CHAPTER - 00
WORK ENERGY POWER

The term ‘work’, ‘energy’ and ‘power’ are frequently used in everyday language. A farmer ploughing the field, a studying for a competitive examination, an artist painting a beautiful landscape, all are said to be working. In physics, however, the word covers a definite and precise meaning

Work

* Work is said to be done whenever a force acts on a body and the body moves through some distance in the direction of the force
* For work to be done, following two conditions must be satisfied

  i) A force must be applied

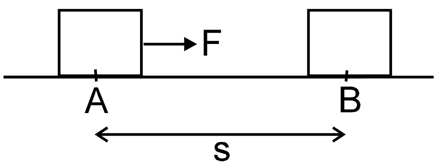
  ii) The point of application of the force move in the direction of the force
* **Work done by a constant force**

Case-1

When applied force and displacement are in same direction

Consider a block which is placed at a point A. Let a horizontal force acts on the body. Then the new position of block is B. The displacement of the body is s.

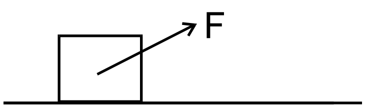
Then the work done by the applied force is given by W = Fs



Case-2

When applied force and displacement are not in same direction.

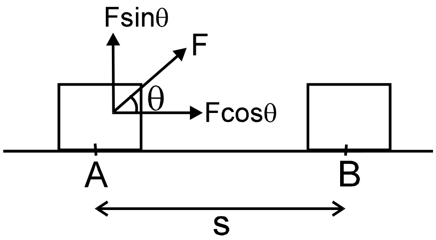
Consider a block which is placed in a horizontal surface. Let F be the force acting on the block at an angle θ with the horizontal.



In this case work is not the product of force and the displacement of the object.

Let us resolve the vector quantity force into its vector components. Then $F\cos\theta$ is along the direction of displacement and $F\sin\theta$ is perpendicular to the direction of displacement.

We can see that the only component of force that produces the displacement is $F\cos\theta$



Then work done is $W = F \times s = F\cos\theta \times s$

$$W = Fs\cos\theta$$

In vector notation, we can write $W = \vec{F}.\vec{s}$

Since the dot product of any two vectors is scalar. work is a scalar quantity.

<u>Types of work</u>

Although work done is a scalar quantity, its value may be positive, negative or zero

<u>Positive work</u>

* If a force acting on a body has a component in the direction of the displacement, then the work done by the force is positive.

* When θ us acute, $\cos\theta$ is positive.

$\therefore W = Fs\cos\theta$ = a positive value

Condition for positive work $0^o \le \theta < 90^o$

Eg.

1) When a body falls freely under gravity the work done by the gravity is positive

2) When a horse pulls a cart, the applied force and displacement are in the same directions, then the work done by the horse is positive

3) When a spring is stretched, work done by the applied force agent is positive

<u>Negative Work</u>

If a force acting on the body has a component in the opposite direction of displacement, the work done is negative.

* When θ is obtuse, $\cos\theta$ is negative

$\therefore W = Fs\cos\theta$ = a negative value

Condition for negative work $90^o < \theta \le 180^o$

Eg.

1) When a body slides against a rough horizontal surface, its displacement is opposite to the force of friction. The work done by the friction is negative.

2) When brakes are applied to a moving vehicle, the work done by the braking force is negative

3) When a body is lifted, the work done by the gravitational force is negative

Zero work

Work done by force is zero, if the body gets displaced along a direction perpendicular to the direction of the applied force.

* Work done is also zero if $\vec{F}$ or $\vec{s}$ or both are zero

Condition for zero work : $\theta = 90^o$ or $\vec{s} = 0$ or $\vec{F} = 0$

Eg.

1) For a body moving in a circular path, the centripetal force and displacement are perpendicular to each other. So the work done by the centripetal force is zero.

2) When a coolie walks on a horizontal platform with a load on his head, he applied a force on it in the upward direction equal to its weight. The displacement of the load is along the horizontal direction. Thus the angle between $\vec{F}$ and $\vec{S}$ is 90^o. Then the work done by the coolie on the load is zero.

3) The tension in the string of a simple pendulum is always perpendicular to its displacement. So the work done by the tension is zero.

4) The work done in pushing an immovable stone is zero, because the displacement of the stone is zero.

* Dimensional formula of work

$W = F \times s$ = $\left[MLT^{-2}\right]\left[L\right]$

= $\left[ML^2T^{-2}\right]$

* **Units of work**

Joule : It is the unit of work in SI.

One Joule of work is said to be done when a force of one Newton displaces a body through a distance of one metre in its own direction.

1 Joule = 1 Newton $\times$ 1 metre

1 J = 1 Nm

Erg : It is the unit of work in CGS system

One erg of work is said to be done if a force of one dyne displaces a body through a distance of one centimetre in its own direction

1 erg = 1 dyne $\times$ 1 cm

Relation between joule and erg

1 Joule = 1Newton $\times$ 1metre

= $10^5 \text{dyne} \times 10^2 \text{cm}$

= 10^7 dyne cm

= 10^7 erg

$$\boxed{1 \text{ Joule} = 10^7 \text{erg}}$$

Gravitational units of work

1. Kilogram metre

One kilogram metre of work is said to be done when a force of one kilogram weight displaces a body through one metre in its own direction.

1 kg m = 1 kg wt $\times$ 1cm

= 9.8N $\times$ 1m

= 9.8 J

2. Gram centimetre

One gram centimetre of work is said to be done when a force of one gram weight displaces a body through one centimetre in its own direction

1 kg cm = 1 g wt $\times$ 1cm

= 980dyne $\times$ 1cm

= 980 erg

Work done in terms of rectangular components

In terms of rectangular components, force and displacement vectors can be written as

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

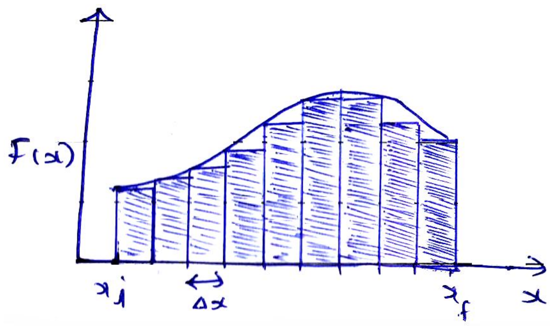
$$\vec{S} = S_x\hat{i} + S_y\hat{j} + S_2\hat{k}$$

$$W = \vec{F}.\vec{S} = \left(F_x\hat{i} + F_y\hat{j} + F_z\hat{k}\right).\left(S_x\hat{i} + S_y\hat{j} + S_2\hat{k}\right)$$

= $F_xS_x + F_yS_y + F_zS_z$

Work done by a variable force

Suppose a variable force F acts on body along the fixed direction, say x axis. Let us calculate the work done when the body moves from initial position x_i to the find position x_f under the force F.



The displacement can be divided into large number of small equal displacements Δx , the force F can be assumed to be constant. Then the work done is

$W \approx F\Delta x =$ Area of rectangle

Adding areas of all rectangles. We get total work done

$W = \sum_{x_i}^{x_f} F\Delta x$ = sum of areas of all rectangles erected over all the small displacement

From the figure, it is seen that when Δx is increases, the area of the curve decreases. Then work done will be lesser than the actual work.

$\therefore$ To set the actual valve of work, Δx should tends to zero.

When $\Delta x \to 0$, the number of rectangles tends to be infinite, then the above summation approaches a definite integral. The value of definite integral is equal to the area under the curve.

$$W = \lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F\Delta x = \int_{x_i}^{x_f} F dx$$

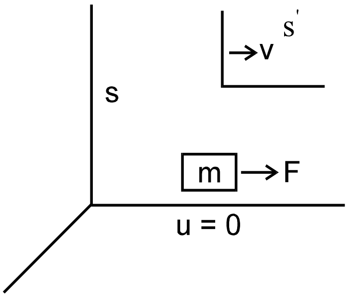
Hence for varying force, the work done is equal to the definite integral of the force over the given displacement.

Work is relative

Work done by a force depends up on the frame of reference.

1) If a person is pushing a box inside a moving train, the work done in the frame of train will $\vec{F}.\vec{s}$ where F is the applied force and s be the displacement of box. In the frame of earth, work will be $\vec{F}.\left(\vec{s} + \vec{s}_0\right)$, where $\vec{s}_0$ is the displacement of the train relative to the ground.

2) Suppose s and s/ are two frames of reference. s/ is moving withy a constant velocity v with respect to s. A block m is placed in s frame which is initially at rest. It is acted upon by a constant force F.



In s frame

Work done by F in time t

W = F.s

$$= F.\left(o.t + \frac{1}{2}\times\frac{F}{m}t^2\right) = \frac{F^2t^2}{2m}$$

In s' frame

Work done by F in time t

$W' = Fs'$

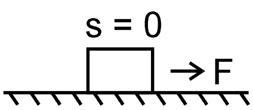
$$= F\left(-vt + \frac{1}{2}\frac{F}{m}t^2\right)$$

Clearly $W \neq W'$

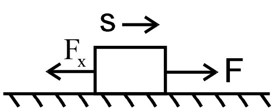
Work done by friction

There is a misconception that the force of friction always does negative work. In reality, the work done by friction may be zero, positive or negative depending up on the situation.

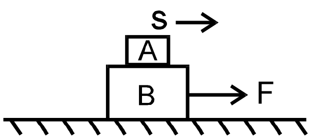
1) When a block is pulled by a force and the block does not move, the work done by friction is zero

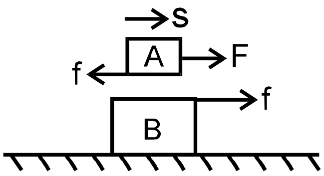


2) When block is pulled by a force F on a stationary surface, the work done by the kinetic friction is negative.



3) Block A is placed on the block B. When the block A is pulled with force F, the friction force does negative work on block A and positive work on block B, which is being accelerated by a force F. The displacement of A relative to the table is in the forward direction. The work done by kinetic friction on block B is positive.





