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# ELECTROSTATICS - I
# ELECTRIC CHARGES AND FIELDS

## ELECTROSTATICS - I

Electricity is the branch of physics which deals with study of charges. The studies are classified into two types

**(i)** **Electrostatics** : This deals with study of charges at rest

**(ii)** **Electrodynamics or current electricity** : This deals with study of charges in motion

## Frictional Electricity

Consider two neutral bodies. Electrical neutrality is due to the equality in the number of electrons and protons. When they are rubbed together, heat energy is developed at their contact surface due to friction. Using this energy, some electrons are transfered from one body to another. Body loosing the electrons gain a positive charge due to deficiency of electrons and the body gaining these electrons gain an equal negative charge due to excess of electrons. These charges are known as frictional electricity.

e.g : When glass rod is rubbed with silk cloth, electrons are transferred from glass to silk. So glass gets a positive charge and silk an equal negative charge.

Usually, electrons are removed from the body in which they are less tightly bound to the nucleus and are transferred to the body in which they are more tightly bound to the nucleus. The minimum energy required to remove an electron from a body is known as work function. So electrons are removed from body with lesser work function and transferred to the body with more work function. So work function is less for glass and more for silk.

The least value of charge that can be transferred between two bodies when they are rubbed together is the charge of a single electron, e = $1.6 \times 10^{-19}$ C. So it is treated as the basic unit of charge. So the positive or negative charge that can appear on a body will be an integer multiple of electronic charge. This fact is called quantisation of charge.

$$Q = \pm ne \; ; \; n = 1, 2, 3, .....$$

Here n is the number of electrons transferred when two bodies are rubbed or the excess number of electrons in a negative body or deficient number of electrons in a positive body.

eg : Consider a body with 1C positive charge. Then deficient number of electrons are

$$n = \frac{Q}{e} = \frac{1}{1.6 \times 10^{-19}} = 6.25 \times 10^{18}$$

If a charge Q flows for a time t, then current established; $I = \frac{Q}{t}$ $\therefore Q = It$

$$\therefore \; ne = Qt$$

Here n is the number of free electrons move through the wire.

Unit of charge : Coulomb (SI)

esu of charge (cgs)

IC = 3 × $10^9$ esu

Dimensions of charge = [AT]

## Properties of charge

(i) Charge is quantised

(ii) For an isolated system, total charge is conserved

(iii) Charge is a scalar

(iv) Charges can flow and the direction of charge flow between two bodies is determined by their potential values.

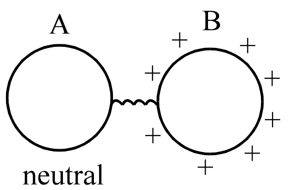
(v) When two identical conducting bodies are made in contact, total charges are equally shared between them. But this is not true for non identical bodies.

(vi) Charges accumulate at the sharp edges of a conducting body.

(vii) Like charges repel while unlike charges attract. But attraction is possible between a charged body and a neutral body. Repulsion happens only between charged bodies. So only repulsion characterise a charge.

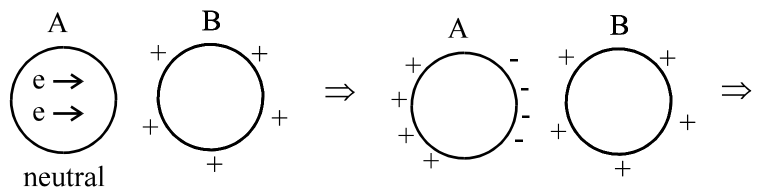
## Charging by conduction

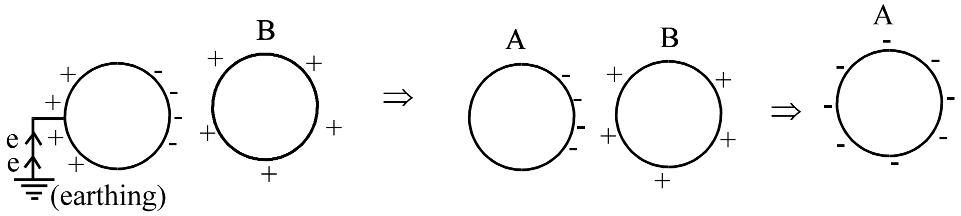
It is a method of charging in a neutral conductor by using a direct contact with a charged body. When a contact is made, free electrons travel from A to B. Hence positive charge on B reduces but due to electron loss an equal positive charge appears on A. Here loss of charge in one body results a gain in charge on the other. Also only similar type of charge can be produced.




## Charging by induction

This is a charging method which does not require a direct contact between bodies.

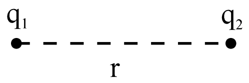



So induction followed by proper earthing is required for charging by induction.

## Coulomb's law

It gives the electrostatic force between two point charges.




$F \propto \frac{q_1 q_2}{r^2}$ or $\boxed{F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}}$

$\varepsilon \rightarrow$ absolute electrical permittivity of the medium between the charges. If free space or vacuum is present between the charges, $\varepsilon$ is used as $\varepsilon_0$, the permittivity of free space. $\varepsilon_0 = 8.85\times10^{-12}\,C^2/Nm^2$.

In such a case; $\boxed{F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}}$ $\frac{1}{4\pi\varepsilon_0} = 9\times10^9$

Unit of $\varepsilon$ is $C^2/Nm^2$.

Dimensions of $\varepsilon = \frac{A^2T^2}{MLT^{-2}L^2} = \left[M^{-1}L^{-3}T^4A^2\right]$

$\varepsilon$ and $\varepsilon_0$ are related as; $\varepsilon = K\varepsilon_0$ or $\varepsilon = \varepsilon_r\varepsilon_0$

$K \rightarrow$ dielectric constant or $\varepsilon_r \rightarrow$ relative permittivity of the medium. It is the property of a medium by which it opposes the passage of electric interaction through it. K values are ;

| **Vacuum** | **air** | **good conductor (metal)** | **Perfect insulator** |
|---|---|---|---|
| K = 1 | $K \approx 1$ | $K = \infty$ | K = 0 |

$\therefore\ F_{air} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$. If a dielectric medium K is filled in the entire spacing between charges.

$F_{med} = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon_0 K}\frac{q_1 q_2}{r^2}$

$\boxed{F_{med} = \frac{F_{air}}{K}}$

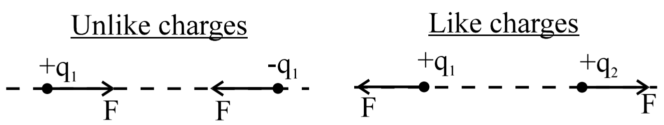
## Characteristics of electric force

(i) It is a conservative force

(ii) It is either repulsive or attractive

(iii) It is a central force. So the direction of electric force between charges always acts along the line joining their centres

(iv) Direction of attractive force on a charge is always towards the other charge and the direction of repulsive force on a charge is always away from the other charge along the line joining their centres.



(v) Force between two charges is independent of presence of other charges but depends on the medium between charges.

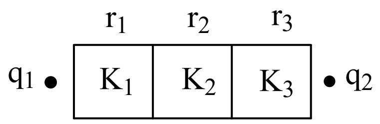
(vi) Electric force is a strong force

(viii) If a medium of dielectric constant K is filled between the charges, force is

$$F=\frac{1}{4\pi\varepsilon_0 K}\frac{q_1q_2}{r^2}=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{\left(r\sqrt{K}\right)^2}=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{d^2}$$

$d=r\sqrt{K}$, is the interactive distance between charges. This idea is used to find the force between charges when more than one dielectrics are present between the charges.

eg :




$$F=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{d^2}$$

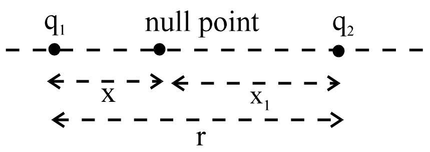
$$d=r_1\sqrt{K_1}+r_2\sqrt{K_2}+r_3\sqrt{K_3}$$

**Null point (Neutral point)**

It is a point in a region of charges where the net electric force is zero.

**e.g. 1 : Two like point charges**

Here null point lies between the charges on the line joining them. Null point will be more close to the charge with lesser magnitude.

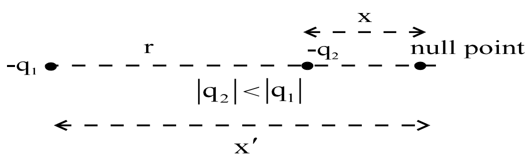



$x=\dfrac{r}{1+\sqrt{\dfrac{q_2}{q_1}}}$ from $q_1$.

$x'=\dfrac{r}{1+\sqrt{\dfrac{q_1}{q_2}}}$ from $q_2$

**e.g. 2 : Two unlike point charges**

Here null point lies outside the system of charges on the line joining them, more close to the charge with lesser magnitude.




$x=\dfrac{r}{1-\sqrt{\dfrac{q_2}{q_1}}}$ from $q_1$

$x'=\dfrac{r}{1+\sqrt{\dfrac{q_1}{q_2}}}$

**e.g. 3** : If equal charges are placed at all the corners of a polygon of equal sides, its geometrical centre is a null point

## ELECTRIC FIELD

It is a region surrounding a charge where the influence due to that charge exist. Charge producing the field is called a source charge and all other charges are test charges. Any point outside this region is called in finity. The word influence means any test charge entering into this region will experience a force of attraction or repulsion.

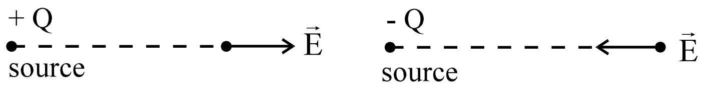
The strength of influence at a point in the field is given by a vector called electric field intensity ( $\vec{E}$ ). At a point it is defined as the force experienced by a unit positive charge placed at that point.

$E = \frac{F}{q_0}$ Unit = $\frac{N}{C}$ or $\frac{V}{m}$ $[E] = [MLT^{-3}A^{-1}]$

Theoretically this test charge ($q_0$) must be selected infinitesimally small so that its field at its position can be neglected.

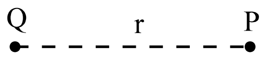
$$\therefore \; E = \lim_{q_0 \to 0} \left( \frac{F}{q_0} \right)$$

Direction of electric field depends on the sign of source charge. If source charge is (+), direction of $\vec{E}$ at a point is directed away from source charge and if source charge is negative, it is directed towards the source charge along the line joining the point to the centre of the source charge.



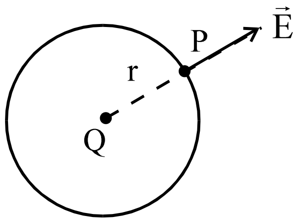

## Electric field due to a point charge

Consider a point P at a distance r from a point charge Q.



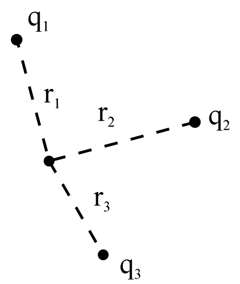

Then field at P, $\boxed{E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}}$

Consider a sphere of radius r with P on the surface and Q at centre. Then $\vec{E}$ acts radially outwards. If Q is negative, $\vec{E}$ is directed radially inwards. That is why the electric field produced by a point charge is called radial electric field.



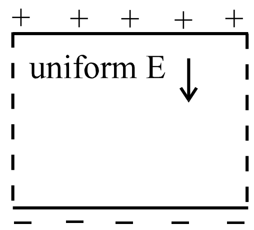

Consider a number of point charges $q_1$, $q_2$, $q_3$...... in a region. P is a point in the region at respective distances $r_1$, $r_2$,....... from the charges.

If $\vec{E}_1, \vec{E}_2, \vec{E}_3$,....... are the electric fields at P due to these charges, then net field at P can be calculated using superposition principle as $\vec{E} = \vec{E}_1 + \vec{E}_2 + .......$




**Uniform Electric field**

It is an electric field where both value and direction of field is same at every point. To produce it, we need to take two identical metallic plates and place them close and parallel to each other. Give a positive charge to one plate and equal negative to the other. Then in the region between the plates field will be uniform, directed from positive to negative plate.

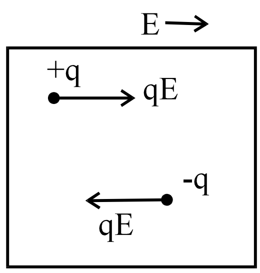



If field varies by value or direction or both from point to point, it is a non uniform $\vec{E}$ field.

**Behaviour of a charge placed in a uniform $\vec{E}$**

Consider a charged particle of mass m and charge q placed at a point in a uniform $\vec{E}$.

When field is switched ON, at once charge experience a force F = qE. If charge is positive, direction of force is same as the direction of field and if charge is negative, direction of force is opposite to the direction of field. Due to this force, charge starts moving with an acceleration a.




