2\pi R^2}$$

$$B_{max} = \frac{\mu_0 I_D}{2\pi R} \rightarrow (r = R)$$

Velocity of EM waves in free space

$$C = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.85 \times 10^{-12}}} = 3 \times 10^8 m/s$$

Velocity of EM waves in a medium

$$V = \frac{1}{\sqrt{\mu \epsilon}} = \frac{1}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \times \frac{1}{\sqrt{\mu_r \epsilon_r}} = \frac{c}{\sqrt{\mu_r \epsilon_r}} = \frac{c}{n}$$

where $\sqrt{\mu_r \epsilon_r} = n =$ refractive index of the medium.

Propagation constant (Angular wave number)

$$K = \frac{2\pi}{\lambda} = \frac{2\pi}{c/\upsilon} = \frac{2\pi\upsilon}{c} = \frac{\omega}{c}$$

$c = \frac{\omega}{K}$ where c = velocity of EM radiations

Average energy density in the electric field

$$U_E = \frac{1}{4}\epsilon_0 E_0^2 = \frac{1}{4}\epsilon_0 \times 2E_{rms}^2 = \frac{1}{2}\epsilon_0 E_{rms}^2$$

$$\left[E_{rms} = \frac{E_0}{\sqrt{2}}\right]$$

Average energy density in the magnetic field

$$U_B = \frac{1}{4}\frac{B_0^2}{\mu_0} = \frac{1}{4\mu_0} 2B_{rms}^2 = \frac{1}{2\mu_0} B_{rms}^2$$

$$\left[B_{rms} = \frac{B_0}{\sqrt{2}}\right]$$

To show that $U_E = U_B$

$$U_E = \frac{1}{4}\epsilon_0 E_0^2 = \frac{1}{4}\epsilon_0 (B_0 c)^2 = \frac{1}{4}\epsilon_0 B_0^2 c^2$$

$$= \frac{1}{4}\epsilon_0 B_0^2 \times \frac{1}{\mu_0 \epsilon_0} = \frac{1}{4\mu_0} B_0^2 = U_B$$

Total average energy density of an EM wave (U)

$$U_{av} = U = 2U_E = 2\times\frac{1}{4}\epsilon_0 E_0^2 = \frac{1}{2}\epsilon_0 E_0^2 = \epsilon_0 E_{rms}^2$$

or

$$U_{av} = U = 2U_B = 2\times\frac{1}{4\mu_0} B_0^2 = \frac{1}{2\mu_0} B_0^2 = \frac{B_{rms}^2}{\mu_0}$$

Poynting vector $(\vec{S})$

The energy transported by EM waves per second per unit area, is represented by a quantity called the Poynting vector.

It is denoted by $\vec{S} = \dfrac{\vec{E}\times\vec{B}}{\mu_0}$

Magnitude of $S = \dfrac{1}{\mu_0} EB$

$$S = \frac{1}{\mu_0} E.\frac{E}{c} = \frac{1}{\mu_0}\frac{E^2}{c}$$

$$S = \frac{1}{\mu_0} BcB = \frac{1}{\mu_0} B^2 c$$

Momentum of EM wave $P = \dfrac{U}{c}$

Momentum delivered by an EM wave to a perfectly absorbing surface $P = \dfrac{U}{c}$

Momentum delivered by an EM wave to a perfectly reflecting surface $= P - (-P) = 2P = \dfrac{2U}{c}$

Intensity of radiation

$$I = \frac{1}{2} \epsilon_0 E_0^2 c$$

$$I = \frac{1}{2\mu_0} B_0^2 c$$

1) $I = Uc$

2) $I = \frac{P}{4\pi d^2}$

where P = power of source
d = distance of the point

3) $I = \frac{S}{2}$

where S = poynting vector

Radiation pressure (ρ)

Absorbing surface $\rho = \frac{S}{c}$

Reflecting surface $\rho = \frac{2S}{c}$

Electromagnetic spectrum

The orderly distribution of EM radiations, according to their wave length or frequency, is called the electromagnetic spectrum

γ-rays, X-rays, u-v-rays, Visible light, IR rays, Microwaves, Radio waves.

γ-rays

Wave length range : 1×10^{-14}m $\rightarrow$ 1×10^{-10} m

Frequency range : 3×10^{22} Hz $\rightarrow$ 3×10^{18} Hz

They are highly energetic radiations and are emitted by radioactive nuclei. They are detected by photographic plate and ionisation chamber. They are neutral and highly penetrating. They can be used to destroy cancer cells.

X-rays

Wave length range : 1×10^{-11} m $\rightarrow$ 3×10^{-8} m

Frequency range : 3×10^{19} Hz $\rightarrow$ 1×10^{16} Hz

They are produced by coolidge X-ray tube. They possess penetrating power and used for diagnosis of deceases.

u-v rays

Wave length range : 1×10^{-8} m $\rightarrow$ 4×10^{-7} m

Frequency range : 3×10^{16} Hz $\rightarrow$ 8×10^{14} Hz

They are part of solar spectrum, they can also be produced by arcs of mercury and iron. They can be detected by photocells and photographic film. u-v rays can produce vitamin D in the skin, but produce extra menalin, tanning the skin. u-v rays can be used for purifying water and also for LASIK (Laser Assisted In Situ Keratomileusis) eye surgery. Ozone layer and also ordinary glass, reduces the intensity of u-v rays.

