CHAPTER - 15
INTEGRATION AND ITS APPLICATION

Definition

If f and g are functions of x such that $g'(x) = f(x)$, then the indefinite integral of f(x) with respect to x is defined and denoted as $\int f(x)dx = g(x) + c$ where c is called the constant of integration. (In the following discussion log x means $\log_e x$, unless or otherwise mentioned)

Basic formula

1. $\int 0dx = c$
2. $\int x^n dx = \frac{x^{n+1}}{n+1} + c, \text{If } n \neq -1$
3. $\int \frac{1}{x}dx = \log|x| + c$
4. $\int e^x dx = e^x + c$
5. $\int a^x dx = \frac{a^x}{\log a} + c$
6. $\int dx = x + c$
7. $\int \cos x dx = \sin x + c$
8. $\int \sin x dx = -\cos x + c$
9. $\int \sec^2 x dx = \tan x + c$
10. $\int \operatorname{cosec}^2 x dx = -\cot x + c$

11. $\int \sec x \tan x dx = \sec x + c$

12. $\int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x + c$

13. $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c \text{ or } -\cos^{-1} x + c$

14. $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c \quad \text{or } -\cot^{-1} x + c$

15. $\int \tan x dx = \log|\sec x| + c$ or $-\log|\cos x| + c$

16. $\int \cot x dx = \log \sin x + c$ or $-\log|\operatorname{cosec} x| + c$

17. $\int \sec x dx = \log|\sec x + \tan x| + c$ or $\log \left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + c$ or $-\ell n|\sec x - \tan x| + c$

18. $\int \operatorname{cosec} x dx = \log|\operatorname{cosec} x - \cot x| + c$ or $\log|\tan x/2| + c$ or $-\ell n|\operatorname{cosec} x + \cot x| + c$

19. $\int \frac{1}{|x|\sqrt{x^2-1}} dx = \sec^{-1} x + c$ or $-\operatorname{cosec}^{-1} x + c$

20. $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1} \frac{x}{a} + c$

21. $\int \frac{1}{\sqrt{a^2+x^2}} dx = \log|x + \sqrt{a^2+x^2}| + c$

22. $\int \frac{1}{\sqrt{x^2-a^2}} dx = \log|x + \sqrt{x^2-a^2}| + c$

23. $\int \frac{1}{a^2-x^2} dx = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + c$

24. $\int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + c$

25. $\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$

26. $\int \sqrt{a^2-x^2} dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + c$

27. $\int \sqrt{a^2+x^2}dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\log|x+\sqrt{a^2+x^2}| + c$

28. $\int \sqrt{x^2-a^2}dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log|x+\sqrt{x^2-a^2}| + c$

29. $\int u(x)v(x)dx = u(x)\int v(x)dx - \int \left[\frac{d(u)}{dx}\int v(x)dx\right]dx$ (Integration by parts)

30. $\int f'(ax+b)dx, a \neq 0 = \frac{1}{a}f(ax+b)+c$

31. $\int \frac{f'(x)}{f(x)}dx = \log|f(x)|+c$

32. $\int [f(x)]^n f'(x)dx = \frac{[f(x)]^{n+1}}{n+1}+c, n \neq -1$

33. $\int f'(g(x))g'(x)dx = f(g(x))+c$

34. $\int \cos mx \cos nx\, dx$, $\int \sin mx \sin nx dx$, $\int \cos mx \sin nx dx$,

use the identities

$$\cos A \cos B = \frac{1}{2}[\cos(A+B)+\cos(A-B)]$$

$$\cos A \sin B = \frac{1}{2}[\sin(A+B)-\sin(A-B)]$$

$$\sin A \sin B = -\frac{1}{2}[\cos(A+B)-\cos(A-B)]$$

$$\sin A \cos B = \frac{1}{2}[\sin(A+B)+\sin(A-B)]$$

35. $\int \sin^m x \cos^n x dx$ if m is odd put cosx = t and; if n is odd put sinx = t

If both m and n are even, use power reducing formulae, $\sin^2 x = \frac{1}{2}(1-\cos 2x)$

$\cos^2 x = \frac{1}{2}(1+\cos 2x)$

If m + n is a negative even integer put tanx = t

36. $\int \frac{a\sin x + b\cos x + c}{p\sin x + q\cos x + r}dx$ write $N^r = A(D^r) + B\frac{d}{dx}(D^r) + C$. Find A,B and C by comparing the coefficients of cosx, sinx and constant term

37. $\int \frac{dx}{a\sin x \pm b\cos x}$ use a = $r\cos\alpha$, b= $r\sin\alpha$ put the integral in the form $\frac{1}{r}\int\frac{dx}{\sin(x\pm\alpha)}$, use formula for $\int \operatorname{cosec} x\,dx$

