CHAPTER - 00

MECHANICAL PROPERTIES OF SOLIDS & FLUIDS

Elasticity

The property of a body by virtue of which it tends to regain the original size and shape after the removal of deforming forces is called elasticity.

Eg. Quartz and phospher-bronze are nearly perfectly elastic bodies

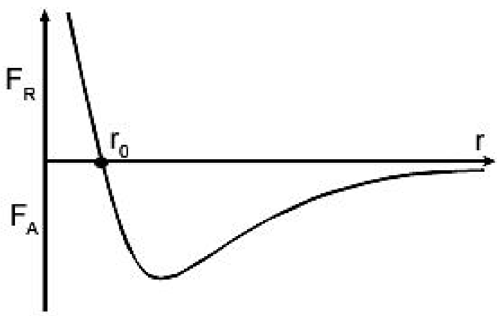
Plasticity

A body which does not regain its original configuration at all removal of deforming forces, whatever small the deforming forces may be is called a plastic body.

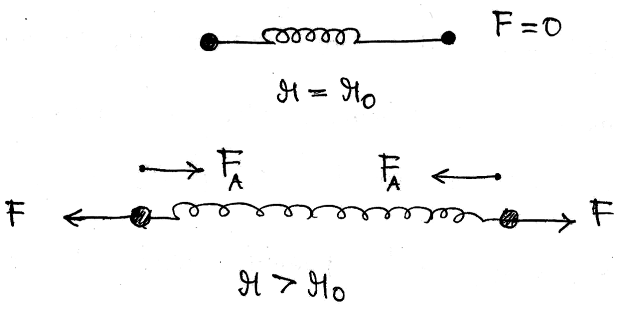
Eg. mud, putty and paraffin wax are nearly perfectly plastic bodies

Elastic behaviour of solids

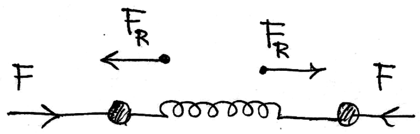
The forces between molecules of a substance are called intermolecular forces and between atoms are called interatomic forces.



* As r decreases the force of attraction will increase to a maximum value and then decreases as the distance decrease beyond r_0, the force becomes repulsive in nature.
* At $r = r_0$, the intermolecular force is zero. r_0 is known as equilibrium distance or mean distance
* When the body is subjected to tensile forces, $r > r_0$, the interatomic attractive forces will act as restoring force



* When the body is subjected to compressive forces, r < r_0. the interatomic repulsive forces will act as restoring forces



Stress (σ)

The restoring force developed per unit area of the body is called stress

$$\boxed{\sigma = \frac{F_{restoring}}{A} = \frac{Load}{Area}}$$

Unit $\rightarrow$ N / m^2
$\Rightarrow$ it is a tensor quantity

Types of Stress

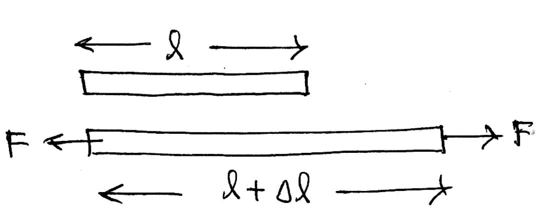
1. Normal stress (σ_n)

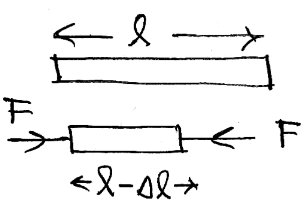
When the deforming force acts normally over an area of the body, the stress developed is called normal stress

$$\boxed{\sigma_n = \frac{F_n}{A}}$$

a) Tensile stress

b) Compressive stress

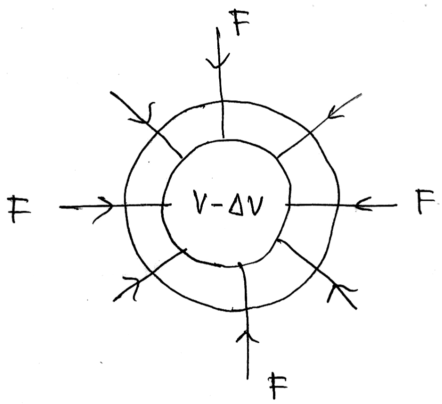




* There is a charge in shape of the body without any change in volume

c) <u>Hydraulic stress</u> (σ_v)

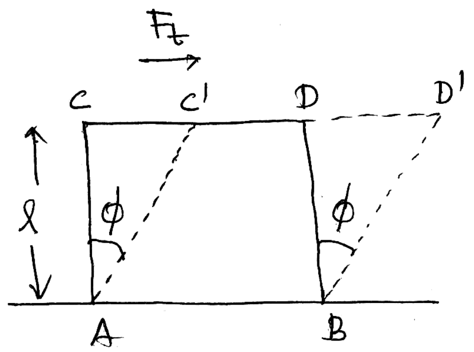
When a solid sphere is placed in a fluid under high pressure it is compressed uniformly on all sides.



$$\boxed{\sigma_V = \frac{F}{A} = \frac{\text{Thrust}}{A} = P}$$

* There is a change in volume of body without any change in shape

2. Shear stress (σ_t)



$$\boxed{\sigma_t = \frac{F_t}{A}}$$

$CC^1 = DD^1 = \Delta x$
$[\text{Lateral displacement}]$

* There is a change in shape of the body without any change in volume

* Strain = $\dfrac{\text{change in dimension}}{\text{Actual dimension}}$

<u>Types of strain</u>

1) Longitudinal strain = $\dfrac{\Delta\ell}{\ell} = e$

2) Volume strain, $e_v = \dfrac{\Delta V}{V}$

3) Shear strain, $\phi = \dfrac{\Delta x}{\ell}$

Hooke's Law

Within elastic limit, stress developed is directly proportional to strain produced in the body

$$\sigma \, \alpha \, e \;\Rightarrow \boxed{\frac{\sigma}{e} = E} \quad E \to \text{modulus of elasticity}$$

Types of modulus of elasticity

1. Young's modulus

$$Y = \frac{\text{longitudinal stress}}{\text{longitudinal strain}} = \frac{\sigma}{e} = \frac{F/A}{\Delta\ell/\ell} = \frac{F\ell}{A\Delta\ell}$$

$$\boxed{\Delta\ell = \frac{F\ell}{AY}} \qquad \boxed{F = \left(\frac{YA}{\ell}\right)\Delta\ell = K\Delta\ell}$$

$K \to$ interatomic force constant

* Young's modulus is involved for solids only

* Steel is more elastic than rubber $\left[Y_S > Y_R\right]$

* For a perfectly rigid body, $Y = \infty$

2. Bulk modulus (B)

$$B = \frac{\text{hydraulic stress}}{\text{volume strain}} = \frac{\Delta P}{\frac{-\Delta V}{V}}$$

* Compressibility, $C = \frac{1}{B}$ $\boxed{\bullet C_g > C_\ell > C_S}$

* for an ideal liquid, $B = \infty$

* B is involved for solids, liquids and gases

Rigidity Modulus (G)

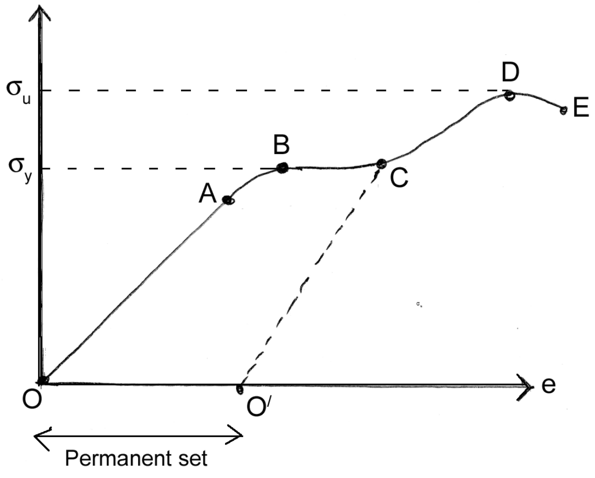
$$G = \frac{\text{shear stress}}{\text{shear strain}} = \frac{\sigma_t}{\phi} = \frac{F_t/A}{\Delta_r/\ell}$$

$$\boxed{G = \frac{F_t\ell}{A\Delta x}}$$

•for most materials, $G = \frac{Y}{3}$

•G is involved for solids only

Stress - Strain curve



A $\rightarrow$ proportional limit
B $\rightarrow$ elastic limit or yield point
D $\rightarrow$ ultimate tensile strength
E $\rightarrow$ fracture point

Ductile Materials

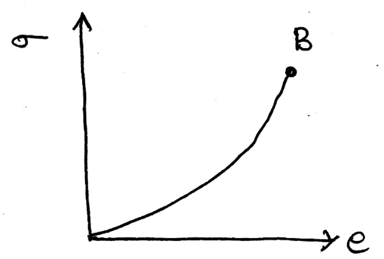
1. The material which exhibit large plastic range beyond the elastic limit such that they can be drawn into wire springs and sheets are known as ductile materials
2. For example, silver, steel, copper, aluminium

Brittle Materials

1. These are those material which show a very small plastic range beyond the elastic limit. The breaking point lies very close to elastic limit. For example, rubber, glass

Elastomers

These materials do not obey Hooke's law. They have no well defined plastic region. The breaking point lies close to elastic limit.



