CHAPTER - 03

LAWS OF MOTION & FRICTION

SYNOPSIS

- **Inertia of rest** is the inability of body to change its state of rest by itself, while inertia of motion is the inability of the body to change its state of motion by itself.
- **Inertia of direction** is the inability of the body to change its direction of motion by itself.
- **Newton's first law** of motion gives the definition of force and the concept of inertia.
- The quantity of motion possessed by a body is called momentum. It is measured by the product of mass and velocity.
- **Newton's second law** says that the rate of change of momentum is directly proportional to the applied force.
- Force is an external effort in the form of push or pull which (i) one produces or tries to produce motion in a body at rest, or (ii) stops or tries to stop a moving body, or (iii) changes or tries to change the direction of motion of the body.
- Frictional force, tensional force, normal force, air resistance force, applied force and spring force are contact forces.
- Gravitational force, electrical force, weak force are action-at-a distance forces.
- The force between two static charges is called electrostatic force.
- The force between two magnetic poles is called magnetic force.
- If $\overline{F_1}, \overline{F_2}, \overline{F_3}$ .... are the concurrent forces acting on the same point, then the point will be in equilibrium if $\overline{F_1} + \overline{F_2} + \overline{F_3} + ... = 0$.
- Impulse is the measure of the degree to which an external force produces a change in momentum of the body. The product of a large force acting on a body for a small interval is called impulse.
- Newton's third law of motion says that to every action has equal and opposite reaction. Action and reaction act on different bodies and they are simultaneous.
- Action and reaction never cancel each other.
- The law of conservation of linear momentum is a logical consequence of Newton's second law.
- If n bullets each of mass m and velocity v are fired from a gun then the average force acting on the gun is mnv.
- Principle of conservation of momentum follows from Newton's first law of motion.

- Newton's third law contains law of conservation of momentum.
- In case of a uniform circular motion, the change in momentum with time is not zero, i.e. $\frac{\Delta\vec{p}}{\Delta t} \neq 0$ but $\Delta p = 0$.
- When the lift is at rest or in uniform motion, then the weight recorded by the spring balance is equal to the actual weight of the body.
- When the lift is accelerated up, then the weight recorded by the spring balance is more than the actual weight of the body.
- When the lift is accelerated down, the weight recorded by the spring balance is less than the actual weight of the body.
- For a lift falling freely, acceleration of the lift is *g* and hence the body will feel weightlessness.
- The spring balance will record an increase in weight for a moment, when the lift starts to move up and then with uniform velocity will record the actual weight of the boy.
- If no external force acts on a system of two or more bodies, then the total momentum of the system remains constant. This is known as the principle of conservation of momentum.
- Thrust on the rocket at any instant is equal to the product of the exhaust speed of the burnt gases and the rate of combustion of fuel at that instant.

  i.e. $F = -u\left(\frac{dM}{dt}\right)$

  -ve sign indicates that thrust on the rocket is in a direction opposite to the direction of escaping gas.
- Velocity of rocket at any instant *t*, when mass of the rocket is *M*, is given by $v = v_0 + u\log_e\left(\frac{M_0}{M}\right)$,

  where v_0 is the initial velocity of the rocket. If the initial velocity of the rocket is zero, then $v = u\log_e\left(\frac{M_0}{M}\right)$

  If effect of gravity is also taken into account, then $v_0 = u\log_e\left(\frac{M_0}{M}\right) - gt$.

Friction

- Force of friction opposes relative motion of the point of contact with respect to the surface.
- Force of friction acts parallel to the surface.
- The number of frictional forces acting on a body depends upon the number of contact surfaces. For every contact surface, there is a frictional force.
- Frictional forces are produced on account of intermolecular interactions between the two bodies or surfaces.
- Frictional force is a self adjusting force which increases with increasing applied force till the body is at rest but on the point of motion.
- For frictional force, the relative motion between two bodies or surfaces is not necessary. In fact, contact

between two bodies or surfaces is necessary.

Type of frictional forces

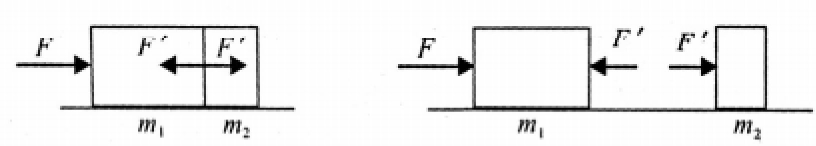
- Static friction: It is a self adjusting force with an upper limit called the limiting friction.
- Dynamic friction/kinetic friction.
- Sliding friction
- Rolling Friction
- Rolling friction < dynamic friction < sliding friction
- If μ_s = coefficient of static friction, R=N=normal reaction, then static frictional force < $\mu_s R$ (body is at rest). Limiting friction= $\mu_s R$ (body is on point of motion).
- If μ_k = coefficient of kinetic friction, then kinetic frictional force = $\mu_k R$ (body is in motion).
- The limiting frictional force between two surfaces in contact with each other depends only on μ (nature of the surfaces) and normal contact force R. It does not depend on their shape, size or surface area.