38. $\int e^x\left(f(x)+f'(x)\right)dx = e^x f(x) + c$

39. $\int e^{ax}\left(f(x)+\frac{f'(x)}{a}\right)dx = \frac{e^{ax}}{a}f(x)+c$

40. $\int e^{ax}\cos(bx+c)dx = \frac{e^{ax}}{a^2+b^2}\left(a\cos(bx+c)+b\sin(bx+c)\right)+D$

41. $\int e^{ax}\sin(bx+c)dx = \frac{e^{ax}}{a^2+b^2}\left(a\sin(bx+c)-b\cos(bx+c)\right)+D$

42. $\int \frac{(x^2+1)dx}{x^4+kx^2+1}$ dividing by x^2 put $x - \frac{1}{x} = t$

43. $\int \frac{(x^2-1)dx}{x^4+kx^2+1}$ dividing by x^2 and put $x+\frac{1}{x}=t$

44. $\int \frac{dx}{a\sin x + b\cos x + c}$ Put t = tanx/2

45. $\int \frac{dx}{a\sin^2 x + b\cos^2 x + c}$ Divide Nr and Dr by $\cos^2 x$ and put $t = \tan\theta$

46. $\int \frac{1}{x(x^n+1)}dx = \frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right| + c$

$\int \sqrt{\frac{x-\alpha}{\beta-x}}dx$ or $\int \sqrt{(x-\alpha)(\beta-x)}dx$ Put $x = \alpha\cos^2\theta + \beta\sin^2\theta$

Reduction formula

If $I_n = \int \tan^n x\,dx$, then $I_n = \frac{\tan^{n-1}x}{n-1} - I_{n-2}, n \ge 2$

If $I_n = \int \cot^n x\,dx$, then $I_n = \frac{-\cot^{n-1}x}{n-1} - I_{n-2}, n \ge 2$

If $I_n = \int \sec^n x dx$, then $I_n = \frac{\tan x \sec^{n-2} x}{n-1} + \frac{n-2}{n-1} I_{n-2}$

If $I_n = \int \operatorname{cosec}^n x dx$, then $I_n = \frac{-\cot x \operatorname{cosec}^{n-2} x}{(n-1)} + \frac{n-2}{n-1} I_{n-2}$

DEFINITE INTEGRAL

f(x) is a continuous function in [a.b] so that $\phi'(x) = f(x)$ for all x in [a,b], then $\int_n^b f(x)dx = \phi(b) - \phi(a)$,

called definite integral of $f(x)$ in[a,b]

Properties of Definite Integral

1. $\int_a^b f(x)dx = \int_a^b f(t)dt$

2. $\int_a^b f(x)dx = -\int_b^a f(x)dx$

3. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

4. $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

5. $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

6. $\int_{-a}^a f(x)dx = 0$, if $f(x)$ is odd function

7. $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$, if $f(x)$ is even

8. $\int_0^{2a} f(x)dx = 0$, if $(2a-x) = -f(x)$

9. $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$, if $f(2a-x) = f(x)$

10. $\int_a^b f(x)dx = 0$, if $f(a+x) = -f(b-x)$

11. If $f(x) \geq g(x)$ on $[a,b]$, then $\int_a^b f(x)dx \geq \int_a^b g(x)dx$

12. $\left|\int_a^b f(x)dx\right| \leq \int_a^b |f(x)|dx$

13. $\int_a^b f(x)dx = (b-a)\int_0^1 |f(b-a)x+a|dx$

14. $\left|\int_a^b f(x)g(x)dx\right|^2 \leq \int_a^b f^2(x)dx\int_a^b g^2(x)dx$ where f(x) and g(x) are integrable in [a,b]

15. If $h(x), f(x)$ and $g(x)$ are three continuous functions on [a,b] such that $h(x) \leq f(x) \leq g(x)\ \forall x \in [a,b]$ then $\int_a^b h(x)dx \leq \int_a^b f(x)dx \leq \int_a^b g(x)dx$ (it is useful, when f(x) is not feasible to integrate)

16. If m and M are absolute (global) minimum and absolute (global) maximum respectively of f(x) in [a,b] then
$m(b-a) \leq \int_a^b f(x)dx \leq M(b-a)$

17. If f(x) is a periodic function with period T, then

i) $\int_0^{nT} f(x)dx = n\int_0^T f(x)dx$

ii) $\int_{nT}^{nT} f(x)dx = (n-m)\int_0^T f(x)dx$

iii) $\int_{a+nT}^{b+nT} f(x)dx = \int_a^b f(x)dx$

18. Leibnitz's rule for differentiation under integration

i) $\frac{d}{dx}\left[\int_{g(x)}^{h(x)} f(t)dt\right] = f(h(x))h'(x) - f(g(x))g'(x)$

ii) $\frac{d}{dx}\left(\int_{\alpha}^{h(x)} f(t)dt\right) = f(h(x))h'(x)$

iii) $\frac{d}{dx}\left(\int_{g(x)}^{h(x)} f(x,t)dt\right) = \int_{g(x)}^{h(x)} \frac{\delta f(x,t)}{\delta x} = f(x,h(x))h'(x) - f(x,g(x))g'(x)$

where $\frac{\delta f(x,t)}{\delta x}$ is derivative of f with respect to x, keeping 't' as a constant

iv) $\frac{d}{dx}\left[\int_a^b f(x,t)\,dt\right] = \int_a^b \frac{\delta t\ f(x,t)}{\delta x}\,dt$

