SETS

Set

A set is a well defined collection of objects

Reprecentation of a set

1. Statement form

2. Roster form (Tabular form)

3. Set builder form (Rule method)

Types of set

1. Empty set (void set) (null set) which contains no element
2. Singleton set
   Which contians exactly one element
3. Finite set
   Which contains finite number of elements
4. Infinite set
   Which contians infinite number of elements

5. Equivalent set

If n(A) = n(B) then A and B are equivalent

Subset and Super set

If every elements in A is an element of B then A is a subset of B and B is a super set of A it is denoted as $A \subseteq B$ and $B \supseteq A$

Equal set

If $A \subseteq B$ and $B \subseteq A$ then A = B

Proper subset

If A is a subset of B and $A \neq B$ then A is a proper subset of B and is denoted as $A \subset B$

Let $A = \{1, 2, 3\}$

Subsets of A are $\phi, \{1\}\{2\}\{3\}\{1, 2\}\{2, 3\}\{1, 3\}\{1, 2, 3\}$

Proper subsets of A are

$\phi, \{1\}\{2\}\{3\}\{1, 2\}\{2, 3\}\{1, 3\}$

Power set

The set of all subsets of a set is called the power set

Let $A = \{1, 2, 3\}$

$P(A) = \{\phi, \{1\}, \{2\}\{3\}, \{1, 2\}\{2, 3\}\{1, 3\}, \{1, 2, 3\}\}$

Number of subsets

If n(A) = n then

1) Number of subsets of A = 2^n

2) Number of proper subsets of $A = 2^n - 1$

3) Number of elements in p(A) = 2^n

Open interval

If $a < x < b$ then $x \in (a, b)$

Clossed interval

If $a \leq x \leq b$ then $x \in [a.b]$

Universal set

In any operation in set theory we consider a set which is the superset of all sets under consideration is called their universal set

Let $A=\{1,2,3\}$ $B=\{2,3,4,5\}$, $C=\{3,4,5,6,7\}$ $U=\{1,2,3,4,5,6,7\}$

Venn diagram

Most of the operations in set theory can be expressed by the help of digrams they are called venn digram

Operations on set

1) Union

$$A\cup B=\{x:x\in A \text{ or } x\in B\}$$

$$x\in(A\cup B)\Rightarrow x\in A \text{ or } x\in B$$

$$x\notin(A\cup B)\Rightarrow x\notin A \text{ and } x\notin B$$

Let $A=\{1,2,3\}$ $B=\{2,3,4,5\}$

$$A\cup B=\{1,2,3,4,5\}$$

2) Intersection

$$A\cap B=\{x:x\in A \text{ and } x\in B\}$$

$$x\in(A\cap B)\Rightarrow x\in A \text{ and } x\in B$$

$$x\notin(A\cap B)\Rightarrow x\notin A \text{ or } x\notin B$$

Let $A=\{1,2,3,4\}$ $B=\{3,4,5,6,7\}$

$$A\cap B=\{3,4\}$$

3) Difference

$$A-B=\{x:x\in A \text{ and } x\notin B\}$$

$$x\in(A-B)\Rightarrow x\in A \text{ and } x\notin B$$

Let $A=\{1,2,3,4\}$ $B=\{3,4,5,6,7\}$

$A - B = \{1,2\}$ $B - A = \{5,6,7\}$

Symmetric difference

$A \Delta B = (A - B) \cup (B - A)$

$= (A \cup B) - (A \cap B)$

$A = \{1,2,3,4\}$ $B = \{3,4,5,6,7\}$

$A \Delta B = \{1,2,5,6,7\}$

Complement

Let $A = \{1,2,3,4\}$ $U = \{1,2,3,4,5,6,7\}$

$A' = \{x : x \in U \text{ and } x \notin A\}$

$A' = \{5,6,7\} = U - A$

$x \in A' \Rightarrow x \notin A$

$x \in A \Rightarrow x \notin A'$

Important laws in set theory

1) Idempotent laws

$A \cup A = A$

$A \cap A = A$

2) Commutative laws

$A \cup B = B \cup A$

$A \cap B = B \cap A$

3) Associative laws

1) $A \cup (B \cup C) = (A \cup B) \cup C$

2) $A \cap (B \cap C) = (A \cap B) \cap C$

4) Distributive laws

1) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

2) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

5) Absorption laws

1) $A \cup (A \cap B) = A$

2) $A \cap (A \cup B) = A$

6) Demorgan's laws

1) $(A \cup B)' = A' \cap B'$

2) $(A \cap B)' = A' \cup B'$

3) $A - (B \cup C) = (A - B) \cap (A - C)$

4) $A - (B \cap C) = (A - B) \cup (A - C)$

7) Involution laws

$(A')' = A$

8) Identity laws

1) $A \cup \phi = A$

2) $A \cap U = A$

9) Complement laws

1) $A \cup A' = U$

2) $A \cap A' = \phi$

10) Boundedness laws

1) $A \cup U = U$

2) $A \cap \phi = \phi$

Number of elements in a set (Cardinality)

If $A = \{1, 2, 3\}$ then $n(A) = 3$

or $O(A) = 3$

Results

1) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

2) If $n(A \cup B) = n(A) + n(B)$ then A and B are disjoint set

3) $n(A \Delta B) = n(A) + n(B) - 2n(A \cap B)$

4) $n(A - B) = n(A \cap B') = n(A) - n(A \cap B)$

5) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$