Tension

- Tension force always pulls a body.
- Tension can never push a body or rope.
- Tension across a massless pulley or frictionless pulley remains constant.
- Rope becomes slack when tension force becomes zero.
- When a rope is pulled by a force, the rope becomes tight or taut. This conveys the concept of tension.

Contact force-Motion of bodies in contact

(a) Two bodies in contact



A force F is applied on mass m_1 horizontally. Masses m_1 and m_2 are in contact. Contact force between m_1 and m_2 = F. Contact force is that force with which one body presses the other at the point of contact when the two bodies are placed on a frictionless surface.

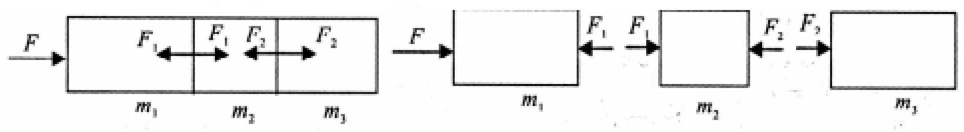
$\therefore$ Acceleration a = $\dfrac{\text{Force applied to system}}{\text{mass of system}}$

$$\therefore a = \frac{F}{(m_1 + m_2)}$$

$\therefore$ Contact force on m_1 and $m_1 a = \dfrac{m_1 F}{(m_1 + m_2)}$

Contact force on m_2=m_2a= $\frac{m_2F}{(m_1+m_2)}$, or $F' = m_2a = \frac{m_2F}{(m_1+m_2)}$

(b) Three bodies in contact



Force applied on the system = F

Mass of the system = m_1+m_2+m_3

Acceleration of the system=a

$\therefore$ Acceleration of each mass(a)= $\frac{F}{(m_1+m_2+m_3)}$

Contact force between m_1 and m_2=F_1

Contact force between m_2 and m_3=F_2

For first body, F–F_1=m_1a

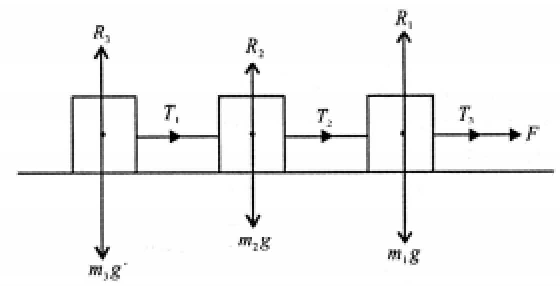
For second body, F_1–F_2=m_2a

For third body, F_2=m_3a

$$\therefore F_1 = F - m_1a = F - \frac{m_1F}{(m_1+m_2+m_3)} = \frac{(m_2+m_3)F}{(m_1+m_2+m_3)}$$

$$\therefore F_2 = m_3a = \frac{m_3F}{(m_1+m_2+m_3)}$$

(c) Motion of bodies connected by strings



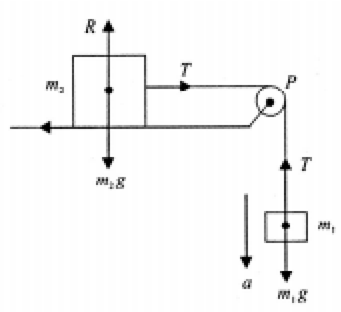
Acceleration in system=a

$\therefore a = \frac{F}{(m_1+m_2+m_3)}$ Tension $T_1 = m_3a = \frac{m_3F}{(m_1+m_2+m_3)}$

Tension $T_2 = (m_2 + m_3)a = \frac{(m_2 + m_3)F}{(m_1 + m_2 + m3)}$

Tension T_3=F

- Consider a body of mass m_2 which rests on a surface which is horizontal. Let a string passing over a pulley connect m_2 with mass m_1 as shown in figure.



a) without friction between m_2 and horizontal table,

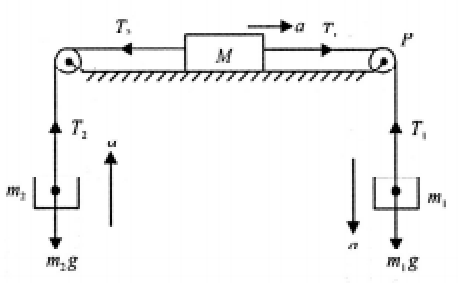
Acceleration, $a = \frac{m_1g}{(m_1 + m_2)}$

Tension, $T = \frac{m_1m_2g}{(m_1 + m_2)}$

b) With friction between body m_2 and table,

Acceleration, $a = \frac{(m_1 - \mu m_2)g}{(m_1 + m_2)}$ Tension, $T = \frac{m_1m_2(1+\mu)g}{(m_1 + m_2)}$

- Two masses are suspended as shown in figure.



