# CHAPTER - 17
# VECTORS

## JEE MAIN - SECTION I

1. 2

$\text{Median} = \hat{2} - \hat{j} + 4\hat{k}$

$\text{length of } = \sqrt{1+1+4^2}$

$\text{Median} = \sqrt{18}$

2. 4

$$\therefore \begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 4 \\ 1 & \alpha & \beta \end{vmatrix} = 0 \Rightarrow 1+\alpha^2+\beta^2 = 3 \Rightarrow \alpha = \pm 1, \beta = 1$$

3. 4

We must have $\lambda(\hat{i} - 3\hat{j} + 5\hat{k}) = \vec{a} + \dfrac{2\hat{k}+2\hat{j}-\hat{i}}{3}$

For $\lambda = \dfrac{22}{35}$, $\vec{a} = \dfrac{41}{105}\hat{i} - \dfrac{88}{105}\hat{j} - \dfrac{40}{105}\hat{k}$

4. 1

$\vec{a}+\vec{b}+\vec{c} = \vec{0} \Rightarrow 2|\vec{b}||\vec{c}|\cos\theta = 49-34 = 15$

$\Rightarrow 2\times5\times3\cos\theta = 15 \Rightarrow \cos\theta = \dfrac{1}{2} \Rightarrow \theta = \dfrac{\pi}{3} = 60^0$

5. 3

$|a+b+c|^2 \geq 0$

$|a|^2+|b|^2+|c|^2+2(a.b+b.c+ca) \geq 0$

$3+6\cos\theta \geq 0$

$\cos\theta \geq -\dfrac{1}{2}$

$\theta = 2\dfrac{\pi}{3}$

6. 4

$\vec{r} = x_1(\vec{a}\times\vec{b}) + x_2(\vec{b}\times\vec{c}) + x_3(\vec{c}+\vec{a})$

$\Rightarrow \vec{r}\cdot\vec{a} = x_2[\vec{a}\ \vec{b}\ \vec{c}]$, $\vec{r}\cdot\vec{b} = x_3[\vec{b}\ \vec{c}\ \vec{a}]$ and $\vec{r}\cdot\vec{c} = x_1[\vec{c}\ \vec{a}\ \vec{b}] = x_1[\vec{a}\ \vec{b}\ \vec{c}]$

$\Rightarrow x_1 + x_2 + x_3 = 4\vec{r}\cdot(\vec{a}+\vec{b}+\vec{c})$

7. 4

$$\overline{OP} = \left(3\cos\frac{\pi}{4}\right)\hat{i} + \left(3\sin\frac{\pi}{4}\right)\hat{j} = \frac{3}{\sqrt{2}}(\hat{i}+\hat{j})$$

$$\overline{OR} = \left(4\cos\frac{3\pi}{4}\right)\hat{i} + \left(4\sin\frac{3\pi}{4}\right)\hat{j} = \frac{4}{\sqrt{2}}(-\hat{i}+\hat{j}) = 2\sqrt{2}(-\hat{i}+\hat{j})$$

Now, $\overline{OP}+\overline{PQ} = \overline{OP}+\overline{OR} = \frac{1}{\sqrt{2}} = \left((-\hat{i}+7\hat{j})\right)$

$$\therefore\ \overline{OM} = \frac{\frac{3}{\sqrt{2}}(\hat{i}+\hat{j}) + \frac{1}{\sqrt{2}}(-\hat{i}+7\hat{j})}{2} = \frac{2\hat{i}+10\hat{j}}{2\sqrt{2}} = \frac{\hat{i}+5\hat{j}}{\sqrt{2}}$$

Now, PT : TR = 1 : 2 $\Rightarrow \overline{OT} = \frac{\sqrt{2}}{3}(\hat{i}+5\hat{j})$

8. 4

Let $|\vec{a}| = |\vec{b}| = |\vec{c}| = \lambda$

We have $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\alpha = \lambda^2\cos\alpha$

