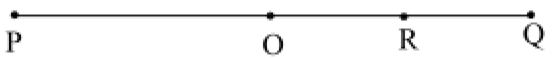
CHAPTER - 10
OSCILLATIONS

SYNOPSIS

If a body repeats the same motion after equal time interval that motion is called periodic motion.

If the periodic motion is a to and fro motion along a st. line on either side of the mean position that periodic motion is called SHM.

All SHM are periodic motion but all periodic motions are not SHM.



Consider a body making SHM along a line PQ on either side of mean position O, with an angular freq. $[\omega]$.

If R is the position of the body after t sec OR is the displacement made in t sec from the mean position.

The maximum displacement from the mean position to the extreme position is called the amplitude [A].

ie, OP = A and OQ = A

Length of the line $\boxed{PQ = OP + OQ = 2A}$

The displacement made in t sec from the mean position, along Y axis is given by

$Y = A\sin\omega t$

Along X axis is given by $x = A\cos\omega t$

If there is phase difference ϕ.

$\boxed{y = A\sin(\omega t \pm \phi)}$ and $\boxed{x = A\cos(\omega t \pm \phi)}$

Velocity of the body t sec after crossing the mean position is

$\boxed{V = A\omega\cos\omega t}$.

Velocity after a displacement y from the mean position is

$\boxed{V = \omega\sqrt{A^2 - y^2}}$.

At the mean position , y = 0

$\therefore V = \omega \times A$, the maximum velocity . ie, $\boxed{V_{max} = \omega \times A}$.

At extreme position, y = A

$\therefore V = 0$.

Acceleration of the body after t sec is,

$\boxed{a = -\omega^2 A \sin \omega t}$.

But A $\sin \omega t = y$.

$\therefore$ Acceleration at a displacement y is,

$\boxed{a = -\omega^2 y}$.

- ve sign means displacement y is away from the men position but acceleration is towards the mean position.

At the mean position, y =0

$\therefore$ acceleration a = 0.

At extreme position , y = A.

Acceleration $a = -\omega^2 A$, the maximum acceleration.

ie, $\boxed{a_{max} = -\omega^2 A}$

If $y = A \sin \omega t$, is the displacement of a body in SHM at t seconds, $\left(\frac{dy}{dt}\right)$ is the velocity and $\left(\frac{d^2y}{dt^2}\right)$ is the acceleration at t seconds

$$\therefore \frac{d^2y}{dt^2} = -\omega^2 A \sin \omega t = -\omega^2 y$$

$$\therefore \frac{d^2y}{dt^2} + \omega^2 y = 0$$

This equation is called the differential equation of SHM

Energy of SHM

$$KE = \frac{1}{2} m v^2,$$

at y displacement , $v = \omega\sqrt{A^2 - y^2}$

$\therefore$ at y displacement, $KE = \frac{1}{2} m\left[\omega\sqrt{A^2 - y^2}\right]^2$

$$\boxed{KE = \frac{1}{2} m\omega^2 \left(A^2 - y^2\right)}$$

$\therefore$ at mean position, y =0,

$\boxed{KE = \frac{1}{2}m\omega^2A^2}$.

At extreme position , y = A.

$\boxed{\therefore K.E = 0}$.

To displace a body of mass m through a distance y, an amount of work $= \frac{1}{2}m\omega^2y^2$, is to be done.

This work will be stored as PE at the displaced position

$\therefore$ PE at a displacement y is,

$\boxed{PE = \frac{1}{2}m\omega^2y^2}$.

At mean position , y = 0 $\therefore PE = 0$.

At extreme position , y = A $\boxed{\therefore PE = \frac{1}{2}m\omega^2A^2}$.

Total energy at any position is the sum of KE and PE at that position.

$\therefore$ TE at any position = KE + PE $= \frac{1}{2}mw^2A^2$, is a constant.