The block on horizontal table moves towards right with acceleration a.

$m_1g–T_1=m_1a$ .... (i)

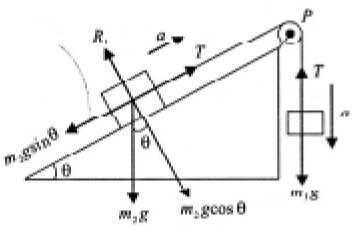
$T_2–m_2g=m_2a$ ....(ii)

$T_1–T_2=Ma$ ....(iii) we get, acceleration, $a = \frac{(m_1 - m_2)g}{(m_1 + m_2 + M)}$

Tension $T_1 = \dfrac{m_1(2m_2 + M)g}{(m_1 + m_2 + M)}$

Tension $T_2 = \dfrac{m_2(2m_1 + M)g}{(m_1 + m_2 + M)}$

- Two masses are suspended over a pulley on an inclined plane as shown in the figure. The mass m_1 descends with an acceleration a. Mass m_2 is on inclined plane.



With friction

Acceleration, a = $\dfrac{[m_1 - m_2(\sin\theta - \mu\cos\theta)]g}{m_1 + m_2}$

Tension T = $m_1 m_2 = \dfrac{(1 - \sin\theta + \mu\cos\theta)g}{m_1 + m_2}$

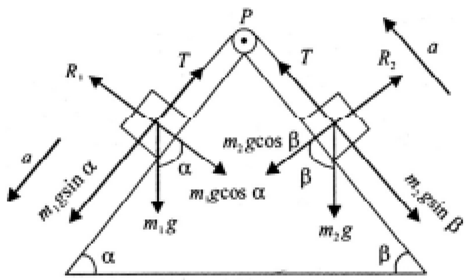
Without friction

Acceleration, $a = \dfrac{(m_1 - m_2\sin\theta)}{(m_1 + m_2)}g; \text{Tension}, T = \dfrac{m_1 m_2(1 + \sin\theta)g}{(m_1 + m_2)}$

- Masses m_1 and m_2 are connected by a string passing over a pulley such that $m_1 > m_2$

Acceleration a = $\dfrac{(m_1\sin\alpha - m_2\sin\beta)g}{m_1 + m_2}$

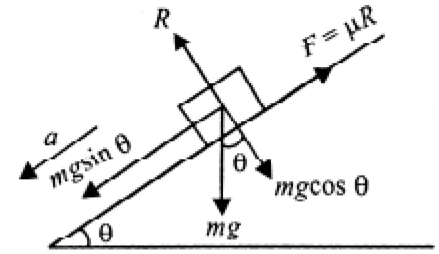
Tension T = $\dfrac{m_1 m_2(\sin\alpha + \sin\beta)g}{m_1 + m_2}$



Motion on an inclined plane

The inclined plane may be rough or smooth. A body may move down the plane or up the plane.

(i) Motion of a body down the rough plane:



Normal reaction $R = mg\cos\theta$

$mg\sin\theta - \mu R = ma$

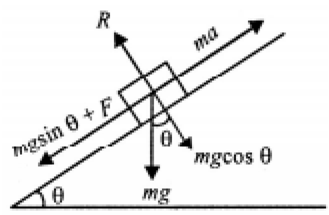
$\therefore mg\sin\theta - \mu mg\cos\theta = ma$ or $a = g(\sin\theta - \mu\cos\theta)$

$\therefore$ Acceleration down the rough plane $= g(\sin\theta - \mu\cos\theta)$

(ii) Motion of a body down the smooth plane : $(\mu = 0)$

$\therefore$ Acceleration of body = $g\sin\theta$.

(iii) Motion of a body up the rough plane :

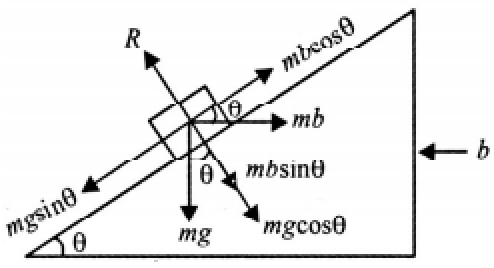


When the body moves up the plane, the force of friction $F = \mu R$ acts down the plane to oppose the motion.

$\therefore ma = mg\sin\theta + F = mg\sin\theta + \mu R$

$= mg\sin\theta + \mu mg\cos\theta = mg(\sin\theta + \mu\cos\theta)$. or acceleration $a = g(\sin\theta + \mu\cos\theta)$

v) Plane is given horizontal acceleration, plane is smooth



$mg\cos\theta - mb\cos\theta = ma \quad \therefore a = (g\sin\theta - b\cos\theta)$

If $a = 0, b = g\tan\theta$.