$\Rightarrow \vec{b}\cdot\vec{c} = |\vec{b}||\vec{c}|\cos\beta = \lambda^2\cos\beta$

$\Rightarrow \vec{c}\cdot\vec{a} = |\vec{c}||\vec{a}|\cos\gamma = \lambda^2\cos\gamma$

Now, $|\vec{a}+\vec{b}+\vec{c}|^2 \geq 0$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}) \geq 0$

$\Rightarrow 3\lambda^2 + 2\lambda^2(\cos\alpha+\cos\beta+\cos\gamma) \geq 0$

$\Rightarrow \cos\alpha+\cos\beta+\cos\gamma \geq -\frac{3}{2}$

9. 2

Given, $\vec{a}\cdot\vec{b} = \vec{a} \Rightarrow \vec{a}$ is perpendicular to $\vec{b}$.

$\vec{a}\cdot\vec{c} = 0 \Rightarrow \vec{a}$ is perpendicular to $\vec{c}$.

$\therefore\ \vec{a}$ is perpendicular to the plane of $\vec{b}$ and $\vec{c}$.

Also $\vec{a}$ is a unit vector.

$\therefore\ \vec{a} = \pm\dfrac{\vec{b}\times\vec{c}}{|\vec{b}\times\vec{c}|}$ .....(1)

But $|\vec{b}\times\vec{c}| = |\vec{b}||\vec{c}|\sin\dfrac{\pi}{6} = 1\cdot 1\cdot\dfrac{1}{2}$

$\therefore$ From (1) we have $\vec{a} = \pm 2(\vec{b}\times\vec{c})\ \therefore\ n = \pm 2$

10. 2

$|\vec{a}| = 3$, $|\vec{a}\times\vec{b}| = |2\hat{i}+2\hat{j}+\hat{k}| = 3\ \Rightarrow\ |\vec{c}-\vec{a}| = 2\sqrt{2}$

$\Rightarrow |\vec{c}-\vec{a}|^2 = 8\ \Rightarrow\ |\vec{c}|^2 + |\vec{a}|^2 - 2(\vec{a}\cdot\vec{c}) = 8$

$\Rightarrow c^2 + 9 - 2c = 8$ $[\because\ \vec{a}\cdot\vec{c} = |\vec{c}|]$

$\Rightarrow\ c^2 - 2c + 1 = 0$ $[\because\ |\vec{c}| = 1]$

$\Rightarrow |(\vec{a}\times\vec{b})\times\vec{c}| = |\vec{a}\times\vec{b}||\vec{c}|\sin(30) = 3\times 1\times\dfrac{1}{2} = \dfrac{3}{2}$

11. C $\overline{AD} = \dfrac{\overline{AB}+\overline{AC}}{2}, \overline{BE} = \dfrac{\overline{BA}+\overline{BC}}{2}, \overline{CF} = \dfrac{\overline{CA}+\overline{CB}}{2}$

12. 3

$(\vec{a}+\vec{b})\times(\vec{a}-\vec{b}) = 2(\vec{b}\times\vec{a})$

$= 2\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -2 \\ 3 & 2 & 2 \end{vmatrix}$

$= 2(8\hat{i} - 8\hat{j} + 4\hat{k})$

Required vector $= \pm 12\dfrac{(2\hat{i}-2\hat{j}-\hat{k})}{3} = \pm 4(2\hat{i}-2\hat{j}-\hat{k})$

13. 2

$4\hat{i} + (3-\lambda_2)\hat{j} + 6\hat{k} = 4\hat{i} + 2\lambda_1\hat{j} + 6\hat{k}$

$\Rightarrow\ 3 - \lambda_2 = 2\lambda_1\ \Rightarrow\ 2\lambda_1 + \lambda_2 = 3$ .... (1)

