# CHAPTER - 00
# OSCILLATIONS

**Periodic Motion OR Harmonic Motion**

Any motion that repeats itself at regular intervals of time is known as periodic motion

Eg. Revolution of earth around the sun

**Oscillatory Motion**

A body is said to be in oscillatory motion if it undergoes a to and fro motion about an equilibrium position or mean position

Eg. Motion of a simple pendulum

* Every oscillatory motion is periodic, but every periodic motion need not be oscillatory

$\Rightarrow$ Example, circular motion is periodic, but not oscillatory

* **Time period (T)**

  * Time for one oscillation

  * SI unit is second(s)

* **Frequency** $(\nu)$

No. of oscillations per second

$$\boxed{\nu = \frac{1}{T}}$$

$\Rightarrow$ SI unit (Hz) or $s^{-1}$

$1Hz = 1s^{-1}$

There is no significant difference between oscillations & vibrations. If frequency is high, we call it as vibration. If frequency is low, it is oscillation

**Displacement**

In oscillatory motion, the term displacement means any physical quantity that changes with time.

Eg. In the case of oscillation of heart, displacement means volume

In oscillatory motion, displacement can be represented by using periodic functions. A function f(t) is said to be periodic if and only if

$$\boxed{f(t) = f(T+t)}$$

$\Rightarrow$ where T is the period of function

eg: $\boxed{\begin{array}{l}\sin\theta = \sin(\theta + 2\pi) \\ \& \\ \cos\theta = \cos(\theta + 2\pi)\end{array}}$

**Simple Harmonic Motion [SHM]**

A motion is said to be simple harmonic, if its acceleration at any instant is directly proportional to displacement from mean position, and acceleration is always directed towards the mean position.

i.e., a $\alpha$ - displacement

[Displacement is always away from mean position & it is always measured from mean position]

a $\alpha$ - y

a = $-\omega^2 y$

$a + \omega^2 y = 0$

i.e., $\boxed{\frac{d^2y}{dt^2} + \omega^2 y = 0}$

This is the equation of a SHM. Solution of this equation gives displacement of SHM, which is given by

$$\boxed{\begin{array}{l} y = A\sin\omega t + \phi \\ \text{OR} \\ y = A\cos(\omega t + \phi)\end{array}}$$

where $A \rightarrow$ Amplitude : Maximum displacement

$\omega \rightarrow$ Angular velocity OR Angular frequency

$$\boxed{\omega = \frac{2\pi}{T} = 2\pi\nu}$$

$\Rightarrow$ SI unit of $\omega$ is rad/s

$(\omega t + \phi) \Rightarrow$ phase

$\phi \Rightarrow$ initial phase OR phase constant OR epoch

If $\phi = 0$

$y = A\sin\omega t$

when $t = 0$

$y = 0$

$\therefore \boxed{y = A\sin\omega t}$, represents oscillation starting from mean position

$\Rightarrow y = A\sin(\omega t + \phi)$ represents particle initially shifted from mean position by an angle $\phi$

$y = A\cos\omega t$

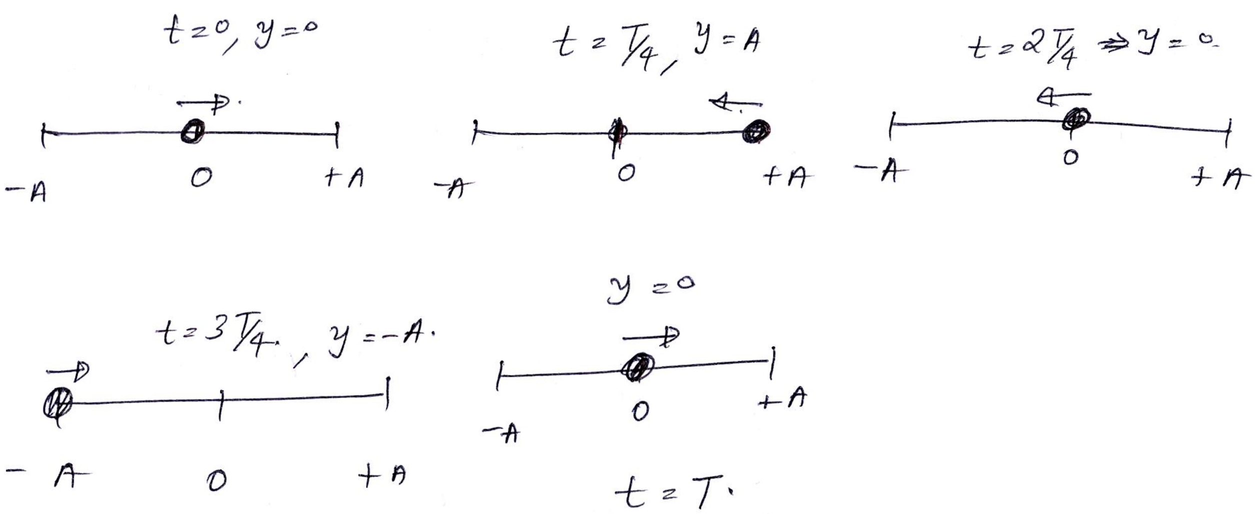
when $t = 0$

$y = A$

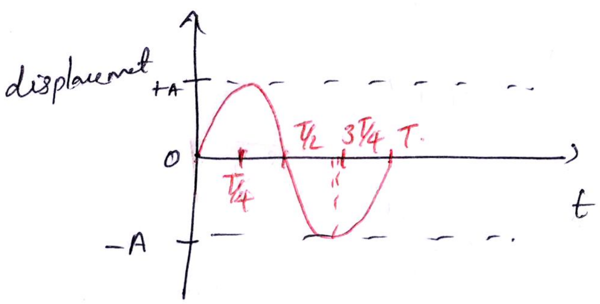
$\therefore \boxed{y = A\cos\omega t}$, represents, oscillation starting from extreme position

$\Rightarrow y = A\cos(\omega t + \phi)$. represents particle initially shifted from extreme position by an angle $\phi$

**Oscillation of a particle along a straight line**




**Draw the displacement - time graph of above motion**




$y = A\sin\omega t \Rightarrow$ starting from mean position.