PART I - (JEEMAIN)

SECTION - I - Straight objective type questions

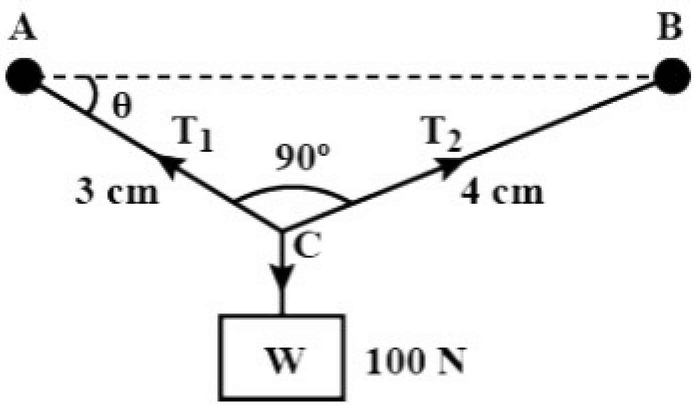
1. A force of 5N gives a mass M_1 an acceleration equal to $8 m/s^2$ and M_2, an accelereation equal to $24 m/s^2$. What is the acceleration if both masses are combined together?

   1) $16\ m/s^2$ 2) $6 m/s^2$ 3) $12\ m/s^2$ 4) $4\ m/s^2$

2. If a bullet of mass 5gm moving with velocity 100m/sec, penetrates the wooden block upto 6cm. Then the average force imposed by the bullet on the block is

   1) 8300N 2) 417 N 3) 830 N 4) zero

3. A and B are two nails on a horizontal line, AC, BC are two uniform strings from which a weight of 100N is suspended. The tension in (N) in AC is



   1) 100 N 2) 80N 3) 60N 4) 50N

4. A light string passing over a smooth light pulley connects two blocks of masses m_1 and m_2 (vertically). If the acceleration of the system is $\frac{g}{8}$ then the ratio of the masses is

1) 8:1 2) 9 : 7 3) 4 : 3 4) 5 : 3

5. A person is standing in an elevator, in which situation he finds his weight less than actual when

1) The elevator moves upward with constant acceleration

2) The elevator moves downward with constant acceleration

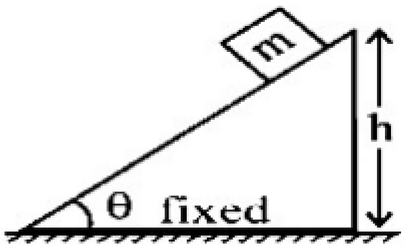
3) The elevator moves upward with uniform velocity

4) The elevator moves downward with uniform velocity

6. A rocket has an initial mass of $20\times10^3\,kg$. If it is to blast off with an initial acceleration of $4ms^{-2}$, the initial thrust needed is ($g = 10ms^{-2}$)

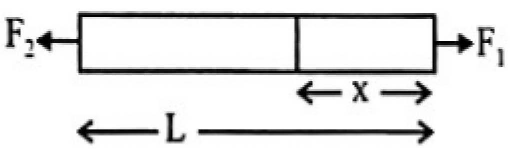
1) $6\times10^4 N$ 2) $28\times10^4 N$ 3) $20\times10^4 N$ 4) $12\times10^4 N$

7. A block is released on smooth inclined plane of inclination θ. The time at which it will reach bottom is



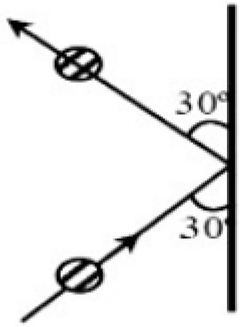
1) $\frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}$ 2) $\frac{2}{\sin\theta}\sqrt{\frac{2h}{g}}$ 3) $\frac{3}{\sin\theta}\sqrt{\frac{2h}{g}}$ 4) $\frac{4}{\sin\theta}\sqrt{\frac{2h}{g}}$

8. The rod of length L and mass M is acted on by two unequal forces F_1 and F_2($F_2 < F_1$) as shown in the following figure. The tension in the rod at a distance x from the end A is given by



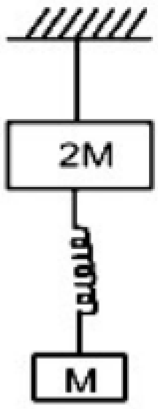
1) $F_1\left[1-\frac{x}{L}\right]+\frac{F_2x}{L}$ 2) $F_2\left[1-\frac{x}{L}\right]+F_1\left[\frac{x}{L}\right]$ 3) $(F_1-F_2)\frac{x}{L}$ 4) None of these

9. A 0.5kg ball moving with speed of 12m/s strikes a hard wall at an angle of 30^0 with the wall, it is reflected with the same speed and at the same angle. If the ball is in contact with the wall for 0.25 seconds, the average force acting on the wall is



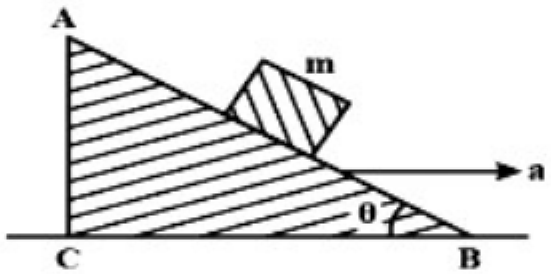
1) 24N　　2) 12 N　　3) 96N　　4) 48 N

10. Two blocks are connected by a spring. The combination is suspended, at rest from a ceiling as shown. The string breaks suddenly. Immediately after the string breaks, what is the initial downward acceleration of the upper block of mass 2M.



