CHAPTER - 8
CIRCLES

THE CIRCLE

Memory module (Basic principles and formulae)

1. **Definition** : Circle is the locus of a point moving in a plane such that its distance from a fixed point in the plane is a constant. The fixed point is the centre and the constant distance is the radius

2. The general second degree equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

   represents a circle if $a = b \neq 0$ and $h = 0$

3. General equation of the circle is $x^2 + y^2 + 2gx + 2fy + c = 0$

4. Circle with centre (h, k) and radius 'r' is $(x - h)^2 + (y - k)^2 = r^2$

5. Circle having centre at (0, 0) and radius 'r' is $x^2 + y^2 = r^2$

6. Centre and radius of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$

   centre is $(-g, -f)$ and radius is $\sqrt{g^2 + f^2 - c}$

Remark

If $g^2 + f^2 - c > 0$, the circle is real

$g^2 + f^2 - c = 0$, the circle reduces to a point

$g^2 + f^2 - c < 0$, the circle is imaginary

7. The circle having the segment joining (x_1, y_1) and $(x_2\ y_2)$ as diameter is

   $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

8. Circle touching the x - axis at origin

   $x^2 + (y \pm r)^2 = r^2$

9. Circle touching the y - axis at origin

   $y^2 + (x \pm r)^2 = r^2$

10. Circle touching the y - axis and having radius r $(x \pm r)^2 + (y \pm k)^2 = r^2$, circle touching x axis and radius r, $(x \pm h)^2 + (y \pm r)^2 = r^2$

11. Circle touching both the axis is $(x \pm r)^2 + (y \pm r)^2 = r^2$

12. Intercept cut off from the x-axis by the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $2\sqrt{g^2 - c}$

13. Intercept cut off from the y - axis by the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $2\sqrt{f^2 - c}$

14. For three noncollinear pts there is one and only one circle through them. The equation of the circle through three given points is obtained by solving the equation

$x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c = 0$

$x_2^2 + y_2^2 + 2gx_2 + 2fy_2 + c = 0$

$x_3^2 + y_3^2 + 2gx_3 + 2gx_3 + 2fy_3 + c = 0$

for g, f and c [Here (x_1, y_1) (x_2, y_2) (x_3, y_3) are the three points given

15. Circle passing through the points (0, 0) (a, 0) and (0, b) is $x^2 + y^2 - ax - by = 0$

16. Parametric form of the equation $x^2 + y^2 = a^2$ is $x = a\cos\theta$, $y = a\sin\theta$

17. Parametric form of the equation $(x - h)^2 + (y - k)^2 = r^2$

is $x - h = r\cos\theta$ $\quad y - k = r\sin\theta$

OR $x = h + r\cos\theta$ $\qquad y = k + r\sin\theta$

18. It S = 0 be a circle 'r' its radius L = 0 be a line d its distances from the centre of the circle then

(a) L = 0 touches s = 0 iff d = r

(b) L = 0 meets s = 0 in two real points if d < r

(c) L = 0 does not meet s = 0 if d > r

(d) L = 0 is a normal to the circle iff it passes through the centre of the circle

19. The line $y = mx + c$ touches circle $x^2 + y^2 = r^2$ iff $c^2 = r^2(1 + m^2)$ OR $C = \pm r\sqrt{1+m^2}$

20. Any tangent to the circle $x^2 + y^2 = r^2$ may be given as $y = mx \pm r\sqrt{1+m^2}$

21. The power of the point (x_1, y_1) w.r.t the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ is S_1 where

$S_1 \equiv x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$

S_1 could be = 0, > 0, or < 0

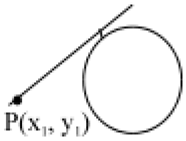
If $S_1 = 0$ P (x_1, y_1) Lies on the circle S = 0

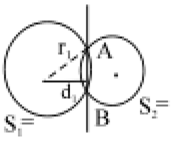
If $S_1 < 0$ P is interior to the circle

If $S_1 > 0$ P is exterior to the circle

22. Length of the tangent from P (x_1, y_1) on the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ is

$$PT = \sqrt{S_1} = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$$

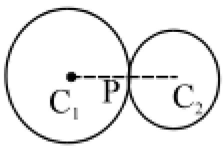


23. 