Visible light (VIBGYOR)

Wave length range : 4×10^{-7} m $\rightarrow$ 8×10^{-7} m

Frequency range : 8×10^{14} Hz $\rightarrow$ 4×10^{14} Hz

They are emitted due to atomic excitation Human eye is sensitive, only to visible part of EM spectrum.

Infrared rays

Wave length range : 8×10^{-7} m $\rightarrow$ 5×10^{-3} m

Frequency range : 4×10^{14} Hz $\rightarrow$ 6×10^{11} Hz

IR rays are heat radiations and therefore all hot bodies including the sun are sources of IR rays. They are detected by thermopiles. IR rays are responsible for “Greanhouse effect”.

Microwaves (Short Radio Waves)

Wave length range : 1×10^{-3} m $\rightarrow$ 1×10^{-1} m

Frequency range : 3×10^{11} Hz $\rightarrow$ 1×10^{9} Hz

Microwaves are produced by oscillating electronic circuits. (Klystron valve, Magnetron valve etc) They are used in RADAR and also in ovens.

Radiowaves

Wave length range : 1×10^{-1} m $\rightarrow$ 1×10^{4} m

Frequency range : 3×10^{9} Hz $\rightarrow$ 3×10^{4} Hz

They are produced by oscillating electronic circuits. Radiowaves are used as carrier waves in radio broadcasting and television transmission.

SECTION - I - Straight objective type questions

1. Displacement current is

A) continuous when electric field is changing in the circuit

B) continuous when magnetic field is changing in the circuit

C) continuous in both types of fields

D) continuous through wires and resistance only

2. Maxwell's modified form of Ampere's circuital law is

A) $\oint_s \vec{B}.\overline{ds} = 0$

B) $\oint_s \vec{B}.\overline{ds} = \mu_0 I$

C) $\oint_l \vec{B}.\overline{d\ell} = \mu_0 I$

D) $\oint_l \vec{B}.\overline{d\ell} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$

3. Which of the following has zero average value in a plane E.M. wave ?

A) Kinetic energy

B) Magnetic field

C) Electric field

D) Both B and C

4. Speed of E.M. waves through a medium depends upon

A) only electric properties of the medium

B) only magnetic properties of the medium

C) mechanical and thermal properties of medium

D) both electric and magnetic properties of the medium

5. Consider an electromagnetic wave that propagates in the Z direction with an electric field strength of 1 v/m pointing in the Y direction. Then the direction and magnitude of magnetic field pulse that travels along with electric field is :

A) 3.33×10^{-9} T in y direction

B) 3.33×10^{-9} T in z direction

C) 3.33×10^{-9} T in –ve direction

D) 6.66×10^{-9} T in x direction

6. A plane EM wave is incident on a material surface. If the wave delivers momentum P and energy E, then

A) P = 0, E = 0

B) $P \neq 0, E \neq 0$

C) $P \neq 0, E = 0$

D) $P = 0, E \neq 0$

7. In an em wave, the amplitude of electric field is 1 V/m. The frequency of wave is 5×10^{14} Hz and is moving along z-axis. What is the average energy density of electric field?

A) 4.4×10^{-12} B) 6.6×10^{-12} C) 2.2×10^{-12} D) 8.8×10^{-12}

8. The total average energy density of e.m waves whose electric field variation is given by $E = (50\ N/c) \sin(\omega t - kx)$ will be nearly

A) $10^{-8}\ J/m^3$ B) $10^{-6}\ J/m^3$ C) $10^{-10}\ J/m^3$ D) $10^{-12}\ J/m^3$

9. The electric part of a plane electromagnetic wave varies with time of amplitude 2v/m propagating along z-axis. The average energy density is (in Jm^{-3})

A) 13.29×10^{-12} B) 8.86×10^{-12}

C) 17.172×10^{-12} D) 4.43×10^{-12}

10. About 5% of the power of a 100 W bulb is converted to visible radiation. The average intensity of the visible radiation at a distance of 10m from the bulb is

A) $0.4\ W/m^2$ B) $4 \times 10^{-5}\ W/m^2$

C) $4 \times 10^{-3}\ W/m^2$ D) $5 \times 10^{-1}\ W/m^2$

11. An electromagnetic wave of energy $3\times10^2\,J$ is incident on a particular surface on a particular time interval. If the surface absorbs 60% and reflects 40% of incident radiation. The total momentum transferred in that particular time is:

A) $6\times10^7\,kg\ m/sec$ B) $8\times10^{-7}\,kg\ m/sec$

C) $1.4\times10^{-6}\,kg\ m/sec$ D) $4\times10^{-6}\,kg\ m/sec$

12. A radar sends the waves towards a distant object and receives the signal reflected by the object. These waves are

A) sound waves B) X-rays C) microwaves D) Radiowaves

13. Green house effect is due to

A) UV radiations B) infrared radiations

C) γ-rays D) X-rays

14. Given below are two statements: In the light of the statements choose the correct option

Statement-I : Microwaves are better carrier of signals than optical waves

Statement-II : Microwaves and optical waves move with the same speed

A) Statement-I and Statement-II are true and Statement-II is the correct explanation of Statement-I.