Energy

Energy of a body is defined as the capacity or ability to do work

- Like work, energy is a scalar quantity
- Unit and dimension of energy is same as that of work

SI unit - Joule

CGS unit - erg

Dimension - $\left[ML^2T^{-2}\right]$

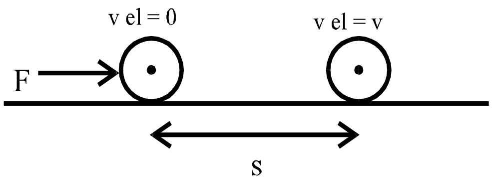
Some other units of work or energy			
Sl.No.	Unit	Symbol	Value in SI
1	erg	erg	10^{-7} J
2	electron volt	eV	1.6×10^{-19} J
3	calorie	cal	4.186 J
4	kilowatt hour	kWh	3.6×10^{6} J

KINETIC ENERGY

The energy possessed by a body by virtue of its motion is called its kinetic energy. A moving object can do work. The amount of work that a moving object can do before coming to rest is equal to kinetic energy

Expression for kinetic energy

The kinetic energy of a body is equal to the amount of work required to bring the body into motion from its state of rest.



Consider a body of mass m which is initially at rest. Let a constant force is applied on the body. If v be the velocity of the body after covering a distance s from initial position

$$v^2 = u^2 + 2as$$

u = 0 $\qquad v^2 = 2as \qquad a = \dfrac{v^2}{2s}$

As the force and displacement are in same direction

W = Fs

= ma.s

$= m \times \dfrac{v^2}{2s} \times s$

$$W = \frac{1}{2}mv^2$$

Thus work done appears as kinetic energy of the body

Relation between KE and linear momentum

$$KE = \frac{1}{2}mv^2$$

multiply and divide by m

$$-\frac{1}{2}mv^2 \times \frac{m}{m}$$

$-\dfrac{1}{2}\dfrac{m^2v^2}{m}$ $\qquad$ Since p = mv

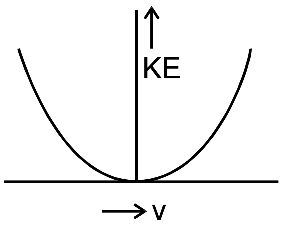
$$p^2 = m^2v^2$$

$$\boxed{KE = \frac{1}{2}\frac{p^2}{m}}$$

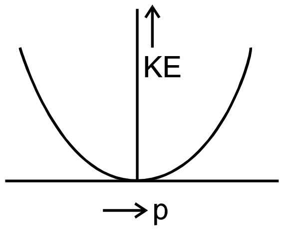
$$\boxed{P = \sqrt{2m(KE)}}$$

Various groups of kinetic energy

1. Graph between KE and velocity, when mass is constant
2. Graph between KE and momentum when mass is constant
3. Graph between KE and mass when momentum is constant
4. Graph between $\sqrt{KE}$ and p when mass is constant

1) 

$KE \ \alpha \ v^2$, $m = \text{constant}$

2) 

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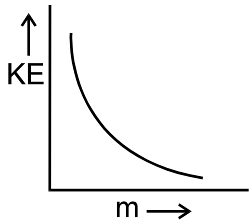
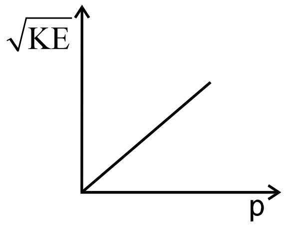
3) 

$KE \ \alpha \ \frac{1}{m}$, $p = \text{constant}$

4) 

$p \ \alpha \ \sqrt{KE}$, $m = \text{constant}$

Work Energy Theorem

It states that the work done by the net force acting on a body is equal to the change produced in the kinetic energy of the body.

Work energy theorem for a constant force

Suppose a constant force F acting on a body of mass m produces acceleration a in it. After covering distance s, suppose the velocity of the body changes from u to v. The equation of motion

$$v^2 = u^2 + 2as$$

$$v^2 - u^2 = 2as$$

Multiply both sides by $\frac{1}{2}m$

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}m \times 2as$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = F \times s$$

Change in KE = W

Thus the change in KE of the body is equal to the work done on the body by the net force.

Work Energy Theorem for a variable force

Suppose a variable force $\vec{F}$ acts on a body of mass m and produces displacement $\overrightarrow{ds}$ in its own direction.

The small work done is

$$dw = \vec{F}.\overrightarrow{ds} = Fds\cos 0^o = Fds$$

According to Newton's second law of motion

$$F = ma = m\frac{dv}{dt}$$

$$dw = m\frac{dv}{dt}.ds \qquad \frac{ds}{dt} = v$$

= $mdv.\frac{ds}{dt}$

= mdv.v

dw = mv.dv

If the applied force increases the velocity from u to v, then the total work done on the body will be

$$W = \int dw = \int_u^v mv\,dv = m\int_u^v vdv$$

= $m\left(\frac{v^2}{2}\right)_u^v = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

W = $KE_f - KE_i$

Stopping a vehicle by Retarding force

If a vehicle move with some initial velocity and due to some retarding force it stops after covering some distance after some time.

Stopping distance

W = change in KE

W = $KE_i - KE_f$

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$KE_f = \frac{1}{2}mv^2 - 0$$

$$W = \frac{1}{2}mv^2$$

$$F \times s = \frac{1}{2}mv^2$$

$$s = \frac{1}{2}\frac{mv^2}{F}$$

Stopping time

By the impulse-momentum theorem

$F \times t = \Delta p$

$F \times t = mu - mv$

But mv = 0

$F \times t = mv$

$t = \dfrac{mv}{F}$

Two vehicles of masses m_1 and m_2 are moving with velocities v_1 and v_2 respectively. When they are stopped by the same retarding force (F). Then

The ratio of their stopping distance $\dfrac{x_1}{x_2} = \dfrac{\left(\dfrac{\frac{1}{2}mv_1^2}{F}\right)}{\left(\dfrac{\frac{1}{2}m_2v_2^2}{F}\right)} = \dfrac{m_1v_1^2}{m_2v_2^2}$

The ratio of stopping time $\dfrac{t_1}{t_2} = \dfrac{\left(\dfrac{m_1v_1}{F}\right)}{\left(\dfrac{m_2v_2}{F}\right)} = \dfrac{m_1v_1}{m_2v_2}$

- If vehicle is stopped by friction, then

stopping distance = $\dfrac{\frac{1}{2}mv^2}{F} = \dfrac{\frac{1}{2}mv^2}{ma} = \dfrac{\frac{1}{2}mv^2}{m \times \mu g}$

$\boxed{s = \dfrac{v^2}{2\mu g}}$ since $a = \mu g$

Stopping time $t = \dfrac{mv}{F} = \dfrac{mv}{ma} = \dfrac{mv}{m\mu g} = \dfrac{v}{\mu g}$

$\boxed{t = \dfrac{v}{\mu g}}$

- If vehicle possess same velocities

  $v_1 = v_2$

$$\frac{x_1}{x_2} = \frac{\dfrac{\frac{1}{2}m_1v^2}{F}}{\dfrac{\frac{1}{2}m_2v^2}{F}} = \frac{m_1}{m_2}$$

$$\frac{t_1}{t_2} = \frac{\dfrac{m_1v}{F}}{\dfrac{m_2v}{F}} = \frac{m_1}{m_2}$$

- If vehicle possess same momentum

$$\frac{x_1}{x_2} = \frac{\dfrac{p_1^2}{2m_1 \times F}}{\dfrac{p_2^2}{2m_2 \times F}} = \frac{m_2}{m_1}$$ Since $p_1 = p_2$

$$\frac{t_1}{t_2} = \frac{\dfrac{p_1}{F}}{\dfrac{p_2}{F}} = 1$$ since $p_1 = p_2$

- If vehicle possess same kinetic energy

$$\frac{x_1}{x_2} = \frac{\dfrac{KE_1}{F}}{\dfrac{KE_2}{F}} = 1$$ since $KE_1 = KE_2$

$$\frac{t_1}{t_2} = \frac{p_1}{p_2} = \frac{\sqrt{2m_1(KE)}}{\sqrt{2m_2(KE)}} = \sqrt{\frac{m_1}{m_2}}$$

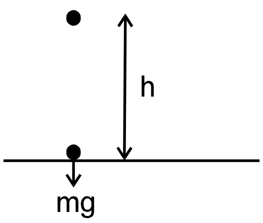
Potential energy

Potential energy of a body is defined as the energy possessed by the body by virtue of its position or state of strain.

Gravitational potential energy

It is the energy possessed by the body by virtue of its position above the surface of the earth

Consider a body of mass m lying on the surface of the earth. Let the body is raised vertically to a height h. Then work has to be done against the force of gravity.



$W = F \times s$

= $mg \times h$

W = mgh

The work done to raise the object through a height h is stored as energy. This energy is called potential energy.

If the surface of earth h = 0, $W = mg \times 0 = 0$

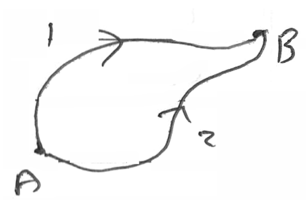
$\therefore$ Gravitational potential energy at the surface of earth is zero

$\boxed{W = PE = mgh}$

Conservative Force

- A force is conservative, if the work done by the force in displacing a particle from one point to another is independent of the path followed by the particle and depends only on the end point.

  Suppose a particle moves from point A to point B along either path 1 or 2. If a conservative force F acts on the particle, then the work done on the particle is same along two points

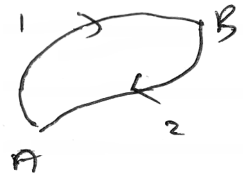


Mathematically W_{AB} (along path 1) = W_{AB} (along path 2)

- Also, a force is said to be conservative, work done by the force to move a particle along a closed path is zero

  Now suppose the particle moves in a round trip, from point A to point B along path 1 and then back to point A along path 2. For a conservative force,

  Work done on the particle along the path 1 from A to B = – work done on the particle along the path 2 from B to A



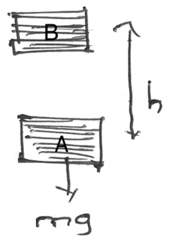
W_{AB} (along path 1) = $-W_{BA}$(along path 2)

$W_{closed\ path} = 0$

Conservative nature of gravitational force

Suppose a body of mass m is raised to a height h vertically upwards from position A to B. The work done against gravity $W = F \times s$

$= mg \times h$



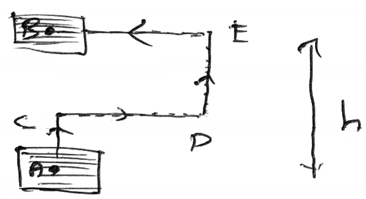
Now suppose the body is taken from position A to B along the path ACDEB as shown in figure. During horizontal path CD and EB, the force of gravity is perpendicular to the displacement, so work done is zero. Work is done along vertical paths AC and DE. The total work done is

$W = W_{AC} + W_{CD} + W_{DE} + W_{EB}$

$= mg \times AC + 0 + mg \times DE + 0$

= mg(AC + DE)   Ac + DE = h

W = mgh



Suppose a ball is thrown vertically upward. As it rises, the gravitational force does negative work on it, decreasing its kinetic energy. As the ball descends, the gravitational force does positive work on it, increasing its kinetic energy. The ball falls back to the point of projection with the same velocity and KE with which it was thrown up. The net work done by the gravitational force on the ball during the round trip is zero. This again shows that the gravitational force is a conservative force.