F = ma

$$\therefore \ ma = qE \Rightarrow \boxed{a = \frac{qE}{m}} \text{(uniform)}$$

So after time t ;

Velocity V = u + at

$$\Rightarrow \ V = \frac{qEt}{m}$$

displacement, $S = ut + \frac{1}{2}at^2 \ \Rightarrow \ S = \frac{qEt^2}{2m}$

KE gained = $\frac{1}{2}mV^2 = \frac{q^2E^2t^2}{2m} = qES$

Here initial KE is zero for particle. But after a time t, it gains a KE. So KE of a charged particle changes in an electric field.

Then using work-energy theorem we can conclude that, the electric field can do work on a charge given by ;

$$W = \vec{F}.\vec{S} \qquad \vec{F} = q\vec{E}$$

$$\boxed{W = q\left(\vec{E}.\vec{S}\right)} \qquad \vec{S} \rightarrow \text{displacement}$$

So using work-energy theorem,

work done = change in KE

$$\boxed{q\left(\vec{E}.\vec{S}\right) = \frac{1}{2}m\left(V^2 - u^2\right)}$$

u $\rightarrow$ initial speed

V $\rightarrow$ final speed

Power delivered by $\vec{E}$ to change at any instant

$$P = \vec{F}.\vec{V}$$

$$\boxed{P = q\left(\vec{E}.\vec{V}\right)}$$

Hence an $\vec{E}$ can do work and deliver power to a charge. It can change the speed and also KE of charge. But magnetic field cannot do any of these things on a charged particle. Both electric field and magnetic field can deflect a charge.

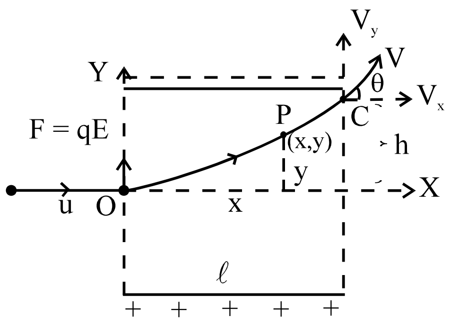
## Charge Entering a uniform $\vec{E}$

Consider a charged particle of charge q and mass m entering a uniform electric field E with a speed u

**(ii) Antiparallel Entry**

Here the above cases are just reversed. That is when a (+) charge enter antiparallel to the field, work done by field on the charge is either negative or zero. When a (-) charge enter antiparallel to the field, work done by field on the charge is always positive. During parallel and antiparallel entry, charge follows straight line paths.

**(iii) Perpendicular Entry**




Due to F, charge deflects into a curved path as shown. Let it reaches a point P after time t.

**X - direction**

$U_x = U, a_x = 0$

$S_x = x$

$S_x = U_x t + \frac{1}{2} a_x t^2$

$x = Ut$

$t = \frac{x}{U} \rightarrow (1)$

**Y- direction**

$U_y = 0,\ a_y = \frac{qE}{m},\ S_y = y$

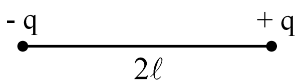
$S_y = U_y t + \frac{1}{2} a_y t^2$

$y = 0 + \frac{1}{2}\frac{qE}{m}\frac{x^2}{U^2}$

$\boxed{y = \frac{qEx^2}{2mU^2}}$ $y \propto x^2$

## Electric Dipole

A pair of equal and opposite point charges separated by a short vector distance is called an electric dipole.




Vector separation is known as dipole length. Its direction is from - q to + q.

A dipole is characterised by its electric dipole moment $(\vec{P})$. It is the product of magnitude of one of the charges and dipole length

$$\vec{P} = q \times 2\vec{\ell}$$ Unit : (cm or Debye (D))

It is a vector whose direction is from - q to + q.

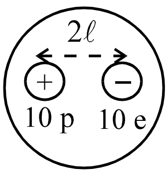
Dipoles are usually found in the molecular world. Consider an HCl molecule. An electron pair is shared between them. Since Cl atom is more electronegative, the shared electron pair will be more shifted towards Cl atom. So Cl atom gets a partial negative charge and H atom an equal partial positive charge.

$$\overset{\delta^+}{H} \ :\overset{\delta^-}{Cl}$$

So it behaves like a molecule of permanent dipole moment. Such molecules with permanent electric dipole moment are known as polar molecules.

**eg** : HCl, HBr, HF, $H_2O$, $NH_3$ etc.

If we consider a water molecule, the ten protons in it generates a (+) charge centre and ten electrons a (-) charge centre. At normal state, these charge centres lie at a small separation, giving a net dipole moment to the water molecule.




P = Charge of 10 protons × 2 $\ell$

But there are certain molecules which do not posess permanent electric dipole moment. Such molecules are called non polar molecules.

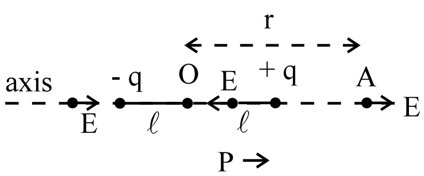
**eg** : $CO_2$, $BeF_2$, $CH_4$ etc.

If we consider a non polar molecule, the (+) and (-) charge centres coincide so that dipole length is zero. So dipole moment is zero. But if we apply a strong $\vec{E}$ to such a molecule, (+) and (-) charge centres tend to seperate and hence they gain a electric dipole moment. Such a dipole is called an induced dipole.

## Electric field due to a dipole

Consider a dipole of charge q, dipole length $2\ell$ and dipole moment p.

**(i) At an axial point**



Field at axial point A; $E_{ax} = \frac{1}{4\pi\varepsilon_0}\frac{2pr}{\left(r^2-\ell^2\right)^2}$

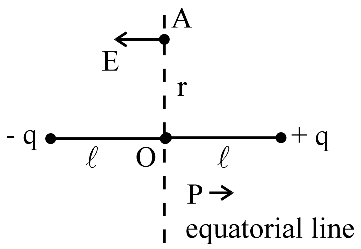
For a short dipole $\ell << r.$ Then

$$\boxed{E_{ax} = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}}$$

Direction of electric field at an axial point is same as the direction of dipole moment. But if the point lies between the charges, direction of electric field will be opposite to that of dipole moment.

**(ii) <u>At an equatorial point</u>**

An equatorial line is a perpendicular bisector to the dipole.




At equatorial point A;

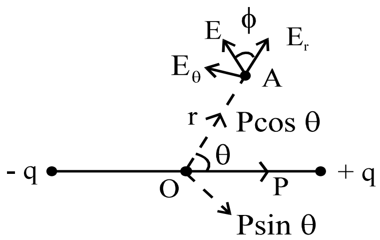
$$E_{eq} = \frac{1}{4\pi\varepsilon_0}\frac{P}{\left(r^2+\ell^2\right)^{3/2}}$$

For a short dipole; $\ell << r.$ Then

$$\boxed{E_{eq} = \frac{1}{4\pi\varepsilon_0}\frac{P}{r^3}}$$

Direction of electric field at an equatorial point is opposite to the direction of dipole moment

**(iii) <u>At any point surrounding dipole</u>**



For $\vec{p}\cos\theta$ component, A is an axial point and for $\vec{p}\sin\theta$ component, A is an equatorial point. Let they produce fields $E_r$ (radial component) and $E_\theta$ (angular component) at point A respectively.

$$E_r = \frac{1}{4\pi\varepsilon_0}\frac{2P\cos\theta}{r^3}; \quad E_\theta = \frac{1}{4\pi\varepsilon_0}\frac{P\sin\theta}{r^3}$$

net field at A

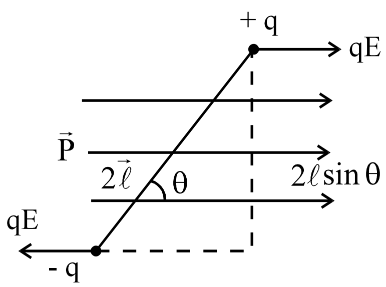
$$E = \sqrt{E_r^2 + E_\theta^2} \quad \Rightarrow \boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{P}{r^3}\sqrt{3\cos^2\theta + 1}}$$

Let, net field at $A\left(\vec{E}\right)$ makes an angle $\theta$ with $\vec{r}$. Then $E_r = E\cos\phi$ and $E_\theta = E\sin\phi$.

$$\therefore \frac{E_\theta}{E_r} = \tan\phi \quad \Rightarrow \frac{\left(\frac{1}{4\pi\varepsilon_0}\frac{P\sin\theta}{r^3}\right)}{\left(\frac{1}{4\pi\varepsilon_0}\frac{2P\cos\theta}{r^3}\right)} = \frac{\tan\theta}{2}$$

$$\Rightarrow \boxed{\tan\phi = \frac{1}{2}\tan\theta}$$

**Dipole placed in a uniform $\vec{E}$**




Since equal and opposite forces are acting on the dipole, net force on the dipole is zero. But these equal and opposite forces constitute a couple and produce a torque.

$\tau = qE \times$ perpendicular distance between forces

$$= qE \times 2\ell\sin\theta = \left(q \times 2\ell\right)E\sin\theta$$

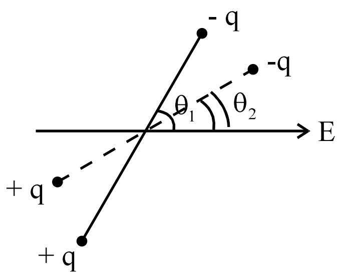
$$\boxed{\tau = PE\sin\theta} \text{ or } \vec{\tau} = \vec{P} \times \vec{E}$$

If dipole is perpendicular to field $\left(\theta = 90^0\right)$, torque is maximum. $\tau_{max} = PE$

If dipole is either parallel or perpendicular to field, torque will be zero.

## Work done to rotate a dipole

When a dipole is placed at an angle $\theta$ to a uniform electric field, it experience a torque. Work done by an external agent to rotate the dipole through a small angle $d\theta$ against this torque




$$dW = \tau d\theta = PE\sin\theta d\theta$$

Then total workdone to rotate the dipole from an initial angle $\theta_1$ to final angle $\theta_2$ with the field

$$W = \int dW = \int_{\theta_1}^{\theta_2} PE\sin\theta d\theta$$

$$= -PE(\cos\theta)\theta_1$$

$$\boxed{W = PE(\cos\theta_1 - \cos\theta_2)}$$

Since $\vec{E}$ is conservative this work done appears as change in potential energy stored in the dipole.

$$\Delta U = PE(\cos\theta_1 - \cos\theta_2)$$

To find the absolute potential energy of a dipole at a particular position with the field, let us assume that, potential energy is zero when dipole is placed perpendicular to field. From there, let dipole is rotated to a position where it makes an angle $\theta$ with the field.

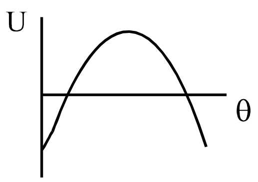
$$U_2 - U_1 = PE(\cos\theta_1 - \cos\theta_2)$$

When $\theta_1 = 90^0,\ U_1 = 0,\ \theta_2 = \theta,\ U_2 = U$

$$U - 0 = PE(\cos 90 - \cos\theta)$$

$\boxed{U = -PE\cos\theta}$ Or $U = -\vec{P}\cdot\vec{E}$

So potential energy - $\theta$ graph is a cosine curve



## Equilibrium of a dipole in a uniform field

Consider different positions of a dipole rotating in a uniform electric field

**(i) When dipole is parallel to field ($\theta = 0^0$)**

$\tau = PE\sin 0 = 0$, U = - PEcos0 = - PE(minimum)

So here dipole is in equilibrium with least potential energy and hence is in stable equilibrium.

**(ii) When dipole is perpendicular to field ( $\theta = 90^0$)**

$\tau = PE\sin 90$ = PE(maximum), U = - PEcos90 = 0

Here dipole is not in equilibrium

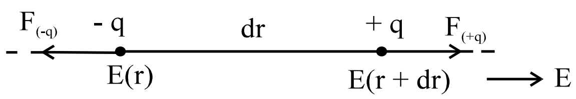
**(iii) When dipole is antiparallel to field** ( $\theta = 180^0$)

$\tau = PE\sin 80 = 0$, U = - PEcos180 = + PE (maximum)

So here also dipole is in equilibrium but with maximum potential energy and hence is in unstable equilibrium

So in a uniform electric field, dipole is always in translational equilibrium. In two positions, dipole is in rotational equilibrium also. So when a dipole rotates in a uniform electric field, there are two positions of equilibrium, one is stable and the other is unstable.

