CHAPTER - 7
STRAIGHT LINE

I. RECTANGULAR AXES

1. Distance formula:- If $A(x_1, y_1)$ and $B(x_2, y_2)$ be two points then

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

In particular the distance of a point P(x,y) fromt he origin O(0,0) is $|OP| = \sqrt{x^2 + y^2}$

2. Section formula: The point which divides the join of the points (x_1, y_1) and (x_2, y_2) in the ratio $l : m$ internally is $\left(\frac{lx_2 + mx_1}{l+m}, \frac{ly_2 + my_1}{l+m}\right)$ and externally is $\left(\frac{lx_2 - mx_1}{l-m}, \frac{ly_2 - my_1}{l-m}\right)$

The point which divides the join of the points (x_1, y_1) and (x_2, y_2) in the ratio l : 1 is $\left(\frac{\lambda x_2 + x_1}{\lambda+1}, \frac{\lambda y_2 + y_1}{\lambda+1}\right)$.

The mid-point of the segment joining (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

3. The area of the triangle ABC whose vertices are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is equal to

$$\frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\left|\Sigma x_1(y_2 - y_3)\right| = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

In particular if one vertex is at the origin (0, 0) then the area of the triangle formed by (0,0), (x_1, y_1) and (x_2, y_2) = $\frac{1}{2}|(x_1y_2 - x_2y_1)|$

Particular cases:

A) If two vertices be on the x-axis say (a,0) and (b,0) and the third vertex is (h, k) then area = ½ base x altitude = ½(a-b)k.

B) If two vertices be on the y-axis say (0, c) and (0, d) and the third vertex is (h, k) then area = ½(c-d)h

C) Area of Δ OAB when O(0, 0), A(a, 0) and B(0, b) = ½ab

4. **Standard points with respect to a triangle:**

i. Centroid:-

The point of concurrence of the medians of a triangle is called the centroid of the triangle denoted by G. The centroid of the triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is $G\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$. G divides each median in the ratio 2:1

ii. Incentre:-

The point of concurrence of the internal bisectors of the angles of a triangle is called the incentre of the triangle denoted by I. The incentre I has co-ordinates $\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$ where a, b, c are the length of the sides BC, CA, AB respectively.

iii. Circumcentre:-

The point of concurrence of the right bisectors of the sides of the triangle is called the circumcentre of the triangle denoted by S. It is the centre of the circle that passes through the vertices of the triangle ie the circumcircle. This is a point which is equidistant from the three vertices of the triangle. If A, B, C are the vertices of the triangle then SA = SB = SC.

iv. Orthocentre:-

The meeting point of the altitudes of the triangle is called the orthocentre of the triangle denoted by O. The orthocentre of a right angled triangle is the right vertex and the circumcentre is the mid-point of the hyptenuse of the triangle.

v. The points S,G,O are collinear and G divides SO in the ratio 1:2

vi. In an equilateral triangle the points G,O,I coincide.

Straight line (First degree) equations:-

1. The equation Ax + By + C = 0 represents the general equation of straight line
2. x = 0, y - axis
3. y = 0, x - axis
4. x = a, parallel to y-axis
5. y = b, parallel to x - aixis
6. y = mx + c. The equation of the line which has y-intercept c and slope m.
7. y = mx any line through the origin
8. $\frac{x}{a}+\frac{y}{b}=1$. Intercept form. Here a and b are the intercept on the axis of x and y respectively. The length

of the intercept between the axis is $\sqrt{a^2+b^2}$.

9. $y-y_1 = m(x - x_1)$. Point-slope form

The equation of a line through a given point (x_1, y_1) and having slope m.

Note:- $m=\frac{y-y_1}{x-x_1}$. If m = 0 the line is parallel to the x-axis and the equation is y - y = 0.

10. $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}$. Two point form. The equation of a line passing through two given points (x_1, y_1) and (x_2, y_2).

ie, $y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$; Its slope $m=\frac{y_2-y_1}{x_2-x_1}=\frac{\text{Difference of ordinates}}{\text{Difference of abscissae}}$

11. xcosa + ysina = P. Perpendicular form.

