CHAPTER - 17

WAVE OPTICS

1. 2 $I_P = I + 9I + 2\sqrt{I.9I}\cos {\pi}/{2} = 10I$

$I_Q = I + 9I + 2\sqrt{I.9I}\cos\pi$
$= I + 9I + 2.3I \times -1 = 4I$
So, $I_P - I_Q = 10I - 4I = 6I$

2. 4 Given : $\dfrac{I_2}{I_1} = 2x$

So if $I_2 = 2x$ then $I_1 = 1$

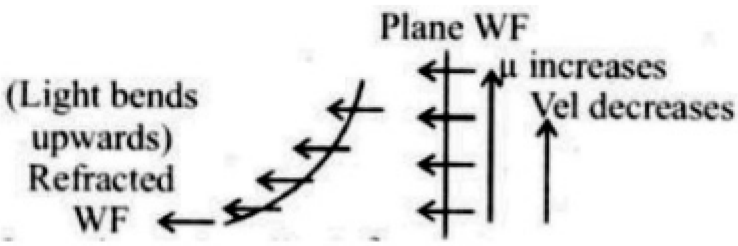
$$\therefore \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{(\sqrt{I_2} + \sqrt{I_1})^2 - (\sqrt{I_2} - \sqrt{I_1})^2}{(\sqrt{I_2} + \sqrt{I_1})^2 + (\sqrt{I_2} - \sqrt{I_1})^2}$$

$$= \frac{(\sqrt{2x} + 1)^2 - (\sqrt{2x} - 1)^2}{(\sqrt{2x} + 1)^2 + (\sqrt{2x} - 1)^2} = \frac{4\sqrt{2x}}{4x + 2} = \frac{2\sqrt{2x}}{2x + 1}$$

3. 3 Optical path for first ray which travels a path L_1 through a medium of refractive index $n_1 = n_1 L_1$
Optical path for second ray which travels a path L_2 through a medium of refractive index $n_2 = n_2 L_2$
Path difference $= n_1 L_1 - n_2 L_2$
Now, phase difference

$$= \frac{2\pi}{\lambda} \times \text{path difference} = \frac{2\pi}{\lambda} \times (n_1 L_1 - n_2 L_2)$$

4. 2



5. 2 Frings width, $\beta = 12\text{ mm}$

Refractive index of water, $\mu = \frac{4}{3}$

The frings width is given by,

$$\beta = \frac{D\lambda}{d} \qquad \text{..... (i)}$$

Here, λ is wavelength of light.
D is distance between screen and source.
d is distance between coherent source.

If the entire arrangement is placed in water then fringes width becomes $\beta' = \frac{D\lambda'}{d}$ ..... (ii)

Dividing equation (ii) by (i), we have

$$\Rightarrow \frac{\beta'}{\beta} = \frac{\lambda'}{\lambda}$$

$$\Rightarrow \beta' = \frac{12 \times 3}{4} \qquad \left(\because \mu = \frac{\lambda}{\lambda'}\right)$$

$$\Rightarrow \beta' = 9\text{ mm}$$

6. 4 Let n_1 fringes are visible with light of wavelength λ_1 and n_2 with light of wavelength λ_2. Then

$$\frac{n_1 D\lambda_1}{d} = \frac{n_2 D\lambda_2}{d}$$

$$\Rightarrow \frac{n_2}{n_1} = \frac{\lambda_1}{\lambda_2} \Rightarrow n_2 = \frac{700}{400} \times 16 = 28$$

7. 4 For 'n' number of maximas

$d \sin\theta = n\lambda$

$0.32 \times 10^{-3} \sin 30° = n \times 500 \times 10^{-9}$

$\therefore \quad n = \frac{0.32 \times 10^{-3}}{500 \times 10^{-9}} \times \frac{1}{2} = 320$

Hence total no. of maximas observed in angular range –

$30° \leq \theta \leq 30°$

$= 320 + 1 + 320 = 641$

8. 1 Here, $x_1 = 2d$ and $x_2 = \sqrt{5d}$

For, first minima, $\Delta x = \frac{\lambda}{2}$

$\therefore \Delta x = x_2 - x_1 = \sqrt{5}d - 2d = \frac{\lambda}{2} \Rightarrow d = \frac{\lambda}{2(\sqrt{5}-2)}$

