CHAPTER - 09

CONIC SECTIONS

JEE MAIN - SECTION I

1. 1 The equation can be written as $(3x-1)^2 = -4(9y+2)$.

Hence the vertex is $\left(\frac{1}{3}, -\frac{2}{9}\right)$.

2. 3 director circle of parabola is $x+11=0 \quad \therefore r=12$

3. 1 Focus of parabola $y^2 = 2px$ is $(p/2, 0)$ .....(i)

$\therefore$ Radius of circle whose centre is $(p/2, 0)$ and touching $x+(p/2)=0$ is p.

Equation of circle is $\left(x-\frac{p}{2}\right)^2 + y^2 = p^2$ .....(ii)

From (i) and (ii), we get the point of intersection $\left(\frac{p}{2}, p\right)$.

4. 3 $(y-0)^2 = k\left(x-\frac{8}{k}\right)$; Vertex $V\left(\frac{8}{k}, 0\right)$

Latus rectum $4a=k, \quad a=k/4$

Equation of directrix $x=\frac{8}{k}-\frac{k}{4}=1$; $32-k^2=4k$; $k^2+4k-32=0$

$(k+8)(k-4)=0, k=-8,4$

5. 3 Put y = 1-x in $S=0$

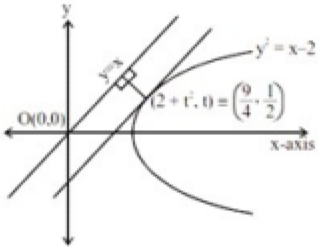
$1-x-x+x^2=0,\ x^2-2x+1=0, (x-1)^2=0. \quad x=1$ only

Hence $L=0$ touches the parabola at (1,1)

6. 1 We have, $2y\cdot\frac{dy}{dx}=1 \Rightarrow \left.\frac{dy}{dx}\right]_{P(2+t^2,t)} = \frac{1}{2t}=1$

$\Rightarrow t=\frac{1}{2} \Rightarrow P\left(\frac{9}{4}, \frac{1}{2}\right)$

So, shortest distance = $\frac{\left|\frac{9}{4}-\frac{2}{4}\right|}{\sqrt{2}} = \frac{7}{4\sqrt{2}}$



7. 3

Given $y^2 = 4x$ .... (1)

and $x^2 + y^2 = 5$ .... (2)

By (1) and (2), $x = 1$ and $y = 2$

Equation of tangent at (1, 2) to $y^2 = 4x$ is $y = x + 1$

8. 2

The line $y = mx + \frac{1}{m}$, touches $y^2 = 4x$ for all $m \neq 0$. This passes through $(1,4)$, so

$4 = m + \frac{1}{m}$; $m^2 - 4m + 1 = 0$;

$m_1 + m_2 = 4,\ m_1 m_2 = 1$;

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \sqrt{\frac{16-4}{2}} = \sqrt{3},\ \theta = \frac{\pi}{3}$$

9. 2

$$\frac{x^2}{2-r} + \frac{y^2}{r-5} + 1 = 0 \Rightarrow \frac{x^2}{r-2} + \frac{y^2}{5-r} = 1$$

Hence r > 2 and and $r < 5 \Rightarrow 2 < r < 5$.

10. 1

The ellipse is $4(x-1)^2 + 9(y-2)^2 = 36$

Therefore, latus rectum $= \frac{2b^2}{a} = \frac{2 \cdot 4}{3} = \frac{8}{3}$.

11. 4

By symmetry the quadrilateral is a rhombus. So area is four times the area of the right angled triangle formed by the tangent and axes in the I^{st} quadrant.

Now, $ae = \sqrt{a^2 - b^2} \Rightarrow ae = 2$

$\Rightarrow$ Tangent (in first quadrant) at end of latus rectum $\left(2, \frac{5}{3}\right)$ is

$\frac{2}{9}x + \frac{5}{3}\frac{y}{5} = 1$ i.e., $\frac{x}{9/2} + \frac{y}{3} = 1$

Area $= 4 \cdot \frac{1}{2} \cdot \frac{9}{2} \cdot 3 = 27$ sq. unit.

12. 2

Focus of an ellipse is given as $(\pm ae, 0)$

Distance between them = 2ae

According to the question, $2ae = \frac{b^2}{a}$.

