CHAPTER - 03

QUADRATIC EQUATIONS

1. 1 $a\alpha^2 + b\alpha + c = 0 \Rightarrow \alpha(a\alpha + b) = -c$

$$\therefore \frac{1}{a\alpha + b} = \frac{-\alpha}{c} \;\&\; \frac{1}{a\beta + b} = \frac{-\beta}{c}$$

$\therefore$ sum of roots = $-\left(\frac{\alpha+\beta}{c}\right) = \frac{b}{ac}$

Product of roots = $\frac{\alpha\beta}{c^2} = \frac{c}{ac^2} = \frac{1}{ac}$

2. 4 $\sec\theta + \tan\theta = \frac{-b}{a}, \sec^2\theta - \tan^2\theta = 1 \Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

3. 1 $1 = \frac{c(a-b)}{a(b-c)} \Rightarrow ab - ac = ac - bc$

$2ac = b(a+c); \quad b = \frac{2ac}{a+c}; \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

4. 3 $\alpha + \beta + \gamma = 0 \Rightarrow \alpha + \beta = -\gamma$

$$\Rightarrow (\alpha+\beta)^{-1} = \frac{-1}{\gamma}$$

$$\therefore (\alpha+\beta)^{-1} + (\beta+\gamma)^{-1} + (\gamma+\alpha)^{-1} = \frac{-1}{\alpha} + \frac{-1}{\beta} + \frac{-1}{\gamma}$$

$$= -\left[\frac{\alpha\beta + \beta\gamma + \alpha\gamma}{\alpha\beta\gamma}\right]$$

$$= \frac{-(+4)}{-1} = 4$$

5. 4 $x^4+\left(2-\sqrt{3}\right)x^2+\left(2+\sqrt{3}\right)=(x-\alpha_1)(x-\alpha_2)(x-\alpha_3)(x-\alpha_4)$

Put x = 1

6. 4 Let $y=\dfrac{x}{x^2-5x+9}\Rightarrow x^2y-5xy-x+9y=0$

$x^2y-(5y+1)x+9y=0$

for real x, $\Delta\geq 0\Rightarrow(5y+1)^2-4y.9y\geq 0$

$(5y+1)^2-36y^2\geq 0\Rightarrow(y-1)(11y+1)\leq 0$

$\Rightarrow y=\left[\dfrac{-1}{11},1\right]$

7. 3 $\alpha^2-6\alpha-2=0\Rightarrow\alpha^{10}-6\alpha^9-2\alpha^8=0$

$\beta^2-6\beta-2=0\Rightarrow\beta^{10}-6\beta^9-2\beta^8=0$

$\therefore\left(\alpha^{10}-\beta^{10}\right)-6\left(\alpha^9-\beta^9\right)-1\left(\alpha^8-\beta^8\right)=0$

$a_{10}-6a_9-2\ a_8=0$

$\therefore a_{10}-2a_8=6a_9$

8. 2 $\left(2+\sqrt{3}\right)^{x^2-2x+1}+\left(2-\sqrt{3}\right)^{x^2-2x-1}=\dfrac{4}{\left(2-\sqrt{3}\right)}\Rightarrow\left(2+\sqrt{3}\right)^{x^2-2x}+\left(2-\sqrt{3}\right)^{x^2-2x}=4$

Put $\left(2+\sqrt{3}\right)^{x^2-2x}=t\Rightarrow t+\dfrac{1}{t}=4$

9. 3 $\alpha=\dfrac{1-i}{1+i}=\dfrac{(1-i)^2}{2}=-i$

10. 3 $\left|\sqrt{x}-2\right|+\sqrt{x}\left(\sqrt{x}-4\right)+2=0,\left|\sqrt{x}-2\right|+\sqrt{x}^2-4\sqrt{x}+2=0$

$\left|\sqrt{x}-2\right|^2+\left|\sqrt{x}-2\right|-2=0$

$\left(\left|\sqrt{x}-2\right|+2\right)\left(\left|\sqrt{x}-2\right|-1\right)=0,\left|\sqrt{x}-2\right|=1$