	At mean position	**At extreme position**	**At y displacement**
1)	displacement y = 0	displacement y = A, the maximum	displacement 0 < y < A
2)	$v = \omega \times A$, the max	v = 0	$v = \omega\sqrt{a^2 - y^2}$
3)	acceleration a = 0	$-\omega^2 A$ the max	$\boxed{a = -\omega^2 y}$
4)	$KE = \frac{1}{2}m\omega^2A^2$, the max	KE = 0	$KE = \frac{1}{2}m\omega^2\left(a^2 - y^2\right)$
5)	PE = 0	$PE = \frac{1}{2}m\omega^2A^2$, the max	$PE = \frac{1}{2}m\omega^2y^2$
6)	$TE = \frac{1}{2}m\omega^2A^2$	$TE = \frac{1}{2}m\omega^2A^2$	$TE = \frac{1}{2}m\omega^2A^2$

$\therefore$ TE = KE at mean position = PE at extreme position. = $\frac{1}{2}m\omega^2A^2$

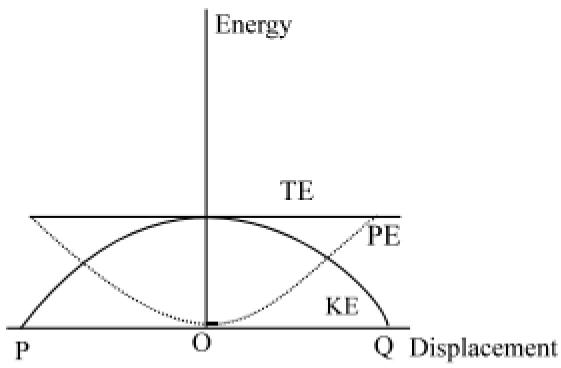
Time taken to complete one SHM is called the time period [T] given by

$T = \frac{2\pi}{\omega}$ or $\omega = \frac{2\pi}{T}$.

Number SHM completed in one second is called frequency (υ) given by

$\upsilon = \frac{1}{T}$ or $\upsilon = \frac{\omega}{2\pi}$.

Displacement energy graph.



When the body moves from extreme left P to extreme right Q, KE increases first from zero to maximum and then decreases to zero.

But PE first decreases from maximum to zero and then increases to maximum. Increase in KE is equal to decrease in PE and vice versa. Therefore TE is a constant at every position.

Simple Pendulum

Consider a simple pendulum of length *l*, with a spherical bob of mass m and radius r, at a place where acceleration due to gravity is g. Length of the pendulum is the distance from point of suspension to the centre of gravity (G) of the bob, If the centre of gravity of the sphere (G) and centre of sphere are same point.

If the bob is slightly displaced to one side and then released it makes SHM on either side of equilibrium position with a time period. $\boxed{T = 2\pi\sqrt{\frac{\ell}{g}}}$

Laws of simple pendulum :-

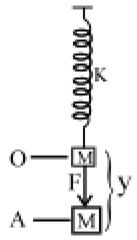
Ist Law - For small amplitude, T is independent of mass or size or shape of the bob.

2nd Law - For small amplitude $T \propto \sqrt{\ell}$.

3rd Law - For small amplitude, $T \propto \frac{1}{\sqrt{g}}$

In simple pendulum experiment, graph between *l* and T^2 is a straight line passing through the origin.

Oscillations of a spring



Mass M is suspended from the spring O is the equilibrium position of M. When M is pulled downward by a small force F it displaces y downwards.

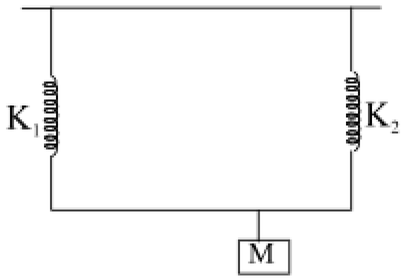
If the mass M is displaced, by y downwards, length of the spring also increases by y. $\therefore$ Displacement of mass M is equal to increase in length of spring.

$F \; \alpha \; y$

$F = K \times y$, K is the spring constant given by $K = \frac{\text{Force}}{\text{Displacement}} = \frac{F}{y}$

When mass M kg is suspended from spring and put into SHM, time period is given by $\boxed{T = 2\pi\sqrt{\frac{M}{K}}}$

Parallel combination



Let K_1 and K_2 be the constants of two springs in parallel carrying mass M. Each spring is hanging from the support and each spring is carrying mass M.

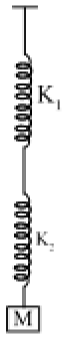
If the mass M is displaced by y down wards length of each spring increases by y.