## Force on a dipole in a non uniform field




Consider a dipole of charge q and dipole length dr placed in a non uniform field as shown. Let $E_r$ and E(r + dr) are the electric fields at positions of - q and + q. So forces are ;

F - q = $qE_r$;        $F_{+q}$ = q E(r + dr)

net force;        F = $F_{-q}$ - $F_{+q}$ = q[$E_r$ - E(r + dr)]

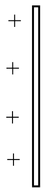
F = q        $dE \times \frac{dr}{dr} = q \times dr\left(\frac{dE}{dr}\right)$        q × dr = P

$$\boxed{F = P\left(\frac{dE}{dr}\right)}$$

## Charged bodies with continuous charge distribution

Coulomb's law can be used only to find electric field due to point charges. But there are bodies with larger size like sheet, ring, sphere etc. over which charges are continuously distributed. To find electric field due to such bodies, we cannot use coulomb's law. For such bodies we can define three type of charge densities.

**(i) Linear charge density** $(\lambda)$



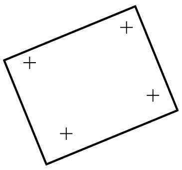
Let a charge Q is distributed along a line of length L. Then its charge per unit length is the linear charge densities.

$$\lambda = \frac{Q}{L} \quad \text{or} \quad \lambda = \frac{dq}{d\ell}$$

For bodies like thin conductor, circular arc, ring etc we define $\lambda$

**(ii) Surface charge density** $(\sigma)$

Let a charge Q is distributed on the surface of body having area A. Then its charge per unit area is called surface charge density $(\sigma)$

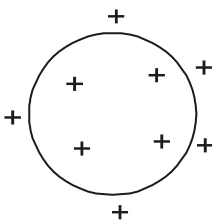


$$\sigma = \frac{Q}{A}$$

For bodies like thin sheet, thin disc etc, we define $\sigma$

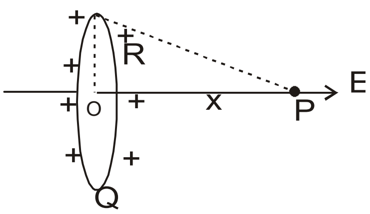
**(iii) Volume charge density** $(\rho)$

Let a charge Q is distributed over the entire volume of a body V. Then its charge per unit volume is called volume charge density $(\rho)$



$$\rho = Q/V$$

**IV. Field due to charged circular ring**



**At axial point (P)**

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{\left(R^2 + x^2\right)^{3/2}}$$

**At centre (O)**

X = 0 $\boxed{E = 0}$

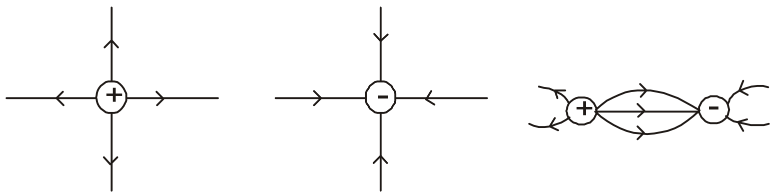
## Electric field lines

If we place a (+) charge at a point, large number of invisible lines are sprayed out of it with the help of these lines charge produces an electric field in its surrounding. If we draw a tangent at any point on this line it gives the direction of $\vec{E}$ at that point.

An electric field line is a line or a curve surrounding a charge such that tangent drawn at any point gives the direction of electric field at that point.

## Properties

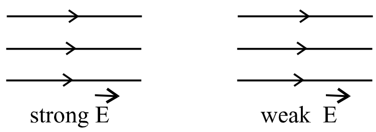
(i) Field lines start from a(+) charge and terminate on (-) charge.




(ii) Field lines are continuous but they never form closed loops

(iii) Number of field lines surrounding a charge is directly proportional to the magnitude of charge

(iv) In a strong field, lines are closely spaced and in a weak field, they are far separated




If d is the separation between field lines, then $E \propto \frac{1}{d}$

(v) In a uniform $\vec{E}$, lines are parallel and equally spaced

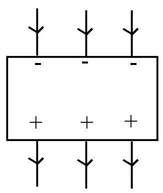
(vi) Two field lines never intersect

(vii) Lines between two unlike charges contract longitudinal causing an attraction

(viii) Lines from two like charges exert lateral pressure on one another causing a repulsion

(ix) Electric field lines always start or terminate perpendicular to the surface of a conducting body

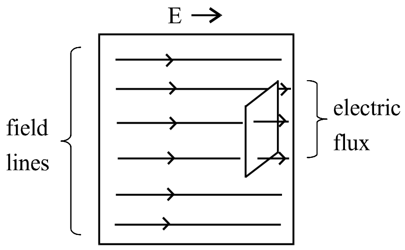
(x) Field lines never penetrate into the inside regions of a conducting body

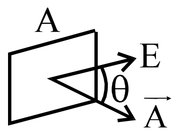



conducting body

## Electric flux $(\phi)$

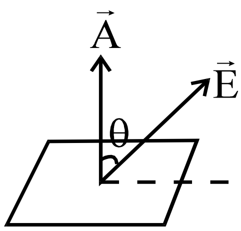
Flux is generally an amount of a vector field passing normally through a surface. Consider a plane surface of area A placed in a uniform field $\vec{E}$. Then electric flux of that surface is the total number of field lines passing normally through that surface




$$\boxed{\begin{array}{l}\phi = EA\cos\theta \\ \phi = \vec{E}.\vec{A}\end{array}}$$

For calculating flux, area is taken as a vector, whose direction is perpendicular to the plane of surface outwards. Here $\theta$ is the angle between $\vec{E}$ and $\vec{A}$




$\theta = 90^0$ - angle between $\vec{E}$ and plane of surface.

**Case 1** : If plane of surface is perpendicular to field

$$\theta = 90 - 90 = 0^0$$

$$\phi = EA\cos 0 = EA \text{ (maximum)}$$

**Case 2** : If plane of surface is parallel to field

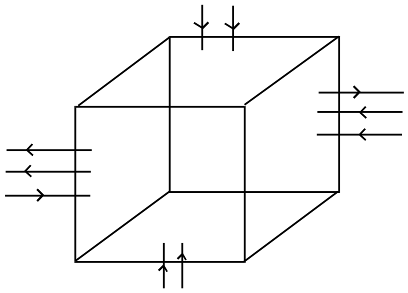
$$\theta = 90 - 0 = 90^0$$

$$\phi = EA\cos 90 = 0 \text{ (minimum)}$$

## Electric flux for a volume enclosing surface

For a two dimensional planar surface, flux is the total number of field lines passing through it. But for a three dimensional volume enclosing surface, flux can be defined in three ways.

(i) flux entering (ii) flux leaving (iii) net flux

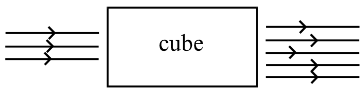


consider a cubical surface in an electric field. Lines can enter and leave the cube through all six faces. Flux entering and flux leaving gives the total number of lines entering and leaving the surface. Then net flux is; $\phi_{net} = \phi_{leaving} + \phi_{entering}$

$\phi_{leaving} = (+), \phi_{entering} = -$

$\therefore \quad \boxed{\phi_{net} = \phi_{leaving} - \phi_{entering}}$

**Case 1 :**

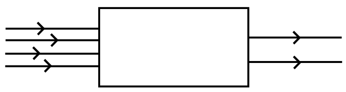



Here $\phi_{leaving} > \phi_{entering}$

$\therefore \phi_{net} = (+)$

This indicates that a net positive charge is present inside the cube.

**Case 2 :**



Here $\phi_{leaving} < \phi_{entering}$

$\therefore \phi_{net} = (-)$

This indicates that a net negative charge is present inside the cube.

**Case 3 :**

Here $\phi_{leaving} = \phi_{entering}$

$\therefore \phi_{net} = 0$

So net charge inside is zero. If a volume enclosing surface is placed in a uniform $\vec{E}$, then $\phi_{net} = 0$ for it.

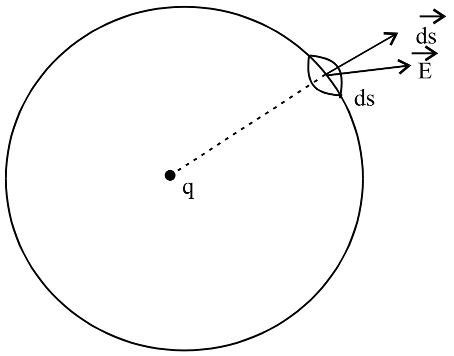
**Gauss's law**

Consider a charge (q) placed at a point. Let it is placed inside a closed spherical surface. Consider a small elementary area ds on its surface. If $\vec{E}$ is the electric field over the element, then flux through the element

$d\phi = \vec{E}.\overrightarrow{ds}$

Net outward flux through the surface

$\phi = \int_S \vec{E}.\overrightarrow{ds}$

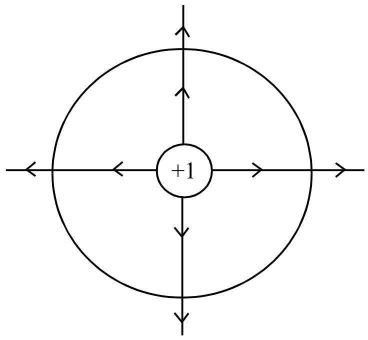



According to Gauss, the net outward flux from a charge enclosing surface is equal to $\frac{1}{\varepsilon_0}$ times the net charge enclosed by the surface.

$$\boxed{\phi = \frac{1}{\varepsilon_0} q_{net}}$$

$$\boxed{\oint \vec{E}.\overrightarrow{ds} = \frac{1}{\varepsilon_0} q_{net}}$$

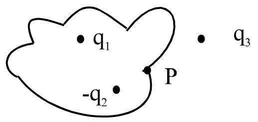
To find the number of field lines coming from a (+ 1C) charge, we can use this law. For that let us first enclose this (+ 1C) inside a closed surface.



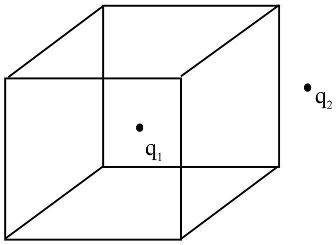

Then it is equal to net outward flux from that closed surface.

$$\phi = \frac{1}{\varepsilon_0} q_{net} = \frac{1}{8.85 \times 10^{-12}} \times 1 = 1.13 \times 10^{11}$$

**Case 1** : Flux of a closed surface depends only on the charges enclosed. But if we consider any point on the surface, the electric field will be due to all the charges in the region. Flux of this surface depends only on $q_1$ and $-q_2$. But if we find field at P, it is due to all the charges $q_1$, $-q_2$ and $q_3$.

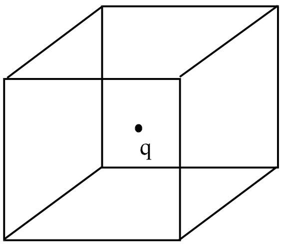



**Case 2** :




Here total flux of cube is due to $q_1$ only. But if we consider flux of right face, it depends on both $q_1$ and $q_2$

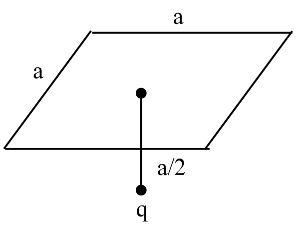
**Case 3** : Let a point charge is placed at the centre of a cube.



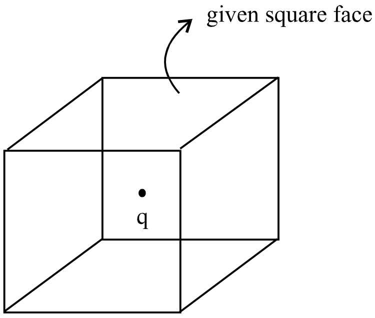

Total flux of cube = $\frac{1}{\varepsilon_0}q$. Since q is at centre, this flux will be equally shared by all 6 faces. $\therefore$ flux through one face = $\frac{q}{6\varepsilon_0}$

**Case 4** : Let a dipole is placed inside a closed surface. Since $q_{net} = 0,\ \phi_{net} = 0$

**Case 5** : Let a point charge is placed at a distance of a/2 below the centre of a square face of side a.



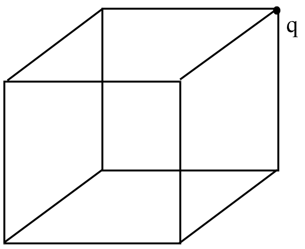

Here we cannot directly apply Gauss's law because q is not enclosed inside a closed surface. So let us consider five other similar faces to complete a cube such that q will be at the centre of cube. Then out of the total flux $q/\varepsilon_0$ through entire face, $\frac{1}{6}$ will come through the given square face.