$\Rightarrow 2a^2e = b^2 = a^2(1-e^2)$

$\Rightarrow 2e = 1-e^2 \Rightarrow (e+1)^2 = 2$

$\Rightarrow e = \sqrt{2}-1$.

13. 1

Given equation of ellipse can be written as $\frac{x^2}{6}+\frac{y^2}{2}=1$

$\Rightarrow a^2 = 6, b^2 = 2$

Now, equation of any variable tangent is

$y = mx \pm \sqrt{a^2m^2+b^2}$ ..... (1)

where 'm' is slope of the tangent

So, equation of perpendicular line drawn from centre to tangent is

$y = \frac{-x}{m}$ .... (2)

Eliminating 'm', we get

$(x^4+y^4+2x^2y^2) = a^2x^2+b^2y^2$.

$\Rightarrow (x^2+y^2)^2 = a^2x^2+b^2y^2$

$\Rightarrow (x^2+y^2)^2 = 6x^2+2y^2$.

14. 1 Distance between foci = 2ae and sum of focal distances from a point = 2a

$\therefore 2ae < 2a \Rightarrow e < 1$

Both statements are true, statement-II is correct explanation for statement-I

15. 1

$OS+OS' = 2a \quad 13+25 = 38, \qquad 0(0,0)$

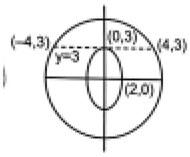
$a = 19 \qquad S(5,12)$

$2ae = ss' = \sqrt{19^2+5^2}$

$2ae = \sqrt{386}, \; e = \frac{\sqrt{386}}{38}$

16. 2 $e = 2$ and $e' = 3$

17. 1



The minimum length of intercept will be possible when

$y=3$ or $y=-3 \Rightarrow AB=8$

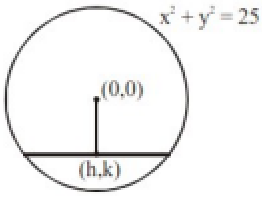
18. 4

Equation of chord $y-k=-\frac{h}{k}(x-h)$

$ky-k^2=-hx+h^2$

$hx+ky=h^2+k^2$

$y=-\frac{hx}{k}+\frac{h^2+k^2}{k}$



Tangent to $\frac{x^2}{9}-\frac{y^2}{16}=1$

$$\left(\frac{h^2+k^2}{k}\right)^2=9\left(-\frac{h}{k}\right)^2-16$$

$(x^2+y^2)^2=9x^2-16y^2$

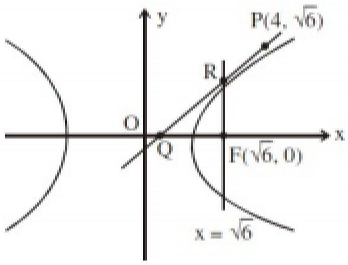
19. 3

$\frac{x^2}{4}-\frac{y^2}{2}=1 \Rightarrow e=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{\frac{3}{2}}$

$\therefore$Focus $F(ae,0) \Rightarrow F(\sqrt{6},0)$

Equation of tangent at P at the hyperbola is $2x-y\sqrt{6}=2$.

tangent meet x-axis at Q(1, 0)



and latus rectum $x=\sqrt{6}$ at $R\left(\sqrt{6},\frac{2}{\sqrt{6}}(\sqrt{6}-1)\right)$

$\therefore$Area of $\Delta_{QFR}=\frac{1}{2}\left(\sqrt{6}-1\cdot\frac{2}{\sqrt{6}}\right)(\sqrt{6}-1)$ = $\frac{7}{\sqrt{6}}-2$.

