CHAPTER - 22
LOGARITHM

Section-1: Single Correct Answer Type

1. (C)

Let $N = 3^{40}$

$\log_{10} N = 40\log_{10} 3 = 40(0.4771)$

$\log_{10} N = 19.084$

$\therefore$ N contains 20 digits

2. (A)

$\log_{10}(\tan 30^0 \cdot \tan 31^0 ..... \tan 45^0 .... \tan 60^0)$

$\log_{10} 1 = 0$

3. (B)

$N = 2^{-1000}$

$\log_{10} N = -1000\log_{10} 2$

$= -1000(0.3010) = -3010000.$

$\log_{10} N = -301$

$\log_{10} N = 300 + 1$

$\therefore$ In N after decimal point there are 300 zero's

4. (B)

$1 + abc = 1 + \log_{24} 12 \cdot \log_{48} 36 \cdot \log_{36} 24$

$= 1 + \log_{24} 12 \cdot \log_{36} 24 \cdot \log_{48} 36$

$= 1 + \log_{48} 12 = \log_{48} 48 + \log_{48} 12.$

$1 + abc = \log_{48} 576$

$1 + abc = \log_{48} 24^2 = 2\log_{48} 24$

$= 2\log_{36} 24 \cdot \log_{48} 36 = 2bc$

5. (D)

$$\log_4\left(1+\frac{1}{4}\right)+\log_4\left(1+\frac{1}{5}\right)+\log_4\left(1+\frac{1}{6}\right)+....+\log_4\left(1+\frac{1}{255}\right)$$

$$\log_4\left(\frac{5}{4}\right)+\log_4\left(\frac{6}{5}\right)+\log_4\left(\frac{7}{6}\right)+....+\log_4\left(\frac{256}{255}\right).$$

$$\log_4\left(\frac{5}{4}\times\frac{6}{5}\times\frac{7}{6}\times.....\times\frac{256}{255}\right)$$

$$\log_4\frac{256}{4} = \log_4 64 = 3.$$

6. (B)

$2\log_3(2^x-5)=\log_3 2+\log_3\left(2^x-\frac{7}{2}\right)$

$(2^x-5)^2=2\left(2^x-\frac{7}{2}\right)$

$(t-5)^2=2\left(t-\frac{7}{2}\right)$ $(\because t=2^x)$

$t^2-10t+25=2t-7$.

$t^2-12t+32$

$(t-8)(t-4)=0$

$t=8;t=4$

$2^x=2^3;2^x=2^2$

$x=3,x=2$ but $x\neq 2$ $(\because\ 2^x-5<0)$

$\therefore\ x=3$.

7. (D)

$\frac{1}{\log_a a+\log_a bc}+\frac{1}{1+\log_b ca}+\frac{1}{\log_c c+\log_c ab}$.

$\frac{1}{\log_a abc}+\frac{1}{\log_b abc}+\frac{1}{\log_c abc}$

$\log_{abc} a+\log_{abc} b+\log_{abc} c$

$\log_{abc}(abc)=1$.

8. (C)

$\log_{0.3}(x-1)<\log_{(0.3)^2}(x-1)^1$

$\log_{0.3}(x-1)<\frac{1}{2}\log_{0.3}(x-1)$

$\log_{0.3}(x-1)^2<\log_{0.3}(x-1)$.

$(x-1)^2<x-1$

$x^2-2x+1-x+1<0$

$x^2-3x+2<0$

$(x-2)(x-1)<0$

$x\in(1,2)$ …. (A)

and $\log_{0.3}(x-1)$ exist for

$x-1>0, x>1$ …. (3)

$\therefore\ A\cap B\Rightarrow x\in(1,2)$.

9. (B)

$\log_{\sqrt{3}}300=\log_{\sqrt{3}}3\times 5^2\times 2^2$

$=\log_{\sqrt{3}}3+2\log_{\sqrt{3}}5+2\log_{\sqrt{3}}2$

$=2+2a+2b$

$=2(a+b+1)$

10. (D)

$\log_2 2^2 \times 5 \cdot \log_2 2^4 \times 5 - \log_2 5 \cdot \log_2 2^6 \times 5$

$(2+\log_2 5)(4+\log_2 5) - \log_2 5(6+\log_2 5)$

$8 + 6\log_2 5 + (\log_2 5)^2 - 6\log_2 5 - (\log_2 5)^2$.

11. (B)

$\sum\left(\frac{\log a}{\log b}\cdot\frac{\log a}{\log c} - \frac{\log a}{\log a}\right) = 0$.

$\sum\left[(\log a)^3 - \log a \cdot \log b \cdot \log c\right] = 0$

$(\log a)^3 + (\log b)^3 + (\log c)^3 - 3\log a \cdot \log b \cdot \log c = 0$.