Given $\vec{a}\cdot\vec{c} = 0$

$\Rightarrow 6 + 6\lambda_1 + 3(\lambda_3 - 1) = 0\ \Rightarrow\ 2\lambda_1 + \lambda_3 = -1$ ..... (2)

Now, $(\lambda_1, \lambda_2, \lambda_3) = (\lambda_1, 3-2\lambda_1, -1-2\lambda_1)$

Now check the options, option (2) is correct

14. 4

Projection of $\vec{b}$ on $\vec{a} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|} = |\vec{a}|$

$\Rightarrow b_1 + b_2 = 2$ ...... (1)

and $(\vec{a}+\vec{b}) \perp \vec{c} \Rightarrow (\vec{a}+\vec{b})\cdot\vec{c}$

$\Rightarrow 5b_1 + b_2 = -10$ ..... (2)

From (1) and (2) $\Rightarrow b_1 = -3$ and $b_2 = 5$ then

$|\vec{b}| = \sqrt{b_1^2 + b_2^2 + 2} = 6$

15. 3

$\vec{b}\times\vec{c} - \vec{b}\times\vec{a} = \vec{0}$

$\vec{b}\times(\vec{c}-\vec{a}) = \vec{0}$

$\vec{b} = \lambda(\vec{c}-\vec{a})$ ..... (1)

$\vec{a}\cdot\vec{b} = \lambda(\vec{a}\cdot\vec{c} - \vec{a}^2)$

$4 = \lambda(0-6) \Rightarrow \lambda = \frac{-4}{6} = \frac{-2}{3}$

From (1) $\vec{b} = \frac{-2}{3}(\vec{c}-\vec{a})$

$\vec{c} = \frac{-3}{2}\vec{b} + \vec{a} = \frac{-1}{2}(\hat{i}+\hat{j}+\hat{k})$

$\boxed{\vec{b}\cdot\vec{c} = \frac{-1}{2}}$

16. 2

Vector perpendicular to plane containing the vectors $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+3\hat{k}$ is parallel to vector

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = \hat{i} - 2\hat{j} + \hat{k}$$

$\therefore$ Required magnitude of projection

$$= \frac{|(2\hat{i}+3\hat{j}+\hat{k})\cdot(\hat{i}-2\hat{j}+\hat{k})|}{|\hat{i}-2\hat{j}+\hat{k}|}$$

$$= \frac{|2-6+1|}{|\sqrt{6}|} = \frac{3}{\sqrt{6}} = \sqrt{\frac{3}{2}}$$

17. 4

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 2 & x\\ 1 & -1 & 1\end{vmatrix}=(2+x)\hat{i}+(x-3)\hat{j}-5\hat{k}$$

$$|\vec{a}\times\vec{b}|=\sqrt{4+x^2+4x+x^2+9-6x+25}=\sqrt{2x^2-2x+38}$$

$$\Rightarrow |\vec{a}\times\vec{b}|\ge\sqrt{\frac{75}{2}}\Rightarrow |\vec{a}\times\vec{b}|\ge 5\sqrt{\frac{3}{2}}$$

18. 1

Angle bisector is $x-y=0$

$$\Rightarrow \frac{|\beta-(1-\beta)|}{\sqrt{2}}=\frac{3}{\sqrt{2}}$$

$\Rightarrow |2\beta-1|=3$

$\Rightarrow \beta=2$ or $-1$

19. 4

We know that the unit vector along bisector of unit vectors $\vec{u}$ and $\vec{v}$ is $\dfrac{\vec{u}+\vec{v}}{2\cos\frac{\theta}{2}}$, where $\theta$ is the angle between vectors $\vec{u}$ and $\vec{v}$.

Also, in an isosceles $\Delta ABC$ in which AB = AC, the median and bisector from A must be same line.