1) 0　　2) $\frac{3g}{2}$　　3) g　　4) 2g

11. A block of mass m is placed on a smooth inclined wedge ABC of inclination θ as shown in the figure. The wedge is given an acceleration 'a' towards the right. The relation between a and θ for the block to remain stationary on the wedge is



1) $a = \frac{g}{\operatorname{cosec}\theta}$　　2) $a = \frac{g}{\sin\theta}$　　3) $a = g\tan\theta$　　4) $a = g\cos\theta$

12. A 5000kg rocket is set for vertical firing. The exhaust speed is $800ms^{-1}$. To give an initial upward acceleration of 20 ms^{-2}, the amount of gas ejected per second to supply the needed thrust will be ($g = 10ms^{-2}$)

1) 127-5 kg s^{-1} 2) 187. 5 kg s^{-1} 3) 185.5 kg s^{-1} 4) 137.5 kg s^{-1}

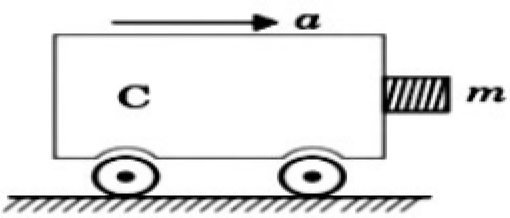
13. A block of mass 5kg is on a rough horizontal surface and is at rest. Now a force of 24N is imparted to it with negligible impulse. If the coefficient of kinetic friction if 0.4 and g = 9.8 m/s^2, then the acceleration of the block is

1) 0.26 m/s^2 2) 0.39 m/s^2 3) 0.69 m/s^2 4) 0.88 m/s^2

14. A vehicle of mass m is moving on a rough horizontal road with momentum P. If the coefficient of friction between the tyres and the road be μ, then the stopping distance is

1) $\frac{P}{2\mu mg}$ 2) $\frac{P^2}{2\mu mg}$ 3) $\frac{P}{2\mu m^2 g}$ 4) $\frac{P^2}{2\mu m^2 g}$

15. A block of mass m is in contact with the cart C as shown in the figure. The coefficient of static friction between the block and the cart is μ. The acceleration α of the cart that will prevent the block from falling satisfies

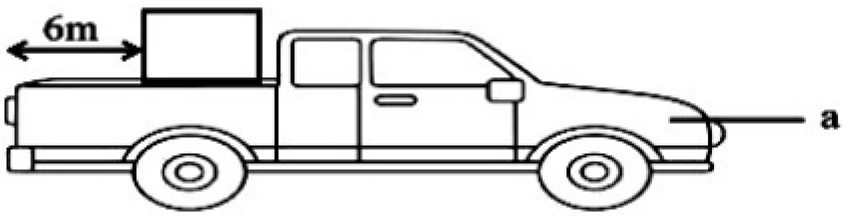


1) $\alpha \geq \frac{g}{\mu}$ 2) $\alpha < \frac{g}{\mu}$ 3) $\alpha > \frac{mg}{\mu}$ 4) $\alpha > \frac{g}{\mu m}$

16. The minimum force required to start pushing a body up along a rough (frictional coefficient μ) inclined plane is F_1 while the minimum force needed to prevent it from sliding dowm is F_2. If the inclined plane makes an angle θ from the horizontal such that $\tan\theta = 2\mu$, then the ratio $\frac{F_1}{F_2}$ is

1) 1 2) 2 3) 3 4) 4

17. A block of mass 2kg is placed on a truck as shown in figure. The coefficient of kinetic friction between the block and surface is 0.5. The truck starts from rest and moves with acceleration $8m/s^2$. After how much time the block fall off the truck.



