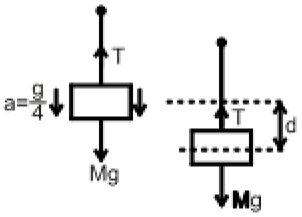
CHAPTER - 04

WORK ENERGY POWER & CIRCULAR MOTION

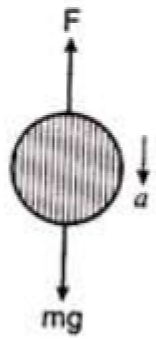
1. 2 Let tension in string be T, then work done by tension T = – Td
Applying newton's second law on the bucket

$$Mg - T = M\left(\frac{g}{a}\right) \quad \text{or} \quad T = \frac{3}{4}Mg$$



∴ required work done = $-\frac{3}{4}$ Mgd (∵ tension and displacement are in opposite direction W = Td cos180° = –Td)

2. 2 The motion of the body is shown in the figure. The following two forces are acting on the body :



(i) Weight mg is acting vertically downward
(ii) The push of the air is acting upward.
As the body is accelerating downward, the resultant force is (mg – F)
Work done by the resultant force to fall through a vertical distance of 20m = $(mg - F) \times 20$ joule.

Gain in the kinetic energy $=\frac{1}{2}mv^2$

Now the work done by the resultant force is equal to the change in kinetic energy, ie.,

$$(mg-F)20=\frac{1}{2}mv^2$$

or $(50-F)20=\frac{1}{2}\times 5\times(10)^2$

or $50-F=12.5$ or $F=50-12.5$

$\therefore F=37.5N$

Work done by the force = -37.5×20

= – 750 joule.

(The negative sign is used because the push of the air is upwards while the displacement is downwards).

3. 2 Displacement vector, $d\vec{s}=dx\hat{i}+dy\hat{j}$

Given $\vec{F}=-k(y\hat{i}+x\hat{j})$

$\therefore$ Work done $W=\int\vec{F}.d\vec{s}$

$$=\int -k\left(y\hat{i}+x\hat{j}\right).\left(dx\hat{i}+dy\hat{j}\right)$$

$$=-k\int_{(0,0)}^{(a,a)}(ydx+xdy)=-k\int_{(0,0)}^{(a,a)}d(xy)$$

$$=-k\left|(xy)\right|_{0,0}^{a,a}=k(a\times a)=-ka^2.$$

4. 1 From the definition of acceleration

$$a=\frac{dV}{dt}=\frac{d\left(2\sqrt{x}\right)}{dx}.\frac{dx}{dt}$$

$$=\frac{2}{2\sqrt{x}}.2\sqrt{x} \qquad a=2m/s^2$$

F = $ma=1\times 2=2N$

W = Fx = max = 1 × 2 × 2 = 4J

5. 1 Particle comes to rest only where friction is present i.e., on horizontal surface. Loss in PE is equal to work done by friction

mgh = μ mgd

d = 4.5 m

ie 3 m forward and 1.5 m backward

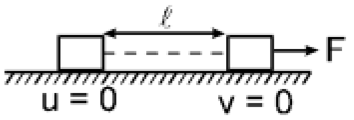
6. 4 Workdone = $\frac{1}{2}k(x_2^2 - x_1^2)$; here x_1 = x and x_2 = x+y

$$= \frac{1}{2}K[(x + y)^2 - x^2] = \frac{1}{2}K(x + y + x)(x + y - x) = \frac{1}{2}K(2x + y)y$$

7. 1 The minimum speed imparted to the particle should be such that it just reaches x = $\frac{2}{3}$ from there on it shall automatically reach x = 0 as both are equilibrium points

$$= \frac{W}{t} = \frac{Fs}{t} mv^2 = -\int_{4}^{2/3} F\,dx = -\int_{-4}^{2/3} x(3x-2)\,dx = \frac{1300}{27} \quad \text{or} \quad v = \sqrt{\frac{2600}{27}} \text{ m/s}$$

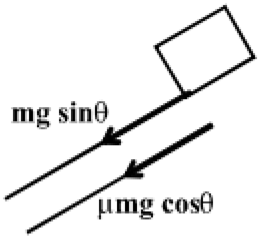
8. 2 Applying work energy theorem on block



$F\ell - \frac{1}{2}k\ell^2 = 0$ $\quad \therefore \ell = \frac{2F}{k}$ $\quad \therefore$ maximum positive work done is = $F\ell = \frac{2F^2}{k}$

after extension $\ell = \frac{2F}{k}$ body starts moving left side. So work done is negative.