$\sqrt{x}-2=\pm 1$

$\Rightarrow \sqrt{x}-2=1, \sqrt{x}-2=-1;\ \sqrt{3}=3, \sqrt{x}=1$

$x=9, x=1;\ \therefore \text{sum}=9+1=10$

11. 2 $\alpha+\beta=4\sqrt{2}k$

$\alpha\beta=2.e^{4\log k}-1=2k^4-1$

$\alpha^2+\beta^2=66 \Rightarrow (\alpha+\beta)^2-2\alpha\beta=66$

$\left(4\sqrt{22}\right)^2-2\left(2k^2-1\right)=66$

$k=\pm 2$ but $k \neq -2\ \therefore k=2$

$\alpha^3+\beta^3=280\sqrt{2}$

12. 1 Case I $x<0$

$-x^2-5x-6=0 \ \rightarrow x^2+5x+6=0$

$x=-2,-3$

Case II $x \geq 0$

$x^2-5x-6=0 \Rightarrow (x-6)(x+1)=0$

$x=6, x=-1$

but $x \neq = 1$

Roots are $-2,-3,+6$

Product = 36

13. 4 $p(x)=ax^2+bx+c$

$p(x)=0 \Rightarrow x=\dfrac{-6\pm\sqrt{b^2-4ac}}{2a}$

So $\therefore b=0$ as roots are purely imaginary

So the equation will be $ax^2+c=0$

$p(p(x))=0 \Rightarrow a(p(x))^2+c=0 \Rightarrow p(x)=\pm\sqrt{\dfrac{-c}{a}}$

$x^2=\dfrac{-c}{a}\pm\sqrt{\dfrac{-c}{a}}$

= real $\pm$ imaginary

14. 1

15. 1

16. 3 $ax^2+b\ x+c=0\ \{\alpha,\beta\}$ ....(1)

$$ax^2 - bx(x-1) + c(x-1)^2 = 0$$

$$a\left(\frac{x}{x-1}\right)^2 - b\left(\frac{x}{x-1}\right) + c = 0 \ldots\ldots \Rightarrow a\left(\frac{x}{1-x}\right)^2 + b\left(\frac{x}{1-x}\right) + c = 0 \ldots\ldots(2)$$

from (1) and (2)

$$\alpha = \frac{x}{1-x}, \beta = \frac{x}{1-x}$$

$$x = \frac{\alpha}{\alpha+1} \therefore x = \frac{\beta}{\beta+1}$$

17. 2

18. 4 $\left(5+2\sqrt{6}\right)^{x^2-3} + \left(5-2\sqrt{6}\right)^{x^2-3} = 10$

$$x^2 - 3 = \pm 1$$

19. 4 $\frac{-D}{4a} = 1 \ \ D = -4a$

$$D = b^2 - 4ac = -4 = -4a = 4 \Rightarrow a = 1$$

$$\frac{-b}{2a} = -4 \ \ b = 8$$

$$b^2 - 4ac = -4 \Rightarrow c = 17$$

$$a + b + c = 26$$

20. 3 Using wavy curve method

SECTION II (NUMERICAL)

21. 7 $\Delta = b^2 - 4a$, for real roots $\Delta \geq 0$

a =1, $\Delta = b^2 - 4 \geq 0 \Rightarrow b = 2,3,4$

a = 2 $\Delta = b^2 - 8 \geq 0 \Rightarrow b = 3,4$

a = 3 $\Delta = b^2 - 12 \geq 0 \Rightarrow b = 4$

a = 4 $\Delta = b^2 - 16 \geq 0 \Rightarrow b = 4$

$\therefore$ numberof possible equation are? $2+2+3+4=11$

22. 3 roots are $\alpha, \alpha^5 \Rightarrow \alpha.\alpha^5 = \frac{c}{5} = \alpha^6$

