CHAPTER - 19
THEORY OF PROBABILITY

JEE MAIN - SECTION I

1. 3

Digits = 3, 3, 4, 4, 4, 5, 5

Total 7 digit numbers = $\dfrac{7!}{2!2!3!}$

Number of 7 digit number divisible by 2

Last digit = 4.

						4

3 , 3 , 4 , 4 , 5 , 5

Now 7 digit numbers which are divisible by 2

$= \dfrac{6!}{2!2!2!}$

Required probability = $\dfrac{\dfrac{6!}{2!2!2!}}{\dfrac{7!}{3!2!2!}} = \dfrac{3}{7}$.

2. C

W	M	No.of committees
4	0	$5C_4 \times 10C_0 = 5$
3	1	$5C_3 \times 10C_1 = 100$
2	2	$5C_2 \times 10C_2 = 450$
1	3	$5C_1 \times 10C_3 = 600$

Total number of committees = 1155

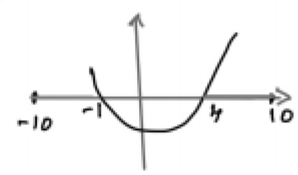
Number of favourable = 5+100=105

3. A $P(B/A\cup B')=\dfrac{P(A\cap(A\cup B'))}{P(A\cup B')}=\dfrac{P(A\cap B)}{P(A)+P(B')-P(AB')}$

$=\dfrac{P(A)-P(AB')}{0.7+0.6+0.5}=\dfrac{0.7-0.5}{0.8}=\dfrac{1}{4}$

4. 4 $b^2-4ac=q^2-3q-4\geq 0$

$(q+1)(q-4)\geq 0$



Favourable $\Rightarrow -10\leq q\leq 1\cup 4\leq q\leq 10$

$P=\dfrac{17}{21}$

5. 2 Total cases = 4^{2n-1}

Favourable

1	2	3	4		2n
2	3 or 5	2	3 or 5		3 or 5

No of favourable cases = 2^n

$P=\dfrac{2^n}{4^{2n-1}}=\dfrac{2^n}{2^{4n}}\times 4=4\times 2^{-3n}$

6. 3 Total ways = ${}^{2n+1}C_3 = \frac{(2n+1)\cdot 2n\cdot(2n-1)}{1\cdot 2\cdot 3} = \frac{n(4n^2-1)}{3}$

Let the three numbers a, b, c are drawn, where $a < b < c$ and given a, b and c are in AP.

$\therefore 2b = a + c$ ..... (1)

It is clear from eqs. (1) that a and c both are odd or both are even.

$\therefore$ Favourable ways = ${}^{n+1}C_2 + {}^{n}C_2 = \frac{(n+1)n}{1\cdot 2} + \frac{n(n-1)}{1\cdot 2} = n^2$

$\therefore$ Required probability = $\frac{n^2}{\frac{n(4n^2-1)}{3}} = \frac{3n}{(4n^2-1)}$.

$\Rightarrow$ Statement-II is false,

In statement-I, $2n+1 = 21 \Rightarrow n = 10$

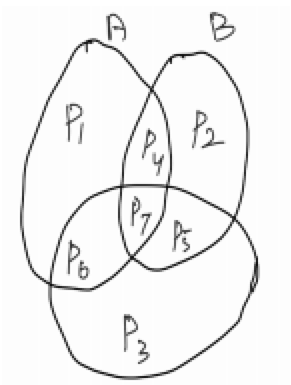
$\because$ Required probability = $\frac{3\times 10}{4(10)^2-1} = \frac{30}{399} = \frac{10}{133}$

$\therefore$ Statement-I is true

7. 3

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$
$$0.8 = 0.6 + 0.4 - P(A\cap B)$$
$$P(A\cap B) = 0.2$$
$$P(A\cup B\cup C) = \sum P(A) - \sum P(A\cap B) + P(A\cap B\cap C)$$
$$\alpha = 1.5 - (0.2 + 0.3 + \beta) + 0.2$$
$$\alpha = 1.2 - \beta \in [0.85, 0.95] \text{ (where } \alpha \in [0.85, 0.95])$$
$$\beta \in [0.25, 0.35].$$