- Examples of conservative force : Gravitational force, electrostatic force, elastic force of a spring, magnetic force, centripetal force

Nonconservative force

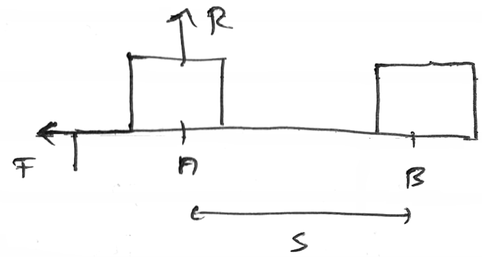
If the amount of work done in moving an object against a force from one point to another point depends on the path along which the body moves, then such a force is called a non-conservative force.

Also if a force is said to be non conservative force work done by the force to move a particle along a closed path or round trip is zero.

Examples of nonconservative force: Forces of friction and viscosity,

If a body is moved from position A to another position B on a rough table, work done against frictional force shall depends on the length of the path between A and B and not only the position A and B

$$W_{AB} = \mu mgs$$



Further if the body is brought back to its initial position A, work has to be done against the frictional force, which always opposes the motion. Hence the work done against the friction over a round trip is not zero.

$$W_{BA} = \mu mgs$$

$$W_{net} = W_{AB} + W_{BA} = \mu mgs + \mu mgs = 2\mu mgs$$

Therefore the friction is a nonconservative force

Relation between potential energy and conservative force

Potential energy comes into play when a work is done against conservative force

When we does work on a particle against conservative force, the change in potential energy of particle is equal to the negative of the work done by the conservative force

$$\Delta U = -w$$

But $W = \int F(x)dx$

$$\Delta U = -\int F(x)dx$$

On differentiating the above equal

$$\frac{d}{dx}(\Delta U) = -F(x)$$

$$F(x) = \frac{-d(\Delta U)}{dx}$$

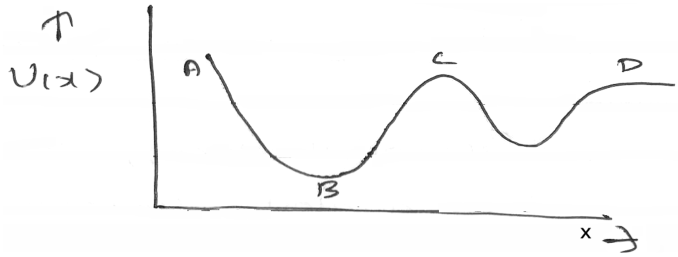
Hence we may define conservative force is equal to the negative gradient of potential energy.

Potential Energy curve

A graph plotted between the potential energy of a particle and its displacement from the centre of force is called potential energy curve. Figure shows a graph of potential energy function U(x) for one dimensional motion. The negative gradient of the potential energy gives force

$$F = \frac{-dv}{dx}$$

Nature of force



1) Attractive force

On increasing x, if U increases

$\frac{dU}{dx}$ = positive, then F is in negative direction

i.e. force is attractive in nature. In graph this is represented in region BC

2) Repulsive force

On increasing x, if U decreases $\frac{dU}{dx}$ = negative, then F is in positive direction.

i.e. Force is repulsive in nature. In graph, this is represented in region AB

3) Zero force

On increasing x, if U does not change $\frac{dU}{dx}$ = 0

then F is zero.

i.e. no force works on the particle. Point B,C and D represents the point of zero force or these points can be termed as position of equilibrium

Types of equilibrium

If the net force acting on a particle is zero, it is said to be in equilibrium

For equilibrium $\frac{dU}{dx}$ = 0, but the equilibrium of particle can be three types

1) Stable equilibrium

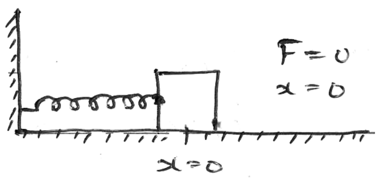
2) Unstable equilibrium

3) Neutral equilibrium

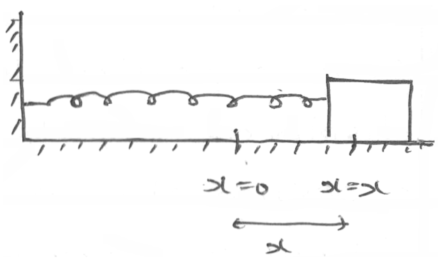
Stable	**Unstable**	**Neutral**
When a particle is displaced slightly from a position, then a foce acting on it brings it back to the initial position, it is said to be in stable equilibrium position	When a particle is displaced slightly from a position, then a force acting on it tries to displace the particle further away from the equilibrium position it is said to be in unstable equilibrium	When a particle is slightly displaced from a position then it does not experience any force acting on it and continues to be in equilibrium in the displaced position, it is said to be in neutral equilibrium
Potential energy is minimum	Potential energy is maximum	Potential energy is constant
$F = \frac{-dU}{dx} = 0$	$F = \frac{-dU}{dx} = 0$	$F = \frac{-dU}{dx} = 0$
$\frac{d^2U}{dx^2} = \text{positive}$	$\frac{d^2U}{dx^2} = \text{negative}$	$\frac{d^2U}{dx^2} = 0$
i.e., rate of change of $\frac{dU}{dx}$ is positive	i.e., rate of change of $\frac{dU}{dx}$ is negative	i.e., rate of change of $\frac{dU}{dx}$ is zero

<u>Potential energy of a spring</u>

Consider an elastic spring of negligible small mass with its one end attached to a rigid support. It's other end is attached to a block of mass m which can slide over a smooth horizontal surface. The position x = 0 is the equilibrium position as shown in figure.



When the spring is stretched or compressed by pulling or pushing the block a spring.



Force F_s begin to act to the spring towards the equilibrium position. According to Hooke's law, the spring force F_s is proportional to the displacement of the block from the equilibrium position

$$F_s \alpha\, x \quad \text{or} \quad F_s = -kx$$

The proportionality constant K is called spring constant. It's SI unit is Nm^{-1}. The negative sign shows that F_s acts in the opposite direction of x,

The work done by the spring force for the small extension dx is $dw_s = F_x dx = -kxdx$

If the block moved from initial displacement x_i to find displacement x_f, the work done by the spring force is

$$w_s = \int dw_s = -\int_{x_i}^{x_f} kx\, dx = -k\left[\frac{x^2}{2}\right]_{x_i}^{x_f}$$

$$W_s = \frac{1}{2}Kx_i^2 - \frac{1}{2}Kx_f^2$$

Properties of spring force

1) Spring force is position dependent (F_s = -Kx)

2) Work done by spring force depends on initial and final positions

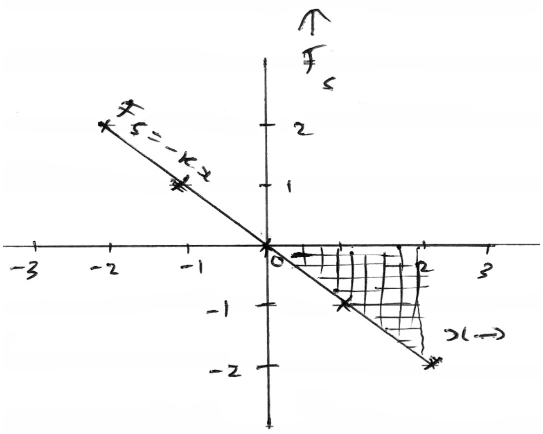
3) Work done by spring force in a cyclic process is zero

- The potential energy of a spring is zero when the black is in equilibrium position. The PE of spring for an extension x will be $U = \frac{1}{2}Kx^2$

PE of an elastic stretched spring by graphical method

According to Hooke's law, the spring force for an extension x_m is $F_s = -kx_m$

x_m	-2	-1	0	1	2
F_s	2	1	0	-1	-2



If k = 1, $F_s = -x_m$

The figure shows the plot of spring force F_s versus displacement x of a block attached to the free end of the spring. Let us consider the area of shaded region

$$= \frac{1}{2}bh = \frac{1}{2} \times F_s x_m$$

$$= \frac{1}{2} \times (-kx_m)\, x_m$$

$$= -\frac{1}{2}kx_m^2$$

$\frac{1}{2}kx^2$ is the work done by the external source

$\therefore$ Area of force displacement graph gives work done by the spring force

Properties of conservative forces

1) A force F is conservative, if it can be defined from the potential energy function by the relation

$$F(x) = \frac{-dU(x)}{dx}$$

2) Work done by a conservative force on an object is path independent and depends only in the end points.

$$W = -\Delta U$$

$$W = -(U_f - U_i) = U_i - U_f$$

3) The work done by the conservative force is zero if the object moving around any closed path returns to its initial position

$$W = \oint F\,dx = 0$$

4) If only conservative forces are acting on body, then its total mechanical energy is conserved

Mechanical energy (ME)

The energy produced by mechanical means is called mechanical energy. It has two forms

1) Kinetic energy (KE)

2) Potential energy (PE)

ME = KE + PE

ME is a scalar quantity

It depends on frame of reference

A body can have ME without having either KE or PE. However, if both PE and KE are zero, ME will be zero KE is always positive, but PE can be positive, and negative. Then negative mechanical energy implies that PE is negative in magnitude and it is more than KE, such a state is called bound state

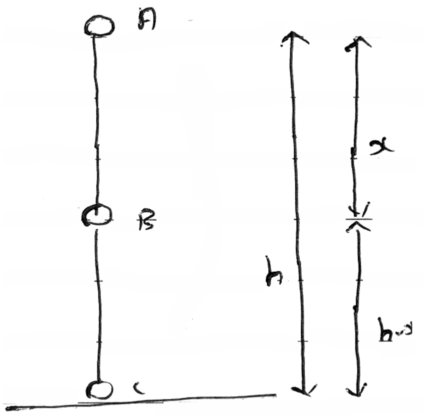
Principle of conservation of mechanical energy

This principle states that if only the conservative force are doing work on a body, then its total mechanical energy is constant

KE + PE = constant

Conservation of mechanical energy in a freely falling body

Consider a body of mass m lying at position A at a height h above the ground. As the body falls, its KE increases at the expense of potential energy. Consider three points A,B and C in the path of freely falling body.