9. 1



The total downward force acting on the block

$$= 0.5\times10\left(\frac{1}{2} + 0.2\times\frac{\sqrt{3}}{2}\right)$$

= 5[0.5 + 0.173] = 3.365 N.

Now the power required to move up along the inclined power = 3.365 × 5 = 16.825 N-m/s

10. 4 r = 144 m, m = 16 kg, T_{max} = 16 N

$$T = \frac{mv^2}{r}$$

$v = \sqrt{\frac{Tr}{M}} = \sqrt{\frac{16 \times 144}{16}}$ = 12 m/s

11. 4 For circular motion in vertical plane normal reaction is minimum at highest point and it is zero, minimum speed of motorbike is

$mg = \frac{mv^2}{R} \Rightarrow v = \sqrt{gR}$

12. 3 $P = Fv = mv\frac{dv}{dt}$

Pdt = mvdv

integrating both side

$Pt = \frac{mv^2}{2}$

$v^2 \propto t$ or $v \propto t^{1/2}$

$\frac{dx}{dt} \propto t^{1/2}$ or $x \propto t^{3/2}$

13. 2 $P = F_t.v = ma_t.v$; $a_t = \frac{d|v|}{dt}$; $a_c = k^2rt^2 \Rightarrow \frac{v^2}{r} = k^2rt^2$

$v = krt \; \frac{d|v|}{dt} = kr$ at = kr $P = mkr.krt = mk^2r^2t$

14. 16 From work-energy theorem,

$x = t^3/3$

$\therefore$ velocity $v = \frac{dx}{dt} = t^2$

At t = 0, $v_i = 0^2 = 0$

At t = 2, $v_f = 2^2 = 4$ m/s

work done $W = \frac{1}{2}m(v_f^2 - v_i^2)$

$= \frac{1}{2} \times 2(4^2 - 0) = 16J$

15. 136

Velocity $= 100$kmp $= 100 \times \frac{5}{8}$ m/s

Force = 3920 N

Wastage of power = 20%

Used power = 80%

Power $= \frac{W}{t} = \frac{Fs}{t}$

80% P = FV

$\frac{80}{100} P = 3920 \times 100 \times \frac{5}{18}$

$\therefore \quad P = \frac{100}{80} \times 3920 \times 100 \times \frac{5}{18}$

$= 136.11 \times 10^3 \text{ w} = 136.11 \text{kw}$

16. 2.5 Mass m = 0.5 kg

Length ℓ = 2 m

$\theta = 60^0$

Work done $W = mg\frac{\ell}{2}(1-\cos\theta)$

$= 0.5 \times 10 \times \frac{2}{2}\,(1-\cos 60^0) = 2.5$ J

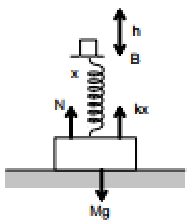
17. 1 For just slip $\Rightarrow \mu mg = m\omega^2 r$

here ω is double then radius is 1/4th

r′ = 1 cm

18. B The block m drops a height h from A compresses the spring and then goes up to B x below the initial level of spring. Applying work energy theorem between A and B

mg (h – x) = $\frac{1}{2}$kx²……………..(1)



Minimum value of x required so that the block bounces is when the N will become zero

N + kx = Mg N = 0 $\Rightarrow x = \frac{Mg}{k}$

$mg\,(h - \frac{Mg}{k}) = \frac{1}{2}k.\frac{M^2g^2}{k^2}$ $\Rightarrow mgh = \frac{M^2g^2}{2k} + \frac{mMg^2}{k}$.

