CHAPTER - 18
THREE DIMENSIONAL GEOMETRY

Important Results

1. Distance formula : $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

2. Section formula : $x=\frac{lx_2+mx_1}{l+m}, y=\frac{ly_2+my_1}{l+m}, z=\frac{lz_2+mz_1}{l+m}$

3. Midpoint x = $\frac{x_1+x_2}{2}, y=\frac{y_1+y_2}{2}, z=\frac{z_1+z_2}{2}$

4. Centroid of the triangle : $x=\frac{x_1+x_2+x_3}{3},\ y=\frac{y_1+y_2+y_3}{3},\ z=\frac{z_1+z_2+z_3}{3}$

5. Distance of P(x, y, z) from the x axis is $\sqrt{y^2+z^2}$, from the y axis $\sqrt{x^2+z^2}$ and from the z axis is $\sqrt{x^2+y^2}$

6. If the projections of PQ on the co-ordinate axes are x, y, z then PQ = $\sqrt{x^2+y^2+z^2}$ and D.Cs of PQ are $\left\langle\frac{x}{\sqrt{x^2+y^2+z^2}},\frac{y}{\sqrt{x^2+y^2+z^2}},\frac{z}{\sqrt{x^2+y^2+z^2}}\right\rangle$

7. Centroid of the tetrahedron x = $\frac{x_1+x_2+x_3+x_4}{4},\ y=\frac{y_1+y_2+y_3+y_4}{4},\ z=\frac{z_1+z_2+z_3+z_4}{4}$

8. Direction cosines of a line making angles α,β,γ with the x, y, z axes respectively are l = Cosα, m =Cosβ, n = Cosγ

9. $l^2+m^2+n^2=1=\text{Cos}^2\alpha+\text{Cos}^2\beta+\text{Cos}^2\gamma$

10. $\text{Sin}^2\alpha+\text{Sin}^2\beta+\text{Sin}^2\gamma=2$

11. D.Cs of the x axis <1,0,0>, y axis <0,1,0> z axis <0,0,1>

12. If $|OP| = r$ and P(x,y,z) then D.Cs of OP are $\left\langle \frac{x}{r}, \frac{y}{r}, \frac{z}{r} \right\rangle$

13. If l, m, n are the D.Cs of $\bar{r}$, then $\bar{r} = |\bar{r}|\left(l\bar{i} + m\bar{j} + n\bar{k}\right)$

14. If $\langle a, b, c\rangle$ are the DRs, then D.Cs are $\left\langle \frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}} \right\rangle$

15. DRs of the line joining (x_1, y_1, z_1) and (x_2, y_2, z_2) are $\langle x_2 - x_1,\ \ y_2 - y_1,\ z_2 - z_1\rangle$

16. Projection of the line joining (x_1, y_1, z_1) and (x_2, y_2, z_2) on another line whose D.Cs are $\langle l,m,n\rangle$ is
    $|(x_2-x_1)l + (y_2-y_1)m + (z_2-z_1)n|$

17. Angle between two lines is given by $\cos\theta = l_1l_2 + m_1m_2 + n_1n_2$ where $\langle l_1,m_1, n_1\rangle$ and $\langle l_2, m_2, n_2\rangle$ are the D.Cs of the lines $\text{Sin}^2\theta = \sum (l_1m_2 - l_2m_1)^2$

18. Acute angle between the diagonals of a cube is $\cos^{-1}(1/3)$ or $\tan^{-1}\left(2\sqrt{2}\right)$

19. If a line makes angles $\alpha, \beta, \gamma, \delta$ with the four diagonals of a cube then

    $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$ and $\sin^2\alpha + \sin^2\beta + \sin^2\gamma + \sin^2\delta = \frac{8}{3}$

20. If the lines are parallel
    i) $l_1 = l_2$, $m_1 = m_2$, $n_1 = n_2$ if D.Cs are given
    ii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ if DRs are given

21. If the lines are perpendicular
    i) $l_1 l_2 + m_1m_2 + n_1n_2 = 0$ if D.Cs are given
    ii) $a_1a_2 + b_1b_2 + c_1c_2 = 0$ if DRs are given

22. General equation of a plane is:
    ax + by + cz + d = 0

23. DRs of the normal to the plane ax + by+cz+d = 0 are <a, b, c>

24. DRs of the normal to XY plane; <0, 0, 1>
    DRs of the normal to YZ plane ; <1,0,0>
    DRs of the normal to ZX plane ; <0,1,0>

