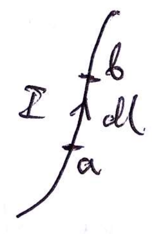
MOVING CHARGES & MAGNETISM

The branch of physics which deals with the magnetism due to electric current or moving charge is called electromagnetism.

The relation between electricity and magnetism was discovered by Oersted in 1820.

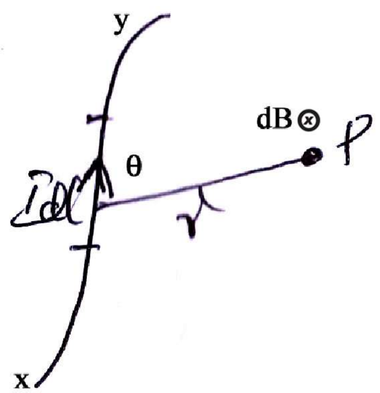
Current Element:

A very small element of length ‘dl’ of a thin conductor carrying current I called current element. Current element is a vector quantity whose magnitude is equal to the product of current and length of small element having the direction of the flow of current



Biot-Savart’s Law

Biot-Savart’s law is used to determine the magnetic field at any point due to current carrying conductor.



According to Biot-Savart’s Law, magnetic field at any point ‘P’ due to the current element Idl is dB.

dB α I, dB α dl, $dB \, \alpha \sin\theta$, $dB \, \alpha \frac{1}{r^2}$

i.e. $dB \, \alpha \frac{Idl\sin\theta}{r^2}$ or $dB = \frac{KIdl\sin\theta}{r^2}$

$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$$ In S.I. unit

In C.G.S. unit k = 1 and In S.I. unit $k = \frac{\mu_0}{4\pi}$

S.I. unit of $\overline{B}$ is Weber / m^2 or Tesla.

In C.G.S unit is Gauss (G) or Maxwell / cm^2

IT = 10^4 gauss

where μ_0 = absolute permeability of air

or vacuum = $4\pi \times 10^{-7} \frac{Wb}{A-m}$ its other unit are

$$\frac{henry}{metre} \text{ or } \frac{N}{A^2} \text{ or } \frac{Tesla-metre}{Ampere}$$

Vector form of Biot-Savart's Law

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\left(\vec{dl}\times\vec{r}\right)}{r^3} = \frac{\mu_0}{4\pi}\times\frac{I\left(\vec{dl}\times\hat{r}\right)}{r^2} = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}\hat{n}$$

where $\hat{n}$ is a unit vector which is perpendicular to both $I\vec{dl}$ and $\vec{r}$ i.e.B is perpendicular to both Idl and r for $\theta = 0$ or 180 $\sin\theta = 0$ i.e. magnetic field on the axis of a current carrying conductor is always zero.

In terms of current density,

$$\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{\left(\vec{J}\times\vec{r}\right)dv}{r^3}$$

J $\rightarrow$ current density

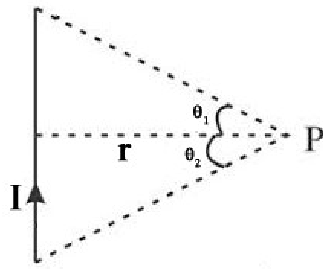
dv $\rightarrow$ small volume element

Direction of magnetic field :

Right hand thumb rule : According to this rule if a straight current carrying conductor is held in the right hand such that the thumb of the hand represents the direction of current flow then the direction of folding fingers will represent the direction of magnetic lines of force.

Application of Biot-Savart's Law

(1) Field due to finite length straight conductor carrying current



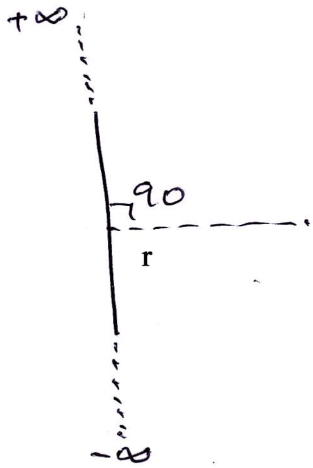
field at point P,

$$B = \frac{\mu_0}{4\pi}\frac{I}{r}\left[\sin\theta_1 + \sin\theta_2\right]$$

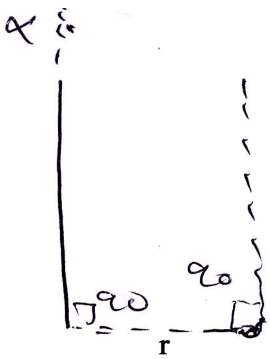
$r \rightarrow$ perpendicular distance to point P from the conductor