$\therefore \alpha = \left(\frac{c}{a}\right)^{1/6}$; $\therefore ax^2 - 3x + c = 0 \Rightarrow a\left(\frac{c}{a}\right)^{\frac{2}{6}} - 3\left(\frac{c}{a}\right)^{1/6} + c = 0$

$\therefore \frac{ac^{2/6}}{a^{2/6}} + c = 3\left(\frac{c}{a}\right)^{1/6}$; $\div \Rightarrow a^{4/6}c^{2/6} + c = 3\left(\frac{c^{1/6}}{a^{1/6}}\right) \div \frac{c^{1/6}}{a^{1/6}} \Rightarrow a^{5/6}.c^{1/6} + c^{5/6}a^{1/6} = 3$

$\left(a^5c\right)^{1/6} + \left(c^5\ a\right)^{1/6} = 3$

23. 18 Let α, β, γ be the root of $x^3 + px^2 + qx + r = 0$

Here $s_1 = \alpha + \beta + \gamma = 2$

$\Rightarrow p = 2$

Also $s_2 = \alpha^2 + \beta^2 + \gamma^2$

$\Rightarrow 6 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$

$\Rightarrow 6 = 4 - 2q$

$\Rightarrow q = -1$

and $-r = \sum\alpha^3 + p\sum\alpha^2 + q\sum\alpha$

$\Rightarrow -r = 8 + 2.6 + (-1).2$

$\Rightarrow r = -14$

Hence, the equatio is

$x^3 + 2x^2 - x - 14 = 0$

$\Rightarrow x^3 = -2x^2 + x + 14$

$\Rightarrow x^4 = -2x^3 + x^2 + 14x$

$\Rightarrow \sum\alpha^4 = -2\sum\alpha^3 + \sum\alpha^2 + 14\sum x$

$\Rightarrow \sum\alpha^4 = -2(8) + (6) + 14(2) \Rightarrow \sum\alpha^4 = 34 - 16 = 18$

Hence the value of $\left(\alpha^4 + \beta^4 + \gamma^4\right)$ is 18

24. 1 We have $\alpha + \beta = -p, \alpha\beta = -q$ and $\gamma + \delta = -p, \gamma\delta = r$

Now $(\alpha - \gamma)(\alpha - \delta) = \alpha^2 - (\gamma + \delta)\alpha + \gamma\delta$; $= \alpha^2 + p\alpha + r\ = q + r$

Also,

$$(\beta-\gamma)(\beta-\delta)=\beta^2-(\gamma+\delta)\beta+\gamma\delta=\beta^2+p\beta+r=(q+r)$$

Hence, the value of; $\dfrac{(\alpha-\gamma)(\alpha-\delta)}{(\beta-\gamma)(\beta-\delta)}=\dfrac{(q+r)}{(q+r)}=1$

25. 6 $\left(|x-3|+2\right)\left(|x-3|=1\right)=0, |x-3|=1, x-3=\pm1, x=4,2$

JEE ADVANCED LEVEL

SECTION III

26. C $x^2+\left(\dfrac{x}{x-1}\right)^2=8$; $\left(x+\dfrac{x}{x-1}\right)^2-2x\left(\dfrac{x}{x-1}\right)=8\Rightarrow\left(\dfrac{x^2}{x-1}\right)^2-2\left(\dfrac{x}{x-1}\right)-8=0$

$\dfrac{x^2}{x-1}=t\Rightarrow t^2-2t-8=0\Rightarrow t=4, t=-2$

$\dfrac{x^2}{x-1}=4\Rightarrow x^2-4x+4=0\Rightarrow x=2$

Put $\dfrac{x^2}{x-1}=-2\Rightarrow x^2+2x=2\Rightarrow(x+1)^2=3$

27. B If $x\geq a$ then given equation reduced to $x^2-4x-2x+3a+2=0$

$x^2-6x+3a+2=0............\rightarrow(1)$

If $x<a\Rightarrow x^2-4x+2x-a+2=0$

$\Rightarrow x^2-2x+2-a=0............\rightarrow(2)$

Given equation has two roots only if (1) has 2 real roots and (2) has imagining roots or viceversa, therefore (discriminant of(1) X(discriminant of (2)) <0