B) Statement-I and Statement-II are true and Statement-II is not the correct explanation of Statement I

C) Statement-I is false but Statement-II is true.

D) Both Statement-I and Statement-II are false.

15. Assertion : If the earth did not have an atmosphere, the day will be warmer and the night will be cooler.
Reason : The earth's atmosphere produce green-house effect.

A) If both Assertion and Reason are true and Reason is the correct explanation of Assertion.

B) If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

C) If Assertion is true but Reason is false.

D) If both Assertion and Reason are false.

SECTION - II - One or more option correct type

16. Which of the following statement(s) are correct about displacement current ?

A) It exist between the plates of a capacitor

B) It is not due to the conduction of electrons

C) It can flow even through vacuum

D) It is due to time varying electric field in between the plates of capacitor

17. Which of the following is / are the property of a monochromatic plane EM wave in free space

A) The electric and magnetic field are vibrates in same phase

B) The energy contribution of both electric and magnetic fields are equal

C) The direction of propagation is in the direction of $\vec{B} \times \vec{E}$

D) The pressure exerted by the wave is the product of its speed and energy density

18. A charged particle oscillates in vacuum about its mean equilibrium position with frequency of 10^9Hz. The EM wave produced

A) will have a frequency of 10^9Hz B) will have speed of 30 m/s

C) will have a wavelength of 0.3m D) fall in the region of infrared waves

19. Which of the following is not transported by the EM wave ?

A) Energy B) Information C) Charge D) Momentum

20. An EM wave enters a medium from air. The electric fields are

$$\vec{E}_1 = E_{01}\hat{x}\cos\left[2\pi\nu\left(\frac{z}{c} - t\right)\right] \text{in air}$$

$$\vec{E}_2 = E_{02}\hat{x}\cos\left[k(2z - ct)\right] \text{in medium}$$

where the wave number k and frequency ν refer to their values in air. The medium is non-magnetic, if ε_{r_1} and ε_{r_2} refer to relative permittivities of air and medium respectively. Which of the following options is correct?

A) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{2}$ B) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = 4$ C) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = 2$ D) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{4}$

21. Given the wave function (in SI unit) for a wave to be $\psi_{(x,t)} = 10^3 \sin\pi\left(3\times10^6 x - 9\times10^{14} t\right)$. The speed of wave is

A) $9\times10^{14}\text{ms}^{-1}$ B) $3\times10^8\text{ms}^{-1}$ C) $3\times10^6\text{ms}^{-1}$

D) $3\times10^7\text{ms}^{-1}$ E) $3\times10^5\text{ms}^{-1}$

22. A plane electromagnetic wave of intensity 6 W/m^2 strikes a small mirror of area 30 cm^2, held perpendicular to the approaching wave. The momentum transferred in kgm/s by the wave to the mirror in each second will be

A) 1.2×10^{-10} B) 2.4×10^{-9} C) 3.6×10^{-8} D) 4.8×10^{-7}

23. An intense light source radiates uniformly in all directions. At a distance 3m from the surface, the radiation pressure on absorbing surface is 9×10^{-11} Pa. Find the total average power output in Watt

A) 1 B) 7 C) 2 D) 3

24. A beam of light travelling along x-axis is described by the electric field $E_y = (600\ Vm^{-1}) \sin \omega (t - x/c)$. Then maximum magnetic force on a charge q=2e, moving along y-axis with a speed of 3×10^7 m/s is (e $=1.6 \times 10^{-19}$ C)

A) 19.2×10^{-17} N B) 1.92×10^{-17} N C) 0.0192 N D) None of these

25. If v_y, v_x and v_m are the speed of gamma rays, x-rays and microwaves respectively in vacuum, then

A) $v_g > v_x > v_m$ B) $v_g < v_x < v_m$ C) $v_g < v_x < v_m$ D) $v_g = v_x = v_m$

26. Which of the following rays is emitted by a human body?

A) x-rays B) visible rays C) IR rays D) UV rays

Section III - Numerical Type Questions

27. A plane electromagnetic wave of frequency 50mHz travels in free space along the positive x-direction. At a particular point in spce and tie, $\vec{E} = 6.3\hat{j}\,V/m$. The corresponding magnetic field $\vec{B}$ at that point is $x \times 10^{-8}\hat{k}\,T$. find the value of a

28. The magnetic field in a travelling electromagnetic wave has a peak value of 20nT. The peak value of electric field strength (in volt m^{-1}) is

29. A new system of unit is evolved in which she values of μ_0 and ε_0 are 2 and 8 respectively. Then, the s peed of light in this system will be

30. The electric field associated with an em wave in vacuum is given by $\vec{E} = 40\cos(k_3 - 6 \times 10^8 t)\hat{i}$, where E, Z and t are in volt/m, meter and seconds respectively. The value of wave vector $\hat{k}$ in (emetre^{-1}) is