At point A

Body is at rest u = 0

$$KE = \frac{1}{2} m \times 0^2 = 0$$

PE = mgh

TE = KE + PE = 0 + mgh = mgh

At point B

Suppose the body falls freely through height x and reaches the point B with velocity v

$$v^2 = U^2 + 2as$$

$$v^2 = 0^2 + 2 \times g \times x$$

$$v^2 = 2gx$$

$$KE = \frac{1}{2} mv^2 = \frac{1}{2} m \times 2gx = mgx$$

$$TE = mgx + mgh - mgx = mgh$$

At point C

Suppose the body finally reaches a point C on the ground with velocity v. Then considering motion from A to C

$$v^2 = U^2 + 2as$$

$$v^2 = 0^2 + 2 \times g \times h$$

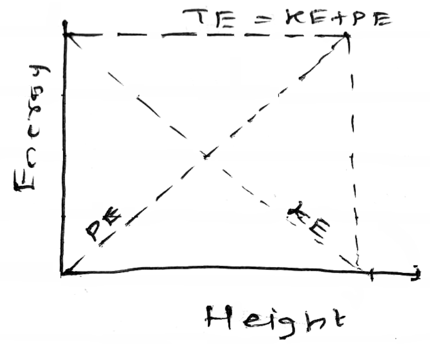
$$v^2 = 2gh$$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m \times 2gh = mgh$$

PE = 0

TE = mgh + 0 = mgh

Clearly, as the body falls, PE decreases and KE increases by an equal amount. However total mechanical energy remains constant at all point. Thus mechanical energy is conserved during the free fall of a body. Figure shows the variation of KE and PE and the constancy of total energy with height.

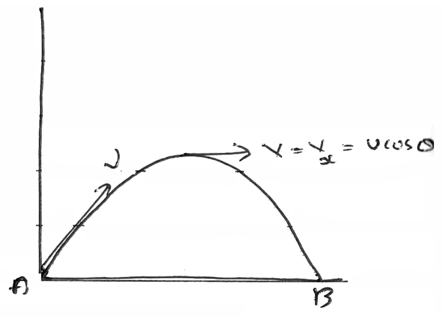


Conservation of mechanical energy in projectile motion

1) PE = mgH

$$H = \frac{u^2 \sin^2 \theta}{2g}$$

$$PE = mg \times \frac{u^2 \sin^2 \theta}{2g} = \frac{1}{2}mu^2 \sin^2 \theta$$



At the highest point, velocity of projectile is along horizontal $v = u\cos\theta$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m \times u^2\cos^2\theta$$

TE = KE + PE = $\frac{1}{2}mu^2\cos^2\theta + \frac{1}{2}mu^2\sin^2\theta = \frac{1}{2}mu^2$

2) **<u>At point of projection</u>**

If the initial velocity of projectile is u

$$KE = \frac{1}{2}mu^2$$

PE = 0

$$TE = \frac{1}{2}mu^2$$

3) **<u>At point B</u>**

Since the landing velocity of projectile of point B is u

$$KE = \frac{1}{2}mu^2$$

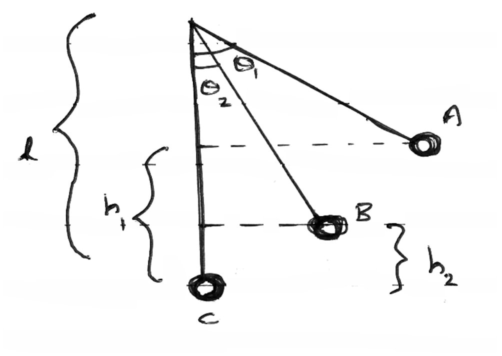
PE = 0

$$TE = \frac{1}{2}mu^2$$

Thus total energy at any point in the path of projectile is constant

<u>Conservation of mechanical energy in oscillating pendulum</u>

Consider an oscillating pendulum A be the extreme position and C be the mean position.



$h_1 = l - l\cos\theta_1$

$h_2 = l - l\cos\theta_2$

$TE_A = KE + PE$

$= 0 + mgh_1$

$TE_C = KE + PE$

$= \frac{1}{2}mv_c^2 + mg \times 0$

$\frac{1}{2}mv_c^2$

$TE_A = TE_C$ $\Rightarrow mgh_1 = \frac{1}{2}mv_c^2$

$\sqrt{2gh_1} = v_c$

$h_1 = \ell - \ell\cos\theta_1$

$h_1 = \ell(1 - \cos\theta_1)$

$v_c = \sqrt{2g\ell(1-\cos\theta_1)}$

$TE_B = \frac{1}{2}mv_B^2 + mgh_2$

$TE_B = (TE)_A$

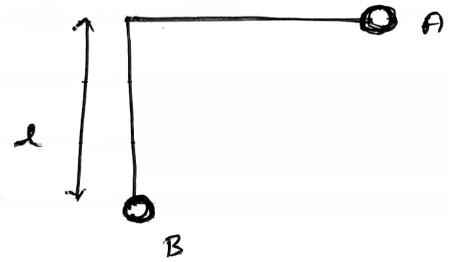
$\frac{1}{2}mv_B^2 + mgh_2 = mgh_1$

$\frac{1}{2}mv_B^2 = mg(h_1 - h_2)$

$v_B = \sqrt{2g(h_1 - h_2)} = \sqrt{2g(\ell - \ell\cos\theta_1 - \ell + \ell\cos\theta_2)}$

$v_B = \sqrt{2g\ell(\cos\theta_2 - \cos\theta_1)}$

- If a pendulum of length ℓ is held horizontal as shown in figure and released



TE_A = KE + PE

= 0 + mg$l$ =mg$l$

TE_B = KE + PE

= $\frac{1}{2}mv^2 + mg \times 0$

= $\frac{1}{2}mv^2$

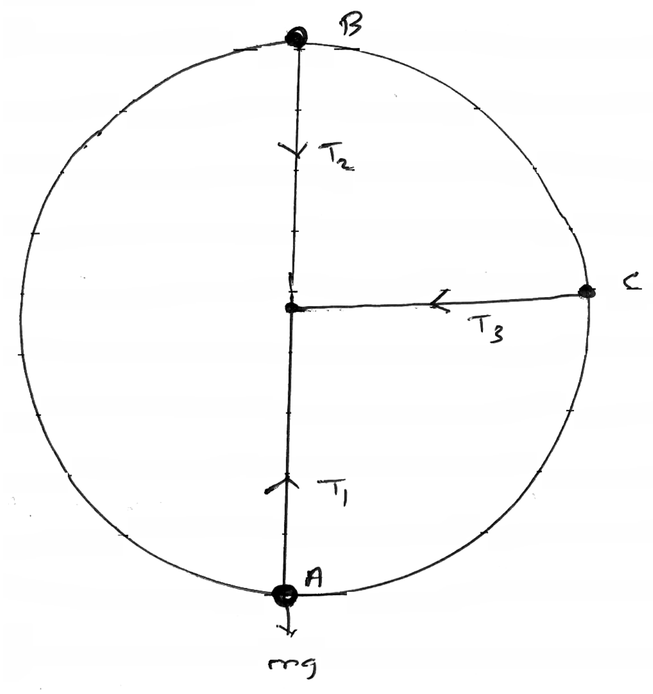
$TE_A = TE_B$

$mg\ell = \frac{1}{2}mv^2$

$\boxed{v = \sqrt{2g\ell}}$

Motion of a particle in a vertical circle

Consider a body tied to a string and whirled in circular path. Consider three points A,B and C on the circle



At point A

$$T_1 - mg = \frac{mv^2}{r}$$

Where m is the mass of body, v_1 is the velocity at bottom, r is the radius and T_1 is the tension at A

At point B

$$mg + T_2 = \frac{mv_2^2}{r}$$

v_2 is the velocity at point B

$$T_1 = \frac{mv_1^2}{r} + mg \text{ .... (1)}$$

$$T_2 = \frac{mv_2^2}{r} - mg \text{ ....(2)}$$

$$T_1 - T_2 = \frac{mv_1^2}{r} + mg - \frac{mv_2^2}{r} + mg$$

$$2mg + \frac{m}{r}\left(v_1^2 - v_2^2\right) \text{ ....(3)}$$

$$TE_A = \frac{1}{2}mv_1^2$$

$$TE_B = \frac{1}{2}mv_2^2 + mg \times 2r$$

TE = constant

$TE_A = TE_B$

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2 + 2mgr$$

$$\frac{1}{2}\left(v_1^2 - v_2^2\right) = 2gr$$

$$v_1^2 - v_2^2 = 4gr \qquad \text{..................(4)}$$

On substituting in (3)

$$T_1 - T_2 = 2mg + \frac{m}{r}\left(4gr\right)$$

$T_1 - T_2 = 6mg$

Let's assume $T_2 = 0$

Then $\boxed{T_1 = 6mg}$

From (2)

$$0 = \frac{mv_2^2}{r} - mg$$

$$mg = \frac{mv_2^2}{r}$$

$$\boxed{v_2 = \sqrt{rg}}$$

Subsiting in (4)