8. 2 $P_1 + P_2 + P_5 + P_6 = 1 - K$



$$P_2 + P_3 + P_4 + P_6 = 1 - 2K$$

$$P_1 + P_3 + P_4 + P_5 = 1 - K$$

$$P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 =$$

$$\frac{(1-K)+(1-2K)+(1-K)+K^2}{2}$$

$$= \frac{2K^2 - 4K + 3}{2} = \frac{2(K-1)^2 + 1}{2}$$

$$\boxed{\frac{2K^2 - 4K + 3}{2} = \frac{2(K-1)^2 + 1}{2}}$$

$$= (K-1)^2 + \frac{1}{2} > \frac{1}{2}$$

9. 1 Since sum of two numbers is even so either both are odd or both are even. Hence number of elements in reduced samples space

$= {}^5C_2 + {}^6C_2$

So, required probability $= \dfrac{{}^5C_2}{{}^5C_2 + {}^6C_2}$.

10. 1 Given E_1, E_2, E_3 are pairwise independent events

So, $P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$ and $P(E_2 \cap E_3) = P(E_2) \cdot P(E_3)$

and $P(E_3 \cap E_1) = P(E_3) \cdot P(E_1)$ and $P(E_1 \cap E_2 \cap E_3) = 0$

Now, $P\left(\dfrac{\overline{E}_2 \cap \overline{E}_3}{E_1}\right) = \dfrac{P[E_1 \cap (\overline{E}_2 \cap \overline{E}_3)]}{P(E_1)}$

$= \dfrac{P(E_1) - [P(E_1 \cap E_2) + P(E_1 \cap E_3) - P(E_1 \cap E_2 \cap E_3)]}{P(E_1)}$

$= \dfrac{P(E_1) - P(E_1).P(E_2) - P(E_1)P(E_3) - 0}{P(E_1)}$.

$= 1 - P(E_2) - P(E_3) = [1 - P(E_3)] - P(E_2) = P(E_3^C) - P(E_2)$

11. 4 Required probability = P(first 9 items contains 3 defective) × P(10^{th} item is defective)

$$\left[{}^{9}C_3 \times \frac{4}{15} \times \frac{3}{14} \times \frac{2}{13}\right] \times \left(\frac{1}{12}\right) = \left(\frac{9 \cdot 8 \cdot 7}{6} \times \frac{1}{15 \times 7 \times 13}\right) = \frac{4}{65}$$

12. 1 Probability that none of $A_1, A_2, ..., A_n$ occur

$$= P(\overline{A}_1 \cap \overline{A}_2 \cap ..... \cap \overline{A}_n) = P(\overline{A}_1) \cdot P(\overline{A}_2) ...... P(\overline{A}_n)$$

$$= [1 - P(A_1)][1 - P(A_2)] .... [1 - P(A_n)]$$

$$= \left[1 - \frac{1}{2}\right]\left[1 - \frac{1}{3}\right]\left[1 - \frac{1}{4}\right] ...... \left[1 - \frac{1}{n+1}\right]$$

$$= \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{3}{4}\right) .... \left(\frac{n}{n+1}\right) = \frac{n!}{(n+1)!}$$

13. 3 $P = P(DGD) + P(GDD) + P(GGG)$

$D = \text{Defective}$ $G = \text{Good}$

14. 2 Experiment will end in 5^{th} throw

$$\left.\begin{array}{l} A = 4'4'4'44 \\ B = 44'4'44 \\ C = 4'44'44 \end{array}\right| \begin{array}{l} 4 = \text{face } 4 \\ 4' = \text{face not4} \end{array}$$

$$P = P(A) + P(B) + P(C) = \frac{175}{6^5}$$

15. 2 $$P = \frac{4C_1 \times 4C_1 + 9C_1 \times 9C_1}{13C_1 \times 12C_1} = \frac{97}{169}$$