1) 4s 2) 3s 3) 2s 4) 1.5s s

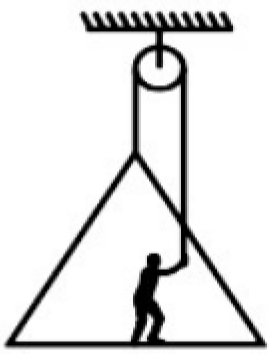
18. A bend in a level road has a radius of 100m. Find the maximum speed which a car turning this bend may have without skidding $(\mu_3 = 0.8)$

1) 25 m/s 2) 26 m/s 3) 28 m/s 4) None

SECTION - II

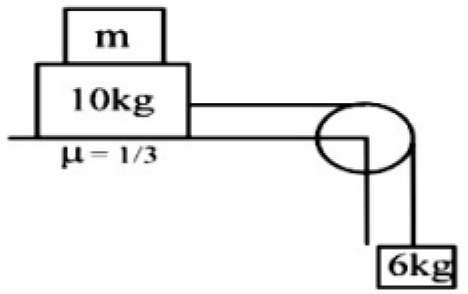
Numerical Type Questions

19. In the figure a painter of mass 100kg standing in a crate of mass 25kg pulls himself up with the crate with an acceleration. If the painter exerts an effective force of 450 N on the floor of the crate. Then the acceleration of the man is. [in m/s^2]



20. The coefficient of static friction between a wooden block of mass 0.5kg and a vertical rough wall is 0.2. The magnitude of horizontal force that should be applied on the block to keep it adhere to the wall will be ....N[$g=10\ ms^{-2}$]

21. Consider the situation shown in the figure. The minimum value of m (in kg) so that block do not move is



Statements Questions

1) Both statement I and statement II are true and statement II is the correct explantion of statement I

2) Both statement I and statement II are true but statement II is not the correct explanation of statement I

3) Statement I is true, Statement II is false

4) Statement I is false and statement II is true

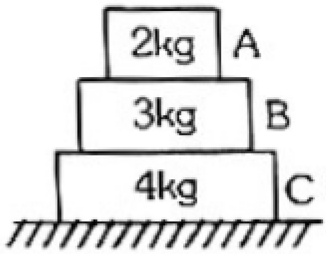
22. Statement I : A cloth covers a table. Some dishes are kept on it. The cloth can be pulled out without disloding the dishes from the table

Statement II : For every action, there is an equal and opposite reaction

PART - II (JEE ADVANCED LEVEL)

SECTION - III (Only one option correct type)

23. Three blocks A, B and Care vertically stagged at rest as shown in the figure. Magnitude of contact force between blocks B and C will be equal to

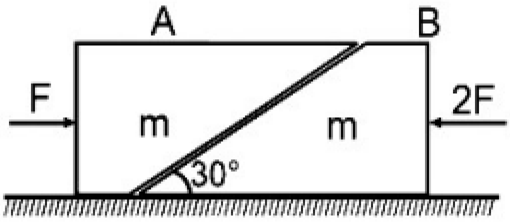


A) 20N B) 70N C) 30N D) 50N

24. A monkey is descending from the branch of a tree with constant acceleration. if the breaking strength of branch is 75% of the weight of the monkey, the minumum acceleration with which monkey can slide down without breaking the branch is

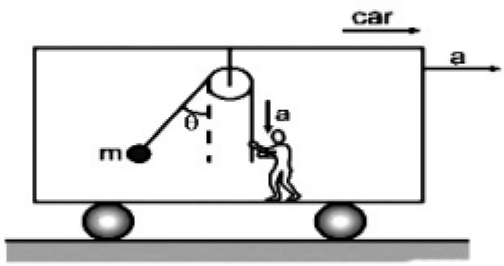
A) g B) 3g/4 C) g/4 D) g/2

25. Two blocks 'A' and 'B' each of mass 'm' are placed on asmooth horizontal surface. Two horizontal forces F and 2F are applied on the two blocks 'A' and 'B' respectively as shown in figure. The block A does not slide on block B. Then the normal reaction acting between the two blocks is:



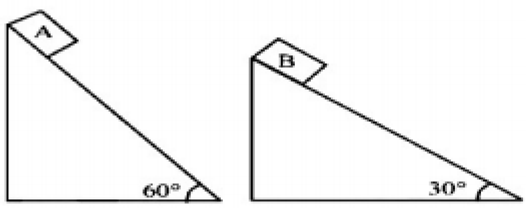
A) F B) F/2 C) $\frac{F}{\sqrt{3}}$ D) 3F

26. A bob is hanging over a pulley inside a car through a string. The second end of the string is in the hand of a person standing in the car. The car is moving with constant acceleration 'a' directed horizontally as shown in figure. Other end of the string is pulled with constant acceleration 'a' (relative to car) vertically. The tension in the string is equal to (assume θ remains constant)



A) $m\sqrt{g^2+a^2}$ B) $m\sqrt{g^2+a^2}-ma$ C) $m\sqrt{g^2+a^2}+ma$ D) $m(g+a)$

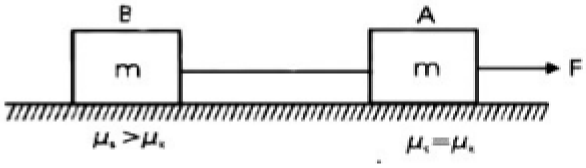
27. Two fixed frictionless inclined planes making an angle 30^0 and 60^0 with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B?