25. Equation of a plane passing through (x_1, y_1, z_1) is $a(x-x_1) + b(y-y_1) + c(z-z_1) = 0$

26. Angle between the planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by:

$$\text{Cos}\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

27. Planes are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ and parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

28. Intercept form of a plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, If a plane makes intercepts OA = a, OB = b, OC = c on the x, y, z axes respectively then $\Delta ABC = \frac{1}{2}\sqrt{(ab)^2 + (bc)^2 + (ca)^2}$

29. Normal form : $lx + my + nz = p$

30. Equation of a plane parallel to $ax + by + cz + d_1 = 0$ is $ax + by + cz + d_2 = 0$

31. Equation of the plane parallel to the planes $ax + by + cz + d_1 = 0$ and $ax + by + cz + d_2 = 0$ and equidistant from them is $ax + by + cz + \frac{d_1 + d_2}{2} = 0$

32. Perpendicular distance of a point from $ax + by + cz + d = 0$ is $\left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$

33. Distance between the parallel planes $ax + by + cz + d_1 = 0$ and $ax + by + cz + d_2 = 0$ is $\frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$

34. Equations of the planes bisecting the angle between two planes are:

$$\frac{a_1x + b_1y + c_1z + d_1}{\sqrt{a_1^2 + b_1^2 + c_1^2}} = \pm \frac{a_2x + b_2y + c_2z + d_2}{\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

35. To distinguish between the bisecting planes;

i) Write both the equations such that the constant terms are positive

ii) If $a_1 a_2 + b_1b_2 + c_1c_2$ is negative then the origin lies in the acute angle. If it is positive then the origin lies in the obtuse angle.

iii) $\frac{a_1x + b_1y + c_1z + d_1}{\sqrt{a_1^2 + b_1^2 + c_1^2}} = + \frac{a_2x + b_2y + c_2z + d_2}{\sqrt{a_2^2 + b_2^2 + c_2^2}}$ bisects the angle between the planes that contains the origin and $\frac{a_1x+b_1y+c_1z+d_1}{\sqrt{a_1^2+b_1^2+c_1^2}}=-\frac{(a_2x+b_2y+c_2z+d_2)}{\sqrt{a_2^2+b_2^2+c_2^2}}$ bisects the angle between the planes that does not contain the origin

36. If (a, b, c) is the foot of the perpendicular from the origin to a plane then the equation of the plane is $ax + by + cz = a^2 + b^2 + c^2$

37. Equation of the plane passing through (x_1,y_1,z_1), (x_2,y_2,z_2) and (x_3,y_3,z_3) is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$$

38. Reflection of the plane $a'x+b'y+c'z+d'=0$ in the plane $ax + by + cz + d = 0$ is

$$2(aa'+bb'+cc')(ax+by+cz+d)=(a^2+b^2+c^2)(a'x+b'y+c'z+d')$$

39. Two points (x_1, y_1, z_1) and (x_2, y_2, z_2) lie on the same side of the plane $ax + by + cz + d = 0$ if $ax_1 + by_1 + cz_1 + d$ and $ax_2 + by_2 + cz_2 + d$ are of the same sign.

The points lie on opposite sides of theplane if $ax_1 + by_1 + cz_1 + d$ and $ax_2 + by_2 + cz_2 + d$ are of opposite signs.

40. If $ax + by + cz + d = 0$ is parallel to the x axis then $a = 0$, if it is parallel to y axis then $b = 0$, if it is parallel to z axis then $c = 0$

41. Equation of a plane passing through the line of intersection of the planes P = 0 and Q = 0 is

$P + lQ = 0$

42. Symmetrical form of a line is

i) $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$ if <l, m, n> are the D.Cs

ii) $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$ if <a,b,c> are the DRs

43. To put $a_1x + b_1y + c_1z + d_1 = 0 = a_2x + b_2y + c_2z + d_2$ in symmetrical form find

(i) any point on the line (ii) DRs of the line

44. Equation of the line joining (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$

45. Angle between a line and a plane is given by $\sin\theta=\frac{|al+bm+cn|}{\sqrt{a^2+b^2+c^2}\sqrt{l^2+m^2+n^2}}$

46. If $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$ is parallel to the plane $ax + by + cz + d = 0$ then $al + bm+cn = 0$ and