$(36-4(3a+2))X(4-4(2-a))<0$; $(7-3a)(a-1)<0\Rightarrow(a-1)(a-7/3)>0\Rightarrow a<1, a>7/3$

28. Let $y = \dfrac{ax^2 - 7x + 5}{5x^2 - 7x + a}$

$\Rightarrow \quad 5x^2y - 7xy + ay = ax^2 - 7x + 5$

$\Rightarrow \quad x^2(5y - a) - 7x(y - 1) + ay - 5 = 0$

$\because \quad x$ is real $\quad \therefore \quad D \geq 0$

$\Rightarrow \quad 49(y-1)^2 - 4(5y - a)(ay - 5) \geq 0$

$\Rightarrow \quad 49(y^2 - 2y + 1) - 4(5ay^2 - 25y - a^2y + 5a) \geq 0$

$\Rightarrow \quad y^2(49 - 20a) + 2y(1 + 2a^2) + 49 - 20a \geq 0$

Which is true for all $y \in R$

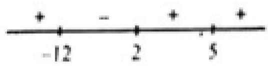
$\therefore \quad D \leq 0$ & leady coefficient $\quad 49 - 2a > 0$

$4(1 + 2a^2)^2 - 4(49 - 20a)^2 \leq 0$

$(1 + 2a^2 + 41 - 20a)(1 + 2a^2 - 49 + 20a) \leq 0$

$(a^2 - 10a + 25)(a^2 + 10a - 24) \leq 0$

$(a - 5)^2(a + 12)(a - 2) \leq 0$



$a \in [-12, 2] \cup \{5\}$

but $\quad a < \dfrac{49}{2}$

$\therefore \quad a \in [-12, 2]$

Now when $a = -12$

$$y = \frac{-12x^2 - 7x + 5}{5x^2 - 7x - 12} = \frac{-(12x^2 + 7x - 5)}{5x^2 - 7x - 12} = -\frac{(12x-5)(x+1)}{(5x-12)(x+1)}$$

Here $(x + 1)$ is a common factor in numerator and denominator

$\therefore \quad y$ does not take all real numbers.

Similarly for $a = 2$ numerator and denominator contains a common linear factor and again y does not take all real numbers.

Hence, $a \in (-12, 2)$

29. B $\quad \alpha^2 = \alpha + 1$

$\beta^2 = \beta + 1$

$a_n = p\alpha^n + q\beta^n$

$= p\left(\alpha^{n-1} + \alpha^{n-2}\right) + q\left(\beta^{n-1} + \beta^{n-2}\right)$

$= a_{n-1} + a_{n-2}$

$\therefore a_{12} = a_{11} + a_{10}$

30. D

$$\alpha=\frac{1+\sqrt{5}}{2},\beta=\frac{1-\sqrt{5}}{2}$$

$$a_4=a_3+a_2$$
$$=2a_2+a_1$$
$$=3a_1+2a_0$$
$$28=p(3\alpha+2)+q(3\beta+2)$$
$$28=(p+q)\left(\frac{3}{2}+2\right)+(p-q)\left(\frac{3\sqrt{5}}{2}\right)$$
$$\therefore p-q=0 \text{ and } (p+q)\times\frac{7}{2}=28$$
$$\Rightarrow p+q=8\Rightarrow p=q=4$$
$$\therefore p+2q=12$$

SECTION IV (More than one correct)

31. A,D

Given, x_1 and x_2 are roots of $\alpha x^2-x+\alpha=0$.