2) If the conductor is infinitely long, then $\theta_1 = \frac{\pi}{2}$ and $\theta_2 = \frac{\pi}{2}$

$$B = \frac{\mu_0 I \times 2}{4\pi r} = \frac{\mu_0 I}{2\pi r}$$



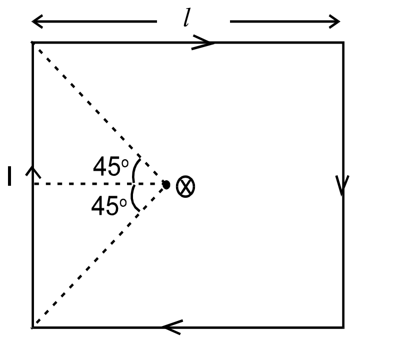
3) Magnetic field due to special semi infinite length wire at point P



$\theta_1 = 90^\circ, \theta_2 = 0$

$$B = \frac{\mu_0 I}{4\pi r}$$

Magnetic field at the centre of a square loop



Field due to one side at centre

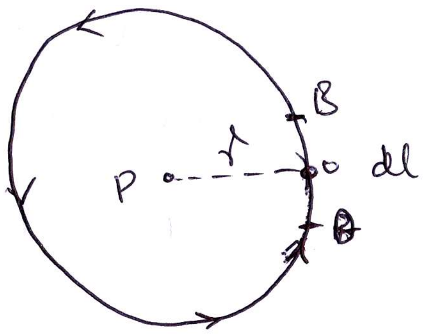
$$B_1 = \frac{\mu_0}{4\pi}\frac{I}{\ell/2}\left[\sin 45^o + \sin 45^o\right]$$

$$= \frac{2\mu_0 I}{4\pi\ell}\ \frac{2}{\sqrt{2}}$$

Net field at centre

$$B = 4B_1 = \frac{2\sqrt{2}\mu_0 I}{\pi\ell}$$

Magnetic field at the centre of a circular current carrying loop



$$dB = \frac{\mu_0}{4\pi}\times\frac{Idl\sin 90}{r^2} = \frac{\mu_0}{4\pi}\frac{Idl}{r^2}$$

$$\therefore B = \int_0^{2\pi r} \frac{\mu_0}{4\pi} \frac{I}{r^2} dl$$

$$\boxed{B = \frac{\mu_0}{2} \frac{I}{r}}$$

For a coil of N turns, $B = \frac{\mu_0}{2} \frac{NI}{r}$

The direction of the magnetic field at the centre of the circular coil can be obtained by using right hand thumb rule. If the fingers are curled along the current, then the stretched thumb will point towards the magnetic field.

Current loop as a 'magnetic dipole'

The face of the coil in which current appears to flow anticlockwise acts as magnetic north pole.

The face of the coil in which current appears to flow clockwise acts as magnetic south pole.

Magnetic field due to current carrying circular segment subtending an angle θ at the centre.

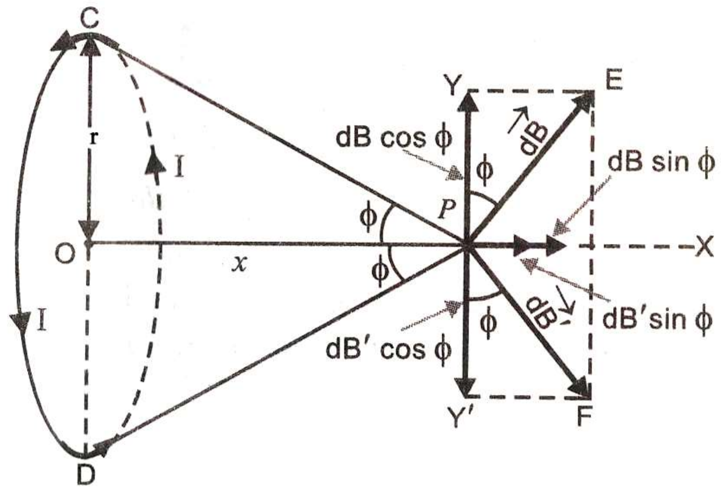
$$dB = \frac{\mu_0}{4\pi} \frac{I\ell}{r^2}$$