20. A $\quad \vec{b}-2\vec{c}=\lambda\vec{a}\Rightarrow\vec{b}=2\vec{c}+\lambda\vec{a}\Rightarrow|\vec{b}|^2=|2\vec{c}+\lambda\vec{a}|^2\Rightarrow 16=4|\vec{c}|^2+\lambda^2|\vec{a}|^2+4\lambda\vec{a}.\vec{c}$

$$\Rightarrow 16=4+\lambda^2+4\lambda\frac{1}{4}\Rightarrow\lambda^2+\lambda-12=0\Rightarrow\lambda=3,-4$$

**SECTION II (NUMERICAL)**

21. 3 $\quad 6-2\vec{a}\cdot\vec{b}-2\vec{b}\cdot\vec{c}-2\vec{c}\cdot\vec{a}=9$

$$\Rightarrow(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{d})=\frac{-3}{2}\Rightarrow|\vec{a}+\vec{b}+\vec{c}|^2\ge 0$$

$$\Rightarrow 3+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})\ge 0$$

$$\Rightarrow \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\ge\frac{-3}{2}.$$

Since, $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=\dfrac{-3}{2}$

$$\Rightarrow|\vec{a}+\vec{b}+\vec{c}|=0\Rightarrow\vec{a}+\vec{b}+\vec{c}=0$$

$$\Rightarrow|2\vec{a}+5(-\vec{a})|=|3\vec{a}|=3$$

22. 1

If $\vec{d} = \alpha\vec{a} + \beta\vec{b} + \gamma\vec{c}$ by symmetry $\alpha = \beta = \gamma = k$

$\therefore\ \vec{d} = k(\vec{a}+\vec{b}+\vec{c}) \Rightarrow \vec{d}\cdot\vec{a} = k(1+\vec{b}\cdot\vec{a}+\vec{c}\cdot\vec{a})$

$\cos\alpha = k(1+2\cos\theta)$ ..... (1)

$\vec{d}\cdot\vec{d} = 3K\cos\alpha \Rightarrow 3K\cos\alpha = 1$ ......(2)

From (1) and (2), $3\cos^2\alpha = 1+2\cos\theta$

$$\Rightarrow 3\left(\frac{1+\cos 2\alpha}{2}\right) = 1+2\cos\theta$$

$\Rightarrow 3+3\cos 2\alpha = 2+4\cos\theta \Rightarrow 4\cos\theta - 3\cos 2\alpha = 1$.

23. 3

$\vec{V}$ is coplanar with $\vec{V}_1$ and $\vec{V}_2$ and perpendicular to $\vec{V}_3$.

Let $\vec{V} = \lambda(\vec{V}_1\times\vec{V}_2)\times\vec{V}_3 = -\lambda[\vec{V}_3\times(\vec{V}_1\times\vec{V}_2)]$

$= -\lambda[(\vec{V}_3\cdot\vec{V}_2)\vec{V}_1 - (\vec{V}_3\cdot\vec{V}_1)\vec{V}_2] = -\lambda[-5(\hat{i}+\hat{j}-2\hat{k})-(-2)(\hat{i}-2\hat{j}+\hat{k})$

24. 5

$|\vec{a}| = |\vec{b}| = 1,\ \vec{a}\cdot\vec{b} = 0$

Let $\vec{\ell} = (\vec{a}\times\vec{b})\times(\vec{a}-2\vec{b}) = (\vec{a}\times\vec{b})\times\vec{a} - 2(\vec{a}\times\vec{b})\times\vec{b}$

$= |\vec{a}|^2\vec{b} - (\vec{a}\cdot\vec{b})\vec{a} - 2(\vec{a}\cdot\vec{b})\vec{b} + 2|\vec{b}|\vec{a} = \vec{b}+2\vec{a}$

$\Rightarrow (2\vec{a}+\vec{b})\cdot\vec{\ell} = |2\vec{a}+\vec{b}|^2 = 5$.