20. 1

$$\sqrt{3}x - y = 4\sqrt{3}k \;-(1)$$

$$k\sqrt{3}x + ky = 4\sqrt{3} - (2)$$

from (2) $k = \dfrac{4\sqrt{3}}{\sqrt{3}x + y}$; substitute (1)

$$\sqrt{3}x - y = 4\sqrt{3}\left(\frac{4\sqrt{3}}{\sqrt{3}x + y}\right)$$

$$3x^2 - y^2 = 48$$

$$\frac{x^2}{16} - \frac{y^2}{48} = 1$$

$$e = \sqrt{\frac{16+48}{16}} = 2$$

21. 2

$$(\lambda+1)x + \lambda y = 4$$

$$\lambda x + (1-\lambda)y + \lambda = 0$$

Vertex A is on y axis

$$x = 0$$

$$y = \frac{4}{\lambda}, \quad y = \frac{\lambda}{\lambda-1}$$

$$\frac{4}{\lambda} = \frac{\lambda}{\lambda-1}$$

$$\lambda = 2$$

A is $(0,2)$ Let $C(\alpha, 2\alpha+2)$

$$\left(\frac{2\alpha}{\alpha-1}\right)\left(\frac{-3}{2}\right) = -1$$

$$\alpha = -\frac{1}{2}$$

C is $\left(-\frac{1}{2}, 1\right)$

22. 1 $x^2 = b$ and $\frac{b}{16} + \frac{3}{b} = 1$

$b = 4$ and 12

$b = 12$ Possible

Hence points of intersection are

$(\pm\sqrt{12}, \pm 6) \Rightarrow$ area $= \underline{\underline{432}}$

23. 3

$e_1 = \frac{5}{4}$

$e_1 e_2 = 1$

$e_2 = \frac{4}{5}$

ellipse is passing through $(\pm 5, 0)$

$a = 5$ and $b = 3$

ellipse is $\frac{x^2}{25} + \frac{y^2}{9} = 1$

End point of chord are $\left(\pm \frac{5\sqrt{5}}{3}, 2\right)$

Length $= \frac{10\sqrt{5}}{3}$.

SECTION II (NUMERICAL)

24. 39

$$\frac{a}{e} = 8 \quad \text{—— (1)}$$

$$ae = 2 \quad \text{—— (2)}$$

$$\cancel{ae}$$

$$8e = \frac{2}{e}$$

$$e = \frac{1}{2}$$

$$a = 4$$

$$b^2 = 12$$

$$\frac{x\cos\theta}{4} + \frac{y\sin\theta}{2\sqrt{3}} = 1$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = 30^\circ$$

$$P(2\sqrt{3}, \sqrt{3}) \text{ and } Q\left(\frac{8}{\sqrt{3}}, 0\right)$$

25. 2

$$2ae = |1+\sqrt{2} - (1-\sqrt{2})| = 2\sqrt{2}$$

$$ae = \sqrt{2}$$

$$a = 1$$

$$L.R = \frac{2b^2}{a}$$

$$= 2$$

26. 25 $y^2 = 4ax = 16x \Rightarrow a = 4$

$A(1,4) \Rightarrow 2\cdot 4\cdot t_1 = 4 \Rightarrow t_1 = \frac{1}{2}$

$\therefore$ Length of focal chord $= a\left(t + \frac{1}{t}\right)^2$

$= 4\left(\frac{1}{2} + 2\right)^2 = 4\cdot\frac{25}{4} = 25$

27. 2

Since the distance between the focus and directrix of the parabola is half of the length of the latus rectum (L.R.). Therefore,

L.R. = 2 (Length of the perpendicular from (3, 3) on

$3x - 4y - 2 = 0$ i.e., $\left|\frac{9-12-2}{\sqrt{9+16}}\right| = 2$.

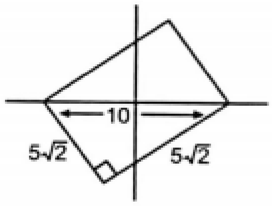
28. 1

Since sides of the square are tangent and perpendicular to each other, so the vertices lie on director circle

$$x^2 + y^2 = 16 + b^2 = \left(\frac{10}{2}\right)^2$$

$$\Rightarrow \quad b = 3$$

$$\frac{A}{\pi} = \frac{\pi(4)(3)}{\pi} = 12$$



29. 18

Given equation of hyperbola is,

$9x^2 - 16y^2 + 72x - 32y - 16 = 0$

$\Rightarrow 9(x^2 + 8x) - 16(y^2 + 2y) - 16 = 0$

$\Rightarrow 9(x + 4)^2 - 16(y + 1)^2 = 144$

$\Rightarrow \frac{(x+4)^2}{16} - \frac{(y+1)^2}{9} = 1$

Therefore, latus rectum $= \frac{2b^2}{a} = 2 \times \frac{9}{4} = \frac{9}{2}$.