Equation of the common chord of the circles $S_1 = 0$ and $S_2 = 0$ is $S_1 - S_2 = 0$

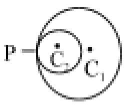
Length of the common chord AB is $A = 2\sqrt{r_1^2 - d_1^2} = 2\sqrt{r_2^2 - d_2^2}$ where r_1, r_2 are the radii and d_1 d_2 the distances of the centres from AB

24. Two circles $S_1 = 0$ and $S_2 = 0$
with centres C_1 and C_2 and radii r_1 and r_2



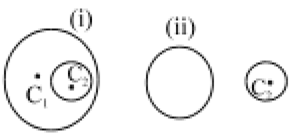
(a) Touch externally if $|C_1C_2| = r_1 + r_2$ these the point of contact P divides C_1C_2 internally in the ratio r_1:r_2

(b) The circles touch internally iff
$|C_1C_2| = |r_1 - r_2|$



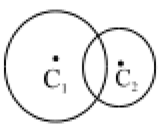
Here P the point of contact divides C_1C_2 externally in the ratio r_1r_2

(c) The circles do not intersect if



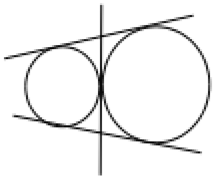
$|C_1C_2| < |r_1 - r_2|$ or $|C_1C_2| > r_1 + r_2$

(d) Intersect in two distinct points



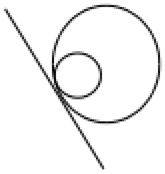
Iff $|r_1 - r_2|$ |<| $|C_1C_2| < r_1 + r_2$

25. Number of common tangents
(a) when circles touch externally



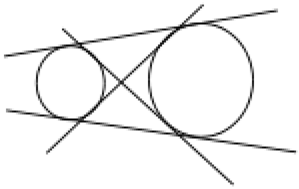
the number of common tangents is 3

(b) when two circles touch in internally (one inside the other)

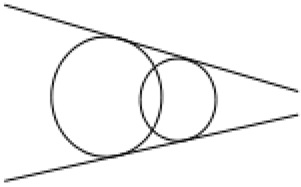


the number of common tangents is just one

(c) When two circles do not intersect and one is outside the other the number of common tangents is 4

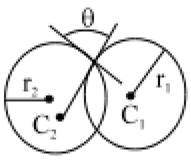


(d) when two circles intersect the number of common tangents is 2



(e) when two circles are non intersecting and one inside the other there is no common tangent

26. Angle between two intersecting circles is the angle between their tangents at a point of intersection of the two circles and

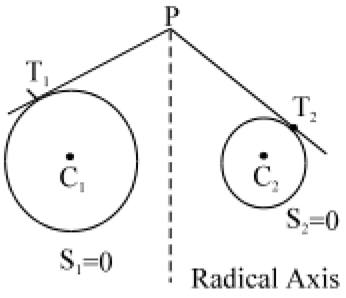


$$\cos\theta = \frac{r_1^2 + r_2^2 - C_1C_2^2}{2r_1 r_2}$$

27. **Two circles are said to be orthogonal:** If the angle between their tangents is a right angle (ie $\theta = \pi/2$) Here $C_1C_2^2 = r_1^2 + r_2^2$

28. The circles $x^2 + y^2 + 2g_1x + 2f_1 y + c_1 = 0$ and $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ are orthogonal iff $2g_1g_2 + 2f_1f_2 = c_1 + c_2$

29. **Radical axis:** of two circles is the locus of points P from which the tangents to the two circles are equal OR Locus of 'p' such that $PT_1 = PT_2$



30. Equation of the R.A of the circles S_1 = O & $S_2 = 0$ is $S_1 - S_2 = 0$

    ie $2(g_1 - g_2)x + 2(f_1 - f_2)y + C_1 - C_2 = 0$. Radical axis is a line

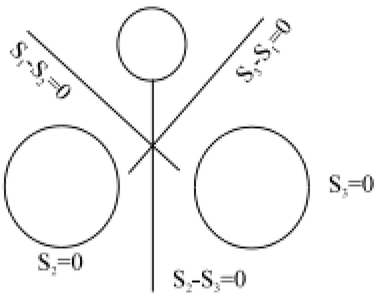
31. Radical axis of two intersecting circles is same as their common chord

32. (a) If $S_1 = 0$ $S_2 = 0$ & $S_3 = 0$ are three circles their radical axes taken two by two are