9. 3 $\beta_{\text{diffrac}} = 2\left(\frac{D\lambda}{a}\right)$, a = slit width

$\beta_{\text{interfer}} = \frac{D\lambda}{d}$ $\qquad \therefore \frac{2D\lambda}{a} = \frac{D\lambda \times n}{d}$

$\Rightarrow \frac{2}{a} = \frac{n}{d} \Rightarrow n = \frac{2d}{a} = \frac{2 \times 6.1a}{a} \simeq 12$

10. 1 Fringe width $B = \frac{D}{d}\lambda$

And number of fringes observed in the field of view is obtained by $\frac{d}{\lambda}$

11. 4 Let a_1 be the amplitude of light from first slit and a_2 be the amplitude of light from second slit.

$a_1 = a$, Then $a_2 = 2a$

Intensity $I \propto (\text{amplitude})^2$

$\therefore \quad I_1 = a_1^2 = a^2$

$I_2 = a_2^2 = 4a^2 = 4I$

$I_r = a_1^2 + a_2^2 + 2a_1a_2\cos\phi = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

$I_r = I_1 + 4I_1 + 2\sqrt{4I_1^2}\cos\phi$

$\Rightarrow \quad I_r = 5I_1 + 4I_1\cos\phi \qquad \ldots(1)$

Now, $I_{max} = (a_1 + a_2)^2 = (a + 2a)^2 = 9a^2$

$I_{max} = 9I_1 \Rightarrow I_1 = \frac{I_{max}}{9}$

Substituting in equation (1)

$I_r = \frac{5I_{max}}{9} + \frac{4I_{max}}{9}\cos\phi \quad \Rightarrow I_r = \frac{I_{max}}{9}[5 + 4\cos\phi]$

$\Rightarrow I_r = \frac{I_{max}}{9}\left[5 + 8\cos^2\frac{\phi}{2} - 4\right]$

$\Rightarrow I_r = \frac{I_{max}}{9}\left[1 + 8\cos^2\frac{\phi}{2}\right] = \frac{I_m}{9}\left(1 + 8\cos^2\frac{\phi}{2}\right)$

12. 1 For path difference of λ, the phase difference is 2π

For path difference of $\frac{\lambda}{6}$, the phase difference is

$\frac{2\pi \times \lambda/6}{\lambda} = \frac{\pi}{3}$

Resultant intensity

$I = I_0\cos^2\left(\frac{\pi/3}{2}\right) = I_0\cos^2\left(\frac{\pi}{6}\right) = \frac{3}{4}I_0$

So, $\frac{I}{I_0} = \frac{3}{4}$

13. 2 For constructive interference path difference (As $\sin\theta \le 1$)

$d \sin\theta = n\lambda$

Given $d = 2\lambda$

$\therefore 2\lambda \sin\theta = n\lambda \Rightarrow \sin\theta = \frac{n}{2}$

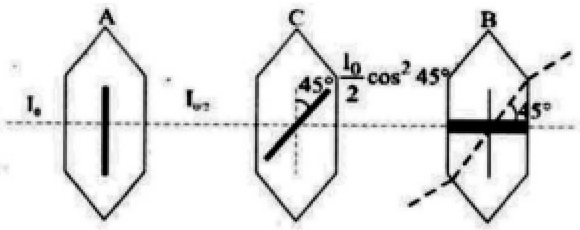
$n = 0, 1, -1, 2, -2$ hence five maxima are possible.

14. 2

15. 2 Angular width between first and second diffraction minima $\theta \simeq \frac{\lambda}{a}$ and angular width of fringe due to double slit is $\theta' = \frac{\lambda}{d}$.

$$\text{So, number of fringes} = \frac{\theta}{\theta'} = \left(\frac{\frac{\lambda}{a}}{\frac{\lambda}{d}}\right) = \left(\frac{d}{a}\right) = \frac{19.44}{4.05} = 4.81 \simeq 5$$

16. 3



$$\frac{I_0}{2}\cos^2 45°\cos^2 45° = \frac{I_0}{2} \times \frac{1}{4} = \frac{I_0}{8}$$