$$v_1^2 - rg = 4gr$$

$$\boxed{v_1 = \sqrt{5gr}}$$

$$TE_C = \frac{1}{2}mv_3^2 + mgr$$

$$TE_C = TE_A$$

$$\frac{1}{2}mv_3^2 + mgr = \frac{1}{2}mv_1^2$$

$$\frac{1}{2}mv_3^2 + mgr = \frac{1}{2}m \times 5gr$$

$$v_3^2 = 3gr$$

$$\boxed{v_3 = \sqrt{3gr}}$$

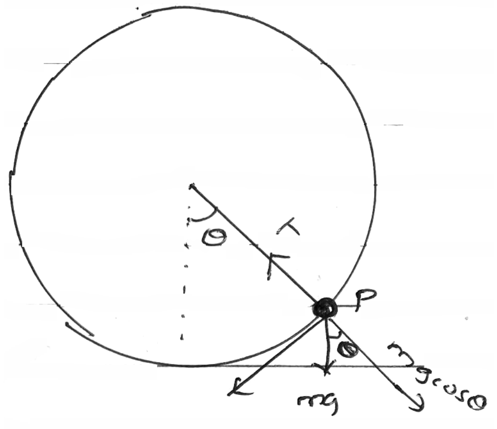
At point C

$$T_3 = \frac{mv_3^2}{r}$$

$$T_3 = m \times \frac{3gr}{r} = 3mg$$

Where T_3 is the tension at point C

- Consider another position of body in its vertical circular path



TE at point P

$$T - mg\cos\theta = \frac{mv^2}{r}$$

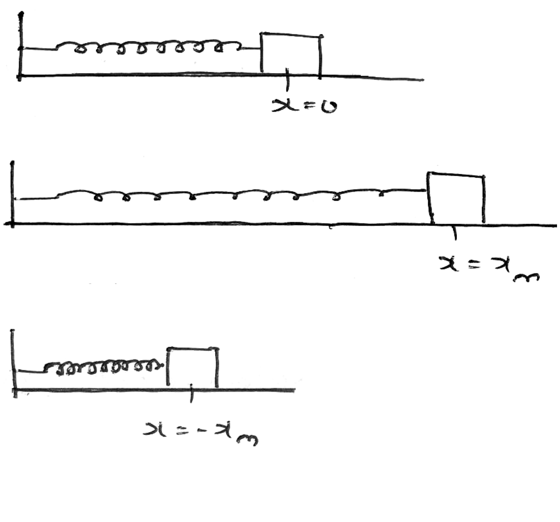
Where v is the velocity of particle point P

$$T = \frac{mv^2}{r} + mg\cos\theta$$

Conservation of energy in an elastic spring

When a spring stretch to a distance x_m, its PE is $\frac{1}{2}kx_m^2$. When it is released, it begins to move under the spring force till it reaches the equilibrium position x = 0, where it has maximum velocity. All the PE is converted into KE. Due to inertia of motion, the body overshoots the x = 0 position. It's velocity decreases until it momentarily stops at position x = –x_m, where all the KE is converted into PE. The spring force again pulls the body towards the position x = 0. Thus the body keeps on oscillating. Total mechanical energy remain constant.

At extreme positions



$x = \pm x_m$, velocity is zero

$$KE = \frac{1}{2}m \times 0^2 = 0$$

$$PE = \frac{1}{2}kx_m^2$$

At equilibrium position

x = 0 $$PE = \frac{1}{2}k_2 0^2 = 0$$

$$KE = \frac{1}{2}mv_m^2$$ v_m = maximum value of velocity

$$TE = \frac{1}{2}mv_m^2$$

To find Maximum speed v_m

$$\frac{1}{2}kx_m^2 = \frac{1}{2}mv_m^2$$

$$\boxed{v_m = \sqrt{\frac{k}{m}}x_m}$$

At any intermediate Position x

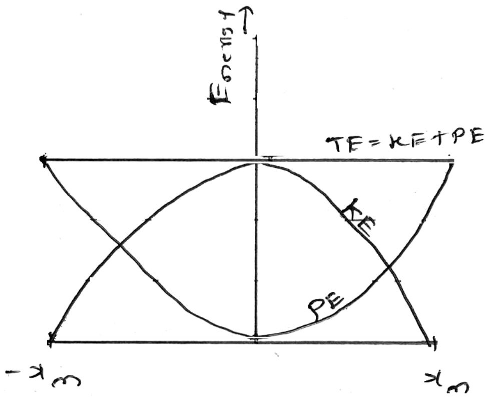
TE at extreme position = TE of that point

$$\frac{1}{2}kx_m^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$mv^2 = k\left(x_m^2 - x^2\right)$$

$$v = \sqrt{\frac{k}{m}\left(x_m^2 - x^2\right)}$$

- The variations of KE, PE and total energy with displacement x are shown in figure

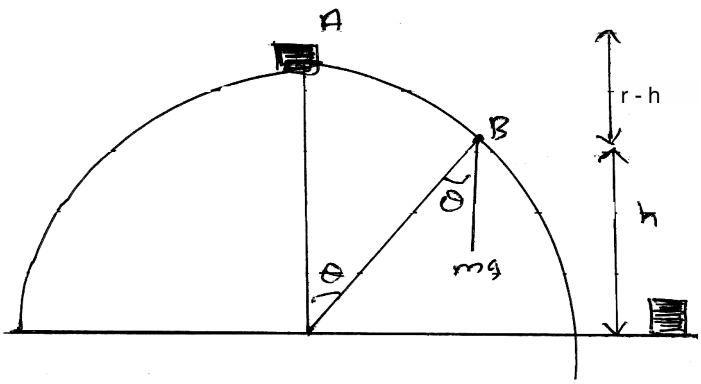


As both KE & PE depend on x^2, their graphs are parabolic. Total total = KE +PE remain constant, so its graph is a straight line parallel to displacement axis

Motion of a block on frictionless hemisphere

A small block of mass m slides down from the top of a frictionless hemisphere of radius r. At a certain point, the block lose contact with the surface of the sphere. We are going to find the height and angle at which the block loses its contact with the spherical surface.

Let us assume that at point B, the block loses its contact. Let v be the velocity of block at B.



At point B

$$mg\cos\theta = \frac{mv^2}{r} \text{ .... (1)}$$

$TE_A = TE_B$

$$mgr = \frac{1}{2}mv^2 + mgh$$

$$mgr - mgh = \frac{1}{2}mv^2$$

$2mg(r-h)=mv^2$ ....(2)

Substitute (2) in (1)

$mg\cos\theta=\dfrac{2mg(r-h)}{r}$ $\cos\theta=\dfrac{h}{r}$ ....(3)

$mg\times\dfrac{h}{r}=\dfrac{2mg(r-h)}{r}$

h = 2r – 2h

3h = 2r $h=\dfrac{2r}{3}$ ....(4)

Block loses contact at the height of $\dfrac{2}{3}r$ from the ground

From (4) $\dfrac{h}{r}=\dfrac{2}{3}$

Substitute in (3)

$\cos\theta=\dfrac{2}{3}$ $\theta=\cos^{-1}\left(\dfrac{2}{3}\right)$

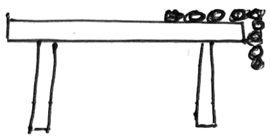
The angle at which block loss contact is $\cos^{-1}\left(\dfrac{2}{3}\right)$

Work done in pulling the chain against gravity

Case-1

Work done in pulling the chain with half of its length hanging over the edge

Consider a chain of mass m and length L held on a frictionless table with half of its length hanging over the edge



W = change in PE

W = $U_i - U_f$

= $\dfrac{M}{2}g\dfrac{L}{4}-Mg\times 0$

Note that surface of table as a reference level (zero point energy)

Then $W = \frac{mgL}{8}$

<u>Case-2</u>

Work done in pulling the chain on table with $\left(\frac{1}{n}\right)^{th}$ of its length hanging over the edge

Again reference level is the surface of table

W = $U_i - U_f$

$$W = \frac{M}{n} g \frac{L}{2n} - Mg \times 0$$

$$W = \frac{mgL}{2n^2}$$

- Velocity of chain while leaving the table

TE of chain when $\left(\frac{1}{n}\right)^{th}$ length hanging from the edge = KE + PE

= $0 + \left(\frac{-MgL}{2n^2}\right)$

TE of chain when it leaves the table

= $\frac{1}{2}Mv^2 + \left(-Mg\frac{L}{2}\right)$

TE = constant

$$-\frac{MgL}{2n^2} = \frac{1}{2}Mv^2 + \left(-Mg\frac{L}{2}\right)$$

$$\frac{-MgL}{2}\left(\frac{1}{n^2} - 1\right) = \frac{1}{2}mv^2$$

$$gL\left(1 - \frac{1}{n^2}\right) = v^2$$

$$\boxed{v = \sqrt{gL\left(1 - \frac{1}{n^2}\right)}}$$

<u>Power</u>

- Power is defined as the rate of doing work.

If an agent does work W in time t, then its average power is given by

$$P_{av} = \frac{W}{t}$$

Shorter is the time taken by a person or a machine in performing a particular task, larger is the power of that person or machine.

- Power is a scalar quantity. It is the dot product of two vectors
- Dimension of power

$$[P] = \frac{[W]}{[t]} = \frac{[ML^2T^{-2}]}{[T]} = [ML^2T^{-3}]$$

- Units of power

  SI unit is watt (W)

  1 watt = $\frac{1\text{ joule}}{1\text{ second}}$ $\quad 1\text{ W} = 1\text{ Js}^{-1}$

- Bigger units of power are kilowatt (kW) and horse power (hP)

  1 kilowatt = 1000 watt or 1kW = 10^3 W

  1 horse power = 746 watt or 1 hP = 746 W

Instantaneous power

The power of an agent may not be constant during a time interval. The instantaneous power is defined as the limiting value of the average power as the time interval approaches zero. If ΔW work is done in a small time interval Δt, then the instantaneous power is given by

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

Power as dot product

Instantaneous power, $P = \frac{dw}{dt}$

$$dw = \vec{F}.\overrightarrow{ds}$$

$$P = \vec{F}.\frac{\overrightarrow{ds}}{dt}$$

$$P = \vec{F}.\vec{v}$$

Power of an agent at any instant is equal to the dot product of its force and velocity vectors at that instant.