16. 2 $\text{Mean} = E(x) = \sum x_i P(x_i)$

x	0	1	2	3	4	5	...
$P(x_i)$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3^2}$	$\frac{1}{3^3}$	$\frac{1}{3^4}$	$\frac{1}{3^5}$	...

$$\text{Mean} = 0 \times \frac{1}{2} + 1 \times \frac{1}{3} + 2 \times \frac{1}{3^2} + 3 \times \frac{1}{3^3} + ...$$

$$= 0 + \frac{1}{3} + 2 \times \frac{1}{3^2} + 3 \times \frac{1}{3^3} + ...$$

$$= \frac{1}{3}\left[1 + 2 \times \frac{1}{3} + 3 \times \frac{1}{3^2} + ...\right]$$

$$= \frac{1}{3}\left[\frac{1}{1-\frac{1}{3}} + \frac{1 \times \frac{1}{3}}{\left(1-\frac{1}{3}\right)^2}\right]$$

$$= \frac{1}{3}\left[\frac{3}{2} + \frac{1}{3}\,\frac{9}{4}\right] = \frac{1}{3}\left[\frac{3}{2} + \frac{3}{4}\right]$$

$$= \frac{1}{3} \times \frac{9}{4} = \frac{3}{4} \Rightarrow \text{mean} = \frac{3}{4}$$

$X \Rightarrow 0\ 1\ 2\ 3\ 4\ 5\ 6$ ....

(X +ive and even) $= 2, 4, 6, 8, ...$

$P[x \text{ +ive and even}] = P(2) + P(4) + P(6) + P(8) + ...$

$$= \frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6} + \frac{1}{3^8} + ... = \frac{1}{3^2}\left[1 + \frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^8} + ...\right]$$

$$= \frac{1}{9}\left[\frac{9}{1-r}\right] = \frac{1}{9}\left[\frac{1}{1-\frac{1}{3^2}}\right] = \frac{1}{9}\left[\frac{1}{1-\frac{1}{9}}\right]$$

$$= \frac{1}{9} \times \frac{9}{8} = \frac{1}{8}$$

17. 1 Let the coin be tossed n times

$P(H)=P(T)=\frac{1}{2}$

P(7 heads) = ${}^{n}C_7\left(\frac{1}{2}\right)^{n-7}\left(\frac{1}{2}\right)^{7}=\frac{{}^{n}C_7}{2^n}$

P(9 heads) = ${}^{n}C_9\left(\frac{1}{2}\right)^{n-9}\left(\frac{1}{2}\right)^{9}=\frac{{}^{n}C_9}{2^n}$.

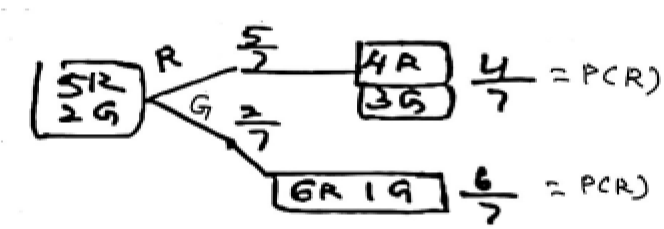
P(7 heads) = P(9 heads)

${}^{n}C_7={}^{n}C_9 \Rightarrow n=16$

P(2 heads) = ${}^{16}C_2\left(\frac{1}{2}\right)^{14}\left(\frac{1}{2}\right)^{2}=\frac{15\times 8}{2^{16}}$

P(2 heads) = $\frac{15}{2^{13}}$.

18. 1



$P(R)=\frac{5}{7}\times\frac{4}{7}+\frac{2}{7}\times\frac{6}{7}=\frac{32}{49}$

19. 4 Let A be the event of drawing red and E be the event that the person will say that the ball is red.