* **Velocity in SHM**

Let $y = A\sin(\omega t + \phi)$

$$v = \frac{dy}{dt} = A\omega\cos(\omega t + \phi)$$

$$v = A\omega\sqrt{1 - \sin^2(\omega t + \phi)}$$

$$v = \omega\sqrt{A^2 - A^2\sin^2(\omega t + \phi)}$$

$\boxed{v = \omega\sqrt{A^2 - y^2}}$ $\Rightarrow$ Relation between velocity & displacement in SHM

$\Rightarrow$ At mean position (y = 0)

$\therefore \boxed{v_{max} = \omega A}$

$\Rightarrow$ At extreme position ( y = A)

$\boxed{v = 0}$

* **Acceleration in SHM**

We have $y = A\sin(\omega t + \phi)$

$$v = \omega A\cos(\omega t + \phi)$$

$$a = -\omega^2 A\sin(\omega t + \phi)$$

i.e., $\boxed{a = -\omega^2 y}$

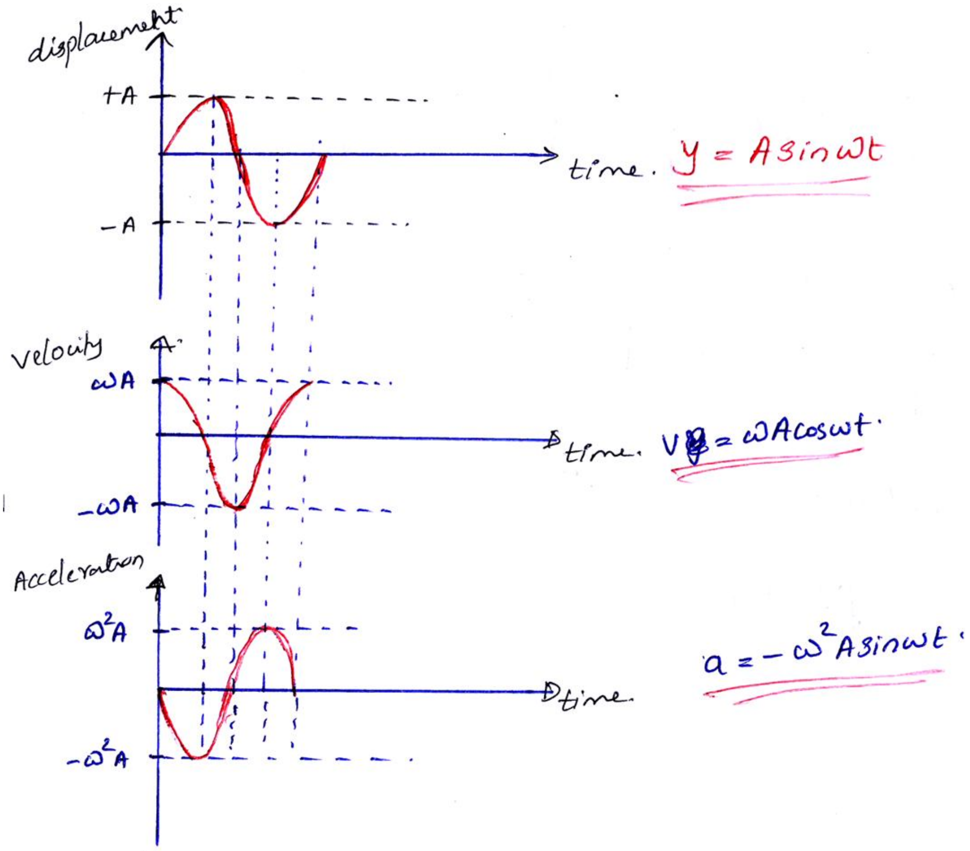
**At mean position (y = 0)**

$\boxed{a = 0}$

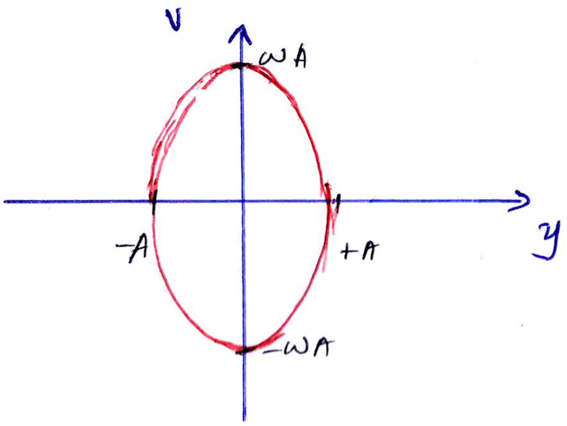
At extreme position (y = A)

$\boxed{a_{max} = \omega^2 A}$

Comparison of displacement-time graph, velocity-time graph, acceleration-time graph




**Displacement - Velocity graph**



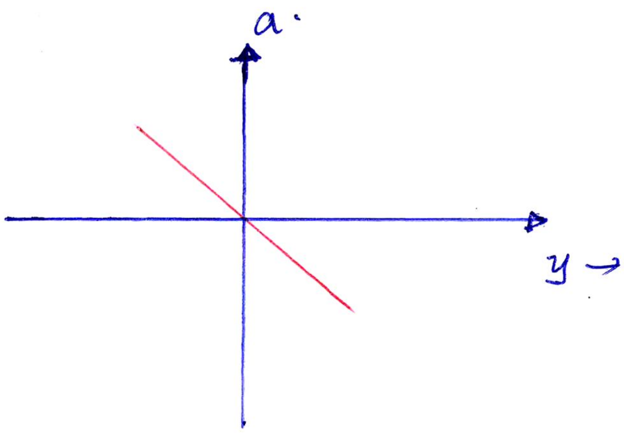

$$v = \omega\sqrt{A^2 - y^2}$$

$$\frac{v^2}{\omega^2} = A^2 - y^2$$

$$y^2 + \frac{v^2}{\omega^2} = A^2$$

$$\frac{y^2}{A^2} + \frac{v^2}{\omega^2 A^2} = 1$$ Equation of ellipse

## Acceleration - displacement graph




$a = -\omega^2 y$

$\underline{slope = -\omega^2}$

Force in SHM

Let ‘m’ be the mass of a particle in SHM. Then force acting on it is

F = ma

where $a = -\omega^2 y$

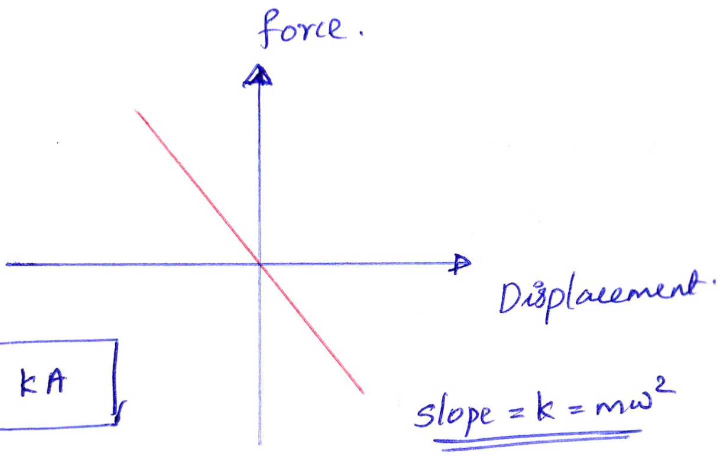
$\boxed{F = -m\omega^2 y}$ where $\boxed{m\omega^2 = k}$ force constant

