CHAPTER - 00
FRICTION

* Frictional force opposes the relative motion between two surfaces in contact.

* Static friction opposes the tendency of one surface to slip over another surface.

* The maximum possible value of static friction is called limiting friction.

* $\left(f_s\right)_{max} = f_L = \mu_s N$

* μ_s is called coefficient of static friction between two surfaces in contact

* N is the normal contact force between two surfaces

* Static friction is a self adjusting force. It has any value between zero and limiting friction.

* Kinetic friction opposes the actual relative slipping between two surfaces in contact.

* Kinetic friction is not a self adjusting force

* $f_K = \mu_K N$

* μ_K is the coefficient of kinetic friction between two surfaces in contact.

* $\mu_K < \mu_s$

* $f_K < f_L$

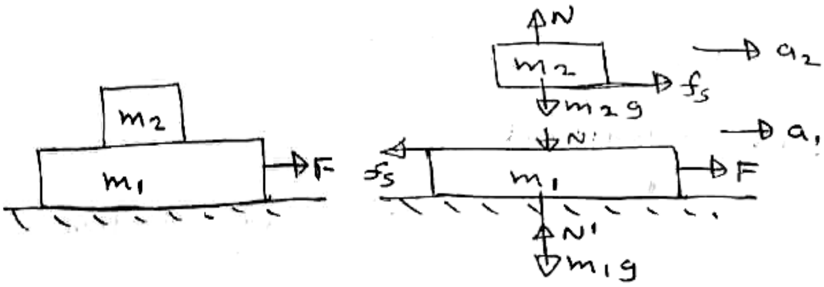
* The value of kinetic friction is slightly less than limiting friction hence the force required to keep a body in uniform motion is less than the force required to start the motion.

* According to old view friction is due to the interlocking between surface irregularities

* According to modern view friction is due to the molecular attraction between two surfaces in contact (Adhesive force)

* Friction is independent of area of contact

e. g $\Rightarrow$ In the following figure floor is smooth μ_s is the coefficient of static friction between m_1 and m_2 and μ_K is the coefficient of kinetic friction between them. For what maximum value of F two block move together with out relative slipping between them.



The direction of static frictional force applied by m_2 on m_1 is opposite to a_{12}. where a_{12} is the relative acceleration of m_1 as seen by an observer on m_2 in the absence of friction between them. If there is no slipping between m_1 and m_2 then $a_1 = a_2 = a$