Flux through given square face $= \dfrac{q}{6\varepsilon_0}$

**Case 6** : Let a point charge q is placed at one corner of a cube. Consider 7 other similar cubes sharing that corner. Then q will be at the centre of a closed surface formed by 8 cubes. So the net charge enclosed by one cube $q_{net}$ = q/8

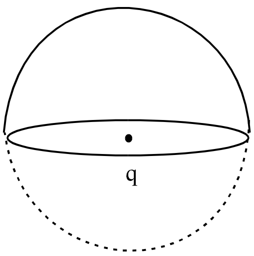
$\therefore$ Net flux from cube = $\dfrac{1}{\varepsilon_0}\left(\dfrac{q}{8}\right) = \dfrac{q}{8\varepsilon_0}$




If we consider the 3 faces which contain the corner in which charge is placed, no flux will be associated. So the total flux will be equally shared by 3 other faces.

$\therefore \phi_{\text{each face}} = 0$ OR $\phi_{\text{each face}} = \dfrac{1}{3}\left(\dfrac{q}{8\varepsilon_0}\right) = \dfrac{q}{24\varepsilon_0}$

**Case 7** : Let a point charge q is placed at the centre of the base of a hemisphere.




Imagine a similar hemisphere to complete a sphere with q at centre. Then out of total flux $q/\varepsilon_0$ through entire sphere, half $(q/2\varepsilon_0)$ will come through given hemisphere.

**Applications of Gauss's theorem**

Coulomb's theorem can be used to find the electric field due to point charges only. To find the field produced by charged bodies of larger size, we use Gauss's theorem. This theorem is applied in two steps.

**Step 1** : Imagine a closed surface called Gaussian surface enclosing the large. For this we can use the points

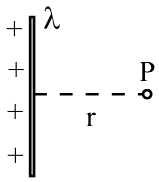
(i) The point at which field is to be calculated must appear on the surface of the Gaussian surface

(ii) It should not pas through discrete charges but can pass through continuous charges

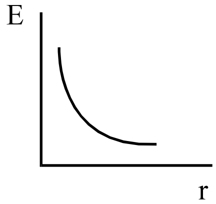
(iii) It is better to use symmetric Gaussian surface. It is a surface over which field value is the same at every point

**Step 2** : Apply the equation $\oint \vec{E}.\overrightarrow{ds} = \frac{1}{\varepsilon_0}$ $q_{net}$ over the surface and find E

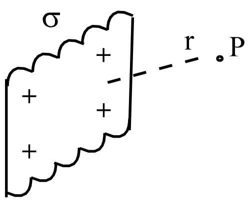
## I. Field due to infinite line of charge




Field at P; $\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$ $E \propto \frac{1}{r}$

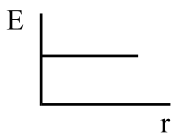



## II. Field due to infinite thin conducting sheet




field at P $\boxed{E = \frac{\sigma}{2\varepsilon_0}}$

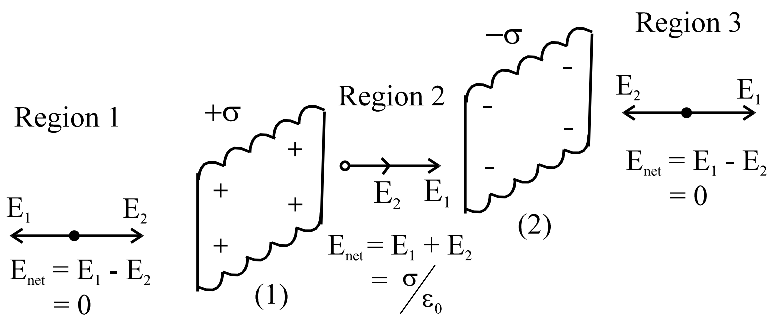
E is independent of r




For a thick sheet; $\boxed{E = \frac{\sigma}{\varepsilon_0}}$

For a non conducting sheet; $E = \frac{\sigma}{2\varepsilon_0}$

## PRODUCTION OF UNIFORM $\vec{E}$




Consider two infinite thin conducting sheet placed parallel and are given with equal and opposite surface charge densities. Field due to the sheet are, $E_1 = E_2 = \dfrac{\sigma}{2\varepsilon_0}$

So in the region between the plates, field is uniform with a value $E = \dfrac{\sigma}{\varepsilon_0}$ and direction from the positive plate to negative plate. Beyond the plates field is zero.

### Charge distribution over conducting bodies

A conducting body contains a large number of free electrons inside. So if a charge is given to any inside point of a conducting body, free electrons will take these charges to its outer surface.

(i) Charges do not stay inside a conducting body

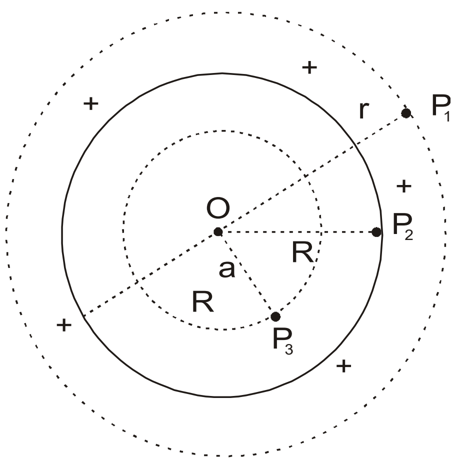
(ii) If a charge is given to a conducting body, these charges will be distributed only over its outer surface

(iii) For an isolated regular conducting sphere, such a charge will be uniformly distributed over its outer surface. But if surface is irregular or any surrounding charges are present, distribution will become nonuniform

### III. Electric field due to conducting sphere/hollow sphere

For a spherical charge distribution, Gaussian surface is a concentric sphere with the point where the field is to be calculated on its surface.

Let a charge Q is given to an isolated conducting sphere of radius R. It will be uniformly distributed only over its outer surface. So we can define only surface charge density ($\sigma$) and not volume charge density ($\rho$).



$\sigma = \frac{Q}{4\pi R^2}$

To find the field at a point, imagine a Gaussian sphere through that point. Then only the charges enclosed by that sphere will produce field at that point.

**Outside point** **($P_1$)**

$$\boxed{E_{out} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}}$$

$E_{out} \propto 1/r^2$

**Surface point** ($P_2$)

$r = R$

$$\boxed{E_{surface} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}}$$

$Q = 4\pi R^2\sigma$

$$\boxed{E_{surface} = \frac{\sigma}{\varepsilon_0}}$$

**Inside point** **($P_3$)**

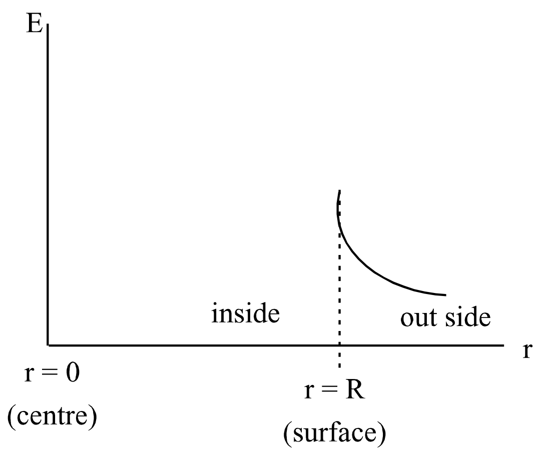
Gaussian sphere through $P_3$ encloses no charge. So

$$\boxed{E_{in} = 0}$$

**Centre (0)**

$E = 0$

Not only for a conducting sphere, for every conducting body, net electric field at an inside point is always zero



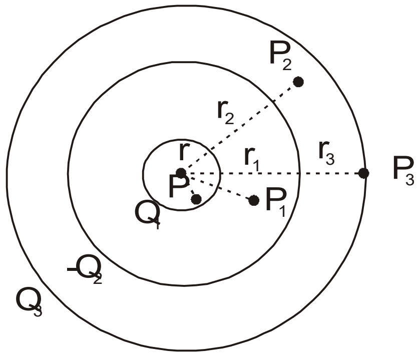

**Field due to concentric thin conducting spheres**

To find field at a point, imagine a concentric Gaussian sphere through that point only the charges enclosed by that sphere constitute field at that point and is given by

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q_{net}}{R^2}$$

$Q_{net} \rightarrow$ net charge enclosed by the gaussian sphere

$R \rightarrow$ distance to that point from centre




**Point P**

Gaussian sphere through P encloses no charge

$\therefore Q_{net} = 0$

$\therefore E_P = 0$

**Point $P_1$**

Gaussian sphere through $P_1$ enclose a charge $Q_1$

$\therefore \; Q_{net} = Q_1, \; R = r_1$

$$E_{P_1} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r_1^2}$$

**Point $P_2$**

Gaussian sphere through $P_2$ encloses charges $Q_1$ and - $Q_2$

$\therefore \; Q_{net} = Q_1 - Q_2, \; R = r_2$

$$E_{P_2} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1 - Q_2}{r_2^2}\right)$$

**Point $P_3$**

Gaussian sphere through $P_3$ encloses all charges

$\therefore \; Q_{net} = Q_1 - Q_2 + Q_3$

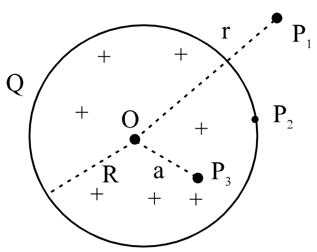
$R = r_3$

$$E_{P_3} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1 - Q_2 + Q_3}{r_3^2}\right)$$

## IV Field due to nonconducting sphere of uniform density

A nonconducting body lacks free electrons. So if a charge is given to any inside point of such a body, that charge will bound to that point. Let a charge Q is uniformly distributed over the entire volume (V). Then its volume charge density

$$P = \frac{Q}{\left(\frac{4}{3}\pi R^3\right)}$$



**Outside point ($P_1$)**

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}}$$

$E_{out} \propto 1/r^2$

**Surface point ($P_2$)**

$$\boxed{E_{surface} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}}$$

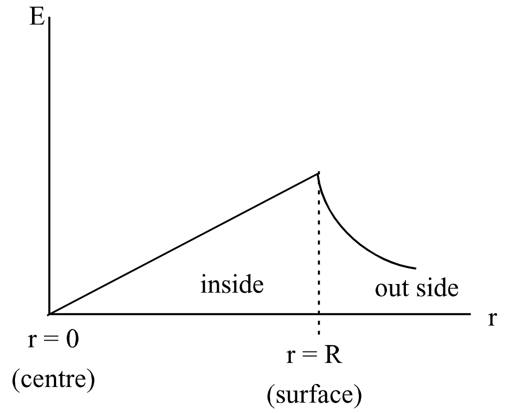
**Inside point ($P_3$)**

$$\boxed{E_{in} = \frac{\rho a}{3\varepsilon_0}}$$

$E_{in} \propto a$

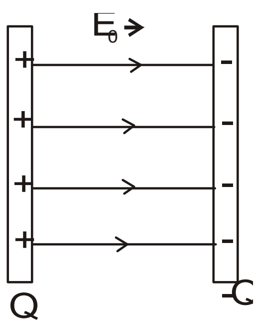
**Centre (0)**

E = 0

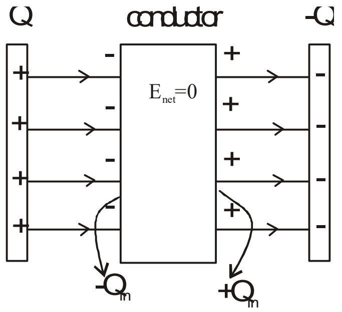



**Electrostatic shielding**

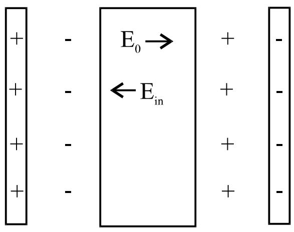
Consider a uniform electric field ($E_0$) applied between two identical metallic plates. If we consider the field lines between the plates, all the field lines starting from the (+) plate will terminate all the (-) plate.




Now a conducting sheet $(K = \infty)$ is inserted in the region between the plates. Then induction happens and induced charges are accumulated as shown. Then all the electric field lines starting from the (+) plate will terminate at the (-) ve induced charge. But an equal number of field lines start from the (+) induced charge on the other side of the conductor and terminate on the (-) plate. So no field lines penetrate into the inside regions of the conducting sheet. So net electric field at the inside region of the conducting sheet is zero. Hence we can conclude that the inside regions of the conducting sheet is free from the electric effects outside. This is called electrostatic shielding and such a conducting cavity is called a Faraday's cage.




In actual practice $E_0$ will surely enter the region inside the conductor. Then an another electric field is developed inside the conductor due to the induced charges accumulated at the surface.




Direction of this field $\left(\vec{E}_{in}\right)$ is opposite to the direction of $\vec{E}_0$ so that net field in the region,

$E_{net} = E_0 - E_{in}$, where $E_{in} = E_0\left(1-\frac{1}{K}\right)$

for a conductor, $K = \infty$ $\therefore E_{in} = E_0 \;\; \therefore E_{net} = E_0 - E_0 = 0$

Hence $E_{in}$ cancels $E_0$ before it make any effects.