17. 1 After passing through the first sheet

$$I_1 = \frac{I}{2}$$

After passing through the second sheet

$$I_2 = I_1 \cos^2(45^\circ) = \frac{I}{4}$$

After passing through n^{th} sheet

$$I_n = \frac{I}{2^n} = \frac{I}{64}$$

$$n = 6$$

18. 2 Given :

Intensity, $I_0 = 3.3\ Wm^{-2}$

Area, $A = 3 \times 10^{-4} m^2$

Angular speed, $\omega = 31.4$ rad/s

Average energy $= I_0 A < \cos^2\theta > T$

$\because < \cos^2\theta > = \frac{1}{2}$

$\therefore$ Average energy $= \frac{(3.3)(3\times10^{-4})}{2} \times \frac{2\pi}{\omega} \approx 10^{-4}$ J

19. 1 Polariser A and B have same alignment of transmission axis.

Lets assume polariser c is introduced at θ angle

$$\frac{1}{2}\cos^2\theta \times \cos^2\theta = \frac{1}{3} \Rightarrow \cos^4\theta = \frac{2}{3} \Rightarrow \cos\theta = \left(\frac{2}{3}\right)^{1/4}$$

20. 4 From the Brewster's law, angle of incidence for total polarization is given by $\tan\theta = n$

$\Rightarrow \theta = \tan^{-1} n$

Where n is the refractive index of the glass.

21. 9 In young's double slit experiment, intensity at a point is given by

$$I = I_0 \cos^2 \frac{\phi}{2} \quad ...(i)$$

where, ϕ = phase difference,

Using phase difference, $\phi = \frac{2\pi}{\lambda} \times$ path difference

For path difference λ, phase difference $\phi_1 = 2\pi$

For path difference, $\frac{\lambda}{6}$, phase difference $\phi_2 = \frac{\pi}{3}$

22. 1.2 $a \sin\theta = n\lambda$

23. 30 Initialy polaroids have angle of 0° between them.
From the law of Malus,

$$I = \frac{I_0}{2}\cos^2\theta$$

Here I = resultant intensity on screen

$$\therefore \frac{I}{2}\cos^2\theta = \frac{3I}{8}$$

$$\Rightarrow \cos^2\theta = \frac{3}{4} \Rightarrow \cos^2\theta = \frac{\sqrt{3}}{4} \Rightarrow \theta = 30°$$

24. 198 For obtaining secondary minima at a point path difference should be integral multiple of wavelength

$$\therefore d\sin\theta = n\lambda \quad \Rightarrow \sin\theta = \frac{n\lambda}{d}$$

For n to be maximum $\sin\theta = 1$

$$n = \frac{d}{\lambda} = \frac{6 \times 10^{-5}}{6 \times 10^{-7}} = 100$$

Total number of minima on one side = 99
Total number of minima = 198.

PART - II (JEE ADVANCED LEVEL)

SECTION - III (One correct answer)

25 C $I = I_{max}\cos^2\left(\frac{\phi}{2}\right) \quad \frac{I_{max}}{4} = I_{max}\cos^2\frac{\phi}{2}$

$\cos\frac{\phi}{2}=\frac{1}{2}$ or $\frac{\phi}{2}=\frac{\pi}{3}$

$\phi=\frac{2\pi}{3}=\left(\frac{2\pi}{\lambda}\right)\cdot\Delta x$ ...(1) where $\Delta x = d\sin\phi$

Substituting in Eq. (1), we get

$\sin\phi=\frac{\lambda}{3d}$ or q = $\sin^{-1}\left(\frac{\lambda}{3d}\right)$

correct answer is (C).