$$\therefore\ x_1+x_2=\frac{1}{\alpha} \text{ and } x_1x_2=1$$

Also, $|x_1-x_2|<1$

$$\Rightarrow |x_1-x_2|^2<1 \Rightarrow (x_1-x_2)^2<1$$
or $(x_1+x_2)^2-4x_1x_2<1$
$$\Rightarrow \frac{1}{\alpha^2}-4<1 \text{ or } \frac{1}{\alpha^2}<5$$
or $5\alpha^2-1>0$ or $(\sqrt{5}\alpha-1)(\sqrt{5}\alpha+1)>0$

$$\therefore\ \alpha\in\left(-\infty,-\frac{1}{\sqrt{5}}\right)\cup\left(\frac{1}{\sqrt{5}},\infty\right) \quad ...(i)$$

Also, $D>0$

$$\Rightarrow\ 1-4\alpha^2>0 \text{ or } \alpha\in\left(-\frac{1}{2},\frac{1}{2}\right) \quad ...(ii)$$

From Eqs. (i) and (ii), we get

$$\alpha\in\left(-\frac{1}{2},\frac{-1}{\sqrt{5}}\right)\cup\left(\frac{1}{\sqrt{5}},\frac{1}{2}\right)$$

32. A,B

$1 \le x^2 + x + 2$

$x^2 + x + 1 \ge 1$

$x^2 + x \ge 0$

$x \le -1, x \ge 0 \quad ...(1)$

$x^2 + x + 1 < 2$

$x^2 + x - 1 < 0$

$\frac{-1-\sqrt{5}}{2} < x < \frac{\sqrt{5}-1}{2}$

$\therefore x \in \left(\frac{-1-\sqrt{5}}{2}, \frac{\sqrt{5}-1}{2}\right) ...(2)$

from (1) and (2)

$\frac{-1-\sqrt{5}}{2} < x \le -1, 0 \le x < \frac{\sqrt{5}-1}{2}$

33. B,D

$x^2 + bx - 1 = 0$

$x^2 + x + b = 0$

$\overline{}$

$(b-1)x = 1 + b$

$\text{Common Root} = \frac{1+b}{b-1}$

$\Rightarrow b^2 + 2b + 1 = b^2 - b^3 + 1 - b$

$3b = -b^3$

$b = \pm i\sqrt{3}$

34. A,C

α, β Roots of $x^2 + ax + bc = 0$

$\alpha + \beta = -a, \alpha\beta = bc$

β, γ roots of $x^2 + bx + ca = 0$

$\beta + \gamma = -b, \beta\gamma = ca$

γ, α roots of $x^2 + cx + ab = 0$

$+a = -c, \gamma\alpha = ab$

$\Rightarrow \alpha + \beta + \gamma = -\frac{(a+b+c)}{2}$

$\Rightarrow \alpha^2 + \beta^2 + \gamma^2 = a^2b^2c^2$

$\Rightarrow \alpha\beta\gamma = abc$

35. B,C,D

$f(x) = ax^2 + 2bx + 4c - 16$

$f(-2) = 4a - 4b + 4c - 16 > 0$

$f(0) > 0 \Rightarrow c > 4$

36. B,C Let $\log_{10} x + 2 = a$

And $\log_{10} x - 1 = b$

$\therefore a + b = 2\log_{10} x + 1$

$\therefore$ given $a^3 + b^3 = (a+b)^3$

$\therefore 3ab(a+b) = 0]$

$(\log_{10} x + 2).(\log_{10} x - 1)(2\log_{10} x + 1) = 0$

$\therefore x = \frac{1}{100}$ or $x = 10$ or $x = \frac{1}{\sqrt{10}}$

SECTION V - (Numerical type)

37. C,D $\alpha + \alpha^2 = \frac{-p}{3}, \alpha^3 = 1 \therefore \alpha = 1$

$\alpha = \frac{-1 + i\sqrt{3}}{2}$, or $\alpha = \frac{-1 - i\sqrt{3}}{2}$

(i) If $\alpha = 1$, then $p = -6$ so that the equation is $3x^2 - 6x + 3 = 0$ roots 1,1