# ELECTROSTATICS - 2
# ELECTRIC POTENTIAL AND CAPACITANCE

## ELECTRIC POTENTIAL (V)

Just like electric field intensity ($\vec{E}$), electric potential is a scalar physical quantity which is used to characterise an electric field. It referes to an ability to do work to move a charge in an electric field. It has many roles in electricity.

(i) The concept of potential helps us to find the work to be done to move a charge in an electric field.

(ii) The direction of charge flow between two connected bodies is determined by their potential values. Positive charge always flow from higher to lower and negative charge from lower to higher potential and this charge flow will continue untill both the bodies attain a common potential (Electrostatic Equilibrium).

(iii) If two connected bodies are in electrostatic equilibrium, their potentials will be equal.

(iv) Electric potential can be used to accelerate or decelerate charges.

## Electric Potential at a Point

Consider a point (P) in an electric field. To find the potential at this point, place a test charge q at infinity and then an external agent has to bring this charge from infinity to that point P, slowly. If W is the work done for this, then potential at that point; then $\boxed{V = \frac{W}{q}}$ If q = IC, then V = W.

Hence electric potential at a point in an electric field is defined as the work done to bring a unit positive charge from infinity to that point without an acceleration.

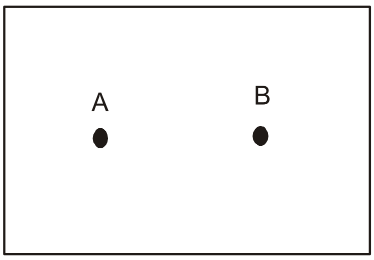
Unit : Volt (SI) or esu of potential (cgs)

1esu = 300V

$[V] = [ML^2T^{-3}A^{-1}]$

## Electric Potential difference between the points

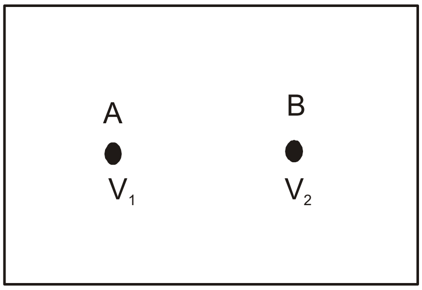
Consider two points in an electric field A and B. Let a charge q is moved from point A to point B without an acceleration. If W is the work done for this process, then potential difference between these points;



$\boxed{dV = \frac{W}{q}}$ $dV = V_{final} - V_{initial}$ [here $dV = V_B - V_A$]

If q = IC, dV = W. Hence potential difference between two points can be defined as the work done to move a unit positive charge between the two points without an acceleration.

**Work done to move a charged particle in an E field**




Consider two points in an electric field A and B where potentials are $V_1$ and $V_2$. Let a charged particle of charge q is moved from point A to point B very slowly without an acceleration. Then work done by the external agent for this motion;

W = charge to be moved × p.d. between the two points here

$V_{initial} = V_1$

$V_{final} = V_2$

$$\boxed{W = q\left(V_{final} - V_{initial}\right)}$$

Since this work is done in a conservative electric field, this is stored as change in potential energy.

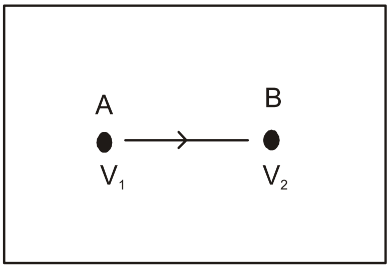
$$\boxed{\Delta U = q\left(V_{final} - V_{initial}\right)}$$

Using the idea, work done by conservative field = - change in PE

$\therefore$ Work done by electric field = - $\Delta$ PE

$$\boxed{WE.field = -q\left(V_{final} - V_{initial}\right)}$$

**Work-energy theorem applied in an E field while a charge is moved**




Let a charged particle of charge q and mass m is moved by an external agent from point A to point B in an electric field. Then both external agent and electric field performs work on charge. So total work done on charge

$W = W_{ext} + W_{E\text{-field}}$

Using work - energy theorem; W = change in KE

$W_{ext} + W_{E.field} = \Delta KE$

but $W_{E.field} = -\Delta PE$

$W_{ext} - \Delta PE = \Delta KE \quad + W_{ext} = \Delta PE + \Delta KE$

but $\Delta PE = q\left(V_{final} - V_{initial}\right)$

$$\boxed{W_{ext} = q\left(V_{final} - V_{initial}\right) + \Delta KE}$$

When charged particle is moved from one point to another, we can consider two situations.

(i) If charge is moved without an acceleration speed does not change. $\Delta KE = 0$

$W_{ext} = q\left(V_{final} - V_{initial}\right)$

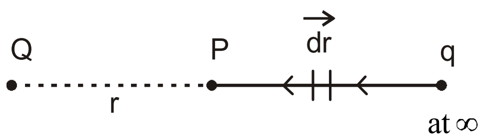
(ii) If charge is moved with an acceleration speed changes and $\Delta KE \neq 0$. Then

$W_{ext} = q\left(V_{final} - V_{initial}\right) + \Delta KE$

**Relation between electric field (E) and electric potentials (V)**

Consider a source charge Q producing an electric field in the surrounding. P is a point at a distance r from Q.

Let a charged particle




q is moved by an external agent from infinity to point P without an acceleration. Consider a small element $\overrightarrow{dr}$ in the path where the electric field is $\vec{E}$. Small work done to move the particle across the element

$dW = -\vec{F}.\overrightarrow{dr} \qquad F = q\vec{E}$

$dW = -q\left(\vec{E}.\overrightarrow{dr}\right) \Rightarrow \dfrac{dW}{q} = -\vec{E}.\overrightarrow{dr}$

But $\dfrac{dW}{q} = dV$, the p.d. across the element

$$\boxed{dV = -\vec{E}.\overrightarrow{dr}}$$

between two points of position vectors $r_1$ and $r_2$

$$\boxed{dV = -\int_{r_1}^{r_2} \vec{E}.\overrightarrow{dr}}$$

Let initial point is at infinity where potential is zero. Let potential is V at point, P

i.e., $r_1 = \infty, V_1 = 0; r_2 = r, V_2 = V$

$$V_2 - V_1 = -\int_{\infty}^{r} \vec{E}.\overrightarrow{dr}$$

$$V = -\int_{\infty}^{r} \vec{E}.\overrightarrow{dr}$$

So potential difference between two points is the negative line integral of electric field. Practically, let

$$\vec{E} = Ex\,\hat{i} + Ey\,\hat{j} + Ez\,\hat{k}$$
$$\overrightarrow{dr} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

$$\vec{E}.\overrightarrow{dr} = Exdx + Eydy + Ezdz$$

Let $\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}; \quad \vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

$$\therefore \boxed{V_2 - V_1 = -\left[\int_{x_1}^{x_2} Ex\,dx + \int_{y_1}^{y_2} Ey\,dy + \int_{z_1}^{z_2} Ez\,dz\right]}$$

Now electric field can be written as

$$\boxed{\vec{E} = -\frac{dV}{dr}}$$

Hence electric field is the negative gradient of potential.

In three dimensions, gradient is a vector differential operator given by $\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$

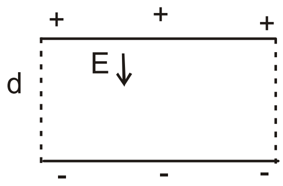
$\therefore E = -\nabla V$ or

$$\boxed{\therefore E = -\left[\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right]}$$

Negative sign in the relation between $\vec{E}$ and $\vec{V}$ shows that, the direction of electric field is same as the direction in which potential is reducing.

Consider a uniform electric field E between two parallel plates, separated by a distance d. Then potential difference between the plates

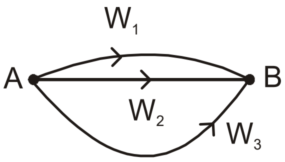
$$\boxed{V = Ed}$$




## Conservative nature of electrostatic field

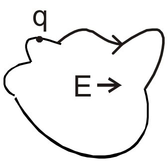
Electrostatic field, since conservative, posses the following properties.

(i) Work done to move a charge between two points in an electrostatic field depend on initial and final positions only and is independent of path through which it is moved.




Let a charge q is moved from point A to B through three different paths as shown. Then work done $W_1 = W_2 = W_3$

(ii) Work done to move a charge once around closed loop in an electrostatic field is zero.




$W = q\,dV = 0$

(iii) Potential difference over a closed loop in an electrostatic field is zero.

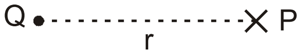
dV = 0 but $dV = -\oint \vec{E}.\overrightarrow{dr}$

$\therefore \oint \vec{E}.\overrightarrow{dr} = 0$

So the line integral of electrostatic field over a closed loop is zero, which indicates the conservative nature of electrostatic field.

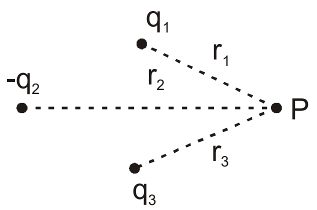
## Potential due to a point charge

Consider a point P at a distance r from point charge Q.




Potential at P; $\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}}$

Since potential is a scalar, sign of charge must be substituted in all the equations in which potential are calculated. Consider a system of charges,

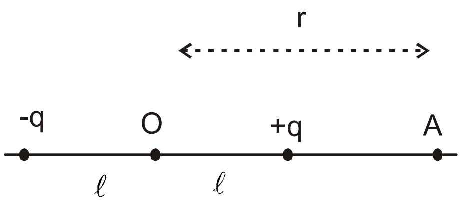



Potential at P; $V=\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1}+\frac{1}{4\pi\varepsilon_0}\left(\frac{-q_2}{r_2}\right)+\frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3}$

## Potential due to a dipole

Consider a dipole of charge q, dipole length $2\ell$ and moment P

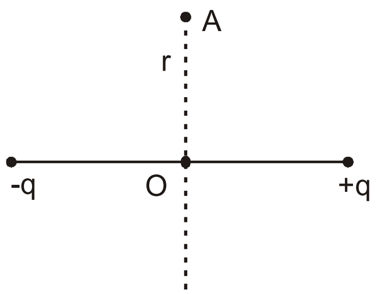
(i) **Axial point**




Potential at A $V=\frac{1}{4\pi\varepsilon_0}\frac{P}{r^2-\ell^2}$ for a short dipole $\ell^2 << r^2$

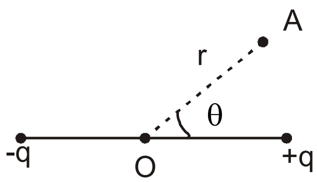
$$\boxed{V=\frac{1}{4\pi\varepsilon_0}\frac{P}{r^2}}$$

(ii) **Equatorial Point**




Potential at A; $\boxed{V=0}$

(iii) **Any general point**

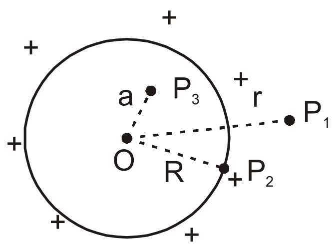


Potential at A $V = \frac{1}{4\pi\varepsilon_0}\frac{P\cos\theta}{r^2 - \ell^2\cos^2\theta}$ for a short dipole $V = \frac{1}{4\pi\varepsilon_0}\frac{P\cos\theta}{r^2}$ or

$$V = \frac{1}{4\pi\varepsilon_0}\frac{\vec{P}.\vec{r}}{r^3}$$

## Potential due to a conducting sphere

Consider a conducting sphere of charge Q and radius R. Charge is uniformly distributed over its outer surface. So its surface charge density $\sigma = \frac{Q}{4\pi R^2}$




**$P_1$ (outside point)**

$$\boxed{V_{out} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}} \quad Q = 4\pi R^2\sigma$$

or $\boxed{V_{out} = \frac{\sigma R^2}{\varepsilon_0 r}}$

**$P_2$ (surface point)**

$\boxed{V_{surface} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}}$ or

$$\boxed{V_{surface} = \frac{\sigma R}{\varepsilon_0}}$$

**$P_3$ (inside point)**

$$\boxed{V_{in} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}}$$

or

$$\boxed{V_{in} = \frac{\sigma R}{\varepsilon_0}}$$

**O (centre)**

$$\boxed{V_{centre} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}}$$

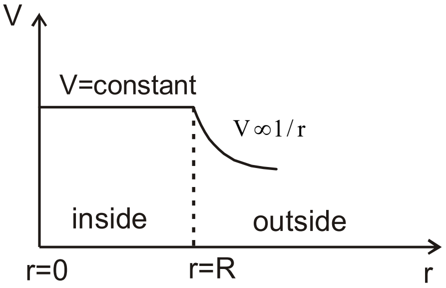
or

$$\boxed{V_{centre} = \frac{\sigma R}{\varepsilon_0}}$$

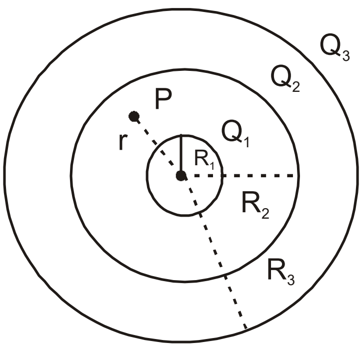
So for a conducting sphere;

$$\boxed{V_{centre} = V_{in} = V_{surface}}$$

This is because, electric field inside a conducting sphere is zero. So to bring IC positive charge from infinite to inside point, work is need to be done up to surface only




## Potential due to Concentric thin conducting spheres




When we calculate, potential due to concentric thin conducting spheres, there is no need to consider the charges produced by induction. We will get same answers even if we take or donot take induction. Consider a point P at a distance r from common centre. To find potential at P, find potential due to each sphere at P and take their sum. When we calculate potential due to a sphere, take that sphere only and identify the position of the point P for that sphere, outside, surface, inside or centre. Use the corresponding equation. Remember that, if the point lies inside for the sphere, potential value at the surface of sphere must be taken.

