CHAPTER - 07

GRAVITATION

PART I - (JEEMAIN LEVEL)

SECTION - I

1. 4 Acceleration due to gravity at height h

$$g' = \frac{g}{\left[1+\frac{h}{R}\right]^2}$$

So weight at given height

$$mg' = \frac{mg}{\left[1+\frac{h}{R}\right]^2} = \frac{18}{\left[1+\frac{1}{2}\right]^2} = 8N$$

2. 2
3. 3
4. 1
5. 2
6. 2
7. 4
8. 2

9. 2 $v \propto \frac{1}{\sqrt{r}}$.

% increase in speed = $\frac{1}{2}$ (% decrease in radius)

$= \frac{1}{2}(1\%) = 0.5\%$

i.e. speed will increase by 0.5%

10. 3 Areal velocity of the planet remains constant.

If the areas A and B are equal then $t_1 = t_2$.

11. 2 $\frac{T^2}{r^3} = \text{constant} \Rightarrow T^2r^{-3} = \text{constant}$

SECTION - II

Numerical Type Questions

12. 11.2

13. 22

14. 8 Given $v_0 = v_e / 2$

$$\left(\frac{GM}{R+h}\right)^{1/2} = \frac{1}{2}\left(\frac{2GM}{R}\right)^{1/2}$$

On solving, $h = R$.

From the law of conservation of energy,

$$-\frac{GM}{(R+h)} = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

or, $\frac{1}{2}mv^2 = \frac{GMm}{R} - \frac{GMm}{2R} = \frac{GMm}{2R}$

or, $v = \sqrt{\frac{GM}{R}} = \sqrt{gR}$

$$= \left[(10)(6.4\times10^6)\right]^{1/2} = 8\,km/\sec.$$

15. 26 : G.T. $\overline{E_g} = 5\hat{i} + 12\hat{j}\,N/Kg$

$\Rightarrow E_g = \sqrt{5^2 + 12^2} = 13N/Kg$

$\Rightarrow$ Magnitude of the gravitational force, $F_g = m(E_g) \Rightarrow F_g = 2(13) \Rightarrow F_g = 26N$

PART - II (JEE ADVANCED LEVEL)

SECTION - III (One correct answer)

16. B

$C \rightarrow$ cavity, $T \rightarrow$ Total, $R \rightarrow$ remaining

$$F_1 = \frac{GMm}{(2R)^2}$$

$$F_R = F_2 = F_T - F_C = F_1 - F_C$$

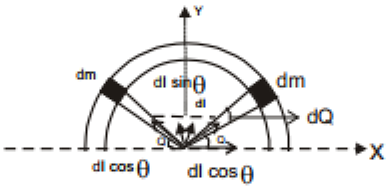
or $$F_2 = \frac{GMm}{(2R)^2} - \frac{G\left(\frac{M}{8}\right)m}{(3R/2)^2}$$

or $$F_2 = \frac{14\ GMm}{72R^2}$$

From Eqs. (i) and (ii) we get,

$$\frac{F_2}{F_1} = \frac{7}{9}$$

17. D



$$dI = \frac{Gdm}{R^2} = \frac{G}{R^2}\left(\frac{M}{\ell}\right)dx$$

$$dx = RdQ$$

$$dI = \frac{GM}{R^2\ell}RdQ = \frac{GMdQ}{R\ell}$$

$$Q = 180$$

$$I = \int_{Q=0}^{Q=180} dI \sin\theta$$

$$= \frac{GM}{R\ell}\int_0^{180} \sin\theta dQ$$

$$= \frac{GM}{R\ell} - [\cos\theta]_0^{180}$$

$$= \frac{-GM}{R\ell}[-1-1] = \frac{2GM}{R\ell}$$

$$But\,\pi R = \ell$$

$$R = \frac{\ell}{\pi}$$

$$I = \frac{2GM\,\pi}{\ell \times \ell} = \frac{2\pi GM}{\ell^2}$$

18. B $-\frac{dV}{dr} = -\frac{k}{r^2} \Rightarrow \int_{10}^{V} dV = \int_{2}^{3} \frac{k}{r^2} dr$

$$V - 10 = k\left[\frac{1}{2} - \frac{1}{3}\right]$$

$$V - 10 = \frac{k}{6} \Rightarrow V = 11 \text{ volts}$$

19. B $-\frac{GMm}{r_0} = \frac{1}{2}mv^2 - \frac{GMm}{R}$

$$\frac{1}{2}mv^2 = \frac{GMm}{R} - \frac{GMm}{r_0} \quad \frac{v^2}{2} = \frac{GM}{R} - \frac{GM}{r_0}$$