Effective constant (K) of the combination is the sum of constants of all springs

$\boxed{\therefore K = K_1 + K_2}$.

$$\boxed{T = 2\pi\sqrt{\frac{M}{K_1 + K_2}}}$$

Series Combination



Let K_1 and K_2 be the constants of two springs in series carrying mass M. Any one of the given springs is hanging from the support and another spring is carrying mass M.

When M is displaced by y downwards length of each spring increases, but total increase in length of all springs will be the displacement y of mass M.

Reciprocal of effective constant K is the sum of the reciprocals of constants of all springs.

ie, $\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2}$

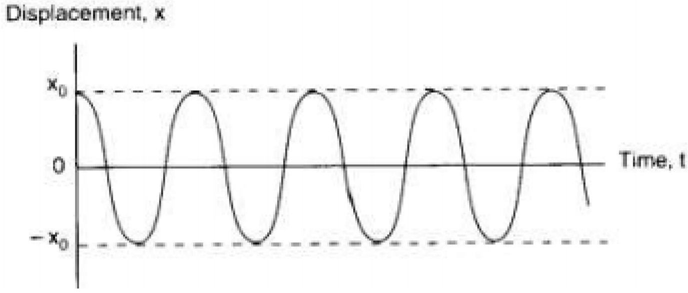
$$K = \frac{K_1.K_2}{K_1 + K_2}$$

$$T = 2\pi\sqrt{\frac{M(K_1 + K_2)}{K_1.K_2}}$$

[In parallel combination one end of each spring is connected to the support and other and of each spring is connected to the mass. In series combination one end of one spring is connected to the support and one end of another spring is connected to the mass.]

Free oscillation

The oscillation of a particle with fundamental frequency under the influence of restoring force are defined as free oscillations. The amplitude, frequency and energy of oscillation remains constant. Frequency of free oscillation is called natural frequency .



Damped oscillations

Differential equation for damped oscillators and its solution. In a real oscillator in a fluid, the damping force is proportional to the velocity vector v of the oscillator.

$F_d = -bv$

Where b is damping constant which depends on the characteristics of the fluid and the body that oscillates in it. The negative sign indicates that the damping force opposes the motion.

$\therefore$ Total restoring force $F = -kx - bv$

or $m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$ or $m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$

This is the differential equation for damped S.H.M. The solution of the equation is

$x(t) = ae^{-bt/2m}\cos(\omega' t + \phi)$

The amplitude of the damped S.H.M. is

$a' = ae^{-bt/2m}$

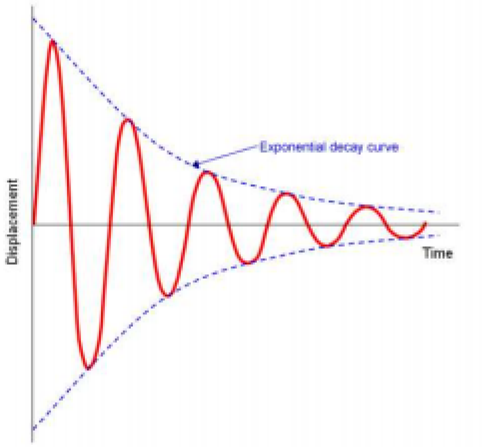
Where a is amplitude of undamped S.H.M. Clearly, a' decreases exponentially with time.

The angular frequency of the damped oscillator is

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

mechanical energy of the damped oscillator at any instant t will be

$$E(t) = \frac{1}{2}ka'^2 = \frac{1}{2}ka^2e^{-bt/m}$$



Note:

Small damping means that the dimensionless ratio $\left(b/\sqrt{km}\right) << 1$.

Forced oscillation

The oscillation in which a body oscillates under the influence of an external periodic force is known as forced oscillation

Resonance : When the frequency of external force is equal to the natural frequency of the oscillator, the body oscillates with maximum amplitude. This state is known as resonance. And this frequency is known as resonance frequency.

Differential equation of force oscillator

Let F(t) = $F_0 \cos \omega_d t$ is an external force applied to a damped oscillator.

