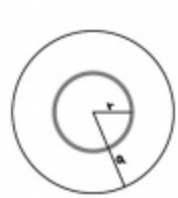
CHAPTER - 06
RIGID BODY DYNAMICS

LEVEL - I

1. A



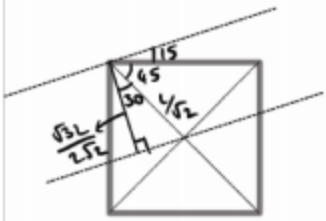
$$\int_0^a [\sigma \cdot 2\pi r dr] \cdot r^4$$

$$= \int_0^a [A + Br] 2\pi r dr \cdot r^4$$

$$= 2\pi A \frac{a^6}{6} + 2\pi B \frac{a^5}{5}$$

$$= 2\pi a^4 \left[\frac{A}{6} + \frac{Ba}{5}\right] \rightarrow \boxed{A} \checkmark$$

2. B



$$\frac{ML^2}{12} + M\left[\frac{\sqrt{3}L}{2\sqrt{2}}\right]^2$$

$$= ML^2\left[\frac{1}{12} + \frac{3}{8}\right]$$

$$= ML^2 \frac{2+9}{24} = \frac{11}{24} ML^2 \rightarrow \boxed{B} \checkmark$$

3. B

$$\left\{\frac{MR^2}{2} + 6\left[\frac{MR^2}{2} + M\cdot(2R)^2\right]\right\} + 7M\cdot(3R)^2$$

$$= MR^2\left[\frac{1}{2} + 6\cdot\frac{9}{2} + 63\right]$$

$$= \frac{181}{2} MR^2 \rightarrow \boxed{B} \checkmark$$

4. A

$$\tau_p = 5 \times 20 = 100$$

$$\tau_0 = 0$$

$$\rightarrow \boxed{A} \checkmark$$

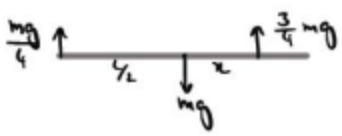
5. C

$$\frac{l}{2}\cdot mg + l\cdot mg = \left[\frac{ml^2}{3} + ml^2\right]\alpha$$

$$\frac{3}{2}g = l\alpha\cdot\frac{4}{3}$$

$$\alpha = \frac{9}{8}\frac{g}{l} \rightarrow \boxed{C}$$

6. A

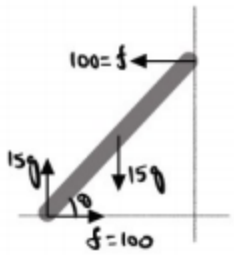


$$\frac{l}{2}\cdot\frac{mg}{4} = x\cdot\frac{3}{4}mg$$

$$\therefore x = \frac{l}{6}$$

$$\therefore \frac{l}{2} + \frac{l}{6} = \frac{4l}{6} = \frac{2l}{3} \rightarrow \boxed{A}$$

7. C



$$\frac{1}{2}\cdot 15g = l\sin\theta\cdot f$$

$$\therefore f = \frac{15g}{2}\cdot\frac{4}{3} = 100$$

8. B

$$\sqrt{2}\cdot(1+1) = 2\sqrt{2} \rightarrow \boxed{B}$$

9. C

$$Mr^2 \cdot \omega = \left[Mr^2 + 2mr^2\right]\Omega$$

$$\therefore \Omega = \frac{M\omega}{M+2m} \rightarrow \boxed{C} \checkmark$$

10. A

$$\frac{R}{2} \cdot \frac{M}{2} v = \left[\frac{MR^2}{2} + \frac{M}{2}R^2\right]\omega$$

$$\frac{R}{4} Mv = MR^2\omega$$

$$\therefore \omega = \frac{v}{4R} \rightarrow \boxed{A} \checkmark$$

11. C

$$\frac{\sqrt{3}}{2} a \cdot F = \frac{a}{2} \cdot mg$$

$$\therefore F = \frac{mg}{\sqrt{3}}$$

$$\therefore \mu N \geq \frac{mg}{\sqrt{3}}$$

$$\therefore \mu \geq \frac{1}{\sqrt{3}} \rightarrow \boxed{C} \checkmark$$

12. A

$$mg\frac{l}{4} = \frac{1}{2}\left[\frac{ml^2}{12} + m\frac{l^2}{16}\right]\omega^2$$