$ax_1 + by_1 + cz_1 + d \ne 0$

If the line and the plane are perpendicular then $\frac{l}{a} = \frac{m}{b} = \frac{n}{c}$

47. If the line lies in the plane then

i) $al + bm + cn = 0$ and (ii) $ax_1 + by_1 + cz_1 + d = 0$

48. Equation of a plane containing the line $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ is $a(x-x_1) + b(y-y_1) + c(z-z_1) = 0$ where $al + bm + cn = 0$

49. Equation of the plane containing line $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ and the point (x_2, y_2, z_2) not lying on the line is $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l & m & n \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \end{vmatrix} = 0$

50. Equation of the plane containing the lines $\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ and $\frac{x-x_2}{\ell_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$ is

$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$ or $\begin{vmatrix} x-x_2 & y-y_2 & z-z_2 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$ and the condition for the above lines to be coplanar is $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$

51. If the lines $a_1x + b_1y + c_1z + d_1 = 0 = a_2x + b_2y + c_2z + d_2$ and $a_3x + b_3y + c_3z + d_3 = 0 = a_4x + b_4y + c_4z + d_4$ intersect then $\begin{vmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \end{vmatrix} = 0$

52. Equation of the plane through the line $\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{m_1}$ and parallel to the line $\frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$ is $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$

53. If S.D = 0 the lines intersect or they are coplanar

54. Intersection of 3 planes

Consider

$a_1x \ b_1y + c_1z + d_1 = 0$

$a_2x + b_2y + c_2z + d_2 = 0$

$a_3x + b_3y + c_3z + d_3 = 0$

$$\Delta_1 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \ \Delta_2 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}, \ \Delta_3 = \begin{vmatrix} a_1 & c_1 & d_1 \\ a_2 & c_2 & d_2 \\ a_3 & c_3 & d_3 \end{vmatrix}, \ \Delta_4 = \begin{vmatrix} b_1 & c_1 & d_1 \\ b_2 & c_2 & d_2 \\ b_3 & c_3 & d_3 \end{vmatrix}$$

If $\Delta_1 \neq 0$ the planes intersect in a point. If $\Delta_1 = 0$ and $\Delta_2 = 0$ or $\Delta_3 = 0$ or $\Delta_4 = 0$ the planes intersect in a line. If $\Delta_1 = 0$ and neither of $\Delta_2, \Delta_3, \Delta_4$ is zero then the planes form a triangular prism.

55. If $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ represents a pair of planes then $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$

Vector forms:-

1. Equation of a line passing through a fixed point A(**a**) and parallel to a vector **b** is **r = a** + l**b**

2. Equation of a line passing through two points A(**a**) and B(**b**) is **r** = **a** + l(**b-a**)

3. Angle between the lines **r** = $\mathbf{a_1}$+l$\mathbf{b_1}$ and **r** = $\mathbf{a_2}$ +m$\mathbf{b_2}$ is given by: $\text{Cos}\theta = \dfrac{\mathbf{b_1.b_2}}{|\mathbf{b_1}||\mathbf{b_2}|}$

   If the lines are perpendicular $\mathbf{b_1}$. $\mathbf{b_2}$ = 0 and if the lines are parallel then $\mathbf{b_1}$ = l$\mathbf{b_2}$

4. The distance of the point A ($\mathbf{a_2}$) from the line **r** = $\mathbf{a_1}$ + l$\mathbf{b_1}$ is $\dfrac{|(\mathbf{a_2} - \mathbf{a_1}) \text{ x } \mathbf{b_1}|}{|\mathbf{b_1}|}$, Foot of the perpendicular from $A(\bar{a}_2)$ to the line $\bar{r} = \bar{a}_1 + \lambda\bar{b}$ is $\bar{a}_1 + \dfrac{(\bar{a}_2 - \bar{a}_1).\bar{b}_1\bar{b}_1}{|\bar{b}_1|^2}$