Consider $(m_1 + m_2)$ system

$a = F/(m_1 + m_2)$

Consider m_2

$$a = \frac{f_s}{m_2}$$

$$f_s = m_2 a$$

$$* \; f_s = \frac{Fm_2}{m_1 + m_2}$$

To prevent slipping between m_1 and m_2

$f_s \le \mu_s N$ $\qquad$ $f_s \le f_L$

$$f_s \le \mu_s m_2 g$$

$$\left(\frac{m_2 F}{m_1 + m_2}\right) = \mu_s m_2 g$$

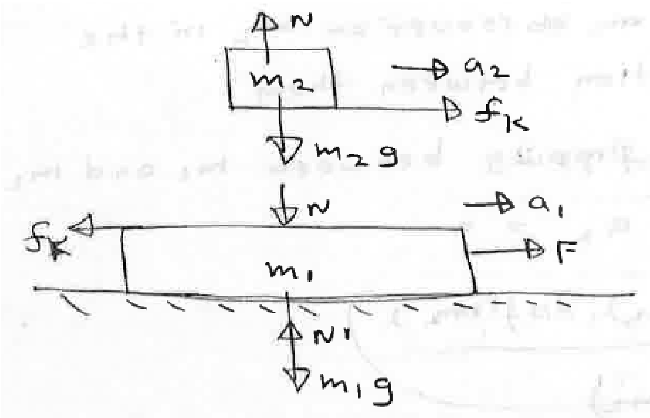
$$\boxed{F \le \mu_s (m_1 + m_2) g}$$

$$\boxed{F_{max} = \mu_s (m_1 + m_2) g}$$

If $F \ge \mu_s (m_1 + m_2) g$

then $a_2 < a_1$

m_2 will slip over m_1



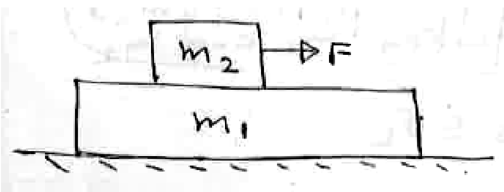
* $f_K = \mu_K N = \mu_K m_2 g$

* $f_K = \mu_K m_2 g$

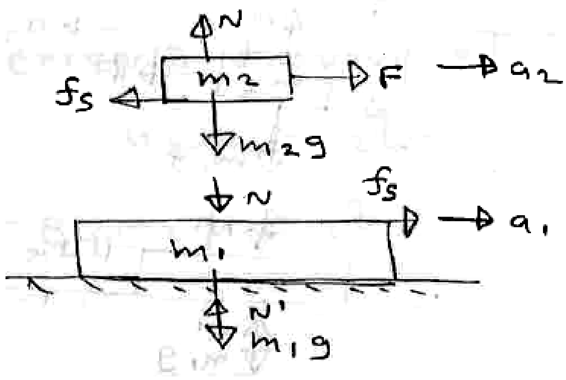
consider $m_1 \Rightarrow \boxed{a_1 - \frac{F - f_K}{m_1} = \frac{F - \mu_K m_2 g}{m_1}}$

consider $m_2 \Rightarrow \boxed{a_2 - \frac{f_K}{m_2} = \frac{\mu_K m_2 g}{m_2} = \mu_K g}$

Case 2



If there is no slipping between m_1 and m_2 then $a_1 = a_2 = a$



Consider (m_1 +m_2) system

$$a = F/(m_1 + m_2)$$

Consider m_1

$$a = \frac{f_s}{m_1}$$

* $f_s = m_1 a$

* $f_s = \dfrac{m_1 F}{m_1 + m_2}$

* To prevent slipping

* $f_s \le f_L$

* $f_s \le \mu_S N$

* $f_s \le \mu_s m_2 g$

* $\dfrac{m_1 F}{m_1 + m_2} \le \mu_s m_2 g$

* $F \le \dfrac{\mu_s (m_1 + m_2) m_2 g}{m_1}$

* $\boxed{F_{max} = \dfrac{\mu_s (m_1 + m_2) m_2 g}{m_1}}$

* If $F > \dfrac{\mu_s (m_1 + m_2) m_2 g}{m_1}$

then m_2 will slip over m_1.

$a_2 > a_1$

friction between blocks is kinetic

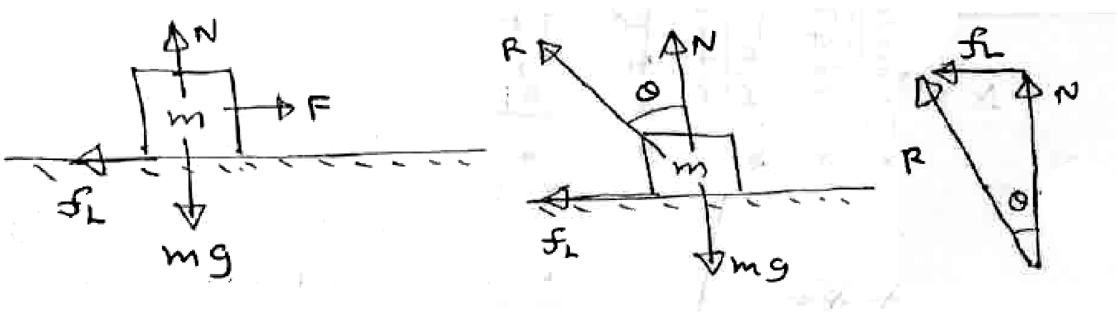
$\boxed{f_K = \mu_K N = \mu_K m_2 g}$

Consider $m_1 \Rightarrow a_1 = \dfrac{f_K}{m_1} = \dfrac{\mu_K m_2 g}{m_1}$

Consider $m_2 \Rightarrow a_2 = \dfrac{F - f_K}{m_2} = \dfrac{F - \mu_K m_2 g}{m_2}$

Angle of friction

Angle of friction is the angle between resultant contact force (The resultant of normal reaction force and friction) and normal reaction. When body shows maximum tendency to slip under the action of an applied force.



