CHAPTER - 02
KINEMATICS

SYNOPSIS

1. **Point Object** :- An object is said to be a point object if it changes its position by distances which are much greater than its size.
2. **State of rest** :- An object or particle is said to be in a state of rest if it does not change its position with time with respect to its surroundings
3. **State of motion** :- An object or particle is said to be in a state of motion if it changes its position with respect to its surroundings
4. **Motion in one dimension** :- It is that motion in which a particle or a body moves in one particular direction w.r.to a point of reference.
5. **Distance** :- It is the length of actual path traversed by a body during motion in a given interval of time. Distance is a scalar quantity. The distance travelled by a body can never be zero or negative.
6. **Displacement** :- Displacement of a body in a given time is defined as the change in position of the body in a particular direction.

   1) Displacement is a vector quantity

   2) The unit of displacement is that of length

   3) Displacement can be positive, zero or negative

   4) The value of displacement can never be greater than the distance travelled

   5) When a moving body returns to its starting point then its effective displacement is zero
7. **Speed** :- The speed of a particle or body is defined as the distance travelled by it in unit time

   Speed = $\frac{\text{distance}}{\text{time}}$ is a scalar quantity
8. **Uniform speed** :- A body is said to move with uniform speed, if it covers equal distances in equal intervals of time, howsoever small these intervals may be
9. **Variable speed** :- A body is said to be moving with a variable speed if it covers equal distances in unequal intervals of time or unequal distances in equal intervals of time.
10. **Average speed** :- When a body is moving with a variable speed, then the average speed of the body is defined as the ratio of total distance travelled by the body to the total time taken.

    Average speed = $\frac{\text{total distance travelled}}{\text{total time taken}}$
11. **Instantaneous speed** :- When the body is moving with variable speed, then the speed of the body at a given instant of time is called its instantaneous speed.

12. **Velocity** :- Velocity of a particle or body is defined as the rate of change of displacement

 1) Velocity is a vector quantity

 2) Velocity of a body can be zero, negative or positive

 3) Velocity can never be greater than the speed of the body

13. **Uniform velocity** :- A body is said to be moving with uniform velocity, if it undergoes equal displacements in equal intervals of time; howsoever these intervals may be.

14. **Average velocity** :- When a body is moving with a variable velocity, the average velocity of the body in a given time is defined as the ratio of the total displacement to the total time taken.

 Average Velocity = $\frac{\text{total displacement}}{\text{total time taken}}$

15. **Instantaneous Velocity :-**

 1) When a body is moving with a variable velocity the velocity of the body at a given instant of time is called instantaneous velocity.

 2) If at an instant t, a body while moving covers a displacement $\overline{\Delta r}$ in a small interval of time Δt, so that $\Delta t \to 0$, then instantaneous velocity = $\underset{\Delta t \to 0}{Lt} \frac{\vec{\Delta} r}{\Delta t} = \frac{dr}{dt}$

 ($\frac{dr}{dt}$ is the first derivative of displacement $\overline{r}$ w.r.t. time)

16. **Acceleration** :- Acceleration of a particle or body is defined as the rate of change of velocity.

 $\text{acceleration} = \frac{\text{Change in velocity}}{\text{time taken}}$

 1) Acceleration is a vector quantity and its S.I.unit is m/s^2.

 2) Acceleration can be positive negative or zero

 3) Negative acceleration is called retardation or deceleration

17. **Uniform acceleration** :- A object is said to be moving with a uniform acceleration, if its velocity changes by equal amounts in equal intervals of time.

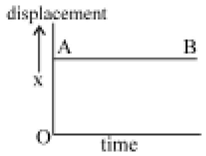
 Average acceleration = $\frac{\text{total change in velocity}}{\text{total time taken}}$

18. **Instantaneous acceleration** : If at an instant 't' a body while moving with variable acceleration then the acceleration of the body at a given instant of time is called instantaneous acceleration.

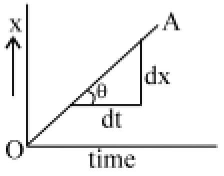
 instantaneous acceleration = $\underset{\Delta t \to 0}{Lt} \frac{\vec{\Delta} v}{\Delta t} = \frac{d\overline{v}}{dt}$

 ($\frac{dv}{dt}$ is the first derivative of velocity w.r. to time)

19. **Displacement time graph** :- A graph drawn with time along the x-axis and the displacement along the y-axis.

 1) 

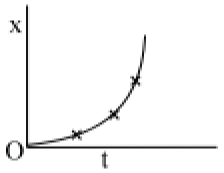
For a stationary body, the time displacement graph AB, is a straight line parallel to the time axis

2) 

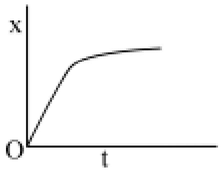
Uniform motion. Equal displacement take place at equal intervals of time.

Velocity = $\frac{dx}{dt}$ = slope of the straight line OA.

$V = \frac{dx}{dt} = \tan\theta$. Where θ is the angle made by the straight line OA with the time axis.