30. 5 The condition for the line $y = mx + c$ will touch the hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $c^2 = a^2m^2 - b^2$

Here $m = -1$, $c = \sqrt{2}p$, $a^2 = 9, b^2 = 4$

$\therefore$ We get $2p^2 = 5$.

JEE ADVANCED LEVEL

SECTION III

31. B The vertex and focus are (1,1) and (2,2).

The find the equation of the directrix and apply $SP = PM$.

32. D Foot to perpendicular from focus upon tangent is say (a,b).

So $\frac{\alpha+1}{2} = \frac{\beta+1}{-1} = \frac{-(-3+1-8)}{3^2+(-1)^2} = 1$

Images of $(7,13)$ and $(-1,-1)$ w.r.t. $(2,-2)$ will lie on respectively the axis and the directrix of the parabola. The two points are respectively $(-3,-17)$ and $(5,-3)$.

Slope of axis $= \frac{-1+17}{-1+3} = 8$. So equation of directrix : $y+3 = \frac{1}{8}(x-5)$

i.e., x+8y+19=0

33. B If (h,k) be the point of intersection of tangents then $ky = 2a(x+h)$ is same as normal $y = mx - 2am - am^3$. Comparing and eleminating m we get the required locus as $y^2(x+2a) + 4a^3 = 0$

34. A Let the tangents be drawn at the points 't_1' and 't_2' and (h,k) be their point of intersection.

$\therefore h = at_1t_2, k = a(t_1+t_2)$ .....(i)

Also their equations are

$t_1y = x + at_1^2, t_2y = x + at_2^2$

Also their slopes are $\frac{1}{t_1}$ and $\frac{1}{t_2}$ .....(ii)

If they include an angle α then

$$\tan\alpha = \frac{\frac{1}{t_1} - \frac{1}{t_2}}{1 + \frac{1}{t_1t_2}} = \frac{(t_2-t_1)}{(t_1t_2+1)} \text{ or } \tan^2\alpha(1+t_1t_2)^2 = \{(t_1+t_2)^2 - 4t_1t_2\}$$

or $\tan^2\alpha\left(1+\frac{h}{a}\right)^2=\left\{\frac{k^2}{a^2}-\frac{4h}{a}\right\}$ or $\tan^2\alpha(h+a)^2=\left(k^2-4ah\right)$

Hence the required locus is $(x+a)^2\tan^2\alpha=y^2-4ax$

35. B Major axis is y = 15
Distance between foci 2ae = 16, ae = 8
Same x-axis touching the ellipse we have b = 15 so that

$15^2=a^2\left(1-b^2\right)$

$(ab)^2=a^2-b^2$

$64=a^2-15^2,\ a=17,\quad e=8/17$

$2a=34$

36. C $f\left(a^2-5\right)>f(4a)\Rightarrow a^2-5<4a\Rightarrow a\in(-1,5)$

37. C We can write $x^2+4y^2=4$ as $\frac{x^2}{4}+\frac{y^2}{1}=1$ ..(1)

equation of the tangent to the ellipse (1) is $\frac{x}{2}\cos\theta+y\sin\theta=1$ .....(2)

equation of the ellipse $x^2+2y^2=6$ can be written as $\frac{x^2}{6}+\frac{y^2}{3}=1$ .....(3)

suppose (2) meets the ellipse (3) at P and Q and the tangent at P and Q to the ellipse (3) intersect at (h,k) w.r.t. the ellipse (3) and thus its equation is

$\frac{hx}{6}+\frac{ky}{3}=1$ .....(4)

since (2) and (4) represents the same line $\frac{h/6}{\cos\theta/2}=\frac{k/3}{\sin\theta}=1$