5. S.D = $\left|\dfrac{(\mathbf{a_2} - \mathbf{a_1}).\mathbf{b_1} \text{x} \mathbf{b_2}}{|\mathbf{b_1} \text{x} \mathbf{b_2}||}\right|$ or $\left|\dfrac{(\mathbf{a_2} - \mathbf{a_1}, \mathbf{b_1}, \mathbf{b_2}}{|\mathbf{b_1} \text{x} \mathbf{b_2}|}\right|$, Image of $A(\bar{a}_2)$ in the line $\bar{r} = \bar{a}_1 + \lambda\bar{b}_1$ is

$$2\overline{a}_1 + \frac{2(\overline{a}_2 - \overline{a}_1).\overline{b}_1\overline{b}_1}{|b_1|^2} - \overline{a}_2$$

6. The lines $\overline{r} = \overline{a}_1 + t\overline{b}_1$ and $\overline{r} = \overline{a}_2 + s_2\overline{b}_2$ lie in a plane if $(a_2 - \overline{a}_1).\overline{b}_1 \times \overline{b}_2 = 0$

7. S.D. between two parallel lines is $\left|\frac{(\mathbf{a}_2 - \mathbf{a}_1)x\mathbf{b}_1}{|\mathbf{b}_1|}\right|$

8. Vector equation of a plane in the normal form: **r**.**n** = d where **n** is perpendicular to the plane

9. Equation of a plane through $\overline{a}$ and perpendicular to $\overline{n}$ is $(\overline{r} - \overline{a}).\overline{n} = 0$ ie $\overline{r}.\overline{n} = \overline{a}.\overline{n}$

10. Equation of the plane containing A(**a**), B(**b**) and C(**c**) is: **r**.[**b** x **c** + **c** x **a** + **a** x **b**] = (**a**, **b**, **c**)

11. Angle between the planes $\mathbf{r}.\mathbf{n}_1 = d_1$ and $\mathbf{r}.\mathbf{n}_2 = d_2$ is given by $\text{Cos}\theta = \frac{\mathbf{n}_1.\mathbf{n}_2}{|\mathbf{n}_1||\mathbf{n}_2|}$

12. Equation of a plane passing through A(**a**) and parallel to **b** and **c** is **r** = **a** + l**b** + m**c**. The above equation in scalar product form is (**r**, **b**, **c**) = (**a**, **b**, **c**)

13. Equation of a plane parallel to $\mathbf{r}.\mathbf{n} = d_1$ is $\mathbf{r}.\mathbf{n} = d_2$

14. The line of intersection of the planes $\overline{r}.\overline{n}_1 = d_1$ and $\overline{r}.\overline{n}_2 = d_2$ is parallel to $\overline{n}_1 \times \overline{n}_2$

15. Equation of a plane passing through the line of intersection of the planes $\mathbf{r}.\mathbf{n}_1 = d_1$ and $\mathbf{r}.\mathbf{n}_2 = d_2$ is

    $(\mathbf{r}.\mathbf{n}_1 - d_1) + l(\mathbf{r}.n_2 - d_2) = 0$

16. Length of the perpendicular from a point A(**a**) to the plane **r**.**n** = d is given by $p = \frac{|\mathbf{a}.\mathbf{n} - d|}{|\mathbf{n}|}$

17. Equation of planes bisecting the angles between $\mathbf{r}.\mathbf{n}_1 = d_1$ and $\mathbf{r}.\mathbf{n}_2 = d_2$ are $\frac{\mathbf{r}.\mathbf{n}_1 - d_1}{|\mathbf{n}_1|} = \pm \frac{\mathbf{r}.\mathbf{n}_2 - d_2}{|\mathbf{n}_2|}$

18. If q is the angle between the line **r** = **a** + l**b** and the plane **r**.**n** = d then Sinq = $\frac{\mathbf{b}.\mathbf{n}}{|\mathbf{b}||\mathbf{n}|}$. If the line and the plane are parallel then $\overline{b}.\overline{n} = 0$. If the line lies in the plane then $\overline{b}.\overline{n} = 0$ and $\overline{a}.\overline{n} = d$

19. If the lines $\mathbf{r} = \mathbf{a}_1 + l\mathbf{b}_1$ and $\mathbf{r} = \mathbf{a}_2 + m\mathbf{b}_2$ are coplanar then $(\mathbf{a}_1, \mathbf{b}_1, \mathbf{b}_2) = (\mathbf{a}_2, \mathbf{b}_1, \mathbf{b}_2)$ and the plane containing the lines is $(\mathbf{r}, \mathbf{b}_1, \mathbf{b}_2) = (\mathbf{a}_1, \mathbf{b}_1, \mathbf{b}_2)$ or $(\mathbf{r}, \mathbf{b}_1, \mathbf{b}_2) = (\mathbf{a}_2, \mathbf{b}_1, \mathbf{b}_2)$