19. If f(x) is an odd function, then $\int_0^x f(t)\,dt$ is even

20. If f(x) is an even function, then $\int_0^x f(t)\,dt$ is odd

21. Mean value of function f(x) defined in [a,b] is $\frac{1}{b-a}\int_a^b f(x)\,dx$

22. If f(x) is continuous in $[a,\infty]$ then $\int_a^\infty f(x)\,dx$ is called an improper integral, calculated by $\lim_{b\to\infty}\int_a^b f(x)\,dx$.

Similarly $\int_{-\infty}^b f(x)\,dx = \lim_{a\to\infty}\int_a^b f(x)\,dx$. If it provides finite limit, then it is convergent otherwise it is divergent

Definite integral as limit of sum

23. $\int_{-\infty}^b f(x)\,dx = \lim_{a\to-\infty}\int_a^b \left[f(a)+f(a+b)+.....f(a+(n-1)h\right]$ where $h = \frac{b-a}{n} = \lim_{h\to 0} h\sum_{r=0}^{n-1} f(a+rh)$

24. $\lim_{a\to\infty}\frac{1}{n}\sum_{r=g(n)}^{r=h(n)} f\left(\frac{r}{n}\right) = \int_\alpha^\beta f(x)\,dx$ when $\alpha = \lim_{n\to\infty}\frac{g(n)}{n}$ and $\beta = \lim_{n\to\infty}\frac{h(n)}{n}$

25. $\lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{n-1} f\left(\frac{r}{n}\right) = \int_0^1 f(x)\,dx$

26. $\int_0^{\frac{\pi}{2}} \sin^n x\,dx = \begin{cases} \frac{n-1}{n},\frac{n-3}{n-2}............\frac{1}{2}.\frac{\pi}{2} \text{ if n is even} \\ \frac{n-1}{n},\frac{n-3}{n-2}..........\frac{2}{3}. \text{ If n is odd} \end{cases}$

27. $\int_0^{\frac{\pi}{2}} \cos^n x\,dx = \int_0^{\frac{\pi}{2}} \sin^n x\,dx$

28. $\int_0^{\frac{\pi}{2}} \sin^m x \cos^n x dx = \frac{[(m-1)(m-3).....1 \text{ or } 2][(n-1)(n-2)......(1 \text{ or } 2)]}{(m+n)(m+n-2)..........(1 \text{ or } 2)}$

Where k is $\frac{\pi}{2}$ if m and n are both even, otherwise k is 1.

Some useful results

1. $\int_0^{n} \sin x \ dx = 2$

2. $\int_0^{\frac{\pi}{2}} \frac{f(\sin x)}{f(\sin x)+f(\cos x)} dx = \frac{\pi}{4}$

3. $\int_0^{1} \frac{x^n - 1}{\cos x} dx = \log(n+1)$

4. $\int_0^{\frac{\pi}{2}} \log \sin x \, dx = \frac{-\pi}{2} \log 2 = \frac{\pi}{2} \log\left(\frac{1}{2}\right)$

5. $\int_0^{\pi} \frac{dx}{1+\sin x} = 2$

6. $\int_0^{\frac{\pi}{2}} x f(\sin x) dx = \pi \int_\theta^{\frac{\pi}{2}} f(\sin x) dx$

7. $\int_0^{\frac{\pi}{2}} \frac{dx}{a^2 \sin^2 x + b^2 \sin^2 x} = \frac{\pi}{2ab}$

8. $\int_0^{\frac{\pi}{4}} \log(1+\tan\theta) = \frac{\pi}{8} \log 2$

9. $\int_0^{\infty} \frac{dx}{\left(x+\sqrt{1+x^2}\right)^n} = \frac{n}{n^2-1}$

10. $\int_0^{\pi} \frac{dx}{a^2 - 2a\cos x + 1} = \begin{cases} \frac{\pi}{a^2-1}; & a > 1 \\ \frac{\pi}{1-a^2} & 0 < a < 1 \end{cases}$

11. $\int_0^{\pi} \frac{dx}{(a-\cos x)^2} = \frac{\pi a}{(a^2-1)^{3/2}}$

12. $\int_0^{h} \frac{dx}{\sqrt{(x-a)(b-x)}} = \pi$

13. $\int_a^{b} \sqrt{(x-a)(b-x)}dx = (b^2-a^2)\frac{\pi}{8}$

14. $\int_{\alpha}^{\beta} (ax^2+bx+c)dx = \frac{-a}{6}(\beta-\alpha)^3$, where α, β are the roots of $ax^2+bx+c=0$