$$d\ell = rd\theta$$

$$dB = \frac{\mu_0}{4\pi} \frac{Ird\theta}{r^2} = \frac{\mu_0}{4\pi} \frac{Id\theta}{r}$$

$$B = \int_0^{\phi} \frac{\mu_0}{4\pi} \frac{Id\theta}{r} = \frac{\mu_0 I}{4\pi r}\theta$$

Magnetic field on the axis of a circular coil



$$dB = \frac{\mu_0}{4\pi} \frac{Id\ell \sin\theta}{a^2}$$

$$\therefore dB = \frac{\mu_0}{4\pi} \frac{Id\ell}{a^2} \qquad (\theta = 90^\circ)$$

$$\therefore B = \oint dB \sin\phi = \oint \frac{\mu_0}{4\pi} \frac{Id\ell}{a^2} \sin\phi = \frac{\mu_0 I}{4\pi a^2} \sin\phi \times 2\pi r$$

$$B = \frac{\mu_0 Ir}{2a^2} \times \frac{r}{a} = \frac{\mu_0 Ir^2}{2a^3} = \frac{\mu_0 Ir^2}{2\left(r^2 + x^2\right)^{3/2}}$$

It act along the axis of a circular coil. If the coil consist of N turns then

$$B = \frac{\mu_0 NIr^2}{2\left(r^2 + x^2\right)^{3/2}}$$

Special Cases

1) at the centre of the loop x = 0

$$B = \frac{\mu_0 NIr^2}{2r^2} \text{ or } \frac{\mu_0 NI}{2r}$$

2) If the observation point is far away from the coil i.e. $r << x$

$$B = \frac{\mu_0 N I r^2}{2x^3} = \frac{\mu_0}{2\pi}\frac{NIA}{x^3} = \frac{\mu_0}{4\pi} \times \frac{2NIA}{x^3} = \frac{\mu_0}{4\pi} \times \frac{2m}{x^3}$$

The quantity NIA is known as the magnetic dipole moment M of the current loop.

Current loop as a magnetic dipole

Based on the fact that the magnetic field of a current loop is identical with that of a magnetic dipole, it was speculated by ampere in 1820 that all magnetism is due to current loops and this speculation is indeed correct.

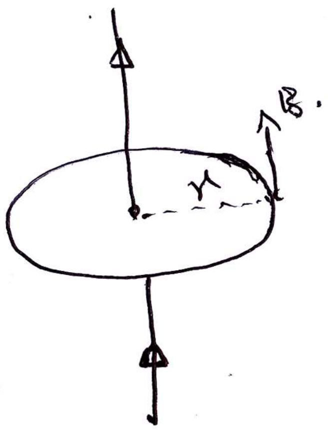
Ampere's circuital law

The line integral $\oint B.d\ell$ for a closed curve is equal to μ_0 times the net current I through the area bounded by the curve.

$$\boxed{\text{i.e.} \oint B.d\ell = \mu_0 I}$$

Applications of Ampere's Law

Magnetic field due to an infinite current carrying conductor



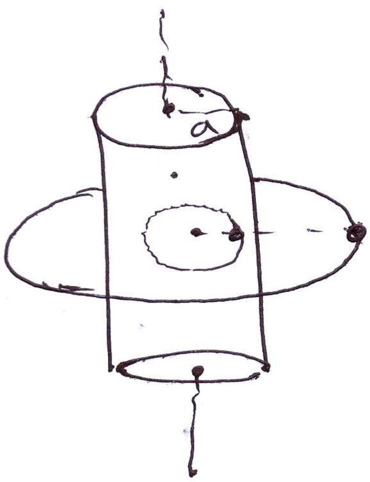
$$\oint B.d\ell = \mu_0 I$$

$$B \times 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

Magnetic field produced by a current along a circular cylinder of infinite length

1. When observation point is outside the cylinder



i.e. r > a

$$\oint B.d\ell = \mu_0 I$$

$$B \times 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r} \qquad \text{i.e. } B\,\alpha\,\frac{1}{r}$$