$\boxed{F = -ky}$

**–ve sign indicates that force is restoring**

Here also, force maximum at extreme position

$\boxed{F_{max} = KA}$




$\boxed{F_{max} = kA}$

$\underline{slope = k = m\omega^2}$

## Energy in SHM

1) Kinetic energy

$$KE = \frac{1}{2}mv^2$$

where $v = \omega\sqrt{A^2 - y^2}$

$$KE = \frac{1}{2}m\omega^2\left[A^2 - y^2\right]$$

$$\boxed{KE = \frac{1}{2}k\left[A^2 - y^2\right]}$$

**At mean position (y = 0)**

$$\therefore k_{max} = \frac{1}{2}kA^2$$

At extreme position (y = A)

KE = 0

2) Potential Energy

Potential energy of an oscillating particle at a point is defined as work done by an external agent to bring the body from mean position to that point

Let 'y' be the displacement from mean position, then potential energy is given by

$$\boxed{U = \frac{1}{2}ky^2}$$

At mean position (y = 0)

$\therefore$ PE = 0

At extreme position (y = A)

$$\boxed{PE_{max} = \frac{1}{2}KA^2}$$

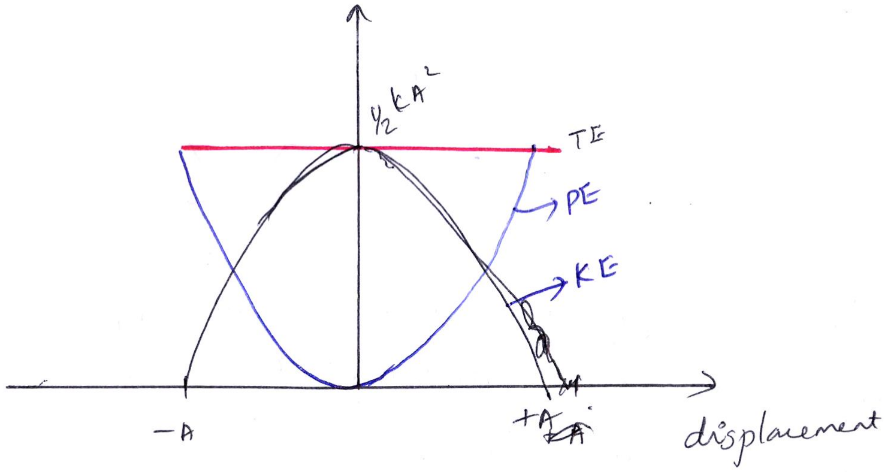
Total Energy

E = KE + PE

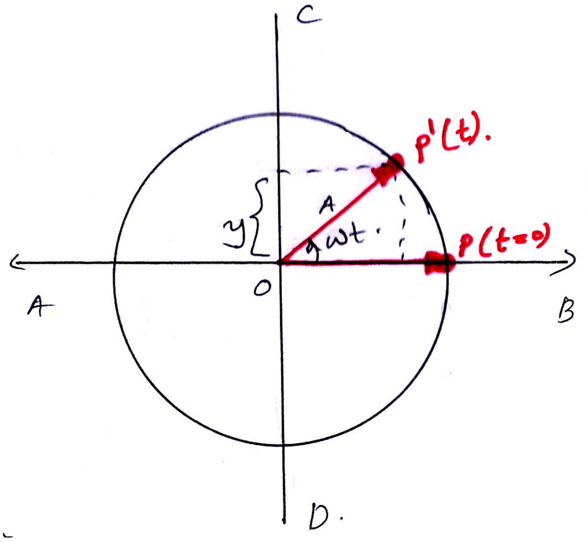
$$E = \frac{1}{2}m\omega^2\left[A^2 - y^2\right] + \frac{1}{2}m\omega^2A^2$$

$$E = \frac{1}{2}m\omega^2A^2$$

$\boxed{E = \frac{1}{2}kA^2}$ Total Energy of the oscillating particle always remains a constant




**Relation between SHM & uniform circular motion**




$\Rightarrow$ Consider a uniform circular motion

A $\rightarrow$ Radius of circular path

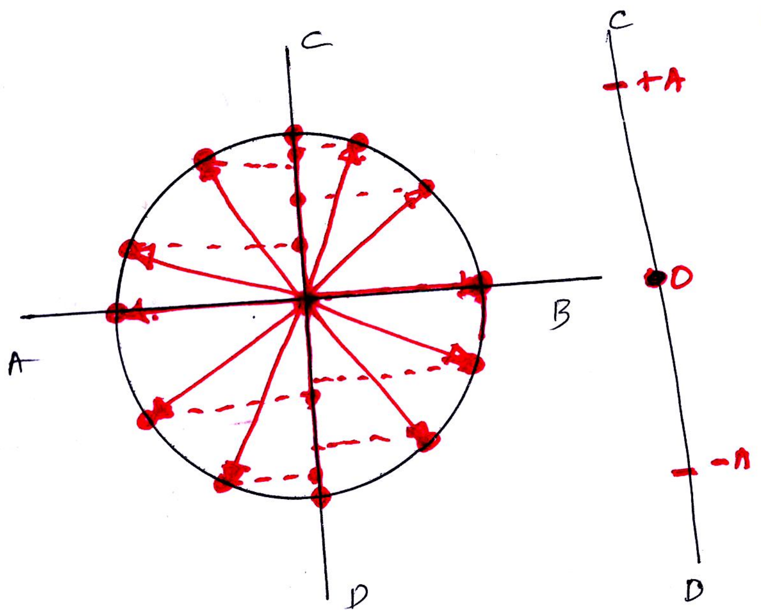
B $\rightarrow$ Angular velocity of circular motion

Let 'p' be the position of the particle at t = 0, and p$^{/}$ be the position at time t.

Projection of position vector op$^{/}$ on the diameter CD is

$\boxed{y = A\sin\omega t}$

This is displacement equation of SHM



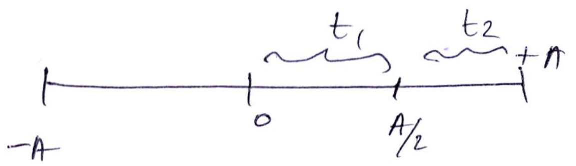

From the figure it is clear that particle only execute circular motion. But foot of the perpendicular drawn from the particle to the diameter CD will execute a SHM. Therefore SHM can also be defined as the projection of a uniform circular motion, on any of the diameter.

