CHAPTER - 17
VECTORS

Important Results

1. Triangle law of vectors. If $\overline{OA} = \bar{a}$, $\overline{AB} = \bar{b}$ then $\overline{OA} + \overline{AB} = \overline{OB}$ ie., $\overline{OB} = \bar{a} + \bar{b}$
2. If O is a fixed point and P any point then $\overline{OP} = \bar{r}$ represents the position vector of P.
3. $\overline{AB}$ = Position vector of B - Position vector of **A**
4. $\bar{a}$ and $m\bar{a}$ are collinear where m is a scalar
5. If $\bar{a}, \bar{b}, \bar{c}$ are coplanar then any one of them can be expresed as a linear combination of the other two. ie. $\bar{a} = x\bar{b} + y\bar{c}$
6. If $\bar{a}, \bar{b}, \bar{c}$ are non coplanar then any $\bar{r} = x\bar{a} + y\bar{b} + z\bar{c}$
7. If $\bar{a}, \bar{b}, \bar{c}$ are non coplanar and $x\bar{a} + y\bar{b} + z\bar{c} = \bar{0}$ then x = y = z = 0
8. To prove A, B, C are collinear, find $\overline{AB}, \overline{BC}$ and show that one of them is a scalar multiple of the other
9. To prove A, B, C, D are coplanar, find $\overline{AB}, \overline{AC}, \overline{AD}$ and show that these are coplanar.
10. Section formula : If $\bar{a}$ and $\bar{b}$ are the position vectors of A and B then position vector of a point dividing AB in the ratio l : m is given by $\bar{r} = \dfrac{l\bar{b} + m\bar{a}}{l + m}$
11. P.V. of the midpoint of $AB = \dfrac{\bar{a} + \bar{b}}{2}$
12. P.V. of the centroid of triangle ABC is $\dfrac{\bar{a} + \bar{b} + \bar{c}}{3}$ where $\bar{a}, \bar{b}, \bar{c}$ are the position vectors of A, B, C respectively.
13. Dot product or Scalar product $\bar{a}.\bar{b} = |\bar{a}|\,|\bar{b}|\cos\theta$ where θ is the angle between $\bar{a}$ and $\bar{b}$
14. $\cos\theta = \dfrac{\bar{a}.\bar{b}}{|\bar{a}|\,|\bar{b}|}$
15. Scalar projection of $\bar{b}$ in the direction of $\bar{a} = \dfrac{\bar{a}.\bar{b}}{|\bar{a}|}$, scalar projection of $\bar{a}$ in the direction of $\bar{b} = \dfrac{\bar{a}.\bar{b}}{|\bar{b}|}$

$$\frac{\text{Projection of } \bar{b} \text{ on } \bar{a}}{\text{Projection of } \bar{a} \text{ on } \bar{b}} = \frac{|\bar{b}|}{|\bar{a}|}$$

16. Vector projection of $\bar{b}$ in the direction of $\bar{a} = \left(\frac{\bar{a}.\bar{b}}{|\bar{a}|}\right)\frac{\bar{a}}{|\bar{a}|}$

Component of $\bar{r}$ in the direction of $\bar{a} = \frac{\bar{r}.\bar{a}}{|\bar{a}|^2}\bar{a}$ and perpendicular to $\bar{a} = \bar{r} - \frac{\bar{r}.\bar{a}}{|\bar{a}|^2}\bar{a}$

17. $\bar{a}.\bar{b} = \bar{b}.\bar{a}$

18. If $\bar{a}$ and $\bar{b}$ are non zero vectors and $\bar{a}.\bar{b} = 0$ then $\bar{a}$ and $\bar{b}$ are perpendicular, if $\bar{a}.\bar{b} < 0$ then θ is obtuse, if $\bar{a}.\bar{b} > 0$, θ is acute

19. $(\bar{a})^2 = \bar{a}\,.\,\bar{a} = a^2$

20. $\left(\bar{a}+\bar{b}\right)^2 = a^2 + b^2 + 2\bar{a}.\bar{b}$; $\quad \left(\bar{a}-\bar{b}\right)^2 = a^2 + b^2 - 2\bar{a}.\bar{b}$; $\quad \left(\bar{a}+\bar{b}\right).\left(\bar{a}-\bar{b}\right) = |\bar{a}|^2 - |\bar{b}|^2 = a^2 - b^2$