2. When observation point is on the surface of the cylinder $(r = a)$

$$B = \frac{\mu_0 I}{2\pi a}$$

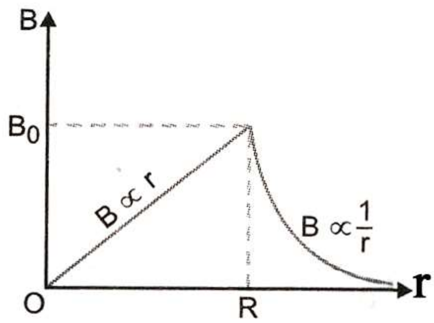
3. When observation point is inside the cylinder

$$\oint B.d\ell = \mu_0 I^1 \qquad \text{where } I^1 = \frac{I}{\pi a^2} \times \pi r^2$$

$$B \times 2\pi r = \mu_0 \frac{I}{\pi a^2} \times \pi r^2$$

$$\boxed{B = \frac{\mu_0 I r}{2\pi a^2}}$$

Variation of B with r



Solenoid

A solenoid is used to generate magnetic field. A long solenoid is one whose length is very large, compared to its radius.

A solenoid consists of a long metallic insulated wire wound in the form of a helix, where the neighbouring, turns are closely spaced. Each turn can be regarded as a cicular loop.

Field inside the solenoid, at centre.

$$B = \mu_0 n I$$

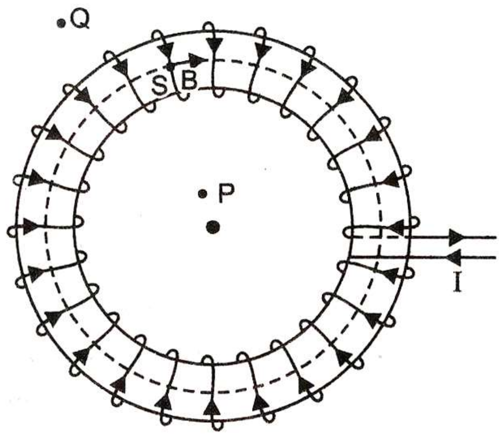
where, $n = \frac{N}{\ell}$ (number of turns per unit length)

at end point

$$B = \frac{\mu_0 nI}{2}$$

For a long solenoid the inside field is almost uniform, and out side field is near to zero.

Toroid



i) The field at a point such as P is zero. This is because the circle through P does not encloses any current

ii) The field at a point such as 'Q' is also zero. This is because each turn of the winding passes twice through the area enclosed by the circle through r, carrying equal currents in opposite directions. So the net current enclosed by this circle is zero.

iii) Inside the solenoid point such as S

$$\oint B.d\ell = \mu_0 I_0$$

$$B \times 2\pi r = \mu_0 NI$$

$$B = \frac{\mu_0 NI}{2\pi r} = \mu_0 nI$$

LORENTZ MAGNETIC FORCE

Consider a positive change 'q' moving in a uniform magnetic field $\vec{B}$ with a constant velocity v. The change 'q' will experience a force 'F_m' known as Lorentz magnetic force. It is given by

$$F_m = q\left(\vec{V} \times \vec{B}\right) = Bqv\sin\theta\hat{n}$$

$$\left|F_m\right| = Bqv\sin\theta$$

The direction of magnetic force is perpendicular to both $\vec{v}$ and $\vec{B}$

$\therefore$ Work done by magnetic force = 0

K.E. of the particle remains constant and magnitude of velocity remains constant

Note

1. If the charge is at rest (v = 0) then $F_m = 0$ So, a stationary charge in a magnetic field experiences no magnetic force.
2. If $\theta = 0^\circ$ or 180° i.e. If the change moves parallel to the direction of the magnetic field then $F_m = 0$
3. If the charge moves perpendicular to the direction of the magnetic field i.e. $\theta = 90^\circ$ then $F_m = Bqv$

The direction of F_m can be determined by Fleming's Left hand rule.

Statement : Stretch the middle finger, fore finger and thumb of the left hand in mutually perpendicular directions. If the fore finger points in the direction of the magnetic field, the middle finger points in the direction of motion of the +ve charge, then the thumb gives the direction of the force.