25. 1

$[\vec{a}\ \vec{b}\ \vec{c}] = 0 \Rightarrow [\vec{a}\ \vec{b}\ \vec{c}]^2 = 0$

$$\Rightarrow \begin{vmatrix} \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c} \\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c} \\ \vec{c}\cdot\vec{a} & \vec{c}\cdot\vec{b} & \vec{c}\cdot\vec{c} \end{vmatrix} = \begin{vmatrix} 1 & \cos\gamma & \cos\beta \\ \cos\gamma & 1 & \cos\alpha \\ \cos\beta & \cos\alpha & 1 \end{vmatrix} = 0$$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma - 2\cos\alpha\cos\beta\cos\gamma = 1$

## SECTION - II

**JEE ADVANCED LEVEL**

26. A Let $\vec{r} = x_1\hat{a} + x_2\hat{b} + x_3\left(\hat{a}\times\hat{b}\right) \Rightarrow \vec{r}.\hat{a} = x_1 + x_2\hat{a}.\hat{b} + x_3\hat{a}.\left(\hat{a}\times\hat{b}\right) = x_1$

Also, $\vec{r}.\hat{b} = x_1\hat{a}.\hat{b} + x_2 + x_3\hat{b}.\left(\hat{a}+\hat{b}\right) = x_2$ and $\vec{r}.\left(\hat{a}\times\hat{b}\right) = x_1\hat{a}.\left(\hat{a}\times\hat{b}\right) + x_2\hat{b}.\left(\hat{a}\times\hat{b}\right)$

$+x_3\left(\hat{a}\times\hat{b}\right).\left(\hat{a}\times\hat{b}\right)=x_3 \qquad \Rightarrow \vec{r}=\left(\vec{r}.\hat{a}\right)\hat{a}+\left(\vec{r}.\hat{b}\right)\hat{b}+\left(\vec{r}.\left(\hat{a}\times\hat{b}\right)\right)\left(\hat{a}\times\hat{b}\right)$

27. C $\quad \vec{a}\times\left(\hat{i}+2\hat{j}+\hat{k}\right)=\hat{i}-\hat{k}=\left[\hat{j}\times\left(\hat{i}+2\hat{j}+\hat{k}\right)\right]\Rightarrow\left(\vec{a}-\hat{j}\right)\times\left(\hat{i}+2\hat{j}+\hat{k}\right)=\vec{0}$

$\Rightarrow \vec{a}-\hat{j}=\lambda\left(\hat{i}+2\hat{j}+\hat{k}\right) \qquad \Rightarrow \vec{a}=\lambda\hat{i}+(2\lambda+1)\hat{j}+\lambda\hat{k},\lambda\in R$

28. B

$\left|\vec{a}\times\vec{b}-\vec{a}\times\vec{c}\right|=\left|\vec{a}\times\left(\vec{b}-\vec{c}\right)\right|^2=\left|\vec{a}\right|^2\left|\vec{b}-\vec{c}\right|^2-\left(\vec{a}.\left(\vec{b}-\vec{c}\right)\right)^2=\left|\vec{b}-\vec{c}\right|^2=\left|\vec{b}\right|^2+\left|\vec{c}\right|^2-2\left|\vec{b}\right|\left|\vec{c}\right|\cos\frac{\pi}{3}=1$

29. C $\quad 1+9\left(\vec{a}.\vec{b}\right)^2-6\left(\vec{a}.\vec{b}\right)+4\left|\vec{a}\right|^2+\left|\vec{b}\right|^2+9\left|\vec{a}\times\vec{b}\right|^2+4\vec{a}.\vec{b}=47$

$\Rightarrow 1+4+4+36-4\cos\theta=47\Rightarrow\cos\theta=-\frac{1}{2}\Rightarrow$ Angle between $\vec{a}$ and $\vec{b}$ is $\frac{2\pi}{3}$

