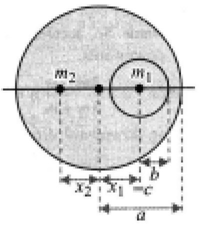
CHAPTER - 05

CENTRE OF MASS, CONSERVATION OF MOMENTUM & COLLISIONS

1. B



Let M = mass of the disc

Then, mass per unit area, $\sigma = \frac{M}{\pi a^2}$

$$m_1 = \sigma\left(\pi b^2\right) = \frac{Mb^2}{a^2}$$

$$m_2 = \sigma\pi\left(a^2 - b^2\right) = \frac{M\left(a^2 - b^2\right)}{a^2}$$

Coordinate of CM of disc, $x_{cm} = 0$

Now, $x_{cm} = \frac{m_1 x_1 + m_2\left(-x_2\right)}{m_1 + m_2}$

$$\Rightarrow \quad 0 = \frac{\left(Mb^2 / a^2\right)c + \left[M\left(a^2 - b^2\right)/a^2\right]\left(-x_2\right)}{m_1 + m_2}$$

$$\therefore \quad x_2 = \frac{cb^2}{\left(a^2 - b^2\right)}$$

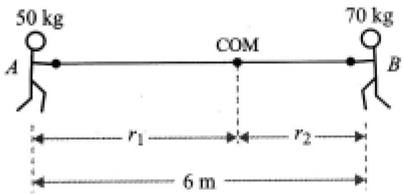
2. A

Acceleration, $a = \frac{3m - m}{3m + m}g = \frac{g}{2}$

Considering downward direction as positive, we get

$$a_{cm} = \frac{(3m)a - ma}{3m + m} = \frac{a}{2} = \frac{g}{4}$$

3. C



Since, external force on the system is zero, $a_{cm} = 0$

$\Rightarrow v_{cm} =$ constant = 0. (since initially, $v_{cm} = 0$

The two skaters will meet at COM.

Now, $r_1 + r_2 = 6$ and $\frac{r_1}{r_2} = \frac{70}{50}$

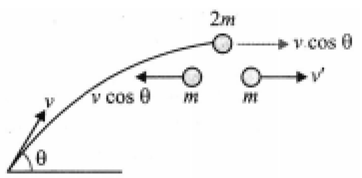
$\therefore\ r_1 = 3.5m$ and $r_2 = 2.5m$

4. B Whether the bomb explodes or not, COM will hit the ground at the same location, i.e., at a distance of range equal to $\frac{u^2 \sin 2\theta}{g}$

5. A Along y-axis, $v_{cm} = 0 \ \Rightarrow m_1 y_1 + m_2 y_2 = 0$

$\Rightarrow \ \frac{m}{4} \times 15 + \frac{3m}{4} y_2 = 0 \quad \therefore y_2 = -5cm$

6. A



Velocity of shell at highest point = v cos θ

Since, the first piece retraces the path, its speed must be v cos θ backwards.

Let $v' =$ speed of the other piece.

By cons. of linear momentum.

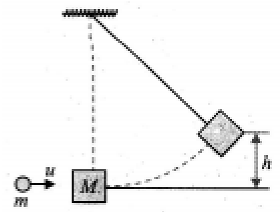
$2m \times v \cos\theta = m(-v\cos\theta) + m'$

$\therefore\ v' = 3v\cos\theta$

7. D Transfer of energy will be maximum when mass m_1 comes to rest. This will happen when $m_1 = m_2$ so that velocities interchange.

8. D Since, the masses are equal and collision is elastic, the velocities will interchange. The velocities after collision will be – 5 m/s and + 3 m/s.

9. B



By cons. of linear momentum,

$mu + 0 = (M + m)\sqrt{2gh}$

$\therefore\ u = \frac{M+m}{m}\sqrt{2gh}$

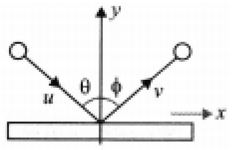
10. B

11. D

Height after nth rebound = he^{2n}

$\Rightarrow\ \frac{h}{2} = he^{2\times3}\ \Rightarrow\ e^6 = \frac{1}{2}\quad \therefore\ e = (1/2)^{1/6}$

12. B



Consider x and y-axes as shown.

Since, there is no collision along x-axis, we have

$u_x = v_x$

Also, since the plate is fixed $v_y = -eu_y$

$\Rightarrow u\sin\theta = v\sin\phi$ and $v\cos\phi = eu\cos\theta$

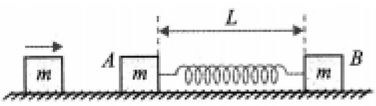
13. A

After collision, the velocity of block A is v.

At the time of maximum compression, velocity of A and B shall be equal,. say v'.

By cons. of linear momentum.

$mv = mv' + mv'\qquad \therefore\ v' = v/2$



Let x_0 = max. compression in spring.