- Force on a moving charge in uniform electric and magnetic fields

$$\vec{F} = q\vec{E} + q\left(\vec{V} \times \vec{B}\right) = q\left(\vec{E} + \vec{V} \times \vec{B}\right)$$

- S.I unit of magnetic induction 'B' is Tesla and its c.g.s and is gauss.

IT = 10^4 gauss

Charged particle moving in a uniform magnetic field

Force on a charged particle moving in a uniform magnetic field

$$\vec{F}_m = q\left(\vec{V} \times \vec{B}\right) = BqV \sin\theta\, \hat{n}$$

Case I : When the charged particle is moving parallel or antiparallel to the magnetic field

i.e. $\theta = 0^\circ$ or 180°

Particle moves along a straight line path

$$F_m = BqV \sin 0 \text{ or } BqV \sin 180 \text{ or } F_m = 0$$

Case II : When changed particle enter perpendicular to the magnetic field

$$F_m = q\left(V \times B\right) = BqV \sin 90 = BqV$$

The direction of force is given by Fleming's left hand rule. In this case the centripetal force is provided by the Lorentz magnetic force. So, the changed particle follows a circular path

$$BqV = \frac{mv^2}{r} \quad \therefore r = \frac{mV}{Bq}$$

and time period of revolution $T = \dfrac{2\pi r}{V}$

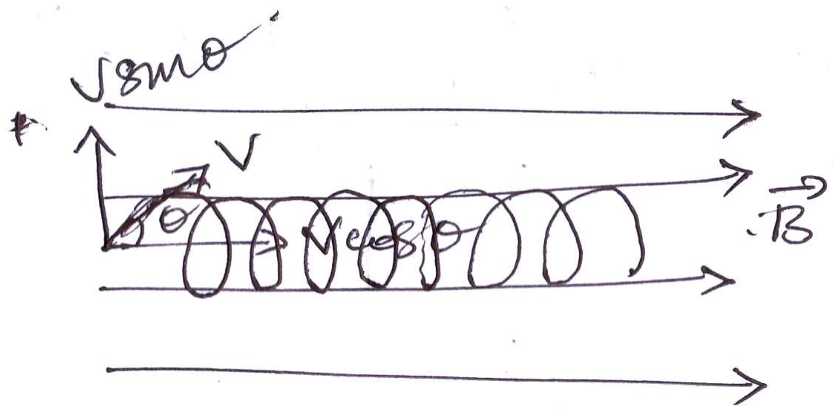
$= \frac{2\pi}{V}\frac{mv}{Bq} = \frac{2\pi m}{Bq} = \frac{2\pi}{B\left(q/m\right)}$

frequency $\nu = \frac{Bq}{2\pi m}$

Angular frequency $\omega = 2\pi\nu = 2\pi \times \frac{Bq}{2\pi m}$

or $\omega = \frac{Bq}{m}$

Case III : When a changed particle moves at an angle θ to a uniform magnetic field B such that $\theta \neq 0, \neq 90$ and $\theta \neq 180^{\circ}$



Then the charged particle will follow a helical path

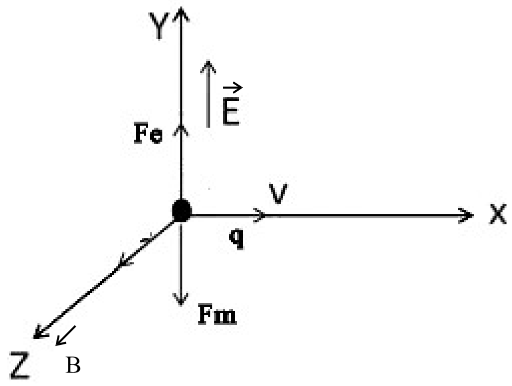
Pitch of the helical path. It is the linear distance covered by the changed particle in the magnetic field in a time during which the changed particle covers one revolution of its circular path

i.e. pitch of the helix = $V\cos\theta \times T$

$= V\cos\theta \times \frac{2\pi m}{Bq}$	Time period $T = \frac{2\pi m}{Bq}$
radius $r = \frac{mV\sin\theta}{Bq}$	and frerquency $\nu = \frac{Bq}{2\pi m}$