15. $\int_0^{n} [x]dx = \frac{n(n+1)}{2}$

16. $\int_0^{n} \{x\}dx = n\int_0^{1} \{x\}dx = \frac{n}{2}; n \in N$

17. $\int_0^{n} [x]dx = -n$

18. $\int_{-n}^{n} [f(x)]dx = -n$, if $f(x)$ is odd function

19. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \, dx = 2$

20. $\int_0^{\infty} e^{-x}dx = 1$

21. $\int_0^{\frac{\pi}{2}} \log \tan x \, dx = 0$

22. $\int_0^{\frac{\pi}{2}} \frac{dx}{\sin x + \cos x} = \sqrt{2}\log(\sqrt{2}+1)$

23. $\int_0^{\frac{\pi}{2}} f(\sin x)dx = \int_0^{\frac{\pi}{2}} f(\cos x)dx$

24. $\int_0^{\frac{\pi}{2}} \frac{dx}{(a^2\sin^2 x + b^2\cos^2 x)^2} = \frac{\pi(a^2+b^2)}{4a^3b^3}$

25. $\int_0^{\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)} = \frac{\pi}{2ab(a+b)}$

26. $\int_0^{\frac{\pi}{2}} \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx = \sqrt{2}\pi$

27. $\int_0^{\pi} \frac{dx}{a - \cos x} = \frac{\pi}{\sqrt{a^2-1}}; a > 1$

28. $\int_0^1 x^m (1-x)^n \, dx = \frac{m!n!}{(m+n+1)!}, m, n \in N$

29. $\int_a^b \sqrt{\frac{x-a}{b-x}} dx = (b-a)\frac{\pi}{2}$

30. Gamma functions: $\int_0^{\infty} e^{-x} x^{n-1} dx, n$ is a positive rational number is known as gamma function denoted by $\sqrt{n}$

31. $\sqrt{n} = (n-1) r(n-1)$

32. $\sqrt{n} = (n-1)!$ if $n \in N$

33. $r(1) = 1$

34. $r_{\left(\frac{1}{2}\right)} = \sqrt{\pi}$

Area of the region bounded by graph of a function

1. f(x) is a function defined over an interval [a,b] and f(x) is non-negative in this interval. Then the area A, bounded by the x-axis, the lines x = a and x = b and the graph $y = f(x)$ is given by $A = \int_a^b f(x) dx$:

2. If $f(x) \le 0$, for all $x \in [a,b]$, then $A = -\int_a^b f(x) dx$ or $A = \left|\int_a^b f(x) dx\right|$

3. f(x) is continuous function in [a,b] and f(a) f(b) < 0, then by intermediate value theorem f(x) = 0 has root x = c and area bounded by and x = a and x = b is

$$A = \int_a^c f(x) dx + \left|\int_c^b f(x) dx\right| \text{if } f(x) > 0 \text{ in } (a,c) \,\&\, f(x) < 0 \text{ is } (c,b)$$

4. Area bounded by the curve $x=f(y)$, the axis of y and abscissea $y=c$ and $y=d$ is given by

$$\int_c^d x\,dy=\int_c^d f(y)\,dy \text{ if } f(y)\geq 0\ \forall\ y\in[c,d]$$

5. If $y=f(x)$ and $y=g(x)$ are two continuous functions, both above the x-axis for $x\in[a,b]$ such that $f(x)\geq g(x)$ then area $A=\int_a^b (f(x)-g(x))dx$

6. Area included between the two curves $y=f(x)$ and $y=g(x)$ is $\left|\int_c^b f(x)-g(x)\right|dx$ where c,d are x-coordinates of the points of intersection of $y=f(x)$ and $y=g(x)$

Some specified areas (with out using definite integral)

1. Area of the circle $x^2+y^2=a^2$ is $A=\pi a^2$
2. Area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $A=\pi ab$
3. Area bounded by $\frac{(x-\alpha)^2}{a^2}+\frac{(y-\beta)^2}{b^2}=1$ is $A=\pi ab$
4. Area bounded by $y^2=4ax$ and $y=mx, A=\frac{8a^2}{3m^3}$
5. Area bounded by $y^2=4ax$ and $x=c$ is $A=\frac{8c\sqrt{ac}}{3}, a>0, c>0$
6. Area bounded by $y^2=4ax$ and its latus rectum $x=a$ is $\frac{8a^2}{3}$
7. Area bounded by $y^2=4ax$ and $x^2=4by$ is $A=\frac{16ab}{3}$
8. Area bounded by $y^2=4ax, x^2=4ay(a>0)$, is $A=\frac{5a^2}{4}$
9. The area of smaller region bounded by a circle $x^2+y^2+2gx+2fy+c=0$ and its chord $y=mx+c$, which substand 90° at the centre $(\pi-2)\frac{r^2}{4}, r=\sqrt{g^2+f^2-c}$
10. Area bounded by $y=ax^2+bx+c$ and $x-\text{axis}$ is $\left|\frac{a}{6}(\alpha-\beta)^3\right|$ where α,β are the roots of $ax^2+bx+c=0$