Then P(E) = $P(A)\cdot P(E/A)+P(\bar{A})\cdot P(E/A)$

$=\left(\frac{1}{5}\times\frac{3}{4}\right)+\left(\frac{4}{5}\times\frac{1}{4}\right)=\frac{7}{20}$.

20. 1 Let B_1 be the event where Box-I is selected and B_2 where box-II selected

$$P(B_1)=P(B_2)=\frac{1}{2}$$

Let E be the event where selected card is non-prime
For B_1: Prime numbers: {2, 3, 5, 7, 11, 13, 17, 19, 23, 29}
For B_2: Prime numbers: {31, 37, 41, 43, 47}

$$P(E)=P(B_1)\times P\left(\frac{E}{B_1}\right)+P(B_2)\times P\left(\frac{E}{B_2}\right)=\frac{1}{2}\times\frac{20}{30}+\frac{1}{2}\times\frac{15}{20}.$$

Required probability: $P\left(\frac{B_1}{E}\right)=\dfrac{\frac{1}{2}\times\frac{20}{30}}{\frac{1}{2}\times\frac{20}{30}+\frac{1}{2}\times\frac{15}{20}}=\dfrac{\frac{2}{3}}{\frac{2}{3}+\frac{2}{4}}=\frac{8}{17}.$

SECTION II (NUMERICAL)

21. 2 Let p = probability (white) $=\frac{a}{a+b}$ and q = probability (black) $=\frac{b}{a+b}$ Given P(A) : P(B) = 3:1 $\Rightarrow$ 1:q = 3:1

$$\Rightarrow\frac{1}{q}=\frac{3}{1}\Rightarrow\frac{a+b}{b}=3\Rightarrow\frac{a}{b}=2$$

22. 1 $ax^2+bx+c=0$

$$\left.\begin{matrix}\alpha+\beta=\frac{-b}{a} & \alpha\beta=\frac{c}{a}\\ \alpha^2+\beta^2=\frac{-b}{a} & \alpha^2\beta^2=\frac{c}{a}\end{matrix}\right\}\Rightarrow\left(\frac{c}{a}\right)^2=\left(\frac{c}{a}\right);c=0\text{ or }c=a$$

$$\boxed{c=0\Rightarrow\alpha\beta=0}$$

$$(\alpha+\beta)^2=\alpha^2+\beta^2+2\alpha\beta$$

$$\left(\frac{b}{a}\right)^2=\frac{-b}{a}+0\Rightarrow\left(\frac{b}{a}\right)^2+\left(\frac{b}{a}\right)=0$$

$$\frac{b}{a}\left(1+\frac{b}{a}\right)=0\Rightarrow b=0\Rightarrow\frac{b}{a}=-1\Rightarrow b=-a$$

$b=0$ or $b=-a$

$c=a\Rightarrow\alpha\beta=1$

$(\alpha+\beta)^2=\alpha^2+\beta^2+2\alpha\beta$

$$\left(\frac{b}{a}\right)^2=\frac{-b}{a}+2$$

$$\left(\frac{b}{a}\right)^2+\left(\frac{b}{a}\right)-2=0\Rightarrow\frac{b^2}{a^2}+\frac{b}{a}-2=0$$

$$\frac{b^2+ab-2a^2}{a^2}=0\Rightarrow b^2+ab-2a^2=0 \text{ Q.E in b}$$

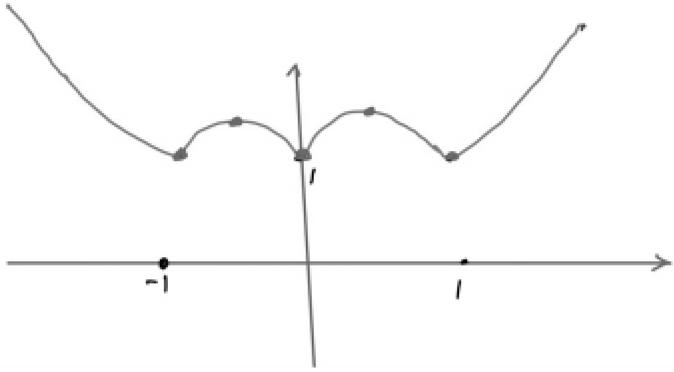
$$b=\frac{-a\pm\sqrt{a^2+8a^2}}{2}=\frac{-a\pm 3a}{2}$$