The equation of a line in terms of the length of the perpendicular P from the origin and a the angle which the perpendicular makes with the positive direction of the x - axis.

Here (coefficient of x)^2 + (coefficient of y)^2 = 1.

12. $\frac{x-x_1}{\cos\theta}=\frac{y-y_1}{\sin\theta}=r$. Distance Equations [Symmetric form]

The line makes an angle q with the x-axis and r is the distance btweeen the points (x_1, y_1) and (x, y). Any point on this line is $(x_1$ + rcosq, y_1 + rsinq)

13. Slope and intercepts of Ax + By + C = 0 are $m=-\frac{A}{B}=-\frac{\text{coefficient of x}}{\text{coefficient of y}}$ and x - intercept = $-\frac{C}{A}$ and y - intercept = -C/B

14. Point of intersection of two given lines is obtained by solving the equation of the lines.

15. Condition for collinearity. Three points are collinear if the third point lies on the line joining the first two points or if the three points be A, B, C then show that slope of AB = Slope of BC

16. Angle between two lines.

$$\tan\theta=\frac{m_1-m_2}{1+m_1m_2}=\frac{A_1B_2-A_2B_1}{A_1A_2+B_1B_2}$$

17. Two lines are parallel if $m_1 = m_2$ or $\frac{A_1}{A_2}=\frac{B_1}{B_2}$

18. Any line parallel to Ax + By + C = 0 is

Ax + By + K = 0

19. Two lines are perpendicular if $m_1m_2 = -1$ or

$A_1A_2 + B_1B_2 = 0$

20. Any line perpendicular to $Ax + By + C = 0$ is

    $Bx - Ay + K = 0$.

21. The general equation of a line through the intersection of two given lines $P = 0$ and $Q = 0$ is $P + lQ = 0$. The value of l is to be found by an additional condition given

22. Length of the perpendicular from a given point (x_1, y_1) to a given line $ax + by + c = 0$ is $\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

    Length of the perpendicular from the origin to the given line is $\frac{|c|}{\sqrt{a^2 + b^2}}$

23. Position of two points $L(x_1, y_1)$ and $M(x_2, y_2)$ with respect to a given line $ax + by + c = 0$.

    If $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ are of the same sign L and M are on same side and if $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ are of opposite signs L and M are on opposite sides

    Origin and non-origin side:-

    The origin and (x_1, y_1) lie on the same side if c and $ax_1 + by_1 + c$ have the same sign (origin side) . If they have opposite signs the points (0, 0) and (x_1, y_1) lie on opposite sides (non-origin side).

24. Equation of bisectors of given lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are

    $$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

    To find the bisector of acute or obtuse angle find tan Q is where Q is the angle between one of the given lines and one bisector. If |tan Q| < 1 it is the bisector of the acute angle and if |tan Q| > 1 it is the bisector of the obtuse angle.

25. Distance between two parallel lines is $\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$

26. If three lines $a_1x + b_1y + c_1 = 0$,

    $a_2x + b_2y + c_2 = 0$ and $a_3x + b_3y + c_3 = 0$ are concurrent then $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$

    Find the point of intersection of two lines and show that it satisfies the third also.

27. Change the Axes:-

    Translation:- If the origin is shifter to the point (h, k) without changing the direction of the axes then the equation of transformation are $x = X + h$ and $y = Y + k$ where (x, y) are the co-ordinates referred to the original axes and (X, Y) the co-ordinates referred to the new axes of any point in the plant. Also $X = x - h$ and $Y = y - k$.

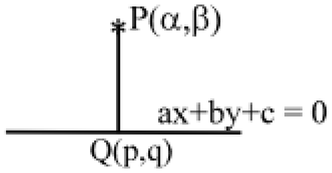
    After shifting the original equation $f(x, y) = 0$ becomes $f(X+h, Y + k) = 0$

28. Locus:- The locus of a point in the plane is the path traced out by that point under certain given

condition or conditions.

