CHAPTER - 20

LINEAR INEQUILITIES

JEE MAIN - SECTION I

1. 4 $|x+3| \geq 10$

$x \geq 7$ or $x \leq -13$

$x \in (-\infty, -13] \cup [7, \infty)$

2. 4 $A = \ell b$ and $P = 2(\ell + b)$; $(\ell + b)^2 \geq 4\ell b$

$\left(\frac{P}{2}\right)^2 \geq 4A$; $P^2 \geq 16A$

3. 1 $\frac{2x-1}{3} \geq \frac{5(3x-2)-4(2-x)}{20}$; $20(2x-1) \geq 3(19x-18)$; $x \leq 2$; $x \in (-\infty, 2]$

Also, $-3 \leq 4 - \frac{7x}{2} \leq 18$; $-4 \leq x \leq 2$; $x \in [-4, 2]$

4. 2 $\frac{x+3}{x-2} - 2 \leq 0$; $\frac{7-x}{x-2} \leq 0 \Rightarrow \frac{x-7}{x-2} \geq 0$; $x \in (-\infty, 2) \cup [7, \infty]$

5. 4 $2(2x+3) - 10 < 6(x-2)$

$4x - 4 < 6x - 12$; $-2x < -8$; $x > 4$

Also, $\frac{2x-3}{4} + 6 \geq 2 + \frac{4x}{3}$; $\frac{2x-3+24}{4} \geq \frac{6+4x}{3}$; $\frac{2x-3+24}{4} \geq \frac{6+4x}{3}$; $39 \geq 10x$

$x \leq \frac{39}{10}$; $\therefore x > 4$ and $x \leq 3.9$

6. 1 $\frac{3-|x|}{4-|x|} \geq 0$; $(3-|x|)(4-|x|) \geq 0$; $(|x|-3)(|x|-4) \geq 0$

$|x|-3 \geq 0$ and $|x|-4 > 0$ or $|x|-3 \leq 0$ and $|x|-4 < 0$

$x \in (-\infty, -4) \cup (4, \infty) \cup [-3, 3]$

7. 4 $x^2-1\le 0$ and $x^2-x-2\ge 0$; $(x+1)(x-1)\le 0$ and $(x-2)(x+1)\ge 0$

$-1\le x\le 1$ and $x\in(-\infty,-1]\cup[2,\infty)$; $\therefore x\in\{-1\}$

8. 2 $8-x^3>0\Rightarrow x^3<8\Rightarrow x<2$; Also $|x+2|-|x|\ge 0$

$|x+2|\ge|x|\Rightarrow x\ge -1$; $\therefore x\in[-1,2)$

9. 3 $75\le A.M<80$

$$75\le\frac{95+72+73+83+x}{5}<80;375\le 323+x<400;52\le x<77$$

10. 4 $$\frac{2x+3}{x^2+x-12}<\frac{1}{2};\frac{2x+3}{x^2+x-12}-\frac{1}{2}$$

$$\frac{(x-6)(x+3)}{(x+4)(x-3)}>0;x\in(-\infty,-4)\cup(-3,3)\cup(6,\infty)$$

SECTION II (NUMERICAL)

11. 8 $A=(-\infty,1)\cup(3,\infty);$

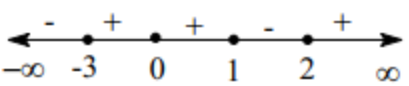
$B=(-\infty,-2)\cup(2,\infty)$

$C=(-\infty,-2]\cup[6,\infty)$

$A\cap B\cap C=(-\infty,-2)\cup[6,\infty)$

$Z\cap(A\cap B\cap C)^c=\{-2,-1,0,1,2,3,4,5\}$

12. 1 $$\frac{(x+3)(x-1)}{x^2(x-2)^3}\le 0$$



$x\in(-\infty,-3]\cup[1,2)$

13. 2 $-6\le a+3x\le 6$

$$\frac{-6-a}{3}\le x\le\frac{6-a}{3}$$

$$\frac{6-a}{3}=\frac{4}{3}\Rightarrow a=2$$

14. 4

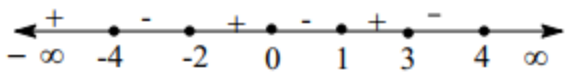
$$\frac{2x-3}{3} \geq \frac{x-1+6}{6}$$

$$4x-6 \geq x+5$$

$$3x \geq 11; x \geq \frac{11}{3} \approx 3.66$$

15. 0

$$\frac{(x+2)(x-1)(x-3)(x+4)}{x(x-4)} \geq 0$$



$$x \in (-\infty,-4] \cup [-2,0) \cup [1,3] \cup (4,\infty)$$

PART - II (JEE ADVANCED LEVEL)