The total force acting on the damped oscillator, F = – bv – kx + $F_0 \cos \omega_d t$

where –bv is the damping force and –kx is the linear restoring force

F + bv + kx = $F_0 \cos \omega_d t$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0 \cos \omega_d t$$

This is the differential equation of an oscillator of mass m on which a periodic force of (angular) frequency d is applied. The oscillator initially oscillates with its natural frequency ω. When we apply the external periodic force, the oscillation with the natural frequency die out, and the body oscillates with the (angular) frequency of the external periodic force.

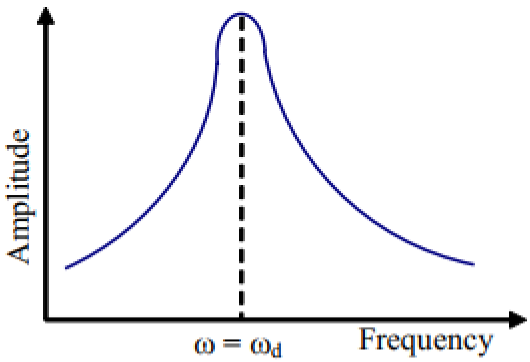
Motion of forced oscillator

The solution (displacement) of the above differential equation of forced oscillator can be written as

$$x(t) = A\cos(\omega_d t + \phi)$$

where A = $\dfrac{F_0}{\left\{m^2\left(\omega^2 - \omega_d^2\right)^2 + \omega_d^2 b^2\right\}^{1/2}}$ and $\tan\phi = \dfrac{-v_0}{\omega_d x_0}$

where m is the mass of the particle. v_0 and x_0 are the velocity and the displacement of the particle at time t = 0, (which is the moment when we apply the periodic force).



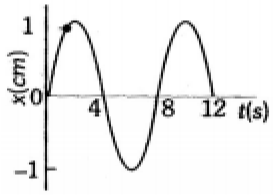
PART 1 - JEE MAIN

Section 1 - Straight objective type questions

1. A particle executes simple harmonic motion with a time period of 16s. At time t = 2s, the particle crosses the mean position while at t = 4s, its velocity is $4 ms^{-1}$. The amplitude of motion in metre is

   1) $\sqrt{2}\,\pi$ 2) $16\sqrt{2}\,\pi$ 3) $24\sqrt{2}\,\pi$ 4) $32\sqrt{2}/\pi$

2. The x-t graph of a particle undergoing simple harmonic motion is shown below. The acceleration of the particle at t = 4/3 s is



   1) $\frac{\sqrt{3}}{32}\pi^2 cm/s^2$ 2) $\frac{-\pi^2}{32} cm/s^2$ 3) $\frac{\pi^2}{32} cm/s^2$ 4) $-\frac{\sqrt{3}}{32}\pi^2 cm/s^2$

3. A body performs SHM with an amplitude A. At a distance $A/\sqrt{2}$ from the mean position, the correct relation between KE and PE is

   1) $KE = \frac{PE}{2}$ 2) $KE = \sqrt{2}\,PE$ 3) KE = PE 4) $KE = \frac{PE}{\sqrt{2}}$

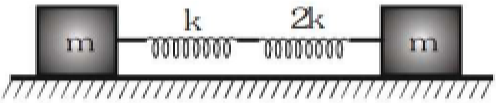
4. A horizontal spring block of mass M executes simple harmonic motion with an amplitude A when the block is passing through its equilibrium position an object of mass m is put on it and the two move together. Find new amplitude of vibration.

   1) $A\sqrt{\frac{M}{M+m}}$ 2) A 3) $\sqrt{\frac{M+m}{M}}$ 4) $\sqrt{\frac{(M+m)^2}{M}}A$

5. A spring of spring constant 150 N/m is divided into 5 equal parts, 3 of then are connected in parallel with a block of mass 5kg. Then time period of oscillation of the system is:

1) $\frac{5}{\sqrt{2}}$ second 2) $\frac{2}{3\sqrt{5}}$ second 3) $5\sqrt{2}$ second 4) $\frac{3\sqrt{2}}{5}$ second

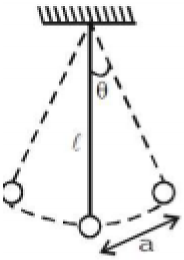
6. A system is shown in the figure. The time period for small oscillations of the two blocks will be