$$mg\frac{l}{4} = \frac{ml^2}{2}\,\frac{7}{48}\,\omega^2$$

$$\omega^2 l = \frac{24}{7} g \rightarrow \boxed{A} \checkmark$$

13. A

$$mgh = \frac{1}{2}\left[\frac{mR^2}{2} + mR^2\right]\omega^2$$

$$mgh = \frac{1}{2}\,\frac{3}{2}\,mv^2$$

$$mg \cdot v = \frac{3}{4} m \cdot 2va$$

$$\therefore a = \frac{2g}{3} \rightarrow \boxed{A} \checkmark$$

14. B

$$mg\frac{l}{2} = \frac{1}{2}\,\frac{ml^2}{3}\omega^2$$

$$\omega = \sqrt{\frac{3g}{l}}$$

$$l\omega = \sqrt{3gl} \rightarrow \boxed{B} \checkmark$$

15. A

$$\frac{\frac{1}{2}mv^2}{\frac{1}{2}\left[1+\frac{2}{5}\right]mv^2} = \frac{5}{7} \rightarrow \boxed{A} \checkmark$$

16. D

$$\frac{1}{2}(1+\beta)mv^2 = mg\cdot\frac{3v^2}{4g}$$

$$1+\beta = \frac{3}{2}$$

$$\beta = \frac{1}{2} \rightarrow \boxed{D}$$

17. A

$$a = \frac{g\sin\theta}{1+\beta} = \frac{g\sin\theta}{2}$$

$$\frac{h}{\sin\theta} = 0 + \frac{1}{2}\cdot\frac{g\sin\theta}{2}\cdot t^2 \Rightarrow t = \frac{1}{\sin\theta}\sqrt{\frac{4h}{g}} \rightarrow \boxed{A}$$

18. 12

$$\left.\begin{aligned}\omega_0^2 - \left(\frac{\omega_0}{2}\right)^2 &= 2\alpha\cdot[36\cdot 2\pi]\\ \left(\frac{\omega_0}{2}\right)^2 - 0^2 &= 2\alpha\cdot[n\cdot 2\pi]\end{aligned}\right\} \; 3 = \frac{36}{n} \Rightarrow n = 12 \rightarrow \boxed{12}$$

19. 20

$$10 + \frac{1}{2}\omega = 20 \Rightarrow \omega = 20 \rightarrow \boxed{20}$$

20. 16

$$R\cdot 40 = MR^2\alpha$$

$$\alpha = \frac{40}{MR} = \frac{40}{5\times\frac{1}{2}} = 16 \rightarrow \boxed{16}$$

LEVEL - II

21. D

$mg\frac{l}{2}(1-\phi) = \frac{1}{2}\cdot\frac{ml^2}{3}\cdot\omega^2 \Rightarrow \omega^2 l = 3g(1-\phi)$

$\frac{1}{2}\cdot mg\phi = \frac{ml^2}{3}\kappa \Rightarrow l\kappa = \frac{3}{2}g\phi$

$f = m\left[\frac{l}{2}\kappa\phi - \omega^2\frac{l}{2}\phi\right] = \frac{3}{4}mg\phi\phi - \frac{3mg}{2}(1-\phi)\phi$

$= \frac{3mg}{2}\left[\frac{\phi\phi}{2} - \phi + \phi\phi\right]$

$= \frac{3mg}{2}\left[\frac{3}{2}\phi\phi - \phi\right] \rightarrow$ (D) ✓

22. B

$R\cdot MV_0 + \frac{2}{5}MR^2\cdot\frac{V_0}{2R} = \left[1+\frac{2}{5}\right]RMV$

$\frac{6}{5}V_0 = \frac{7}{5}V$

$\therefore V = \frac{6}{7}V_0 \rightarrow$ (B) ✓

23. A

$2J = Mu_g$

$\therefore u_g = \frac{2J}{M}$

$\therefore T = \frac{2u_g}{g} = \frac{4J}{Mg}$

$2(L\cdot j) = \frac{M\cdot 4L^2}{12}\omega$

$\therefore \omega = 6\frac{j}{ML}$

$\therefore \frac{6j}{ML}\cdot\frac{4J}{Mg} = \pi$

$jJ = M^2\frac{\pi gL}{24}$

$\left[\sqrt{J^2+j^2}\right]_{min} = \sqrt{2}\cdot M\sqrt{\frac{\pi gL}{24}} = M\sqrt{\frac{\pi gL}{12}} = \frac{M}{2}\sqrt{\frac{\pi gL}{3}} \rightarrow$ (A) ✓