Motion of a charge particle in combined electric and magnetic fields

(1) Velocity selector



The charge particle experiences both electric and magnetic forces

The electrostatic force

$$\vec{F}_e = qE\hat{j}$$

The magnetic force

$$\vec{F}_m = q\left(\vec{v}\times\vec{B}\right) = q\left(v\hat{i}\times B\hat{k}\right)$$

$$\vec{F}_m = -qvB\hat{j}$$

$\therefore$ Net force

$$\vec{F} = \vec{F}_e = +\vec{F}_m$$

$$\vec{F} = q\left(E - vB\right)\hat{j}$$

The electric and magnetic forces are in opposite directions

When, $F_e = F_m$

Net fofce = 0

qE = qvB

$$v = \frac{E}{B}$$

This condition can be used to select charged particles of a particular velocity out of a beam containing charges moving with different speeds

- **Force on a current-carrying conductor in a uniform magnetic field**

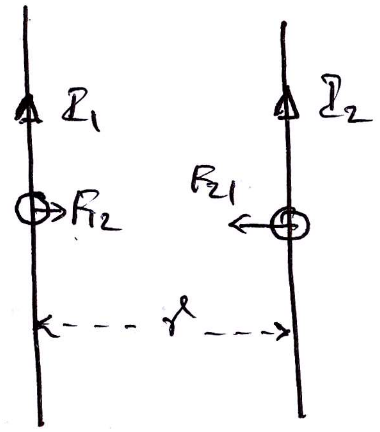
$$\vec{F} = I\left(\vec{\ell}\times\vec{B}\right)$$

The magnitude of force F = $BI\ell\sin\theta$

- The direction of the force is given by Fleming's Left hand Rule

  Statement : If the forefinger, middle finger and thumb of the left hand are held in the mutually perpendicular directions such that the forefinger shows the direction of the magnetic field, the middle finger shows the direction of the current, then the thumb will points in the direction of the force on the current carrying conductor.

  Forces between two parallel current carrying conductors



Force on first conduction due to

Second one $= F_{12} = B_2 I_1 \ell \sin 90$

$$= \frac{\mu_0 I_2}{2\pi r} I_1 \times \ell$$

$$F_{12}/\ell = \frac{\mu_0 I_1 I_2}{2\pi r}$$

Force on second due to first one

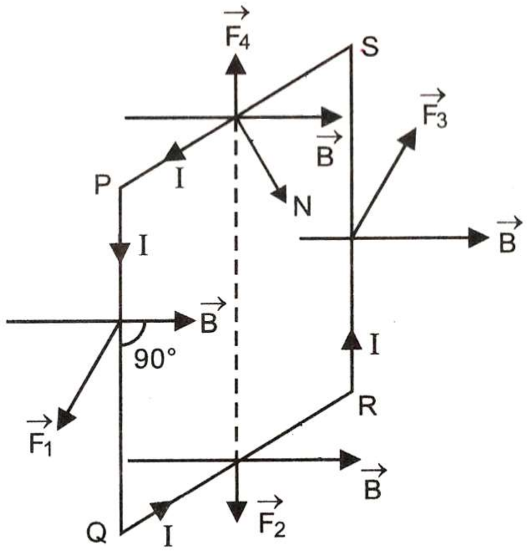
$$= F_{21} = B_1 I_2 \ell \sin 90 = \frac{\mu_0 I_1 I_2 \ell}{2\pi r}$$

$$F_{21}/\ell = \frac{\mu_0 I_1 I_2}{2\pi r}$$

Note :

1. Two parallel current carrying conductors attract each other if the currents are in the same direction and repel each other if the currents are in the opposite direction.
2. The force between two parallel current carrying conductor is proportional to the product of the current strengths and the length of the conductor under consideration and varies inversely as the distance between them.

Torque on a current carrying coil in a magnetic field



The net magnetic force on a current loop in a uniform magnetic field is zero but a torque may acting on the loop.

$\vec{\tau} = \vec{M} \times \vec{B}$, the magnitude of torque is given by,

$\tau = MB\sin\theta$

θ is the angle between normal to the loop and direction of field.