1) $2\pi\sqrt{\frac{3m}{k}}$ 2) $2\pi\sqrt{\frac{3m}{2k}}$ 3) $2\pi\sqrt{\frac{3m}{4k}}$ 4) $2\pi\sqrt{\frac{3m}{8k}}$

7. In each of the following questions, a statement of Assertion (A) is given followed by a corresponding statement of Reason (R) just below it. Of the statements mark the correct answer as

Statement - I: The motion of a simple pendulum is simple harmonic only for $a << \ell$



and

Statement 2 - Motion of a simple pendulum is SHM for small angular displacement

1) Statement -1 is True, Statement - 2 is True; Statement - 2 is a correct explanation for Statement - I

2) Statement - 1 is True, Statement - 2 is True; Statement - 2 is NOT a correct explanation for Statement - I

3) Statement - 1 is True, Statement - 2 is False

4) Statement - 1 is False, Statement - 2 is True

8. The frequency of a simple pendulum is n oscillations per minute while that of another is (n + 1) oscillations per minute. The ratio of length of first pendulum to the length of second is

1) $\frac{n}{n+1}$ 2) $\left(n+\frac{1}{n}\right)^2$ 3) $\left(\frac{n+1}{n}\right)^2$ 4) $\left(\frac{n}{n+1}\right)^2$

9. Statement - I: When a girl sitting on a swing stands up, the periodic time of the swing will increase

and

Statement -2: In standing position of a girl, the length of the swing will decrease

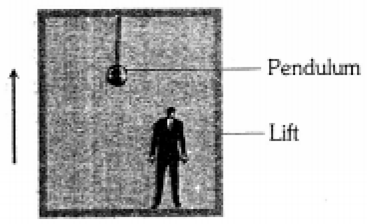
1) Statement -1 is True, Statement - 2 is True; Statement - 2 is a correct explanation for Statement - I

2) Statement - 1 is True, Statement - 2 is True; Statement - 2 is NOT a correct explanation for Statement - I

3) Statement - 1 is True, Statement - 2 is False

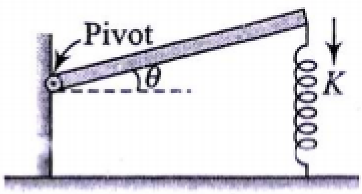
4) Statement - 1 is False, Statement - 2 is True

10. A man measures the period of a simple pendulum inside a stationary lift and finds it to be T s. If the lift accelerates upwards with an acceleration g/4, then the period of the pendulum will be



1) T 2) $\frac{T}{4}$ 3) $\frac{2T}{\sqrt{5}}$ 4) $2T\sqrt{5}$

11. A horizontal rod of mass m and length L is pivoted at one end. The rod's other end is supported by a spring of force constant k. The rod is displaced by a small angle θ from its horizontal equilibrium position and released. The angular frequency of the subsequent simple harmonic motion is



1) $\sqrt{\frac{3k}{m}}$ 2) $\sqrt{\frac{k}{3m}}$ 3) $\sqrt{\frac{3k}{m}+\frac{3g}{2L}}$ 4) $\sqrt{\frac{k}{m}}$

12. A uniform disc of radius 5 cm and mass 200g is fixed at its centre to a metal wire the other end of which is fixed with a clamp. The hanging disc is rotated about the wire through an angle and is released. If the disc makes torsional oscillations with time period 0.2s find torsional constant of wire

1) 25 kg m^2/s^2 2) 2.5 kg m^2/s^2

3) 0.25 kg m^2/s^2 4) 250 kg m^2/s^2

13. A moving particle of mass has one - dimensional potential energy $U(x) = ax^2 + bx^4$, where 'a' and 'b' are positive constants. The angular frequency of small oscillations about the minima of the potential energy is equal to

1) $\pi\sqrt{\frac{a}{2b}}$ 2) $2\sqrt{\frac{a}{m}}$ 3) $\sqrt{\frac{2a}{m}}$ 4) $\sqrt{\frac{a}{2m}}$

14. A particle is performing S.H.M. with acceleration $a = 8\pi^2 - 4\pi^2 x$ where x is coordinate of the particle w.r.t. the origin. The parameters are in S.I. units. The particle is at rest at x = –2 at t = 0