$h = \frac{(M^2 + 2mM)g}{2km}$

19. B If the particle is released at the origin, it will try to go in the direction of force.

Here $\frac{du}{dx}$ is positive and hence force is negative, as a result it will move towards –ve x-axis.

20. A As long as the block of mass m remains stationary, the block of mass M released from rest comes down by $\frac{2Mg}{K}$ (before coming it rest momentarily again).

Thus the maximum extension in spring is

$x = \frac{2Mg}{K}$

for block of mass m to just move up the incline

$kx = mg \sin\theta + \mu \, mg \cos\theta$

$2Mg = mg \times \frac{3}{5} + \frac{3}{4} mg \times \frac{4}{5}$ or $M = \frac{3}{5} m$

21. A (a) At equilibrium

$mg = kx_0$

or $x_0 = \frac{mg}{k}$

(b) The maximum distance the block can be taken up for the spring not to get slack is $x_0 = \frac{mg}{k}$ for conservation of energy,

$\frac{1}{2} kx_0^2 + \frac{1}{2} mu^2 = mg\, x_0$

$\Rightarrow \frac{1}{2} mu^2 = (mg - \frac{1}{2} kx_0)\, x_0 = \frac{mg\, x_0}{2}$

or $u = \sqrt{\frac{mg^2}{k}}$

22. D At the moment m_2 stops, extension in the spring must be able to produce enough force to move m_1 or

$kx = \mu_1 m_1 g \quad \Rightarrow \quad x = \frac{.4 \times 5 \times 10}{100} = 20\text{cm}.$

As it is equal to displacement of m_2 also, applying work-energy theorem on m_2

$\Delta K = 0 - \frac{1}{2} mv^2 = W_S + W_f = -\frac{1}{2} kx^2 - \mu_2 m_2 g.\, x$ (W_s = work done by spring W_f = work done by friction)

$\Rightarrow \frac{1}{2} m_2 v^2 = \frac{1}{2} \times 100 \times 0.2 \times 0.2 + 0.2 \times 2 \times 10 \times 0.2$

$v^2 = 2 + 0.8 \Rightarrow v = \sqrt{2.8}$ m/s

23. B $K = \frac{1}{2} mv^2 = as^2 \quad \Rightarrow \quad v^2 = \frac{2as^2}{m}$

$$a_c = \frac{v^2}{R} = \frac{2as^2}{mR} \Rightarrow a_t = v\frac{dv}{ds} = \frac{2as}{m}$$

$$a = \sqrt{\left(\frac{2as^2}{mR}\right)^2 + \left(\frac{2as}{m}\right)^2} = \frac{2as}{m}\left(1+\frac{s^2}{R^2}\right)^{1/2}$$

Total force = ma = $2as\left(1+\frac{s^2}{R^2}\right)^{1/2}$

24. ABD $U = \frac{\alpha}{x^2} - \frac{\beta}{x}$

$$F = -\frac{dU}{dx} = -\left[\frac{-2\alpha}{x^3} + \frac{\beta}{x^2}\right]$$

for speed to be maximum, U should be minimum.

Hence $-\frac{2\alpha}{x^3} + \frac{\beta}{x^2} = 0 \Rightarrow x = \frac{2\alpha}{\beta} = 2x_o$

at $x = 2x_o$, $\frac{d^2U}{dx^2} > 0$, hence $x = 2x_o$ is a stable equilibrium point.

Again $\frac{1}{2}mv_{max}^2 = \left[\frac{\alpha}{(x_o)^2} - \frac{\beta}{x_o}\right] - \left[\frac{\alpha}{(2x_o)^2} - \frac{\beta}{2x_o}\right] = 0 + \frac{1}{4}\frac{\alpha}{x_o^2}$

Hence $v_{max} = \frac{1}{x_o}\sqrt{\frac{\alpha}{2m}}$, as U = 0 at $x = x_o$ and $x = \infty$, the particle will reach to infinity with zero speed.

