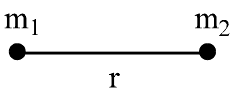
GRAVITATION

NEWTON'S LAW OF GRAVITATION

In this universe every object attracts every other objects and this force is directly proportional to product of their massed and inversely proportional to square of the distance between them.



$F \propto m_1 m_2$ $\qquad$ $F \propto \frac{1}{r^2}$

$F \propto \frac{m_1 m_2}{r^2}$ $\qquad$ $\boxed{F = \frac{Gm_1 m_2}{r^2}}$

G $\rightarrow$ Universal gravitational constant

$\boxed{G = 6.67 \times 10^{-11}\ Nm^2 / kg^1}$

Dimension $\rightarrow$ $M^{-1} L^3 T^{-2}$

Properties of Gravitational force

$\rightarrow$ Always attractive in nature.

$\rightarrow$ It is independent of the medium between two masses

$\rightarrow$ Always conservative in nature

$\rightarrow$ Gravitational force between two objects is not affected by the presence or absence of other masses.

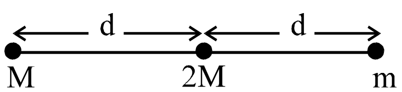
$\rightarrow$ Gravitational force on a mass due to number of other masses are the vector sum of individual forces.

Note :

The above equation for gravitational force is applicable only for point masses. It is not applicable for extended objects.

In the case of spherical bodies we can apply the above equation and we assume that whole mass of the body is located at their centre and 'r' is the distance between their centres.

Example :



Find the net force acting on 'm' ?

Solution :

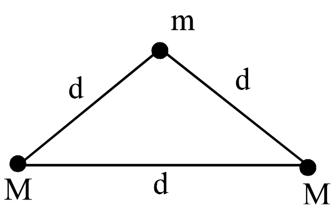
$$F_1 = \frac{GMm}{(2d)^2} = \frac{GMm}{4d^2}\ (\text{left})$$

$$F_2 = \frac{G(2M)m}{d^2} = \frac{2GMm}{d^2}\ (\text{left})$$

Net force acting is $F_{net} = F_1 + F_2$

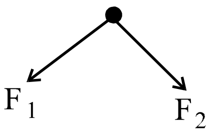
$$= \frac{9GMm}{4d^2}$$

Example :

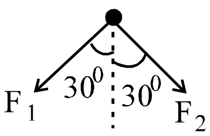


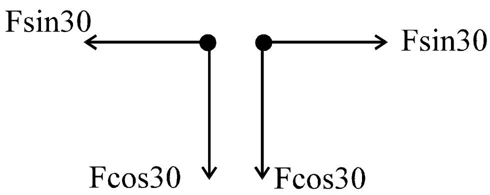
Find the net force acting on 'm' ?

Solution :



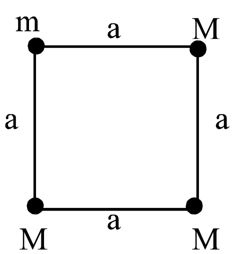
$F_1 = \frac{GMm}{d^2} = F$; $\qquad F_2 = \frac{GMm}{d^2} = F$





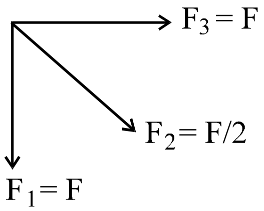
Net f = $F\cos 30 + F\cos 30 = \sqrt{3}F \ = \sqrt{3}\frac{GMm}{d^2}$

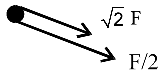
Example :



Find net force on 'm' ?

$F_1 = \frac{GMm}{a^2}$ $F_2 = \frac{GMm}{2a^2}$ $F_3 = \frac{GMm}{a^2}$

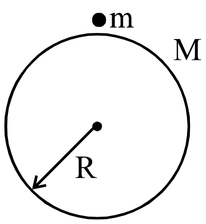




Net force = $F/2 + \sqrt{2}F$

$$F_{net} = \left(1/2 + \sqrt{2}\right)\frac{GMm}{a^2}$$

Acceleration due to Gravity



Radius of Earth = R   Mass of earth = M

Force acting on m is given by

$$F = \frac{GMm}{R^2}$$

$$ma = GMm/R^2$$

a $\rightarrow$ acceleration due to gravity (g)

$$\boxed{g = GM/R^2}$$

This is the acceleration due to gravity at the surface of the earth.