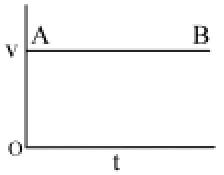
3) 

Body moving with constant acceleration. The graph is a curve which bends upwards.

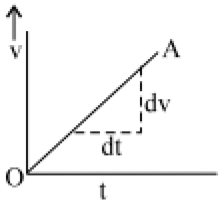
4) 

Body moving with constant deceleration

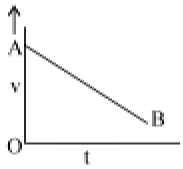
20. **Velocity time graph** :- A graph drawn with velocity along the y-axis and time along the x-axis.

1) 

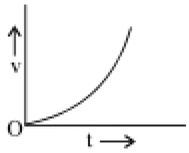
Body moving with constant velocity the velocity time graph is a straight line AB parallel to the time axis.

2) 

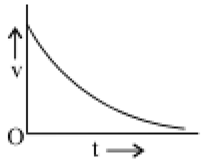
Body moving with constant acceleration and its initial velocity is zero. It is a straight line inclined to the time axis. The slope of the line OA gives the acceleration. The area under the v-t graph gives the displacement.

3) 

Body moves with constant retardation and its initial velocity is not zero the velocity time graph is an oblique. Straight line AB not passing through origin.

4) 

Body moving with increasing accelerations. The v-t graph is a curve which bend upwards.

5) 

Body moving with negative acceleration

21. **Equations of motion**

u = initial velocity of body
a = Uniform acceleration of the body
v = velocity after a time t
s = distance travelled by the body in time t
s_n = distance travelled by the body in the n^{th} second

1) $V = u + at$
2) $V^2 = u^2 + 2as$
3) $S = ut + 1/2\ at^2$
4) $S_n = u + \frac{a}{2}(2n-1)$

22. **Relative velocity** :-

1) The relative velocity of one body w.r.t another body is the velocity with which one body moves w.r.t another body
2) The relative velocity of one body w.r.t. another body is defined as the time rate of change of relative position w.r.t. to another.
3) When two bodies A and B are in relative motion, the relative velocity of body A w.r.t. to body B can be obtained by imposing equal and opposite velocity of B on both A and B, so that B is brought to rest. The resultant of the two velocities of A gives the relative velocity of body A w.r.t body B.

   i.e., Relative velocity of A w.r.t B is $\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$

   Relative velocity of B w.r.t A is $\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$
4) When two bodies are moving along parallel straight lines in the same direction, the relative velocity of A w.r.t. B is

   $V_{AB} = V_A - V_B$ is the direction of $\vec{V}_A$
5) When two bodies are moving along parallel straight lines in opposite directions, the relative velocity of A w.r.t. B is
   $V_{AB} = V_A + V_B$ in the direction of $\vec{V}_A$

   Relative velocity of B w.r.t. A is $V_{AB} = V_B + V_A$ in the direction of $\vec{V}_B$

VECTORS

A vector is a quantity having both magnitude and direction.

- Representation of a vector

  $\vec{r} = r\,\hat{n}$ where $r = |\vec{r}|$ gives the magnitude and $\hat{n}$ is a unit vector that gives direction

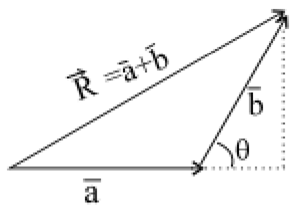
- Unit vectors along the three coordinate axes are called orthogonal unit vectors. They are named $\hat{i}, \hat{j}, \hat{k}$ respectively
- Magnitude of a vector

  If a vector is represented as $\vec{r} = x\,\hat{i} + y\hat{j} + z\hat{k}$, its magnitude is given by $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$

- Types of vectors
  - $\rightarrow$ Equal vectors - two vectors having equal magnitude and same direction
  - $\rightarrow$ Parallel vectors - vectors along the same direction
  - $\rightarrow$ Antiparallel vectors - vectors in opposite directions
  - $\rightarrow$ Collinear vectors - vectors along the same line
  - $\rightarrow$ Zero vector - a vector having zero magnitude
  - $\rightarrow$ Coplanar vectors - vectors over the same plane

VECTOR ADDITION

1. Triangle law of vector addition

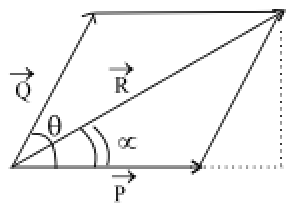


If $\bar{a}, \bar{b}$ are the consecutive sides of a triangle then the resultant is given by the closing side of the triangle taken in the opposite order. Magnitude of the resultant is given by,

$$|\vec{R}| = \sqrt{a^2 + b^2 + 2ab\cos\theta}$$

2. Parallelogram law of vector addition

If $\vec{P}$ & $\vec{Q}$ represent the adjacent sides of a parallelogram, the resultant vector is given by the diagonal of the parallelogram drawn from the meeting point of tails of $\vec{P}$ and $\vec{Q}$



Magnitude of the resultant vector is given by, $|\vec{R}| = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

