CHAPTER - 07
GRAVITATION

SYNOPSIS

Newton's Law of Gravitation

$$F = G\frac{m_1m_2}{r^2}$$

G is universal constant $G = 6.67\times10^{-11}\,Nm^2kg^{-2}$

Gravitational force is attractive, central, conservative, weakest and independent of medium. It also obeys inverse square law and principle of superposition.

$[G] = M^{-1}L^3T^{-2}$

Acceleration due to gravity 'g'

On the surface of earth $g = \frac{GM}{R^2}$

At a height h from the surface of earth $g_h = \frac{GM}{(R+h)^2}$

If $h << R$; $g_h = \left(1-\frac{2h}{R}\right)g \Rightarrow \frac{\Delta g}{g} = \frac{-2h}{R}$

At a depth d from the surface of earth, $g_d = g\left(1-\frac{d}{R}\right)$

$$\frac{\Delta g}{g} = \frac{-d}{R}$$

The value of 'g' is maximum at the poles and minimum at the equator 'g' also varies due to latitude.

$$g_\lambda = g_p - R\omega^2\cos^2\lambda$$

At the equator $\lambda = 0$ $\quad g_e = g_p - R\omega^2 \Rightarrow g_p - g_e = R\omega^2 = 0.034m/s^2$

Weightlessness due to rotation of earth

Apparent weight of a body decreases due to rotation of earth. It becomes weightless at equator when

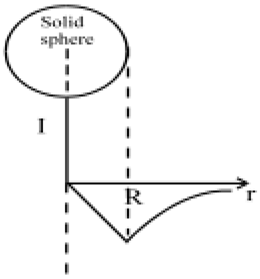
$$g - R\omega^2 = 0 \qquad \omega = \sqrt{\frac{g}{R}} \qquad R = 6400\,\text{km}$$

$$\omega = \frac{2\pi}{T} \qquad \omega = 1.25\times10^{-3}\ \text{rad / sec} \qquad T = 1.4\ \text{hours}$$

Gravitational field: Force experienced by unit mass $I = \frac{\vec{F}}{m}$

Uniform solid sphere

Outside the surface $(r > R)$ $I = \frac{GM}{r^2}$



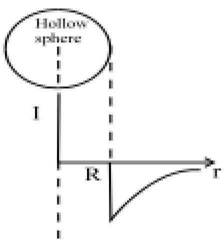
On the surface $(r = R)$ $I = \frac{GM}{R^2}$

Inside the sphere $(r < R)$ $I = \frac{GMr}{R^3}$

Intensity due to spherical shell

Outside the surface $(r > R)$ $I = \frac{GM}{r^2}$

On the surface $(r = R)$ $I = \frac{GM}{R^2}$



Inside the shell $(r < R)$ $I = 0$

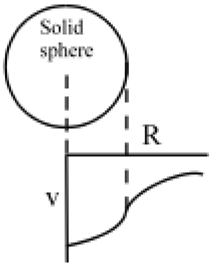
Gravitational Potential

Gravitational potential at a point V is defined as negative of work done per unit mass in shifting from the given point to infinity.

$$V = -\frac{W}{m} = -\frac{\int \vec{F}.d\vec{r}}{m} = -\int \vec{I}.d\vec{r}$$

$I = \frac{-dv}{dr}$

Potential due to a uniform solid sphere



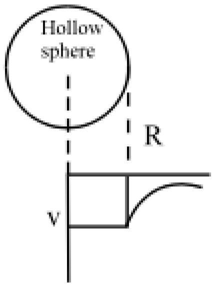
Outside the surface $(r > R) = \frac{-GM}{r}$

On the surface $(r = R) = \frac{-GM}{R}$

Inside the sphere $(r < R) = \frac{-GM}{2R}\left[3 - \left(\frac{r}{R}\right)^2\right] = -\frac{GM}{2R^3}\left[3R^2 - r^2\right]$

At the centre of the sphere = $-\frac{3}{2}\frac{GM}{R} = \frac{3}{2}V_{surface}$

Potential due to a spherical shell



Outside the surface $(r > R) = \frac{-GM}{r}$

On the surface $(r = R) = \frac{-GM}{R}$

Inside the sphere $= -\frac{GM}{R}$

Gravitational potential energy

P.E. of a mass m at a distance r from centres of earth $U = -\frac{GMm}{r}$

On the surface = $-\frac{GMm}{R} = -mgR$

When a mass m is taken from the surface of earth to a height h = nR. The increase in P.E = $\frac{n}{n+1}mgR$

Average density of earth $\rho = \dfrac{3g}{4\pi RG} \approx 5500\ \text{kg/m}^3$

Satellites

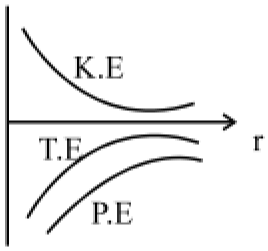
The necessary centripetal force is provided by gravitational force of attraction

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \qquad v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{R+h}} \quad v \propto \frac{1}{\sqrt{r}}$$

v does not depend on mass of satellite.

