CHAPTER - 01

UNITS AND MEASUREMENTS

SYNOPSIS

1. Measurement of any physical quantity involves comparison with a certain basic, arbitrarily chosen internationally accepted reference standard called **unit**
2. Physical quantities that are independent of others are called fundamental quantities. Physical quantities that can be defined in terms of the base quantities are called derived quantities. The units for the fundamental or base quantities are called **fundamental or base units.** The units of all other physical quantities can be expressed as a combination of the base units. Such units obtained for the derived quantities are called **derived units**
3. A system of units is a family of units of fundamental and derived physical quantities. The system of units which is at present internationally accepted for measurement is the "system international d units" abbreviated as SI.

   Other system of units are

   a) C.G.S (Centimetre, Gram, Second)

   b) F.P.S (Foot, Pound, Second)

   c) M.K.S (Metre, Kilogram, Second)

SI base quantities and units

Base quantity	SI unit	
	Name	symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K or θ
Amount of substance	mole	mol
Luminous intensity	candela	cd

Besides these seven base units (given above) there are two more units that are defined for plane angle and solid angle. The **unit for Plane angle is radian (rad)** and unit for the **solid angle is steradian (sr).**

4. **Practical units of Lengths**

1 fermi = 1f = 10^{-15} m; 1 angstrom = $1\overset{0}{A}$ = 10^{-10}m

1 astronomical unit = 1Au mean (distance of the sun from the earth) = 1.496 x 10^{11}m

1 light year = 1ly = 9.46 x 10^{15}m (distance that light travels with velocity of 3 x 10^{8} ms^{-1} in 1 year)

1 par sec = 3.26 light years = 3.08 x 10^{16} m

(Note that parsec is an abbreviation of parallactic second)

Dimensions of physical quantities

1. The dimensions of a physical quantity are the powers to which the base quantities are raised to represent that quantity.
2. The nature of a physical quantity is described by its dimensions
3. All the physical quantities represented by derived units can be expressed in terms of some combination of seven fundamental or base quantities.
4. Length has the dimension [L], mass [M], time [T], electric current [A], thermodynamic temperature [K] luminous intensity [Cd] and amount of substance [mol]
5. Using the square brackets [ ] around a quantity means that we are dealing with the dimensions of that quantity
6. In mechanics all the physical quantities can be written in terms of dimensions [L], [M] and [T]

   eg: Force = mass x acceleration = mass x (length) /(time)^2

   ie the dimensions of force are $\frac{[M][L]}{[T]^2}$ = [MLT^{-2}]

   ie force has one dimension in mass, one dimension in length and –2 dimensions in time. The dimensions in all other base quantities are zeros

   In this type of representation, the magnitudes are not considered.
7. The dimensional equations are the equations which represent the dimensions of a physical quantity in terms of the base quantities

   for eg: the dimensional eqn of volume [V] may be expressed as

   [V] = [$M^0L^3T^0$]
8. Only those physical quantities can be added or subtracted which have the same dimensions. ie velocity cannot be added to force
9. The arguments of special functions, such as the trignometric, logarithmic and exponential functions are dimensionless. A pure number, ratio of similar physical quantities has no dimensions

   Dimensionless quantities

   Angle, solid angle, trigonometrical ratios, relative density, relative permittivity, relative permeability, Poisson's ratio, strain, refractive index, mechanical equivalent of heat (Joules constant), π, angular displacement, emissivity, thermo-dynamic constant $\left(\gamma = \frac{C_p}{C_v}\right)$, limit of resolution of telescope (angle), Power factor $(\cos\phi)$, form factor, quality factor (Q), amplication factor (μ), and efficiency have no dimensions and is equal to $M^0L^0T^0A^0\theta^0cd^0mol^0$.
10. **Different quantities having same dimensions**

    a.Work, energy, heat, torque, couple, moment of force have same dimensions viz $\left[ML^2T^{-2}\right]$. Units of energy like erg, joule, calorie, electron volt and kWh also have the same dimensions. Potential energy (mgh), kinetic energy $\left(\frac{1}{2}mv^2 or \frac{1}{2}I\omega^2\right)$, energy contained in an inductance $\left(\frac{1}{2}LI^2\right)$, electrostatic energy of condenser $\left(\frac{1}{2}QV, \frac{1}{2}CV^2, \frac{Q^2}{2C}\right)$ and energy spent in a resistance $\left(I^2Rt, VIt, \frac{V^2}{R}t\right)$ also have the dimensions $\left[ML^2T^{-2}\right]$.