To find the equation of the locus of a point P take the co-ordinates of P as (h, k) and eliminate the parameter or parameters involved to get the relation between h and k. Now generalise by replacing h by x and k by y to get the required equation of the locus.

29. <u>Foot of the perpendicular from (a, b) to the line ax + by + c = 0:-</u>



Let Q(p, q) be the foot of the perpendicular from P(a, b) to the line ax + by + c = 0. Slope of PQ = $\frac{q-\beta}{p-\alpha}$.

Slope of ax + by + c = 0 is $\frac{-a}{b}$. PQ perpendicular to the given line

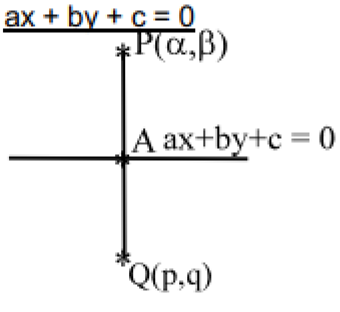
$\therefore \left(\frac{q-\beta}{p-\alpha}\right)\left(\frac{-a}{b}\right)=-1$; $\therefore \frac{p-\alpha}{a}=\frac{q-\beta}{b}=\frac{a(p-\alpha)+b(q-\beta)}{a^2+b^2}$

$= \frac{ap+bq-(a\alpha+b\beta)}{a^2+b^2}$; Q(p, q) lies on ax + by + c = 0

\ ap + bq + c = 0; ap + bq = -c ;

$\therefore \frac{p-\alpha}{a}=\frac{q-\beta}{b}=\frac{-(a\alpha+b\beta+c)}{a^2+b^2}$

30. <u>The image of the point (a, b) on the line ax + by + c = 0</u>



$$\frac{p-\alpha}{a}=\frac{q-\beta}{b}=\frac{-2(a\alpha+b\beta+c)}{a^2+b^2}$$

PART I - (JEEMAIN)

SECTION - I - Straight objective type questions

1. The intercept of a line between the coordinate axes is divided by point (-5,4) in the ratio 1:2. The equation of the line will be

1) $5x-8y+60=0$ 2) $8x-5y+60=0$ 3) $2x-5y+30=0$ 4) $8x+5y-60=0$

2. The straight line passing through the point of intersection of the straight lines $x-3y+1=0$ and $2x+5y-9=0$ and having infinite slope and at a distance of 2 units from the origin, has the equation\
1) x = 2 2) $3x+y-1=0$ 3) $y=1$ 4) $y=5$

3. For the straight lines given by the equation $(2+k)x+(1+k)y=5+7k$, for different values of k which of the following statements is true
1) Lines are parallel 2) Lines pass through the point (-2,9)
3) Lines pass through the point (2,-9) 4) Lines passes through (1,2)

4. One side of a rectangle lies along the line 4x+7y+5=0. Two of its vertices are (–3, 1) and (1, 1). Then the equations of other three sides are
1) $7x-4y+25=0, 4x+7y=11$ and $7x-4y-3=0$
2) $7x+4y+25=0, 7y+4x-11=0$ and $7x-4y-3=0$
3) $4x-7y+25=0, 7x+4y-11=0$ and $4x-7y-3=0$
4) $7x-4y-25=0, 4x+7y+11=0$ and $7x+4y+3=0$

5. The lines ax+by+c=0, where 3a+2b+4c=0 are concurrent at the point
1) (1/2,3/4) 2) (1,3) 3) (3,1) 4) (3/4,1/2)

6. The point (4, 1) undergoes the following three transformations successively
(i) Reflection about the line $y = x$
(ii) Translation through a distance 2 units along the positive direction of x-axis
(iii) Rotation through an angle $\pi/4$ about the origin in the anti-clockwise direction.
The final position of the point is given by the coordinates
1) $\left(\frac{1}{\sqrt{2}},\frac{7}{\sqrt{2}}\right)$ 2) $\left(-\sqrt{2},7\sqrt{2}\right)$ 3) $\left(-\frac{1}{\sqrt{2}},\frac{7}{\sqrt{2}}\right)$ 4) $\left(\sqrt{2},7\sqrt{2}\right)$