(ii) If $\alpha = w$ or w^2, then $P = -3(\alpha + \alpha^2) = -3(w + w^2) = 3$ and hence $P = 3$. So that the equation is $3x^2 + 3x + 3 = 0$, where roots are w, w^2

38. 5 $\lambda = 5 + \cfrac{1}{4 + \cfrac{1}{5 + \ldots\ldots\infty}}$, $\lambda = 5 + \cfrac{1}{4 + \cfrac{1}{\lambda}} \Rightarrow \lambda = 5 + \frac{\lambda}{4\lambda + 1}$, $4\lambda^2 - 20\lambda - 5 = 0$

$\lambda = \frac{20 \pm \sqrt{480}}{8} = \frac{20 \pm 4\sqrt{30}}{8} = \frac{5 \pm \sqrt{30}}{2} = \frac{5 + \sqrt{30}}{2}$

$2\lambda - \sqrt{30} = 5$

39. 40.48 $\lambda x^2 + (1-\lambda)x + 5 = 0, \alpha + \beta = \frac{\lambda - 1}{\lambda}$ $\alpha\beta = \frac{5}{\lambda}$

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{4}{5}, \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{4}{5}, \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{4}{5}$

$\cfrac{\frac{(\lambda-1)^2}{\lambda^2} - \frac{10}{\lambda}}{\frac{5}{\lambda}} = \frac{4}{5}$ $\frac{(\lambda^2 - 2\lambda + 1 - 10\lambda)}{\lambda^2}\frac{\lambda}{5} = \frac{4}{5}$

$\lambda^2 - 12\lambda + 1 = 4\lambda$, $\lambda^2 - 16\lambda + 1 = 0$ $\lambda_1 + \lambda_2 = 16$

$$\frac{\lambda_1}{\lambda_2^2}+\frac{\lambda_2}{\lambda_1^2}=\frac{\lambda_1^3+\lambda_2^3}{\lambda_1^2\lambda_2^2}=\frac{(\lambda_1+\lambda_2)^3-3\lambda_1\lambda_2(\lambda_1+\lambda_2)}{(\lambda_1\lambda^2)^2}=16^3-48$$

$$=4096-48=4048$$

SECTION VI - (Matrix match type)

40. A $\quad A\to q, B\to p, C\to s, D\to r$

$f(x)=x^2-2px+p^2-1$

(A) Both roots of $f(x)=0$ are less then 4

$\therefore af(4)>0$ & $\frac{-b}{2a}<4$

$\therefore 1\times(16-8p+p^2-1)>0$ & $\frac{2p}{p}<4$

$\Rightarrow (p-3)$ or $(p-5)>0$ & $P<4$ ...(i)

$P<3$ or $p>5$ ...(ii)

From (i) and (ii) $p\in(-\infty,3)$

(B) Both roots are greater then -2

$\therefore af(-2)>0$ & $\frac{-b}{2a}>-2$

$\Rightarrow 1(4+4.p+p^2-1)>0, \frac{2p}{2a}>-2\Rightarrow p>-2$

$\therefore (p+1)(p+3)>0, p>-2$

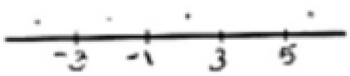
$p<-3$ or $p>-1$ & $p>-2$

$\therefore p\in(-1,\infty)$

(C) Exactly one root lies between $(-2,4)$

$\Rightarrow f(-2)f(4)<0\Rightarrow(4+4p+p^2-1)(16-8p+p^2-1)<0$

$\Rightarrow (p+1)(p+3)(p-3)(p-5)<0$



$\therefore p\in(-3,-1)\cup(3,5)$

(D) 1 lies between the root

$\therefore af(1)<0$

$\Rightarrow 1(1-2p+p^2-1)<0\Rightarrow p(p-2)<0$

$\Rightarrow P\in(0,2)$