b. Linear momentum and linear impulse have same dimensions ie, $\left[MLT^{-1}\right]$

c. Angular momentum, angular impulse and Planck's constant have same dimensions ie, $\left[ML^2T^{-1}\right]$

d. Stress, pressure, Young's modulus, rigidity modulus, bulk modulus of elasticity and energy density have the same dimensions ie, $\left[ML^{-1}T^{-2}\right]$

e. Surface tension, surface energy, spring constant and force gradient have same dimensions ie, $\left[ML^0T^{-2}\right]$

f. Thermal capacity, entropy, Boltzmann's constant have same dimensions ie, $\left[ML^2T^{-2}\theta^{-1}\right]$

g. Velocity gradient, frequency and angular speed have same dimensions ie, $\left[T^{-1}\right]$

h. Latent heat and square of velocity have dimension $\left[M^0L^2T^{-2}\right]$

i. Speed and velocity have dimension $\left[M^0LT^{-1}\right]$

j. Distance, displacement, amplitude, wavelength and focal length have dimension $\left[M^0L^1T^0\right]$

k. Time, L/R (Inductance/Resistance), CR (Capacitance × resistance) and $\sqrt{LC}$ have dimension $\left[M^0L^0T^1\right]$

l. Dimension of frequency, $R/L, \frac{1}{CR}$ and $\frac{1}{\sqrt{LC}}$ are denoted by $\left[M^0L^0T^{-1}\right]$

m. Dimension of velocity, $\sqrt{\text{Latent Heat}}$ and $\frac{1}{\sqrt{\mu_0 \in_0}}$ are denoted by $\left[M^0LT^{-1}\right]$. where μ_0 is magnetic permeability and $\in_0$ is the electrical permittivity.

11. A dimensionally correct equation need not be actually a correct equation, but dimensionally wrong, incorrect or inconsistent equation must be wrong.

12. Uses of dimensions

    i) Conversion of units from one system of units to another

    ii) To check the dimensional correctness of a given physical equation

    iii) To establish the relation among various physical quantities

13. Limitations

If a physical quantity depends on more than three factors, the relation among them cannot be established because we can have only three equation by establishing the powers of M,L and T.

14. Principle of Homogeneity of Dimensions

This principle states that the dimensions of all the terms in a physical expression should be the same. For eg, in the physical expression $s = ut + 1/2\ at^2$, the dimensions of s, ut, $1/2at^2$ all are same. The

physical quantities separated by the symbols
+, –, =, >, < etc have the same dimensions.

Least count

The smallest division on the scale of a measuring instrument is called its least count.

Significant figures

Significant figures indicate, the precision of the measurement which depends on the least count of the measuring instrument.

The reliable digits plus the first uncertain digit are known as the significant digits or significant figures.

Common rules for counting significant figure.

1. A choice of change of different units does not change the number of significant digits or figure in a measurement
2. All the non zero digits are significant
3. All the zeros between two non - zero digits are significant, no matter where the decimal point is, if at all
4. If the number is less than 1, the zeros on the right of decimal point but to the left of the first non zero digit are not significant
5. The trailing zeros in a number with a decimal point are significant
6. The terminal or trailing zeros in a number without a decimal point are not significant.

Rules for Arithmetic operations with significant figures

1. In multiplication and devision, the final result should retain as many significant figures as there are in the original number with the least significant figures.
2. In addition and subtraction, the final result should retain as many decimal places as there are in the number with the least decimal places.