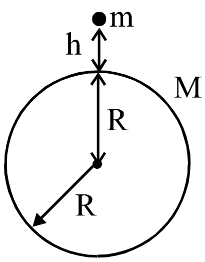
$$M = \rho \times 4/3\,\pi R^3$$

then $g = G/R^2 \times \rho \times 4/3\,\pi R^3$

$$\boxed{g = 4/3\,\pi\,\rho GR}$$

g = 9.8 m/s^2

Variation of g with height



Force acting on m is given by

$$F = \frac{GMm}{(R+h)^2}$$

$$m \times a = \frac{GMm}{(R+h)^2}$$

$$a \to g'$$

$$\boxed{g' = \frac{GM}{(R+h)^2}}$$

$$g' = \frac{GM}{R^2\left(1+{h}/{R}\right)^2}$$

$$\boxed{g' = \frac{g}{\left(1+{h}/{R}\right)^2}}$$

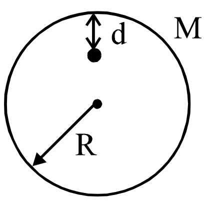
if h << R

$$g' = g\left(1+{h}/{R}\right)^{-2} \approx g\left(1-{2h}/{R}\right)$$

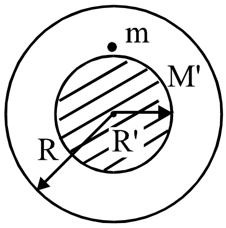
$\therefore$ If h < < R

$$g' = g\left(1-{2h}/{R}\right)$$

Variation of 'g' with depth



We need to find 'g' at a depth d from the surface of the earth.



$$M' = \rho \times \frac{4}{3}\pi (R')^3 \qquad \rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3}$$

$$M' = \frac{M}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi (R')^3 \qquad M' = M\frac{(R')^3}{R^3} \qquad R' = R - d$$

force between them is given by

$$F = \frac{GM'm}{(R')^2} = \frac{G}{(R')^2} \times \frac{M(R')^3}{R^3} \times m$$

$$mg' = \frac{GM(R')}{R^3} m$$

$$g' = \frac{GM}{R^3}(R-d) \qquad g' = \frac{GM}{R^2}\left(\frac{R-d}{R}\right)$$

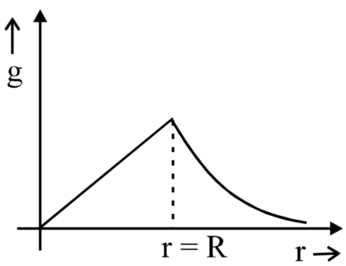
$$\boxed{g' = g\left(1 - \frac{d}{R}\right)}$$

at the centre of the earth d = R

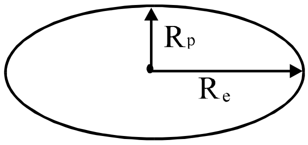
$$g' = g\left(1 - \frac{R}{R}\right) = g(1-1) = 0$$

Graph between g and r

r $\rightarrow$ distance from centre of Earth



Variation of g due to shape of Earth



$g = GM/R^2$ from figure $R_e > R_p$

$g \propto 1/R^2$ $\therefore g_{equator} < g_{poles}$

When we are moving from equator to pole weight of the body will increase.

Gravitational Field Intensity (I)

Gravitational field intensity at a point is defined as the force acting on a unit mass placed at that point.

Force acting on m_0 mass = F

then force acting on unit mass = F/m_0

$$\boxed{I = F/m_0}$$

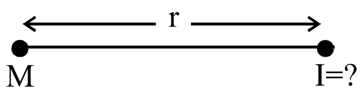
Unit : N/kg

It is a vector quantity.

Note :

Gravitational field intensity at a point due to number of charges is the vector sum of individual gravitational field.