20. If the lines $\bar{r} = \bar{a} + \lambda(\bar{b} \times \bar{c})$ and $\bar{r} = \bar{b} + \mu(\bar{c} \times \bar{a})$ intersect then $\bar{b}.\bar{c} = \bar{c}.\bar{a}$

21. Volume of the tetrahedron ABCD = $\frac{1}{6}\left[\overline{AB}, \overline{AC}, \overline{AD}\right]$

22. $\bar{r} = \lambda\bar{i}$ represents the x axis, $\bar{r} = \lambda\bar{j}$ represents the y axis, $\bar{r} = \lambda\bar{k}$ represents the z axis.

23. $\bar{r} = \lambda\bar{i} + \mu\bar{j}$ represents z = 0, $\bar{r} = \lambda\bar{j} + \mu\bar{k}$ represents x = 0, $\bar{r} = \lambda\bar{i} + \mu\bar{k}$ represents y = 0

PART I - (JEEMAIN)

SECTION - I - Straight objective type questions

1. Which of the following set of points are non- collinear
(1) (1, −1, 1), (−1, 1, 1), (0, 0, 1)
(2) (1, 2, 3), (3, 2, 1), (2, 2, 2)
(3) (−2,4, −3), (4, −3, −2), (−3, −2, 4)
(4) (2, 0, −1), (3, 2, −2), (5, 6, −4)

2. If O is the origin and OP = 3 with direction ratios $-1, 2, -2$, then co-ordinates of P are
(1) (1, 2, 2) (2) $(-1, 2, -2)$ (3) (–3, 6, –9) (4) $(-1/3, 2/3, -2/3)$

3. If $\frac{x-1}{\ell} = \frac{y-2}{m} = \frac{z+1}{n}$ is the equation of the line through (1, 2, −1) and (−1, 0, 1), then (ℓ, m, n) is
(1) (−1, 0, 1) (2) (1, 1, −1) (3) (1, 2, −1) (4) (0, 1, 0)

4. If A, B, C, Dare the points (2, 3, −1),(3, 5, −3), (1, 2, 3), (3, 5, 7) respectively, then the angle between AB and CD is
(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{6}$

5. The line passing through the points (5,1,a) and (3,b,1) crosses the yz - plane at the point $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$. Then,
1) a = 2, b = 8 2) a = 4, b = 6 3) a = 6, b = 4 4) a= 8, b = 2

6. If a point $R(4, y, z)$ lies on the line segment joining the points $P(2,-3,4)$ and Q(8, 0, 10), then the distance of R from the origin is:

(1) $2\sqrt{14}$ (2) 6 (3) $\sqrt{53}$ (4) $2\sqrt{21}$

7. A plane P meets the coordinate axis at A, B and C respectively. The centroid of ΔABC is given to be (1, 1, 2). Then the equation of the line through this centroid and perpendicular to the plane P is:

(1) $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$ (2) $\frac{x-1}{2}=\frac{y-1}{2}=\frac{z-2}{1}$

(3) $\frac{x-1}{2}=\frac{y-1}{1}=\frac{z-2}{1}$ (4) $\frac{x-1}{1}=\frac{y-1}{1}=\frac{z-2}{2}$

8. The shortest distance between the lines $\frac{x-3}{3}-\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$ is

(1) $\frac{7}{2}\sqrt{30}$ (2) $3\sqrt{30}$ (3) 3 (4) $2\sqrt{30}$

9. The length of the perpendicular from the point $(2,-1,4)$ on the straight line, $\frac{x+3}{10}=\frac{y-2}{-7}=\frac{z}{1}$ is

(1) less than 2 (2) greater than 3 but less than 4

(3) greater than 4 (4) greater than 2 but less than 3

10. Two lines $\frac{x-3}{1}=\frac{y+1}{3}=\frac{z-6}{-1}$ and $\frac{x+5}{7}=\frac{y-2}{-6}=\frac{z-3}{4}$ intersect at the point R. The reflection of R in the xy-plane has coordinates:

(1) (2, 4, 7) (2) $(-2,4,7)$ (3) $(2,-4,-7)$ (4) $(2,-4,7)$

11. If the lines $x=ay+b$, $z=cy+d$ and $x=a'z+b'$, $y=c'z+d'$ are perpendicular, then

(1) $cc'+a+a'=0$ (2) $aa'+c+c'=0$ (3) $ab'+bc'+1=0$ (4) $bb'+cc'+1=0$

12. The S.D. between the lines

$$\bar{r}=\left(\bar{i}+2\bar{j}+\bar{k}\right)+\lambda\left(2\bar{i}+\bar{j}+2\bar{k}\right)$$

and $\bar{r}=2\bar{i}-\bar{j}-\bar{k}+\mu\left(2\bar{i}+\bar{j}+2\bar{k}\right)$ is

1) 0 unit 2) $\frac{\sqrt{101}}{3}$ units 3) $\frac{3}{\sqrt{101}}$ units 4) $\frac{101}{3}$ units

13. If for some $\alpha \in R$, the lines $L_1:\frac{x+1}{2}=\frac{y-2}{-2}=\frac{z-1}{1}$ and $L_2:\frac{x+2}{\alpha}=\frac{y+1}{5-\alpha}=\frac{z+1}{1}$ are coplanar, then the line L_2 passes through the point:

(1) $(-2,10,2)$ (2) $(10,2,2)$ (3) $(10,-2,-2)$ (4) $(2,-10,-2)$

14. A line passes through a point A with position vector $3\hat{i}+\hat{j}-\hat{k}$ and is parallel to the vector $2\hat{i}-\hat{j}+2\hat{k}$. If P is a point on this line such that AP = 15 units, then the position vector of the point P is /are

1) $13\hat{i}+4\hat{j}-9\hat{k}$ 2) $13\hat{i}-4\hat{j}+9\hat{k}$ 3) $7\hat{i}-6\hat{j}+11\hat{k}$ 4) $7\hat{i}+6\hat{j}+11\hat{k}$

15. The angle between the straight lines $x-1=\frac{2y+3}{3}=\frac{z+5}{2}$ and x = 3r + 2; y = – 2r –1; z = 2, where r is a parameter, is

1) $\frac{\pi}{4}$ 2) $\cos^{-1}\left(\frac{-3}{\sqrt{182}}\right)$ 3) $\sin^{-1}\left(\frac{-3}{\sqrt{182}}\right)$ 4) $\frac{\pi}{2}$

16. If the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3},\frac{x-1}{3}=\frac{y-2}{-1}=\frac{z-3}{4}$ and $\frac{x+k}{3}=\frac{y-1}{2}=\frac{z-2}{h}$ are concurrent then

1) $h=-2, k=-6$

2) $h=\frac{1}{2}, k=2$

3) $h=6, k=2$

4) $h=2, k=\frac{1}{2}$

17. The point of intersection of the lines $\overline{r}=7\overline{i}+10\overline{j}+13\overline{k}+S(2\overline{i}+3\overline{j}+4\overline{k})$ and

$\overline{r}=3\overline{i}+5\overline{j}+7\overline{k}+t(\overline{i}+2\overline{j}+3\overline{k})$ is

1) $\overline{i}+\overline{j}+\overline{k}$ 2) $2\overline{i}-\overline{j}+4\overline{k}$ 3) $\overline{i}-\overline{j}+\overline{k}$ 4) $\overline{i}-\overline{j}-\overline{k}$

18. The acute angle between the lines whose d.c's are given by $l+m-n=0,\ l^2+m^2-n^2=0$ is

1) 0 2) $\frac{\pi}{6}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{3}$

19. A line passes through a point A with position vector $3\hat{i}+\hat{j}-\hat{k}$ and is parallel to the vector $2\hat{i}-\hat{j}+2\hat{k}$. If P is a point on this line such that AP = 15 units, then the position vector of the point P is /are

1) $13\hat{i}+4\hat{j}-9\hat{k}$ 2) $13\hat{i}-4\hat{j}+9\hat{k}$ 3) $7\hat{i}-6\hat{j}+11\hat{k}$ 4) $-7\hat{i}+6\hat{j}-11\hat{k}$

20. **Statement-I:** If the vectors $\vec{a}$ and $\vec{c}$ are non-collinear then the lines $\vec{r} = 6\vec{a} - \vec{c} + \lambda(2\vec{c} - \vec{a})$ and $\vec{r} = \vec{a} - \vec{c} + \mu(\vec{a} + 3\vec{c})$ are coplanar

Statement-II: There exist λ and μ such that the two values of $\vec{r}$ in Statement-I becomes same.