$$b=\frac{-a+3a}{2}=a \text{ or } b=\frac{-4a}{2}=-2a$$

$b=a$ or $b=-2a$

a	b	c	Equation		b^2-4ac
a	0	0	$ax^2=0$	$x^2=0$	$b^2-4ac=0$
a	-a	0	$ax^2-ax=0$	$x^2-x=0$	$1+4\neq 0$
a	a	a	$ax^2+ax+a=0$	$x^2+x+1=0$	$1-4\neq 0$
a	-2a	a	$ax^2-2ax+a=0$	$x^2-2x+1=0$	$4-4=0$

23. 5



Number of local maximum points = 2 $P=\frac{2}{5};\therefore 5P=5$

Numbe of local minimum points = 3

Total numberof extreme points = 5

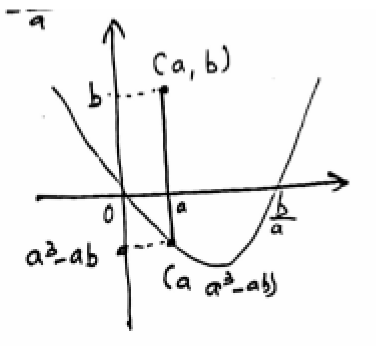
24. 6 Let $P(E_1)=a\ P(E_2)=b\ P(E_3)=c$

$\alpha=a(1-b)(1-c)\ \beta=b(1-a)(1-c)$

$\gamma=c(1-a)(1-b)\ P=(1-a)(1-b)(1-c)$

Now put α,β,γ and P in the given equations

25. 10 $y=ax^2-bx=x(ax-b)\Rightarrow x=0\ x=\frac{b}{a}$



$a^3-ab<b\Rightarrow a^3<b+ab$

$$a^3 < b(a+1) \Rightarrow b > \frac{a^3}{a+1}$$

$$\text{Total} \Rightarrow 9C_1 \times 9C_1 = 81$$

$$(a,b) \in \{1,2,3,4,5,6,7,8,9\}$$

$$P = \frac{19}{81}$$

$$81P - 9 = 19 - 9 = 10$$

a	$\frac{a^3}{a+1}$		
1	0.5	1 2 3 4 5 6 7 8 9	9
2	2.7	3, 4, 5, 6, 7, 8, 9	7
3	6.7	7, 8, 9	3
4	12		
5		NIL	
6			
7			
8			
9			

$= 19$

JEE ADVANCED LEVEL
SECTION III

26. C $f'(x) \geq 0 \Rightarrow 3x^2 + 2ax + b \geq 0$ for all x, ie a^2-3b$\leq$0, which happens for the orderpairs (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6), and (4, 6) hence the required probability = 4/9.

27. C Value of determinant = –4

(Use sarrus method and $e^{i\theta} = (\cos\theta + i\sin\theta)$

$\therefore n = -4 \Rightarrow -n = 4$; $P(x) = \frac{nC_x}{2^n}$

$$P(3) = \frac{4C_3}{2^4} = \frac{1}{4}$$

28. B s_1, s_2 denote the events that he solves the 1st and 2nd problems.

$$P(s_1)=\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times\frac{3}{5}+\frac{1}{3}\times\frac{4}{5}=\frac{19}{30}$$

$$P(s_2)=\frac{1}{3}\times\left(\frac{1}{2}\right)^2+\frac{1}{3}\times\left(\frac{3}{5}\right)^2+\frac{1}{3}\times\left(\frac{4}{5}\right)^2=\frac{125}{300}\text{; now } P\left(\frac{s_2}{s_1}\right)=\frac{25}{38}$$