Breaking stress (σ_b)

$\sigma_b = \dfrac{F_b}{A}$, but breaking stress in a fixed value for a given material

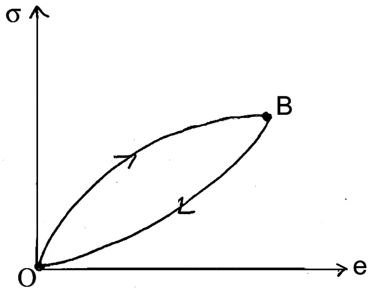
$F_b = \sigma_b A$

$F_b \alpha A \Rightarrow$ Breaking force depends on area of cross section

Elastic hysteries

Elastic hysteries is the difference between the strain energy required to generate a given stress in a material.

* The area of the loop represents energy dissipated as heat per unit volume in one cycle



Elastic potential energy (U)

When a wire is stretched, some work is done against internal restoring forces. This work done appears as elastic potential energy of the wire.

$$W = F_{av} \times \Delta\ell = \left(\frac{O+F}{2}\right)\Delta\ell$$

$$\boxed{W = \frac{1}{2}F\Delta\ell = U}$$

Elastic energy density (U)

$$U = \frac{U}{vol.} = \frac{1}{2}\sigma e = \frac{1}{2}Ye^2 = \frac{1}{2Y}\sigma^2$$

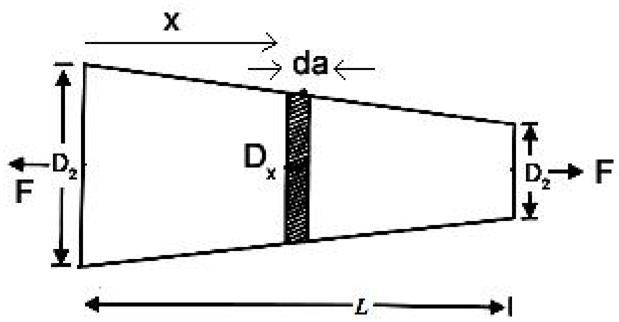
Poisson's ratio (μ)

$$\mu = \frac{\text{Lateral strain}}{\text{Long strain}} = \frac{-\Delta d/d}{\Delta\ell/\ell} -$$

Theoretical value lies between –1 to $1/2$

Practical value lies between 0 to $1/2$

Elongation of a unifomly tapering circular rod



$$K = \frac{\text{Loss of diameter}}{\text{Length}} = \frac{D_1 - D_2}{L}$$

$$D_x = D_1 - k_x$$

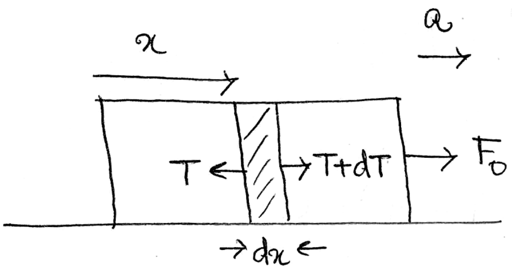
$$A_x = \frac{\pi}{4} D_r^2 = \frac{\pi}{4}\left(D_1 - k_x\right)^2$$

Elongation of dx = $\dfrac{Fdx}{A_x Y} = \dfrac{4Fdx}{\pi\left(D_1 - k_x\right)^2 Y}$

$\therefore$ Total elongation, $\Delta\ell = \dfrac{4F}{\pi Y}\displaystyle\int_0^L \dfrac{1}{\left(D_1 - k_x\right)^2} dx = \dfrac{4FL}{\pi Y D_1 D_2}$

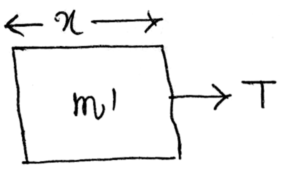
Elongation of a rod subjected to horizontal acceleration

1. Smooth surface



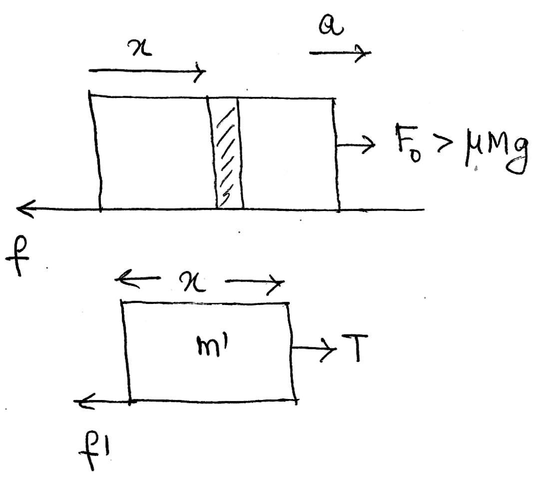
$$T = m'a = \frac{Mx}{L}\frac{F_0}{M} = \frac{F_0 x}{L}$$

$$\text{elongation of } dx = \frac{Tdx}{AY}$$



Total elongation, $\Delta\ell = \displaystyle\int_0^L \frac{Tdx}{AY} = \frac{F_0 L}{2AY}$

2. Rough surface



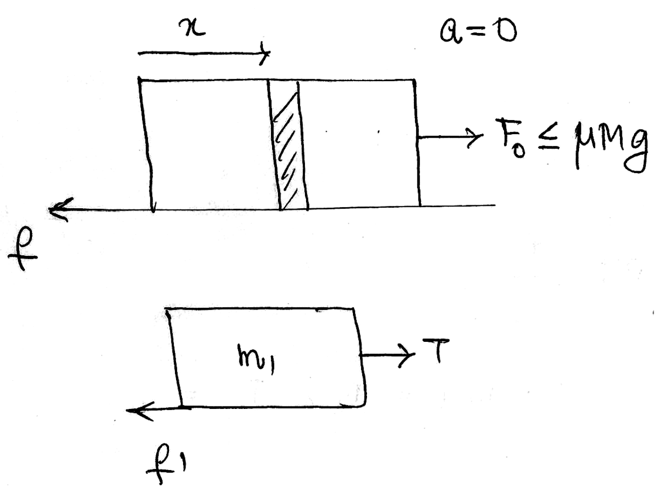
$$T - f' = m'a$$

$$T - f' = \frac{Mx}{L}\left[\frac{F_0 - f}{M}\right]$$

$$T = \frac{F_0 x}{L}$$

$$\therefore \Delta\ell = \frac{F_0 L}{2AY}$$

3. **Rough surface**

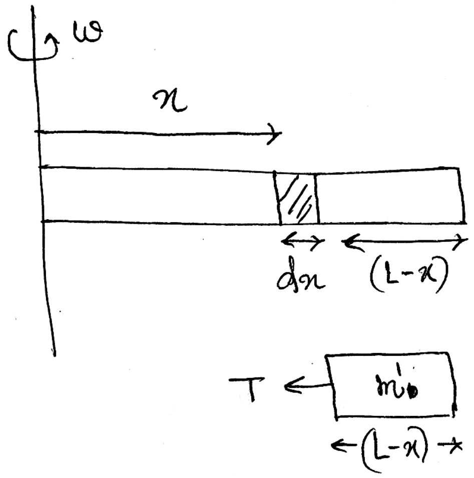


$$T - f' = 0$$

$$T = f' = \frac{fx}{L} = \frac{F_0 x}{L}$$

$$\therefore \Delta\ell = \frac{F_0 L}{2AY}$$

Elongation of a rod subjected to rotation



$$T = m^1\omega^2 r_{cm}$$
$$= \frac{M}{L}(L-x)10^2\left(\frac{L+x}{2}\right)$$
$$T = \frac{M\omega^2}{2L}\left(L^2 - x^2\right)$$

total elongation = $\int_0^L \frac{Tdx}{AY} = \frac{M\omega^2}{2LAY}\int_0^L \left(L^2 - x^2\right)dx$

$$\Delta\ell = \frac{\delta\omega^2 L^3}{3Y}$$

* If a gas undergoes a process, PV^x = constant

$$\frac{\Delta P}{P} + x\frac{\Delta V}{V} = 0$$

$$\boxed{xP = \frac{\Delta P}{\frac{-\Delta V}{V}} = B}$$

* for isothermal process x = 1

B_{iso} = P

* for adiabatic process $x = \gamma$ where $\gamma = {C_p}/{C_p}$

$$B_{adia} = \gamma P$$

Variation of density with pressure

$$\rho_1 = \frac{m}{v}$$

$$\rho_2 = \frac{m}{V - \Delta V} = \frac{\rho_1 V}{V\left[1 - \frac{\Delta V}{V}\right]}$$

If $\frac{\Delta P}{B} << 1$

$$\boxed{\rho_1 = \rho_1\left[1 + \frac{\Delta P}{B}\right]}$$

Relationship between elastic moduli

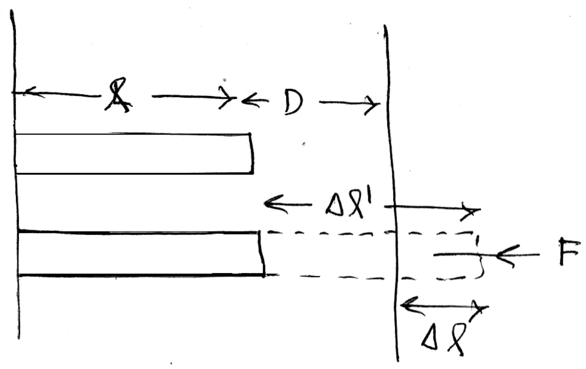
1) $Y = 3B[1 - 2\mu]$

2) $Y = 2G[1 + \mu]$

3) $\frac{9}{Y} = \frac{1}{B} + \frac{3}{G}$

Thermal Stress

If a body is allowed to expand or contracts freely with rise or fall in temperature no stresses are developed in it. If the expansion or contraction is prevented thermal stresses are developed in it.