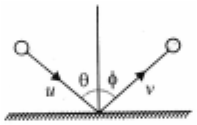
By cons. of mechanical energy,

$$\frac{1}{2}mv^2 + 0 = \left(\frac{1}{2}mv'^2 + \frac{1}{2}mv'^2\right) + \frac{1}{2}kx_0^2$$

$$\Rightarrow\ x_0^2 = \frac{m}{k}\left(v^2 - 2v'^2\right) = \frac{m}{k}\left[v^2 - 2\left(\frac{v}{2}\right)^2\right] = \frac{mv^2}{2k}$$

$$\therefore\ x_0 = v\sqrt{\frac{m}{2k}}$$

14. B



Here, $\theta = 45°$, $e = ½$

Along horizontal direction, there is no change in velocity.

So, $u \sin\theta = v \sin\phi$

Along vertical direction, $v \sin\phi = eu \cos\theta$

$$\Rightarrow \tan\phi = \frac{\tan\theta}{e} = \frac{\tan 45°}{1/2} = 2 \Rightarrow \sin\phi = \frac{2}{\sqrt{5}}$$

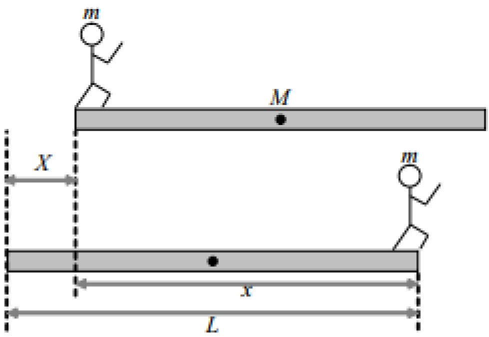
Since, $v \sin\phi = u \sin\theta$

$$\Rightarrow v = \frac{u \sin\theta}{\sin\phi} = \frac{u \times 1/\sqrt{2}}{2/\sqrt{5}} = u\sqrt{\frac{5}{8}}$$

$\therefore$ Friction of KE lost

$$= \frac{\frac{1}{2}mu^2 - \frac{1}{2}mv^2}{\frac{1}{2}mu^2} = 1 - \left(\frac{v}{u}\right)^2 = 1 - \left(\sqrt{\frac{5}{8}}\right)^2 = \frac{3}{8}$$

15. B



Let m = mass of man, M = mass of boat

Then, m = 50 kg, M = 450 kg

Let x = displacement of man w.r.t. ground

X = displacement of boat w.r.t. ground

Then, $x + X = L$ and $mx = MX$

$\Rightarrow$ $x + X = 10$ and $50x + 450X$

On solving, we get $X = 1m$

16. B

Velocity of the composite mass.

$$v = \frac{20 \times 10 + 0}{20 + 5} = 8 \text{ m/s}$$

$$KE = \frac{1}{2} \times (20 + 5) \times 8^2 = 800 \text{ J}$$

17. A

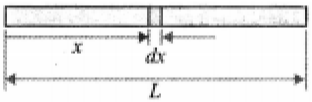
Let $u_1 = u$ Here, $u_2 = 0$, $v_2 = 2v_1$

Now, $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$\Rightarrow mu + 0 = mv_1 + m(2v_1)$ $\Rightarrow v_1 = u/3$

$$\therefore \quad e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{2v_1 - v_1}{u - 0} = \frac{v_1}{u} = \frac{1}{3}$$

18. B



Consider an element of length dx at distance x from the origin.

It mass will be $dm = \lambda\, dx = \lambda_0\, xdx$

$$\therefore \quad x_{cm} = \frac{1}{M}\int_0^L x\,dm$$

$$= \frac{\int_0^L x\lambda_0 xdx}{\int_0^L \lambda_0 xdx} = \frac{x^3/3\big|_0^L}{x^2/2\big|_0^L} = \frac{2L}{3}$$

19. A

Here, $m_1 = 3kg$, $m_2 = 6kg$, $v_1 = -1\,\text{m/s}$

$v_2 = 2\,\text{m/s}$, $k = 200$ N/m

Let x_0 = max. extension in the spring

v = velocity of both blocks when extension is max.

By cons. of linear momentum,

$m_1v_1 + m_2v_2 = m_1v + m_2v$

$\Rightarrow \quad 3\times(-1) + 6\times 2 = (3+6)v \Rightarrow v = 1\,\text{m/s}$

By cons. of mechanical energy,

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 + \frac{1}{2}kx_0^2$$

$\Rightarrow \quad 3\times(-1)^2 + 6\times 2^2 = (3+6)\times 1^2 + 200v_0^2$

$\therefore\ x_0 = 0.3m = 30cm$

PART - II (JEE ADVANCED LEVEL)

SECTION - III (One correct answer)