Period of satellite $T = 2\pi\sqrt{\dfrac{r^3}{GM}}$ Thus $T \propto r^{3/2}$

Energy of satellite



$$P.E = -\frac{GMm}{r}$$

$$K.E = \frac{GMm}{2r}$$

$$T.E = -\frac{GMm}{2r}$$

Binding energy of satellite = $+\dfrac{GMm}{2r}$

First cosmic velocity

If the height of the satellite is very small for a closest orbit $v_0 = \sqrt{\dfrac{GM}{R}} = \sqrt{Rg} = 7.9\ \text{km/s}$ (1st cosmic velocity)

Escape Velocity

The escape velocity of a body at the surface of earth $V_e = \sqrt{\dfrac{2GM}{R}} = \sqrt{2Rg} = 11.2\ \text{km/sec}$

It is also called 2nd cosmic velocity

If a body is projected with velocity greater than escape velocity $(v > v_e)$ then by conservation energy.

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = \frac{1}{2}mv'^2 + 0\ ;\ \ v^2 - \frac{GM}{R} = v'^2\ ;\ \ v' = \sqrt{v^2 - v_e^2}$$

The body will move in interstellar space with velocity $\sqrt{v^2 - v_e^2}$.

Kepler's laws of Planetary motion

(1) The law of orbits: Every planet moves around sun in an elliptical orbit with sun at one of the focii.

(2) Law of area: The line joining the sun to the planet sweeps out equal areas in equal interval of time ie, areal velocity is constant.

$\frac{dA}{dt}=\frac{L}{2m}$ where L - Angular momentum

(3) Law of periods: The square of period of revolution of any planet around sun is directly proportional to the cube of semi-major axis of the orbit. $T^2 \propto a^3$

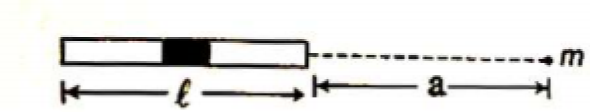
PART I - (JEEMAIN)

SECTION - I - Straight objective type questions

1. The weight of a body at the surface of earth is 18N. The weight of the body at an altitude of 3200 km above the earth's surface is (given, radius of earth R_e=6400 km)

   1) 9.8 N 2) 4.9 N 3) 19.6 N 4) 8 N

2. A mass m is at a distance 'a' from one end of a uniform rod of length 'ℓ' and mass M. The gravitational force on the mass due to the rod is



   1) $\frac{GMm}{(a+\ell)^2}$ 2) $\frac{GMm}{a(\ell+a)}$ 3) $\frac{GMm}{a^2}$ 4) $\frac{GMm}{2(a+\ell)^2}$

3. A point mass m is placed inside a spherical shell of radius R and mass M at a distances $\left(\frac{R}{2}\right)$ from the centre of the shell. The gravitational force exerted by the shell on the point mass is

   1) $\frac{GMm}{R^2}$ 2) $\frac{2GMm}{R^2}$ 3) zero 4) $\frac{4GMm}{R^2}$

4. Acceleration due to gravity on moon is $1/6$ of the acceleration due to gravity on earth. If the ratio of densities of earth (ρ_e) and moon (ρ_m) is $\frac{\rho_e}{\rho_m}=\frac{5}{3}$ then radius of moon (R_m) in terms of (R_e) will be

   1) $\frac{5}{18}R_e$ 2) $\frac{1}{6}R_e$ 3) $\frac{7}{18}R_e$ 4) $\frac{1}{2\sqrt{3}}R_e$

5. At what depth from the surface of earth the time period of a simple pendulum 0.5% more than that on the surface of the earth ? (Radius of earth is 6400 km)

   1) 32 km 2) 64 km 3) 96 km 4) 128 km

6. Acceleration due to gravity 'g' and density of the earth 'ρ' are related by which of the following relations?

   [where G $\rightarrow$ gravitational constant, R $\rightarrow$ radius of earth]

   1) $\rho=\frac{4\pi GR}{3g}$ 2) $\rho=\frac{3g}{4\pi GR}$ 3) $\rho=\frac{3G}{4\pi gR}$ 4) $\rho=\frac{4\pi gR}{3G}$