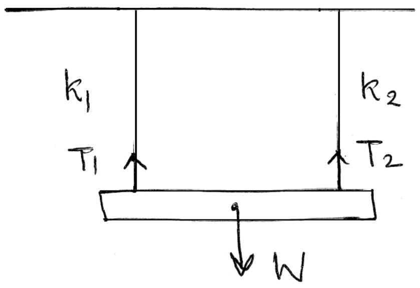
$$\Delta \ell^1 - \Delta \ell = D$$

$$\ell \alpha \Delta T - \frac{F\ell}{AY} = D$$

$$\boxed{\frac{F}{A} = Y\frac{[\ell \alpha \Delta T - D]}{\ell} = \sigma}$$

Combination of Wires

Parallel combination



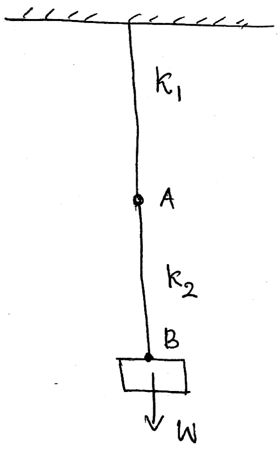
- $\Delta\ell_1 = \Delta\ell_2$
- $W = T_1 + T_2$
- $K_P = K_1 + K_2$

$$K_{eq} = K_1 + K_2 \Rightarrow \frac{Y_{eq}A_{eq}}{L_{eq}} = \frac{Y_1A}{L} + \frac{Y_2A}{L}$$

$A_{eq} = 2A$

$L_{eq} = L$ $\quad\quad \therefore Y_{eq} = \dfrac{Y_1 + Y_2}{2}$

Series



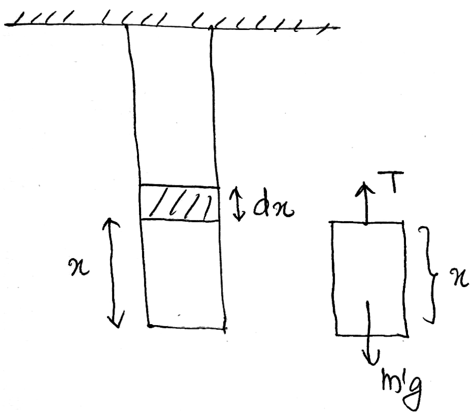
- $\Delta\ell = \Delta\ell_1 + \Delta\ell_2$
- $T_1 = T_2 = W$
- $\dfrac{1}{K_S} = \dfrac{1}{K_1} + \dfrac{1}{K_2}$

$$\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2}$$

$$\frac{L_{eq}}{K_{eq}} = \frac{L}{Y_1A} + \frac{L}{Y_2A}$$

$$\boxed{Y_{eq} = \frac{2Y_1Y_2}{Y_1 + Y_2}}$$

Elongation due to self weight



$$T = mg = \rho Axg$$

$$\Delta\ell \int_0^L \frac{Tdx}{AY} = \frac{\rho g L^2}{2Y}$$

Strain energy due to self weight

$$T = \rho Axg \Rightarrow \sigma = \frac{T}{A} = \rho gx$$

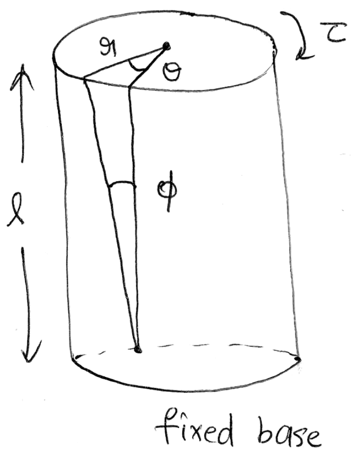
$$\frac{dU}{d_{vol}} = \frac{1}{2Y}\sigma^2 = \frac{1}{2Y}\rho^2 g^2 x^2$$

$$\int dU = \int \frac{1}{2Y}\rho^2 g^2 x^2 Adx$$

$$U = \frac{1}{2Y}\rho^2 g^2 A \int_0^L x^2 dx = \frac{\rho^2 g^2 AL^3}{6Y}$$

Twisting of cylindrical wire

$$\boxed{\ell\phi = r\theta}$$



ϕ = Angle of shear

θ = Angle of twist

τ = Torsional couple

c = torsional rigidity or couple per unit twist

$$\tau \alpha \theta$$

$$\boxed{\tau = c\theta}$$

For solid cylinder

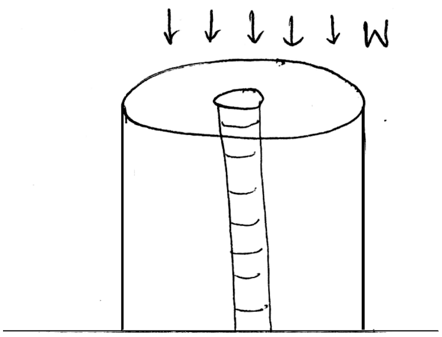
$$C_S = \frac{\pi G r^4}{2\ell}$$

For hollow cylinder

$$C_h = \frac{\pi G\left[r_2^4 - r_1^4\right]}{2\ell}$$

Analysis of Composite section

Consider a column made up of 2 different materials. The sum of the restoring force in the two materials should balance the external applied force



$$W = F_1 + F_2 = \sigma_1 A_1 + \sigma_2 A_2 ....(1)$$

$$\Delta\ell_1 = \Delta\ell_2$$

$$\text{i.e.} \frac{\sigma_1}{Y_1} = \frac{\sigma_2}{Y_2} ....(2)$$

Solve (1) and (2) to find σ_1 and σ_2

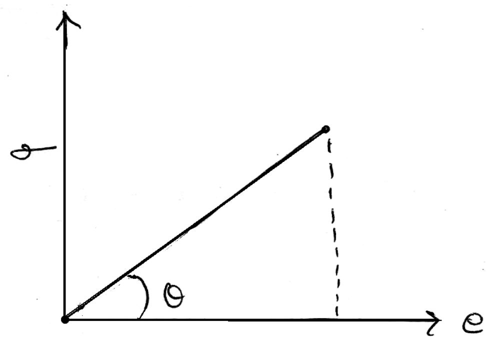
Elastic after effect

It is the temporary delay in regaining the original configuration of the body after the removal of deforming forces

Elastic fatique

When a body is subjected to alternative deforming forces for a long perod of time, the elasticity of body decreases called elastic fatigue.

Analysis of stress strain curve



$$\text{slope} = \tan\theta = \frac{\alpha}{e} = Y$$

$$\text{Area} = \frac{1}{2}bh = \frac{1}{2}\sigma e = \text{energy density}$$

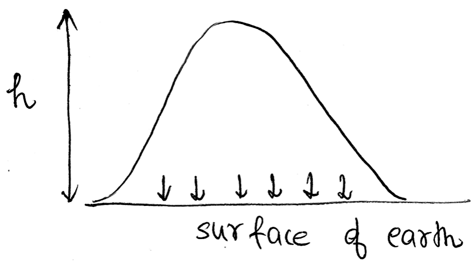
* If poisson ratio, $\mu = \frac{1}{2}$ then volume of the body remains constant

* If $\mu = 0$, then the body suffers no lateral strain

* Factor of safety

$$FOS = \frac{\text{Breaking stress}}{\text{Working stress}} > 1$$

Maximum height of a mountain



Density of rock = ρ

Presence at base of mountain = $h\rho g$

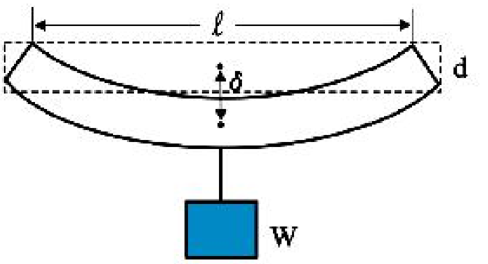
Elastic limit of surface of earth = σy

$\therefore h\rho g \le \sigma y$

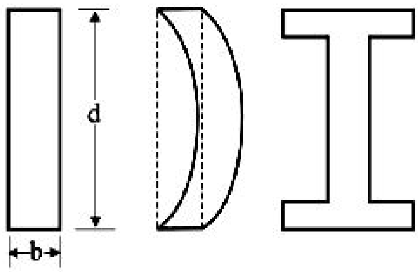
$$\boxed{h \le \frac{\sigma y}{\rho g}}$$

Bending of beams

$\delta = Wl^3/(4bd^3Y)$



A beam supported at the ends and loaded at the centre.