**NOTE 1 :**

1) Time taken to displace a particle from 0 to $A/2$ is $T/12$

2) Time taken to displace the particle from $A/2$ to A is $T/6$

**Proof**




$\underline{0 \to A/2}$

$y = A\sin\omega t_1$

$\frac{A}{2} = A\sin\omega t_1 \qquad \Rightarrow \sin\omega t_1 = 1/2$

$\omega t_1 = \pi/6$

$$\boxed{t_1 = \frac{\pi}{6\omega} = \frac{\pi T}{6 \times 2\pi} = \frac{T}{12}}$$

$$\underline{\frac{A}{2} \to A}$$

$$\boxed{t_2 = \frac{T}{4} - t_1 = \frac{T}{4} - \frac{T}{12} = \frac{3T - T}{12} = \frac{T}{6}}$$

**NOTE : 2**

If two SHM are represented by

$y_1 = A_1 \sin(\omega_1 t + \phi_1)$ &

$y_2 = A_2 \sin(\omega_2 t + \phi_2)$

Then phase difference between them is

$$\boxed{\Delta\phi = (\omega_2 t + \phi_2) - (\omega_1 t + \phi_1)}$$

$\Rightarrow$ If $\Delta\phi$ is +ve, $y_2$ leads $y_1$

$\Rightarrow$ If $\Delta\phi$ –ve, $y_1$ leads $y_2$

$$\boxed{\begin{array}{l} \Rightarrow \text{If } \Delta\phi = 0, 2\pi, 4\pi, ........ \\ \Rightarrow \text{Then they are said to be in} - \text{phase} \end{array}}$$

$$\left\{\begin{array}{l} \text{If } \Delta\phi = \pi, 3\pi, 5\pi, ...... \\ \text{Then they are said to be out of phase} \end{array}\right\}$$

**NOTE : 3**

If a SHM is represented by

$y = A_1 \sin \omega t + A_2 \sin(\omega t + \phi)$

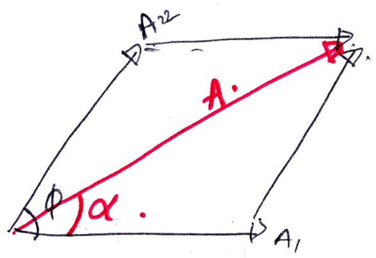
Then it can be represented as

$$\boxed{y = A \sin(\omega t + \alpha)}$$

where resulted amplitude

$$\boxed{A = \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos\phi}}$$

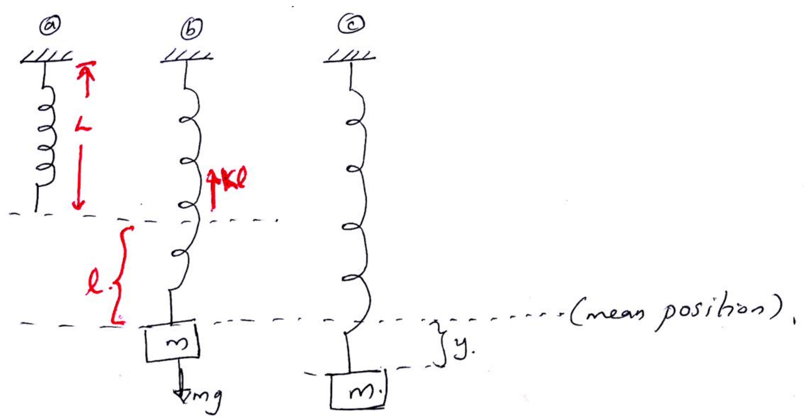
where $\boxed{\tan\alpha = \dfrac{B\sin\theta}{A + B\cos\theta}}$




## Applications of SHM

1) Oscillations of a spring

Consider a light and elastic spring suspended vertically from a rigid support. Let a body of mass 'm' be attached to the lower end of the spring




$\ell \rightarrow$ elongation or extension

From figure (B), according to Hooke's Law, restoring force is directly proportional to elongation

i.e. $F \alpha \ell$ OR

$F = -k\ell$ | $k \rightarrow$ spring constant

–ve sign indicates that force is restoring

At equilibrium

$mg = k\ell$

$\therefore \boxed{k = \dfrac{mg}{\ell}}$

Now the body be pulled further down through a small distance ‘y’ and released, it starts vertical oscillations.

From fig. (c)

$$F_{net} = mg - k(\ell + y)$$

$$ma = mg - k\ell - ky$$

i.e. ma = –ky

$$\boxed{a = -\left(\frac{k}{m}\right)y}$$

This equation of the form $a = -\omega^2 y$. $\therefore$ Motion of spring is SHM

By comparing $\omega = \sqrt{\frac{k}{m}}$ & $k = m\omega^2$

$\therefore$ Time period of oscillation of a spring-block system is

$$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}}$$

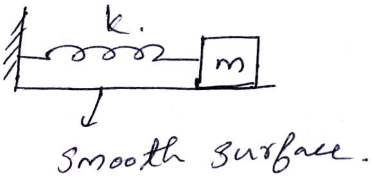
$\Rightarrow$ Spring constant ‘k’ is independent of acceleration due to gravity g

**<u>NOTE 1 :</u>**

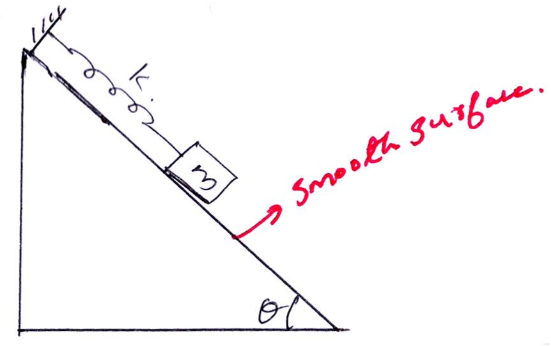
If the spring is not light, but has a mass $m_5$, then time period of oscillation of spring block system is

$$T = 2\pi\sqrt{\frac{m + \frac{1}{3} + m_s}{k}}$$

**NOTE 2 :**

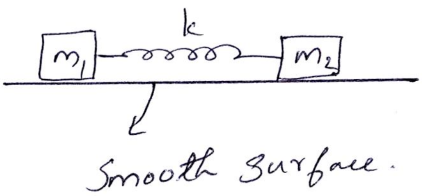



$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

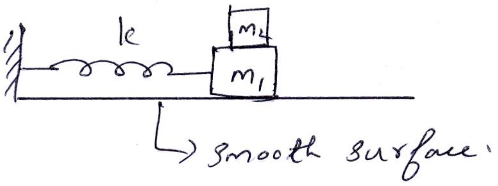



$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

**NOTE : 3**




**NOTE : 4**

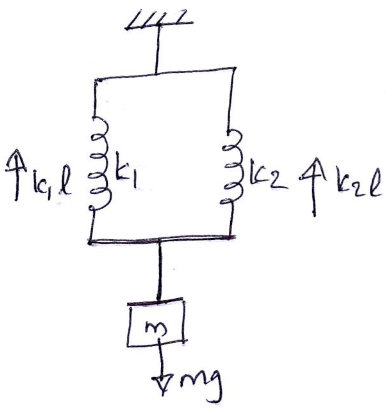



$$\boxed{T = 2\pi\sqrt{\frac{(m_1 + m_2)}{k}}}$$

$\Rightarrow$ Where restoring force is less than frictional force between $m_1$ & $m_2$

**Combination of Springs**

a) Parallel combination



$\Rightarrow$ For parallel combination both springs have same elongation.