7. Infinite number of bodies, each of mass 2 kg, are situated on x-axis at distance 1m, 2m, 4m, 8m .... respectively, from the origin. The resulting gravitational potential due to this system at the origin will be:

1) –G 2) $\frac{-4}{3}G$ 3) $\frac{-8}{3}G$ 4) –4G

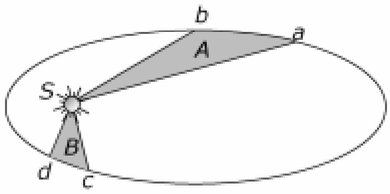
8. A body of mass m is lifted up from the surface of earth to a height three times the radius of earth. The change in potential energy of the body is

1) mgR 2) $\frac{3mgR}{4}$ 3) $\frac{mgR}{3}$ 4) $\frac{2mgR}{3}$

9. A satellite is moving around the earth with speed 'v' in a circular orbit of radius 'r'. If the orbit radius is decreased by 1%, its speed will

1) Increase by 1% 2) Increase by 0.5%
3) Decrease by 1% 4) Decrease by 0.5%

10. The figure shows the motion of a planet around the sun in an elliptical orbit with sun at the focus. The shaded areas A and B are also shown in the figure which can be assumed to be equal. If t_1 and t_2 represent the time for the planet to move from 'a' to 'b' and 'd' to 'c' respectively, then



1) $t_1 < t_2$ 2) $t_1 > t_2$ 3) $t_1 = t_2$ 4) $t_1 \le t_2$

11. According to Kepler, the period of revolution of a planet (T) and its mean distance from the sun (r) are related by the equation

1) T^3r^3 = constant 2) T^2r^{-3} = constant
3) Tr^3 = constant 4) T^2r = constant

SECTION - II - Numerical Type Questions

12. The escape velocity for a body projected vertically upwards from the surface of the earth is 11.2 km/s. If the body is projected in a direction making an angle of 45° with the vertical, the escape velocity will be (in km/s)

13. The escape velocity from earth is about 11 km per second. The escape velocity from a planet having twice the radius and same mean density as earth is in km/s

14. An artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of the escape velocity from the earth. If the satellite is stopped suddenly in its orbit and allowed to fall freely on the earth, Find the speed (in km/s) with it hits the surface of earth

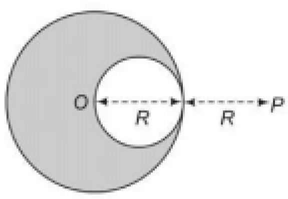
($g = m/\sec^2$ and $R = 6400\,km$)

15. The gravitational field in a region is given by $\overline{E_g} = 5\hat{i} + 12\hat{j}\,N/Kg$, then the magnitude of the gravitational force acting on a particle of mass 2 Kg. placed at the origin, (in N) will be

PART - II (JEE ADVANCED)

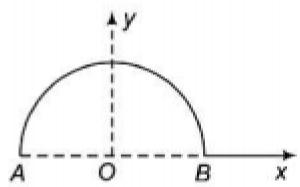
SECTION - III (One option correct type question)

16. A solid sphere of uniform density and radius R applies a gravitational force of attraction equal to F_1 on a particle placed at P. distance 2R from the centre 0 of the sphere. A spherical cavity of radius R/2 is now made in the sphere as shown in figure. The sphere with cavitty now applies a gravitational force F_2 on same particle at P. The ratio F_2/F_1 will be



A) 1/2 B) 7/9 C) 3 D) 7

17. Gravitational field at the centre of a semicircle formed by a thin wire AB of mass M and length l is



A) $\frac{GM}{l^2}$ along x-axis

B) $\frac{GM}{\pi l^2}$ along y-axis

C) $\frac{2\pi GM}{l^2}$ along x-axis

D) $\frac{2\pi GM}{l^2}$ along y-axis

18. If the gravitational field in the space is given as $\left(-\frac{K}{r^2}\right)$. Taking the refrerence point to be at r=2cm with gravitational potential V=10 J/kg. Find the gravitational potential at r=3cm in SI unit (Given, that K=6J cm/kg)

A) 9 B) 11 C) 12 D) 10

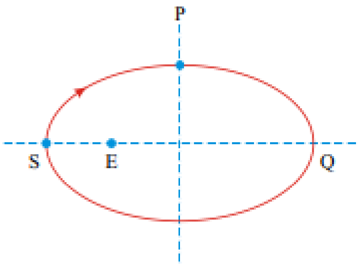
19. A body starts from rest from a point at a distance r_0 from the centre of the earth. It reaches the surface of the earth whose radius is R. The velocity acquired by the body is