Rounding off the uncertain digit

1. Preceding digit is raised by 1 if the insignificant digit to be dropped is more than 5, and is left unchanged if the latter is less than 5.
2. If the insignificant digit is 5 and if the preceding digit is even, the insignificant digit is simply dropped and if it is odd, the preceding digit is raised by 1.

Error

5. The result of every measurement by any measuring instrument is an approximate number, which contains some uncertainty. This uncertainty is called error

The accuracy of a measurement is a measure of how close the measured value is to the true value of the quantity Precision tells us to what resolution or limit the quantity is measured.

In general, the error in measurement can be broadly classified as (a) systematic errors and (b) random errors.

a) Systematic errors: are those errors that tend to be in one direction either positive or negative. some of the sources are

i) Instrumental errors (due to imperfect design or calibration, zero error)

ii) Imperfection in experimental technique or procedure (due to changes in external condition)

iii) Personal errors

c) Random errors

These are those errors which are irregular and thus random in nature with respect to their sign or size. The causes of these errors are not known. The same person may get different readings for the same measurement on two occasions.

Types of errors

a) Absolute error

a = actural value

a_i = i^{th} observed value

absolute error, $\Delta a_i = a_i - a$

b) Relative or fractional error

The relative error in the ith reading is = $\frac{\Delta a_i}{a}$

= $\frac{\text{absolute error}}{\text{actual value}}$

c) Percentage error

= $\frac{\Delta a_i}{a} \times 100$

Determination of the actual value

No instrument can give the actual value because every instrument has a least count ie a certain precision. So arithmetic mean of all readings is assumed to be the actual value a.

Note: The arithmetic mean of all the absolute errors is taken as the final mean absolute error of the value of the physical quantity a.

Combination of errors

a) Error of a sum or a difference

Rule: when two quantities are added or subtracted, the absolute error in the final result is the **sum of the absolute errors** in the individual quantities.

b) Error of a product or a quotient

Rule: When two quantities are multipled or divided, the maximum fractional error in the result is the sum of the **fractional errors in the individual quantities**

c) Error due to the power of a measured quantity, fractional error or relative error in a quantity raised to power (n) is n times fractional / relative error in the quantity itself.

if $Z = \frac{A^P B^q}{C^r}$ $\Delta Z / Z = P(\Delta A / A) + q(\Delta B / B) + r(\Delta C / C)$

Rule: The fractional error in a physical quantity raised to some power is the power times the fractional error in the individual quantities

Eg: Find the fractional error in Z, If Z = $A^4 B^{1/3} / CD^{3/2}$

Ans: The fractional error in Z is,

$$\frac{\Delta Z}{Z}=4\left(\frac{\Delta A}{A}\right)+\left(\frac{1}{3}\right)\left(\frac{\Delta B}{B}\right)+\left(\frac{\Delta C}{C}\right)+\frac{3}{2}\left(\frac{\Delta D}{D}\right)$$

PART I - JEE MAIN

SECTION - I - Straight objective type questions

1. Par sec is the unit of

   1) Time 2) Distance

   3) Frequency 4) Angular acceleration

2. The advantage(s) of choosing the wavelength of a particular light radiation as a standard of length is/are

   I. it can be easily and accurately reproduced

   II. it is not affected by environmental conditions

   III. it is very sensitive to environmental conditions

   IV. it is independent of other physical quantities

   Which of the above statement(s) is/are correct?