26. A Path difference at 'C"

$$\Delta x=t_1\left({}_\omega\mu_g-1\right)-t_2\left({}_\omega\mu_g-1\right)={}_\omega\mu_g\left(t_1-t_2\right)-\left(t_1-t_2\right)$$

$$\Delta x=\left(t_1-t_2\right)\left({}_\omega\mu_g-1\right)$$

$$\Delta x=\left(t_1-t_2\right)\left(\frac{\mu_g}{\mu_\omega}-1\right)=\left(2.5-1.25\right)\left[\frac{1.4}{4}\times3-1\right]$$

$$=1.25\times\left[\frac{42}{40}-1\right]=1.25\times\frac{2}{40}=\frac{2.5}{40}$$

$$\Delta x=\frac{25}{400}=\frac{1}{16}\mu m.$$

Phase difference $(\phi)=\frac{2\pi}{\lambda_\omega}\times\Delta x$

$$\phi=\frac{2\pi}{\mu_a\lambda_a}\times\mu_\omega\times\Delta x=\frac{2\pi\times4}{1\times5000\times10^{-10}\times3}\times\frac{1}{16}\times10^{-6}$$

$$\phi=\frac{\pi\times10^{-6}}{30\times10^{-7}}=\frac{\pi\times10^{-6}}{3\times10^{-6}}=\frac{\pi}{3}$$

intensity at 'C'

$$I_C=4I_0\cos^2\left(\phi/2\right)=4I_0\cos^2\left(\pi/6\right)=4I_0\times\frac{3}{4}$$

$$I_C=3I_0$$

maximum intensity $I_{\max}=4I_0$

$$\frac{I_C}{I_{\max}}=\frac{3I_0}{4I_0}\Rightarrow\frac{I_C}{I_{\max}}=\frac{3}{4}$$

27. A Path difference due to slab should be integral multiple of l or $\Delta x = n\lambda$

or $(\mu-1)t=n\lambda,\ n=1,2,3$ or $t=\frac{n\lambda}{\mu-1}$

for minimum value of t, n = 1; $t=\frac{\lambda}{\mu-1}=\frac{\lambda}{1.5-1}=2\lambda$

28. D $x=(2n-1)\frac{\lambda D}{2d}$ 32. $n_1\lambda_1=n_2\lambda_2$ 33. $\theta=\frac{\lambda}{d}$

29. D $a\sin\theta = n\lambda$; $\theta = \frac{\lambda}{a}\frac{n\pi}{180} rad$

39. $Y = \frac{\lambda D}{a}$; w = 2Y

30. A Resolving power of eye $= \left(\frac{1}{60}\right)^0$

$$\frac{1}{60}\times\frac{\pi}{180} = \frac{d}{11000}$$

31. C $I_1 = \frac{I_0}{2}$; $I_2 = I_1\cos^2\theta$

$$I_3 = \frac{I_0}{2}\cos^2\theta\sin^2\theta$$

SECTION - IV (More than one correct answer)

32. A,B,C,D

Path difference at 'O' is $\Delta x = d$, which is maxima

(A) If $d = \frac{7\lambda}{2}$, the point 'O' will be minima.

(B) If $d = \lambda$, The point 'O' will be maxima

(C) If $d = 4.8\lambda$, then a total 10 minimas can be observed on screen 5 above 'O' and 5 below 'O', which corresponding to

$$\Delta x = \pm\frac{\lambda}{2}, \pm\frac{3\lambda}{2}, \pm\frac{5\lambda}{2}, \pm\frac{7\lambda}{2}, \pm\frac{9\lambda}{2}.$$

(D) If $d = \frac{5\lambda}{2}$, then 'O' will be minimum and hence intensity is minimum.

33. D Any where on the screen because there is no relation b/n θ & μ.

34. A Total path difference $\Delta x = (\mu - 1)t - d\sin\theta$. For central maxima $\Delta x = 0$ hence $(\mu - 1)t = d\sin\theta$

35. B Phase difference

$$\delta = (2n-1)\pi = [2(5)-1]\pi = 9\pi; \qquad \phi = 9\pi$$

SECTION - V (Numerical Type - Upto two decimal place)

36. 2 $D_2 - D_1 = 50cm$; $D_1 = 1.5m = 150cm$

$\lambda = 6000Å = 6000\times10^{-8}cm$; $d = 0.15mm$

Change in fringe width $\Delta\beta = \beta_2 - \beta_1$

$$\Delta\beta = (D_2 - D_1)\frac{\lambda}{d} = \frac{50\times6000\times10^{-8}}{0.15\times10^{-1}}$$

$$\Delta\beta = \frac{20\times10^{-4}}{10^{-3}} = 2\times10^{-1}cm = 2mm \qquad \Delta\beta = 2mm$$