1) coordinate of the particle w.r.t. origin at any time t is $2 - 4\cos 2\pi t$

2) coordinate of the particle w.r.t. origin at any time t is –2 + 4 $\sin 2\pi t$

3) coordinate of the particle w.r.t. origin at any time t is –4 + 2 $\cos 2\pi t$

4) the coordinate cannot be found because mass of the particle is not given

Section 2 - Integer type questions

15. A particle performs S.H.M. with time period T. The time taken by the particle to move from half the amplitude to the maximum displacement is T/n. Find n

16. In a certain oscillatory system (particle is performing SHM), the amplitude of motion is 5m and the time period is 4s. If the minimum time taken by the particle for passing between points, which are at distances of 4m and 3m from the centre and on the same side of it is t. Find 90t.

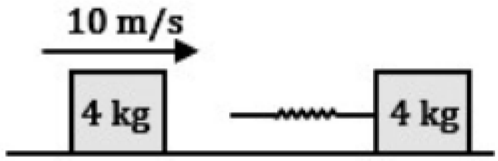
PART - II (JEE ADVANCED)

SECTION - III (Only one option correct type questions)

17. A particle is performing SHM on x-axis about mean position x = 0 having amplitude A and angular frequency ω. At t = 0 particle is at $x = \frac{A}{2}$ and moving in +ve x direction. If equation of SHM is x = **A** $\sin(\omega t + \phi)$ then ϕ is:

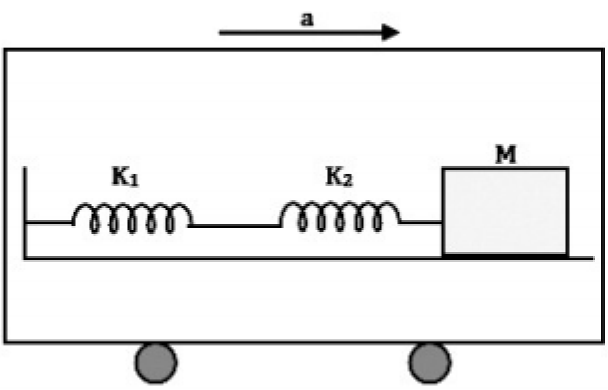
A) $\frac{\pi}{6}$ B) $\frac{\pi}{3}$ C) $\frac{2\pi}{3}$ D) $\frac{5\pi}{6}$

18. A 4kg block moving with 10 m/s strikes a spring of constant $\pi^2\ N/m$ attached to 4 kg block at rest kept on a smooth floor. The time for which rear moving block remain in contact with spring will be:



A) $\sqrt{2}\,\text{sec}$ B) $\frac{1}{\sqrt{2}}\text{sec}$ C) 1 sec D) $\frac{1}{2}\text{sec}$

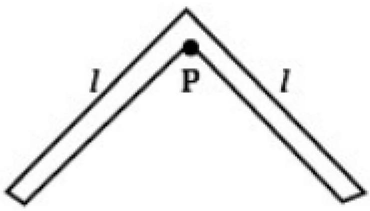
19. As shown, inside a cart that is accelerating horizontally at acceleration a there is a block of mass M connected to two light springs of force constants k_1 and k_2. The block can move without friction horizontally. Find the vibration frequency of the block.



A) $\frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{M}}$ B) $\frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{M}-a}$

C) $\frac{1}{2\pi}\sqrt{\frac{k_1k_2}{(k_1+k_2)M}}$ D) $\frac{1}{2\pi}\sqrt{\frac{k_1k_2}{(k_1+k_2)M}+a}$

20. A system of two identical rods (L-shaped) of mass m and length *l* are resting on a peg P as shown in the figure. If the system is displaced in its plane by a small angle θ, find the period of oscillations.



A) $2\pi\sqrt{\frac{\sqrt{2}\,\ell}{3g}}$ B) $2\pi\sqrt{\frac{2\sqrt{2}\,\ell}{3g}}$ C) $2\pi\sqrt{\frac{2\ell}{3g}}$ D) $3\pi\sqrt{\frac{\ell}{3g}}$

SECTION - IV (More than one correct answer)

21. The amplitude of a particle executing SHM about O is 10 cm. Then :

A) When the *K.E* of 0.64 of its max. *K.E.* Its displacement is 6cm from O.