SECTION - III (One correct answer)

16. C

$|r| < 1,\ r > -1$ or $r < 1$

$a = 5(1-r)\ \therefore\ a = 5 - 5r$

$\therefore\ 5r = 5 - a,\ r = \frac{5-a}{5}$

$\frac{5-a}{5} > -1\ \therefore\ 5 - a > -5\ \therefore\ a < 10$ ....(i)

$\frac{5-a}{5} < 1\ \therefore\ 5 - a < 5\ \therefore\ a > 0$ ....(ii)

From (i) and (ii) $\therefore\ 0 < a < 10$

17. C

18. C

$$\frac{4x-1}{3x+1} \geq 1$$

$4x - 1 \geq 3x + 1$

$x \geq 2,\ x \neq -\frac{1}{3},\ x \neq \frac{1}{4}$

19. C

$|2x-3| < |x+5|$ ... (i)

Let $x \geq \frac{3}{2}$, then $2x-3 \geq 0$ and $x+5 \geq 0$

Thus, $2x-3 < x+5 \Rightarrow x < 8$

Thus $x \in \left[\frac{3}{2}, 8\right)$ satisfies the above inequality for $x \geq \frac{3}{2}$

Now, let $-5 \leq x < \frac{3}{2}$

Then, $2x-3 < 0$ and $x+5 \geq 0$

So, $3-2x < x+5 \Rightarrow 3x > -2 \Rightarrow x > -2/3$

But, $-5 \leq x < 3/2$

Thus, $x \in (-2/3, 3/2)$ satisfies the given inequality (i)

Also, let $x < -5$. Then, $2x-3 < 0$

Also, $x+5 < 0$

$3-2x < -5-x \Rightarrow x > 8$ ... (ii)

However, $x < -5$

$\Rightarrow$ The above inequality (ii) does not hold.

$\Rightarrow$ We have checked all the required intervals.

$\Rightarrow$ $x \in [3/2, 8) \cup (-2/3, 3/2) \Rightarrow x \in (-2/3, 8)$.

Above is the required answer.

20. C $(a-b)^2+(b-c)^2+(c-a)^2 \geq 0$

$1 \geq ab+bc+ca$

$(a+b+c)^2 \geq 0$

$(a+b+c)^2 \geq 0$

$cab+bc+ca \geq -\frac{1}{2}, \left[\frac{-1}{2}, 1\right]$

21. A $\frac{(a+b)+(c+d)}{2} \geq \sqrt{(a+b)(c+d)}$

$1 \geq \sqrt{m}, m \leq 1, m > 0$

$0 < m \leq 1$]

22. A $AM \geq GM,\ 2\tan\alpha$

23. C

24. B For S_1 we have $\Rightarrow \frac{(x+2)(x^2+3x+5)}{x^2-3x+2} \leq 0$

$\Rightarrow x \in (-\infty, -2] \cup (1, 2)$

For S_2 we have , $B-A = R-(-2,5)3^x(3^x-3)-3^2(3^x-3) \leq 0$

For $S_2, x \in [1,2]$

$\Rightarrow S_1 \cup S_2 = (-\infty,-2] \cup [1,2]$

25. A $\quad A = \{m \in R, x^2 - (m+1)x + m + 4 = 0 \text{ has real roots}\}$

$D \geq 0 \Rightarrow (m+1)^2 - 4(m+4) \geq 0 \Rightarrow m^2 - 2m - 15 \geq 0$; $\therefore A = \{(-\infty,-3) \cup [5,\infty)\}$

$B = [-3,5) \Rightarrow A - B = (-\infty,-3) \cup [5,\infty)$