CHAPTER - 23
LINEAR PROGRAMMING

Programming means systematic planning or decision making. It is a technique for solving optimization (maximization or minimization) problems subject to certain **constraints.** Out of all permissible allocations of resources, we have to decide one which will minimize the total cost or maximize the total profits. ***Linear Programming*** *is a device which is used in decision making in business for obtaining* ***optimum*** *values of quantities subject to certain constraints when the relationships involved in the problem are linear.*

12.3.2. An illustration.

Let us assume that a firm produces two products I and II. Each product has to undergo three operations, cutting, mixing and packaging before it takes the final shape. As usual, the main object of this firm is to find how much of products I and II should be produced with the maximum capacity available for each operation so as to maximize the total net revenue. The revenue per unit and the time required per unit of each product in each operation is enlisted below :

Department	Time taken by each unit of products (in hours) I	and II	Maximum capacity available in each operation
Cutting	11	6	136 hrs
Mixing	3	10	70 hrs
Packaging	7	14	55 hrs
Net revenue per unit of product	Rs 6	Re 1	—

Let us assume that the firm can minimize the total net revenue by producing x and y units of products I and II respectively. The above table can now be transformed in the form of inequalities describing constraints, generally known as **structural constraints.** Such constraints are because of technological limitations, or capacity and resource limitations.

Since each unit of product I requires 11 hours of cutting facility, the total time required for x units in this department will be $11x$ hrs. In the same way y units of product II will require $6y$ hrs of cutting facility. Hence, the production of $x + y$ units of the two products will require $11x + 6y$ hours of cutting facility. The total capacity in the cutting department at the maximum is 136 hours, therefore, we must have $11x + 6y \leq 136$. So structural constraint in the cutting department is described in terms of the above inequality. Exactly with the same reasoning we can form inequalities describing the constraints in the mixing and the packaging departments.

Thus, we have three structural constraints in our problem :

(1) $11x + 6y \leq 136$

(2) $3x + 10y \leq 70$

(3) $7x + 14y \leq 55.$

Along with these structural constraints we have **'non-negativity'** constraints which assume that negative values of the variables x and y are not possible in the solution of L.P. (linear programming) problems. In terms of inequalities these constraints are described as

(4) $x \geq 0, y \geq 0.$

Let Rs f be the total revenue that x, y units of products I and II together fetch, then $f = 6x + y$. So we are to maximize f under the constraints (1), (2), (3) and (4). The function f is called the **objective function.**

In L.P. problems prices are to be assumed constant during a given time period. Hence, prices are not considered as variables.

As seen above, every L.P. problem in its standard form involves three parts :

(*i*) Objective Function

(*ii*) Structural constraints and

(*iii*) Non-negativity constraints.

Hence, our problem (given in the form of a table) can be stated in the standard form as follows :

Find two real numbers x and y such that $11x + 6y \leq 136$, $3x + 10y \leq 70$, $7x + 14y \leq 55$ and $x \geq 0$, $y \geq 0$, for which the function $f = 6x + y$ is maximum.

12.3.3. Methods of Solving Linear Programming Problems.

There are two methods of solving a linear programming problems :

(1) *Graphical Method* *and*

(2) *Simplex Method.*

The simplex method is beyond the scope of this book. We shall explain the graphical method of solving a linear programming problem. For this, we need the following back ground :

(*i*) *A set S of points in a plane is said to be* **convex**, *if the line segment joining any two points in it, lies in it completely i.e.* if we take any two points A and B in the set S, then the segment [AB] lies in S. A circle, a square, a rhombus, a polygon are examples of convex sets. In fig. 12.11., (*i*) and (*iii*) are convex sets, whereas (*ii*) is not a convex set.

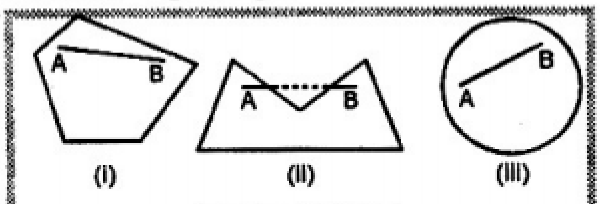


FIGURE 12.11

(*ii*) *If we have a system of linear inequations in two variables, then the set of points (x, y) for which all the inequations of the system hold true, is either empty, or is a convex region bounded by straight lines* (**a convex polygon**) *or an unbounded region with straight line boundaries.*

(*iii*) *The set of points, whose co-ordinates satisfy the constraints of a linear programming problem, is said to be the* ***feasible region.***

(*iv*) *The* **optimum value (maximum** *or* **minimum)** *of the objective function is obtained at a vertex of the feasible region (if it is bounded). If there are more than one points (vertices) where the objective function is optimum (maximum or minimum), then every point on the line segment joining any two such vertices optimizes the objective function.*

(*v*) In case the feasible region is unbounded and minimum value of objective function $f = ax + by$ at a corner point is m, then this is actually minimum only if the graph of $ax + by < m$ has no common point with feasible region, otherwise f has no minimum value.