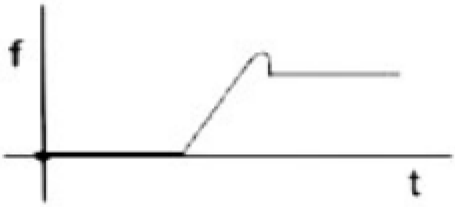


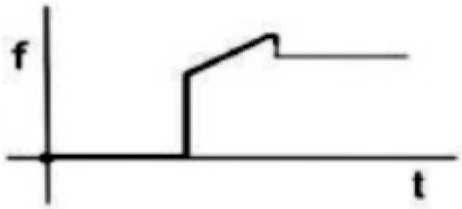
A) 4.9 ms^{-2} B) 9.8 ms^{-2} in vertical direction

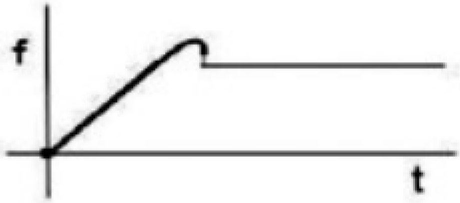
C) zero D) 4.9 ms^{-2} in vertical direction

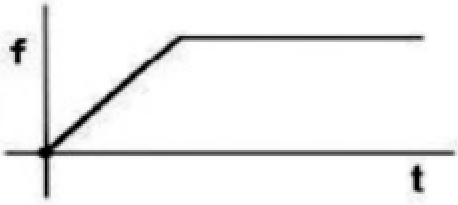
28. A force F= t is applied to block A as shown in figure. The force is applied at t = 0 seconds when the system was at rest and string is just straight without tension. Which of the following graph is gives the friction force between B and horizontal surface as a functin of time 't'.



A) 

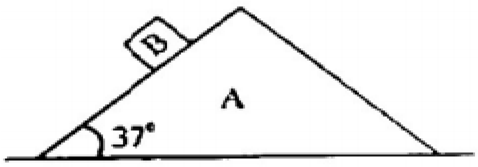
B) 

C) 

D) 

SECTION - IV (More than one correct answer)

29. Block A of mass 1kg is place on a smooth horizontal surface. A smooth block B of mass 1kg starts to move on that inclined plane. Then



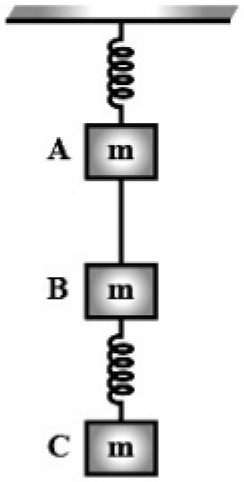
A) acceleration of block A is $\frac{6g}{17}$

B) acceleration of block B is with respect to A is $\frac{15g}{17}$

C) if the acceleration of block B makes an angle θ with horizontal, then $|\tan\theta = 1.5|$

D) acceleration of block A is $\frac{4g}{15}$

30. The system shown in the figure is in equilibrium. All masses are equal to m.



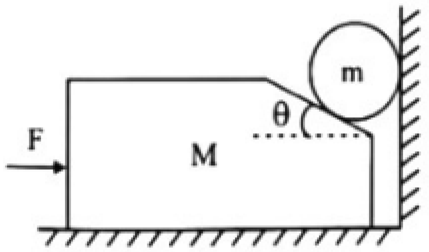
A) When spring between A and celing is cut, $a_A = a_B = 15\,m/s^2\ a_C = 0$ and tension in the string $\frac{mg}{2}$

B) When the string between A and B is cut, then $a_A = 2g$, $a_B = 2g$ and $a_C = 0$

C) When the spring between B and C is cut, $a_C = a_A = g/2, a_C = g$

D) When the spring between B and C is cut, then the tension in the string is $3mg/2$

31. A wedge of mass M loaded with a sphere of mass -m. If a horizontal force F is applied on the wedge it remains in equilibrium, then



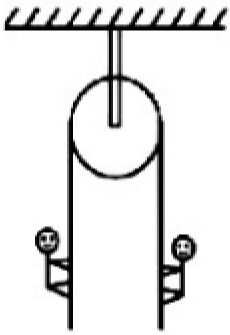
A) Normal reaction between wedge and sphere is $mg\sec\theta$

B) $F = mg\tan\theta$

C) Normal reaction between sphere and vertical wall is $mg\tan\theta$

D) Normal force between ground and wedgge Mg + mg

32. Two men of unequal masses hold on to the two sections of a light rope passing over a smooth light pulley. Which of the following is/are possible



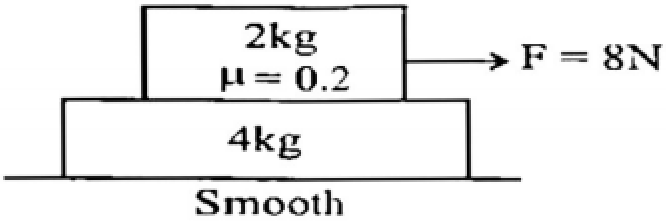
A) The lighter man is stationary while the heavier man slides down with some acceleration

B) The heavier man is stationary while the lighter man climbs up with some acceleration

C) The heavier man is stationary while the lighter man slips down with some acceleration

D) The lighter man is stationary while the heavier man climbs up with some acceleration

33.