    $S_2 - S_3 = 0$ $S_3 - S_1 = 0$ and $S_1 - S_2 = 0$

    These three lines are concurrent and the point of concurrence is known as the radical centre

(b) The length of the tangent from the radical centre to the three circles are equal



33. Equation of any circle passing through the intersection of a circle , S = 0 and a line L = 0 is S + kL= 0, k being arbitrary

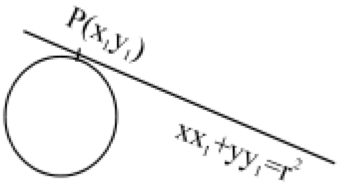
34. Equation of any circle passing through the intersection of two circles

    $S_1 = 0$ and $S_2 = 0$ is given by $S_1 + K(S_1 - S_2) = 0$

35. Two or more circles having the same centre is known as concentric circles

36. Four or more points which lie on the same circle are known as concyclic points

37. Equation of the tangent to the circle

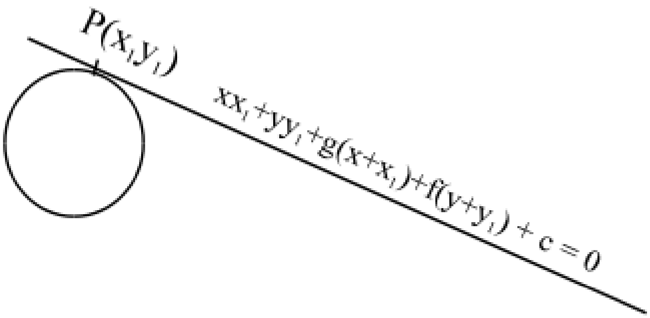


$x^2 + y^2 = r^2$ at a point

P $(x_1 y_1)$ on it is

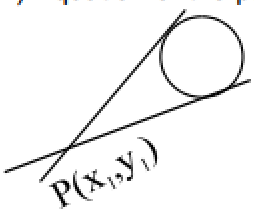
$x x_1 + y y_1 = r^2$

38. 1) Equation of the tangent at P $(x_1 y_1)$ on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$



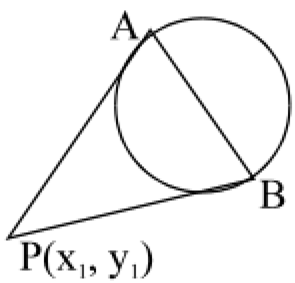
is $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

2) Equation of the pair of tangents from the point (x_1, y_1) is given by $T_1^2 = SS_1$

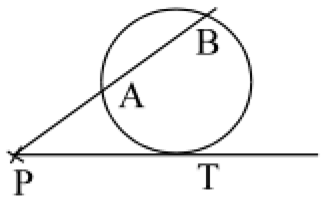


39. Equation of the chord of contact of tangents from $(x_1 y_1)$ ie, equation of AB is given by $T_1 = 0$

ie $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$



40. If a line through an external point P meets a circle at A and B and PT tangent from P to the circle PA.PB= PT^2



PART I - (JEEMAIN)

SECTION - I - Only one option correct type

1. The equation of the circle touching the x axis at (5,0) and the line y = 10 is

1) $x^2+y^2-10x-10y+25=0$ 2) $x^2+y^2-10x-10y-25=0$

3) $x^2+y^2-5x-5y-5=0$ 4) $x^2+y^2-5x-5y+5=0$

2. The circle passing through the point (–1, 0) and touching the Y-axis at (0, 2) and also passes through the point:

1) $\left(\frac{-3}{2},0\right)$ 2) $\left(\frac{-5}{2},2\right)$ 3) $\left(\frac{-3}{2},\frac{5}{2}\right)$ 4) $(-4,0)$

3. A variable circle passes through the fixed point A (p,q) and touches the x axis. The locus of the other end of the diameter through A is

1) $(y + q)^2 = 4px$ 2) $(x – q)^2 = 4py$ 3) $(y – p)^2 = 4qx$ 4) $(x – p)^2 = 4qy$

4. If one of the diameters of the circle $x^2+y^2-2x-6y+6=0$ is a chord to the circle with centre (2,1) then the radius of the circle is

1) 3 2) 2 3) 3/2 4) 2/3

5. Tangents drawn from the point P(1, 8) to the circle $x^2+y^2-6x-4y-11=0$ touch the circles at the points A and B. The equation of the circumcircle of the triangle PAB is