* $F \Rightarrow$ applied force

* $F_L \Rightarrow$ limiting friction

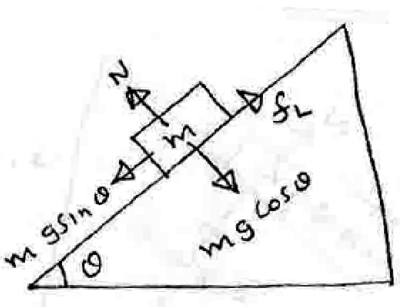
* $R \Rightarrow$ resultant contact force

* $Q \Rightarrow$ angle of friction

* $\tan\theta = \dfrac{f_L}{N} = \dfrac{\mu_g N}{N}$

* $\boxed{\theta = \tan^{-1}(\mu_s)}$

* Angle of repose is the minimum inclination of a rough inclined plane such that a body placed on it shows maximum tendency to slide down.



$\theta \Rightarrow$ angle of repose

$N = mg\cos\theta$

$f_L = \mu_s N = \mu_s mg\cos\theta$

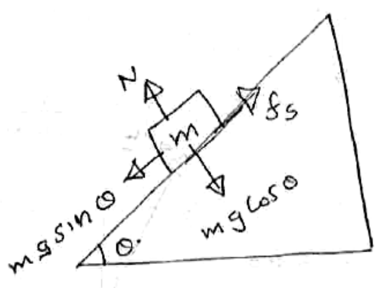
$mg\sin\theta = f_L$

$mg\sin\theta = \mu_s mg\cos\theta$

$\boxed{\tan\theta = \mu_s}$

$\boxed{\theta = \tan^{-1}\mu_s}$

* Case : 1



* $\theta < \tan^{-1}(\mu_s)$

* friction between body and inclined plane is static

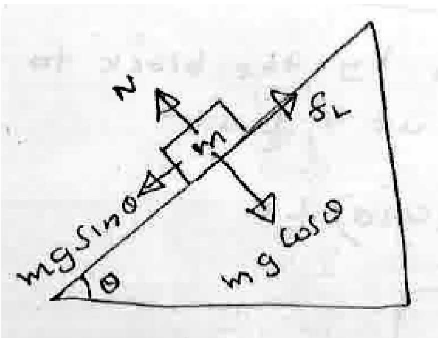
* body remains in equilibrium

* F_{net} = o

* $N = mg\cos\theta$

* $\boxed{f_s = mg\sin\theta}$

Case 2



* $\theta = \tan(\mu_s)$

* friction between body and inclined plane is limiting

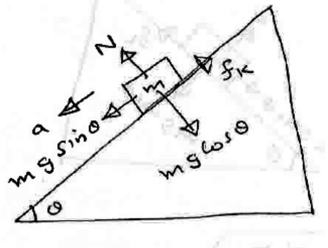
* $f_L = \mu_s N = mg\cos\theta$

* Body remains in equilibrium

* F_{net} = 0

* $\boxed{mg\sin\theta = f_L}$ $\boxed{mg\sin\theta = \mu_s mg\cos\theta}$

* Case 3



* $\theta > \tan^{-1}(\mu_s)$

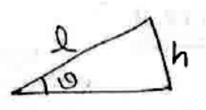
* Body accelerates down along the inclined plane

* Friction between body and inclined plane is kinetic

* $f_K = \mu_K N = \mu_K mg\cos\theta$

$$a = \frac{mg\sin\theta - f_K}{m} = \frac{mg\sin\theta - \mu_K mg\cos\theta}{m}$$

$$\boxed{a = g[\sin\theta - \mu_K\cos\theta]}$$



$$\sin\theta = \frac{h}{\ell}$$

* $\ell = \dfrac{h}{\sin\theta}$

Let V be the velocity of block when it reaches of the bottom

* $V^2 = u^2 + 2a\ell$

$$V^2 = \frac{0 + 2g(\sin\theta - \mu_K\cos\theta)h}{\sin\theta}$$

$$\boxed{V = \sqrt{\frac{2g(\sin\theta = \mu_K\cos\theta)h}{\sin\theta}}}$$

Let t be the time taken by the block to reach the bottom

* $\ell = ut + \dfrac{1}{2}at^2$

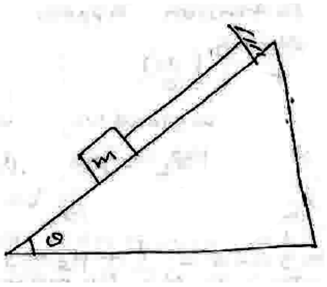
* $\dfrac{h}{\sin\theta} = 0 + \dfrac{1}{2}g(\sin\theta - \mu_K\cos\theta)t^2$