I. Section prevents Buckling

MECHANICAL PROPERTIES OF FLUIDS

$\rightarrow$ **Mass Density,** $\rho - \frac{m}{v}$

$\rightarrow \rho_{\omega} = 1000 \text{ kg}/\text{m}^3 - 1 \text{ g}/\text{cm}^3$

$\rightarrow \rho_{Hg} = 13.6 \times 10^3 \text{Kg}/\text{m}^3 = 13.6 \text{ g}/\text{cm}^3$

$$\rho_{mix} = \frac{\text{Total mass}}{\text{Total volume}}$$

$$\boxed{\rho_{mix} = \frac{M_1 + M_2}{V_1 + V_2}}$$

* If $V_1 = V_2$, then $\rho_{mix} = \frac{\rho_1 + \rho_2}{2}$

* If $M_1 = M_2$, then $\rho_{mix} = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2}$

→ **Relative density / Specific gravity**

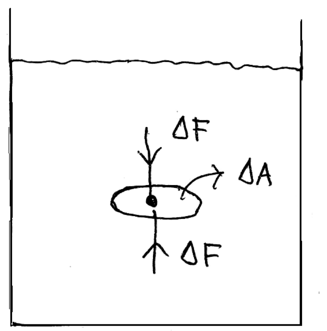
$$(\mathrm{RD})_{\mathrm{solid}} = \frac{\sigma_{\mathrm{solid}}}{\rho_\omega} = \frac{\omega_a}{\omega_a - \omega_w}$$

$$(\mathrm{RD})_\ell = \frac{\rho_\ell}{\rho_\omega} = \frac{\omega_a - \omega_\ell}{\omega_a - \omega_w}$$

$$(\mathrm{RD})_{\mathrm{Hg}} = \frac{\rho_{\mathrm{Hg}}}{\rho_\omega} = \frac{13.6\times10^3}{10^3} = 13.6$$

RD = mass density in CGS

→ **Pressure at a point**



$$\mathrm{Pressure} = \frac{\mathrm{Thrust}}{\mathrm{Area}} = \frac{\Delta F}{\Delta A}$$

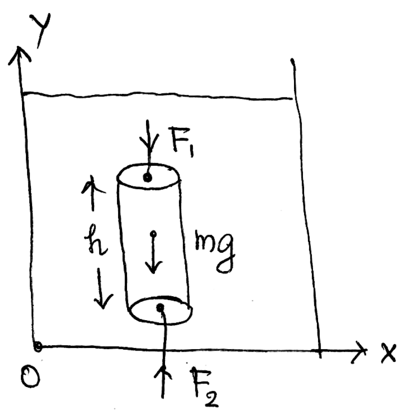
$$\text{Pressure at a point} = \underset{\Delta A\to0}{\mathrm{Lt}} \frac{\Delta F}{\Delta A} = \frac{dF}{dA}$$

Unit → N/m^2 or Pa

It is a scalar quantity

→ Pressure is a scalar quantity because no definite direction associated with fluid pressure

→ **Variation of liquid pressure with depth [Hydrostatic law]**



$$F_1 + mg = F_2$$

$$P_1A + \rho Ahg = P_2A$$

$$P_2 - P_1 = h\rho g = \Delta P$$

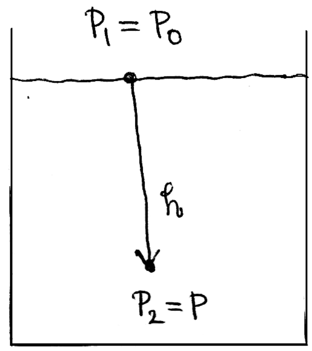
* When the points are on the same horizontal level, h = 0

$\therefore P_1 = P_2$, the side thrusts cancel out each other

If $P_1 = P$ $P_2 = P + dP$ h = dy

then $\frac{dP}{dy} = -\rho g$ [negative sign indicator pressure decreases with height]

$$\frac{dP}{dx} = 0$$

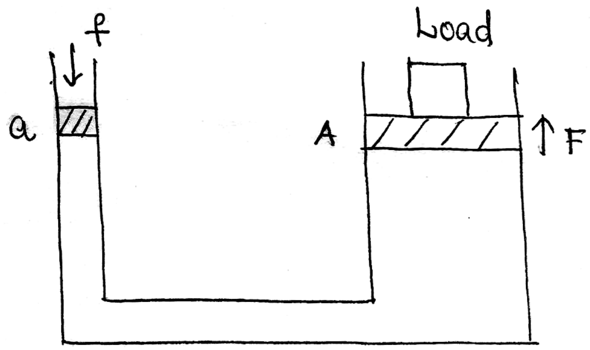


$$P - P_0 = h\rho g \left[\text{gauge pressure}\right]$$
$$P = P_0 + h\rho g \left[\text{Absolute pressure}\right]$$

Pascal's Law

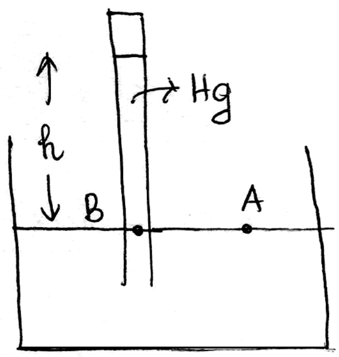
Whenever an external pressure is applied to a fluid enclosed in a vessel it is transmitted equally and undiminished in all directions

Hydraulic Life



$$\frac{f}{a} = \frac{F}{A}$$
$$\boxed{F = \left(\frac{A}{a}\right)f}$$
$$A >> a$$
$$\therefore F >> f$$

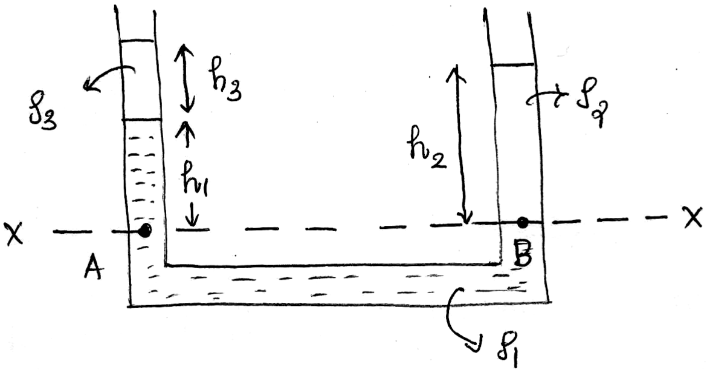
Atmospheric pressure



$$h = 76\text{cm of hg}$$
$$P_A = P_B$$
$$P_0 = h\rho g = 0.76 \times 10^3 \times 13.6 \times 9.8$$
$$\boxed{P_0 = 1.013 \times 10^5\, N/m^2}$$
$$1\text{bar} = 10^5\, Pa$$
$$\boxed{13.6\text{cm of water} = 1\text{cm of hg}}$$

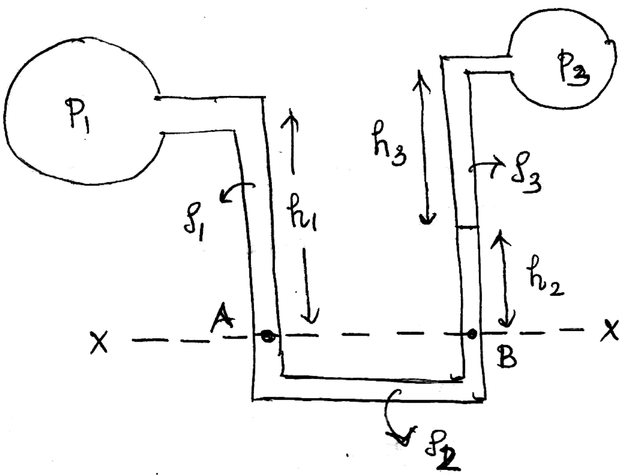
Manometer

Open tube Manometer



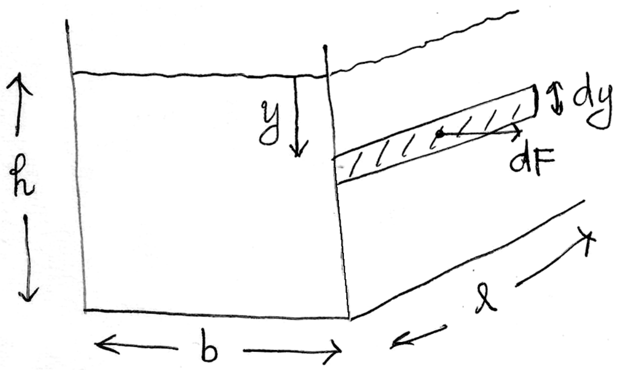
$$P_A = P_B$$
$$P_0 + h_3\rho_3 g + h_1\rho_1 g$$
$$= P_0 + h_2\rho_2 g$$
$$h_3\rho_3 g + h_1\rho_1 = h_2\rho_2$$