   1) I only 2) I and II 3) I and III 4) I, II and IV

3. Which of the following quantities has not been expressed in proper unit ?

   a) Torque : Newton metre

   b) Stress : Newton metre^{-2}

   c) Modulus of elasticity : Newton metre^{-2}

   d) Power : Newton metre second

   e) Surface tension : Newton metre^{-2}

   1) a,b 2) a,c 3) d,e 4) b,c

4. A force is given by $F = at + bt^2$, where t is time. What are the dimension of a and b

   1) MLT^{-3} and ML^2T^{-4} 2) MLT^{-3} and MLT^{-4} 3) MLT^{-3} and MLT^0 4) MLT^{-4} and MLT^1

5. A dimensionless quantity

   1) never has a unit 2) always has a unit 3) may have a unit 4) does not exist

6. If E, M, J and G, respectively, denotes energy, mass, angular momentum and gravitational constant, EJ^2/M^5G^2 has the dimensions of

   1) length 2) angle 3) mass 4) time

7. The dimensions of $(\mu_0\varepsilon_0)^{-(1/2)}$ are

   1) $[L^{1/2}T^{1/2}]$ 2) $[L^{1/2}T^{-1/2}]$ 3) $[L^{-1}T]$ 4) $[LT^{-1}]$

8. A student writes four different expressions for the displacement y in a periodic motion as a function of time t, a as amplitude, T as time period. Which of the following can be correct?

1) $y = aT\sin\frac{2\pi t}{T}$ 2) y = a sin Vt

3) $y = \frac{a}{T}\sin\frac{t}{a}$ 4) $y = \frac{a}{\sqrt{2}}\left[\sin\frac{2\pi t}{T} + \cos\frac{2\pi t}{T}\right]$

9. From the point of view of significant figures, which of the following statements are correct

i) 10.2 cm + 8 cm = 18.2 cm ii) 2.53 – 1.2 m = 1.33 m

iii) 4.2 m × 1.4 m = 5.88 m^2 iv) $3.6\,m \div 1.75\,s = 2.1\,m/s$

1) i only 2) i and iv 3) i and ii 4) iv only

10. In simple pendulum experiment percentage error in length and time period are 2% and 1% respectively. The percentage error in the calculation of g is :

1) 1% 2) 2% 3) 3% 4) 4%

11. A public park, in the form of a square, has an area of $(100 \pm 0.2)m^2$. The side of park is:

1) $(10 \pm 0.01)m$ 2) $(10 \pm 0.1)m$ 3) $(10 \pm 0.02)m$ 4) $(10 \pm 0.2)m$

12. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{L/g}$. Measured value of L is 10 cm known to 1mm accuracy and time for 100 oscillations of the pendulum is found to be 50s using a wrist watch of 1s resolution.What is the accuracy in the determination of g?

1) 2% 2) 3% 3) 4% 4) 5%

SECTION - II

Numerical Type Questions

13. In a new system of units, the unit of mass is 100 g, unit of length is 4 m and unit of time is 2 s. Find the numerical value of 10 J in this system.

14. The length of a wire is 2.17 cm and radius is 0.46 cm. Find number of significant digits in the value of volume of wire.

PART - II (JEE ADVANCED)

SECTION - III (One correct answer)

15. If the constant of gravitational (G), Planck's constant (h), and the velocity of light (c) be chosen as fundamental units. The dimension of the radius of gyration is

A) $h^{\frac{1}{2}}c^{\frac{-3}{2}}G^{\frac{1}{2}}$ B) $h^{\frac{1}{2}}c^{\frac{1}{2}}G^{\frac{1}{2}}$ C) $h^{\frac{1}{2}}c^{\frac{-3}{2}}G^{\frac{-1}{2}}$ D) $h^{\frac{-1}{2}}c^{\frac{-3}{2}}G^{\frac{1}{2}}$

16. A, B, C and D are four different physical quantities having different dimensions. None of them is dimensionless. However, we know that the equation AD=C ln (BD) holds true. Then, which of the following combination is not a meaningful quantity?