$\Rightarrow h=3\cos\theta,k=3\sin\theta$ locus of (h,k) is $x^2+y^2=9$

38. A Homogenisation and $a+b=0$ $\therefore P=\frac{ab}{\sqrt{b^2-a^2}}$

The line touches the circle $\Rightarrow d=r\Rightarrow p=r$

39. A $\frac{x^2}{4}+\frac{y^2}{3}=1\quad e=\frac{1}{2}$

Confocal, same focus $(\pm ae,0)=(\pm1,0)$

$1=a^2+b^2,\quad a=\sin\theta$

$b^2=\cos^2\theta$

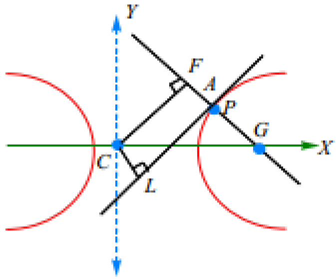
$\frac{x^2}{\sin^2\theta}-\frac{y^2}{\cos^2\theta}=1,\quad x^2\operatorname{cosec}^2\theta-y^2\sec^2\theta=1$

40. B Clearly PFCL is a rectangle

$\therefore PF = CL$

= length of the perpendicular from C (0,0) to the tangent at P

$$\therefore PF = \frac{1}{\sqrt{\frac{\sec^2\theta}{a^2}+\frac{\tan^2\theta}{b^2}}} = \frac{ab}{\sqrt{\frac{a^2}{b^2}PG^2}} \Rightarrow PF.PG = b^2$$



SECTION IV (More than one correct)

41. A,C Equation of tangent in term of slope of the parabola $y^2 = 8x$ is $y = mx + \frac{2}{m}$ .....(i)

$\because$ angle between eq. (i) and $y = 3x + 5$ is 45^0, then $\Rightarrow \left|\frac{m-3}{1+3m}\right| = \tan 45^0 = 1 \Rightarrow \pm(m-3) = 1+3m$

taking '+' sign, then $m = -3 = 1 + 3m$

$\therefore m = -2$ and taking '–' sign, then

$-m + 3 = 1 + 3m \qquad \therefore m = \frac{1}{2}$

Now, from eq. (i) equation of tangents are $y = -2x - 1$ and $y = \frac{x}{2} + 4$ or $2x + y = 1 = 0$ and $x - 2y + 8 = 0$

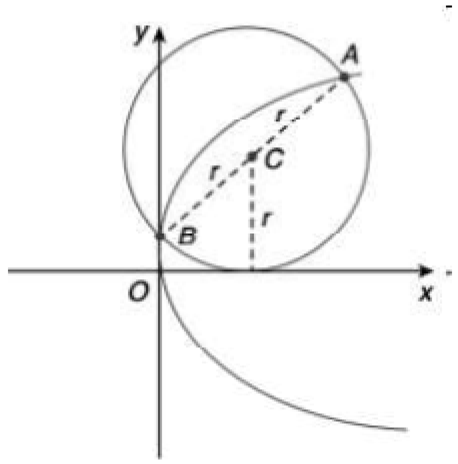
42. C,D $A\left(t_1^2, 2t_1\right), B\left(t_2^2, 2t_2\right), \left(t_1 \neq t_2\right)$

Slope of $AB = \frac{2}{t_1 + t_2}$

C is the centre of the circle describes on AB as a diameter

$\left(\frac{t_1^2 + t_2^2}{2}, \left(t_1 + t_2\right)\right)$

$\left|t_1 + t_2\right| = r$; Slope $AB = \pm\frac{2}{r}$



43. A,D Let P be $\left(at^2, 2at\right)$. Tangent and normal at P are $ty = x + at^2$ and $tx + y = 2at + at^3$ respectively. Thus, T is $\left(-at^2, 0\right)$ and N is $\left(2a + at^2, 0\right)$

If centroid be $G(h,k)$ then $h = \dfrac{2a + at^2}{3}, k = \dfrac{2at}{3} \Rightarrow \left(\dfrac{3h - 2a}{a}\right) = \dfrac{9k^2}{4a^2}$

$\Rightarrow$ Required parabola is $\dfrac{9y^2}{4a^2} = \dfrac{(3x - 2a)}{a} \Rightarrow y^2 = \dfrac{4a}{3}\left(x - \dfrac{2a}{3}\right)$

Vertex $\equiv \left(\dfrac{2a}{3}, 0\right)$; Focus $\equiv (a, 0)$

44. A,B,C,D

Since $0 < \alpha < \pi/4, \cos^2\alpha > \sin^2\alpha$

$\Rightarrow$ eccentricity of the two given ellipse, are same.