25. ABD As there are no external forces acting on the 'A + B' system, its total momentum is conserved. If the masses of A and B are 2m and m respectively, and v is the final common velocity,

$mu = (m + 2m)v$

or $v = u/3$.

Word done against friction = loss in KE $\frac{1}{2}mu^2 - \frac{1}{2}(3m)v^2$

$$= \frac{1}{2}mu^2 - \frac{1}{2}(3m)\frac{u^2}{9}$$

$$= \frac{1}{2}mu^2\left[1 - \frac{1}{3}\right] = \frac{2}{3} \times \frac{1}{2}mu^2.$$

The force of friction between the blocks is μmg.

Acceleration of A (to the right) $= a_1 = \frac{\mu mg}{2m} = \frac{\mu g}{2}$.

Acceleration of B (to the left) $= a_1 = \frac{\mu mg}{m} = \mu g$.

Acceleration of A relative to B $= a_1 - (-a_2) = \frac{3}{2}\mu g$.

26. AD

$T\cos\theta_0 = mg$ ...(i)

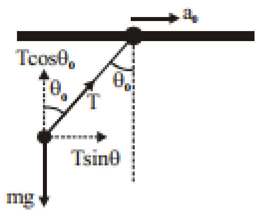
$T\sin\theta_0 = ma_0$ ...(ii)

(ii) / (i)

$\tan\theta_0 = \frac{a}{g}$

$\theta_0 = 30°$

$T = \frac{mg}{\cos 30°} = \frac{2mg}{\sqrt{3}}$.



27. BCD Work done against friction on ice is zero and work done against friction on the road is $(\mu mg)l$. So, average work done is $\frac{0+(\mu mg)l}{2} = (\mu mg)\frac{l}{2}$

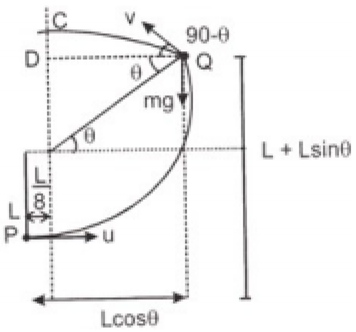
Thus indicating that the effective length of the sledge that has to be dragged so that it just gets completely on the road is $\frac{l}{2}$.

Distance covered by the sledge on the road before coming to rest is $\frac{v_0^2}{2\mu g}$.

So total distance moved by the sledge is $\left(\frac{v_0^2}{2\mu g} + \frac{l}{2}\right)$

Distance covered by the sledge on the road is $l - \left(\frac{v_0^2}{2\mu g} + \frac{l}{2}\right) = \left(\frac{v_0^2}{2\mu g} - \frac{l}{2}\right)$

28. 4



Now, we have following equations

(1) $T_Q = 0$ Therefore, $mg\sin\theta = \dfrac{mv^2}{L}$ .... (1)

(2) $v^2 = u^2 - 2gh = u^2 - 2gL(1+\sin\theta)$ .... (2)

(3) $QD = \dfrac{1}{2}(range)$ .... (3)

$$u = \sqrt{gL\left(2 + \frac{3\sqrt{3}}{2}\right)}$$

29. 8 $W_F + W_{fr} + W_{mg} = \Delta KE = 0$

$W_F = -W_{mg} - W_{fr}$

Where $W_{fr} = -\mu mgl$

$W_F = -(-mgh) - (-\mu mgl) = mgh + \mu mgl = 8\,J$

30. A - p,r, B - q,s, C - q,r, D - p

The displacement of A shall be less than displacement L of block B.

Hence work done by friction on block A is positive and its magnitude is less than μmgL.

And the work done by friction on block B is negative and its magnitude is equal to μmgL.

Therefore workdone by friction on block A plus on block B is negative its magnitude is less than μmgL.

Work done by F is positive. Since F> 2μmg, magnitude of work done by F shall be more than 2μmgL.

So, (A)→ p, r (B)→ q, s (C)→ q, r (D)→ p