* $\boxed{t = \sqrt{\dfrac{2h}{g(\sin\theta - \mu_K \cos\theta)}}}$

If inclined plane is smooth then $\mu_K = 0$

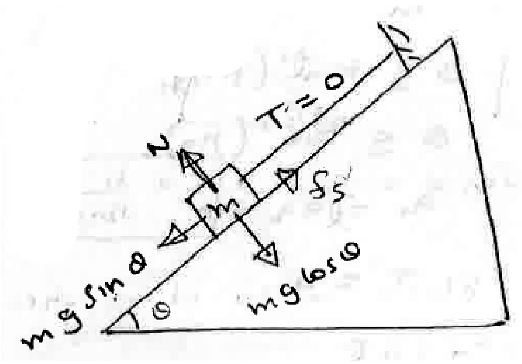
$\boxed{a = g\sin\theta}$ $\boxed{v = \sqrt{2gh}}$ $\boxed{t = \dfrac{1}{\sin\theta}\sqrt{\dfrac{2h}{g}}}$

example $\Rightarrow$



Case 1

* $\theta < \tan^{-1}(\mu_s)$



F_{net} = 0

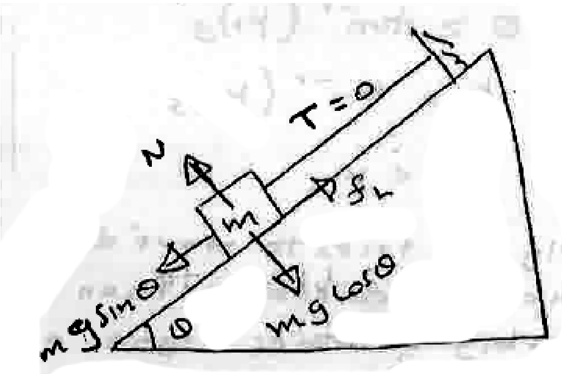
T = 0

$N = mg\cos\theta$

* $f_s = mg\sin\theta$

Case 2

* $\theta = \tan^{-1}(\mu_s)$



* $f_s = f_L = \mu_s N = \mu_s mg\cos\theta$

* T = 0

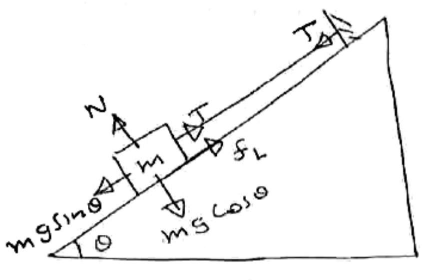
* $F_{net} = 0$

* $f_s = mg\sin\theta$

* $\mu_s mg\cos\theta = mg\sin\theta$

Case 3

* $\theta > \tan^{-1}(\mu_s)$



In this case string is tight so tension appears in the string.

$F_{net} = 0$

$N = mg\cos\theta$

$$f_s = f_L = \mu_s N = \mu_s \ mg \cos\theta$$

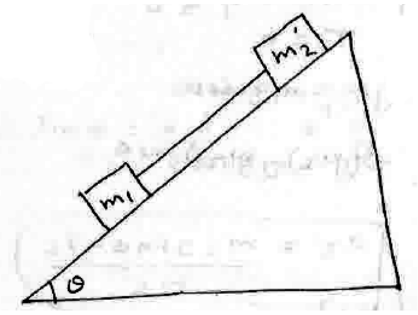
$$mg \sin\theta = T + f_L$$

$$mg \sin\theta = T + \mu_s mg \cos\theta$$

$$\boxed{T = mg \sin\theta - \mu_s mg \cos\theta}$$

example

Case I



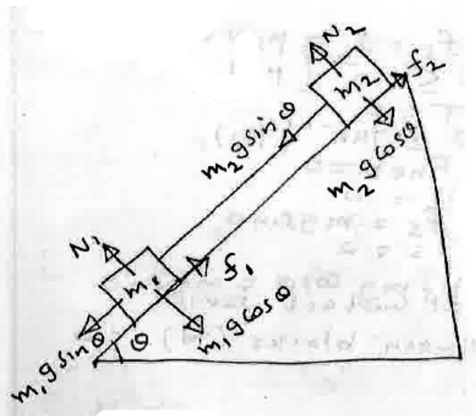
$$\theta \le \tan^{-1}(\mu_1)_s$$

$$\theta \le \tan^{-1}(\mu_2)_s$$

$$a_1 = a_2 = 0$$

T = 0

Case 2



$\theta > \tan^{-1}(\mu_1)s$

$\theta > \tan^{-1}(\mu_2)s$

$\mu_1 < \mu_2$

Initially m_1 tries to move down with more acceleration than m_2. But string becomes tight and tension appears in the string hence both blocks move down along the inclined plane with same acceleration.