7. A ray of light coming from the point (1, 2) is reflected at a point A on the x–axis and then passes through the point (5, 3). The coordinates of the point A are
1) $\left(\frac{13}{5},0\right)$ 2) $\left(\frac{5}{13},0\right)$ 3) $(-7,0)$ 4) (1,2)

8. The graph of the function y = $\cos x \cos(x+2) - \cos^2(x+1)$ is

1) A straight line passing through $\left(0, -\sin^2 1\right)$ with slope 2

2) A straight line passing through (0, 0)

3) A parabola with vertex $\left(1, -\sin^2 1\right)$

4) A straight line passing through the point $\left(\frac{\pi}{2}, -\sin^2 1\right)$ and parallel to the x–axis

9. If $x_1, x_2 x_3$ and $y_1, y_2 y_3$ are both in G.P. with the same common ratio, then the points (x_1, y_1), (x_2, y_2) and (x_3, y_3)

1) Lie on a straight line
2) Lie on an ellipse
3) Lie on a circle
4) Are vertices of a triangle

10. If the lines ax+2y+1=0,bx+3y+1=0 and cx+4y+1 = 0 are concurrent, then a, b, c are in

1) A. P.
2) G. P.
3) H. P.
4) A.G.P

11. The equation of perpendicular bisectors of the sides AB and AC of a triangle ABC are $x - y + 5 = 0$ and $x + 2y = 0$ respectively. If the point A is (1,-2) , then the equation of line BC is

1) 23x+14y-40 = 0
2) 5x-4y+7 = 0
3) 3x-2y+3 = 0
4) 14x + 23y-40 = 0

12. If the equation of base of an equilateral triangle is 2x-y =1 and the vertex is (-1,2), then the length of the side of the triangle is

1) $\sqrt{\frac{20}{3}}$
2) $\frac{2}{\sqrt{15}}$
3) $\sqrt{\frac{8}{15}}$
4) $\sqrt{\frac{15}{2}}$

13. A line 4x+y = 1 passes through the point A(2,-7) meets the line BC whose equation is $3x - 4y + 1 = 0$ at the point B. The equation to the line AC so that AB = AC, is

1) 52x+89y+519 = 0
2) 52x+89-519 = 0
3) 89x+52y+519 = 0
4) 89x+52y-519 = 0

14. The area enclosed within the lines $|x| + |y| = 1$ is

1) $\sqrt{2}$
2) 1
3) $\sqrt{3}$
4) 2

15. The line joining two points A(2,0), B(3,1) is rotated about A in anti-clockwise direction through an angle of 15^0. The equation of the line in the new position, is

1) $\sqrt{3}x-y-2\sqrt{3}=0$ 2) $x-3\sqrt{y}-2=0$ 3) $\sqrt{3}x+y-2\sqrt{3}=0$ 4) $x+\sqrt{3}y-2=0$

16. Let C be the centroid of the triangle with vertices $(3,-1)$, (1, 3) and (2, 4). Let P be the point of intersection of the lines x+3y-1= 0 and 3x-y+1= 0 . Then the line passing through the points C and P also passes through the point:

1) (7, 6) 2) (-9,-6) 3) (-9,-7) 4) (9, 7)

17. If a straight line passing through the point P(-3,4) is such that the portion between the coordinate axes is bisected at P, then its equation is:

1) $x-y+7=0$ 2) $3x-4y+25=0$ 3) $4x+3y=0$ 4) $4x-3y+24=0$

18. If the line 3x+4y-24 = 0 intersects x-axis at the point A and the y-axis at the point B, then the incentre of the triangle OAB, where O is the origin, is