A) $\frac{C}{BD}-\frac{AD^2}{C}$ B) $A^2-B^2C^2$ C) $\frac{A}{B}-C$ D) $A-\frac{C}{D}$

17. Two full turns of the circular scale of a screw gauge cover a distance of 1 mm on its main scale. The total number of divisions on the circular scale is 50. For their, it is found that the screw gauge has a zero error of -0.03mm while measuring the diameter of a thin wire, a student notes the main scale reading of 3mm and the number of circular scale divisions in line with the main scale as 35. The diameter of the wire is

A) 3.32 mm B) 3.73 mm C) 3.67 mm D) 3.38 mm

18. A student uses a simple pendulum of exactly 1m length to determine g, the acceleration due to gravity. He uses a stop watch with the least count of 1s for this and records 40s for 20 oscillations. For, this observation, which of the following statement(s) is (are) true?

I. Error ΔT in measuring *T*, the time period , is 0.05s

II. Error ΔT in measuring *T*, the time period, is 1 second

III. Percentage error in the determination of g is 5%

IV. Percentage error in the determination of g is 2.5%

A) I and III B) I and IV C) II and III D) II and IV

SECTION - IV (More than one correct answer)

19. Identify the correct statement/statements from the following :

A) Zero error has to be subtracted from the observed reading to get actual reading

B) Zero error has to be added to the observed reading to get actual reading

C) Zero correction has to be subtracted from the observed reading to get actual reading

D) Zero correction has to be added to the observed reading to get the actual reading

20. The diameter of a cylinder is measured using a vernier callipers with no zero error. It is found that the zero of the vernier scale lies between 5.10 cm and 5.15 cm of the main scale. The vernier scale has 50 divisions equivalent to 2.45 cm. The 24th division of the vernier scale exactly coincides with one of the main scale divisions.

A) The diameter of the cylinder is 5.148 cm

B) The diameter of the cylinder is 5.124 cm

C) The least count of the vernier callipers is 0.001 cm

D) The least count of the vernier callipers is 0.01 cm

21. Which of following pairs have the same dimensions?

(L= inductance, C = capacitance, R = resistance)

1) $\frac{L}{R}$ and CR 2) LR and CR 3) $\frac{L}{R}$ and $\sqrt{LC}$ 4) RC and $\frac{1}{LC}$

22. Consider three quantities : $x=\frac{E}{B}, y=\frac{1}{\sqrt{\mu_0\varepsilon_0}}$ and $z=\frac{\ell}{CR}$. Here, ℓ is the length of a wire, C is the capacitance, R is a resistance, E is electric field and B is magnetic field. All other symbols have usual meanings. Then

1) x and y have the same dimensions

2) x and z have the same dimesnions

3) y and z have the same dimensions

4) None of the above three pairs have the same dimensions

SECTION - V (Numerical Type - Upto two decimal place)

23. The parallax of distant planets as measured from two diametrically opposite ends of earth is 1 minute. The distance of planet from the earth is $n \times 10^{10}$ m. The value of n is (given radius of the earth = 6400 km)

24. If there is a positive error of 50% in the measurement of velocity of a body, then the error in the measurement of kinetic energy is $n \times 10^2$ %. The value of n is :

25. A student measured the diameter of a small steel ball using a screw gauge of least count 0.001 cm. The main scale reading is 5 mm and zero of circular scale division coincides with 25 division above the reference level. If screw gauge has a zero error of –0.004 cm, the correct diameter of the ball in cm is :

SECTION - VI (Matrix Matching)

26. In a system of units, unit of mass is α kg, the unit of length is βm and unit of time is γ second. Then match the two columns

Column I		Column II	
i.	1 N in new system	a.	$\alpha^{-1}\beta^{-2}\gamma^{2}$
ii.	1 J in new system	b.	$\alpha^{-1}\beta^{-1}\gamma^{2}$
iii.	1 Pascal (SI unit of pressure) in new system	c.	$\alpha^{-1}\beta\gamma^{2}$
iv.	1 Watt in new system	d.	$\alpha^{-1}\beta^{-2}\gamma^{3}$

A) i-b, ii-a, iii-c, iv-d

B) i-c, ii-b, iii-d, iv-a

C) i-a, ii-b, iii-d, iv-c

D) i-c, ii-d, iii-a, iv-b