PART I - (JEEMAIN)

SECTION - I - Straight objective type questions

1. $\int \sqrt{1+\sin\left(\frac{x}{4}\right)}dx, 0<x<2\pi$

(1) $8\left(\sin\frac{x}{8}-\cos\frac{x}{8}\right)+c$ (2) $\left(\sin\frac{x}{8}+\cos\frac{x}{8}\right)+c$

(3) $\frac{1}{8}\left(\sin\frac{x}{8}-\cos\frac{x}{8}\right)+c$ (4) $8\left(\cos\frac{x}{8}-\sin\frac{x}{8}\right)+c$

2. $\int \frac{1+x+\sqrt{x+x^2}}{\sqrt{x}\ \ +\sqrt{1+x}}dx=$

(1) $\frac{1}{2}\sqrt{1+x}+c$ (2) $\frac{2}{3}(1+x)^{3/2}+c$ (3) $\sqrt{1+x}+c$ (4) $2(1+x)^{3/2}+c$

3. $\int \sqrt{\frac{a+x}{a-x}}dx=$

(1) $a\cos^{-1}\frac{x}{a}+\sqrt{a^2-x^2}+c$ (2) $a\cos^{-1}\frac{x}{a}-\sqrt{a^2-x^2}+c$

(3) $-a\cos^{-1}\frac{x}{a}+\sqrt{a^2-x^2}+c$ (4) $-a\cos^{-1}\frac{x}{a}-\sqrt{a^2-x^2}+c$

4. $\int \frac{1}{x^2(x^4+1)^{3/4}}dx=$

(1) $\frac{(x^4+1)^{1/4}}{x}+c$ (2) $-\frac{(x^4+1)^{1/4}}{x}+c$ (3) $\frac{3}{4}\frac{(x^4+1)^{3/4}}{x}+c$ (4) $\frac{4}{3}\frac{(x^4+1)^{3/4}}{x}+c$

5. $\int \frac{dx}{\cos^3 x\sqrt{2\sin 2x}}$ is equal to

(1) $\sqrt{\tan x}+\frac{\tan^{5/2} x}{5}+c$

(2) $\sqrt{\tan x}+\frac{2}{5}\tan^{5/2} x+c$

(3) $2\sqrt{\tan x}+\frac{2}{5}\tan^{5/2} x+c$

(4) None of these

6. $\int e^{2x}\left(\frac{\sin 4x-2}{1-\cos 4x}\right)dx=$

(1) $\frac{1}{2}e^{2x}\cot 2x+c$ (2) $-\frac{1}{2}e^{2x}\cot 2x+c$ (3) $-2e^{2x}\cot 2x+c$ (4) $2e^{2x}\cot 2x+c$

7. $\int \frac{x^2-1}{x^4+x^2+1}dx$ is equal to

(1) $\log(x^4+x^2+1)+c$

(2) $\frac{1}{2}\log\frac{x^2-x+1}{x^2+x+1}+c$

(3) $\frac{1}{2}\log\frac{x^2+x+1}{x^2-x+1}+c$

(4) $\log\frac{x^2-x+1}{x+x+1}+c$

8. For $x>1$, $\int \frac{1}{x(x^4-1)}\ dx=$

(1) $\log\frac{x^4-1}{x^4}+K$ (2) $\frac{1}{4}\log\frac{x^4-1}{x^4}+K$ (3) $\log\frac{x^4-1}{x}+K$ (4) $\frac{1}{4}\log\frac{x^4-1}{x}+K$

9. $\int \frac{dx}{\cos(x-a)\cos(x-b)}=$

(1) $\text{cosec}(a-b)\log\frac{\sin(x-a)}{\sin(x-b)}+c$

(2) $\text{cosec}(a-b)\log\frac{\cos(x-a)}{\cos(x-b)}+c$

(3) $\text{cosec}(a-b)\log\frac{\sin(x-b)}{\sin(x-a)}+c$

(4) $\text{cosec}(a-b)\log\frac{\cos(x-b)}{\cos(x-a)}+c$

10. If $I=\int e^{x}\sin 2x\,dx$, then for what value of K, $KI=e^{x}(\sin 2x-2\cos 2x)+\text{constant}$

(1) 1 (2) 3 (3) 5 (4) 7

11. If $\int\frac{\cos x-\sin x}{\sqrt{8-\sin 2x}}dx=a\sin^{-1}\left(\frac{\sin x+\cos x}{b}\right)+c$, where c is a constant of integer, then the ordered pair (a,b) is equal to

(1) $(-1,3)$ (2) (3, 1) (3) (1, 3) (4) $(1,-3)$

12. **Statement-I:** $\int\left(\frac{1}{1+x^{4}}\right)dx=\tan^{-1}(x^{2})+C$

Statement-II: $\int\frac{1}{1+x^{2}}dx=\tan^{-1}x+C$.

(1) If both Statement-I and Statement-II are true and the reason is the correct explanation of the statement-I.