1) If both statement-I and statement -II are true and the reason is the correct explanation of the statement -I

2) If both statement-I and statement -II are true but reason is not the correct explanation of the statement-I

3) If statement-I is true but statement-II is false

4) If statement-I is false but statement-II istrue

SECTION - II

Numerical Type Questions

21. If the length of the perpendicular from the point $(\beta, 0, \beta)(\beta \neq 0)$ to the line, $\frac{x}{1} = \frac{y-1}{0} = \frac{z+1}{-1}$ is $\sqrt{\frac{3}{2}}$, then $|\beta|$ is equal to

22. If (a, b, c) is the image of the point $(1, 2, -3)$ in the line $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$, then a + b + c is equal to

23. The lines $\frac{x-2}{2} = \frac{y}{-2} = \frac{z-7}{16}$ and $\frac{x+3}{4} = \frac{y+2}{3} = \frac{z+2}{1}$ intersect at the point P. If the distance of P from the line $\frac{x+1}{2} = \frac{y-1}{3} = \frac{z-1}{1}$ is l, then $14l^2$ is equal to......

24. Let O be the origin, and M and N be the points on the lines $\frac{x-5}{4} = \frac{y-4}{1} = \frac{z-5}{3}$ and $\frac{x+8}{12} = \frac{y+2}{5} = \frac{z+11}{9}$ respectively such that MN is the shortest distance between the given lines. Then $\overline{OM}.\overline{ON}$ is equal to ....

25. If the line $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1}$ intersects the curve $xy = c^2$ and $z = 0$, then the value of $[|c|]$ is (where, [.] denotes the greatest integer function)

PART - II (JEE ADVANCED)

SECTION - III (One correct answer)

26. Equation of the line $x-y+2z=5$,

$3x+y+z=6$ in symmetrical of form is

A) $\frac{x-1}{-3}=\frac{y+1}{5}=\frac{z-2}{4}$ B) $\frac{x-3}{1}=\frac{y-1}{-1}=\frac{z-1}{2}$

C) $\frac{4x-11}{-3}=\frac{4y+9}{5}=\frac{z-0}{1}$ D) $\frac{4x-11}{-3}=\frac{4y+9}{5}=\frac{z}{4}$

27. The distance of the point $(-1,-5,-10)$ from the point of intersection of the line, $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane, $x-y+z=5$, is

A) 10   B) 11   C) 12   D) 13

28. The distance of the point $(1,-2,3)$ from the plane $x-y+z=5$ measured parallel to the line, $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$ is

A) 1   B) 6/7   C) 7/6   D) 1/6

29. The distance of the point $P(3,8,2)$ from the line $\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-2}{3}$ measured parallel to the plane $3x+2y-2z+17=0$ is

A) 2   B) 3   C) 5   D) 7

30. The plane which bisects the line joining the points $(4,-2,3)$ and $(2,4,-1)$ at right angle is

A) $x-3y+3z-3=0$ B) $2x-6y+2z-2=0$

C) $x-3y+2z-2=0$ D) $x-3y+4z-4=0$

31. The equation of a plane passing through the line of intersection of the planes $x+2y+3z=2$ and $x-y+z=3$ and at a distance $\frac{2}{\sqrt{3}}$ from the point $(3,1,-1)$ is

A) $5x-11y+z=17$

B) $\sqrt{2}x+y=3\sqrt{2}-1$

C) $x+y+z=\sqrt{3}$

D) $x-\sqrt{2}y=1-\sqrt{2}$

32. Equation of the plane which passes through the point of interseciton of lines $\frac{x-1}{3}=\frac{y-2}{1}=\frac{z-3}{2}$ and $\frac{x-3}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ and at greatest distance from orgin is

A) $4x+5y+3z=20$ B) $4x+3y+5z=50$ C) $x+y=1$ D) $x+3z-1=0$

SECTION - IV (More than one correct answer)

33. Plane $ax+by=0$ is rotated about its line of intersection with the plane z=0, through an angle θ then its equation in new position may be