$x^2/\cos^2\alpha + y^2/\sin^2\alpha = \dfrac{x^2}{\sin^2\alpha} + \dfrac{y^2}{\cos^2\alpha}$

$\Rightarrow \left(1/\cos^2\alpha - 1/\sin\alpha\right)x^2 + \left(\dfrac{1}{\sin^2\alpha} - \dfrac{1}{\cos^2\alpha}\right)y^2 = 0$

Since sum of the coefficients of x^2 & y^2 is zero.

POR & QOS are perpendicular and equations of these lines are $y = \pm x$.

$\therefore P(\cos\alpha\sin\alpha, \sin\alpha\cos\alpha), S(\cos\alpha\sin\alpha, -\sin\alpha\cos\alpha)$

$\therefore PS = \sin 2\alpha$, and hence B, D also correct

45. A,B,C $2\left(x^2 - 6x + 4\right) + 3\left(y^2 + 2y + 1\right) - 8 - 3 + 5 = 0$

$\Rightarrow \dfrac{(x-2)^2}{3} + \dfrac{(y+1)^2}{2} = 1$

Equation of the auxilary circle: $(x-2)^2 + (y+1)^2 = 3$

Equation of director circle: $(x-2)^2 + (y+1)^2 = 3 + 2$

Director circle always passes through $=(4,-2)$

46. B,C

Given : $\frac{x^2}{9}+\frac{y^2}{4}=1$

Equation of tangent

$y = mx + \sqrt{a^2.m^2 + b^2}$ .....(1)

Equatoin of tangent at $A : x = +3$ .....(2)

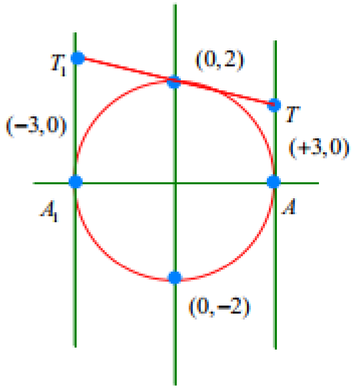
Equation of tagnent at $A^1 : x = -3$ .....(3)

$T\left(3, 3m+\sqrt{9m^2+4}\right) : T^1\left(-3, -3m+\sqrt{9m^2+4}\right)$

Equation of circle TT^1 as ends of diametre:-

$(x-3)(x+3)+\left(y-\left(3m+\sqrt{9m^2+4}\right)\right)\left(y-\left(-3m+\sqrt{9m^2+4}\right)\right)=0$

Above equation of the circle always passes through Focii of the ellipse.



47. A,C Eccentricity of the ellipse $e=\sqrt{1-\frac{16}{25}}-\frac{3}{5}$

$\therefore$ Eccentricity of the hyperbola $e_1=\frac{5}{3}$

Foci of the ellipse are $(\pm 3,0)$. Clealry these are the vertaices of the hyperbola, whose equation is then $\frac{x^2}{9}-\frac{y^2}{b^2}=1$. Now $b^2=9\left(\frac{25}{9}-1\right)=16$

So, the equation of the hyperbola is $\frac{x^2}{9}-\frac{y^2}{16}=1$

Foci of the hyperbola are $(\pm 5,0)$

48. B,D $\frac{x^2}{\frac{1}{4}}+\frac{y^2}{\frac{1}{9}}=1$. Tangent at (x_1, y_1) is $4xx_1+9yy_1=1$

which is parallel to $9y = 8x$ $\quad \frac{-4x_1}{9y_1} = \frac{8}{9}, x_1 = 2y_1$

$x_1 = -2y, \quad 16y_1^2 + 9y_1^2 = 1.y_1 = \pm\frac{1}{5}, \quad x_1 = \pm\frac{2}{5}$

SECTION V - (Numerical type)

49. 0.60

$y + xt = 2at + at^3$ ......(1)

$y = mx = c$ ....(2)

$\frac{m}{t} = \frac{-1}{1} = \frac{c}{-2at - at^3}$

$t = -m, c = 2at + at^3 = -2am - am^3$

$c + 2am + am^3 = 0, \quad a = 2$

$p = 4, q = 2 \quad \frac{p+q}{10} = 0.60$

50. 5 Equation of tangent to parabola at P is $2(y+9) = 6x \Rightarrow y + 9 = 3x$