(2) If both Statement-I and Statement-II are true but reason is not the correct explanation of the statement-I.

(3) If Statement-I is true but Statement-II is false.

(4) If Statement-I is false but Statement-II is true.

13. $\int_{0}^{2\pi}\sqrt{1+\sin\frac{x}{2}}\,dx=$

(1) 0 (2) 2 (3) 8 (4) 4

14. $\int_{0}^{1}\sqrt{\frac{1-x}{1+x}}\,dx$ equals

(1) $\left(\frac{\pi}{2}-1\right)$ (2) $\left(\frac{\pi}{2}+1\right)$ (3) $\frac{\pi}{2}$ (4) $(\pi+1)$

15. The value of $\int_{-1}^{3}\tan^{-1}\left(\frac{x}{x^{2}+1}\right)+\tan^{-1}\left(\frac{x^{2}+1}{x}\right)dx$ is

(1) 2π (2) π (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{4}$

16. $\int_{-1}^{1} x|x|\,dx =$

(1) 1 (2) 0 (3) 2 (4) – 2

17. The value of $\int_0^{\pi/2} \frac{dx}{1+\tan^3 x}$ is

(1) 0 (2) 1 (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{4}$

18. The value of $\int_0^{\sqrt{2}} [x^2]dx,$ where [.] is the greatest integer function

(1) $2-\sqrt{2}$ (2) $2+\sqrt{2}$ (3) $\sqrt{2}-1$ (4) $\sqrt{2}-2$

19. $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n} e^{\frac{r}{n}}$ is

(1) $e+1$ (2) $e-1$ (3) $1-e$ (4) e

20. If $f(x) = \int_0^x t\sin t\,dt,$ then $f'(x) =$

(1) $\cos x + x\sin x$ (2) $x\sin x$ (3) $x\cos x$ (4) None of these

21. $\int_0^{\pi/2} \frac{dx}{2+\cos x} =$

(1) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (2) $\sqrt{3}\tan^{-1}\left(\sqrt{3}\right)$ (3) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (4) $2\sqrt{3}\tan^{-1}\left(\sqrt{3}\right)$

22. $\int_0^1 \sin\left(2\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right)dx =$

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{2}$ (4) π

23. The value of $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}dx$ is

(1) 1 (2) 0 (3) – 1 (4) $\frac{1}{2}$

24. If $(n-m)$ is odd and $|m| \neq |n|$, then $\int_0^{\pi} \cos mx \sin nx\, dx$ is

(1) $\frac{2n}{n^2-m^2}$ (2) 0 (3) $\frac{2n}{m^2-n^2}$ (4) $\frac{2m}{n^2-m^2}$

25. The value of $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$ is

(1) 1 (2) 0 (3) – 1 (4) None of these

26. The value of the integral $\int_{-\pi}^{\pi} (\cos ax - \sin bx)^2\, dx$, ($a$ and b are integer) is

(1) $-\pi$ (2) 0 (3) π (4) 2π

27. $\lim\limits_{n\to\infty} \frac{1}{1^3+n^3} + \frac{4}{2^3+n^3} + ... + \frac{1}{2n}$ is equal to

(1) $\frac{1}{3}\log_e 3$ (2) $\frac{1}{3}\log_e 2$ (3) $\frac{1}{3}\log_e \frac{1}{3}$ (4) None of these

28. The value of the integral $\int\limits_0^1 x\cot^{-1}(1-x^2+x^4)dx$ is

(1) $\frac{\pi}{4} - \frac{1}{2}\log_e 2$ (2) $\frac{\pi}{2} - \log_e 2$ (3) $\frac{\pi}{2} - \frac{1}{2}\log_e 2$ (4) $\frac{\pi}{4} - \log_e 2$

29. **Statement-I:** The value of the integral $\int\limits_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ is equal to $\frac{\pi}{6}$.

Statement-II: $\int\limits_a^b f(x)dx = \int\limits_a^b f(a+b-x)dx$.

(1) If both Statement-I and Statement-II are true and the reason is the correct explanation of the statement-I.

(2) If both Statement-I and Statement-II are true but reason is not the correct explanation of the statement-I.

(3) If Statement-I is true but Statement-II is false.

(4) If Statement-I is false but Statement-II is true.