1) $x^2+y^2+4x-6y+19=0$ 2) $x^2+y^2-4x-10y+19=0$

3) $x^2+y^2-2x+6y-29=0$ 4) $x^2+y^2-6x-4y+19=0$

6. The locus of the centre of the circle of radius '2' which rolls on the outside of the circle $x^2+y^2+3x-6y-9=0$ is

1) $x^2+y^2+3x-6y+5=0$ 2) $x^2+y^2+3x-6y-31=0$

3) $x^2+y^2+3x-6y+\frac{29}{4}=0$ 4) $x^2+y^2-3x+6y-\frac{29}{4}=0$

7. The locus of the centre of a circle which touches externally the circle $x^2+y^2-6x-6y+14=0$ and also touches the y-axis, is given by the equation

(1) $x^2-6x-10y+14=0$ (2) $x^2-10x-6y+14=0$

(3) $y^2-6x-10y+14=0$ (4) $y^2-10x-6y+14=0$

8. Length of the tangent drawn from any point on the circle $x^2+y^2+2gx+2fy+c_1=0$ to the circle $x^2+y^2+2gx+2fy+c=0$ is (where $c<c_1$)

(1) $\sqrt{c_1-c}$ (2) $\sqrt{c-c_1}$ (3) $\sqrt{c_1+c}$ (4) None of these

9. The value of k so that $x^2+y^2+kx+4y+2=0$ and $2(x^2+y^2)-4x-3y+k=0$ cut orthogonally is

(1) $\frac{10}{3}$ (2) $\frac{-8}{3}$ (3) $\frac{-10}{3}$ (4) $\frac{8}{3}$

10. If a circle, whose centre is (−1, 1) touches the straight line $x+2y+12=0$, then the coordinates of the point of contact are

(1) $\left(\frac{-7}{2},-4\right)$ (2) $\left(\frac{-18}{5},\frac{-21}{5}\right)$ (3) (2,−7) (4) (−2, −5)

11. From any point on the circle $x^2+y^2=a^2$ tangents are drawn to the circle $x^2+y^2=a^2\sin^2\alpha$, the angle between them is

(1) $\frac{\alpha}{2}$ (2) α (3) 2α (4) 3α

12. The equation of the circle which passes through the intersection of $x^2+y^2+13x-3y=0$ and $2x^2+2y^2+4x-7y-25=0$ and whose centre lies on $13x+30y=0$ is

(1) $x^2+y^2+30x-13y-25=0$ (2) $4x^2+4y^2+30x-13y-25=0$

(3) $2x^2+2y^2+30x-13y-25=0$ (4) $x^2+y^2+30x-13y+25=0$

13. If the chord $y=mx+1$ of the circle $x^2+y^2=1$ subtends an angle of measure 45^o at the major segment of the circle then value of m is

(1) 2 (2) − 2 (3) −1 (4) None of these

14. If the length of the chord of the circle, $x^2+y^2=r^2 (r>0)$ along the line, $y-2x=3$ is r, then r^2 is equal to

(1) $\frac{9}{5}$ (2) $\frac{12}{5}$ (3) 12 (4) $\frac{24}{5}$

15. If a line, $y=mx+c$ is a tangent to the circle, $(x-3)^2+y^2=1$ and it is perpendicular to a line L_1, where L_1 is the tangent to the circle, $x^2+y^2=1$ at the point $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$, then

(1) $c^2-6c+7=0$ (2) $c^2+6c+7=0$ (3) $c^2+7c+6=0$ (4) $c^2-7c+6=0$

16. Let the tangents drawn from the origin to the circle, $x^2+y^2-8x-4y+16=0$ touch it at the points A and B. The $(AB)^2$ is equal to:

(1) $\frac{52}{5}$ (2) $\frac{32}{5}$ (3) $\frac{56}{5}$ (4) $\frac{64}{5}$

17. If the angle of intersection at a point where the two circles with radii 5 cm and 12 cm intersect is 90^0, then the length (in cm) of their common chord is:

(1) $\frac{60}{13}$ (2) $\frac{120}{13}$ (3) $\frac{13}{2}$ (4) $\frac{13}{5}$

18. A circle touching the x -axis at (3, 0) and making an intercept of length 8 on the y -axis passes through the point:

(1) (3, 10) (2) (2, 3) (3) (1, 5) (4) (3, 5)