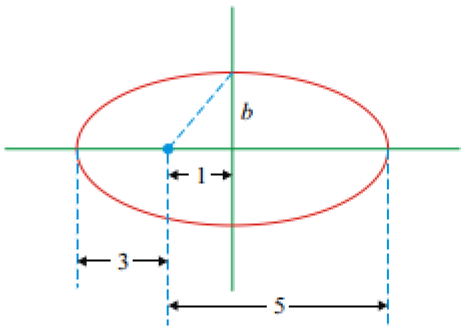
$$v^2 = 2GM\left[\frac{1}{R} - \frac{1}{r_0}\right] \quad v = \sqrt{2GM\left(\frac{1}{R} - \frac{1}{r_0}\right)}$$

20. A $\frac{GMm}{R^2} - \frac{mv^2}{R} = \frac{mg}{2} = \frac{m}{R} \cdot \frac{GM}{R^2} \Rightarrow v = \sqrt{\frac{GM}{2R}}$

As $v_e = \sqrt{\frac{2GM}{R}}$ $\quad v_e = 2v$

21. A From given information, semi-major axis is equal to 4 units. Let e be the eccentricity of ellipse, then

$$ae = 1 \Rightarrow e = \frac{1}{4}$$



Semi-minor axis,

$$b = a\sqrt{1-e^2} = 4\sqrt{1-\frac{1}{16}} \, units = \sqrt{15} \, units.$$

So, required distance $= \sqrt{b^2 + 1^2} = 4 \; units$.

SECTION - IV (More than one correct answer)

22. A,B,C,D
23. ABC
24. A, D

$$r_1 = R + \frac{R}{4} = \frac{5R}{4}$$

$$r_2 = R + \frac{R}{6} = \frac{7R}{6}$$

$$T \propto r^{3/2}$$

$$\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2} = \left(\frac{\frac{5R}{4}}{\frac{7R}{6}}\right)^{3/2} = \left(\frac{15}{14}\right)^{3/2}$$

v of conservation of energy.

$$(U + K)_{\text{surface}} = (U + K)$$

$$-\frac{GMm_1}{R} + \frac{1}{2}m_1 v_1^2 = -\frac{GMm_1}{2\left(R + \frac{R}{4}\right)}$$

$$v_1^2 = \frac{6GM}{5R}$$

Similarly, $-\dfrac{GMm_2}{R} + \dfrac{1}{2}m_2 v_2^2 = -\dfrac{GMm_2}{2\left(R + \frac{R}{6}\right)}$

$\therefore$ $$v_2^2 = \frac{8GM}{7R}$$

$$\frac{v_1^2}{v_2^2} = \frac{\frac{6}{5}}{\frac{8}{7}} = \frac{21}{20} \Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{21}{20}}$$

25. ABCD

In a head on elastic collision between two particles, the kinetic energy becomes minimum and potential energy becomes maximum and potential energy becomes maximum at the instant when they move with a common velocity. The momentum and energy are conserved at energy instant.

Let m and u be the mass and initial velocity of the first particle, 2m be the mass of second particle and v be the common velocity. Then,

$\frac{1}{2}mu^2 = 3J \quad mu = (m + 2m)v$ or $v = \frac{u}{3}$

Minimum kinetic energy of system $\frac{1}{2}(3m)\left(\frac{u}{3}\right)^2 = 1J$

Maximum potential energy of system = 2J

SECTION - V (Numerical Type - Upto two decimal place)

26. 2 $PE = -4MJ \quad TE = -2MJ$

The additional energy required to make the satellite escape $= +2MJ$.

27. 5 $mg' = mg\left(1 - \frac{d}{R}\right)$ where d is the depth $= mg\left(1 - \frac{1}{2}\right) = \frac{mg}{2} = 50\,N$

SECTION - VI (Matrix Matching)

28. A a $\rightarrow$p, q, b $\rightarrow$ p, q, r, (c) $\rightarrow$ r, s, (D) $\rightarrow$ r, s