30. The area bounded by the curves $y^2 = 8x$ and $y = x$ is

(1) $\frac{128}{3}$ sq. unit (2) $\frac{32}{3}$ sq. unit (3) $\frac{64}{3}$ sq. unit (4) 32 sq. unit

31. The area bounded by the curves $y^2 - x = 0$ and $y - x^2 = 0$ is

(1) $\frac{7}{3}$ (2) $\frac{1}{3}$ (3) $\frac{5}{3}$ (4) 1

32. The area of smaller part between the circle $x^2 + y^2 = 4$ and the line $x = 1$ is

(1) $\frac{4\pi}{3} - \sqrt{3}$ (2) $\frac{8\pi}{3} - \sqrt{3}$ (3) $\frac{4\pi}{3} + \sqrt{3}$ (4) $\frac{5\pi}{3} + \sqrt{3}$

33. The area formed by triangular shaped region bounded by the curves $y = \sin x, y = \cos x$ and $x = 0$ is

(1) $\sqrt{2} - 1$ (2) 1 (3) $\sqrt{2}$ (4) $1 + \sqrt{2}$

34. The area of region $\{(x, y) : x^2 + y^2 \le 1 \le x + y\}$ is

(1) $\frac{\pi^2}{5}$ (2) $\frac{\pi^2}{2}$ (3) $\frac{\pi^2}{3}$ (4) $\frac{\pi}{4} - \frac{1}{2}$

35. Area bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$ is

(1) $\frac{8}{9}$ sq. unit (2) $\frac{9}{8}$ sq. unit (3) $\frac{4}{3}$ sq. unit (4) None of these

SECTION - II

Numerical type Questions

36. If $\int \frac{1}{[(x-1)^3 (x+2)^5]^{1/4}} dx = \frac{a}{b}\left(\frac{x-1}{x+2}\right)^{1/4} + c$, then $a + b$ value is__________

37. If $\int \cos^{-3/7} x \sin^{-11/7} x \, dx = \frac{-a}{b} \tan^{-4/7} x + c$, then the $2(a+b)$ value is

38. $\int_{\pi}^{10\pi} |\sin x| \, dx$ is

39. If $I_n = \int_0^{\pi/4} \tan^n \theta d\theta$, then $56(I_8 + I_6)$ is equals to

40. The area of the figure bounded by the curves $y=|x-1|$ and $y=3-|x|$, is _______sq. units

PART - II (JEE ADVANCED)

SECTION - III (Only one option correct type)

41. The value of $\int_{1/e}^{\tan x}\frac{t\,dt}{1+t^2}+\int_{1/e}^{\cot x}\frac{dt}{t\left(1+t^2\right)}=$

A) -1 B) 1 C) 0 D) none of these

42. $\int_0^{\pi/2}\frac{x\sin x\cos x}{\cos^4 x+\sin^4 x}dx=$

A) 0 B) $\frac{\pi}{8}$ C) $\frac{\pi^2}{8}$ D) $\frac{\pi^2}{16}$

43. The area (in sq. units) of the part of the circle $x^2+y^2=36$, which is outside the parabola $y^2=9x$, is:

A) $24\pi+3\sqrt{3}$ B) $12\pi-3\sqrt{3}$ C) $24\pi-3\sqrt{3}$ D) $12\pi+3\sqrt{3}$

44. $\int\frac{\cos ec^2 x-2005}{\cos^{2005}x}dx$

A) $\frac{\cot x}{(\cos x)^{2005}}+c$ B) $\frac{\tan x}{(\cos x)^{2005}}+c$ C) $-\frac{\tan x}{(\cos x)^{2005}}+c$ D) $\frac{-\cot x}{(\cos x)^{2005}}+c$

45. $\int x^5\left(x^{10}+x^5+1\right)\left(2x^{10}+3x^5+6\right)^{1/5}dx$ is

A) $\frac{1}{6}\left(2x^{10}+3x^5+6\right)^{1/5}+c$

B) $\frac{1}{36}\left(2x^{15}+3x^{10}+6x^5\right)^{6/5}+c$

C) $\frac{1}{36}\left(2x^{15}+3x^{10}+6x^5\right)^{11/5}+c$

D) None of these

46. $\int\frac{\sec^2 x dx}{(\sec x+\tan x)^{9/2}}$ is equal to

A) $\frac{-1}{(\sec x+\tan x)^{9/2}}\left(\frac{1}{11}-\frac{1}{7}(\sec x+\tan x)^2\right)+C$

B) $\frac{1}{(\sec x+\tan x)^{11/2}}\left(\frac{1}{11}-\frac{1}{7}(\sec x+\tan x)^2\right)+C$

C) $\frac{-1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}+\frac{1}{7}(\sec x+\tan x)^2\right\}+C$

D) $\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}+\frac{1}{7}(\sec x+\tan x)^2\right\}+C$

47. Let $f(x)=\frac{x}{\left(1+x^n\right)^{1/n}}$ for n>2 and $g(x)=\underbrace{(fofo...of)}_{\text{focus n times}}(x)$. Then $\int x^{n-2}g(x)\,dx$ equals :

A) $\frac{1}{n(n-1)}\left(1+nx^n\right)^{1-\frac{1}{n}}+K$ B) $\frac{1}{n-1}\left(1+nx^n\right)^{1-\frac{1}{n}}+K$

C) $\frac{1}{n(n+1)}\left(1+nx^n\right)^{1+\frac{1}{n}}+K$ D) $\frac{1}{n+1}\left(1+nx^n\right)^{1+\frac{1}{n}}+K$

48. Let $I=\int_0^1\frac{\sin x}{\sqrt{x}}dx$ and $J=\int_0^1\frac{\cos x}{\sqrt{x}}dx$. Then which one of the following is true?