29. D $\frac{5\times 5}{{}^{10}C_2}\times\frac{4\times 4}{{}^{8}C_2}\times\frac{3\times 3}{{}^{6}C_2}\times\frac{2\times 2}{{}^{4}C_2}$

30. D $\text{A.M.}=E(x)=\sum x_i p(x_i)$

SECTION IV (More than one correct)

31. A,C $P(A\cup B)=P(A)+P(B)-P(A\cap B)\geq\frac{3}{4}$

$$\therefore 1\geq P(A)+P(B)-P(A\cap B)\geq\frac{3}{4}$$

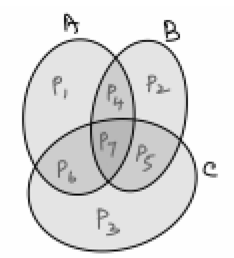
As the minimum value of $P(A\cap B)\geq\frac{1}{8}$, we get

$$P(A)+P(B)-\frac{1}{8}\geq\frac{3}{4}\Rightarrow P(A)+P(B)\geq\frac{1}{8}+\frac{3}{4}=\frac{7}{8}$$

As the maximum value of $P(A\cap B)=\frac{3}{8}$, we get

$$1\geq P(A)+P(B)-\frac{3}{8}\Rightarrow P(A)+P(B)\leq 1+\frac{3}{8}=\frac{11}{8}$$

32. B,C



$$P = P_1 + P_4 + P_6 + P_7\,;\; b = P_2 + P_4 + P_5 + P_7\,;\; c = P_3 + P_5 + P_6 + P_7$$

$$P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 = 0.75\,;\; P_4 + P_5 + P_6 + P_7 = 0.5$$

$$P_4 + P_5 + P_6 = 0.7$$

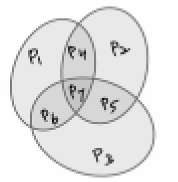
33. A,B $P\left(\frac{X}{Y}\right) = \frac{P(X \cap Y)}{P(Y)} = \frac{1}{2}$ and $\frac{P(X \cap Y)}{P(X)} = \frac{1}{3}$

$P(X \cap Y) = \frac{1}{6} \Rightarrow P(Y) = \frac{1}{3}$ and $P(X) = \frac{1}{2}$

Clearly X and Y are indipendent.

Also, $P(X \cup Y) = \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{2}{3}$

34. B,D



$$\boxed{\begin{array}{cc} P_1 = \frac{9}{32} & P_5 = \frac{3}{32} \\ P_2 = \frac{3}{32} & P_6 = \frac{1}{32} \\ P_3 = \frac{3}{32} & P_7 = \frac{1}{32} \\ \multicolumn{2}{c}{P_4 = \frac{3}{32}} \end{array}}$$

$$P_1 = P\left(x_1 x_1' x_3'\right), P_2 = P\left(x_2 x_1' x_2'\right)$$

$$P_3 = P\left(x_3 x_1' x_2'\right),\; P_4 = P\left(x_1 x_2 x_3'\right);\; P_5 = P\left(x_2 x_3 x_1'\right), P_6 = P\left(x_1 x_3 x_2'\right)$$

$$P_7 = P(x_1\ x_2 x_3)$$

35. B,C,D

A) $P(W)=\Sigma P(ui)P(w/ui)$

$P(ui)=\frac{1}{n}$ $P(w/ui)=\frac{i}{n+1}$

B) $P(ui)=c\Rightarrow c=\frac{1}{n}$

$P(un/w)=\frac{P(un)P(w/un)}{P(w)}$; $P(w)=\frac{1}{2}$ $P(un/w)=\frac{n}{n+1}$; C) $P(E)=\frac{1}{2}$

$$P(W/E)=\frac{P(WnE)}{P(E)}=\frac{P(wnu_2)+P(wnu_4)+....+P(wnun)}{\left(\frac{1}{2}\right)}$$

$$=2\left[P(u_2)P(w/u_2)+P(u_4)P(w/u_4)+.......\right]=\frac{n+2}{2(n+1)}$$

D) $P(ui)=Ki\Rightarrow\Sigma Ki=1\Rightarrow K=\frac{2}{n(n+1)}$

$P(ui)=\frac{2i}{n(n+1)}$ and $P(w)=\Sigma P(ui)P(w/ui)$

SECTION V - (Numerical type)