Also, $\tan\alpha = \dfrac{Q\sin\theta}{P+Q\cos\theta}$

Special cases

1. If the vectors are along the same direction, $\theta = 0^0 \therefore |\vec{R}| = |\vec{P}+\vec{Q}|$

2. If the vectors are in opposite directions, $\theta = 180^0 \therefore |\vec{R}| = |\vec{P}-\vec{Q}|$

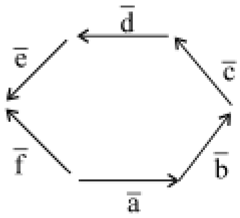
3. If the vectors are perpendicular to each other, $\theta = 90^0 \therefore |\vec{R}| = \sqrt{P^2+Q^2}$

Note

The resultant of vectors $\vec{p}$ and $\vec{Q}$ always lies between $|\vec{P}-\vec{Q}|$ and $|\vec{P}+\vec{Q}|$

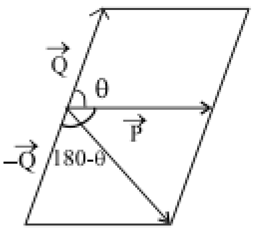
3. Polygon law of vector addition

If $\bar{a}, \bar{b}, \bar{c}, \bar{d}$ and $\bar{e}$ are the successive sides of a polygon, the resultant vector is given by the closing side of the polygon taken in the opposite order.



$\bar{f} = \bar{a} + \bar{b} + \bar{c} + \bar{d} + \bar{e}$

VECTOR SUBTRACTION



$|\vec{P}-\vec{Q}| = \sqrt{P^2+Q^2-2PQ\cos\theta}$

APPLICATIONS

1. Relative velocity

If $\vec{V}_A$ and $\vec{V}_B$ are the velocities of bodies A and B travelling at an angle θ apart, relative velocity of A with respect to B is given by

$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B = \sqrt{V_A{}^2 + V_B{}^2 - 2V_AV_B\cos\theta}$

2. Acceleration

If $\vec{V}_i$ and $\vec{V}_f$ are the initial and final velocities, acceleration is given by $\vec{a} = \dfrac{\vec{V}_f - \vec{V}_i}{t}$

PRODUCT OF VECTORS

I. Dot product (Scalar product)

Dot product of vectors $\vec{A}$ and $\vec{B}$ is defined as

$\vec{A}.\vec{B}=|\vec{A}||\vec{B}|\text{Cos}\theta$ $\quad \theta \rightarrow$ angle between $\vec{A}$ and $\vec{B}$

Note: If $\vec{A}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{B}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$, then $\vec{A}.\vec{B}=a_1b_1+a_2b_2+a_3b_3$

- Properties

1. It is commutative, $\vec{A}.\vec{B}=\vec{B}.\vec{A}$
2. It is distributive, $\vec{A}.(\vec{B}+\vec{C})=\vec{A}.\vec{B}+\vec{A}.\vec{C}$
3. Angle between the vectors $\vec{A}$ and $\vec{B}$, $\theta=\text{Cos}^{-1}\left[\dfrac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}\right]$
4. If two vectors are perpendicular, $\theta=90^0$ $\therefore \vec{A}.\vec{B}=0$
5. $\vec{A}.\vec{A}=A^2$
6. $\hat{i}.\hat{i}=1;\ \hat{j}.\hat{j}=1;\hat{k}.\hat{k}=1$ ie., identical unit vectors when taken dot product give one
7. $\hat{i}.\hat{j}=0\ \ \hat{j}.\hat{k}=0\ ;\ \hat{k}.\hat{i}=0$ ie., unlike unit vectors when taken dot product give zero.

Note: The common situations where the dot product is used are given below:

(i) Power = $\vec{F}.\vec{V}$

(ii) Work = $\vec{F}.\vec{d}$

(iii) Flux = $\vec{E}.\vec{A}$ or $\vec{B}.\vec{A}$

II. Cross product (Vector product)

Cross product of vectors $\vec{A}$ and $\vec{B}$ is defined as

$\vec{A}\times\vec{B}=|\vec{A}||\vec{B}|\ \sin\theta\ \hat{n}$

When $\hat{n}$ represents the direction. Direction is given by right handed screw rule.

If a right handed screw is rotated from $\vec{A}$ to $\vec{B}$, the direction of tip of the screw gives the direction of $\vec{A}\times\vec{B}$.