Again if the maximum value of $f = ax + by$ at a corner point is M (when the feasible region is unbounded) then M is actually maximum only if the graph of $ax + by > M$ has no common point with the feasible region, otherwise f has no maximum value.

Thus, solving a linear programming problem by graphical method involves the following steps :

First step. Plot the graphs of the inequalities describing the various constraints (structural) on the graph paper.

Second step. Find the portion of the graph paper in the first quadrant ($\because$ of non-negativity constraints) which is common to the graphs plotted in the first step. Locate the extreme points (*i.e.* corner points) of this region (known as feasible region).

Third step. Find the value of the objective function corresponding to each corner point. The points which corresponds to the optimum (*i.e.* maximum or minimum) value of the objective function is (are) required solution (s) of the given linear programming problem.

Remark. Though (0, 0) may be a corner point of the feasible region in some problems, but it is not to be examined for the optimum solution.

QUESTIONS

1. Maximum value of $(2x+3y)$ subject to the constraints $x \geq 0, y \geq 0,\ x+y \leq 5$ and $3x+y \leq 9$ is

A) 15 B) 36 C) 60 D) 40 E) 50

2. The maximum value of $z = 5x+3y$ subject to the conditions $3x+5y \leq 15, 5x+2y \leq 10, x, y \geq 0$ is

A) $\frac{235}{19}$ B) $\frac{325}{19}$ C) $\frac{523}{19}$ D) $\frac{532}{19}$ E) $\frac{521}{19}$

3. For the LPP $\min(z) = x_1 + x_2$ such that inequalities $5x_1 + 10x_2 \geq 0, x_1 + x_2 \leq 1, x_2 \leq 4$ and $x_1, x_2 \geq 0$

A) There is a bounded solution B) There is no solution

C) There are finite solution D) There is one solution

4. The region represented by the inequation system $x, y \geq 0, y \leq 6, x+y \leq 3$ is

A) unbounded in the first quadrant

B) unbounded in the first and second quadrant

C) bounded in the first quadrant

D) bounded in the fourth quadrant

5. The constants $-x_1 + x_2 \leq 1, -x_1 + 3x_2 \leq 9, x_1, x_2 \geq 0$ defines on

A) bounded feasible space

B) unbounded feasible space

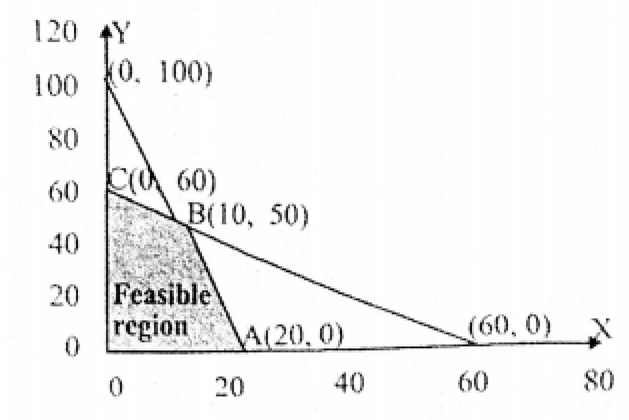
C) Both bounded and unbounded feasible space

D) None of the above

6. A wholesale merchant wants to start the business of cereal with Rs. 24000, wheat is Rs. 400 per quintal and rice is Rs. 600 per quintal, He has capacity to store 200 quintal cereal. He earns the profit 25 per quintal on wheat and Rs. 40 per quintal on rice, if he stores x quintal rice and y quintal wheat, then for maximum profit the objective function is

A) 25x+40y B) 40x+25y C) 400x+600y D) $\frac{400}{40}x + \frac{600}{25} = y$

7. The feasible region for a L.P.P. is shown in the figure below. Let $z = 50x + 15y$ be the objective function, then the maximum value of z is



A) 900 B) 1000 C) 1250 D) 1300 E) 1520

8. Consider the linear programming problem

Maximize z = 10x + 5y

subject to the constraints

$2x + 3y \le 120$

$2x + y \le 60$

$x, y \ge 0$

Then the coordinates of the corner points of the feasible region are

A) $(0,0), (30,0), (0,40)$ and $(15,30)$ B) $(0,0), (60,0), (0,40)$ and $(15,30)$

C) $(0,0), (30,0), (0,60)$ and $(15,30)$ D) $(0,0), (30,0), (0,40)$ and $(30,40)$

E) $(0,0), (60,0), (0,40)$ and $(30,40)$

9. The constrainst of a linear programming problem are $x + 2y \le 10$ and $6x + 3y \le 18$. Which of the following points lie in the feasible region?

A) (0,6) B) (4,3) C) (5,7) D) (1,7) E) (1,3)

10. The maximum value of $z = 7x + 5y$ subject to $2x + y \le 100, 4x + 3y \le 240, x \ge 0, y \ge 0$ is

A) 350 B) 380 C) 400 D) 410 E) 420