CHAPTER - 01

SET, RELATIONS AND REAL FUNCTIONS

1. 3 Since, $y = e^x$ and $y = x$ do not meet for any $x \in R$

$\therefore\ A \cap B = \phi$.

2. 3 $A \cap (A \cup B)' = A \cap (A' \cap B')$, $(\because (A \cup B)' = A' \cap B')$

$= (A \cap A') \cap B'$, (by associative law)

$= \phi \cap B'$, $(\because A \cap A' = \phi)$

$= \phi$.

3. 3 $n(P) = 25\%, n(C) = 15\%$

$n(P^c \cap C^c) = 65\%, n(P \cap C) = 2000$

Since, $n(P^c \cap C^c) = 65\%$

$\therefore n(P \cup C)^c = 65\%$ and $n(P \cup C) = 35\%$

Now, $n(P \cup C) = n(P) + n(C) - n(P \cap C)$

$35 = 25 + 15 - n(P \cap C)$

$\therefore n(P \cap C) = 40 - 35 = 5$. Thus $n(P \cap C) = 5\%$

But $n(P \cap C) = 2000$

$\therefore$ Total number of families $= \dfrac{2000 \times 100}{5} = 40{,}000$

Since, $n(P \cup C) = 35\%$

and total number of families = 40,000

and $n(P \cap C) = 5\%$. $\therefore$ (b) and (c) are correct.

4. 4

For $(a, b), (c, d) \in N \times N$

$(a,b)R(c,d) \Rightarrow ad(b+c) = bc(a+d)$

Reflexive : Since $ab(b+a) = ba(a+b) \forall ab \in N$, $\therefore (a,b)R(a,b)$, $\therefore$ R is reflexive.

Symmetric : For $(a,b),(c,d) \in N \times N$, let $(a,b)R(c,d)$

$\therefore\ ad(b+c) = bc(a+d) \Rightarrow bc(a+d) = ad(b+c)$

$\Rightarrow cb(d+a) = da(c+b) \Rightarrow (c,d)R(a,b)$

$\therefore$ R is symmetric

Transitive : For $(a,b),(c,d),(e,f) \in N \times N$,

Let $(a,b)R(c,d),(c,d)R(e,f)$

$\therefore\ ad(b+c) = bc(a+d),\ cf(d+e) = de(c+f)$

$\Rightarrow adb + adc = bca + bcd$ .....(i)

and $cfd + cfe = dec + def$ .....(ii)

(i) $\times$ ef + (ii) $\times$ ab gives,

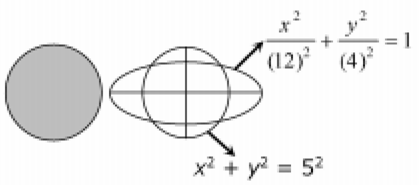
$adbef + adcef + cfdab + cfeab = bcaef + bcdef + decab + defab$

$\Rightarrow adcf(b+e) = bcde(a+f) \Rightarrow af(b+e) = be(a+f) \Rightarrow (a,b)R(e,f)$.

$\therefore$ R is transitive. Hence R is an equivalence relation

5. 4

A = Set of all values (x, y) : $x^2 + y^2 = 25 = 5^2$



$B = \frac{x^2}{144} + \frac{y^2}{16} = 1$ i.e., $\frac{x^2}{(12)^2} + \frac{y^2}{(4)^2} = 1$.

Clearly, $A \cap B$ consists of four points.

6. 1

Let $A = \{1,2,3\}$ and R = {(1, 1), (1, 2)}, S = {(2, 2) (2, 3)} be transitive relations on A.

Then $R \cup S$ = {(1, 1); (1, 2); (2, 2); (2, 3)}

Obviously, $R \cup S$ is not transitive.

Since $(1, 2) \in R \cup S$ and $(2,3) \in R \cup S$ but $(1, 3) \notin R \cup S$.

