CHAPTER - 22
LOGARITHM

LOGARITHM

If $N = a^n$ then 'n' is the logarithm of N with to the base 'a', and 'a' is called base of the logarithm

When a = 10, logarithm is known as common logarithm usually denoted log N, and it is used for calculations.

For theoretical functions a = e, where 'e' is Euler's constant and the logarithm is known as natural logarithm and denoted by '*ln*'. $\ln x = \log_e x$

Exponential series

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + ...\infty$$

Logarithmic series

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ...\infty$$

Properties of logarithm

1. $\log_a 1 = 0$

2. $\log_a a = 1$

3. $\log_a (MN) = \log_a M + \log_a N$

4. $\log_a \left(\frac{M}{N}\right) = \log_a M - \log_a N$

5. $\log_a (M^N) = N \log_a M$

6. $\log_a x = \frac{\log_b x}{\log_b a}$

7. $\log_a x = \log_b x \log_a b$

8. $\log_b a = \dfrac{1}{\log_a b}$

9. $\log_{n^q} m^p = \dfrac{p}{q}\log_n m$

10. $a^{\log_a n} = n, \log_a a^x = x$

11. $a^{\log_c b} = b^{\log_c a}$

12. $a^{(\log_a b)^{\frac{1}{n}}} = b^{(\log_b a)^{1-\frac{1}{n}}}$

13. If 0 < a < 1 and $0 < \alpha < \beta$, then $\log_a \alpha > \log_a \beta$

14. If 0 < a < 1 and $0 < \alpha < 1$ $\log_a \alpha > 0$

15. If a > 1 and $0 < \alpha < 1$, then $\log_a \alpha < 0$

16. If a > 1 and $\alpha > 1$ then $\log_a \alpha > 0$

Common Logarithms and Use of Log Tables

Logarithms to the base 10 are known as common logarithms. The logarithm of a number consists of two parts :

(i) Characteristic : [The integral part of the logarithm]

(ii) Mantissa : [The fractional or decimal part of the logarithm]

For Example : In log 273 = 2.4362, the integral part is 2 and the decimal part is 0.4362; therefore the characteristic = 2 and mantissa = 0.4362.

To find Characteristic

(i) The characteristic of the logarithm of a number greater than one is non-negative and is numerically one less than the number of digits before the decimal point.

For Example : In number 475.8; the number of digits before the decimal point is three,

∴ Characteristic of log 475.8 = 2 [3 – 1 = 2]

similarly,

Characteristic of log 4758 = 3 [4 – 1 = 3]

Characteristic of log 47.58 = 1 [2 – 1 = 1]

Characteristic of log 4.758 = 0 [1 – 1 = 0]

(ii) The characteristic of the logarithm of a number less than one is negative and numerically one more than the number of zeros immediately after decimal point.

For Example : The number 0.004758 is less than one and the number of zeros immediately after decimal point in it is two.

$\therefore$ Characteristic of log 0.004758 = –(2+1) = –3 which is also written as $\bar{3}$.

Similarly, Characteristic of log 0.4352 = –1 = $\bar{1}$

[Since the number of zeros after decimal point = 3 and 3 + 1 = 4]

To Find Mantissa

The mantissa of the logarithm of a number can be obtained from the logarithmic table.

A logarithmic table consists of three parts:

(i) A column at the extreme left contains two digit numbers starting from 10 to 99

(ii) Ten columns headed by digits 0, 1, 2, ...9.

(iii) Nine more columns headed by digits 1, 2, ...9.

1. To find the mantissa of the logarithm of one digit number : Let the number be 3.

$\therefore$ Mantissa of log 3 = value of the number 30 under zero = 0.4771

2. To find the mantissa of the logarithm of two digit number : Let the number be 32.

$\therefore$ Mantissa of log 32 = value of 32 under zero = 0.5051

3. To find the mantissa of the logarithm of three digit number : Let the number be 325.

$\therefore$ Mantissa of log 325 = value of 32 under 5 = 0.5119

4. To find the mantissa of the logarithm of a four digit number : Let the number be 3257.

$\therefore$ Mantissa of log 3257 = value of 32 under 5 plus the difference under 7 = 0.5128 [5119 + 9 = 5128]

Antilogarithms

If log 5274 = 3.7221, then 5274 is called antilogarithm of 3.7221 and we write : antilog 3.7221 = 5274.

We find an antilogarithm from antilogarithm tables. The antilogarithm tables are used in the same way as the logarithm tables. The only difference between the two tables is that column at the extreme left of the log table contains all two digit numbers starting from 10 to 99; whereas an antilog table contains numbers from 0.00 to 0.99 (i.e. all fractional numbers with only two digits after decimal) in the extreme left column of it.

Note

(i) Antilog tables are used only to find the antilogarithm of decimal part.