$V_P$ = V due to inner sphere + V due to middle sphere + V due to outer sphere.

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r} + \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{R_2} + \frac{1}{4\pi\varepsilon_0}\frac{Q_3}{R_3}$$

To find the potential of a particular sphere, take a point on its surface and find potential of that point, using the method discussed above. For example let us find potential of middle sphere. For that mark a point on the surface of middle sphere and find potential there.

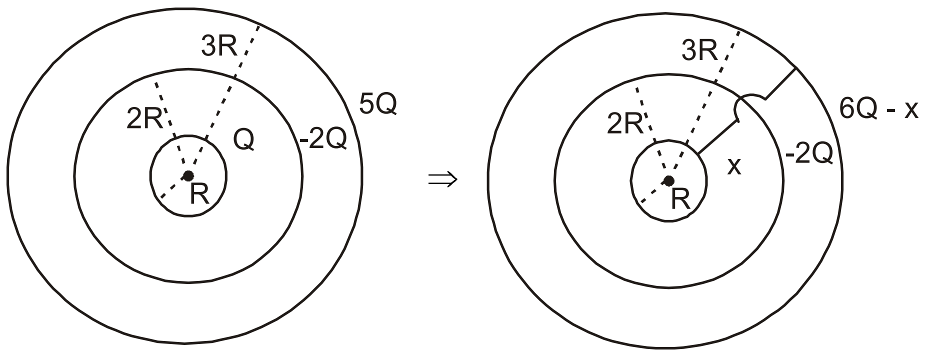
$$V_{\text{middle sphere}} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{R_2} + \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{R_2} + \frac{1}{4\pi\varepsilon_0}\frac{Q_3}{R_3}$$

## Charge sharing between two connected bodies

When two bodies are connected together, a charge flow happens between them. The direction of charge flow between two bodies is decided by their potential values. Positive charge always flow from body with higher potential to the body with lower potential. But negative charge flow from body with lower potential to the body with higher potential.

Charge flow will stop when both attain common potential. So when two bodies are connected together, a charge flow will happen between them untill both attain a common potential. So if two bodies are found to be connected together, their final potentials can be equated.

eg: Consider three concentric conducting thin spherical shells. Of radius R, 2R and 3R given with respective charges Q, -2Q and 5Q. Now the inner and outer spheres are connected using a wire. Find the final charges appearing on all spheres.




Charges given

Final charges after distribution

When inner and outer spheres are inter connected, a charge flow happen between them and soon the potentials of inner and outer spheres become equal. Let x be the final charge on inner sphere (unknown). Then using conservation of charge between inner and outer spheres, final charge on outer sphere is 6Q-x. No changes will happen on the middle sphere.

Total potential on inner sphere (final) = Total potential on outer sphere (final)

$$\frac{1}{4\pi\varepsilon_0}\left[\frac{x}{R}+\frac{-2Q}{2R}+\frac{6Q-x}{3R}\right]=\frac{1}{4\pi\varepsilon_0}\left[\frac{x}{3R}+\frac{-2Q}{3R}+\frac{6Q-x}{3R}\right]$$

On solving; $x=\frac{Q}{2}$

final charge are;

(i) inner sphere = $x=\frac{Q}{2}$

(ii) middle sphere = -2Q

(iii) outer sphere = $6Q-x=\frac{11Q}{2}$

Charge flown through wire = Q/2, from inner to outer sphere.

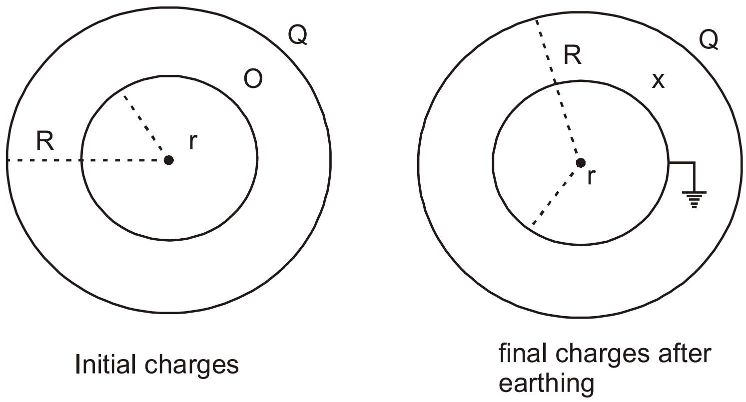
**Earthing**

In electricity, earth is always postulated to be at zero potential. So if a body is connected to earth, a charge flow will happen between that body and earth untill potential of that body becomes zero. So final potential of a earthed body is always zero.

It is a misconcept that, if a charged body is earthed its entire charge flows to earth. Actually, when a body is earthed, it is not the charge, but it is the potential which becomes zero. Its final charge may or may not become zero. If an isolated charged body is earthed, the entire charge on the body will flow to earth. But non isolated body is earthed, final charge on it will not become zero. When a non isolated body is earthed, only the unwanted charges on it will loss to earth. If we consider parallel conducting plates, charges appearing at the two extreme surfaces of the arrangement are unwanted. If we consider concentric conducting spherical shells, charges appearing at the outer surface of the outer most shell are unwanted. If such a body is earthed, that unwanted charge will flow to earth.

eg: Consider a solid conducting sphere of radius r, surrounded by a hollow conducting sphere of radius R. The outer sphere is given a charge Q and inner sphere is earthed. Find the final charge on inner sphere.

Ans:




$x \rightarrow$ charge gained by inner sphere due to earthing.

Since inner sphere is earthed, its final potential is zero ( V = 0)

V due to inner sphere + V due to outer sphere = 0

$$\frac{1}{4\pi\varepsilon_0}\frac{x}{r} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} = 0 \Rightarrow \frac{x}{r} = \frac{-Q}{R}$$

$$x = -\left(\frac{r}{R}\right)Q$$

## Equipotential Surfaces

It is a surface over which potential is the same at every point on the surface. Consider a spherical surface with a point charge at the centre. Then every point on its surface has same potential value. So it is an equipotential surface.

**Properties**

1. Potential difference between two points is zero.
2. Work done to move a charge between two points is zero
3. Electric field is always perpendicular to every point on the surface of a equipotential surface
4. Tangential component of electric field is zero
5. Field lines are always perpendicular to such a surface
6. Two equipotential surfaces never intersect

7. In a uniform electric field, equipotential surfaces are parallel planes with equal separations and equal potential differences.
8. In a strong electric field, separation between neighbouring equipotential surfaces are very small. But in a weak field, they are far separated.

The most important point, we need to consider about an equipotential surface is that, all conducting surfaces are equipotential. If we consider a conducting body of any shape and size, every point on its surface is at the same potential. So all conducting surfaces obey all the properties of equipotential surfaces. So $\vec{E}$ is always perpendicular to a conducting surface. Field lines are also always perpendicular to conducting surface.

The value of this common potential at every point on a conducting surface is given by $V = \frac{\sigma R}{\varepsilon_0}$

R $\rightarrow$ radius of curvature at that point

$\sigma \rightarrow$ surface charge density of the region surrounding that point

Since V = constant, for a conducting surface

$\frac{\sigma R}{\varepsilon_0} = \text{Constant}$ or $\sigma R$ = Constant or $\boxed{\sigma \propto \frac{1}{R}}$

So at points where radius of curvature is less (sharp points), $\sigma$ will be high and charges accumulate more. That is why it is said that, charges accumulate more at the sharp edges of a conducting surface. This fact is utilized in the working of instruments like lightning arrester and Van De Graaff generator.

**Electric Potential Energy**

It is the potential energy developed due to the interaction between charged bodies. Since electrostatic field is conservative, if a work is done by an external agent to move a charged particle slowly in an electric field, that work does not loss but is stored as electrostatic potential energy. If we release this energy, this potential energy converts to kinetic energy and make the charges to move.

**Case 1: Potential energy of a single charged particle placed at a point**

Let a charged particle of charge q is placed at a point. Then its potential energy is defined as the work done to bring that particle from infinity to that point without an acceleration. It is given by U = q V.

So

$$\boxed{\textbf{Potential energy of a charged particle placed at a point} = \textbf{that charge} \times \textbf{Potential of the point where charge is placed}}$$

**Case 2: Potential energy of a system of two point charges**

Consider two point charges $q_1$ and $q_2$ separated by a distance r. Potential energy of this system is defined as the work done to bring these charges from infinite separation to r separation.

$$q_1 \overset{r}{\bullet\text{- - - - - - - -}\bullet} q_2$$

$$\boxed{U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}}$$

For like charges; U is positive and hence force is repulsive. But for unlike charges, U is negative and force is attractive.

**Case 3: Potential energy of a system of more than two point charges**

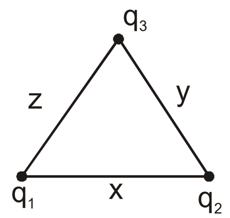
It is also defined as the work done to assemble the system from infinity. It can be calculated using pairing method in 3 steps.

**Step 1:** Split the given charges into all possible pairs

**Step 2:** Find PE of each pair using $\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$

**Step 3:** To find the PE of total system, take the sum of PE of all pairs. To find the PE of a single charge, take the sum of potential energy of pairs containing that charge only.

eg: Let three point charges are placed at the 3 corners of a triangle as shown.




Step 1: Pairs are ($q_1$, $q_2$), ($q_1$, $q_3$), ($q_2$, $q_3$)

Step 2: PE of each pair are $U_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{x}$

$U_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2q_3}{y}$ $\quad$ $U_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_3}{z}$

Step 3: $U_{total} = U_1 + U_2 + U_3$

PE of $q_1 = U_1 + U_3$

PE of $q_2 = U_1 + U_2$

PE of $q_3 = U_2 + U_3$

PE of $q_1$ + PE of $q_2$ + PE of $q_3$ = $2(U_1 + U_2 + U_3)$

$\therefore$ Total PE of system $= \frac{1}{2}$[sum of PE of individual charges]

Note 1: To find the work done to assemble the system, find PE of system

Note 2: To find the work done to break the system to infinity, find the negative of PE of system.

Note 3: To find the work done to bring a single charge to a system from infinity, find the PE of that charge in the system.

Note 4: To find the work done to escape a charge from a system to infinity, find PE of that charge in the system and take its negative.

**Accelerating Potential**

Let a charged particle q and mass m is placed at rest. Let a potential V is applied to it. Then work done by potential on it is W = qV. Due to this, the particle starts moving with entire work done converting into KE of the particle.

KE gained by particle = qV

If u is the speed of the particle, then

$$\frac{1}{2}mu^2 = qV \qquad u = \sqrt{\frac{2qV}{m}}$$

Let 1 V potential difference is applied to an electron. Then KE gained by it

$$KE = qV = 1.6\times10^{-19}\times1 = 1.6\times10^{-19}\,J$$

It is called 1 eV of energy.