Differential Manometer



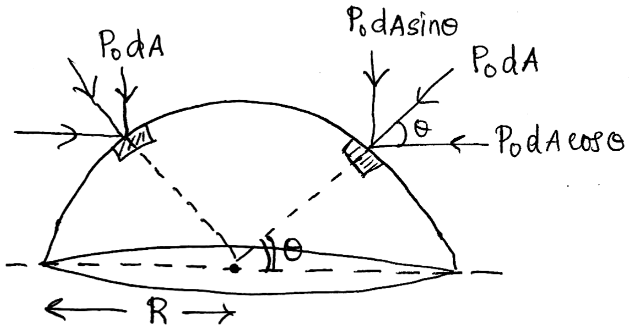
$$P_A = P_B$$
$$P_1 + h_1\rho_1 g = P_3 + h_3\rho_3 g + h_2\rho_2 g$$
$$P_1 - P_3 = h_3\rho_3 g + h_2\rho_2 g - h_1\rho_1 g$$

Hydrostatic thrust on side wall of a vessel



$$dF = pdA$$
$$= \rho gy\,\ell dy$$
$$F = \int_0^h \rho g\ell\ y dy = \frac{\rho g\ell h^2}{2}$$
Also, $F = P_{cm} \times A$

Hydrostatic thrust on curved surface



$F = \int P_0 dA \sin\theta$

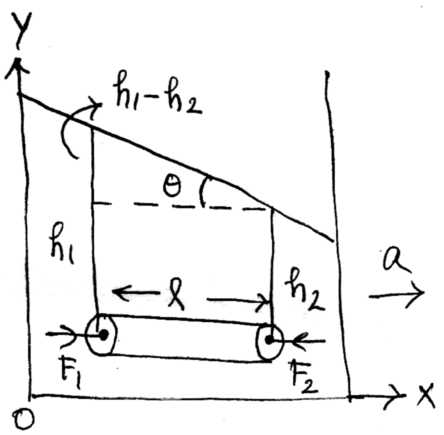
$= P_0 \int dA \sin\theta$

$dA \sin\theta$ = projected area of dA in horizontal plane

$\boxed{\therefore F = P_0 \times \text{total vertical projected area}}$

$F = P_0 \times \pi R^2$

Liquid subjected to horizontal acceleration



$F_1 - F_2 = ma$

$P_1 A - P_2 A = \rho A \ell a$

$\boxed{P_1 - P_2 = \rho \ell a = \Delta P}$

$\tan\theta = \dfrac{h_1 - h_2}{\ell} = \dfrac{a}{g}$

If $P_1 = P + dP$

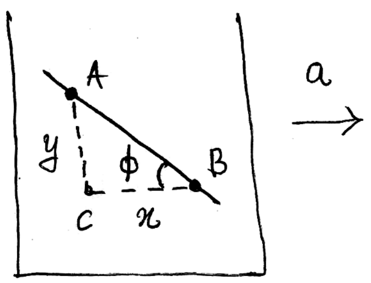
$P_\ell = P$

$\ell = dx$

$P + dP - P = \rho dx\, a$

$\boxed{\dfrac{dP}{dx} = -\rho a}$

Equipressure lines



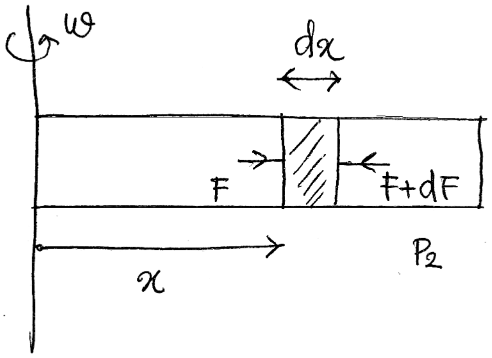
$P_A + y\rho g - \rho x a = P_B$

for equipressure line $P_A = P_B$

$yg = ax$

$\boxed{\dfrac{y}{x} = \dfrac{a}{g} = \tan\phi}$

Liquid subjected to rotation



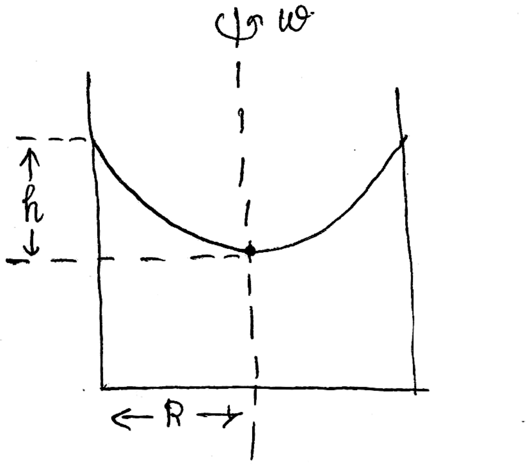
$$dF = dm\omega^2 x$$

$$dPA = \rho A dx\, \omega^2 x$$

$$dP = \rho\omega^2 x dx$$

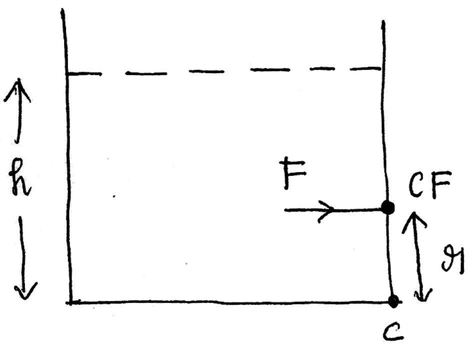
$$\int_{P_1}^{P_2} dP = \int_{x_1}^{x_2} \rho\omega^2 x dx$$

$$\boxed{P_2 - P_1 = \frac{\rho\omega^2}{2}\left[x_2^2 - x_1^2\right]}$$



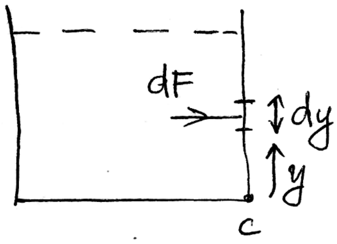
$$h = \frac{\omega^2 R^2}{2g}$$

Centre of Force



$$F = P_{cm}A = \frac{h}{2}\rho g h L = \frac{\rho g L h^2}{2}$$

$$\tau_C = F \times r = \frac{\rho g L h^2}{2} \times r \quad ....(1)$$



$$d\tau_C = dFy = PdAy = \rho g\left(h-y\right)ydyL$$

$$\tau_C = \int_0^h \rho gL\left[hy - y^2\right]dy$$

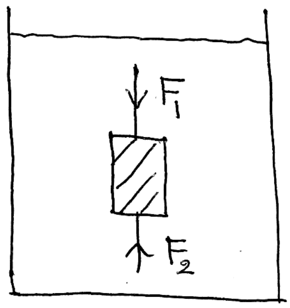
$$\tau_C = \rho gL\left[\frac{hy^2}{2} - \frac{y^3}{3}\right]_0^h$$

$$\tau_C = \rho gL\frac{h^3}{6} \quad ....(2)$$

$$\frac{\rho gLh^2}{2}\times r = \frac{\rho gLh^3}{6} \Rightarrow r = \frac{h}{3}$$

Buoyant Force

When a body is partially or completely immersed in a fluid, it experiences a net upward thrust called buoyant force. The force of buoyancy act through center of gravity of the displaced fluid called centre of buoyancy.



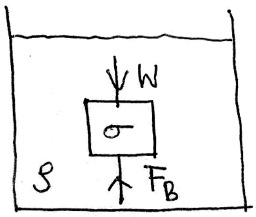
$$F_2 > F_1$$
$$F_{net\ thrust} = F_2 - F_1 = F_B$$

Archimede's Principle

F_B = weight of fluid displaced by the body

$$\boxed{F_B = \rho v^1 g}$$

Case-1 [Sinking body]

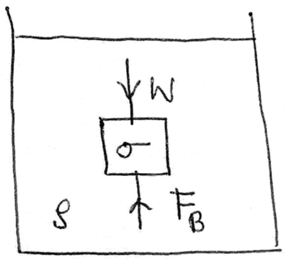


$W > F_B$

$\sigma > \rho$

$W_{app} = W - F_B = W\left[1 - \frac{\rho}{\sigma}\right]$

Case-2 [floating body when completely submerged]

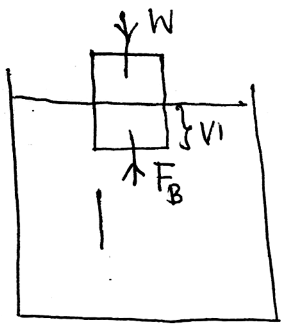


$F_B > W$

$\rho > \sigma$

$-W_{app} = W\left[\frac{\rho}{\sigma} - 1\right]$

Case-3 [floating body]