Gravitational field Intensity due to point mass



Force acting on m_0 placed at that point

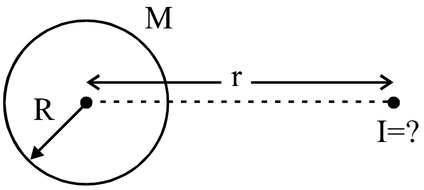
$$F = \frac{GMm_0}{r^2}$$

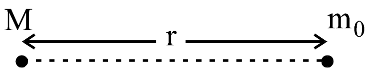
$$F/m_0 = \frac{GM}{r^2}$$

$\boxed{I = \frac{GM}{r^2}}$ direction towards M.

Gravitational field intensity due to uniform solid sphere

i) Outside





$$F = \frac{GMm_0}{r^2}$$

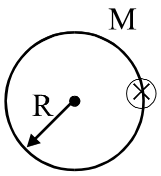
$$F/m_0 = \frac{GM}{r^2}$$

$$\boxed{I = \frac{GM}{r^2}}$$

Direction towards the centre of the sphere.

ii) at the surface

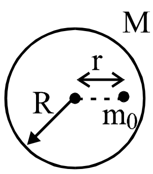
at surface r = R



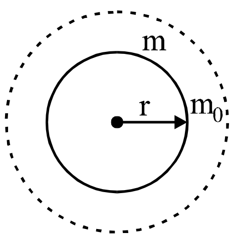
$$\boxed{I = \frac{GM}{R^2}}$$

Direction towards centre of the sphere.

ii) Inside the sphere



$$\boxed{\rho = \frac{M}{\frac{4}{3}\pi R^3}}$$



$m = \rho \times \frac{4}{3}\pi r^3$

force between them is

$m = \dfrac{M}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3$

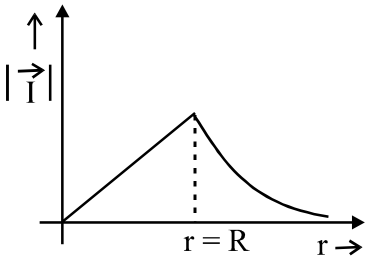
$F = \dfrac{Gmm_0}{r^2}$

$m = M\left(\dfrac{r}{R}\right)^3$

$F/m_0 = \dfrac{Gm}{r^2}$

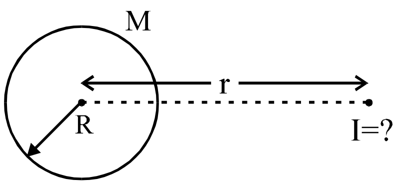
$I = \dfrac{G}{r^2} M \dfrac{r^3}{R^3}$

$\boxed{I = \dfrac{GMr}{R^3}}$ Direction towards the centre of the sphere.



Gravitational field intensity due to spherical shell

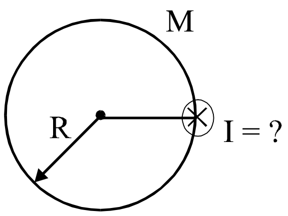
i) Outside point



$\boxed{I = \dfrac{GM}{r^2}}$

Direction towards centre of sphere

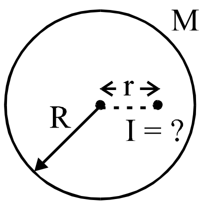
ii) At the surface



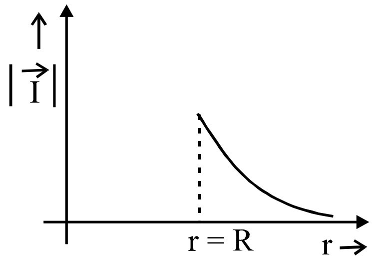
$\boxed{I = \dfrac{GM}{R^2}}$

Direction towards centre of the sphere.

ii) Point inside the sphere



$\boxed{I = 0}$



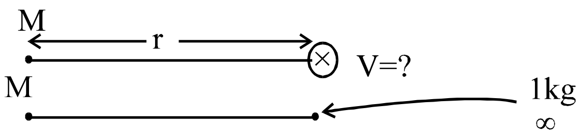
Gravitational Potential

Gravitational potential at a point is defined as the work done to move a unit mass from infinity to that point without any acceleration.

It is a scalar quantity.

So gravitational potential at a point due to number of other charges is algebraic sum of individual potential.