1) (3, 4) 2) (2, 2) 3) (4, 4) 4) (4, 3)

19. A straight line L at a distance of 4 units from the origin makes positive intercepts on the coordinate axes and the perpendicular from the origin to this line makes an angle of 60^0 with the line x+y = 0. Then an equation of the line L is:

1) $\left(\sqrt{3}+1\right)x+\left(\sqrt{3}-1\right)y=8\sqrt{2}$ 2) $\left(\sqrt{3}-1\right)x+\left(\sqrt{3}+1\right)y=8$

3) $\sqrt{3}x+y=8$ 4) $x+\sqrt{3}y=8$

20. The image of the point (3,5) in the line x-y+1 = 0, lies on

1) $(x-2)^2+(y-4)^2=4$ 2) $(x-4)^2+(y+4)^2=16$

3) $(x-4)^2+(y-4)^2=8$ 4) $(x-2)^2+(y-2)^2=12$

SECTION - II - Numerical type Questions

21. The number of integral values of m, for which the x-co-ordinate of the point of intersection of the lines $3x+4y=9$ and $y=mx+1$ is also an integer is

1) 2 2) 0 3) 4 4) 1

22. If the straight line, $2x-3y+17=0$ is perpendicular to the line passing through the points (7,17) and ,$(15,\beta)$ then β equals:

23. If the perpendicular bisector of the line segment joining the points P(1, 4) and Q(k, 3) has y-intercept equal to – 4, then a value of $|k|$ is:

24. If the line 2x + y = k passes through the point which divides the line segment joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then 'k' equals

25. A straight line L with negative slope passes through the point (8, 2) and cuts the positive coordinates axes at points P and Q. As L varies, the absolute minimum value of OP + OQ is (O is origin)

PART - II (JEE ADVANCED LEVEL)

SECTION - III (Only one option correct type)

26. If $\frac{a}{\sqrt{bc}}-2=\sqrt{\frac{b}{c}}+\sqrt{\frac{c}{b}}$ where a,b,c > 0 then family of lines $x\sqrt{a}+y\sqrt{b}+\sqrt{c}=0$ passes through the point

A) (1,1) B) $(1,-2)$ C) $(-1,2)$ D) $(-1,1)$

27. The line $2x+y=4$ meet x-axis at A and y-axis at B. The perpendicular bisector of AB meets the horizontal line through (0,-1) at C. Let G be the centroid of ΔABC. The perpendicular distance from G to AB equals

A) $\sqrt{5}$ B) $\frac{\sqrt{5}}{3}$ C) $2\sqrt{5}$ D) $3\sqrt{5}$

28. If the point $P(\alpha^2,\alpha)$ lies in the region corresponding to the acute angle between the lines $x-3y=0$ and $x-5y=0$ then

A) $\alpha\in(5,15)$ B) $\alpha\in(5,8)$ C) $\alpha\in(4,8)$ D) $\alpha\in(3,5)$

29. Line L has intercepts a and b on the coordinate axes. When the axes are rotated through a fixed given angle keeping the origin fixed, the same line L has intercepts p and q, then

A) $a^2+b^2=p^2+q^2$ B) $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}+\frac{1}{q^2}$ C) $a^2+p^2=b^2+q^2$ D) $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{b^2}+\frac{1}{q^2}$

30. A line through P (3,4) cuts the lines x = 6 and y = 8 at L and M respectively. Q is a variable point on the line such that $\frac{1}{PQ}=\frac{1}{PL}+\frac{1}{PM}$ then the locus of Q is

A) $4x+3y-36=0$ B) $x^2+y^2=36$ C) $3x-4y-36=0$ D) $4x^2-9y^2=36$

SECTION - IV (More than one correct answer)

31. Given three straight lines $2x+11y-5=0, 24x+7y-20=0,$ and $4x-3y-2=0$. Then,

A) They form a triangle

B) They are concurrent

C) One line bisects the angle between the other two

D) Two of them are parallel

32. The vertices of a ΔABC are $A(-5,-2);B(7,9)$ and $C(5,-4)$. Then:

A) Measure of angle B is $\frac{\pi}{4}$

B) Equation of the altitude drawn from the vertex C has the equation, $3x+2y-7=0$