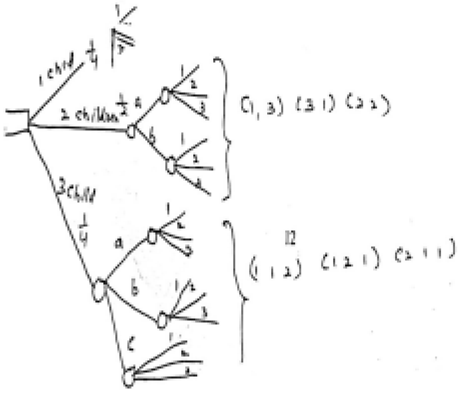
36. 6 For the first draws, following events may occur

E_1 = both balls are white
E_2 = first is white and second is black
E_3 = first is black and second is white
E_4 = both balls are black

Let E represent the event that the third ball is black. Then,

$$P(E)=P(E_1)P\left(\frac{E}{E_1}\right)+...+P(E_4)P\left(\frac{E}{E_4}\right)=\frac{1}{6}\times\frac{3}{2}+\frac{1}{3}\times\frac{3}{4}+\frac{1}{5}\times\frac{3}{4}+\frac{3}{10}\times\frac{4}{6}=\frac{23}{30}$$

37. 0.21

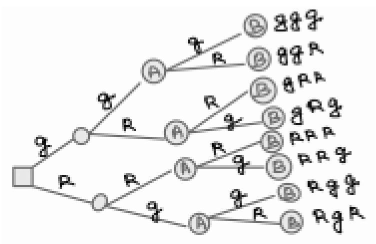


Required probability is

$$P=\frac{1}{2}\left[\frac{1}{4}\times\frac{1}{4}+\frac{1}{4}\times\frac{1}{4}+\frac{1}{2}\times\frac{1}{2}\right]+\frac{1}{4}\left[\frac{1}{4}-\frac{1}{4}\,\frac{1}{2}+\frac{1}{4}.\frac{1}{2}\times\frac{1}{4},\frac{1}{4}\times\frac{1}{4}\times\frac{1}{2}\right]=\frac{27}{128}$$

38. 6 By total prob theorem the req probability $=\frac{3}{5}\times\frac{2}{5}+\frac{2}{5}\times\frac{7}{10}=\frac{13}{25}$

39. 0.87



$$P[\text{Green at origin} \mid \text{Green at B}]=\frac{P[\text{Green at origin and Green at B}]}{P[\text{Green at B}]}$$

$$=\frac{P[\text{ggg or gRg}]}{P[\text{ggg or gRg or RRg or Rgg}]}$$

SECTION VI - (Matrix match type)

40. B Imaginary roots $\Rightarrow b^2-4ac<0$

$P^2-4q<0$

$\therefore P^2<4q$

q	4q	$P^2 < 4q$
1	4	1
2	8	1, 2
3	12	1, 2, 3
4	16	1, 2, 3
5	20	1, 2, 3, 4
6	24	1, 2, 3, 4
7	28	1, 2, 3, 4, 5
8	32	1, 2, 3, 4, 5
9	36	1, 2, 3, 4, 5
10	40	1, 2, 3, 4, 5, 6

Number of favourable = 38

Total = $10C_1 \times 10C_1 = 100$

$$P(\text{Imaginary roots}) = \frac{38}{100} = 0.38$$

$$P(\text{Real roots}) = 1 - 0.38 = 0.62$$

$$P(\text{Real and equal}) = \frac{3}{100} = 0.03$$