Let '$\ell$' be the elongation, then

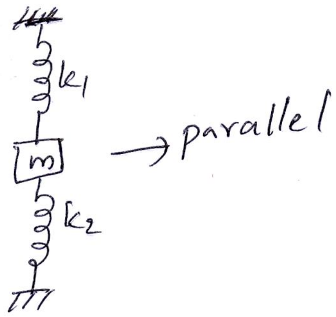
$mg = k_1\ell + k_2\ell$

$= \left(k_1 + k_2\right)\ell$

i.e., $mg = \left(k_p\right)\ell$

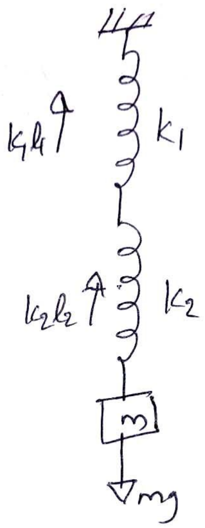
where $\boxed{k_p = k_1 + k_2}$ $\Rightarrow$ Effective spring constant in parallel combination

$\therefore \boxed{T = 2\pi\sqrt{\frac{m}{k_p}}}$




b) Series combination

In series combination force is same in both springs, but elongation is different




For first spring $k_1\ell_1 = F$.

$\ell_1 = \frac{F}{k_1}$

For second spring $k_2\ell_2 = F$

$$\ell_2 = \frac{F}{k_2}$$

$\Rightarrow$ Let '$\ell$' be the total elongation of springs, then $\ell = \ell_1 + \ell_2$

$$\frac{F}{k_s} = \frac{F}{k_1} + \frac{F}{k_2}$$

i.e., $\boxed{\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2}}$

where $k_s \rightarrow$ effective spring constant in series combination

i.e., $\boxed{k_s = \frac{k_1 k_2}{k_1 + k_2}}$

**NOTE : 1**

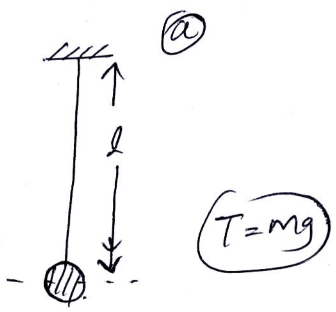
$\therefore \boxed{\text{Spring constant } \alpha \frac{1}{\text{length of spring}}}$

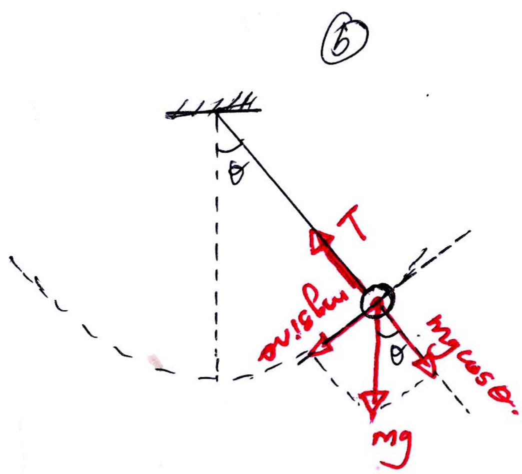
**NOTE : 2**

If a spring of spring constant 'k' is cut into 'n' equal parts, then spring constant of each part becomes nk.

2) Oscillations of a simple pendulum

Simple pendulum consist of a light & inextensible spring connected to a small mass or bob



m $\rightarrow$ mass of bob

$\ell \rightarrow$ length of pendulum

(Distance between point of suspension & centre of mass of bob)

From figure (B)

$T - mg\cos\theta = F_C \rightarrow$ centripetal force

$$\boxed{T = mg\cos\theta + \frac{mv^2}{\ell}}$$

Restoring torque is provided by $mg\sin\theta$

i.e., restoring torque $\tau = -\ell(mg\sin\theta)\sin(90)$ | $-$ ve sign indicates restoring torque

i.e., $I\alpha = -\ell mg\sin\theta$

for small oscillations $\sin\theta \approx \theta$

$\therefore I\alpha = -\ell mg\theta$

$$\boxed{\alpha = -\left(\frac{\ell mg}{I}\right)\theta}$$

$I \rightarrow$ moment of inertia

This is the form of $a = -\omega^2 y$

$\therefore$ It is a simple harmonic motion.

By comparing $\omega = \sqrt{\frac{\ell mg}{I}}$ where $\boxed{I = m\ell^2}$

$$\therefore \boxed{\omega = \sqrt{\frac{\ell mg}{m\ell^2}} = \sqrt{\frac{g}{\ell}}}$$

$\therefore$ Time period of oscillation

$$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\ell}{g}}}$$

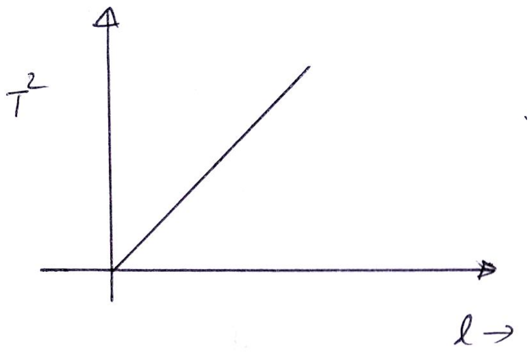
1) Time period is independent of mass of bob