(ii) To find the antilog of 2.368 means to find the number whose log is 2.368

Section1 - Only one option correct type

1. If $\log_{10} 3 = 0.477$, then the number of digits in 3^{40} is

   A) 18 B) 19 C) 20 D) 21

2. $\log_{10} \tan 30^\circ + \log_{10} \tan 31^0 + \log_{10} \tan 32^0 + ... + \log_{10} \tan 60^0 =$

   A) 0 B) 1 C) 2 D) 31

3. If $\log_{10} 2 = 0.30103$, then number of zeros which are between the decimal point and the first significant digit in $\left(\frac{1}{2}\right)^{1000} =$

   A) 103 B) 300 C) 301 D) 130

4. If $a = \log_{24} 12$, $b = \log_{48} 36$ and $c = \log_{36} 24$ then $1+abc$

   A) 2ab B) 2bc C) 2ca D) None of those

5. The value of $\log_4\left(1+\frac{1}{4}\right)+\log_4\left(1+\frac{1}{5}\right)+\log_4\left(1+\frac{1}{6}\right)+.......+\log_4\left(1+\frac{1}{255}\right)=$

   A) 0 B) 1 C) 2 D) 3

6. If $\log_3^2, \log_3(2^x - 5), \log_3(2^x - \frac{7}{2})$ are in AP then the value of x is $\Rightarrow$

   A) 2 B)3 C) 4 D) None of them

7. $\frac{1}{1+\log_a bc}+\frac{1}{1+\log_b ca}+\frac{1}{1+\log_c ab}=$

   A) 0 B) 3 C) 2 D) 1

8. If $\log_{.3}(x-1) < \log_{.09}(x-1)$, then x lies in the internal

   A) $(2, \alpha)$ B) $(-2, -1)$ C) $(1, 2)$ D) None of these

9. If $\log_{\sqrt{3}} 5 = a$ and $\log_{\sqrt{3}} 2 = b$, then $\log_{\sqrt{3}} 300 =$

   A) 2(a+b) B) 2(a+b+1) C) 2(a+b+2) D) a+b+4

10. The value of $\log_2 20 \log_2 80 - \log_2 5 \log_2 320$ is equal to

    A) 5 B) 6 C) 7 D) 8

11. If a, b, c are distinct numbers each being different from one such that

    $(\log_b a \log_c a - \log_a a) + (\log_a b \log_c b - \log_b b) + (\log_a c \log_b c - \log_c c) = 0$, then abc is

    A) 0 B) 1 C) 3 D) 2

12. Let (x_0, y_0) be the solution of the following equations $(2x)^{\ln 2} = (3y)^{\ln 3}, 3^{\ln x} = 2^{\ln y}$. Then X_0 is

    A) $\frac{1}{6}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) 6

Section 2 - More than one correct answer type

13. If $3^x = 4^{x-1}$, then $x =$

A) $\dfrac{2\log_3 2}{2\log_3 2-1}$ B) $\dfrac{2}{2-\log_2 3}$ C) $\dfrac{1}{1-\log_4 3}$ D) $\dfrac{2\log_2 3}{2\log_2 3-1}$

14. If $\log_2(3^{2x-2}+7) = 2+\log_2(3^{x-1}+1)$ then x is

A) 0 B) 1 C) 2 D) None of them

15. Solution of $\log_{(x^2+6x+8)} \log_{(2x^2+2x+3)}(x^2-2x) = 0$ is

A) A natural number B) A negative integer C) –1 D) None of these

16. If $\dfrac{\log x}{b-c} = \dfrac{\log y}{c-a} = \dfrac{\log z}{a-b}$, then which of the following are true

A) $xyz = 1$ B) $x^a y^b z^c = 1$ C) $x^{b+c} y^{c+a} z^{a+b} = 1$ D) $xyz = x^a y^b z^c$

17. The value of $\dfrac{1}{\log_2 n} + \dfrac{1}{\log_3 n} + ... + \dfrac{1}{\log_{43} n}$ is _______

18. The least value of the expression $2\log_{10} x - \log_x(0.01)$ for $x > 1$ is_______

A) 1 B) 2 C) 3 D) 4

19. The number $\log_2 7$ is

A) an integer B) a national number C) an irrational number D) a prime number

20. The equation $x^{[(\log_3^x)^2 - \frac{9}{2}\log_3^x + 5]} = 3\sqrt{3}$ has

A) at least one real solution B) Exactly three real solutions
C) Exactly one irrational solution D) None of these

21. The solution of $\log_4(x-1) = \log_2(x-3)$ is

A) 3 B) 1 C) 2 D) 0

Section 3 - Numerical type

22. $6+\log_{\frac{3}{2}}\left(\dfrac{1}{3\sqrt{2}}\sqrt{4-\dfrac{1}{3\sqrt{2}}\sqrt{4-\dfrac{1}{3\sqrt{2}}\sqrt{4-\dfrac{1}{3\sqrt{2}}\cdots\cdots\cdots}}}\right)$ is

23. If the value of $\log_8 128 - \log_9(\cot\frac{\pi}{3})$ is equal to x, then $[x] = ([\,] = \text{g iv})$ _______

24. The value of $\log_2 \log_2 \log_4 256 + 2\log_{\sqrt{2}} 2 =$ ______

25. The expression $\log_P \log_P \sqrt{P\sqrt{P\sqrt{P\ldots\ldots\ldots\ldots\ldots P\sqrt{P}}}}$ is equal to –K, (total number of p is equal to 2018). The sum of the digits of K is equal to _______