7. 1

S = {1, 2, 3, ..., 100} = Total non-empty subsets-subsets with product of element is odd

$= 2^{100} - 1 - 1[(2^{50} - 1)] = 2^{100} - 2^{50} = 2^{50}(2^{50} - 1)$

8. 3

$A=\{x \in Z : 2^{(x+2)(x^2-5x+6)}=1\}$

$2^{(x+2)(x^2-5x+6)}=2^0 \Rightarrow x=-2,2,3$

$A=\{-2,2,3\}$,

$B=\{x \in Z : -3<2x-1<9\}$

B = {0, 1, 2, 3, 4}

A × B has is 15 elements, so number of subsets of A × B is 2^{15}

9. 4

Let $a^2+b^2 \in Q$ and $b^2+c^2 \in Q$

E.g. $a=2+\sqrt{3}$ and $b=2-\sqrt{3} \Rightarrow a^2+b^2=14 \in Q$

Let $c=(1+2\sqrt{3}) \Rightarrow b^2+c^2=20 \in Q$

But $a^2+c^2=(2+\sqrt{3})^2+(1+2\sqrt{3})^2 \notin Q$ for R_2

Let $a^2=1$, $b^2=\sqrt{3}$ and $c^2=2 \Rightarrow a^2+b^2 \notin Q$ and $b^2+c^2 \notin Q$

But $a^2+c^2 \in Q$.

10. 1

Number of onto function such that exactly three elements in $x \in A$ such that $f(x) = y_2$ is equal to

$= {}^7C_3 \cdot \{2^4 - 2\} = 14 \cdot {}^7C_3$

11. 3

Let S = {1, 2, 3} ⇒ n(S) = 3

Now, P(S) = set of all subsets of S

Total No. of subsets = $2^3 = 8$

∴ n[P(S)] = 8

Now, number of one to one functions from $S \to P(S)$ is

${}^8P_3 = \frac{8!}{5!} = 8 \times 7 \times 6 = 336$.

12. 3

Let $\alpha \in (A \cap B) \Rightarrow \alpha \in A$ and $\alpha \in B$

$\Rightarrow g(\alpha)=0$ and $f(\alpha)=0$

$\Rightarrow \{f(\alpha)\}^2+\{g(\alpha)\}^2=0$

$\Rightarrow \alpha$ is a root of $\{f(x)\}^2+\{g(x)\}^2=0$

Hence, statement-I is true and statement-II is false

13. 2

Statement-I

(a) Reflexive xRy : (x – x) is an integer, which is true,
Hence, it is reflexive.
(b) Symmetric xRy: (x – y) is an integer
$\Rightarrow -(y-x)$ is also an integer
$\therefore$ $(y-x)$ is also an integer $\Rightarrow$ yRx
Hence, it is symmetric.
(c) Transitive xRy and yRz
$\Rightarrow$ $(x-y)$ and $(y-z)$ are integer and
$\Rightarrow$ $(x-y)+(y-z)$ is an integer.
$\Rightarrow$ $(x-z)$ is an integer
$\Rightarrow$ xRz
$\therefore$ It is transitive
Hence, it is equivalence relation

Statement-II

$B = \{(x,y) \in R \times R : x = \alpha y$ for some relational number $\alpha\}$
If $\alpha = 1$, then xRy: x = y (To check equivalence)
(a) Reflexive xRx: x = x (True)
$\therefore$ Reflexive
(b) Symmetric xRy: $x = y \Rightarrow y = x \Rightarrow$ yRx
$\therefore$ Symmetric
(c) Transitive xRy and yRz $\Rightarrow x = y$ and $y = z \Rightarrow x = z \Rightarrow$ xRz
$\therefore$ Transitive
Hence, it is equivalence relation.
$\therefore$ Both are true but statement-II is not correct explanation of statement-I

14. 2

$f(x) = \log(x+\sqrt{x^2+1})$
$f(-x) = \log(-x+\sqrt{x^2+1})$
$f(x)+f(-x) = \log(x+\sqrt{x^2+1}) + \log(-x+\sqrt{x^2+1})$
$= \log[(x+\sqrt{x^2+1})(-x+\sqrt{x^2+1})]$
$= \log(1)$
$= 0$
$\therefore$ f(x) is an odd function.