Equation of circle is $(x-6)^2 + (y-9)^2 + \lambda(y - 3x + 9) = 0$

Put $(0,1) \Rightarrow 36 + 64 + \lambda(10) = 0 \Rightarrow \lambda = -10.$

$\therefore$ Equation of circle is $\quad x^2 + y^2 + 18x + 28y + 27 = 0.$

$\therefore$ Radius $= \sqrt{9^2 + (14)^2 - 27} = 5\sqrt{10}.$

51. 1.4 Product of ordinate is : $b^2 \Rightarrow 4 \times 9 = b^2 \Rightarrow 36 = b^2, 2ae = \sqrt{144 + 25}; 2ae = 13$

$\Rightarrow e = \sqrt{\frac{a^2 - b^2}{a^2}} = \frac{13}{\sqrt{313}} \Rightarrow$ Sum of digits : 7

52. 2.5 Equation of ellipse : $\frac{x^2}{25} + \frac{y^2}{16} = 1$ .....(1)

Equation of mid-point of chord w.r.t $S = 0$ is

$S_1 = S_{11}; \quad \frac{x\left(\frac{1}{2}\right)}{25} + \frac{y\left(\frac{2}{3}\right)}{16} - 1 = \frac{\left(\frac{1}{2}\right)^2}{25} + \frac{\left(\frac{2}{3}\right)^2}{16} - 1$ .....(2)

By using : $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \frac{7}{5}\sqrt{41}$

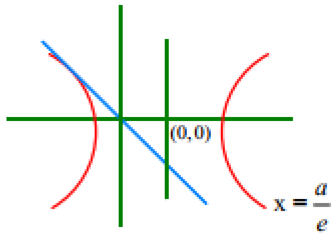
$\therefore$ Sum of digit : 5

53. 2 Since the line $2x + y - 1 = 0$ is tangent

so, $C^2 = a^2m^2 - b^2$

$1 = 4a^2 - b^2 \quad .....(i)$

Also line passes through $\left(-\frac{a}{e}, 0\right)$



So, $2\left(-\frac{a}{e}\right) = 1$

$4a^2 = e^2 \qquad .....(ii)$

Using (i) and (ii) $e = 2$.

SECTION VI - (Matrix match type)

54. A

(a) Equation of mid-point of chord of (0,3) w.r.t

$S = 0$, is $S_1 = S_{11} \Rightarrow i.e.\ y = 3 \quad \Rightarrow \therefore k = 8$

(b) $c^2 = a^2m^2 + b^2; \lambda^2 = 25$ sum of values of $\lambda = 0.$

(c) $\frac{a}{e} - ae = 8 \Rightarrow 2a - \frac{a}{2} = 8 \Rightarrow a = \frac{16}{3} \Rightarrow a^2e^2 = a^2 - b^2 \Rightarrow b = \frac{8}{\sqrt{3}}$

(d) $SP + S^1P = 2b = 8$

55. C

A) Directrix x+2=0, x = -2, $(A) \rightarrow (P), (t)$

B) $y = mx + \frac{a}{m}$ touches $y^2 = 4ax$ at $\left(\frac{a}{m^2} \quad \frac{2a}{m}\right)$

given line $y = -x - 3$, $(a = 3, m = -1)$ and hence it touches the parabola

$y^2 = 12x$ at $\left(\frac{a}{m^2}, \frac{2a}{m}\right) = (3, -6)$

$B \rightarrow (r)$

$y = mx - 21am - am^3$ is a normal to $y^2 = 4a$ at $(am^2 - 2am)$, $m = -4, 3$ and $a = \frac{9}{4}$

$\therefore (am^2, -2am) = \left(\frac{9}{4} \times \frac{16}{3}, \ -2\left(\frac{9}{4}\right)\left(\frac{-4}{3}\right) = (4, 6)\right)$ $C \rightarrow (q)$

The line parallel to $4y - x + 3 = 0$ is $4y - x + c = 0$.

The line with slope m touches the parabola $y^2 = 40$ at the point $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$, $a = \frac{7}{4}, m = \frac{1}{4}$

point of contact $\left(\frac{7}{4} \times 16, \ 2\left(\frac{7}{4}\right) \times 4\right) = (28, 14)$ $D \rightarrow (S)$