Kilowatt hour (KWh)

It is the commercial unit of electrical energy. One kilowatt hour is the electrical energy consumed by an appliance at 1000 watt in one hour

Relation between KWh and Joule

1 KWh = 1000 W $\times$ 1h

= 1000 Js^{-1} × 3600 s

1 KWh = 3.6 $\times$ 10^6J

Position and velocity of an Automobile w.v. to time

An automobile of mass m accelerates, starting from rest, while the engine supplies constant power P, its position and velocity changes w.v. to time

P = Fv

P = (ma) v

$$= m\times\frac{dv}{dt}v$$

$$\frac{P}{m}dt = vdv$$

On integrating

$$\frac{P}{m}t + c_1 = \frac{v^2}{2}$$

At t = 0, v = 0 then c_1 = 0. Then

$$\frac{P}{m}t = \frac{v^2}{2}$$

$$\boxed{v = \sqrt{\frac{2Pt}{m}}}$$

$$v = \frac{ds}{dt} \qquad \frac{ds}{dt} = \left(\frac{2Pt}{m}\right)^{1/2}$$

$$ds = \left(\frac{2Pt}{m}\right)^{1/2} dt$$

$$s = \left(\frac{2P}{m}\right)^{1/2} \int t^{1/2} dt$$

$$s=\left(\frac{2P}{m}\right)^{1/2}\frac{t^{3/2}}{3/2}+c_2$$

$$s=\left(\frac{2P}{m}\right)^{1/2}\frac{2}{3}t^{3/2}+c_2$$

At t = 0, S = 0, C_2 = 0

S = $\left(\frac{8P}{9m}\right)^{1/2} t^{3/2}$

Then $S \alpha t^{3/2}$

- When a machine gun fires in bullets per second, each with velocity v, then power machine gun P = $\frac{\text{Energy}}{\text{Time}}=\frac{n(KE)}{\text{time}}$; P = $n\left(\frac{1}{2}mv^2\right)$
- Force required to hold the machine gun in position F = $\frac{dP}{dt}=\frac{P}{t}=\frac{nP}{1}=n\sqrt{2m(KE)}$
- When a pump delivers mass M of water per second over a height h with a velocity v, then power of the pump P = Mgh + $\frac{1}{2}Mv^2$
- Power of heart P = pressure $\times$ volume of blood pumped per second.
- In blowing a whistle, power of lungs P = $\frac{1}{2}\times \text{mass of air blown / sec} \times (\text{velocity})^2$
- Power dissipated by centripetal force is zero.
- When water is flowing through a pipe with a speed V, then its power is proportional to v^3
- Power of an engine driving a vehicle of mass m with a speed V on a horizontal road is P = μFV, P = μmgV, where μ is the coefficient of friction between the road and the tyres.
- When a vehicle is driven with constant acceleration a, against a constant frictional force F on a level road, then power of engine at time is $P=(ma+F)V$ where V is the velocity of the body at the constant t.
- In moving over a smooth incline of inclination θ, Power P = (mg sinθ)V
- In moving over a rough incline of inclination θ,

  Power P = $(mg\sin\theta+F)V$ = $(mg\sin\theta+\mu mg\cos\theta)V$

  Where F is the friction. μ is the coefficient of friction.

Collision

A collistion is said to occur between two bodies, either it they physically collide against each other or if the path of one is affected by the force exerted by the other. For a collision to take place, actual physical contact is not necessary.

Collision between particles are of different types.

Elastic collision	Inelastic collision
If there is no loss of kinetic energy during a collision, it is called an elastic collision	If there is a loss of KE during collision, it is called inelastic collision
The momentum is conserved	The momentum is conserved
Total energy is conserved	The total energy is conserved
The KE is conserved	The KE is not conserved
Forces involved during the collision are conservative	Some or all the forces involved are non-conservative
The mechanical energy is not converted into heat, light, sound etc	A part of mechanical energy is converted into heat, light, sound etc
Eg: Collision between sub-atomic particles, collision between gas molecules	Eg: Collision between two vehicles, collision between ball and floor.

Perfectly inelastic collision

If two bodies stick together after the collision and move as a single body with a common velocity, then the collision is said to be perfectly inelastic collision. In such collision, loss of KE is maximum. Eg: A man jumping into a moving trolley. A bullet fired into a wooden block and remaining embedded in it.

Super elastic or explosibe collision

In such a collision, there is an increase in KE. This occurs it there is a release of potential energy on an impact. Eg : Bursting of a cracker when it hits the floor forcefully.

Head on collision

It is the collision in which the colliding bodies move along the same straight line path before and after the collision.

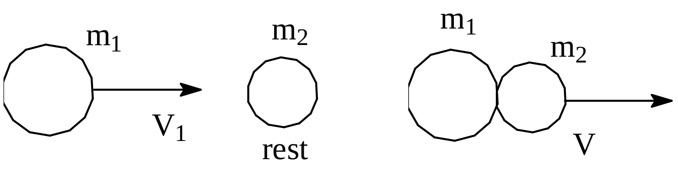
Oblique collision or two dimensional collision

After the collision, the colliding bodies do not move along the same straight line path that followed is called oblique collision

Perfectly inelastic collision in one dimension

Case-1 :

Consider a body of mass m moving with velocity v_1 collides head on with another body of mass m_2 at rest. After collision, the two bodies move together with a common velocity v.



Before collision After collision

As the linear momentum is conserved $m_1v_1 + m_2 \times 0 = (m_1 + m_2)V$; $\boxed{V = \dfrac{m_1v_1}{m_1 + m_2}}$

- Loss in KE = $K_i - K_f$ = $\dfrac{1}{2}m_1v_1^2 - \dfrac{1}{2}(m_1 + m_2)v^2$

$$= \frac{1}{2}m_1v_1^2 - \frac{1}{2}(m_1 + m_2)\frac{m_1^2v_1^2}{(m_1 + m_2)^2} = \frac{1}{2}m_1v_1^2 - \frac{1}{2}\frac{m_1^2v_1^2}{(m_1 + m_2)}$$

$$\Delta K = \frac{1}{2}m_1v_1^2\left(1 - \frac{m_1}{m_1 + m_2}\right) = \frac{1}{2}\frac{m_1m_2v_1^2}{(m_1 + m_2)}$$

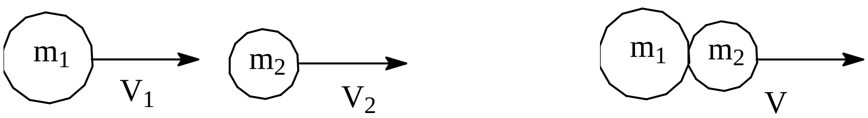
$$\boxed{\Delta K = \frac{1}{2}\frac{m_1m_2}{m_1 + m_2}v_1^2}$$

- $$\frac{K_f}{K_i} = \frac{\frac{1}{2}(m_1 + m_2)\frac{m_1^2v_1^2}{(m_1 + m_2)^2}}{\frac{1}{2}m_1v_1^2} = \frac{m_1}{m_1 + m_2}$$

$$\boxed{\frac{K_f}{K_i} = \frac{m_1}{m_1 + m_2}}$$

<u>Case-2</u>

When two colliding bodies are moving in the same direction.



By the law of conservation of linear momentum $m_1v_1 + m_2v_2 = (m_1 + m_2)V$

$$\boxed{V = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}}$$

- Loss in $KE = \Delta K = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 - \frac{1}{2}(m_1 + m_2) v^2$

$$= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 - \frac{1}{2}(m_1 + m_2) \times \frac{(m_1 v_1 + m_2 v_2)^2}{(m_1 + m_2)^2}$$

$$\Delta K = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 - \frac{1}{2} \frac{(m_1 v_1 + m_2 v_2)^2}{(m_1 + m_2)}$$

$$= \frac{\frac{1}{2} m_1^2 v_1^2 + \frac{1}{2} m_1 m_2 v_1^2 + \frac{1}{2} m_2 m_1 v_2^2 + \frac{1}{2} m_2^2 v_2^2 - \frac{1}{2}(m_1 v_1 + m_2 v_2)^2}{m_1 + m_2}$$

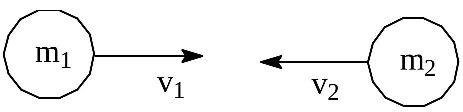
$$= \frac{\frac{1}{2} m_1^2 v_1^2 + \frac{1}{2} m_1 m_2 (v_1^2 + v_2^2) + \frac{1}{2} m_2^2 v_2^2 - \frac{1}{2} m_1^2 v_1^2 - \frac{1}{2} m_2^2 v_2^2 - m_1 m_2 v_2 v_2}{(m_1 + m_2)}$$

$$\Delta K = \frac{\frac{1}{2} m_1 m_2 (v_1^2 + v_2^2 - 2 v_1 v_2)}{(m_1 + m_2)}$$

$$\Delta K = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (v_1 - v_2)^2$$

Case-2

When the colliding bodies are moving in the opposite direction



$v_1 > v_2$

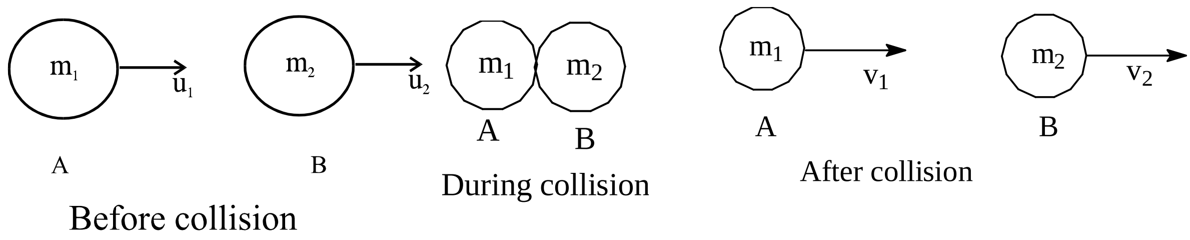
By low of conservation of momentum $m_1 v_1 - m_2 v_2 = (m_1 + m_2) v$

$$\boxed{v = \frac{m_1 v_1 - m_2 v_2}{m_1 + m_2}}$$

$$\Delta K = \text{loss in KE} = \frac{1}{2}\frac{m_1 m_2}{m_1+m_2}(v_1+v_2)^2$$

Elastic collision in one dimension

Consider two perfectly elastic bodies A and B of masses m_1 and m_2 moving along the same straight line with velocities u_1 and u_2 respectively. Let $u_1 > u_2$. After some time, two bodies collide head on and continue moving in the same direction with velocities v_1 and v_2 respectively. If two bodies will separate after the collision if $v_2 > v_1$



As linear momentum is conserved $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \text{.............(1)}$$

Since KE is also conserved $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$

$$m_1(u_1 - v_1)(u_1 + v_1) = m_2(v_2 - u_2)(v_2 + u_2) \text{.............(2)}$$

$$\frac{2}{1} \Rightarrow u_1 + v_1 = v_2 + u_2;\ v_2 = u_1 + v_1 - u_2 \text{ ....(3)}$$

Substitute (3) in (1)

$$m_1u_1 - m_1v_1 = m_2u_1 + m_2v_1 - m_2u_2 - m_2u_2$$

$$u_1(m_1 - m_2) = v_1(m_1 + m_2) - 2m_2u_2$$

$$(m_1 - m_2)u_1 + 2m_2u_2 = (m_1 + m_2)v_1$$

$$v_1 = \frac{(m_1 - m_2)}{m_1 + m_2}u_1 + \frac{2m_2u_2}{m_1 + m_2}$$

$$\boxed{v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2}$$

Interchanging the subscripts 1 and 2 in the above equation, we get

$$\boxed{v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \left(\frac{2m_1}{m_1 + m_2}\right)u_1}$$

<u>Special cases</u>

1. When two bodies of equal masses collide $m_1 = m_2 = m$

   $v_1 = u_2$ and $v_2 = u_1$

   After collision, two bodies exchange their velocities if they have same masses.