15. 3

$$\frac{x^2+14x+9}{x^2+2x+3}=y \Rightarrow x^2+14x+9=x^2y+2xy+3y$$

$$\Rightarrow x^2(y-1)+2x(y-7)+(3y-9)=0$$

Since x is real, $\therefore 4(y-7)^2-4(3y-9)(y-1)>0$

$$\Rightarrow 4(y^2+49-14y)-4(3y^2+9-12y)>0$$

$$\Rightarrow 4y^2+196-56y-12y^2-36+48y>0$$

$$\Rightarrow 8y^2+8y-160<0 \Rightarrow y^2+y-20<0$$

$\Rightarrow (y+5)(y-4)<0$; $\therefore$ y lies between – 5 and 4.

16. D

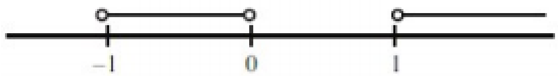
Given $f:(2,3)\to(0,1)$ and $f(x)=x-[x]$

$\therefore f(x)=y=x-2 \Rightarrow x=y+2=f^{-1}(y)\Rightarrow f^{-1}(x)=x+2$.

17. C

$4-x^2\neq 0$; $x^3-x>0$

$x=\pm 2$ $\quad$ $x(x-1)(x+1)>0$



$\therefore\ D_f\in(-1,0)\cup(1,2)\cup(2,\infty)$

18. 3

C. $y=\dfrac{8^{2x}-8^{-2x}}{8^{2x}+8^{-2x}}$

$=\dfrac{8^{2x}-\frac{1}{8^{2x}}}{8^{2x}+\frac{1}{8^{2x}}}$

$=\dfrac{8^{4x}-1}{8^{4x}+1}$

$y\times 8^{4x}+y=8^{4x}-1$

$(y-1)\,8^{4x}=-y-1$

$8^{4x}=\dfrac{-y-1}{y-1}$

$4x=\log_8\left(\dfrac{-y-1}{y-1}\right)$

$$4x = \log_8\left(\frac{1+y}{1-y}\right)$$

$$x = \frac{1}{4}\log_8\left(\frac{1+y}{1-y}\right).$$

19. 3

$f(x) = \log\left(\sqrt{x-4} + \sqrt{6-x}\right)$

$x - 4 \geq 0 \Rightarrow x \geq 4$

$6 - x \geq 0 \Rightarrow x \leq 6$

Domain $= [4, 6]$

20. 3

$y = 5^{\log x}$

$y = x^{\log 5}$

$x = y^{\frac{1}{\log 5}}$

SECTION II (NUMERICAL)

21. 160 $n(C \cup H \cup B) = 224 + 240 + 336 - 64 - 80 - 40 + 24 = 640$

$n(C \cup H \cup B)' = 800 - 640 = 160$

22. 12

Total number of reflexive relations in a set with n elements $= 2^{n^2-n}$

Therefore, total number of reflexive relation set with 4 elements $= 2^{16-4} = 2^{12}$

23. 30 $n(X_i) = 10,\ \bigcup_{i=1}^{50} X_i = T \Rightarrow n(T) = 500$ each element of T belongs to exactly 20 elements of X_i

$\Rightarrow \frac{500}{20} = 25$ distinct elements

So, $\frac{5n}{6} = 25 \Rightarrow$ n = 30.

24. 20 Replace x by x+5

We get $f(x+5)+f(x+15)=f(x+10)+f(x+20)$

$\therefore f(x)=f(x+20)$

25. 29 $n(A)=25$, $n(B)=7$, $n(A\cap B)=3$, $n(A\cup B)=25+7-3=29$

JEE ADVANCED LEVEL

SECTION III

26. C $f(a-(x-a)) = f(a)f(x-a) - f(0)f(x)$ .....(i)

Put $x = 0, y = 0$; $f(0) = (f(0))^2 - [f(a)]^2 \Rightarrow f(a) = 0$

$[\because f(0) = 1]$. From (i), $f(2a - x) = -f(x)$.