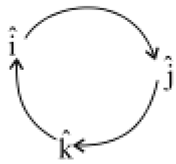
- Properties

1. It is not commutative, ie, $\vec{A}\times\vec{B}\neq\vec{B}\times\vec{A}$
2. It is distributive , ie, $\vec{A}\times(\vec{B}+\vec{C})=\vec{A}\times\vec{B}+\vec{A}\times\vec{C}$
3. Angle between two vectors $\vec{A}$ and $\vec{B}$, $\theta=\sin^{-1}\left[\dfrac{|\vec{A}\times\vec{B}|}{|\vec{A}||\vec{B}|}\right]$
4. If two vectors are parallel, $\theta=0$ $\therefore \vec{A}\times\vec{B}=0$

5. $\vec{A}\times\vec{A}=0$

6. $\hat{i}\times\hat{i}=0;\hat{j}\times\hat{j}=0;\ \hat{k}\times\hat{k}=0$

7. $\hat{i}\times\hat{j}=\hat{k};\hat{j}\times\hat{k}=\hat{i};\ \hat{k}\times\hat{i}=\hat{j}$



Note: 1

The common situations where the cross product is used are listed below:

(i) Torque, $\vec{\tau}=\vec{r}\times\vec{F}$ (ii) Angular momentum, $\vec{L}=\vec{r}\times\vec{P}$ (iii) Linear velocity, $\vec{V}=\vec{\omega}\times\vec{r}$

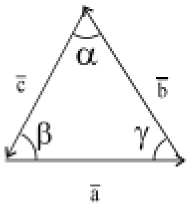
Note : 2

If $\vec{A}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{B}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$

$$\vec{A}\times\vec{B}=\begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$=(a_2b_3-a_3b_2)\hat{i}-(a_1b_3-a_3b_1)\hat{j}+(a_1b_2-a_2b_1)\hat{k}$$

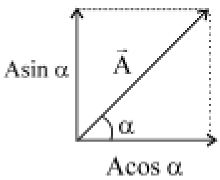
- **Lami's theorem**



In ΔABC with sides $\vec{a},\vec{b},\vec{c}$ $\quad \dfrac{\sin\alpha}{a}=\dfrac{\sin\beta}{b}=\dfrac{\sin\gamma}{c}$

- **Resolution of a vector into components**

(i) Vector in a plane



The vector can be split up into two components, A cos α and A sin α

(ii) Vector in space $\vec{A}=A_x\hat{i}+A_y\hat{j}+A_z\hat{k}$

$\mathrm{Cos}\alpha=\dfrac{A_x}{A}=l$ $\qquad \alpha\rightarrow$ angle between the vector and the x - axis

$\mathrm{Cos}\beta=\dfrac{A_y}{A}=m$ $\qquad \beta\rightarrow$ angle between the vector and the y - axis

$\mathrm{Cos}\gamma=\dfrac{A_z}{A}=n$ $\qquad \gamma\rightarrow$ angle between the vector and the z - axis l, m, n are called direction cosines. Also, $l^2+m^2+n^2=\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

Note: 1

If a body is in equilibrium under a set of non-collinear forces, the minimum number of forces has to be three

Note: 2

(a) If $\vec{A},\vec{B}$ are the adjacent sides of a parallelogram, area of the parallelogram is given by

Area = $\vec{A}\times\vec{B}$

(b) If diagonals $\vec{C}$ and $\vec{D}$ are given,

Area = $\frac{1}{2}(\vec{C}\times\vec{D})$

PROJECTILES

I. OBLIQUE PROJECTILE

A body projected at a particular angle with an initial velocity u

Assumptions

1. Friction due to air is neglected
2. There is no horizontal force acting on the body

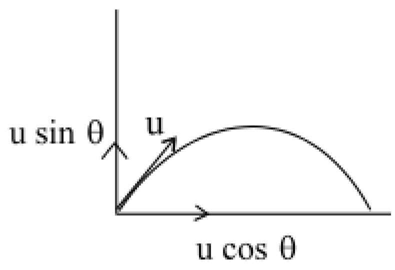
- Projectile motion is a two dimensional motion. The path taken by the body is parabolic and is called trajectory.
- Initial velocity u has two components; horizontal $(u\cos\theta)$ and vertical $(u\sin\theta)$
- As the body starts ascending, vertical component decreases; becomes zero at the topmost point; increases during the descent.
- Horizontal velocity remains $u\cos\theta$ throughout the motion.
- At the topmost point,

a) velocity $= u\cos\theta$

b) momentum $= mu\cos\theta$

c) kinetic energy $= \frac{1}{2}mu^2\cos^2\theta$

d) potential energy $= \frac{1}{2}mu^2\sin^2\theta$



Note :1

If $\theta=45^0$, kinetic energy and potential energy at the topmost point are equal.

Note :2

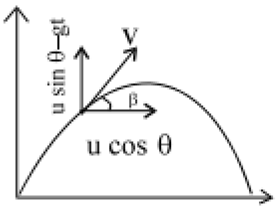
During the motion of a projectile, total energy remains conserved. During the ascent, kinetic energy is converted to potential energy and during descent potential energy is converted back to kinetic energy

Note :3

For a projectile,

a) Velocity of projection = velocity of landing

b) Kinetic energy of projection = kinetic energy of landing



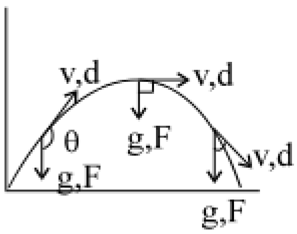
- Equation of trajectory of a projectile is $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$