2) $T \,\alpha\, \sqrt{\ell}$

3) $T \,\alpha\, \frac{1}{\sqrt{g}}$

**Graph**

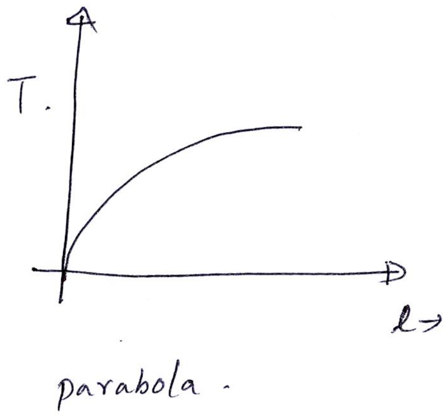
1) $\ell - T^2$




$$\boxed{T^2 = 4\pi^2 \frac{\ell}{g}}$$

$$\text{Slope} = \frac{4\pi^2}{g} \quad T^2 \ \alpha\ \ell$$

2) $\ell - T$ graph

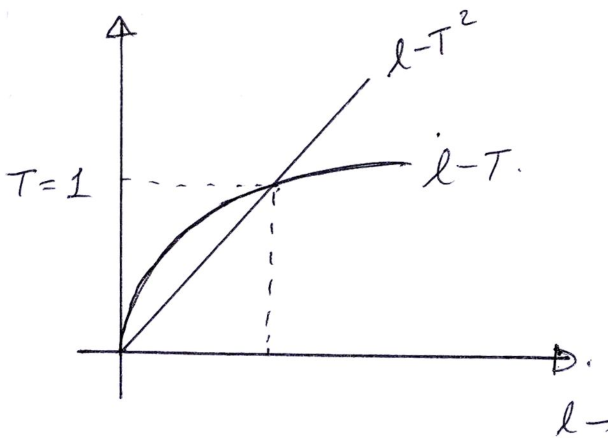



$$T^2 = \frac{4\pi^2}{g}\ell \rightarrow \text{Eqn. of parabola}$$

$$\uparrow \qquad\quad \uparrow$$

$$y^2 = 4ax \rightarrow$$

$\ell - T^2$ & $\ell - T$ graph intersect at T = 1



**NOTE**

Second's pendulum : A pendulum whose time period is 2 second

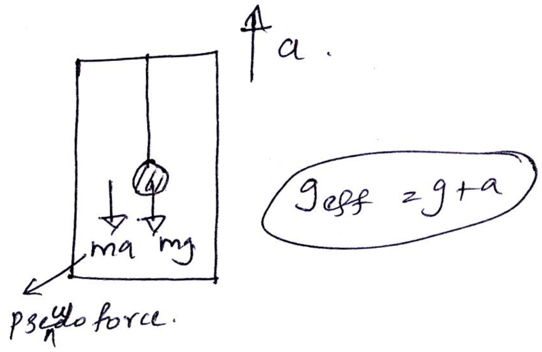
* Length of second's pendulum on the surface of earth is approximately 1m

**Special Cases**

* If a pendulum is suspended from the roof of a lift moving vertically upwards with an acceleration 'a', then effective acceleration on the bob becomes (g + a)

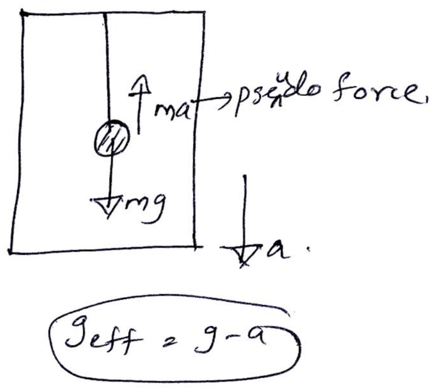
$\therefore$ Time period of oscillation

$$\boxed{T = 2\pi\sqrt{\frac{\ell}{g+a}}}$$



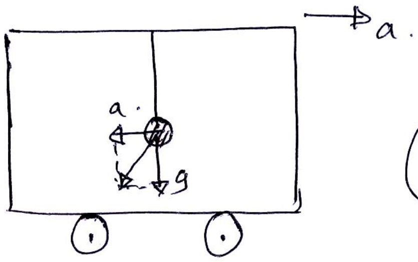

* If a pendulum is suspended from the roof of a lift moving vertically downwards with an acceleration 'a', then effective acceleration on the bob becomes (g – a)

$$\boxed{T = 2\pi\sqrt{\frac{\ell}{g-a}}}$$

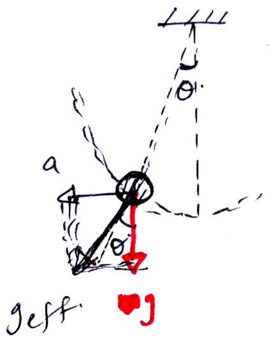



* If a pendulum is suspended on the roof of a car moving horizontally with an acceleration 'a', then effective acceleration on the bob becomes

$$g_{eff} = \sqrt{g^2 + a^2}$$



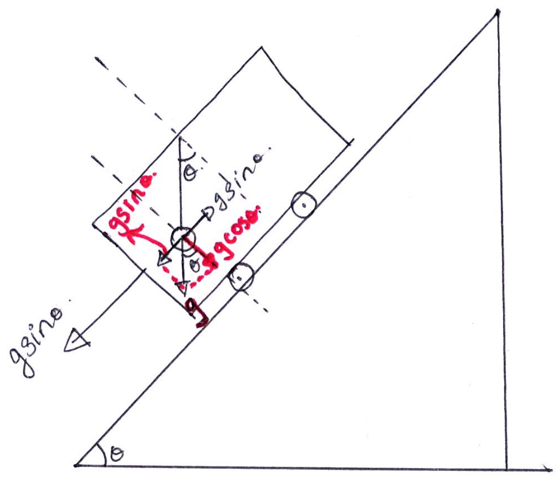

$$\boxed{g_{eff} = \sqrt{g^2+a^2}}$$



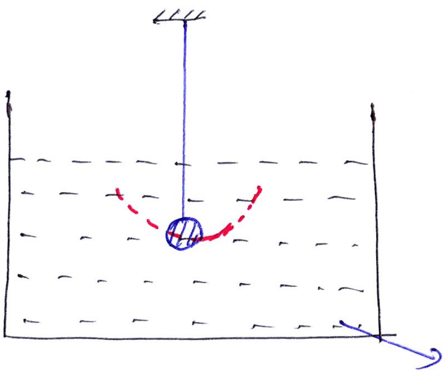

$\Rightarrow$ In such a case mean position of the pendulum is shifted backwards by an angle $\boxed{\tan\theta = \frac{a}{g}}$

* If a pendulum is on the roof of a car moving down an inclined plane of inclination '$\theta$', then effective acceleration on the bob becomes $g\cos\theta$

$\therefore \boxed{T = 2\pi\sqrt{\frac{\ell}{g\cos\theta}}}$




from figure $\boxed{g_{eff} = g\cos\theta}$

* 


Net force on the bob is

$F = mg - F_B$

$mg_{eff}$ = mg – weight of liquid displaced

$V \rightarrow$ volume of bob

$$(V\rho)g_{eff} = (V\rho)g - (V \times \sigma)g$$

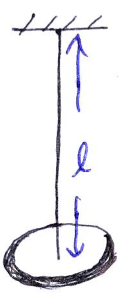
$$\rho g_{eff} = \rho g - \sigma g$$

$$g_{eff} = \left(\frac{\rho - \sigma}{\rho}\right)g \Rightarrow \boxed{g_{eff} = \left(1 - \frac{\sigma}{\rho}\right)g}$$

$$\boxed{T = 2\pi\sqrt{\frac{\ell}{\left(1 - \frac{\sigma}{\rho}\right)g}}}$$

* **Torsional Pendulum**

$\Rightarrow$ It consists of a horizontal circular disc suspended with the help of wire