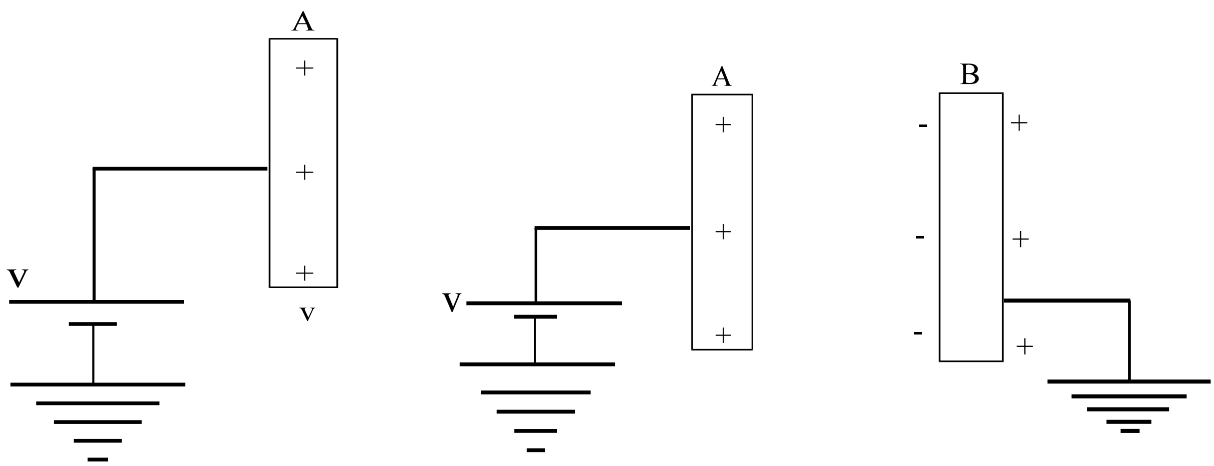
$$1\ eV = 1.6\times10^{-19}\,J$$

## CAPACITOR

Capacitor is a tiny device used to store charge. There are many type of capacitors like parallel plate capacitor, spherical capacitor, cylindrical capacitor etc.

### Principle

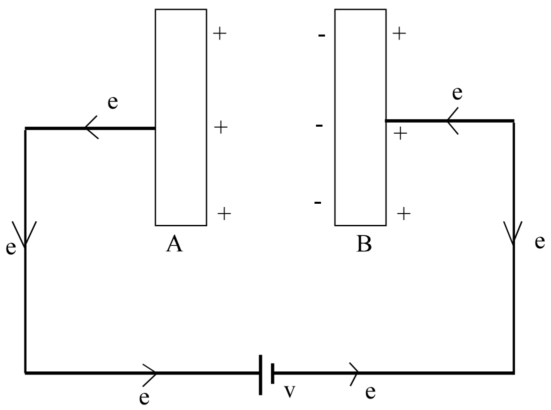
Consider a conductions plate (A) connected to a supply of potential V. Then plate gets charged due to potential V and charging stops when the potential of plate becomes equal to that of supply V. Now an another similar plate (B) is brought near this plate. Then induction happens in the second plate and induced charges are accumulated as shown.



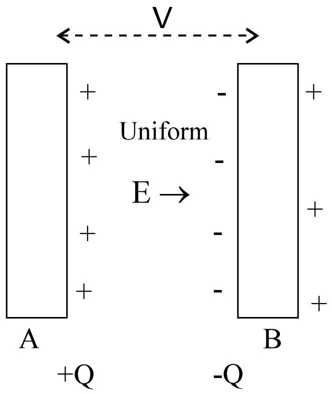

Now right surface of B is earthed. Then all positive induced charges there flows to earth. Due to the remaining negative induced charges on B, a negative potential develops on A. So total potential of A reduce. So to maintain potential at V, more charge is send to plate A. Then charge on plate A increases. This again increases the amount of negative induced charge on B. Hence total potential on A again reduce due to the increased (-) potential on it. So again battery send more charge on it. This process repeats and charge on system goes on increasing. So, an arrangement of two parallel plates can store infinite amount of charge.

**Charging of Capacitor using a battery**

Consider two parallel plates A and B, connected to a supply of potential V as shown.




Then free electrons are attracted from plate A towards (+) of battery. Due to electron loss, (+) charges are accumulated over A at right surface as shown. This causes induction over B. (-) induced charges are accumulated at the left surface of B and (+) at right. Now electrons flow from (-) of battery to B and nullifies the positive induced charges on the right surface. This cycle repeats untill capacitor is fully charged and at that instant, potential difference between the capacitor plates become equal to V.




1. Charging completes when potential difference between capacitor plates becomes equal to the applied potential.

2. If a capacitor has a charge Q, one of its plates has charge +Q and other plate -Q. Usually the plate connected to (+) of battery gets (+) charge and other negative.

3. The equal but opposite charges appearing at the nearby surfaces of the capacitor plates only are considered as the charge stored in the capacitor.

4. A uniform electric field exist between plates.

5. If a charged capacitor is connected to an another battery of higher potential, capacitor further charges. If a charged capacitor is connected to an another battery of lower potential, capacitor discharges. Charging and discharging stops when potential difference between the plates become equal to the supply potential.

6. During charging, electrons are removed from one plate and transferred to the other. The plate losing

electrons get a (+) charge and plate gaining electrons get a (-) charge. So battery does not give any charge to capacitor. Instead, it performs the amount of work required to transfer electrons from one plate to the other. Work done by battery is given by

$$\boxed{W_{bat} = \Delta QV}$$

$\Delta Q \rightarrow$ amount of charge flown through battery.

$V \rightarrow$ Potential of battery

The amount of charge stored in a capacitor is directly proportional to the potential difference between its plates. That is

$Q \propto V$ or $Q = CV$

$\boxed{C = \frac{Q}{V}}$ $C \rightarrow$ Capacitance of capacitor. It is the ability to store charge

Unit : Farad (F) (SI) $[C] = M^{-1}L^{-2}T^4A^2$

egs $\rightarrow$ esu of capacitance

$IF = 9 \times 10^{11}$ esu of capacitance.

eg. Consider a spherical conductor of radius R and charge Q. Then its potential

$V = \frac{1}{4\pi\varepsilon_o}\frac{Q}{R}$. It can be treated as a capacitor of capacitance.

$$C = \frac{Q}{V} = \frac{Q}{\left(\frac{1}{4\pi\varepsilon_o}\frac{Q}{R}\right)}$$

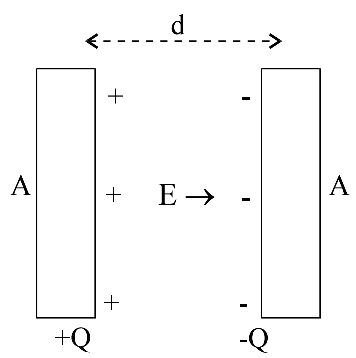
$$\boxed{C = 4\pi\varepsilon_o R} \qquad C \propto R$$

If we consider earth as a spherical capacitor of radius R = 6380 km

$$\boxed{C = 711\,\mu F}$$

Practically $V = 0$ for earth. So C is infinity.

**Parallel plate capacitor**



It consists of two identical metallic plates placed parallel and close to each other Let A be the area of each plate and d is the separation between the plates. Then uniform electric field between the plates,

$E = \frac{\sigma}{\varepsilon_o}$ $\sigma = \frac{Q}{A}$ $E = \frac{Q}{\varepsilon_o A}$. So potential difference between the plates; V = Ed

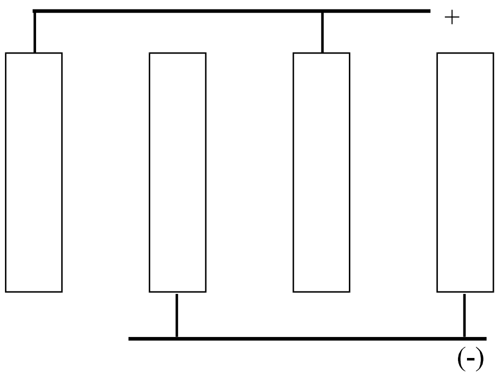
$$V = \frac{Q}{\varepsilon_o A} d$$

$$C = \frac{Q}{V} \Rightarrow \boxed{C = \frac{\varepsilon_o A}{d}} \rightarrow \text{air core capacitor}$$

If entire spacing between plates is filled with a dielectric of constant K, then

$$C = \frac{\varepsilon A}{d} = \frac{K\varepsilon_o A}{d} \quad \therefore \boxed{C^l = KC}$$

Consider a number of identical plates placed in parallel. Let alternate plates are given with like polarity such an arrangement is called a capacitor with N plates.

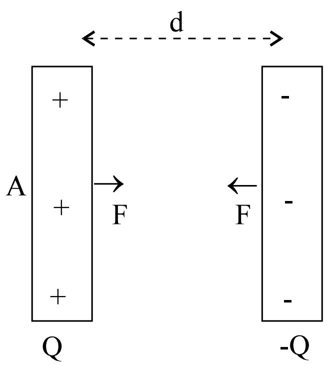



$$\boxed{C = (N-1)\frac{\varepsilon_o A}{d}}$$

## Force between Capacitor plates

An attractive force always exist between the capacitor plates.

Here each plate is placed in the electric field produced by the other, given by




$$E = \frac{\sigma}{2\varepsilon_o}$$

Force on the plates; $F = QE = \frac{Q\sigma}{2\varepsilon_o}\left(\sigma = \frac{Q}{A}\right)$

$$\boxed{F = \frac{Q^2}{2\varepsilon_o A}}$$

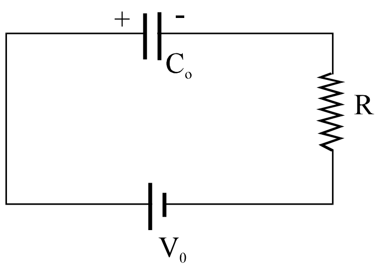
Diving both sides with area A

$$\frac{F}{A} = \frac{Q^2}{2\varepsilon_o A^2} \qquad \frac{F}{A} = Pe; \qquad \frac{Q}{A} = \sigma$$

$\boxed{Pe = \frac{\sigma^2}{2\varepsilon_o}}$ $Pe \rightarrow$ Electrostatic pressure on the plates.

**Energy stored in a capacitor**

Consider a capacitor $C_o$ connected to a battery of potential $V_o$ for charging.




Then, when capacitor is fully charged ($Q_o$), amount of energy stored in the electric field between the plates of the capacitor; given by

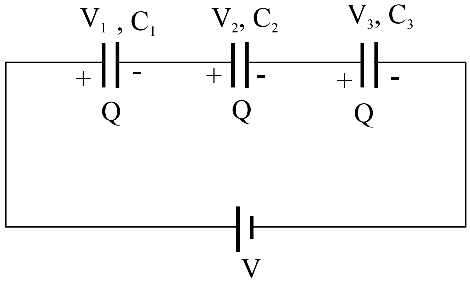
$$U = \frac{Q_o^2}{2C} \quad [Q_o = C_0 V_o] \quad U = \frac{1}{2}C_o V_o^2 \quad U = \frac{1}{2}Q_o V_o$$

work done by the battery; $W_{bat} = Q_o V_o$ of this only $\frac{1}{2}Q_o V_o$ is stored in the capacitor. So the remaining half is lost as heat or em radiations during charging. In actual practice, energy loss during charging is given by

$\Delta H = W_{bat}$ - energy stored in capacitor

**Grouping of capacitors**

**(i) Series:**



$C_1$, $C_2$, $C_3$ are initially uncharged. Since charging currents are equal, the amount of charge gained by all series capacitor are equal. Final charges may or may not be equal. Final charges will be equal only if their initial charges were equal. Here all the three capacitors were initially uncharged. So their final charges are also equal. But voltage drops are different. Using KVL

$$V = V_1 + V_2 + V_3$$

$$\frac{Q}{C_{eff}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \Rightarrow \boxed{\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

multiplying throughout with $\frac{Q^2}{2}$

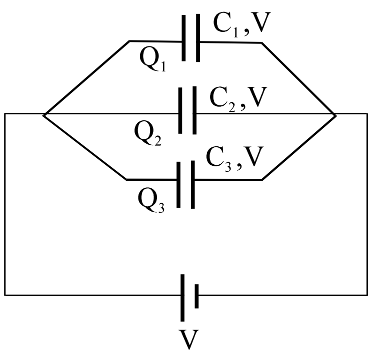
$$\frac{Q^2}{2C_{eff}} = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \frac{Q^2}{2C_3}$$

$$\boxed{U_{total} = U_1 + U_2 + U_3} \rightarrow \text{energy stored}$$

To find the common charge obtained by series connection; $Q = C_{eff} \times \text{applied voltage}$

Individual voltage drops; $V_1 = \frac{Q}{C_1},\ V_2 = \frac{Q}{C_2},\ V_3 = \frac{Q}{C_3}$

**Parallel grouping**




Voltages are some in all but charges are different

$$Q_1 = C_1 V$$

$$Q_2 = C_2 V$$

$$Q_3 = C_3 V$$

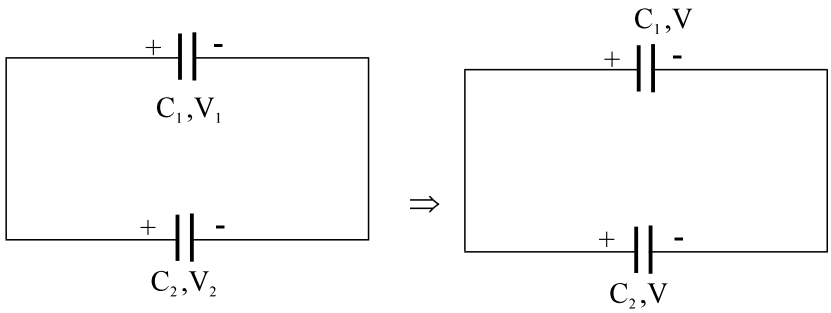
$$Q = Q_1 + Q_2 + Q_3$$

$$C_{eff} V = C_1 V + C_2 V + C_3 V \Rightarrow \boxed{C_{eff} = C_1 + C_2 + C_3}$$