19. If a variable line, $3x+4y-\lambda=0$ is such that the two circles $x^2+y^2-2x-2y+1=0$ and $x^2+y^2-18x-2y+78=0$ are on its opposite sides, then the set of all values of λ is the interval

(1) [12, 21] (2) (2, 17) (3) (23, 31) (4) [13, 23]

20. From the point $P(\sqrt{2},\sqrt{6})$ tangents PA and PB are drawn to the circle $x^2+y^2=4$

Statement-I: Area of the quadrilateral OAPB (O being origin) is 4

Statement-II: Tangents PA and PB are perpendicular to each other and therefore quadrilateral OAPB is a square.

(1) If both Statement-I and Statement-II are true and the reason is the correct explanation of the statement-I.

(2) If both Statement-I and Statement-II are true but reason is not the correct explanation of the statement-I.

(3) If Statement-I is true but Statement-II is false.

(4) If Statement-I is false but Statement-II is true.

SECTION - II

Numerical type Questions

21. Two circles with equal radii are intersecting at the points (0, 1) and $(0,-1)$. The tangent at the point (0, 1) to one of the circles passing through the centre of the other circles. Then the distance between the centres of these circles is

22. The number of common tangents to the circles $x^2+y^2-4x-6x-12=0$ and $x^2+y^2+6x+18y+26=0$, is

23. Let PQ be a diameter of the circle $x^2+y^2=9$. If α and β are the lengths of the perpendiculars from P and Q on the straight line, $x+y=2$ respectively, then the maximum value of $\alpha\beta$ is ____

24. The number of integral values of 'k' for which the line, $3x+4y=k$ intersects the circle, $x^2+y^2-2x-4y+4=0$ at two distinct points is ________

25. If the circle $x^2+y^2+6x-2y+k=0$ bisects the circumference of the circle $x^2+y^2+2x-6y-15=0$, then $|k|$ =

PART - II (JEE ADVANCED)

SECTION - III (Only one option correct type)

26. If $-3l^2-6l-1+6m^2=0$, Then the equation of the circle for which $lx+my+1=0$ is a tangent,is

A) $x^2+y^2=9$ B) $(x-3)^2+y^2=7$ C) $x^2+y^2=6$ D) $(x-3)^2+y^2=6$

27. Consider a triangle Δ whose two sides lie on the x-axis and the line $x+y+1=0$. If the orthocenter of Δ is (1,1), then the equation of the circle passing through the vertices of the triangle Δ is

A) $x^2+y^2-3x+y=0$ B) $x^2+y^2-x+3y=0$

C) $x^2+y^2+2y-1=0$ D) $x^2+y^2+x+y=0$

28. The angle between a pair of tangents drawn from a point P to the circle $x^2+y^2+4x-6y+9\sin^2\alpha+13\cos^2\alpha=0$ is 2α. The equation of the locus of the point P is

A) $x^2+y^2-4x+6y=9$ B) $x^2+y^2+4x-6y+9=0$ C) $x^2+y^2+4x+6y=9$ D) $x^2+y^2-4x-6y-9=0$

29. Let C_1 and C_2 are circles defined by $x^2+y^2-20x+64=0$ and $x^2+y^2+30x+144=0$. The length of the shortest line segment PQ that is tangent to C_1 at P and to C_2 at Q is

A) 15 B) 18 C) 20 D) 24

30. r_1 and r_2 are the radii of smallest and largest circles which passes through (5, 6) and touches the circle $(x-2)^2+y^2=4$, thn r_1r_2 is

A) $\frac{4}{41}$ B) $\frac{41}{4}$ C) $\frac{5}{41}$ D) $\frac{41}{6}$

SECTION - IV (More than one correct answer)

31. A circle is inscribed in a trapezium in which one of the non-parallel sides is perpendicular to the two parallel sides. Then

A) the diameter of the inscribed circle is the geometric mean of the lengths of the parallel sides

B) the diameter of the inscribed circle is the harmonic mean of the lengths of the parallel sides

C) the area of the trapezium is the area of the rectangle having lengths of its sides as the lengths of the parallel sides of the trapezium

D) the area of the trapezium is half the area of the rectangle having lengths of its sides as the lengths of the parallel sides of the trapezium