2. When a body collides against a stationary body of equal mass $m_1 = m_2 = m$ Let $u_2 = 0$

   Then $v_1 = 0,\ v_2 = u_1$

3. When a light body collides against a massive stationary body

   $m_1 << m_2$ and $u_2 = 0$

   $v_1 = \frac{-m_2}{m_2}u_1 = -u_1\ v_1 = -u_1;\ v_2 = 0$

   A light ball on striking a wall rebounds almost with the same speed and the wall remain at rest.

4. When a massive body collides against a light stationary body

   $m_1 >> m_2$; $u_2 = 0$

   $v_1 = u_1$ and $v_2 = 2u_1$

   The velocity at massive body remains almost unchanged while the light body starts moving with twice the velocity of the massive body.

Decrease in KE of projectile

$$AK = \frac{1}{2}m_1u_1^2 - \frac{1}{2}m_1v_1^2 = \frac{1}{2}m_1\left(u_1^2 - v_1^2\right)$$

Fractional decrease in KE

$$\frac{\Delta K}{K} = \frac{\frac{1}{2}m_1\left(u_1^2 - v_1^2\right)}{\frac{1}{2}m_1u_1^2} = \frac{u_1^2 - v_1^2}{u_1^2} = 1 - \frac{v_1^2}{u_1^2}$$

If the target is at rest

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2}$$

Put $u_2 = 0$

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1$$

$$\frac{\Delta K}{K} = 1 - \left(\frac{v_1}{u_1}\right)^2 = 1 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2$$

$$\frac{\Delta K}{K} = \frac{m_1^2 + m_2^2 + 2m_1m_2 - \left(m_1^2 + m_2^2 - 2m_1m_2\right)}{\left(m_1 + m_2\right)^2}$$

$$\frac{\Delta K}{K} = \frac{4m_1m_2}{\left(m_1 + m_2\right)^2}\ ;\ \frac{\Delta K}{K} = \frac{4m_1m_2}{\left(m_1 - m_2\right)^2 + 4m_1m_2}$$

If $m_1 = m_2$ $\quad \frac{\Delta K}{K} = 1$

Then transfer of KE will be maximum

Velocity, momentum and KE of stationary target after head on collision

1) Velocity of target

$$V_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \left(\frac{2m_1}{m_1 + m_2}\right)u_1$$

Put $u_2 = 0$

$$v_2 = \frac{2m_1u_1}{m_1 + m_2}$$

$$v_2 = \frac{2u_1}{1 + \frac{m_2}{m_1}}\ ;\ \text{Let } \frac{m_2}{m_1} = n$$

$$\boxed{v_2 = \frac{2u_1}{1 + n}}$$

2) Momentum of target

$$P_2 = m_2v_2$$

$$m_2 = nm_1\ ;\ P_2 = \frac{m_2 \times 2u_1}{1 + n} = \frac{2m_2u_1}{1 + n}\ ;\ P = \frac{2nm_1u_1}{1 + n}$$

$$P_2 = \frac{2m_1u_1}{\frac{1}{n} + 1}$$

3) KE of target

$$KE_2 = \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_2 \times \frac{4u_1^2}{(1+n)^2} = \frac{1}{2}nm_1\frac{4u_1^2}{(1+n)^2}$$

$$\frac{1}{2}m_1u_1^2 = \text{KE of projectle} = K_1$$

$$KE_2 = \frac{4nK_1}{(1+n)^2};\ KE_2 = \frac{4nK_1}{(1-n)^2+4n}$$

Velocity	$V_2 = \frac{2u_1}{1+n}$	For V_2 to be maximum n must be minimum $n = \frac{m_2}{m_1} \to 0\ \therefore m_2 << m_1$	Target should be very light
Momentum	$P_2 = \frac{2m_1u_1}{1+\frac{1}{n}}$	For P_2 to be maximum $\frac{1}{n}$ must be minimum or n must be maximum $n = \frac{m_2}{m_1} \to \infty \therefore m_2 >> m_1$	Target should be very massive
KE	$K_2 = \frac{4K_1n}{(1-n)^2+4n}$	For K_2 to be maximum $(1-n)^2$ must be minimum $1-n=0, \Rightarrow n=1,$ $n = \frac{m_2}{m_1} = 1\ \therefore m_2 = m_1$	Target and projectile should be of equal mass

Coefficient of restitution (e)

Coefficient of restitution gives a measure of the degree of elasticity of a collision. It is defined at the ratio of magnitude of relative velocity of separation after collision to the magnitude of relative velocity of approach before collision

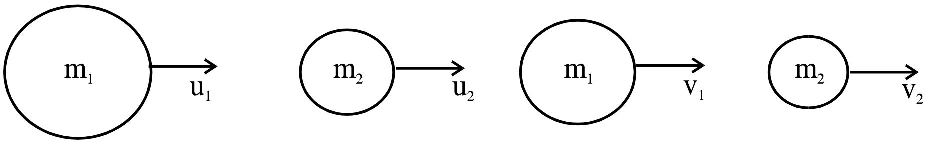
It is given by $e = \frac{|v_1 - v_2|}{|u_1 - u_2|}$

The coefficient of restitution can be used to distinguish between the different types of collisions as follows

* For a perfectly elastic collision e=1, ie relative velocity of separation is equal to the relative velocity of approach
* For on inelastic collision, $0 < e < 1$. The relative velocity of separation is less than relative velocity of approach
* For a perfectly inelastic collision, e =0. Relative velocity of separation is zero
* For super elastic collision $e > 1$ KE increases

Collision	KE	Coefficient of restitution	main domain
Elastic	Conserved	e=1	between atomic particle
Inelastic	Not conserved	0<e<1	between ordinary objects
Perfectly inelastic	Maximum loss of KE	e=0	During shooting
Super elastic	KE Increases	e>1	In explosions

Inelastic head on collision



$$e = \frac{\text{Relative velocity of separates}}{\text{Relative velocity of approach}} = \frac{v_2 - v_1}{u_1 - u_2}$$

$$v_2 - v_1 = e(u_1 - u_2)$$

$$v_2 = v_1 = e(u_1 - u_2) + v_1$$

By the law of conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Substitute the value of V_2

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2(eu_1 - eu_2 + v_1)$$

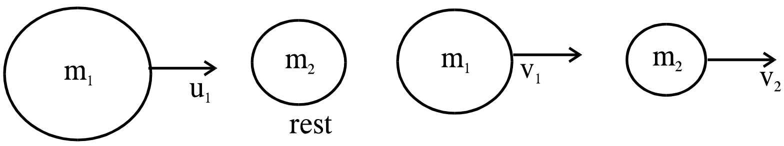
$$u_1(m_1 - m_2e) + u_2(m_2 + m_2e) = v_1(m_1 + m_2)$$

$$\boxed{v_1 = \frac{(m_1 - m_2e)}{m_1 + m_2}u_1 + \frac{m_2(1+e)}{m_1 + m_2}u_2}$$

Interchanging the subscripts 1 and 2 in the above equation, we get v_2

$$\boxed{v_2 = \frac{(m_2 - m_1e)}{m_1 + m_2}u_2 + \frac{m_1(1+e)}{m_1 + m_2}u_1}$$

Inelastic collision when one particle is stationary



$$e = \frac{v_2 - v_1}{u_1}$$

$$eu_1 = v_2 - v_1........(1)$$

$$v_2 = eu_1 + v_1$$

By conservation of mumentum

$$m_1u_1 = m_1v_1 + m_2v_2......(2)$$

$$m_1u_1 = m_1v_1 + m_2eu_1 + m_2v_1$$

$$u_1(m_1 - m_2e) = v_1(m_1 + m_2); \quad v_1 = \frac{(m_1 - m_2e)u_1}{m_1 + m_2}$$

From (1) $v_1 = v_2 - eu_1......(3)$

Substitute (3) in (2)

$$m_1u_1 = m_1v_2 - m_1eu_1 + m_2v_2\ ;\ u_1m_1(1+e) = v_2(m_1+m_2)$$

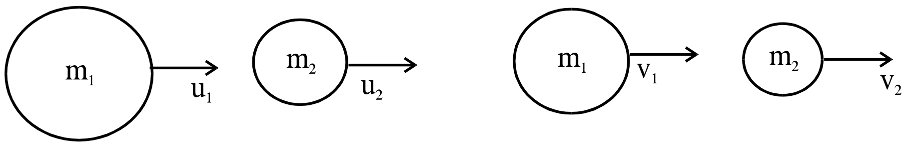
$$\boxed{v_2 = \frac{m_1(1+e)u_1}{m_1+m_2}}$$

$$\frac{v_1}{v_2} = \frac{(m_1 - m_2e)}{m_1(1+e)};\ \text{if } m_1 = m_2$$

$$\boxed{\frac{v_1}{v_2} = \frac{1-e}{1+e}}$$

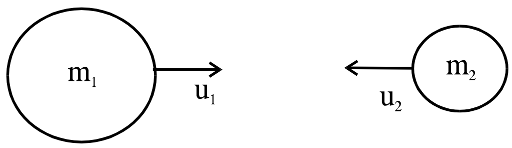
Coefficient of restitution in various situations