## Redistribution of charges between capacitors

Consider two capacitors $C_1$ and $C_2$ charged by potential $V_1$ and $V_2$. Now they are interconnected in two ways as shown

### (i) Like plates connected together




Then a charge flow happen between them until both attain a common potential (V). Using charge conservation; $C_1V_1 + C_2V_2 = C_1V + C_2V$

$$\boxed{V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}}$$

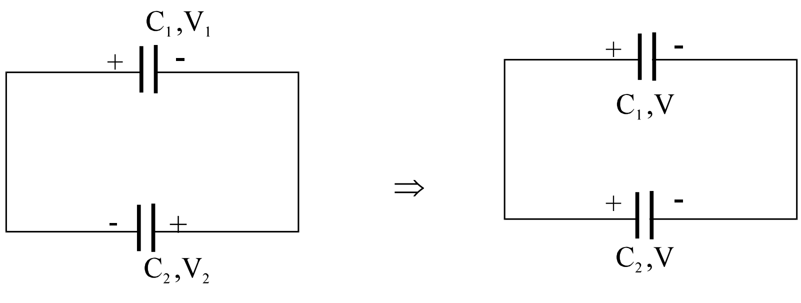
During this redistribution some energy is lost as heat and em radiations.

$$\Delta H = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \left(\frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2\right)$$

on solving

$$\boxed{\Delta H = \frac{1}{2}\left(\frac{C_1C_2}{C_1 + C_2}\right)(V_1 - V_2)^2}$$

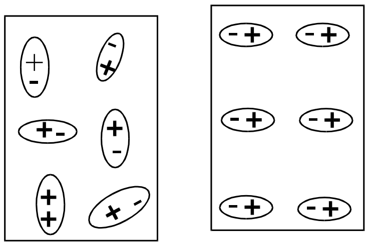
### (ii) Unlike plates connected together




$$\boxed{V = \frac{|C_1V_1 - C_2V_2|}{C_1 + C_2}} \qquad \boxed{\Delta H = \frac{1}{2}\left(\frac{C_1C_2}{C_1 + C_2}\right)(V_1 + V_2)^2}$$

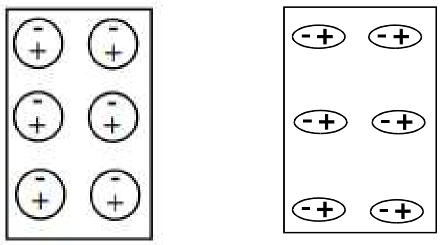
## Dielectric Polarisation

If we consider a polar dielectric, each molecule posess a permanant electric dipole moment. But under the absence of an external field, all the molecular dipoles are randomly oriented so that net dipole moment is zero. But when an external electric field is applied, most of the dipoles are alligned in the direction of field so that dielectric gain a net dipole moment.



$E = 0$ $\quad$ $E \neq 0$

For a nonpolar dielectric, individual molecules do not posess dipole moment so that net dipole moment is zero. This is due to the overlapping of (+) and (-) charge centres. But when a strong electric field is applied, charge centres separate to give a dipole moment to each molecule. Also they allign in the direction of field.



$E = 0$ $\quad$ $E \neq 0$

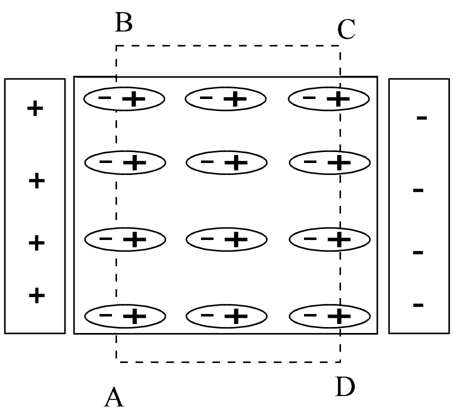
Thus in either case, polar or nonpolar, a dielectric develops a net dipole moment in the presence of external electric field. The dipole moment per unit volume is called polarisation (P).

$P \propto E$ or $\boxed{P = \varepsilon_0 \chi E}$

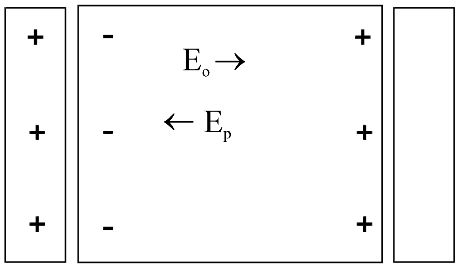
Where x is a constant characterising the dielectric called electric susceptibility. Dielectric constant can be written as $K = 1 + x$

**Introduction of a dielectric between capacitor plates**

Consider a parallel plate capacitor with a uniform electric field $E_o$ between the plates. Now a dielectric is inserted between the plates. Then dielectric polarisation happens.




If we consider the volume element ABCD, equal and opposite charges of the neighbouring dipoles cancel. So polarised charges exist only at the end surfaces.




Due to the polarised charges, a new electric field $E_P$ is developed in the region between the plates in a direction opposite to E. So net electric field in the region

$$E = E_o - E_p$$

So due to the introduction of a dielectric between the capacitor plates, net electric field between the plates reduces. $E_o = \frac{\sigma}{\varepsilon_o}$ $E = \frac{\sigma}{\varepsilon} = \frac{\sigma}{\varepsilon_o K} = \frac{E_o}{K}$

$$\frac{E_o}{K} = E_o - E_p$$

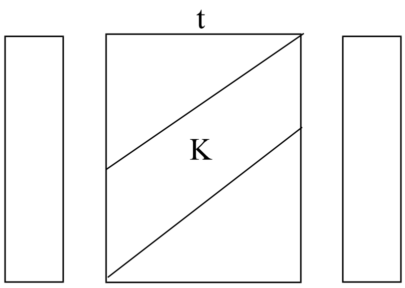
$$E_p = E_o - \frac{E_o}{K} \Rightarrow \boxed{E_p = E_o\left(1 - \frac{1}{K}\right)}$$

Polarised charges; $\boxed{Q_p = Q_o\left(1 - \frac{1}{K}\right)}$

Let a capacitor C is connected to a battery of potential $V_o$. The potential difference between the plates is $V_o$ when charging completes. Now a dielectric is inserted. Then field between the plates reduces to $\frac{E_o}{K}$. Then potential difference also reduces to $\frac{V_o}{K}$. Since battery is still connected, battery will send more charge to the capacitor to take its potential back to $V_o$. Hence capacitor gains more charge and hence capacitance can be said to be increased. So capacitance is said to be increased by the introduction of a dielectric between its plates.

**Equation for Capacitance with dielectric in between**

Consider a parallel plate capacitor with plate area A and plate separation d. Let a dielectric of constant K and thickness t is introduced between the plates.



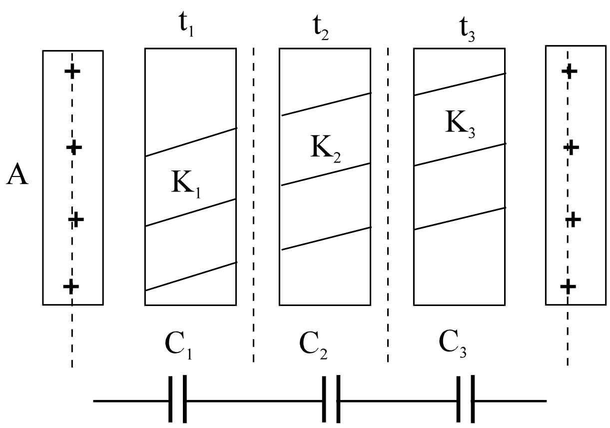
If the entire spacing between the plates is filled with dielectric; then t = d. Then capacitance becomes

$$\boxed{C = K\frac{\varepsilon_o A}{d}}$$

**Introduction of more than one dielectrics**

**(i) Series arrangement of dielectrics**

In this arrangement, plate area will became for the dielectric but plate separation is shared by the dielectrics. The arrangement can be given as below.




Region of each dielectric behaves as a capacitor

$$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \Rightarrow \frac{1}{K_{eff}\frac{\varepsilon_o A}{d}} = \frac{1}{K_1\frac{\varepsilon_o A}{d_1}} + \frac{1}{K_2\frac{\varepsilon_o A}{d_2}} + \frac{1}{K_3\frac{\varepsilon_o A}{d_3}}$$

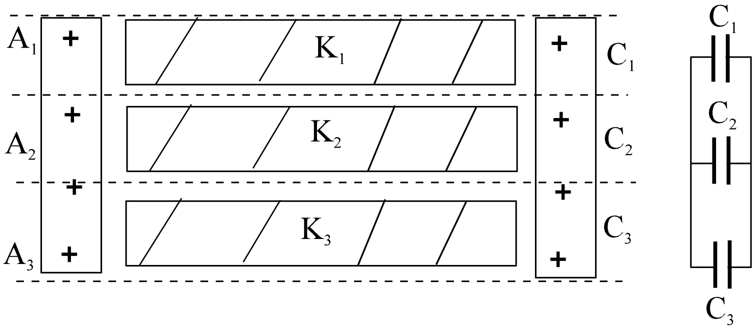
$$\frac{d}{K_{eff}} = \frac{d_1}{K_1} + \frac{t_2}{K_2} + \frac{t_3}{K_3}$$

$$K_{eff} = \frac{d}{\frac{t_1}{K_1} + \frac{t_2}{K_2} + \frac{t_3}{K_3}} \Rightarrow \boxed{K_{eff} = \frac{d}{\Sigma \, t/K}}$$

$$C_{eff} = K_{eff}\left(\frac{\varepsilon_o A}{d}\right)$$

**(ii) Parallel arrangement of dielectrics**

In this all dielectrics have same thickness but they share plate area as shown.




Here also region of each dielectric behaves like an individual capacitor

$$C_{eff} = C_1 + C_2 + C_3$$

$$K_{eff}\frac{\varepsilon_o A}{d} = \frac{K_1\varepsilon_o A_1}{d} + \frac{K_2\varepsilon_o A_2}{d} + \frac{K_3\varepsilon_o A_3}{d}$$

$$K_{eff} = \frac{K_1A_1 + K_2A_2 + K_3A_3}{A}$$

$$\boxed{K_{eff} = \frac{\Sigma KA}{A}} \quad \boxed{C_{eff} = K_{eff}\frac{\varepsilon_o A}{d}}$$

**Methods of changing the capacitance value**

Practically, we can use two methods to change the capacitance value of a parallel plate capacitor.

(i) By changing plate separation d

(ii) By changing the medium between the plates by introducing a dielectric

When we do this, we can approach the system in two ways

(i) With battery still connected

(ii) After disconnecting the battery

If battery is still connected, potential difference between plates (V) will remain constant. But if battery is disconnected, the charge in the capacitor will remain a constant.

In such situations, total work done on the capacitor is; $W_{total} = W_{bat} + W_{ext}$

$W_{bat} \rightarrow$ Work done by battery

$W_{ext} \rightarrow$ Work done by external agent

A part of the total work done is appears as change in energy stored in the capacitor (dU) and remaining part lost as heat and em radiations (dH)

$$\boxed{W_{bat} + W_{ext} = dU + dH}$$

**Work done by battery** $(W_{bat})$

Battery performs work either to charge or discharge the capacitor. It is given by the equations.

$$\boxed{W_{bat} = \Delta Q \times V}$$

$\Delta Q \rightarrow$ amount of charge flown through battery

$V \rightarrow$ potential of battery

Usually, if battery is connected to a capacitor directly, then $\Delta Q$ can be taken as the change in charge stored in the capacitor.

If capacitor is charging, then $\Delta Q = +$, then $W_{bat} = (+)$. Then work is said to be done by battery. If capacitor is discharging, then $\Delta Q_2 = (-)$, then $W_{bat} = -$. Then work is said to be done on the battery.

**Work done by external agent $(W_{ext})$**

External agent performs the required work to do the mechanical process for changing the capacitance value, like changing the plate separation, inserting dielectric etc.

**Change in energy stored in the capacitor (dU)**

dU = Final energy stored - initial energy stored

**Energy loss (dH)**

Energy loss will appear either as joule's heat across resistors or em radiations if charges are accelerated. If resistors are absent heat loss will be zero. If processes are done slowly, charges will not be accelerated, then em radiations will be zero.