24. C

$N = mg = 50g = 500 \Rightarrow f_{max} = (0.3)500 = 150$

$500 - 150 = 50a \Rightarrow a = 7$

$1500 - 3150 = \frac{50 \cdot 3^2}{12}\kappa \Rightarrow \kappa = \frac{1}{3} \Rightarrow 3\kappa = 1$

$\therefore a - 3\kappa = 7 - 1 = 6 \rightarrow$ C ✓

25. B

$N = Mg\cos\theta$

$f = Mg\sin\theta$

$\mu N \geq Mg\sin\theta$

$\therefore \mu \geq \tan\theta$

$\therefore \mu \geq \frac{\kappa}{\sqrt{1-\kappa^2}} \rightarrow$ B ✓

26. A,B,D

$R \cdot MV_0 - \frac{2}{5}MR^2 \cdot \frac{V_0}{2R} = \left(1 + \frac{2}{5}\right)RMV$ C ✗

$\frac{4}{5}V_0 = \frac{7}{5}V$

$\therefore V = \frac{4}{7}V_0 \rightarrow$ B ✓

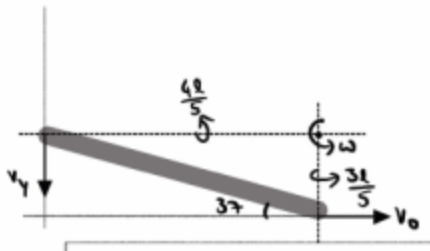
$V_0 - \mu g t = \frac{4}{7}V_0 \Rightarrow t = \frac{3}{7}\frac{V_0}{\mu g} \rightarrow$ A ✓

$K_1 = \frac{1}{2}MV_0^2 + \frac{1}{2}\cdot\frac{2}{5}MR^2\left[\frac{V_0}{2R}\right]^2 = \frac{11}{20}MV_0^2$

$K_2 = \frac{1}{2}\left[1+\frac{2}{5}\right]M\left[\frac{4}{7}V_0\right]^2 = \frac{8}{35}MV_0^2$

$K_1 - K_2 = \frac{9}{28}MV_0^2$ → D

27. A,B,C,D

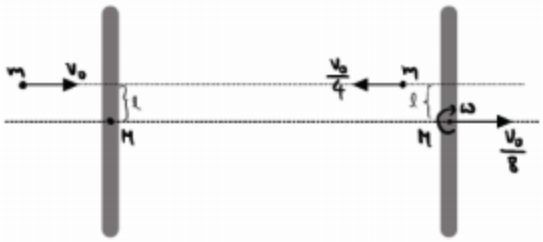


$\omega = \frac{V_0}{\frac{3l}{5}} = \frac{V_Y}{\frac{4l}{5}}$ → B, A

$V_C = \frac{l}{2}\omega = \frac{5}{6}V_0$ → C

$\frac{1}{2}\cdot\frac{ml^2}{3}\cdot\omega^2 = \frac{1}{2}\cdot\frac{m}{3}\cdot\frac{25}{9}V_0^2 = \frac{25}{54}mV_0^2$ → D

28. A,B,C,D



$mV_0 = M\frac{V_0}{8} - m\frac{V_0}{4} \Rightarrow \frac{5m}{4} = \frac{M}{8} \Rightarrow \frac{M}{m} = 10$ → A

$0 = l\cdot M\frac{V_0}{8} - \frac{M\cdot 16l^2}{12}\omega \Rightarrow \omega = \frac{3}{128}\frac{V_0}{l}$ → B