32. Circles of radii 3 and 4 intersect orthogonally. The area common to the two circles is

A) $8\pi+12-7\tan^{-1}\frac{4}{3}$ B) $8\pi-12-7\tan^{-1}\frac{4}{3}$

C) $8\pi-12+7\tan^{-1}\frac{3}{4}$ D) $\frac{9\pi}{2}-12+7\tan^{-1}\frac{3}{4}$

33 If the circle $x^2+y^2-2x-2y+1=0$ is inscribed in a triangle whose two sides are axes and one side has negative slope cutting intercepts a and b on x and y axis. Then

A) $\frac{1}{a}+\frac{1}{b}-1=-\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}$ B) $\frac{1}{a}+\frac{1}{b}>1$

C) $\frac{1}{a}+\frac{1}{b}<1$ D) $\frac{1}{a}+\frac{1}{b}-1=\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}$

34. If the circle $C_1: x^2+y^2=16$ intersects another circle C_2 of radius 5 in such a manner that the common chord is of maximum length and has a slope equal to $\frac{3}{4}$, then the coordinates of the centre of C_2 are

A) $\left(\frac{9}{5},-\frac{12}{5}\right)$ B) $\left(-\frac{9}{5},\frac{12}{5}\right)$ C) $\left(\frac{9}{5},\frac{12}{5}\right)$ D) $\left(-\frac{9}{5},-\frac{12}{5}\right)$

35. Let S_1 be a square of unit area. A circle C_1 is inscribed in S_1, a square S_2 is inscribed in C_1 and so on. n general, a circle C_n is inscribed in the square S_n and then a square S_{n+1} is inscribed in the circle C_n. Let a_n denote the sum of the areas of the circle C_1, C_2, ..., C_n then $\lim_{n\to\infty} a_n$ must be

A) greater than the area of the square S_1 B) smaller than the area of the square S_1

C) $\frac{\pi}{4}$ D) $\frac{\pi}{2}$

36. Let T_1,T_2 be two tangents drawn from (-2,0) to the circle $x^2+y^2=1$, then the equation of the circle touching the given circle and having T_1 and T_2 as their pair of tangents

A) $x^2+y^2-8x+7=0$ B) $3x^2+3y^2+8x+5=0$

C) $x^2+y^2+4x+4\sqrt{3}y+7=0$ D) $x^2+y^2+4x-4\sqrt{3}y+7=0$

37. Let L_1 be a straight line passing through the origin and L_2 be the straight line x+y=1. If the intercepts made by the circle $x^2+y^2-x+3y=0$ on L_1 and L_2 are equal, then which of the following equations can represent L_1

A) x +y = 0 B) x–y=0 C) x+7y+0 D) x–7y=0

38. The circles $x^2+y^2+2x+4y-20=0$ and $x^2+y^2+6x-8y+10=0$

A) are such that the number of common tangents on them is 2

B) are orthogonal

C) are such that the length of their common tangent is $5(12/5)^{1/4}$

D) are such that the length of their common chord is $5\sqrt{\frac{3}{2}}$

39. A tangent to the circle $x^2+y^2=1$ through the point (0,5) cuts the circle $x^2+y^2=4$ at P and Q. If the tangents to the circle $x^2+y^2=4$ at P and Q meet at R, then the co-ordinate of R is

A) $\left(\frac{8\sqrt{6}}{5},\frac{4}{5}\right)$ B) $\left(\frac{8\sqrt{6}}{5},\frac{-4}{5}\right)$ C) $\left(\frac{-8\sqrt{6}}{5},\frac{4}{5}\right)$ D) $\left(\frac{-8\sqrt{6}}{5},\frac{-4}{5}\right)$

SECTION VI - (Matrix match type)

40. Consider the circles c_1, of radius 'a' and c_2 of radius 'b', b > a, both lying in the first quadrant and touching the coordinate axes

Column-I	**Column-II**
a) c_1 and c_2 touch each other	p) $\frac{b}{a}=2+\sqrt{2}$
b) c_2 and c_2 are orthogonal	q) $\frac{b}{a}=2$
c) c_1 and c_2 intersect so that the common chord is longest	r) $\frac{b}{a}=2+\sqrt{3}$
d) c_2 passes through the centre of c_1	s) $\frac{b}{a}=3+2\sqrt{2}$

A) A-S; B-R; C-Q; D-P B) A-P; B-R; C-Q; D-S
C) A-R; B-S; C-Q; D-P D) A-P; B-Q; C-R; D-S