Gravitational potential due to point mass



When 1kg at a distance x from M force acting on it is given by

$$F = \frac{GM(1)}{x^2} = \frac{GM}{x^2}$$

Work done to move it through a distance dx is given by

$$dW = Fdx = \frac{GM}{x^2}dx$$

Total work done $= \int dW$

$$= \int_{\infty}^{r} \frac{GM}{x^2}dx$$

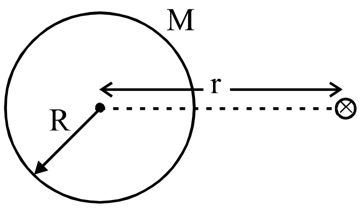
$$= GM\left(\frac{-1}{x}\right)_{\infty}^{r}$$

$$= \frac{-GM}{r}$$

This work done is gravitational potential $V = -\frac{GM}{r}$

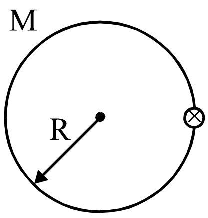
Gravitational Potential due to uniform solid sphere

i) Outside point



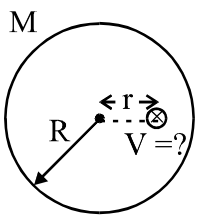
$$\boxed{V = -\frac{GM}{r}}$$

ii) At Surface



$$\boxed{V = -\frac{GM}{R}}$$

iii) Inside point



$$\boxed{V = -\frac{GM}{2R^3}\left(3R^2 - r^2\right)}$$

Note :

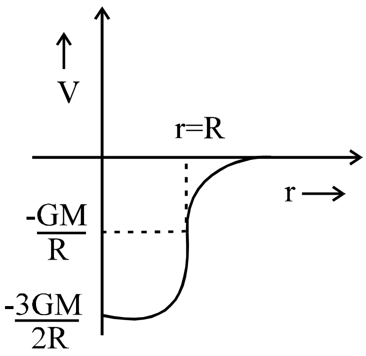
At the centre of the sphere r = 0

$$V_{centre} = -\frac{GM}{2R^3}\left(3R^2 - 0\right)$$

$$\boxed{V_{centre} = -\frac{3GM}{2R^3}}$$

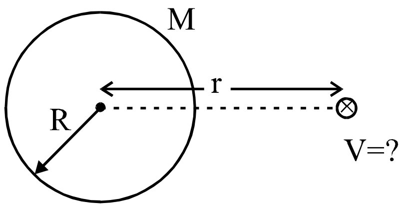
$$V_{centre} = 3/2 \times -\frac{GM}{R}$$

$$\boxed{V_{centre} = 3/2\, V_{surface}}$$



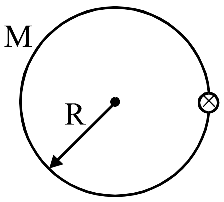
Gravitational potential due to spherical shell

i) **Outside point**



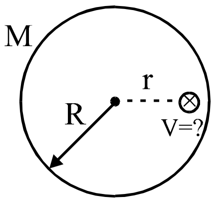
$$\boxed{V = -\frac{GM}{r}}$$

ii) **At Surface**



$$\boxed{V = -\frac{GM}{R}}$$

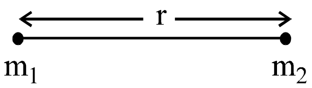
iii) **Inside point**



$$\boxed{V = -\frac{GM}{R}}$$

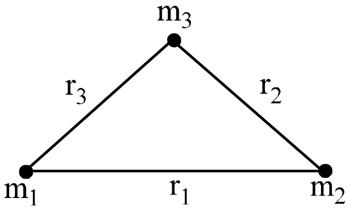
Potential inside the spherical shell is same as that at the surface.

Gravitational potential energy of system of two particles



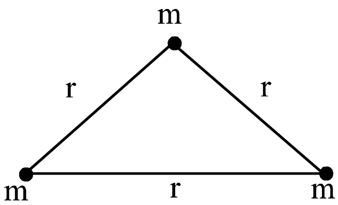
$$\boxed{U = \frac{-Gm_1m_2}{r}}$$

Gravitational potential energy of system of three particle



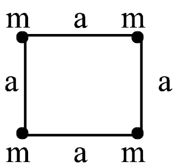
$$\boxed{U = -G\left(\frac{m_1m_2}{r_1} + \frac{m_2m_3}{r_2} + \frac{m_1m_3}{r_3}\right)}$$