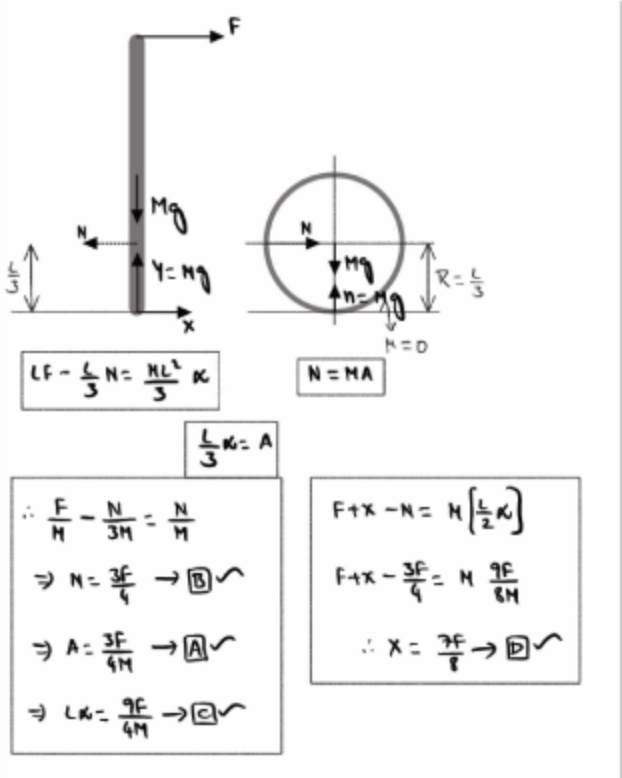
$e = \frac{\frac{V_0}{8} + l\omega + \frac{V_0}{4}}{V_0} = \frac{\frac{3V_0}{8} + \frac{3}{128}V_0}{V_0} = \frac{51}{128}$ → C

$\frac{V_0}{8} + 4l\omega = \frac{V_0}{8} + \frac{3}{32}V_0 = \frac{7}{32}V_0$

$\frac{V_0}{8} - 4l\omega = \frac{V_0}{8} - \frac{3}{32}V_0 = \frac{1}{32}V_0$

→ D

29. A,B,C,D



$LF - \frac{L}{3}N = \frac{ML^2}{3}\alpha$

$N = MA$

$\frac{L}{3}\alpha = A$

$\therefore \frac{F}{M} - \frac{N}{3M} = \frac{N}{M}$

$\Rightarrow N = \frac{3F}{4} \rightarrow$ (B) ✓

$\Rightarrow A = \frac{3F}{4M} \rightarrow$ (A) ✓

$\Rightarrow L\alpha = \frac{9F}{4M} \rightarrow$ (C) ✓

$F + x - N = M\left[\frac{L}{2}\alpha\right]$

$F + x - \frac{3F}{4} = M\,\frac{9F}{8M}$

$\therefore x = \frac{7F}{8} \rightarrow$ (D) ✓

30. A,B,C,D

$\frac{55}{2} \cdot 2x = \frac{1}{2} \cdot 1 \cdot [2v]^2 + \frac{1}{2}\left[1 + \frac{1}{2}\right] 1 \cdot v^2$