A) $ax+by+z\sqrt{a^2+b^2}=0$ if $\theta=45°$

B) $\sqrt{3}ax+\sqrt{3}by-z\sqrt{a^2+b^2}=0$ if $\theta=30°$

C) $ax+by-z\sqrt{a^2+b^2}=0$ if $\theta=45°$

D) $ax+by+\sqrt{3}\sqrt{a^2+b^2}=0$ if $\theta=30°$

34. Plane $ax+by=0$ is rotated about its line of intersection with the plane z=0, through an angle θ then its equation in new position may be

A) $ax+by+z\sqrt{a^2+b^2}=0$ if $\theta=45°$

B) $\sqrt{3}ax+\sqrt{3}by-z\sqrt{a^2+b^2}=0$ if $\theta=30°$

C) $ax+by-z\sqrt{a^2+b^2}=0$ if $\theta=45°$

D) $ax+by+\sqrt{3}\sqrt{a^2+b^2}=0$ if $\theta=30°$

35. In R^3, let L be a straight line passing through the origin. Suppose that all the points on L are at a constant distance from the two planes $P_1: x+2y-z=-1$ and $P_2: 2x-y+z-1=0$. Let M be the locus of the feet of the perpendiculars drawn from the points on L to the plane P_1. Which of the following points lie(s) on M?

A) $\left(0,\frac{-5}{6},\frac{-2}{3}\right)$

B) $\left(\frac{-1}{6},\frac{-1}{3},\frac{1}{6}\right)$

C) $\left(\frac{-5}{6},0,\frac{1}{6}\right)$

D) $\left(\frac{-1}{3},0,\frac{2}{3}\right)$

36. In R^3 consider the planes $P_1 : y = 0$ $P_2 : x + z = 1$. Let P_3 be the plane, different from P_1 and P_2, which passes through the intersection of P_1 and P_2. If the distance of the point (0,1,0) from P_3 is 1 and the distance of a point (α, β, γ) from P_3 is 2, then which of the following relations is (are) true?

A) $2\alpha + \beta + 2\gamma + 2 = 0$ B) $2\alpha - \beta + 2\gamma + 4 = 0$

C) $2\alpha + \beta - 2\gamma - 10 = 0$ D) $2\alpha - \beta + 2\gamma - 8 = 0$

37. A line L passing through the point P(1, 4, 3), is perpendicular to both the lines $\frac{x-1}{2} = \frac{y+3}{1} = \frac{z-2}{4}$, and $\frac{x+2}{3} = \frac{y-4}{2} = \frac{z+1}{-2}$. If the position vector of point Q on L is (a_1, a_2, a_3) such that $(PQ)^2 = 357$, then $(a_1 + a_2 + a_3)$ can be

A) 16 B) 15 C) 2 D) 1

SECTION V - (Numerical type)

38. If the planes $x = cy + bz, y = az + cx$ and $z = bx + ay$ pass through a line then $a^2 + b^2 + c^2 + 2abc$ is equal to

39. Two lines are formed by intersection of plane $2x + 3y + 4z - 1 = 0$ with the planes $x + y + z - 3 = 0$ and $x + y + z + 3 = 0$, then the square of the shortest distance between both the lines is

SECTION VI - (Matrix match type)

40. Match the statements/expressions given in Column I with the values given in Column II

Column I	Column II
A) $L_1 : x = 1 + t, y = t, z = 2 - 5t$ $L_2 : \vec{r} = (2,1,-3) + \lambda(2,2,-10)$	(p) non coplanar lines
B) $L_1 : \frac{x-1}{2} = \frac{y-3}{2} = \frac{z-2}{-1}$ $L_2 : \frac{x-2}{1} = \frac{y-6}{-1} = \frac{z+2}{3}$	(q) lines lie in a unique plane
C) $L_1 : x = -6t, y = 1 + 9t, z = -3t$ $L_2 : x = 1 + 2s, y = 4 - 3s, z = s$	(r) infinite planes containing both the lines
D) $L_1 : \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ $L_2 : \frac{x-3}{-4} = \frac{y-2}{-3} = \frac{z-1}{2}$	(s) lines are not intersecting.

A) A-(R); B-(Q); C-(QS); D-(PS) B) A-(Q); B-(R); C-(QS); D-(PS)

C) A-(R); B-(Q); C-(QS); D-(QS) D) A-(R); B-(S); C-(QS); D-(PS)