$$\boxed{N_1 = m_1 g \cos\theta}$$

$$f_1 = (f_1)_K = (\mu_1)_K m_1 g \cos\theta$$

$$\boxed{N_2 = m_2 g \cos\theta}$$

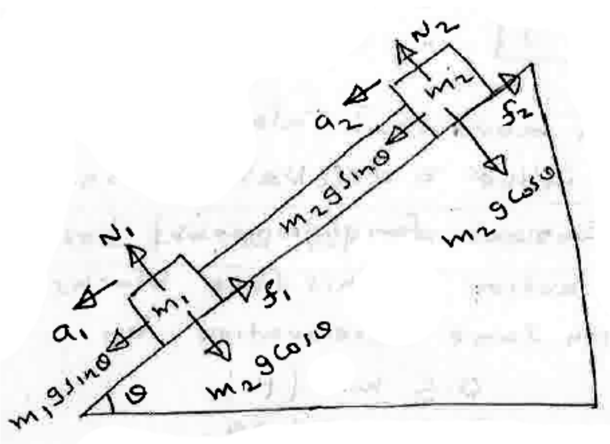
$$f_2 = (f_2)_K = (\mu_2)_K N_2 = (\mu_2)_K m_2 g \cos\theta$$

$$\boxed{a_1 = \frac{m_1 g \sin\theta - f_1 - T}{m_1}}$$

$$\boxed{a_2 = \frac{m_2 g \sin\theta + T - f_2}{m_2}}$$

$a_1 = a_2$

<u>Case 3</u>



$\theta > \tan^{-1}(\mu_1)s$

$\theta > \tan^{-1}(\mu_2)s$

$\mu_1 > \mu_2$

$a_2 > a_1$

T = 0

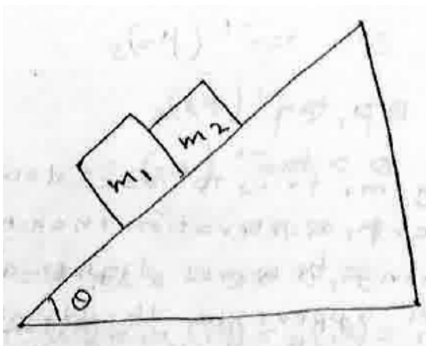
$(f_1) = (f_1)_K = (\mu_1)_K N_1 = (\mu_1)_K m_1 g \cos\theta$

$(f_2) = (f_2)_K = (\mu_2)_K N_2 = (\mu_2)_K m_2 g \cos\theta$

$$\boxed{a_1 = \frac{m_1 g \sin\theta - f_1}{m_1}}$$

$$\boxed{a_2 = \frac{m_2 g \sin\theta - f_2}{m_2}}$$

Case 4



$\theta \le \tan^{-1}(\mu_1)s$

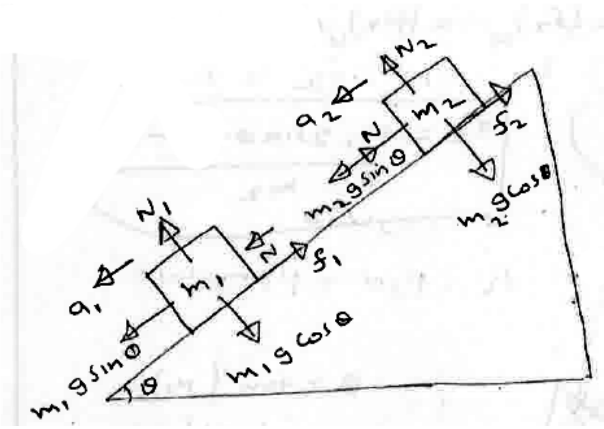
$\theta \le \tan^{-1}(\mu_2)s$

$a_1 = 0$

$a_2 = 0$

normal contact force between blocks (N) = 0

Case 5



$$\theta > \tan^{-1}(\mu_1)s$$

$$\theta > \tan^{-1}(\mu_2)s$$

$$\mu_2 < \mu_1$$

Initially m_2 tries to move down along the inclined plane with more acceleration than m_1. But normal contact force (N) appears between blocks. Due to the action of this force blocks move together with same acceleration by keeping contact.