$\log a + \log b + \log c = 0$

$\log abc = 0$

$abc = 1$

12. (C)

$(2x)^{\log_2} = (3y)^{\log 3}$

Take logs on both side

$\log 2(\log 2 + \log x) = \log 3(\log 3 + \log y)$

$(\log 2)^2 + \log x \cdot \log 2 = (\log 3)^2 + \log 3 \cdot \log y \quad \ldots (1)$

and $3^{\log x} = 2^{\log y}$

Take logs on both side

$$\log x \cdot \log 3 = \log y \cdot \log 2$$

$$\log y = \frac{\log x \cdot \log 3}{\log 2} \text{ put in (1)}$$

$$(\log 2)^2 + \log x \cdot \log 2 = (\log 3)^2 + \log 3 \cdot \frac{\log x \cdot \log 3}{\log 2}$$

$$(\log x)\left[\log 2 - \frac{(\log 3)^2}{\log 2}\right] = (\log 3)^2 - (\log 2)^2 .$$

$$\therefore\ (\log x) = -\log 2 \ \Rightarrow\ x = \frac{1}{2} .$$

Section-II: One or More than one Correct

13. (ABC)

$3^x = 4^{x-1}$

Take logs on both side to the base 2

$\log_2 3^x = \log_2 4^{x-1}$

$x\log_2 3 = (x-1)2$

$x\log_2 3 - 2x = -2$

$x = \dfrac{-2}{\log_2 3 - 2} = \dfrac{2}{2-\log_2 3}$ .... (B)

$3^x = 4^{x-1}$

Take longs on both side to the base 4

$x\log 3 = (x-1) \Rightarrow x = \dfrac{1}{1-\log_4 3}$ .... (C)

$3^x = 4^{x-1}$

Take longs to the both side to the base 3

$x = (x-1)\log_3 4$

$x(1-\log_3 4) = -\log_3 4$

$\Rightarrow x = \dfrac{2\log_3 2}{2\log_3 2 - 1}$ .... (A)

14. (BC)

$\log_2(3^{2x-2}+7) = 2\log_2(3^{x-1}+1)$.

$\log_2\left(\dfrac{3^{2x-2}+7}{3^{x-1}+1}\right) = 2 \Rightarrow \dfrac{(3^{x-1})^2+7}{3^{x-1}+1} = 4$

$t^2+7 = 4t+4$ $(\because t = 3^{x-1})$.

$t^2-4t+3=0$, $t=1,3$

$3^{x-1} = 3^0, 3^1 \Rightarrow x = 1, 2$.

15. (BC)

$\log_{2x^2+2x+3}(x^2-2x) = 1$ where $x^2+6x+8>0, \neq 1$

$x^2-2x = 2x^2+2x+3$.

$x^2+4x+3=0$, $x=-1,-3$

$\therefore$ $x=-1$ satisfied, $x=-3$ not satisfies the given equation

$\therefore$ B & C

16. (ABCD)

17.

$\log_n 43!$

$\log_n 2 + \log_n 3 + ... + \log_n 43$

$\log_n 43!$

18. (D)

$2\log_{10} x - \log_x 10^{-2}$

$2\log_{10} x + 2\log_x 10$

$2\left(\log_{10} x + \frac{1}{\log_{10} x}\right) \geq 4$ ($\because$ AM $\geq$ GM)

$\therefore$ Minimum value $= 4$

19. (C)

20. (ABC)

$x^{(\log_3 x)^2 - \frac{9}{2}(\log_3 x) + 5} = 3^{3/2}$

Take longs on both side to the base 3

$\left[(\log_3 x)^2 - \frac{9}{2}(\log_3 x) + 5\right](\log_3 x) = \frac{3}{2}$

$\left(t^2 - \frac{9}{2}t + 5\right)t = \frac{3}{2}$ $(\because t = \log_3 x)$

$2t^3 - 9t^2 + 10t - 3 = 0$.

$t = 1$, $0 = 0$

$$t = 1 \left| \begin{array}{cccc} 2 & -9 & +10 & -3 \\ 0 & +2 & -7 & +3 \\ \hline 2 & -7 & +3 & \underline{|0} \end{array} \right.$$

$\therefore$ $2t^2 - 7t + 3 = 0$

$2t^2 - 6t - t + 3 = 0$

$2t(t-3) - 1(t-3) = 0$

$t = \frac{1}{2}, t = 3$.

$\log_3 x = 1, \frac{1}{2}, 3$

$x = 3, 3^{1/2}, 3^3$.

21. $x = 5$ is the solution

$\frac{1}{2}\log_2(x-1) = \log_2(x-3)$

$(x-1) = (x-3)^2$.

$x - 1 = x^2 - 6x + 9$

$x^2 - 7x + 10 = 0$

$x = 2, 5$

$x \neq 2, \therefore x = 5$.

Section-III: Numerical Value Type

22. (4)

Let $x = \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{.........\infty}}}}$

$x = \frac{1}{3\sqrt{2}}\sqrt{4-x}$

$18x^2 + x - 4 = 0$

$x = \frac{-1}{2}, x = \frac{4}{9}.$

$\therefore\ 6 + \log_{\frac{3}{2}}\frac{4}{9} = 6 - 2 = 4.$

23. (2)

$\log_{2^3} 2^7 - \log_9 \frac{1}{\sqrt{3}} = x$

$\frac{7}{3} - \log_{3^2} 3^{-1/2} = x$

$\frac{7}{3} + \frac{1}{4} = x.$

$x = \frac{28+3}{12},\ x = \frac{31}{12}$

$[x] = 2$

24. (5)

$\log_2\left(\log_2 \log_{2^2} 2^8\right) + 2\log_{2^{1/2}} 2$

$\log_2 \log_2 4 + 4$

$\log_2 2\log_2 2 + 4$

$1 + 4 = 5$

25. (11)