- Velocity of the body at any instant $V = \sqrt{u^2 + g^2t^2 - 2ugt\sin\theta}$

- Angle made by the velocity vector with the horizontal is given by, $\tan\beta = \dfrac{u\sin\theta - gt}{u\cos\theta}$

Note: In a projectile motion, θ is obtuse while the body ascends; 90^0 at the topmost point and it is acute while the body descends where θ is the angle between

(i) displacement and acceleration
(ii) displacement and force
(iii) velocity and acceleration
(iv) velocity and force



- For a projectile

1. Time of flight, $T = \dfrac{2u\sin\theta}{g}$

2. Maximum height attained $H_{max} = \dfrac{u^2\sin^2\theta}{2g}$

3. Horizontal range $R = \dfrac{u^2\sin 2\theta}{g}$

4. Maximum horizontal range $R_{max} = u^2/g$

Note:1

A projectile attains maximum range when projected at an angle 45^0

Note: 2

R_{max} and H_{max} are related as $R_{max} = 4\,H_{max}$

Note : 3

For two angles of projection, θ and $(90 - \theta)$

1. $R_1 = R_2$; the ranges are equal

2. $\dfrac{T_1}{T_2} = \tan\theta$

3. $T_1T_2 = \dfrac{2R}{g}$

4. $\frac{H_1}{H_2} = \tan^2 \theta$

5. $R = 4\sqrt{H_1 H_2}$

- Change in momentum between

1. The point of landing and point of projection = -2 mu sin θ
2. The topmost point and point of projection = -mu sin θ

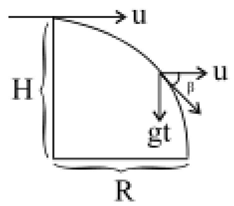
II HORIZONTAL PROJECTILE

A body projected with an initial horizontal velocity. Initial vertical velocity is zero

• For a horizontal projectile,

(a) Time of flight $T=\sqrt{\frac{2H}{g}}$

(b) Horizontal range $R = u\sqrt{\frac{2H}{g}}$



Note :1

Horizontal velocity remains constant throughout the motion; vertical velocity goes on increasing

Note :2

For a body thrown horizontally, time taken to reach the ground is independent of initial velocity of projection.

Velocity at any instant is given by $V = \sqrt{u^2 + g^2t^2}$

Angle made by the velocity vector with the horizontal is given by $\tan\beta = \frac{gt}{u}$

Equation of trajectory of the projectile is $y = \frac{gx^2}{2u^2}$

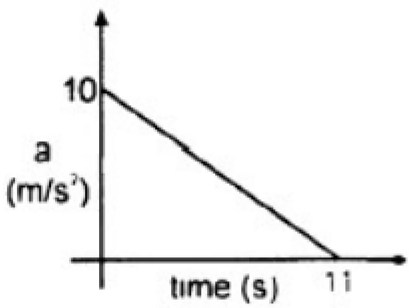
PART I - (JEEMAIN)

SECTION - I

1. A body travels for 15 sec starting from rest with constant acceleration. If it travels distance s_1, s_2 and s_3 in the first five seconds, second five seconds and next five seconds respectively the relation between s_1, s_2 and s_3 is

1) $s_1 = s_2 = s_3$ 2) $Ss_1 = 3s_2 = s_3$ 3) $s_1 = \frac{1}{3}s_2 = \frac{1}{5}s_3$ 4) $s_1 = \frac{1}{5}s_2 = \frac{1}{3}s_3$

2. A particle is initially at rest, it is subjected to a linear acceleration a, as shown in the figure. The maximum speed attained by the particle is



1) 605 m/s 2) 110 m/s 3) 55 m/s 4) 550 m/s

3. The initial velocity of a partilce is given by u (at t = 0) and the acceleration by f, where f = at (here t is time and a is constant). Which of the following relation is valid?

1) $v = u + at^2$ 2) $v = u + \frac{at^2}{2}$ 3) $v = u + at$ 4) v = u

4. Water drops fall at regular intervals from a tap which is 5m above the ground. The third drop is leaving the tap at the instant the first drop touches the ground. How far above the ground is the second drop at that instant? (Take g = 10 ms^{-2})

1) $\frac{5}{4}m$ 2) 4m 3) $\frac{5}{2}m$ 4) $\frac{15}{4}m$

5. A ball is thrown vertically upwards from the foot of a tower. It crosses the top of the tower twice after an interval of 4s and reaches the foot of the tower 8s after it was thrown. What is the height of the tower? Take g = 10 ms^{-2}.