30. C $\quad$ Let $\vec{c}=\left(2\hat{i}+3\hat{j}+4\hat{k}\right)$

$\vec{a}\times\vec{c}=\vec{c}\times\vec{b}$

31. B $\quad \vec{c}=x\vec{a}+x\vec{b}+y\left(\vec{a}\times\vec{b}\right); \qquad \vec{a}.\vec{c}=\vec{b}.\vec{c}=\cos\theta$ [as $\vec{a},\vec{b}$ and $\vec{c}$ are unit vectors]

$\Rightarrow \vec{a}.\vec{c}=x\left|\vec{a}\right|^2\Rightarrow x=\cos\theta$

Also, $\vec{b}.\vec{c}=x\left|\vec{b}\right|^2=\cos\theta$

$\Rightarrow \vec{c}=\cos\theta\left(\vec{a}+\vec{b}\right)+y\left(\vec{a}\times\vec{b}\right)$

Now, $\left|\vec{c}\right|^2=1=\left(\cos\theta\left(\vec{a}+\vec{b}\right)+y\left(\vec{a}\times\vec{b}\right)\right)$

$\left(\cos\theta\left(\vec{a}+\vec{b}\right)+y\left(\vec{a}\times\vec{b}\right)\right)$

$\Rightarrow 1=\cos^2\theta\left(\vec{a}+\vec{b}\right).\left(\vec{a}+\vec{b}\right)+y^2\left(\vec{a}\times\vec{b}\right).\left(\vec{a}\times\vec{b}\right)\Rightarrow 1=\cos^2\theta\left(\left|\vec{a}\right|^2+\left|\vec{b}\right|^2\right)+y^2\left|\vec{a}\right|^2\left|\vec{b}\right|^2$

$\Rightarrow 1=2\cos^2\theta+y^2\Rightarrow\cos^2\theta=\frac{1}{2}-\frac{y^2}{2} \qquad \Rightarrow 0\le\cos^2\theta\le\frac{1}{2}$

$\Rightarrow -\frac{1}{\sqrt{2}}\le\cos\theta\le\frac{1}{\sqrt{2}}\Rightarrow\theta\in\left[\frac{\pi}{4},\frac{3\pi}{4}\right]$

32. D $\quad \left(\left(\vec{a}\times\vec{b}\right)+\left(\vec{a}\times\vec{c}\right)\right)\times\left(\vec{b}\times\vec{c}\right)=\left(\vec{a}\times\vec{b}\right)\times\left(\vec{b}\times\vec{c}\right)+\left(\vec{a}\times\vec{c}\right)\times\left(\vec{b}\times\vec{c}\right)$

$=\left(\left(\vec{a}\times\vec{b}\right).\vec{c}\right)\vec{b}-\left(\left(\vec{a}\times\vec{b}\right).\vec{b}\right)\vec{c}+\left(\left(\vec{a}\times\vec{c}\right).\vec{c}\right)\vec{b}-\left(\left(\vec{a}\times\vec{c}\right).\vec{b}\right)\vec{c}=\left[\vec{a}\ \vec{b}\ \vec{c}\right]\left(\vec{b}+\vec{c}\right)=0$

33. A $\quad \vec{a}\times\left(\vec{b}\times\vec{c}\right)=\left(\vec{a}.\vec{c}\right)\vec{b}-\left(\vec{a}.\vec{b}\right)\vec{c}=5\left(\hat{i}+2\hat{j}+2\hat{k}\right)-6\left(\hat{i}+\hat{j}+2\hat{k}\right)$

$\Rightarrow (1+\alpha)\hat{i}+\beta(1+\alpha)\hat{j}+\gamma(1+\alpha)(1+\beta)\hat{k}=-\hat{i}+4\hat{j}-2\hat{k}$

$\Rightarrow 1+\alpha=-1,\beta=-4$ and $\gamma(-1)(-3)=-2\Rightarrow\gamma=-\frac{2}{3}$

## SECTION IV (More than one correct

34. A,B,C For coplanar vectors, $\begin{vmatrix} 1 & 2 & 3 \\ 0 & \lambda & \mu \\ 0 & 0 & 2\lambda-1 \end{vmatrix} = 0 \Rightarrow (2\lambda-1)\lambda = 0 \Rightarrow \lambda = 0, \frac{1}{2}$