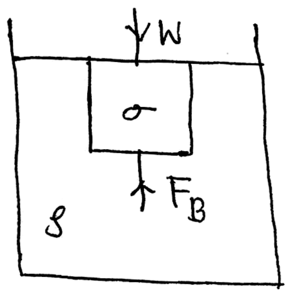
$W = F_B$

$W_{app} = 0$

$\sigma V g = \rho V^1 g$

$\boxed{\frac{V^1}{V} = \frac{\sigma}{\rho}}$

Case-4 [Just floating body]



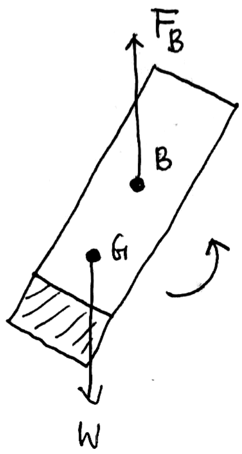
$W = F_B$

Density of body = Density of liquid

$W_{app} = 0$

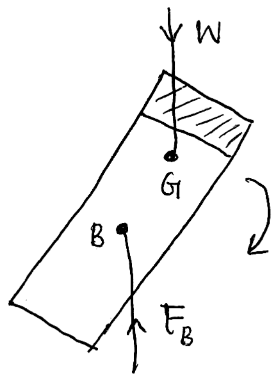
Stability conditions for a completely submerged body

Case I [Stable eqb]



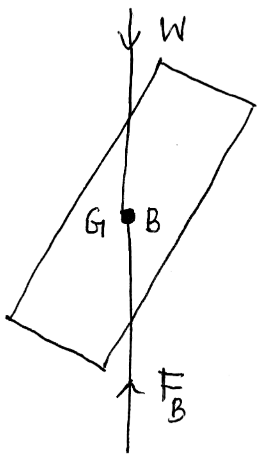
B above G

Case II [unstable eqb]



B below G

Case III [neutral eqb]



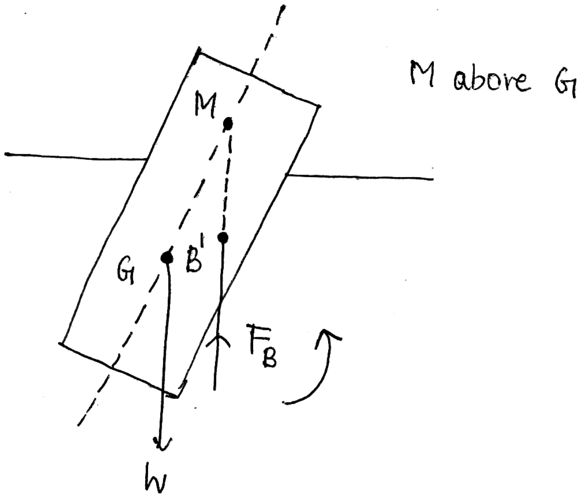
B coincide with G

Meta centre

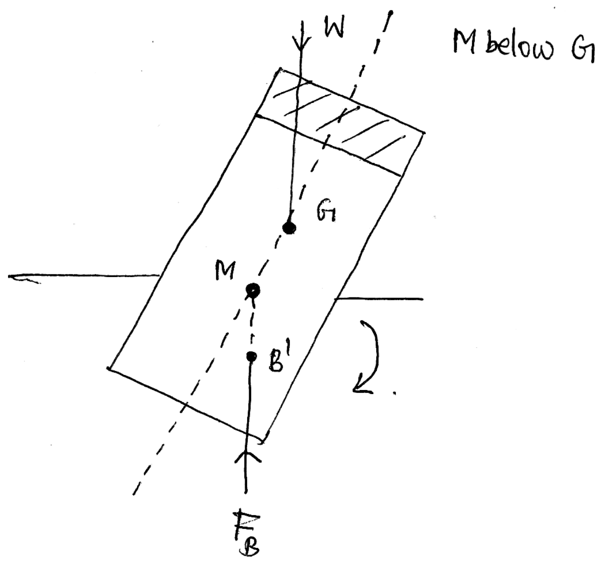
It is defined as point about which a body starts oscillating when body is tilted by a small angle. The metacentre may also be defined as the point at which line of action of buoyant force will meet the normal axis of the body when the body is given a small angular displacement.

Stability of a floating body

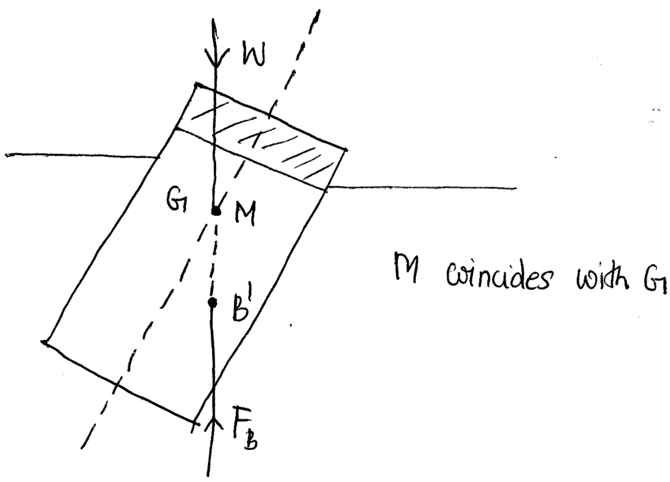
Case I (stable eqb)



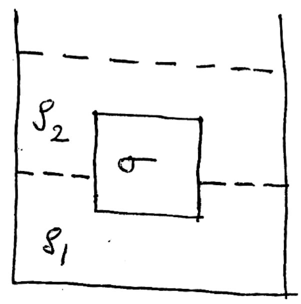
Case II [unstable eqb)



Case III (neutral eqb)



Floatation at interface of 2 liquids



$\rho_1 > \sigma > \rho_2$.

$W = F_{B_1} + F_{B_2}$.

* When a piece of ice floats in a liquid of density ρ_ℓ, and when the ice melt completely the level of liquid will

a) falls, if $\rho_\ell < \rho_W$

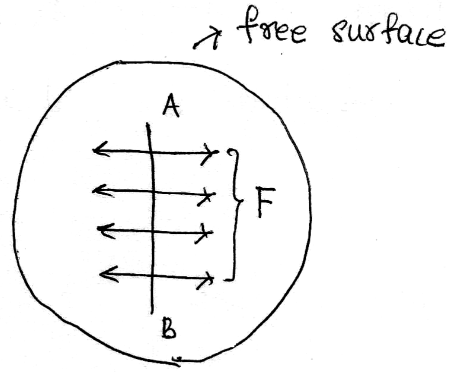
b) rise, if $\rho_\ell > \rho_W$

c) same, if $\rho_\ell = \rho_W$

* When a piece of ice with a stone embedded in it floats in water and when the ice melt completely the water level falls.
* When the piece of ice with a plastic in it floats on water and when the ice melt completely the water level remains the same
* Loss of weight of body = gain of weight of liquid

Surface Tension

It is the property by virtue of which free surface of a liquid at rest behaves like an elastic stretched membrane, tending to contract so as to occupy minimum surface area.

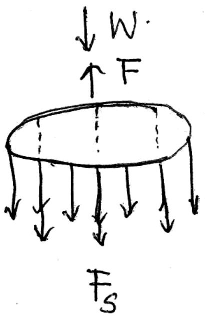


$$\boxed{S = \frac{F}{\ell}}$$

unit $\rightarrow$ N/m

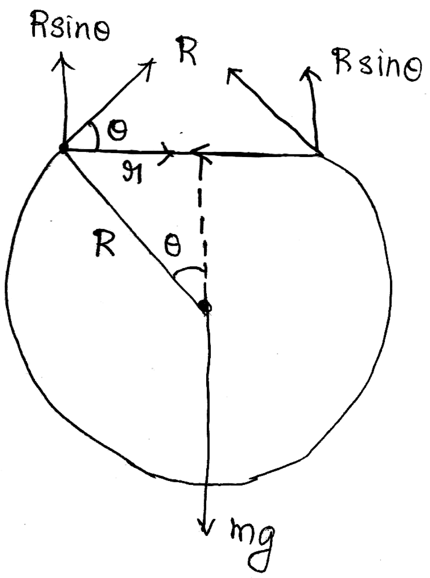
it is a scalar quantity

Force required to lift a disc from surface of a liquid



$$F = W + F_S = W + S \times 2\pi R$$

Radius of detachment



$$\left(R\sin\theta\right)_{total} = mg$$

$$S\times 2\pi r\times\frac{r}{R} = \rho\times\frac{4}{3}\pi R^3 g$$

$$R = \left[\frac{3}{2}\frac{Sr^2}{\rho g}\right]^{1/4}$$

Surface energy

It is the amount of work done in forming a liquid surface against force of surface tension

$$\boxed{\Delta W = \Delta E = S\Delta A}$$

Excess pressure

Whenever the liquid surface is curved, pressure on concave side is greater than pressure on convex side. This difference is pressure is called excess pressure