$\Rightarrow$ If it is twisted by an angle $\theta$ & then released, it executes torsional oscillations

Restoring torque $\tau = -C\theta$ | $C \rightarrow$ restoring torque per unit twist

$I\alpha = -C\theta$

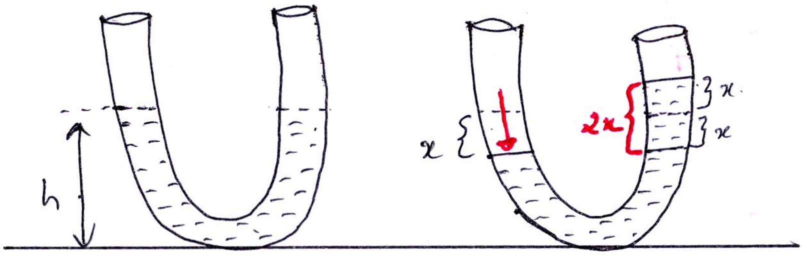
$\boxed{\alpha = -\frac{C}{I}\theta} \rightarrow a = -\omega^2 y$

SHM

By comparing $\omega = \sqrt{\frac{C}{I}}$ | $I \rightarrow$ moment of inertia of disc

$\therefore \boxed{T = 2\pi\sqrt{\frac{I}{C}}}$

**U-Tube Oscillator**




$A \rightarrow$ Area of cross section

$h \rightarrow$ height of liquid in the tube in the state of equilibrium

$d \rightarrow$ density of liquid

If we withdraw applied force, then liquid will oscillate

$\Rightarrow$ Here restoring force is provided by the weight of liquid in '2x' height.

i.e., F = –weight of liquid in 2x height

$= -A(2x)dg$

$M \rightarrow$ Total mass of liquid in the tube

$Ma = -2Adg\, x$

$\boxed{a = -\left(\frac{2Adg}{M}\right)x} \Rightarrow a = -\omega^2 y$

SHM

By comparing $\omega = \sqrt{\dfrac{2Adg}{M}}$

$$\therefore \boxed{T = 2\pi\sqrt{\dfrac{M}{2Adg}}}$$

But $M = (A2h)d$ $\quad\quad \therefore T = 2\pi\sqrt{\dfrac{A2h.d}{2Adg}}$

$$\boxed{T = 2\pi\sqrt{\dfrac{h}{g}}}$$

**<u>Damped Oscillations</u>**

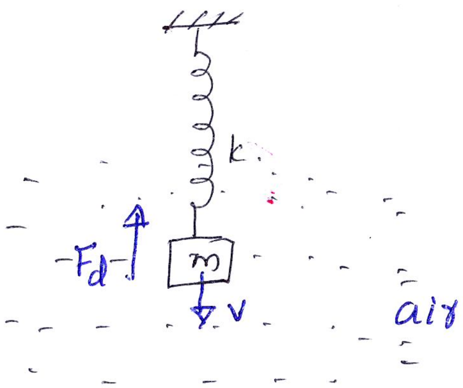
Oscillations in which amplitude decreases gradually with the passage of time are called damped oscillations.

The restoring forces are known as damping forces.

Generally damping force is directly proportional to velocity of the oscillating body.

$F_d \,\alpha\, V$

$$\boxed{\vec{F}_d = -b\vec{V}}$$




$b \rightarrow$ <u>Damping constant</u>, it depends on size and shape of the body and nature of the medium

$\rightarrow$ –ve sign indicates that damping force is opposite to the direction of motion

Net force on the body

F = restoring force + damping force

ma = –ky – bv

ma + bv + ky = 0

$$\boxed{\frac{md^2y}{dt^2}+\frac{bdy}{dt}+ky=0}$$

$\Rightarrow$ This is the equation of damped oscillator. Solution of this equation gives displacement of damped oscillator, which is given by,

$$\boxed{y=Ae^{-\frac{bt}{2m}}\cos\left[\omega^1 t+\phi\right]}$$

where $\boxed{\text{Amplitude}=Ae^{-\frac{bt}{2m}}}$

$A\rightarrow$ initial amplitude

$\omega^1\rightarrow$ Angular velocity of damped oscillator

$$\boxed{\omega^1=\sqrt{\frac{k}{m}-\frac{b^2}{4m^2}}}$$

If $b^2 << km$ [small damping]

$$\boxed{\omega^1=\omega=\sqrt{\frac{k}{m}}}$$

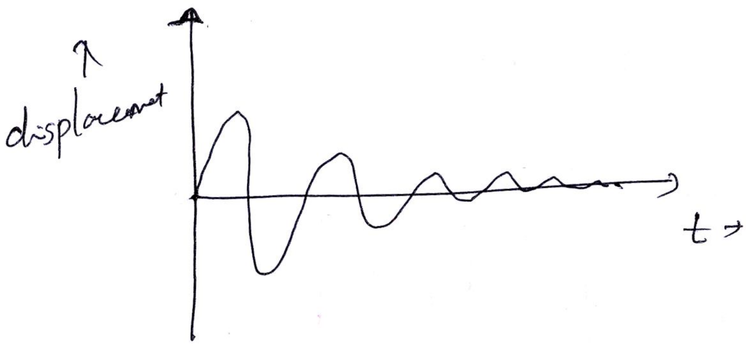
Total Energy of the oscillator $E=\frac{1K}{2}(\text{Amp})^2$

$$E=\frac{1}{2}k.A^2e^{-\frac{bt}{m}}$$

i.e. $\boxed{E=E_0e^{-\frac{bt}{m}}}$

$E_0=\frac{1}{2}kA^2$ initial energy

**Total Energy also decrease with time**




**Forced Oscillation**

A body oscillates under the influence of an external periodic force is known as forced oscillation

Let F(t) be the external periodic force

$$F(t) = F_0 \cos \omega_d t$$

$F_0 \to$ Amplitude of applied force

$\omega_d \to$ Angular velocity of applied force OR driving force

$\therefore$ Net force on the body

$$F = F(t) + F_{res} + F_{dam}$$

$$ma = F_0 \cos \omega_d d - ky - bv$$

$$ma + bv + ky = F_0 \cos \omega_d t$$

$$\boxed{m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = F_0 \cos \omega_d t}$$

$\Rightarrow$ This is the equation of a forced oscillator. Solution of this equation gives displacement of forced oscillator, which is given by

$$\boxed{y = A\cos(\omega_d t + \phi)}$$

where Amplitude $\boxed{A = \dfrac{F_0}{\left\{m^2\left(\omega^2 - \omega_d^2\right)^2 + \omega_d^2 b^2\right\}^{1/2}}}$

Initially the oscillator oscillates with its natural frequency $\omega$. When we apply an external periodic force, oscillations with natural frequency die out & body will oscillates with the frequency of force $(\omega_d)$

**Resonance**

If the frequency of applied force is equal to natural frequency of the body then the body will oscillates with maximum amplitude. This is known as resonance.