35. B,C Let $\bar{\alpha} = \hat{i} + x\hat{j} + 3\hat{k}, \bar{\beta} = 4\hat{i} + (4x-2)\hat{j} + 2\hat{k}$

Given, $2|\bar{\alpha}| = |\bar{\beta}| \quad \Rightarrow 2\sqrt{10+x^2} = \sqrt{20+4(2x-1)^2}$

$\Rightarrow 10 + x^2 = 5 + (4x^2 - 4x + 1) \Rightarrow 3x^2 - 4x - 4 = 0 \Rightarrow x = 2, -\frac{2}{3}$

36. A,B,C $\bar{x}.\bar{y} = \bar{y}.\bar{z} = \bar{z}.\bar{x} = \sqrt{2} \times \sqrt{2} \times \frac{1}{2} = 1.$

Let $\bar{a} = \lambda(\bar{x} \times (\bar{y} \times \bar{z})) = \lambda((\bar{x}.\bar{z})\bar{y} - (\bar{x}.\bar{y})\bar{z}) = \lambda(\bar{y} - \bar{z})$

$\bar{a}.\bar{y} = \lambda$

$\therefore \bar{a} = (\bar{a}.\bar{y})(\bar{y} - \bar{z})$

Similarly $\bar{b} = (\bar{b}.\bar{z})(\bar{z} - \bar{x})$

$\bar{a}.\bar{b} = -(\bar{a}.\bar{y})(\bar{b} - \bar{z})$

## SECTION V - (Numerical type )

37. 5

$|p| = \frac{2\sqrt{2}}{3} \quad |q| = 1$

$d_1 = \bar{a} + \bar{b} = 6p - q$

$d_2 = \bar{a} - \bar{b} = -4p - 5q$

$|d_1|^2 = 36p^2 + q^2 - 12 \times \frac{2\sqrt{2}}{3} \times 1 \times \frac{1}{\sqrt{2}}$

$= 36 \times \frac{8}{9} + 1 - 12 \times \frac{2\sqrt{2}}{3} \times \frac{1}{\sqrt{2}}$

$= 32 + 1 - 8 = 25$

$|d_1| = 5$

38. 3

$\vec{a} = xi + (x-1)j + k$

$\vec{b} = (x+1)i + j + ak$

$\vec{a}\cdot\vec{b} > 0$

$x(x+1) + (x-1) + a > 0$

$x^2 + x + x - 1 + a > 0$

$1x^2 + 2x + a - 1 > 0$

$b^2 - 4ac < 0$

$4 - 4(a-1) < 0$

$1 - a + 1 < 0$

$\underline{\underline{2 < a}}$ $\quad a > 2$

least value $\underline{\underline{a = 3}}$

39. 2

$|\bar{a}| = |\bar{b}| = 1 \quad |\bar{a}+\bar{b}| = \sqrt{3}$

$\bar{c} - \bar{a} - 2\bar{b} = 3(\bar{a}\times\bar{b})$

$(\bar{c} - \bar{a} - 2\bar{b})\cdot\bar{b} = 3(\bar{a}\times\bar{b})\cdot\bar{b}$

$\bar{c}\cdot\bar{b} - \bar{a}\cdot\bar{b} - 2|b|^2 = 0.$

$c\cdot b = \frac{1}{2} + 2|b|^2 = 5/2$

$[c\cdot b] = \left[\frac{5}{2}\right] = 2$

$|a+b|^2 = |a|^2 + |b|^2 + 2\bar{a}\cdot\bar{b}$

$3 = 1 + 1 + 2a\cdot b.$

$\frac{1}{2} = \underline{\underline{a\cdot b}}$

**SECTION V - (Numerical type )**

40. A A-Q,B-S,C-P,D-R