Note :

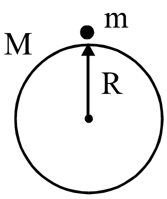


$$U = \frac{-Gm^2}{r} \times 3$$

$$\boxed{U = \frac{-3Gm^2}{r}}$$

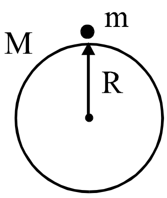


$$\boxed{U = \frac{-Gm^2}{a} \times 4 + \frac{-Gm^2}{\sqrt{2}a} \times 2}$$

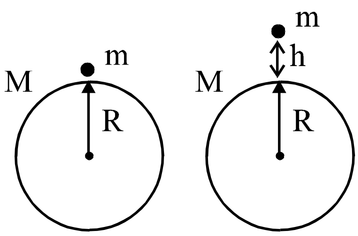


$$\boxed{U = \frac{-GMm}{R}}$$

Example :



Find out the work done to move 'm' to a height h from the surface of the earth.



$$U_i = \frac{-GMm}{R} \qquad U_f = \frac{-GMm}{R+h}$$

$$W = U_f - U_i$$

$$= \frac{-GMm}{R+h} - \frac{-GMm}{R}$$

$$= \frac{GMm}{R} - \frac{GMm}{R+h}$$

$$= GMm\left[\frac{R+h-R}{R(R+h)}\right]$$

$$= \frac{GMmh}{R\ R\left(1+h/R\right)} \qquad = \frac{GM}{R^2}\frac{mh}{1+h/R}$$

$$\boxed{W = \frac{mgh}{\left(1+h/R\right)}}$$

Note :

if h <<R

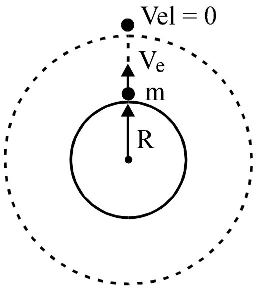
$$h/R \approx 0$$

$$\boxed{W = mgh}$$

Escape Speed

It is the minimum speed of projection required to escape from the gravitational field of Earth.

Escape speed from the surface of Earth



When the body is projected with velocity equal to escape velocity, then when it comes out of gravitational field then its speed will be zero.

TE = Const

$TE_1 = TE_2$

$KE_1 + PE_1 = KE_2 + PE_2$

$$\frac{1}{2}mV_e^2 + \frac{-GMm}{R} = \frac{1}{2}m(0)^2 + \frac{-GMm}{\infty}$$

$$\frac{1}{2}mV_e^2 = \frac{GMm}{R}$$

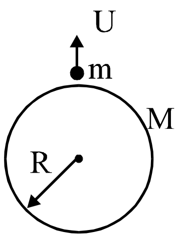
$$V_e^2 = \frac{2GM}{R}$$

$$\boxed{V_e = \sqrt{\frac{2GM}{R}}}$$

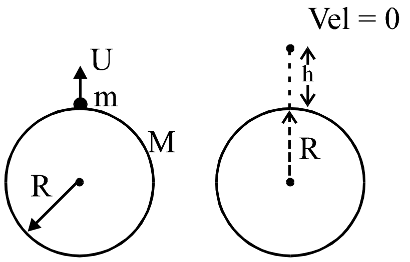
$$V_e\sqrt{\frac{2GM}{R} \times \frac{R}{R}} = \sqrt{2\frac{GM}{R^2}R}$$

$$\boxed{V_e = \sqrt{2gR}}$$

Example :



In this case $U < V_e$ find the maximum height reached by the body.



$TE_1 = TE_2$

$KE_1 + PE_1 = KE_2 + PE_2$

$$\frac{1}{2}mU^2 + \frac{-GMm}{R} = 0 + \frac{-GMm}{R+h}$$

$$\frac{1}{2}mU^2 = \frac{GMm}{R} - \frac{GMm}{R+h}$$

$$\frac{U^2}{2} = GM\left(\frac{R+h-R}{R(R+h)}\right)$$

$$\boxed{\frac{U^2}{2} = \frac{GMh}{R(R+h)}}$$

Solving h can be calculated

if h<<R $\frac{U^2}{2} = \frac{GMh}{R(R)}$

$$\frac{U^2}{2} = gh$$

$$\boxed{h = \frac{U^2}{2g}}$$

Note :

For atmosphere

$$\boxed{\text{rms velocity of gas} < \text{Escape speed}}$$

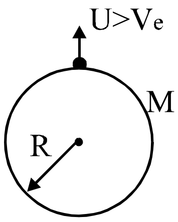
For black holes

$$\boxed{\text{Escape speed} \geq \text{Speed of light}}$$

Escape speed is independent of mass of the body and direction of projection.