37. 4 For λ path difference phase difference $= 2\pi$

$$I = I_0 \cos^2\left(\phi/2\right) = I_0 \cos^2\left(\frac{2\pi}{2}\right) = I_0(+1) = I_0$$

For $\frac{\lambda}{3}$ path differece phase difference

$$\phi^1 = \frac{2\pi}{\lambda} \times \frac{\lambda}{3} = \frac{2\pi}{3}$$

$$I^1 = I_0 \cos^2\left(\phi^1/2\right) = I_0 \cos^2\left(\frac{2\pi}{3\times 2}\right) = I_0 \cos^2\left(\frac{\pi}{3}\right)$$

$$I^1 = \frac{I_0}{4} = \frac{I}{4} = \frac{I}{P} \Rightarrow P = 4$$

38. 2 $n_1\beta_1 = n_2\beta_2 \Rightarrow n_1\frac{\lambda_1 D}{d} = n_2\frac{\lambda_2 D}{d}$

$$n_1\lambda_1 = n_2\lambda_2 \ ; \quad n_b\lambda_b = n_R\lambda_R$$

$$(n+1)\times 5\times 10^{-5} = n\times 7.5\times 10^{-5}$$

$$n+1 = 1.5n \Rightarrow 0.5n = 1 \Rightarrow n = \frac{1}{0.5}$$

$$n = \frac{10}{5} = 2 \Rightarrow n = 2$$

39. 9 As the amplitude are A and 2A then the ratio of intensities is 1:4

$$I_{max} = I_0 = I_1 + I_2 + 2\sqrt{I_1 + I_2} = I + 4I + 2\times 2I$$

$$I_0 = 9I \Rightarrow I = \frac{I_0}{9}$$

Intensity at any point:

$$I^1 = I_1 + I_2 + 2\sqrt{I_1 + I_2}\cos\phi$$

$$I^1 = I + 4I + 2\sqrt{I\times 4I}\cos\phi$$

$$I^1 = 5I + 4I\cos\phi \ ; \quad I^1 = I[5 + 4\cos\phi]$$

$$I^1 = \frac{I_0}{9}[5 + 4\cos\phi] = \frac{I_0}{P}[5 + 4\cos\phi]$$

$$P = 9$$

SECTION - VI (Matrix Matching)

40. A-QR; B-PQR; C-PST; D-QRST

Angular fringe width distance b/n slits

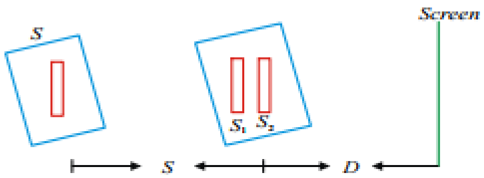
$\omega = \dfrac{\lambda}{d} = \dfrac{\beta}{D}$ $\quad D \to$ distance b/n slits & screen

The arrangement for YDSE is shown in figure. $\quad \lambda \to$ Wavelength of light used

Let the size of source and separation between $\quad \beta \to$ Fringe width

source slit and double slit plane is S. Then for interference fringes to be observed $\dfrac{s}{S} < \dfrac{\lambda}{d}$.

If this condition is not satisfied then interference pattern produced by different parts of the source overlap and fringe pattern disappears.



For p: As 'D' is increased, angular fringe width $\omega = \dfrac{\lambda}{d}$ remains same as it is independnet of D.

As D increase,fringe width increases $\beta = \dfrac{\lambda D}{d}$ $\quad p \to B, C$

For q: When λ is decreased, angular fringe width (ω) and fringe width β decreases and fron the condition as λ decreases and from the condition $\dfrac{s}{S} < \dfrac{\lambda}{d}$ as λ decreases this condition would be failed and fringe pattern disappears. $\quad q \to A, B, D$

For r: 'd' is increasing ω and β both decreases and fringe pattern disappears. $\quad r \to A, B, D$

For s: As the source slit wodth increases, the condition $\dfrac{s}{S} < \dfrac{\lambda}{d}$ would be violated at some instant and fringe pattern disappears but there is no effect on ω and β. $\quad s \to C, D$

For t:As the distance b/n source slit and double slit 'S' is decreasing then fringe pattern disappears but ω and β has no effect. $t \to C, D$