At resonance

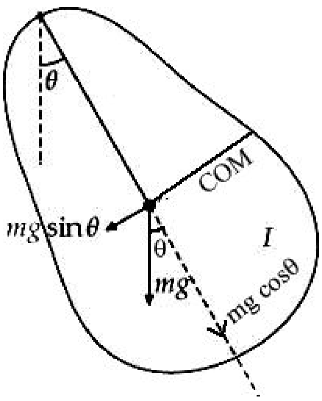
$\omega = \omega_d$

$\therefore$ Amplitude $\boxed{A_{max} = \dfrac{F_0}{\omega_d b}}$

**NOTE :**

$\Rightarrow$ In ideal case of zero damping amplitude of the oscillator at resonance is infinity.

**Compound Pendulum OR Physical Pendulum**




Restoring torque

$\tau = -(mg\sin\theta)d$

$d \rightarrow$ distance between point of suspension and COM

$\therefore I\alpha = -(mgd)\theta$

$\boxed{\alpha = -\left[\dfrac{mgd}{I}\right]\theta}$ S.H.M.

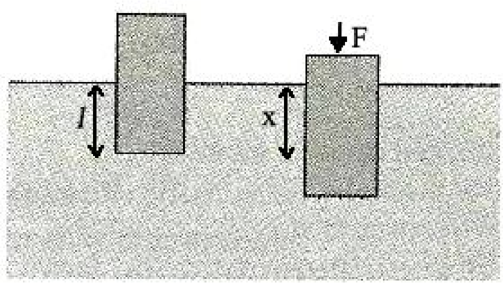
By comparing

$w = \sqrt{\dfrac{mgd}{I}}$

$\therefore \boxed{T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{I}{mgd}}}$

Where $I \rightarrow$ moment of inertia of the given body about the axis of oscillation.

**Oscillation of a cylinder in water**




M $\rightarrow$ mass of cylinder

d $\rightarrow$ density of liquid

A $\rightarrow$ area of cross section

**At equilibrium**

Mg = weight of liquid displaced by the body

i.e. $Mg = (A\ell)dg$

$\therefore M = A\ell g$ ...........(1)

When we displace the cylinder downward & released, it starts vertical oscillations.

Restoring force F = additional upthrust

i.e., Ma = – (Ax)dy

$$\therefore \boxed{a = -\left[\frac{Adg}{M}\right]x}$$

$\therefore$ The cylinder executes SHM

By comparing

$$\omega = \sqrt{\frac{Adg}{M}}$$

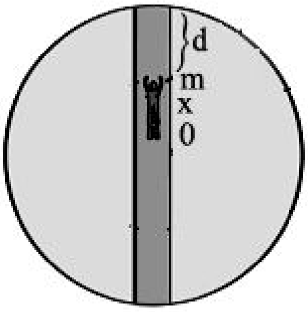
$$\therefore \boxed{T = 2\pi\sqrt{\frac{M}{Adg}}}$$

where $M = A\ell d$

$\therefore T = 2\pi\sqrt{\dfrac{A\ell d}{Adg}}$

$\boxed{T = 2\pi\sqrt{\dfrac{\ell}{g}}}$

**Oscillation of a ball in a tunnel drilled across the diameter of earth**




d $\rightarrow$ depth from the surface of earth.

$\rightarrow$ when the ball is at a distance 'x' from mean position

Restoring force $F = -mg^1$

i.e., $ma = -mg\left[1 - \dfrac{d}{R}\right]$

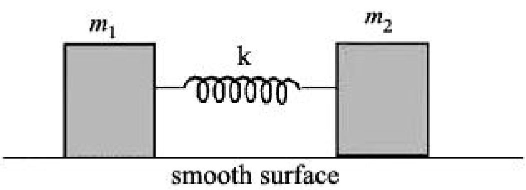
$a = -g\left[R - d\right]$

i.e. $\boxed{a = -\left[\dfrac{g}{R}\right]x}$ SHM

$\omega = \sqrt{\dfrac{g}{R}}$

$\boxed{T = 2\pi\sqrt{\dfrac{R}{g}}}$

**Problem :**

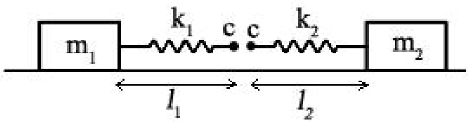


If If the blocks are compressed slightly and released, then time period of oscillation is

**Solution :**

In the absence of any external force present, the centre of mass of the two block system rmain at rest during oscillation.

In figure point 'C' will remain at rest and with respect to this point $M_1$ and $M_2$ will oscilatte independently.




System behaves as two springs are be fixed point 'C' separately.

We have $k \,\alpha\, \frac{1}{\ell}$

$$k_1 \alpha \frac{1}{\ell_1}$$

$$\therefore \frac{k_1}{k} = \frac{\ell}{\ell_1} \Rightarrow \boxed{k_1 = \frac{k\ell}{\ell_1}} \text{ ........... (1)}$$

Similarly

$$\boxed{k_2 = \frac{k\ell}{\ell_2}} \text{ ................ (2)}$$

But

$$\ell_1 + \ell_2 = \ell$$

$$m_1\ell_1 = m_2\ell_2$$

$$m_1\ell_1 = m_2\left[\ell - \ell_1\right]$$

$$\ell_1 = \left[\frac{m_2}{m_1 + m_2}\right]$$

and

$$\ell_2 = \left[\frac{m_1}{m_1 + m_2}\right]\ell$$

$$\therefore k_1 = \frac{k\ell}{\ell_1} = \frac{k\ell(m_1+m_2)}{m_2\ell}$$

$$\text{i.e. } \boxed{k_1 = \left[\frac{m_1+m_2}{m_2}\right]k}$$

$$\& \ k_2 = \left[\frac{m_1+m_2}{m_1}\right]k$$

Now, angular frequency for $m_1$ is,

$$\omega_1 = \sqrt{\frac{k_1}{m_1}} = \sqrt{\frac{(m_1+m_2)k}{m_1 m_2}}$$

and angular frequency for $m_2$ is

$$\omega_2 = \sqrt{\frac{k_2}{m_2}} = \sqrt{\frac{(m_1+m_2)k}{m_1.m_2}}$$

$$\text{i.e., } \boxed{\omega_1 = \omega_2 = \omega}$$

$\Rightarrow$ Both the blocks oscillate with same angular frequency

$\therefore$ Time period of system

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\left[\frac{m_1 m_2}{m_1+m_2}\right]\frac{1}{k}}$$

$$\text{i.e., } \boxed{T = 2\pi\sqrt{\frac{\mu}{k}}}$$

Where $\mu = \dfrac{m_1 m_2}{m_1+m_2} \Rightarrow$ Reduced mass of system