Note :

When a body projected from earth with speed greater than escape speed, then velocity of body when it comes out of gravitational field is v' then v' is given by



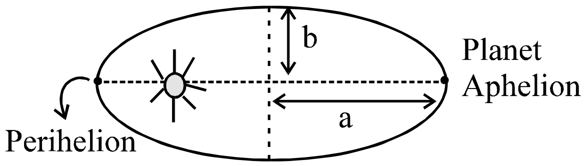
$$\boxed{v' = \sqrt{u^2 - v_e^2}}$$

$$\boxed{\text{Work done by gravity } w = u_i - u_f}$$

KEPLER' S LAW

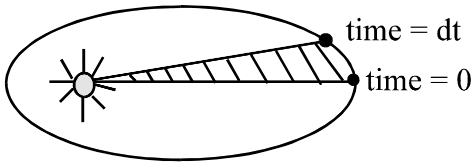
1st law (Law of orbits)

Every planets are revolving around sun in an elliptical orbit.



At Aphelion velocity of body is minimum, Perihelion velocity of body is maximum

2nd law (Law of area)



Line joining the centre of planet and Sun sweeps equal amount of area in equal intervals of time, ie., Aerial velocity is always constant.

Proof:

$$dA = \frac{1}{2} \times b \times h$$

$$dA = \frac{1}{2} \times (vdt) \times r$$

$$dA = \frac{1}{2} vr\,dt \times \frac{m}{m}$$

$dA = \frac{1}{2} L \frac{dt}{m}$; $\frac{dA}{dt} = \frac{L}{2m}$

Torque about focus is zero.

$\tau = 0$

L = Const

$\boxed{\frac{dA}{dt} = \text{Const}}$

<u>3rd law</u> (Law of Orbitals)

/ Square of time period $\propto$ (Semi major axis)^3 of revolution

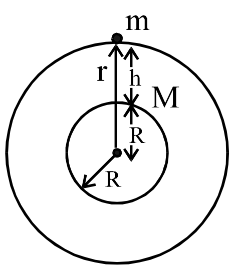
$\boxed{T^2 \propto a^3}$

$\boxed{T \propto a^{3/2}}$

<u>Planet Satellite System</u>

<u>Orbital Velocity</u>

Satellite of mass 'm' revolving around a planet of mass M.



For the satellite necessary centripetal force is provided by gravitational force.

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$v^2 = \frac{GM}{r}$ $\boxed{v = \sqrt{\frac{GM}{r}}}$

$\boxed{v_0 = \sqrt{\frac{GM}{R+h}}}$ $\boxed{v_0 = \sqrt{\frac{gR^2}{R+h}}}$

If the satellite is revolving nearest to the surface of earth then $h \approx 0$

then $R + h \approx R$

$\boxed{v_0\sqrt{\frac{GM}{R}}}$ or $v_0 = \sqrt{\frac{gR^2}{R}}$

This velocity is known as first cosmic velocity.

$\boxed{v_0 = \sqrt{gR}}$

Note :

Relation between or orbital speed and escape speed is $\boxed{v_e = \sqrt{2}\, v_0}$

<u>Time Period</u>

It is time taken by satellite to complete one revolution.