Potential energy of the loop

$u = -MB\cos\theta$	if $\theta = 0^o$	if $\theta = 180^o$
$u = -\vec{M}.\vec{B}$	$\tau = 0$	$\tau = 0$
	$u = -MB$	$u = MB$
	stable equilibrium	unstable equilibrium

Moving coil Galvanometer

Principle : When a current carrying coil is placed in magnetic field, it experience a torque

Theory

Moment of reflecting couple = NBIA

$C \rightarrow$ torsional constant of spring

Moment of restoring couple = $C\theta$

For equilibrium of the coil

NBIA = $C\theta$

or $I = \frac{C\theta}{NBA}$ where $\frac{C}{NBA}$ is the Galvanometer constant.

Current sensitivity of a Galvanometer

The current sensitivity of a galvanometer is the deflection of the meter per unit current

$$I_S = \frac{\theta}{I} = \frac{NBA}{C}$$

Voltage sensitivity of a Galvanometer

It is defined as the deflection of the meter per unit voltage

$$V_S = \frac{\theta}{V} = \frac{\theta}{IR} = \frac{NBA}{CR}$$

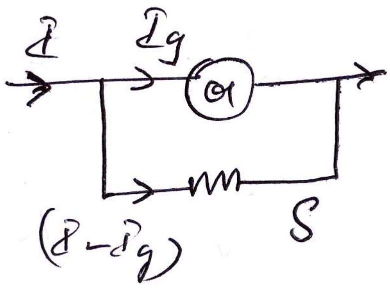
Conversion of Galvanometer to Ammeter

Ammeter is an instrument used specifically for measuring electric current.

Galvanometer can convert to ammeter by connecting a small resistance (s) in parallel with it.

$$I_g \times G = (I - I_g)S$$

$$S = \frac{I_g \times G}{(I - I_g)}$$



$G \rightarrow$ resistance of the galvanometer

The resistance of the ammeter Ra

$\rightarrow$ Resistance of ideal ammeter is zero

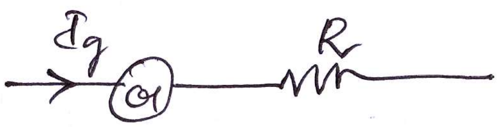
$$R_a = \frac{GS}{G+S}$$

The ammeter is always connected in series in the circuit because it doesn't alter the current due to its small resistance.

Conversion of Galvanometer to voltmeter

Voltmeter is an instrument for measuring potential difference

A galvanometer can be converted in to a voltmeter by connecting a high resistance in series with it.



$$V = I_g(G+R)$$

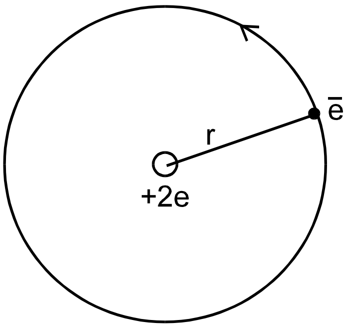
$$R = \frac{V}{I_g} - G$$

$$R_v = R + G$$

$\rightarrow$ Resistance of ideal voltmeter infinity

Voltmeter is always connected in parallel with the circuit because of its high resistance.

The magnetic dipole moment of a revolving electron



The electron performs uniform circular motion around a stationary nucleus.

The current

$$I = \frac{e}{T}$$

time period, $T = \frac{2\pi r}{v}$

$$I = \frac{ev}{2\pi r}$$

the magnetic moment,

$$\mu = IA = \frac{ev}{2\pi r} \times \pi r^2$$

$$\mu = \frac{evr}{2}$$

The angular momentum of the revolving electron

l = mvr

Gyromagnetic ratio = $\frac{\text{magnetic momet}}{\text{angular momentum}}$

$$\frac{\mu}{\ell} = \frac{e}{2m} = 8.8 \times 10^{10}\,C/kg$$

Bohr Magneton

According of Bohr's theory, angular momentum of orbital electron is given by

$L = \frac{nh}{2\pi}$. where n = 1,2,3,... and h is plank's constant

Magnetic moment of orbital electron is given by by $M = \frac{eL}{2m} = n\frac{eh}{4\pi m}$

- If n = 1 then $M = \frac{eh}{4\pi m}$, which is Bohr magneton denoted by μ_3

Definition of (μ_B)

Bohr magneton can be defined as the magnetic moment of orbital electron which revolves in first orbit of an atom.

$$\mu_B = \frac{eh}{4\pi m} = \frac{16 \times 10^{-19} \times 6.6 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31}} = 0.923 \times 10^{-23}\ A.m.^2$$