$$f_1 = (f_1)_K = (\mu_1)_K N_1 = (\mu_1)_K m_1 g \cos\theta$$

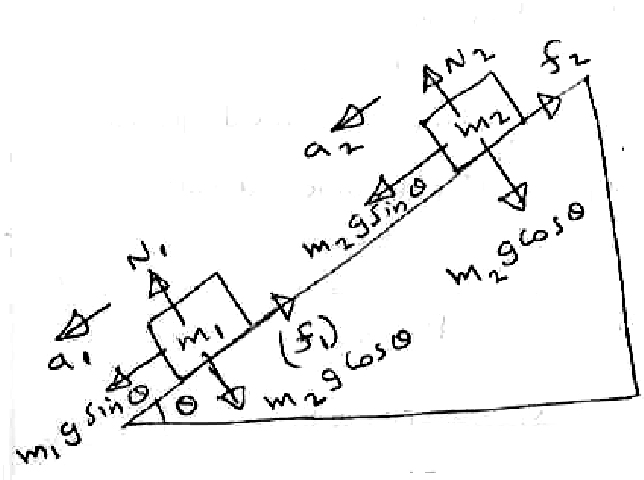
$$f_2 = (f_2)_K = (\mu_2)_K N_2 = (\mu_2)_K m_2 g \cos\theta$$

$a_1 = a_2$

$$a_1 = \frac{m_1 g \sin\theta + N - f_1}{m_1}$$

$$a_2 = \frac{m_2 g \sin\theta - N - f_2}{m_2}$$

case 6



$$\theta > \tan^{-1}(\mu_1)_s$$

$$\theta > \tan^{-1}(\mu_2)s$$

$$\mu_1 < \mu_2$$

$a_1 > a_2$

* So N = 0

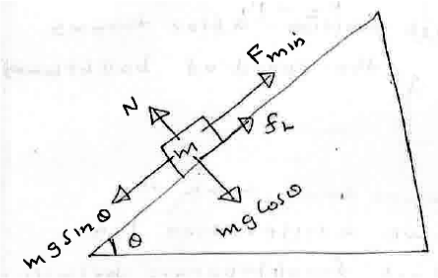
$$f_1 = (f_1)_K = (\mu_1)_K N_1 = (\mu_1)_K m_1 g \cos\theta$$

$$f_2 = (f_2)_K = (\mu_2)_K N_2 = (\mu_2)_K m_2 g \cos\theta$$

$$\boxed{a_1 = \frac{m_1 g \sin\theta - f_1}{g}}$$

$$\boxed{a_2 = \frac{m_2 g \sin\theta - f_2}{g}}$$

Block kept in equilibrium on a rough inclined plane $\left[\theta > \tan^{-1}(\mu s)\right]$ by applying a force F



$$N = mg\cos\theta$$

$$f_L = \mu_s N = \mu_s mg\cos\theta$$

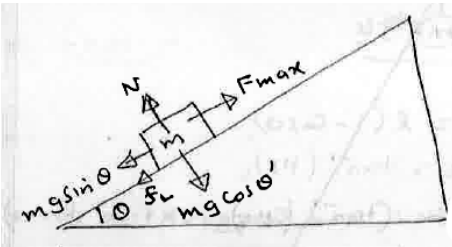
If F= F_{min} then block has maximum tendency to slide down along the inclined plane, hence limiting friction acts on the block up along the inclined plane.

$F_{net} = 0$

* $mg\sin\theta = F_{min} + f_L$

$$mg\sin\theta = F_{min} + \mu_s mg\cos\theta$$

$$\boxed{F_{min} = mg\sin\theta - \mu_s mg\cos\theta}$$



$$N = mg\cos\theta$$

$$f_L = \mu_s N = \mu_s mg\cos\theta$$

If $F = F_{max}$ then block has maximum tendency to slide up along the inclined plane hence limiting friction acts on the block down along the inclined plane.

$$F_{net} = 0$$

* $mg\sin\theta + f_L = F_{max}$

$$\boxed{F_{max} = mg\sin\theta + \mu_s mg\cos\theta}$$