According to Laplace equation $\Delta P = S\left[\frac{1}{R_1}+\frac{1}{R_2}\right]\times n$

a) Liquid drop

n = 1

$R_1 = R_2 = r$ $\boxed{\Delta P = \frac{2S}{r}}$

b) Soap bubble

n = 2

$R_1 = R_2 = r$

$\boxed{\Delta P = \frac{4S}{r}}$

c) Air bubble

n = 1

$R_1 = R_2 = r$ $\boxed{\Delta P = \frac{2S}{r}}$

d) Cylindrical liquid surface

n = 1

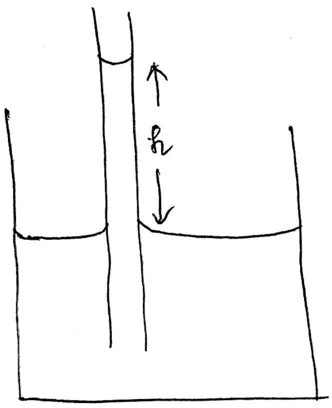
$R_1 = r$ $R_2 = \infty$

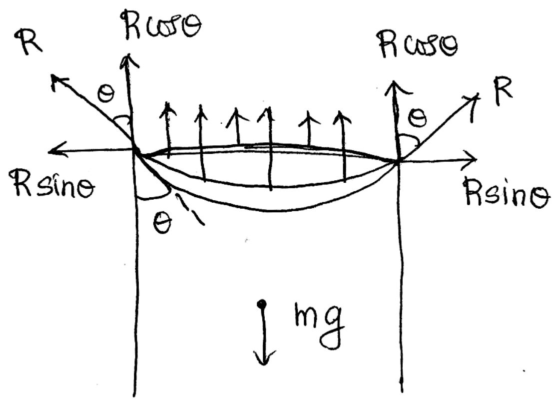
$\boxed{\Delta P = \frac{S}{r}}$

Angle of contact (θ)

Angle of contact between pure water and glass vessel is taken as 0^o and for mercury and glass vessel it is taken as 135^o. Angle of contact is independent of inclination of solid surface to the liquid medium.

Expression for capillarity

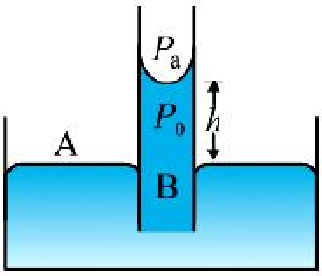




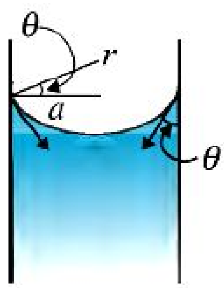
$$\left(R\cos\theta\right)_{total} = mg \rightarrow S \times 2\pi r\cos\theta = \rho\pi r^2 hg$$

$$\boxed{h = \frac{2S\cos\theta}{r\rho g}}$$

Rise of liquid in a tube of insufficient length



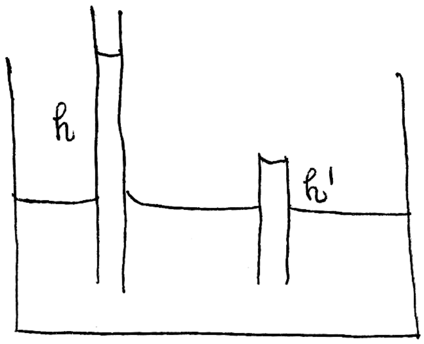
(a)

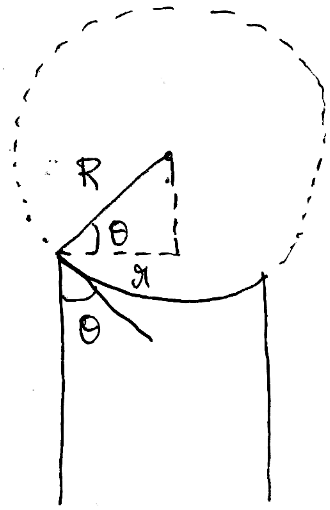


(b)

$(P_i - P_o) = (2S/r) = 2S/(a\sec\theta)$
$= (2S/a)\cos\theta$

$h\rho g = (P_i - P_o) = (2S\cos\theta)/a$





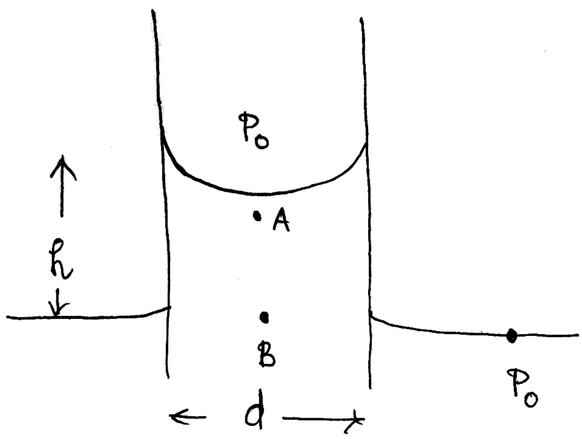
$$\cos\theta = \frac{r}{R}$$

$$h \;\alpha\; \cos\theta \;\alpha\; \frac{1}{R}$$

$$\boxed{hR = \text{constant}}$$

When the tube is of insufficient length, the radius of curvature increases, therefore capillary height decreases in the liquid will not overflow.

Capillarity between two parallel plates



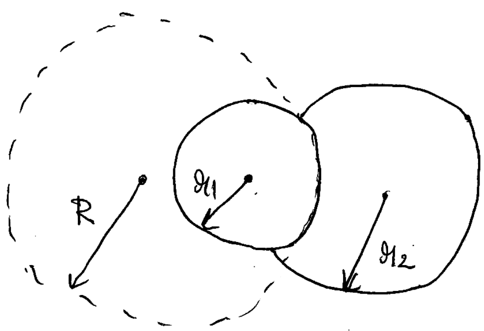
$$P_0 - P_A = \frac{S}{R} = \frac{2S}{d}$$

$$P_B = P_0$$

$$P_0 - \frac{2S}{d} + h\rho g = P_0$$

$$\boxed{h = \frac{2S}{\rho d g}}$$

Radius of curvature of a double bubble



$$\frac{4S}{r_1} - \frac{4S}{r_2} = \frac{4S}{R}$$

$$\boxed{R = \frac{r_1 r_2}{r_2 - r_1}}$$

* When two bubbles combine to form a single bubble under isothermal condition in vacuum

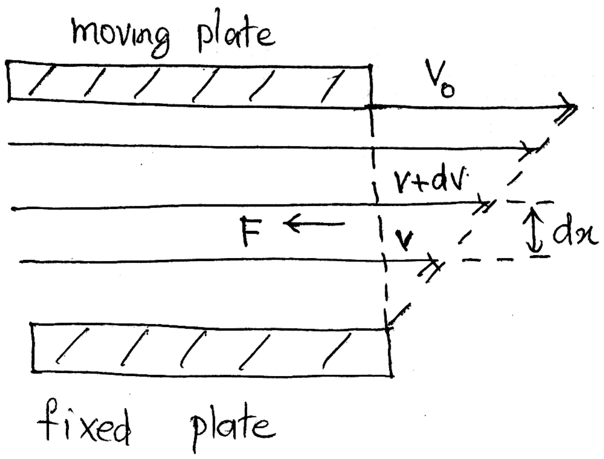
$$r = \sqrt{r_1^2 + r_2^2}$$

* Force required to separate two wet plates in contact

$$F = \frac{2S}{d} \times A$$

Newton's Law of viscosity

It is the property by virtue of which an internal force of friction comes to play whenever there is a relative motion between different layers of a fluid



$$\boxed{F = -\eta A \frac{dv}{dx}}$$

$$\frac{F}{A} = \sigma_t = \eta \frac{dv}{dx}$$

SI unit of viscosity is poiseuille $(P\ell)$

$$1\, P\ell = \frac{1Ns}{m^2} = 10 \text{ poise}$$

Viscosity of liquids decreases with temperature, while for gases viscosity increases with temperature.

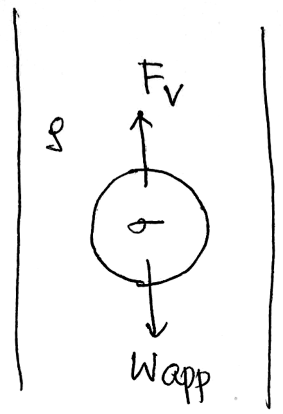
Stoke's Law

When a small spherical body moves through a stationary viscous medium, it experiences a backward dragging called viscous drag.

$$\boxed{F_v = 6\pi\eta rv}$$

Terminal Velocity

When a small spherical body falls through a viscous medium it accelerates initially due to apparent weight. At a certain instant the net force acting on the body becomes zero. The body moves with a maximum constant velocity called terminal velocity.



$$F_v = W_{app}$$

$$6\pi\eta r v = mg\left[1-\frac{\rho}{\sigma}\right]$$

$$\boxed{v = \frac{2}{9}r^2 g\frac{(\sigma-\rho)}{\eta}}$$

Laminar or streamline flow

In this type of flow the fluid particles moves in layers parallel to each other without getting mixed.