A) Acceleration of 2kg with respect to 4kg is $1 m/s^2$

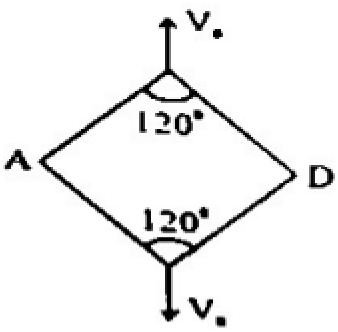
B) Acceleration of 2kg with respect to 4kg after 4 sec is $4 m/s^2$

C) Velocity of 2 kg with respect to 4 kg after 4 sec is 4 m/s

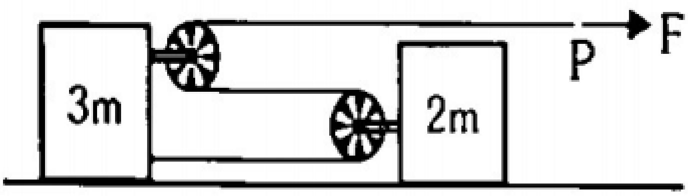
D) Displacement of 2kg with respect to 4kg after 2 sec is 2m

SECTION - V (Numerical Type - Upto two decimal place)

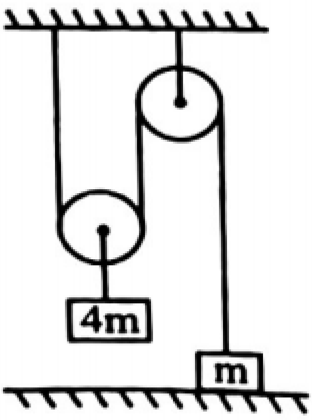
34. The construtcion consists of four light wires each of length l . It is placed on smooth horizontal surface, then the speed of the hinged point D is $\frac{V_0}{\sqrt{n}}$. Then n is



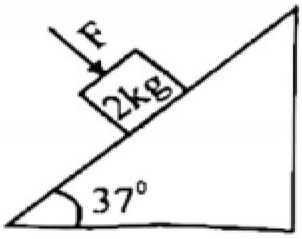
35. In the setup shown, blocks of masses 3m and 2m are placed on frictionless horizontal ground and the free end P of the thread is being pulled by a constant force E. Then the acceleration of the free and P is $\frac{aF}{m}$. Find a



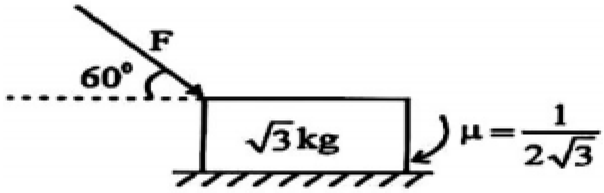
36. In the air arrangement shown in the figure the mass of the body A is 4 times that of body B. The height h = 20 cm. At certain instant the body B is released and the system is set in motion. Maximum height attained by the block is H in cm). Then , $\frac{H}{10}$ is



37. A block of mass 2kg is resting on a rough inclined plane of inclination 30 degree as shown. The coefficient of friction between the block and the plane is $\mu = 0.5$. What minimum force should be applied perpendicular to the plane of the block, so that block doesn't slip on the plane

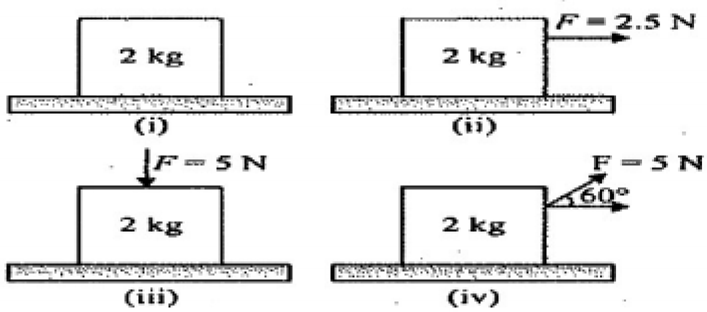


38. What is the maximum value of force F such that the block shown in the arrangement, does not move



SECTION - VI (Matrix Matching)

39. The coefficient of friction between the block and the surface is 0.4 in figure. Match column I with II



Column I	Column II
I. Force ofm friction is zero in	a. Figure i
II. Force of friction is 2.5 N in	b. Figure. Ii
III. Acceleration of the block is zero in	c. Figure iii
IV. Normal force is not equal to 2g in	d. Figure. Iv