1) 60m 2) 80m 3) 100m 4) 120m

6. If the sum of two vectors is a unit vector, then the magnitude of their difference is

1) 3 unit 2) $\sqrt{3}$unit 3) 1/3 unit 4) $1/\sqrt{3}$unit

7. A body has an initial velocity of 3 ms^{-1} and has an acceleration of 1 ms^{-1} normal to the direction of the initial velocity. Then its velocity 4s after the start is

1) 7 ms^{-1} along the direction of initial velocity

2) 7 ms^{-1} along the normal to the direction of initial velocity

3) 7 ms^{-1} midway between the two directions

4) 5 ms^{-1} at an angle $\tan^{-1}\left(\frac{4}{3}\right)$ with the direction of initial velocity

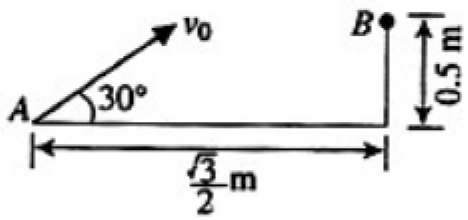
8. A particle is projected such that the horizontal range and vertical height are the same. Then the angle of projection is :

1) $\tan^{-1}(4)$ 2) $\tan^{-1}\left(\frac{1}{4}\right)$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{3}$

9. A particle is projected from the ground with an initial speed of v at an angle θ with horizontal. The average velocity of the particle between its point of projection and highest point of trajectory is

1) $\frac{v}{2}\sqrt{1+2\cos^2\theta}$ 2) $\frac{v}{2}\sqrt{1-2\cos^2\theta}$ 3) $\frac{v}{2}\sqrt{1+3\cos^2\theta}$ 4) $v\cos\theta$

10. A ball is projected from a point A with some velocity at an angle 30^0 with the horizontal shown in figure. Consider the target at point B. The ball will hit the target if it is thrown with a velocity v, equal to



1) 5 ms^{-1} 2) 25 ms^{-1} 3) $\sqrt{5}ms^{-2}$ 4) not possible

11. A ball is projected upwards from the top of a tower with a velocity of 50ms^{-1} making an angle of 30^0 with the horizontal. The height of the tower is 70m. After how many seconds from the instant of throwing will the ball reach the ground?

1) 2s 2) 5s 3) 7s 4) 9s

12. A body A moves with a uniform acceleration a and zero initial velocity. Another body B, starts from the same point, at the same time moves in the same direction with a constant velocity v. The two bodies meet after a time t. The value of t is

1) $\frac{2v}{a}$ 2) $\frac{v}{a}$ 3) $\frac{v}{2a}$ 4) $\sqrt{\frac{v}{2a}}$

13. A car is moving towards east with a speed of 25kmh^{-1}. To the driver of the car, a bus appears to move towards north with a speed of $25\sqrt{3}kmh^{-1}$. What is the actual velocity of the bus?

1) $50kmh^{-1}, 30^0 E$ of N 2) $50kmh^{-1}, 30^0 N$ of E

3) $25kmh^{-1}, 30^0 E$ of N 4) $25kmh^{-1}, 30^0 N$ of E

14. A person standing on a road has to hold his umbrella at 60^0 with the vertical to keep the rain away. He throws the umbrella and starts running at 20 ms^{-1}. He finds that rain drops are hitting his head vertically. Find the speed of the rain drops with respect to

a) the road b) the moving person

1) $\frac{40}{\sqrt{3}}m/s, \frac{20}{\sqrt{3}}m/s$ 2) $20\sqrt{3}\,m/s,\ 20\sqrt{3}m/s$

3) $\frac{20}{\sqrt{3}}m/s, \frac{40}{\sqrt{3}}m/s$ 4) $10\sqrt{3}\,m/s,\ 20\sqrt{3}m/s$

15. A man swims from point A on one bank of a river of width 100m. When he swims perpendicular to the river current, he reaches the other bank 50m downstream. The angle to the bank at which the should swim to reach directly opposite point B on the other bank is

1) 10^0 upstream 2) 20^0 upstream 3) 30^0 upstream 4) 60^0 upstream

SECTION - II
Numerical Type Questions

16. A body walks to his school at a distance of 6km with constant speed of 2.5 km h^{-1} and walks back with a constant speed of 4 kmh^{-1}. His average for round trip expressed in km h^{-1} is, nearly

17. A point initially at rest moves along the x-axis. Its acceleration varies as $a = (6t+5)\,m/s^2$. If starts from origin and the distance covered in 2s in 2x. What is the value of x

18. A particle is projected up an inclined plane of inclination β at an elevation α to the horizontal. Find the ratio between $\tan\alpha$ and $\tan\beta$, if the particle strikes the plane horizontally.

19. At what angle with the horizontal should a ball he thrown so that its range R is related to the time of flight as $R = 5T^2$ [Take g = 10 ms^{-2}] (in degrees)

PART - II (JEE ADVANCED)

SECTION - III (Only one option correct type)

20. A particle starts from rest with uniform acceleration a. It's velocity after n seconds is v. The displacement of the body in the last two seconds is

A) $\frac{2v(n-1)}{n}$ B) $\frac{v(n-1)}{a}$ C) $\frac{v(n+1)}{n}$ D) $\frac{2v(2n+1)}{a}$

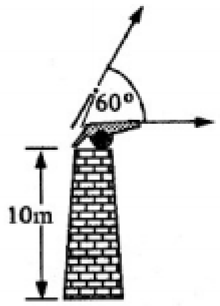
21. A particle of unit mass undergoes one-dimensional motion such that its velocity varies according to $v(x) = bx^{-2n}$ where b and n are constants and x is the position of the particle. The acceleration of the particle as a function of x, is given by:

A) $-2nb^2x^{-4n-1}$ B) $-2b^2x^{-2n-1}$ C) $-2nb^2x^{-4n+1}$ D) $-2nb^2x^{-2n-1}$

22. A particle is thrown upwards from ground. It experiences a constant air resistance which can produce a retardation of 2m/s^2. The ratio of time of ascent to the time of descent is (g = 10 m/s^2)

A) 1 : 1 B) $\sqrt{\frac{2}{3}}$ C) $\frac{2}{3}$ D) $\sqrt{\frac{3}{2}}$

23. Two canons installed at the top of a clift 10m high fire a shot each with speed $5\sqrt{3}\,\text{ms}^{-1}$ at some interval. One canon fires at 60^0 with horizontal whereas the second fires horizontally. The coordinates of point of collision of shots are



A) $3\sqrt{5}\text{m}, 3\text{m}$ B) $\frac{1}{5\sqrt{3}}\text{m}, \frac{1}{5}\text{m}$ C) $\frac{1}{3}\text{m}, \frac{1}{3\sqrt{5}}\text{m}$ D) $5\sqrt{3}\text{m}, 5\text{m}$

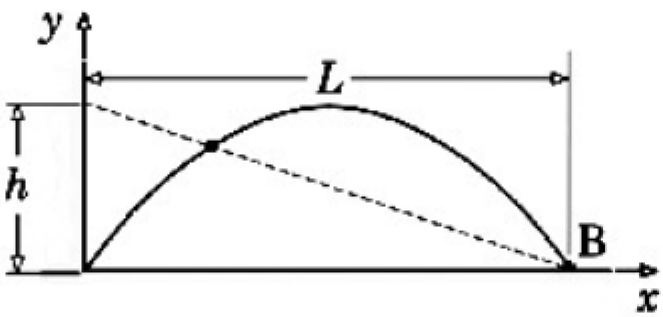
24. A projectile is thrown in the upward direction making an angle of 60^0 with the horizontal direction with a velocity of 150ms^{-1}. Then the time after which its inclination with the horizontal is 45^0 is

A) $15\left(\sqrt{3}-1\right)\text{s}$ B) $15\left(\sqrt{3}+1\right)\text{s}$ C) $7.5\left(\sqrt{3}-1\right)\text{s}$ D) $7.5\left(\sqrt{3}+1\right)\text{s}$

25. A ball is falling down from an initial height of h=20m with a gun held horizontally d = 50m from the trajectory of falling ball, at the height $h' = 40\text{m}$. We are going to shoot at the falling ball. The bullet leaves the gun at a speed of v = 100 m/s. At what time after the start of the fall should the gun be fired in order to hit the falling ball with the bullet? (The air resistance is negligible)

A) 1s B) 2s C) 3s D) 4s

26. A small ball is thrown from foot of a wall with the minimum possible velocity to hit a bulb B on the ground a distance L away from the wall. Find expression for height *h* of shadow of the ball on the wall as a function of time *t*. Acceleration due to gravity is *g*.



A) $h = t\sqrt{\frac{gL}{2}}$ B) $h = t\sqrt{gL}$ C) $h = \frac{t\sqrt{gL}}{2}$ D) $h = t\sqrt{2gL}$

27. When a motorist is driving with a velocity $6\hat{i}+8\hat{j}$, wind appears to come from the direction $\hat{i}$. When he doubles his velocity the wind appears to come from the direction $\hat{i}+\hat{j}$. Then the true velocity of the wind is:

A) $4\hat{i}-8\hat{j}$ B) $4\hat{i}+8\hat{j}$ C) $2\hat{i}+3\hat{j}$ D) $4\hat{i}+4\hat{j}$

28. A boat moves perpendicualr to the river flow with velocity 10kmph relative to water current. The velocity of water is 5 kmph relative to the person on ground. Find the velocity of the Boat relative to the Person and time taken to cross the river, if the width of the river is 3km

A) 11.2 kmph, 18 min B) 10 kmph, 18 min

C) $5\sqrt{3}$ kmph, 16.36 min D) 10 kmph, 16.36 min

SECTION - IV (More than one correct answer)

29. A man standing an a road has to hold his umbrella at 30^0 withthe vertical to keep the rain away. He throws the umbrella and starts running at 10kmph. He finds that raindrops are hitting his head vertically. The speed of the raindrops with respect to the road and with respect to the man respectively.