Turbulent flow

When the fluid velocity exceeds critical value the flow becomes irregular called turbulent flow.

Reynold's number, $R_e = \frac{\rho v d}{\eta}$

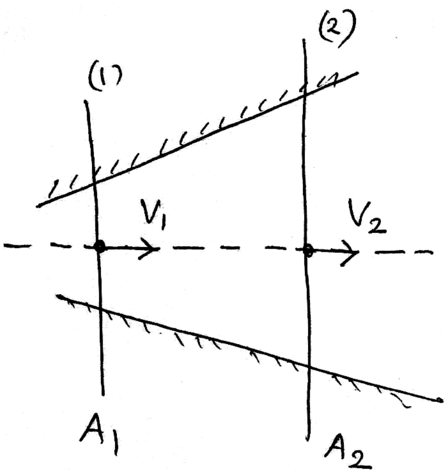
$R_e < 2000 \rightarrow$ Laminar flow

$R_e > 3000 \rightarrow$ Turbulant flow

Ideal liquid

1) incompressible 2) non viscous 3) steady flow 4) irrotational

Equation of continuity



$$\left(\frac{dm}{dt}\right)_1 = \left(\frac{dm}{dt}\right)_2$$

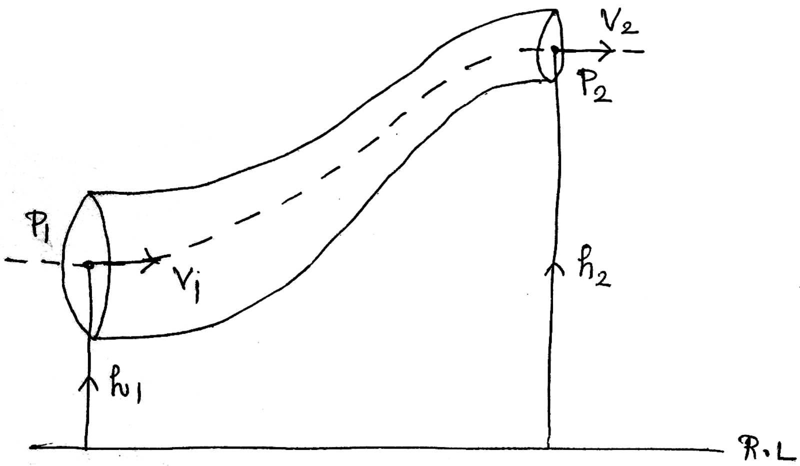
$$\rho A_1 V_1 = \rho A_2 V_2$$

$$\boxed{A_1 V_1 = A_2 V_2}$$

Energies possed by flowing liquid

1) $\frac{KE}{vol} = \frac{1}{2}\rho v^2$ 2) $\frac{v}{vol} = \rho g h$ 3) $\frac{Pr.energy}{vol.} = P$

Bernoulli's theorem



$$(TE)_1 = (TE)_2$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

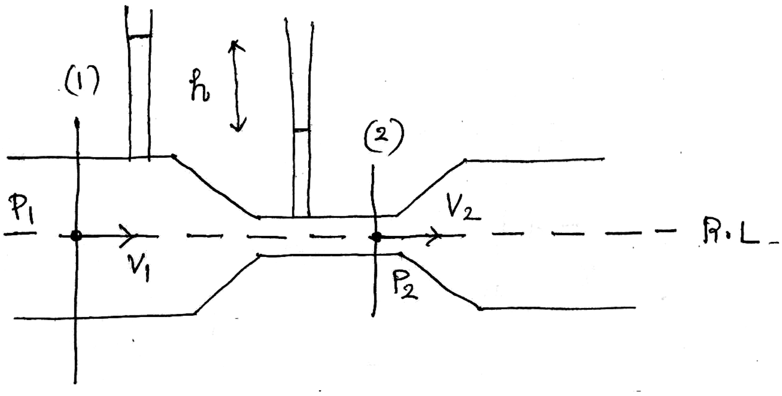
$$\frac{P}{\rho g} + \frac{v^2}{2g} + h = \text{constant}$$

$$\frac{P}{\rho g} = \text{pressure head}$$

$$\frac{v^2}{2g} = \text{velocity head}$$

$$h = \text{datum head}$$

Venturimeter



$$P_1 + \frac{1}{2}\rho v_1^2 + O = P_2 + \frac{1}{2}\rho v_2^2 + O$$

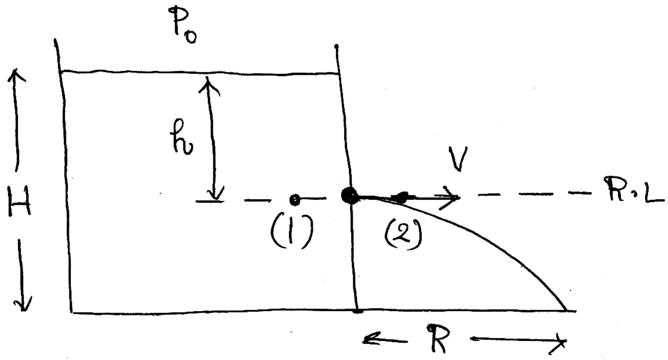
$$\boxed{P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)}$$

$A_1V_1 = A_2V_2$

also $P_1 - P_2 = h\rho g$

$$\boxed{\begin{array}{l}\therefore \text{Rate of flow} \\ Q = A_1A_2\sqrt{\dfrac{2gh}{A_1^2 - A_2^2}}\end{array}}$$

Velocity of efflux



$$P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

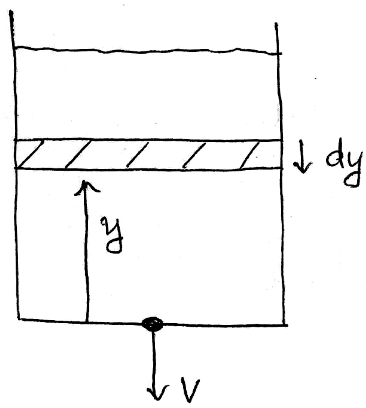
$$P_0 + h\rho g - P_0 = \frac{1}{2}\rho\left[v_2^2 - 0\right]$$

$$\boxed{v = \sqrt{2gh}}$$

$$R = VT = \sqrt{2gh}\sqrt{\frac{2(H-h)}{g}} = 2\sqrt{h(H-h)}$$

for maximum range, $h = \frac{H}{2}$ and $R_{max} = H$

Time for emptying a tank



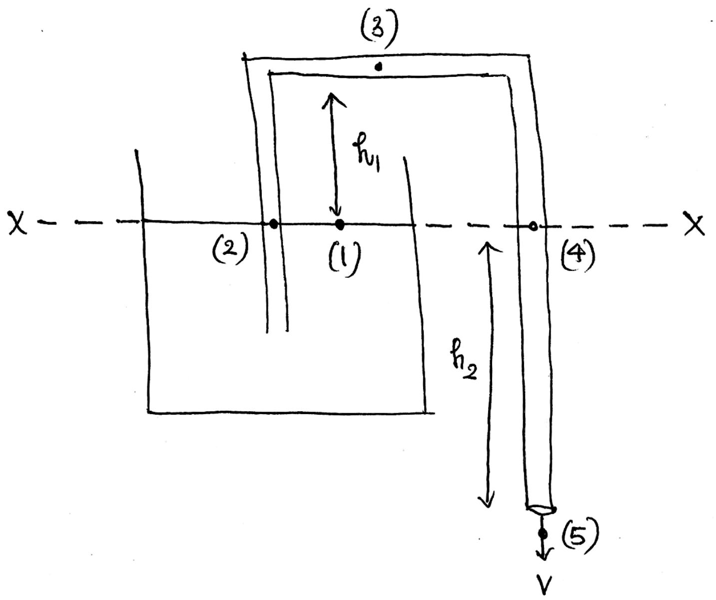
$$A_1V_1 = A_2V_2$$

$$-A\frac{dy}{dt} = a\sqrt{2gy}$$

$$\int_{H_1}^{H_2}\frac{dy}{\sqrt{y}} = \frac{-a\sqrt{2g}}{A}\int_0^T dt$$

$$\boxed{T = \frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{H_1} - \sqrt{H_2}\right]}$$

* **Siphon**



$V_3 = V_4 = V_5 = V$

$V_1 = V_2 = 0$

Applying bernoulli's theorem between (1) and (5)

$$P_0 + 0 + 0 = P_0 + \frac{1}{2}\rho V^2 - \rho g h_2$$

$$V = \sqrt{2gh_2}$$

$$\Rightarrow \boxed{h_2 > 0}$$

between (1) and (3)

$$P_0 + 0 + 0 = P_3 + \frac{1}{2}\rho V^2 + \rho g h_1$$

$$P_3 = P_0 - \rho g (h_1 + h_2)$$

$$P_3 \geq 0 \Rightarrow P_0 - \rho g (h_1 + h_2) \geq 0$$

$$\boxed{\frac{P_0}{\rho g} \geq h_1 + h_2}$$