20. A

If x_1 is the overhang length on second blocks, then,

$$M\left(\frac{\ell}{2}-x_1\right) = \frac{M}{2}(x_1)$$

$$\therefore \quad x_1 = \frac{\ell}{3}$$

Now if x_2 is the overhang from table, then

$$M\left(\frac{\ell}{2}-x_2\right) = \frac{3M}{2}x_2$$

$$\therefore \quad x_2 = \frac{\ell}{5}$$

Now $\quad x = x_1 + x_2 = \frac{\ell}{3}+\frac{\ell}{5}=\frac{8\ell}{15}.$

21. A

$$\vec{P}_1 + \vec{P}_2 = \vec{P}_1' + \vec{P}_2'$$

$$mv\hat{i} + 0 = m\frac{v}{\sqrt{3}}\hat{j} + m\vec{v}_2$$

$$v^2 = \sqrt{v^2 + \left(\frac{v}{\sqrt{3}}\right)^2} = \frac{2v}{\sqrt{3}}.$$

22. A

SECTION - IV (More than one correct answer)

23. A,B

If the man walks along the rails, some velocity say V is imparted to car also. Let M be the mass of car. Then from conservation of linear momentum.

$$M.V = m(v-V)$$

$$\therefore \quad V = \frac{mv}{m+M}$$

$\therefore$ Work done by man

$$= \frac{1}{2}m(v-V)^2 + \frac{1}{2}mV^2$$

$$= \frac{1}{2}\left(\frac{mM}{m+M}\right)v^2 < \frac{1}{2}mv^2$$

24. A,B,C

$$-0.25 \times 200 + 0.25 \times 100 = F.\Delta T \quad \text{... (i)}$$

Block A

$$-1 \times v = T.\Delta t - F.\Delta T \quad \text{... (ii)}$$

Block B

$$2 \times v = T.\Delta t \quad \text{... (iii)}$$

Solving we get, $v = \frac{25}{3}$ m/s.

$$-\frac{1}{2}(1+2)v^2 = -(2)(10)h$$

$$h = \frac{3}{4\times10} \times \frac{25\times25}{3\times3} = \frac{625}{12\times10} = 5.2\,m.$$

25. A,C,D

Clearly, the velocity of centre of mass $= \left(\frac{m}{m+M}\right)v_0$

Initial K.E. in the centre of mass frame (K_{cm})

$$= \frac{1}{2}(m+M)v_{cm}^2$$

$$= \frac{1}{2}(m+M)\left[\left(\frac{m}{m+M}\right)v_0\right]^2$$

$$= \frac{1}{2}\frac{m^2v_0^2}{m+M}$$

The maximum compression (x_m) in spring is given by

$$\frac{1}{2}kx_m^2 = E - K_{cm}$$

or $$\frac{1}{2}kx_m^2 = \frac{1}{2}mv_0^2 - \frac{1}{2}\frac{m^2v_0^2}{m+M}$$

or $$kx = mv_0^2\left(1 - \frac{m}{m+M}\right)$$

$$= mv_0^2\left(\frac{M}{m+M}\right)$$

$$= \left(\frac{mM}{m+M}\right)v_0^2$$

$$\Rightarrow \quad x_m = v_0\sqrt{\left(\frac{mM}{m+M}\right)\frac{1}{k}}$$

26. BD

$$2a = a_r \cos\ \theta$$

$$N \sin\theta = ma$$

Net force on block perpendicular to plane is zero :

Hence, $N + ma\sin\theta = mg\cos\theta$ .... (3)

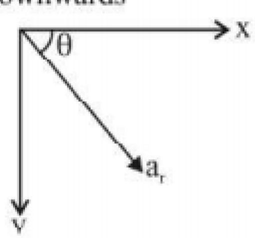
Solving eqs. (1), (2) and (3), we get,

$$a_r = \frac{2g\sin\theta}{1+\sin^2\theta}$$

acceleration of block vertically downwards

$$a_y = a_r \sin\theta$$

$$a_y = \frac{2g\sin^2\theta}{1+\sin^2\theta}$$



$\therefore$ acceleration of COM is

$$a_{com} = \frac{a_y}{2} = \frac{g\sin^2\theta}{(1+\sin^2\theta)}$$

27. ABC

$$\left(\frac{m}{2}\right)u = \left(m+\frac{m}{2}\right).v$$

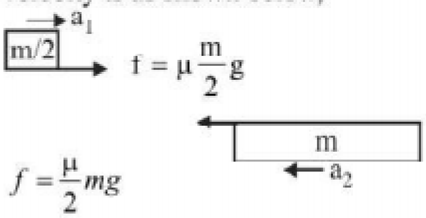
$\therefore \quad v = \frac{u}{3}$

Work done against friction $= E_i - E_f$

$$= \frac{1}{2}\left(\frac{m}{2}\right)u^2 - \frac{1}{2}\left(\frac{3m}{2}\right)\left(\frac{u}{3}\right)^2$$

$$= \frac{1}{6}mu^2 = \frac{2}{3}\left(\frac{1}{4}mu^2\right)$$

Force of friction on the two blocks before the blocks reach a common velocity is as shown below, 16.



$a_1 = \mu g$ and $a_2 = \frac{\mu}{2}g \quad \therefore a_r = \frac{3}{2}\mu g$

SECTION - V (Numerical Type - Upto two decimal place)

28. 25

29. 2.82 km/s

SECTION - VI (Matrix Matching)

30. $a \to r, b \to p, c \to p, d \to s$