A) 20kmph and $10\sqrt{3}$kmph B) 10kmph and $10\sqrt{3}$ kmph

C) 30 kmph and 15 kmph D) $10\sqrt{3}$ kmph and $20\sqrt{3}$ kmph

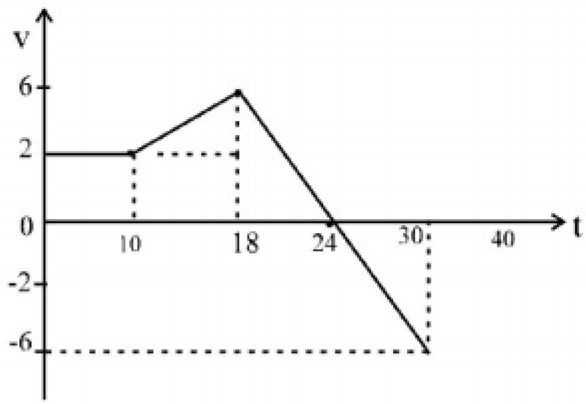
30. Two ships are 10km apart on a line running from south to North. The one farther north is streaming to wards west at 20kmph. The other is streaming towards north at 20kmph. The time taken to reach the position of closest approach is

A) 30 minutes B) 20 minutes C) 15 minutes D) 5 minutes

31. A ball projected vertically upward from the top of a building takes 12s to reach the ground. If the same ball is thrown vertically down with same speed, it reaches ground in 3s ($g=10\ m/s^2$)

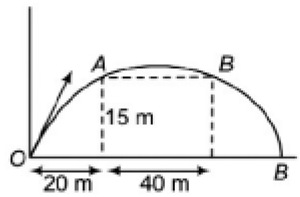
A) Height of the building is 180m
B) If the ball is dropped from the top of the building it reaches ground in 6s
C) Initial velocity of projection 40 m/s
D) The ball hits the ground with speed 75 m/s

32. A particle moves in a straight line with the velocity as shown in graph. At t= 0, x = -16cm



A) The maximum value of the position coordinates of the particle is 54 m
B) The maximum value of the position coordinate of the particle is 36m
C) The particle is at the position of 36m at t= 18s
D) The particle is at the position of 36 m at t = 30s

33. In the projectile motion shown in figure, given $t_{AB} = 2s$ then (g= 10 ms^{-2})



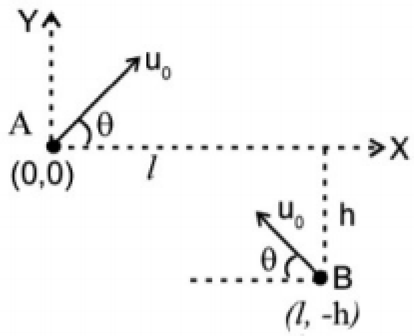
A) particle is at point B at 3s

B) maximum height of projectile is 20m

C) initial vertical component of velocity is 20 ms^{-1}

D) horizontal component of velocity is 20 ms^{-1}

34. Two particles A and B are projected in the vertical plane with same initial speed u_0 from point (0,0) and $(\ell, -h)$ towards each other as shwon in t = 0



A) The path of particle A awith respect to B will be straight line parallel to X-axis

B) the path of particle A with respect to B will be a parabola

C) Minimum distance between A and B is h

D) The time when separation between A and B is minimum is $\ell / 2u_0 \cos\theta$

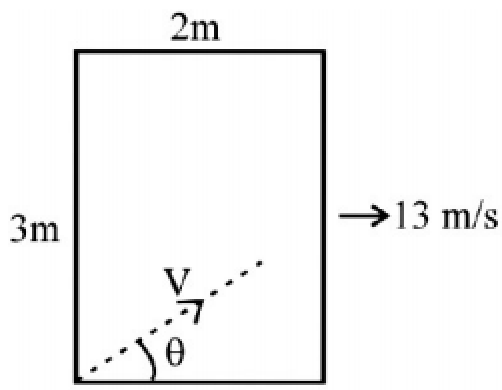
SECTION - V (Numerical Type - Upto two decimal place)

35. A body falling freely under gravity passes two points 30m apart in 1s. Find, from what point above the upper point it began to fall?

36. A 1kg mass is projected into air has velocitites 3 m/s and 4 m/s at two different points. if the two velocities are normal to each other what is the least K.E of the body

37. Six particles situated at the corners of a regular hexagon of side a move at a constant speed V. Each particle maintains a direction towards the particle at the next corner. The six particles will meet at a time $\frac{xa}{v}$. Find x

38. A car 2m long and 3m wide is moving at 13 m/s. A bullet hits it in a direction making an angle $\theta = \tan^{-1}\left(\frac{3}{4}\right)$ with the car as seen from ground. The bullet enters one edge of the corner and passes out of the diagonally opposite corner. Neglecting any interaction between bullet and car, find the time for the bullet to cross the car.



SECTION - VI (Matrix Matching)

39. A ball is projected from the ground with velocity v such that its range is maximum

Column I		Column II	
i.	Velocity at half of the maximum height	**a.**	$\frac{\sqrt{3}v}{2}$
ii.	Velocity at the maximum height	**b.**	$\frac{v}{\sqrt{2}}$
iii.	Change in its velocity when it returns to the ground	**c.**	$v\sqrt{2}$
iv.	Average velocity when it reaches the maximum height	**d.**	$\frac{v}{2}\sqrt{\frac{5}{2}}$

40. The path of a projectile is represented by $y = Ax - Bx^2$

	I		II
1	Range	a	A/B
2	maximum height	b	A
3	Time of flight	c	$A^2/4B$
4	Tangent of angle of projection	d	$\sqrt{\frac{2}{Bg}} \times A$