27. B

$$f(x) = \frac{\sin^{-1}(3-x)}{\log[|x|-2]}$$

Let $g(x) = \sin^{-1}(3-x) \Rightarrow -1 \le 3-x \le 1$

Domain of $g(x)$ is [2, 4] and let $h(x) = \log[|x|-2] \Rightarrow |x|-2 > 0$

$\Rightarrow |x| > 2 \Rightarrow x < -2$ or $x > 2 \Rightarrow (-\infty, -2) \cup (2, \infty)$

we know that $(f/g)(x) = \frac{f(x)}{g(x)} \forall x \in D_1 \cap D_2 - \{x \in R : g(x) = 0\}$

$\therefore$ Domain of $f(x) = (2,4] - \{3\} = (2,3) \cup (3,4]$.

28. B We have $f(x) = \left[\log_{10}\left(\frac{5x-x^2}{4}\right)\right]^{1/2}$ .....(i)

From (i), clearly $f(x)$ is defined for those values of x for which $\log_{10}\left[\frac{5x-x^2}{4}\right] \ge 0$

$\Rightarrow \left(\frac{5x-x^2}{4}\right) \ge 10^0 \Rightarrow \left(\frac{5x-x^2}{4}\right) \ge 1$

$\Rightarrow x^2 - 5x + 4 \le 0 \Rightarrow (x-1)(x-4) \le 0$

Hence domain of the function is [1, 4].

29. C

$$g(x) = x^3 + \tan x + \frac{x^2+1}{P}$$

$$g(-x) = (-x)^3 + \tan(-x) + \frac{(-x)^2+1}{P}$$

$$g(-x) = -x^3 - \tan x + \frac{x^2+1}{P}$$

$g(x) + g(-x) = 0$ because $g(x)$ is a odd function

$$\therefore \left[x^3 + \tan x + \frac{x^2+1}{P}\right] + \left[-x^3 - \tan x + \frac{x^2+1}{P}\right] = 0$$

$$\Rightarrow \frac{2(x^2+1)}{P} = 0 \Rightarrow 0 \le \frac{x^2+1}{P} < 1 \text{ because } x \in [-2,2]$$

$$\Rightarrow 0 \le \frac{5}{P} < 1 \Rightarrow P > 5.$$

30. D

$$f(x) = \begin{cases} \dfrac{x}{x^2+1}; & x \in (1,2) \\ \dfrac{2x}{x^2+1}; & x \in [2,3) \end{cases}$$

$f(x)$ is decreasing function

$$\therefore f(x) \in \left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right].$$

31. A

$$f(x) = \log_e\left(\frac{1-x}{1+x}\right), |x| < 1$$

$$f\left(\frac{2x}{1+x^2}\right) = \ln\left(\frac{1-\dfrac{2x}{1+2x^2}}{1+\dfrac{2x}{1+x^2}}\right)$$

$$= \ln\left(\frac{(x-1)^2}{(x+1)^2}\right) = 2\ln\left|\frac{1-x}{1+x}\right| = 2f(x)$$

32. A

Solve for x, the system of simulteneous inequations $(2\{x\}-1)(3\{x\}-2)\le 0$ and $(3[x]-4)(2[x]-8)\le 0$, where [.] is GIV function and { } is fractional part of function

Given in inequations $(2\{x\}-1)(3\{x\}-2)\le 0$ ....(1)

And $(3[x]-4)(2[x]-8)\le 0$ ....(2)

From (1), $\frac{1}{2}\le\{x\}\le\frac{2}{3}$ ....(3)

From (2), $\frac{4}{3}\le[x]\le 4\Rightarrow[x]\in\{2,3,4\}$

$$\left[2+\frac{1}{2}, 2+\frac{2}{3}\right]\cup\left[3+\frac{1}{2}, 3+\frac{2}{3}\right]\cup\left[4+\frac{1}{2}, 4+\frac{2}{3}\right]$$

$$\left[\frac{5}{2}, \frac{8}{3}\right]\cup\left[\frac{7}{2}, \frac{11}{3}\right]\cup\left[\frac{9}{2}, \frac{14}{3}\right]$$

$$|(a+c+e)-(b+d+f)| = \left|\frac{5}{2}+\frac{7}{2}+\frac{9}{2}-\left(\frac{8}{3}+\frac{11}{3}+\frac{14}{3}\right)\right|$$

$$= \left|\frac{21}{2}-\frac{33}{3}\right|$$

$$= \frac{1}{2}$$

SECTION - IV (More than one correct answer)