A) $2GM\sqrt{\frac{1}{R}-\frac{1}{r_0}}$ B) $\sqrt{2GM\left(\frac{1}{R}-\frac{1}{r_0}\right)}$ C) $GM\sqrt{\frac{1}{R}-\frac{1}{r_0}}$ D) $\sqrt{GM\left(\frac{1}{R}-\frac{1}{R_0}\right)}$

20. A spherical uniform planet is rotating about its axis. The velocity of a point on its equator is v. Due to rotation of planet about its axis the acceleration due to gravity g at equator is $\frac{1}{2}$ of g at poles. The escape velocity of a particle on the pole of planet in terms of v_e:

A) $v_e = 2v$ B) $v_e = \sqrt{3}v$ C) $v_e = v$ D) $v_e = v/2$

21. The satellite is moving in an elliptical orbit about the earth as shown in figure:



The minimum and maximum distance of satellite from earth are 3 units and 5 units, respectively. The distance of satellite from the earth when it is at P is equal to

A) 4 units B) 3 units C) 3.75 units D) none of these

SECTION - IV (More than one correct answer)

22. Two objects of masses m and 4m are at rest at an infinite separation. They move towards each other under mutual gravitational attraction. If G is the universal gravitational constant, then at separation 'r'

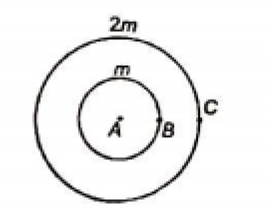
A) The total energy of the two objects is zero

B) Their relative velocity of approach is $\left(\frac{10Gm}{r}\right)^{1/2}$ in magnitude

C) The total kinetic energy of the objects is $\frac{4Gm^2}{r}$

D) The net angular momentum of both the particles is zero about any inertial point

23. Two spherical shells have masses m and 2m as shown. Choose the correct options



A) between A & B gravitational field strength is zero

B) between A & B gravitational potential is constant

C) There will be two points one lying between B and C and another lying between C and infinity where gravitational field strengths are same

D) There will be point between B and C where gravitational potential will be zero

24. Two satellites S_1 and S_2 are to be set into the orbits of heights $\frac{R}{4} and \frac{R}{6}$ (above the surface of earth), respectively. They revolve around the earth in coplanar circular orbits in opposite sense (where, R = radius of earth). Which of the following statement(S) is / are correct ?

A) The ratio of their time periods is $\left(\frac{15}{14}\right)^{3/2}$

B) The ratio of their time periods is $\left(\frac{14}{15}\right)^{3/2}$

C) The ratio of their speeds of projection from the surface of earth is $\sqrt{\frac{23}{15}}$

D) The ratio of their speeds of projection from the surface of earth is $\sqrt{\frac{21}{20}}$

25. A particle mooving with kinetic energy 3 J makes an elastic collison (head - on) with a stationary particle which has twice its mass, During impact :

A) The minimum kinetic energy potential of system is 1 J

B) The minimum elastic potential energy of the system is 2 J

C) Momentum and total energy are conserved at energy instant

D) The ratio of kinetic energy to potential energy of the system first decreases and then increases.

SECTION - V (Numerical Type - Upto two decimal place)

26. The gravitational potential energy of a satellite revolving around the earth in circular orbit is - 4 MJ. Find the additional energy (in MJ) that should be given to the satellite so that it escapes from the gravitational field of the earth.

27. Earth is a sphere of uniform mass density. If the weight of the body is 10n N half way down the centre of earth find the value of n . The body weighed 100 N on the surface.

SECTION - VI (Matrix Matching)

28. Match the Column I with Column II :

	Column – I		Column – II
a)	If a particle escapes the gravitational pull of earth, then path can be	(p)	Parabolic
b)	If a satalite is thrown in the space with velocity $\upsilon > \upsilon_{orbital}$ about earth then it can follow	(q)	Hyperbolic
c)	For a satellite revolving around planet of mass M, its time period is given as $T^2 = 4\pi^2 r_0^3 / GM$ then path is	(r)	elliptical
d)	A satellite of mass m is revolving around planet of mass M its total energy is given as $-\dfrac{GMm}{2r_0}$ then it may follow	(s)	circular

A) a $\rightarrow$p, q, b $\rightarrow$ p, q, r, (c) $\rightarrow$ r, s, (D) $\rightarrow$ r, s

B) a $\rightarrow$ p,q, r, b $\rightarrow$ p, (C) $\rightarrow$r, s (D) q, r, s

C) a $\rightarrow$ p, r, b $\rightarrow$ p, q, r $\rightarrow$ (c) $\rightarrow$s, (D) r, s

D) a $\rightarrow$ q, r, b $\rightarrow$ r, s $\rightarrow$ c $\rightarrow$p, q (D) r, s