C) Orthocentre of the triangle does not lie inside the ΔABC

D) Distance between centroid and circumcentre of the ΔABC is $\frac{4\sqrt{13}}{3}$

33. The straight line $x-y=2$ cuts the co-ordinate axes in A and B. On AB a square is constructed away from the origin, p denotes the $\perp$ distance from (0,0) to a side of the square, then

A) Maximum value of p is $3\sqrt{2}$

B) Area of square is 8 (square units)

C) $x+y=2$ is the equation to one of the sides of the square

D) None of these

34. Let L_1 : 3x+4y = 1 and

L_2 = 5x-12y+2 = 0 be two given lines. Let the image of every point on L_1 w.r.to L lies on L_2, then the possible equation of L can be

A) 14x+112y-23 = 0 B) 64x-8y-3 = 0

C) 11x-4y = 0 D) 52y-45x =7

35. Point P(-1,7) lies on the line 4x+3y=17. Then the coordinates of the points farthest from the line which are at a distance of 10 units from the point P are

A) (7,13) and (–9,1) B) (5,15) and (–1,–7)

C) (–1,5) and (15,–7) D) (15,5) and (–7,–1)

36. A straight line L through the point (3,-2) is inclined at an angle 60^0 with the line $\sqrt{3}x+y=1$ then equation of L is

A) $y=-2$ B) $y-\sqrt{3}x+2+3\sqrt{3}=0$

C) $y+\sqrt{3}x+2-3\sqrt{3}=0$ D) $x=3$

37. Let $a, \lambda, \mu \in R$, consider the system of equations ax $ax + 2y = \lambda$ and $3x - 2y = \mu$ which of the following statement(s) is/are correct

A) If $a = -3$, then the system has infinitely many solutions for all values of λ and μ

B) If $a \neq -3$, then the system has a unique solution for all values of λ and μ

C) If $\lambda + \mu = 0$, then the system has infinitely many solutions for $a = -3$

D) If $\lambda + \mu \neq 0$, then the system has no solution for $a = -3$

38. If the area of the triangle formed by the intersection of line parallel to X-axis and passing through (h,k) with the lines y = x and x+y = 2 is $4h^2$, then the locus is/are

A) $2x + y + 1 = 0$ B) $2x + y - 1 = 0$ C) $2x - y + 1 = 0$ D) $2x - y - 1 = 0$

SECTION - V- Assertion Reason Type

39. The line $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R

Statement 1: The ratio PR:RQ equals $2\sqrt{2} : \sqrt{5}$

Statement 2: In any triangle, bisector of an angle divides the triangle into two similar triangles

A) Statement - 1 is true, Statement -2 is true; Statement 2 is correct explanation for statement 1

B) Statement -1 is true, Statement -2 is true; Statement 2 is not a correct explanation for statement -1

C) Statement -1 is true; Statement -2 is false

D) Statement -1 is false, Statement -2 is true

SECTION VI - (Matrix match type)

40.

	Column I		Column II
a)	If a, b, c are in A.P, then lines ax+by+c = 0 are concurrent at	p)	$(-4,-7)$
b)	A point on the line x + y = 4 which lies at a unit distance from the line 4x + 3y = 10 is	q)	$(-7,11)$
c)	Orthocentre of triangle made by lines x + y = 1, x- y + 3 = 0, 2x + y = 7 is	r)	$(1,-2)$
d)	Two vertices of a triangle are (5, -1) and (-2,3). If orthocentre is the origin then coordinate of the third vertex are	s)	$(-1,2)$
		t)	(3,1)

A) A-R;B-Q,T;C-S;D-P B) A-Q;B-R;C-S;D-P

C) A-P;B-S;C-T;D-P D) A-P;B-Q,T;C-S;D-R