(1) Colliding bodies moving in same direction before and after the collision

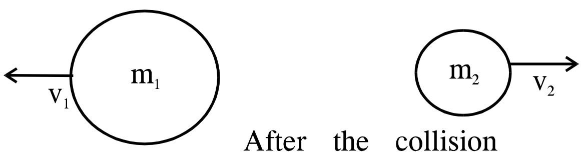


$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

(2) Colliding bodies moving in opposite direction before and after the collision



Before collision

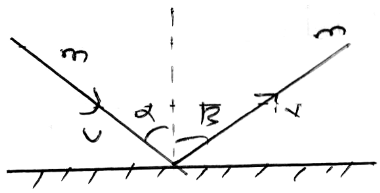


After the collision

$$e = \frac{v_1 + v_2}{u_1 + u_2}$$

(3) Oblique impact on a fixed plane

For a body of mass m moving with velocity u making an angle α with normal to a fixed horizontal floor. After collision, the body is deflected with a velocity V, making an angle β with normal. Impact take place along normal $e = \dfrac{\text{velocity of separation}}{\text{velocity of approach}}$



$$\boxed{e = \dfrac{v\cos\beta}{u\cos\alpha}}$$

(4) Rebounding of a ball after collision with ground

If a ball is dropped from a height h_o on a horizontal floor, then it strikes with a speed v_o

$$v^2 = u^2 + 2gh$$

$$v_o^2 = 0^2 + 2\times g\times h_o$$

$$v_o = \sqrt{2gh_o}$$

If it rebound from the floor with a speed v, to a height $h_1, v^2 = u^2 + 2gh$

$$0^2 = v_1^2 - 2\times g\times h_1$$

$$v_1 = \sqrt{2gh_1}$$

$$e = \dfrac{\text{R.V of separation}}{\text{R.V of approach}} = \dfrac{v_1}{v_0} = \sqrt{\dfrac{h_1}{h_o}}$$

$$\boxed{e = \left(\dfrac{h_1}{h_o}\right)^{\frac{1}{2}}}$$

$$e = \dfrac{v_1}{v_o} \quad v_1 = ev_o = e\sqrt{2gh_o}$$

First height of rebound $h_1 = \dfrac{v_1^2}{2g} = e^2 h_o$

$$\boxed{h_1 = e^2 h_o}$$

* For the second rebound, v_1 is the velocity of approach v_2 is the velocity of separation

$e = \frac{v_2}{v_1}$; $v_2 = ev_1$

But $v_1 = ev_o$

$v_2 = e^2 v_o$

Similarly for n^{th} rebound

$\boxed{v_n = e^n v_o}$

Height of second rebound $h_2 = \frac{v_2^2}{2g} = \frac{e^4 v_o^2}{2g}$

But $\frac{v_o^2}{2g} = h_o$

$h_2 = e^4 h_o$

Similary for n^{th} rebound, the height

$\boxed{h_o = e^{2n} h_o}$

* Distance travelled by the ball before it stops bounding

$M = h_o + 2h_1 + 2h_2 + ...$

$= h_o + 2e^2 h_o + 2e^4 h_o + 2e^6 h_o, ...$

$= h_o \left(1 + 2e^2 + 2e^4 + 2e^6 + ...\right)$

$= h_o \left(1 + 2e^2 \left(1 + e^2 + e^4 + ...\right)\right.$

$$1+e^2+e^4+.....=\frac{1}{1-e^2}$$

$$=h_o\left(1+2e^2\times\frac{1}{1-e^2}\right);\ =h_o\frac{\left(1-e^2+2e^2\right)}{1-e^2}$$

$$\boxed{M=h_o\frac{\left(1+e^2\right)}{\left(1-e^2\right)}}$$

* Total time taken by the ball to stop bounding

$$T=t_o+2t_1+2t_2+2t_3+......$$

$$T=\sqrt{\frac{2h_o}{g}}+2\sqrt{\frac{2h_1}{g}}+2\sqrt{\frac{2h_2}{g}}+....$$

$$T=\sqrt{\frac{2h_o}{g}}+2\sqrt{\frac{2\times e^2h_o}{g}}+2\sqrt{\frac{2e^4h_o}{g}}+.....$$

$$T=\sqrt{\frac{2h_o}{g}}\left(1+2e+2e^2+......\right)$$

$$T=\sqrt{\frac{2h_o}{g}}\left(1+2e\left(1+e+e^2+e^3+....\right)\right)$$

$$T=\sqrt{\frac{2h_o}{g}}\left(1+2e\times\frac{1}{1-e}\right)$$

$$T=\sqrt{\frac{2h_o}{g}}\frac{\left(1-e+2e\right)}{1-e}$$

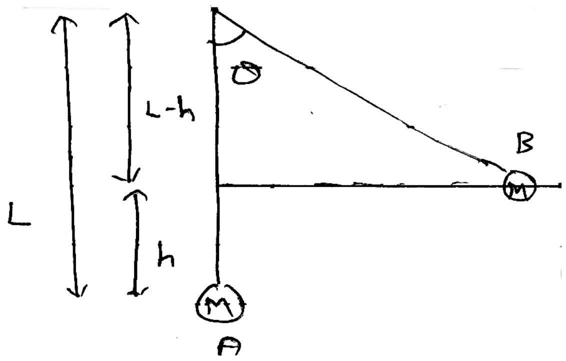
$$\boxed{T=\sqrt{\frac{2h_o}{g}}\left(\frac{1+e}{1-e}\right)}$$

Collision between bullet and vertically suspended block

A bullet of mass m is fired horizontally with velocity v in block of mass M suspended by vertical thread. Let the combined system raised up to height h and the string makes an angle θ witht he vertical

Velocity of the system

Let V be the velocity of the system



By momentum conservation $mv + 0 = (m+M)V$

$$\boxed{V = \frac{mv}{m+M}} .....(1)$$

Velocity of the bullet

$$TE_A = \frac{1}{2}(M+m)V^2$$

$$TF_B = (M+m)gh$$

$$\frac{1}{2}(M+m)V^2 = (M+m)gh$$

$$V = \sqrt{2gh} ......(2)$$

substitute (2) in (1)

$$\sqrt{2gh} = \frac{mv}{M+m}$$

$$\boxed{V = \sqrt{2gh}\,\frac{(m+M)}{m}}$$

Loss in KE

$$Ak = \frac{1}{2}mv^2 - \frac{1}{2}(M+m)V^2$$

$$= \frac{1}{2}mv^2 - \frac{1}{2}(M+m) \times \frac{m^2v^2}{(m+M)^2}$$

$$=\frac{1}{2}mv^2-\frac{1}{2}\frac{m^2v^2}{m+M}$$

$$\Delta K=\frac{1}{2}mv^2\left(1-\frac{m}{m+M}\right)$$

$$\boxed{\Delta KE=\frac{1}{2}mv^2\times\frac{M}{m+M}}$$

Angle of string from the vertical

$$V=\sqrt{2gh}\,\frac{(m+M)}{m}$$

$$V^2=2gh\times\frac{(m+M)^2}{m^2}$$

$$h=\frac{V^2}{2g}\left(\frac{m}{m+M}\right)^2$$

$$\cos\theta=\frac{L-h}{L}=1-\frac{h}{L}=1-\frac{v^2}{2gL}\left(\frac{m}{m+M}\right)^2$$

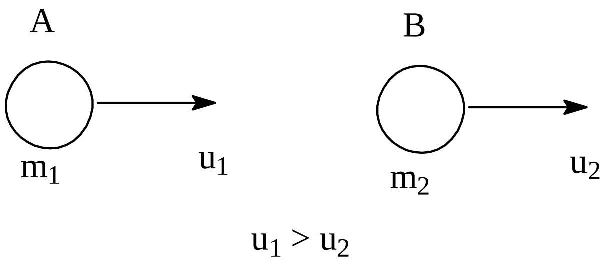
$$\theta=\cos^{-1}\left(1-\frac{v^2}{2gL}\left(\frac{m}{m+M}\right)^2\right)$$

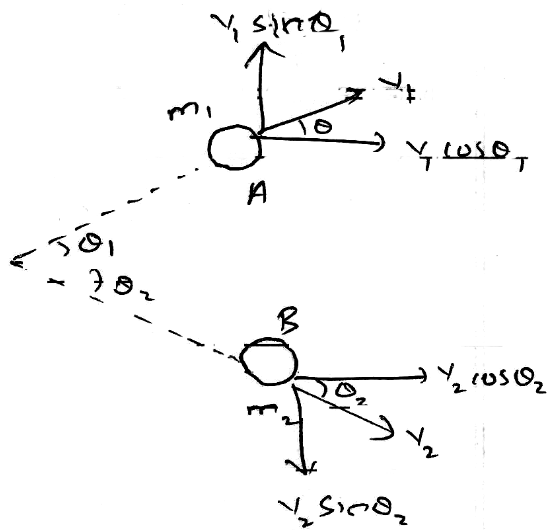
Elastic collision in two dimension

Suppose a particle of mass m moving along X axis with velocity u_1 collides wit another particle of mass m_2 at rest. After the collision, let the two particles move with velocities v_1 and v_2, making angles θ_1 and θ_2 with X axis

Applying the law of conservation of momentum along X axis

$$m_1u_1+m_2u_2=m_1v_1\cos\theta_1+m_2v_2\cos\theta_2$$





* Applying the law of conservation of moment along r axis

$$m_1 \times 0 + m_2 \times 0 = m_1 v_1 \sin\theta_1 + m_2 \times -v_2 \sin\theta_2$$

$$m_1 v_1 \sin\theta_1 = m_2 v_2 \sin\theta_2$$

* Conservation of kinetic energy yields

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

If the particle B is initially at rest

* Conservation of momentum along X axis

$$m_1u_1 = m_1v_1\cos\theta_1 + m_2v_2\cos\theta_2$$

* Conservation of momentum along Y axis

$$o + o = m_1v_1\sin\theta_1 - m_2v_2\sin\theta_2$$

* Conservation of KE

$$\frac{1}{2}m_1u_1^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

Impact parameter

It is the perpendicualr distance between initial velocity vectors of the two particles that take part in collision. It is denoted by b