$55x = 2v^2 + \frac{3}{4}v^2$

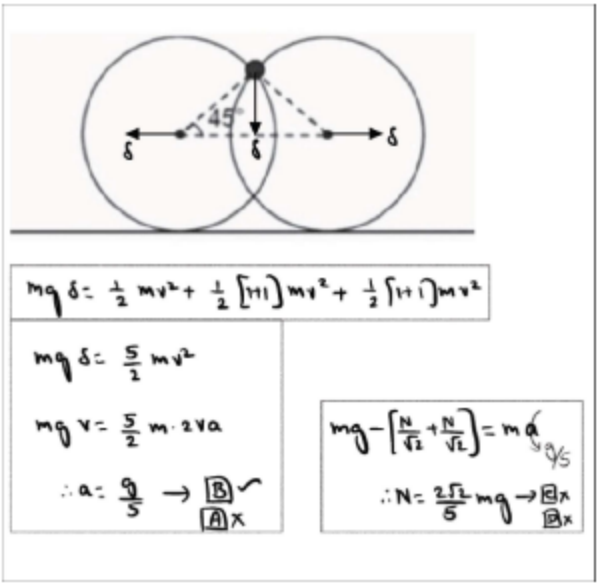
$55\,v = \frac{11}{4} \cdot 2v\,a$

$\therefore a = 10 \rightarrow$ (A) ✓ (D) ✓

$\frac{55}{2} - f_1 = 1 \cdot 20 \Rightarrow f_1 = 7.5 \rightarrow$ (B) ✓

$7.5 - f_2 = 1 \cdot 10 \Rightarrow f_2 = -2.5 \rightarrow$ (C) ✗

31. A,B



$mg\delta = \frac{1}{2}mv^2 + \frac{1}{2}(m)mv^2 + \frac{1}{2}(m)mv^2$

$mg\delta = \frac{5}{2}mv^2$

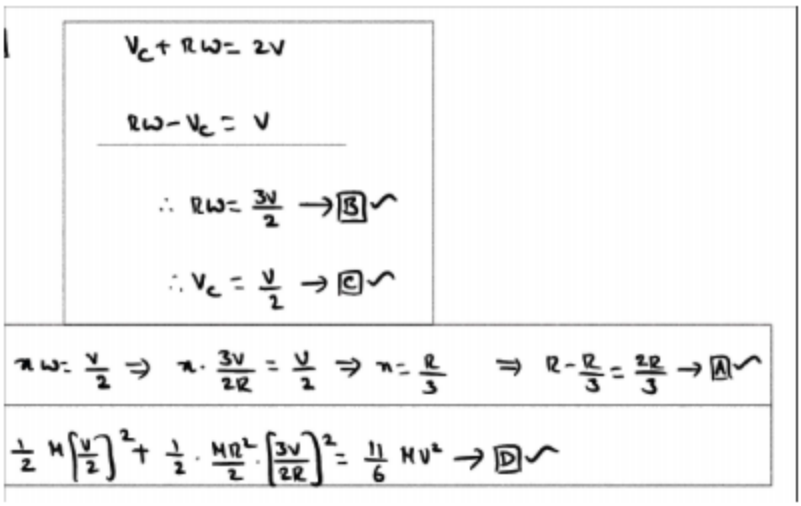
$mgv = \frac{5}{2}m \cdot 2va$

$\therefore a = \frac{g}{5} \rightarrow$ B ✓ A ✗

$mg - \left[\frac{N}{\sqrt{2}} + \frac{N}{\sqrt{2}}\right] = ma$

$\therefore N = \frac{2\sqrt{2}}{5}mg \rightarrow$ C ✗ D ✗

32. A,B,C,D



$V_c + R\omega = 2V$

$R\omega - V_c = V$

$\therefore R\omega = \frac{3V}{2} \rightarrow$ B ✓

$\therefore V_c = \frac{V}{2} \rightarrow$ C ✓

$r\omega = \frac{V}{2} \Rightarrow r \cdot \frac{3V}{2R} = \frac{V}{2} \Rightarrow r = \frac{R}{3} \Rightarrow R - \frac{R}{3} = \frac{2R}{3} \rightarrow$ A ✓

$\frac{1}{2}M\left[\frac{V}{2}\right]^2 + \frac{1}{2} \cdot \frac{MR^2}{2} \cdot \left[\frac{3V}{2R}\right]^2 = \frac{11}{6}MV^2 \rightarrow$ D ✓

33. 3

$mR^2 \cdot 5 = 2mR^2 \cdot \omega$

$\therefore \omega = \frac{5}{2}$

$\int_0^R r \cdot \mu\sigma 2\pi r\,dr \cdot g = \frac{MR^2}{2}\alpha$

$\mu\sigma 2\pi g \frac{R^3}{3} = \frac{MR^2}{2}\alpha$

$\mu \cdot \frac{M}{\pi R^2} \cdot 2\pi g \cdot \frac{R^3}{3} = \frac{MR^2}{2}\alpha$

$R\alpha = \frac{4\mu g}{3}$

$\alpha = \frac{4}{3} \times \frac{\frac{1}{8} \times 10}{2}$

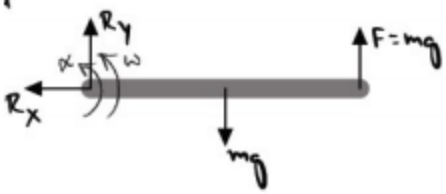
$\alpha = \frac{5}{6}$

$\frac{5}{2} = 5 - \frac{5}{6}t$

$\frac{5}{6}t = \frac{5}{2}$

$t = 3 \rightarrow \boxed{3}$ ✓

34. 15



$R_Y + mg - mg = m\left[\frac{l}{2}\alpha\right]$

$R_Y = \frac{3mg}{4}$

$= \frac{3}{4} \times 2 \times g$

$= 15 \rightarrow \boxed{15}$ ✓

$lmg - \frac{l}{2}mg = \frac{ml^2}{3}\alpha$

$l\alpha = \frac{3g}{2}$