A) $I<\frac{2}{3}$ and J>2 B) $I>\frac{2}{3}$ and J<2 C) $I>\frac{2}{3}$ and J>2 D) $I<\frac{2}{3}$ and J<2

49. The value of the integral $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\left(x^2+\log\frac{\pi-x}{\pi+x}\right)\cos x\,dx$ is

A) 0 B) $\frac{\pi^2}{2}-4$ C) $\frac{\pi^2}{2}+4$ D) $\frac{\pi^2}{2}$

50. The value of $\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}}\frac{x\sin x^2}{\sin x^2+\sin\left(\ln 6-x^2\right)}dx$ is

A) $\frac{1}{4}\ln\frac{3}{2}$ B) $\frac{1}{2}\ln\frac{3}{2}$ C) $\ln\frac{3}{2}$ D) $\frac{1}{6}\ln\frac{3}{2}$

51. The area bounded by the curves $y=6x-x^2$ and $y=x^2-2x$ is (in sq. units)

A) $\frac{32}{3}$ B) $\frac{64}{3}$ C) $\frac{16}{3}$ D) $\frac{8}{3}$

SECTION - IV (More than one correct answer)

52. If $\int\frac{\cos x+\sin 2x}{\left(2-\cos^2 x\right)(\sin x)}dx=\int\frac{A}{\sin x}dx+B\int\frac{\sin x}{1+\sin^2 x}dx+C\int\frac{dx}{1+\sin^2 x}$

A) A+B+C=4 B) A+B+C=2 C) A+BC=-1 D) A+B+C=5

53. If $\int\frac{\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}}=\frac{A}{\pi}\left[\sqrt{x-x^B}-(C-Dx)\sin^{-1}\sqrt{x}\right]-x+c$ then

A) A+B+C+D=7 B) (A+B).(C+D)=12 C) (A+B).C=4 D) A+B+C=5

54. $I_1 = \int_1^e (1+x)(x+\log_e x)^{100} dx$ & $I_2 = \int_{\sin^{-1}\left(\frac{1}{e}\right)}^{\frac{\pi}{2}} (1+e\sin x+\log_e \sin x)^{101} \cos x dx$.

If $I_1 + \frac{e}{101} I_2 = \frac{e(1+e)^{101} - k}{101}$ then k is greater than or equal to

A) 0 B) 1 C) 2 D) -1

55. If $I_1 = \int_x^1 \frac{dt}{1+t^2}$ and $I_2 = \int_1^{1/x} \frac{dt}{1+t^2}, x > 0$ the

A) $I_1 = I_2$ B) $I_1 > I_2$ C) $I_2 > I_1$ D) $I_2 = \frac{\pi}{4} - \tan^{-1} x$

56. If $I(m,n) = \int_0^1 t^m (1+t)^n dt$, then the expression for I(m, n) in terms of I(m+1, n-1) is

A) $\frac{2^n}{m+1} - \frac{n}{m+1} I(m+1, n-1)$

B) $\frac{n}{m+1} I(m+1, n-1)$

C) $\frac{2^n}{m+1} + \frac{n}{m+1} I(m+1, n-1)$

D) $\frac{m}{m+1} I(m+1, n-1)$

SECTION - V (Numerical Type)

57. If $\int \frac{dx}{(\sec x + \tan x + \operatorname{cosec} x + \cot x)^2} = \frac{x}{a} + \frac{\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)}{b} + \frac{\cos 2x}{c} + d$ then the value of $\frac{|a+b+c|}{3}$ is

58. If $I = \int \frac{dx}{x^3+1}$ then $\int \frac{dx}{(x^3+1)^2} = P.I + \frac{x}{Q(x^3+1)}$ then find product PQ =

59. $\int_0^2 \frac{x\sin^2 \pi x}{x^2-2x+3} dx + \int_1^2 \frac{(x^2-2x+1)}{x^2-2x+3} \sin^2 \pi x dx = \frac{k}{2} =$ ___

SECTION VI - (Matrix match type)

60. In the following [ ] represents G.I.F

Column-I	Column-II
A) $\int_{0}^{\frac{\pi}{2}}\left[\frac{1+2\sin^2 x}{1+\sin^2 x}\right]dx$	P) $\frac{\pi}{4}$
B) $\int_{\frac{\pi}{2}}^{\frac{\pi}{2}}\left[\cos x-\cos^2 x\right]dx$	Q) 0
C) $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\left[\frac{\sec x+\cos ecx-\sec x\cos ecx}{2}\right]dx$	R) π
D) $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\left[\frac{\cot x-\tan x}{2}\right]dx =$ ____	S) $\frac{\pi}{2}$
	T) $\frac{\pi}{3}$

A) A-S,B-Q,C-Q,D-Q
B) A-Q,B-Q,C-Q,D-S
C) A-Q,B-Q,C-Q,D-Q
D) A-S,B-S,C-Q,D-Q