B) When the displacement is 5 *cm* from O its *K.E.* is 0.75 of its max. *P.E.*

C) Its total energy at any point is equal to its maximum K.E.

D) Its velocity is half the maximum velocity when its displacement is half the maximum displacement

22. The postion vector of a particle that is moving in space is given by

$\bar{r}=(1+2\cos 2\omega t)\hat{i}+(3\sin^2 \omega t)\hat{j}+(3)\hat{k}$ in the ground frame. All units are in SI. Choose the correct statement(s):

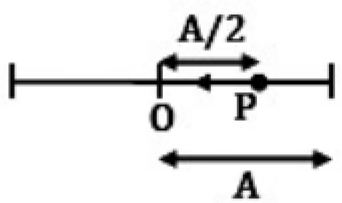
A) The particle executes SHM in the ground frame about the mean position $\left(1,\frac{3}{2},3\right)$

B) The particle executes SHM in a frame moving along the z-axis with a velocity of 3 m/s.

C) The amplitude of the SHM of the particle is $\frac{5}{2}\text{m}$.

D) The direction of the SHM of the particle is given by the vector $\left(\frac{4}{5}\hat{i}-\frac{3}{5}\hat{j}\right)$

23. A particle starts from a point P at a distance of A/2 from the mean position O and travels towards left as shown in the figure. If the time period of SHM, executed about O is T and amplitude A then the equation of motion of particle is:



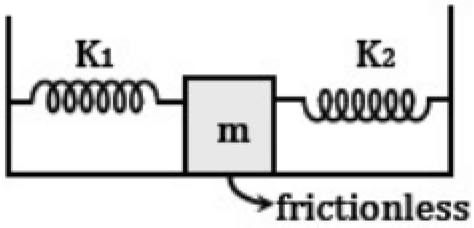
A) $x = A\sin\left(\frac{2\pi}{T}t+\frac{\pi}{6}\right)$

B) $x = A\sin\left(\frac{2\pi}{T}t+\frac{5\pi}{6}\right)$

C) $x = A\cos\left(\frac{2\pi}{T}t+\frac{\pi}{6}\right)$

D) $x = A\cos\left(\frac{2\pi}{T}t+\frac{\pi}{3}\right)$

24. Two springs with negligible masses and force constant of K_1 = 200 Nm^{-1} and K_2 = 160 Nm^{-1} are attached to the block of mass m = 10 kg as shown in the figure. Initially the blocks is at rest, at the equilibrium position in which both springs are neither stretched nor compressed. At time t = 0, a sharp impulse of 50 Ns is given to the block with a hammer.



A) Period of oscillations for the mass m is $\frac{\pi}{3}s$.

B) Maximum velocity of the mass m during its oscillation is 5 ms^{-1}

C) Data are insufficient to determine maximum velocity

D) Amplitude of oscillation is 0.42 m

Paragraph

A particle can move along x-axis under influence of a conservative force. The potential energy the particle is given by $U = 5x^2 - 20x + 2$ joule, where x is co-ordinate of the particle expressed in meter. The particle is released from rest at x = -3m

25. Amplitude of the particle is :

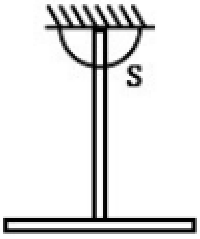
A) 3m B) 2m C) 5m D) None of these

26. The maximum x-coordinate of the particle:

A) -3m B) 7m C) 5m D) None of these

SECTION - V -Numerical Type questions

27. Two identical rods each of mass m and length L, are rigidly joined and then suspended in a vertical plane so as to oscillate freely about an axis normal to the planbe of paper passing through 'S' (point of suspension). Time period of such small oscillations $= \frac{2\pi}{3}\sqrt{\frac{\alpha\ell}{\beta g}}$, then find the value of $\alpha + \beta$.



28. A spring (spring constant k) having one end attached to rigid wall and other end attached to a block of mass m kept on a smooth surface as shown in figure. Initially spring is in its natural length at x = 0, now spring is compressed to x = -a and released. (Coefficient of restitution (e) = $\frac{1}{2}$). If velocity opf block just after first collision is $a\sqrt{\frac{nk}{16m}}$. Find the value of n.