Time period $T = \frac{2\pi r}{v}$

$$T = \frac{2\pi r}{\sqrt{\frac{GM}{r}}} \qquad \boxed{T = \frac{2\pi}{\sqrt{GM}} r^{3/2}}$$

$$r = R + h \qquad \boxed{T = \frac{2\pi}{\sqrt{GM}} (R+h)^{3/2}}$$

$$GM = gR^2 \qquad \boxed{T = \frac{2\pi}{\sqrt{gR^2}} (R+h)^{3/2}}$$

If the satellite is rotating nearer to the surface of Earth, then

$R + h \approx R$

$$T = \frac{2\pi}{\sqrt{gR}} (R)^{3/2}$$

$$T = 2\pi \frac{\sqrt{R}}{\sqrt{g}} \qquad \boxed{T = 2\pi\sqrt{R/g}}$$

<u>Kinetic Energy of the Satellite</u>

$$KE = \frac{1}{2} m v_0^2$$

$$= \frac{1}{2} m \frac{GM}{r} \qquad \boxed{KE = \frac{GMm}{2r}}$$

Potential Energy of Satellite

$$PE = \frac{-Gm_1m_2}{r} \qquad \boxed{PE = \frac{-GMm}{r}}$$

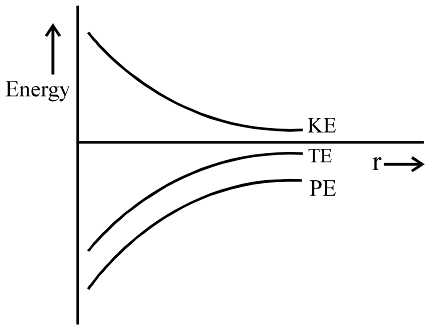
Total Energy of Satellite

$$TE = PE + KE$$

$$= \frac{-GMm}{r} + \frac{GMm}{2r}$$

$$= \frac{-GMm}{2r}$$

$$\boxed{TE = \frac{-GMm}{2r}}$$



GEOSTATIONARY SATELLITES

A geostationary satellites is an earth orbiting satellite placed at an altitude of 35,800 km from the surface of the earth.

They revolves in the same direction as that of Earth. Time period is also same as that of Earth.

Geostationary satellites revolutionized the global communication.

Uses : Television Broadcasting

Weather forecasting

Also play important role in defense and intelligence

POLAR SATELLITES

Geostationary satellites are launched in the equatorial plane.

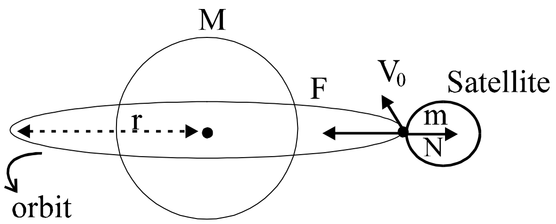
These satellites are revolving in plane containing axis of rotation of Earth.

They go round the poles of earth in north south direction.

Polar satellites are launched at low altitudes of around 500-800 km.

Time period revolution is around 100 minute.

Weightlessness inside a satellite

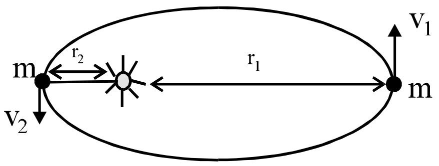


Net force towards centre = Centripetal force

F - N = $\frac{mv^2}{r}$; $v = v_0 = \sqrt{\frac{GM}{r}}$

$\frac{GMm}{r^2} - N = \frac{m}{r} \times \frac{GM}{r}$ $\therefore \boxed{N = 0}$

Note :



At Aphelion Vel-v_1; At Perihelion Vel - v_2

$\tau = 0$; L = const

$L_1 = L_2$

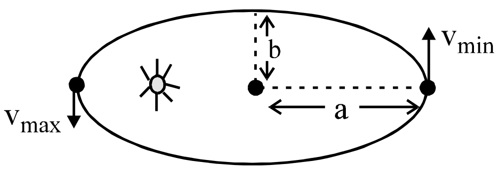
$mv_1 r_1 = mv_2 r_2$

$v_1 r_1 = v_2 r_2$

vr = const

$\boxed{v \propto \frac{1}{r}}$

So at farthest point is velocity is minimum and at the nearest point velocity is maximum.



$e = \sqrt{1 - \frac{b^2}{a^2}}$

$\boxed{v_{max} = \sqrt{\frac{GM}{a}\left(\frac{1+e}{1-e}\right)}}$ $\boxed{v_{min} = \sqrt{\frac{GM}{a}\left(\frac{1-e}{1+e}\right)}}$ $\boxed{\frac{v_{max}}{v_{min}} = \frac{1+e}{1-e}}$