33. A,B,C,D

$$f(x)=|x-1|+|x-2|+|x-3|$$

$$\Rightarrow f(x)=\begin{cases}-3x+6, x<1\\ -x+4, 1\le x\le 2\\ x, 2<x\le 3\\ 3x-6, x>3\end{cases}$$

34. A,C

$$f(x)=\frac{x^2+5x-6}{2x^2+7x-9}=\frac{(x-1)(x+6)}{(x-1)(2x+9)}=\frac{x+6}{2x+9}, x\ne 1$$

$\therefore$ Domain of $f(x)=R-\left\{\frac{-9}{2},1\right\}$; Range of $f(x)=R-\left\{\frac{1}{2},\frac{7}{11}\right\}$

35. A,B,C From fig, by transformation methods\

36. A,B,C,D $f(x)=\begin{cases}x^2-1, x<0\\4-x^2, x\geq 10\end{cases}$ & $g(x)=\begin{cases}x+1, x<0\\2-x, x\geq 0\end{cases}$

$$g(f(x))=\begin{cases}f(x)+1, f(x)<0\\2-f(x), f(x)\geq 0\end{cases}$$

$$g[f(x)]=\begin{cases}x^2, x\in(-1,0)\\5-x^2, x\in(2,\infty)\\3-x^2, x\in(-\infty,-1)\\x^2-2, x\in[0,2]\end{cases}$$

SECTION - V (Numerical Type - Upto two decimal place)

37. 34

$$f(x)=ax^7+bx^5+cx-5$$

$$f(-7)=a(-7)^7+b(-7)^5+c(-7)-5...(1)$$
$$f(7)=a(7)^7+b(7)^5+c(7)-5....(2)$$

$$(1)+(2)\Rightarrow f(7)+f(-7)=-10$$

$$f(7)=-10-7=-17$$

$$f(7)+17\cos x=-17(\cos x-1)\in[-34,0]$$

38. 0

$f(x)=\cos(\log x)$

Now let $y=f(x)\cdot f(4)-\frac{1}{2}\left[f\left(\frac{x}{4}\right)+f(4x)\right]$

$\Rightarrow y=\cos(\log x).\cos(\log 4)-\frac{1}{2}\left[\cos\log\left(\frac{x}{4}\right)+\cos(\log 4x)\right]$

$\Rightarrow y=\cos(\log x)\cos(\log 4)-\frac{1}{2}[\cos(\log x-\log 4)+\cos(\log x+\log 4)]$

$\Rightarrow y=\cos(\log x)\cos(\log 4)-\frac{1}{2}[2\cos(\log x)\cos(\log 4)]\Rightarrow y=0$.

39. 2

$$f^{-1}og^{-1} = (gof)^{-1}$$

$$gof = 2(x^3+3)+1$$

$$(gof)^{-1} = \left(\frac{x-7}{2}\right)^{1/3}$$

$$\therefore (gof)^{-1}(23) = 8^{1/3} = 2$$

40. 3

$$f(x) = \frac{a-x}{a+x},\ x \in R - \{-a\} \to R$$

$$f(f(x)) = \frac{a-f(x)}{a+f(x)} = \frac{a-\left(\frac{a-x}{a+x}\right)}{a+\left(\frac{a-x}{a+x}\right)}$$

$$f(f(x)) = \frac{(a^2-a)+x(a+1)}{(a^2+a)+x(a-1)} = x$$

$$\Rightarrow (a^2-a)+x(a+1) = (a^2+a)x + x^2(a-1)$$

$$\Rightarrow a(a-1)+x(1-a^2)-x^2(a-1) = 0 \Rightarrow a = 1$$

$$f(x) = \frac{1-x}{1+x},\ f\left(\frac{-1}{2}\right) = \frac{1+\frac{1}{2}}{1-\frac{1}{2}} = 3.$$