21. $\bar{a}.\left(\bar{b}+\bar{c}\right) = \bar{a}.\bar{b} + \bar{a}.\bar{c}$

22. If $\bar{i}, \bar{j}, \bar{k}$ are unit vectors along the co-ordinate axes then
$\bar{i}.\bar{i} = \bar{j}.\bar{j} = \bar{k}.\bar{k} = 1,\ \bar{i}.\bar{j} = \bar{j}.\bar{i} = \bar{j}.\bar{k} = \bar{k}.\bar{j} = \bar{k}.\bar{i} = \bar{i}.\bar{k} = 0$

23. If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ then $|\bar{r}| = \sqrt{x^2 + y^2 + z^2}$

24. If P is (x, y, z) then the position vector of $P = x\bar{i} + y\bar{j} + z\bar{k}$

25. If $\bar{a} = a_1\bar{i} + a_2\bar{j} + a_3\bar{k},\ \bar{b} = b_1\bar{i} + b_2\bar{j} + b_3\bar{k}$ then $\bar{a}.\bar{b} = a_1b_1 + a_2b_2 + a_3b_3$

26. In the above, $\cos\theta = \frac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{\sum a_1^2} \times \sqrt{\sum b_1^2}}$

27. If $\bar{a}$ and $\bar{b}$ are perpendicular then $a_1b_1 + a_2b_2 + a_3b_3 = 0$

28. Cross product or vector product $\bar{a} \times \bar{b} = |\bar{a}||\bar{b}|\sin\theta\bar{n}$ where $\bar{n}$ is a unit vector perpendicular to the plane containing $\bar{a}$ and $\bar{b}$ such that $\bar{a}, \bar{b}, \bar{n}$ form a right handed triad.

29. Area of the parallelogram whose adjacent sides are $\bar{a}$ and $\bar{b}$ is $|\bar{a} \times \bar{b}|$

30. Area of $\Delta ABC = \frac{1}{2}|\overline{AB} \times \overline{AC}|$

31. If $\bar{a} = a_1\bar{i} + a_2\bar{j} + a_3\bar{k},\ \bar{b} = b_1\bar{i} + b_2\bar{j} + b_3\bar{k}$ then $\bar{a} \times \bar{b} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

32. A unit vector perpendicular to the plane of $\bar{a}$ and $\bar{b}$ is $\pm\left(\frac{\bar{a}\times\bar{b}}{|\bar{a}\times\bar{b}|}\right)$

33. $\bar{a}\times\bar{b}\neq\bar{b}\times\bar{a}$ and $\bar{b}\times\bar{a}=-\bar{a}\times\bar{b}$

34. $\bar{a}\times\bar{a}=\bar{0}$

35. If $\bar{a}$ and $\bar{b}$ are collinear then $\bar{a}\times\bar{b}=\bar{0}$

36. $\bar{a}\times(\bar{b}+\bar{c})=\bar{a}\times\bar{b}+\bar{a}\times\bar{c}$

37. $\bar{i}\times\bar{i}=\bar{j}\times\bar{j}=\bar{k}\times\bar{k}=\bar{0}$ and $\bar{i}\times\bar{j}=\bar{k},\ \bar{j}\times\bar{k}=\bar{i},\ \bar{k}\times\bar{i}=\bar{j}$ and $\bar{j}\times\bar{i}=-\bar{k}\quad \bar{k}\times\bar{j}=-\bar{i},\ \bar{i}\times\bar{k}=-\bar{j}$

38. $\sin\theta=\dfrac{|\bar{a}\times\bar{b}|}{|\bar{a}||\bar{b}|}$

39. If $\bar{a},\bar{b},\bar{c}$ are the vertices of a ΔABC then $\Delta=\frac{1}{2}|\bar{b}\times\bar{c}+\bar{c}\times\bar{a}+\bar{a}\times\bar{b}|$. If the points $\bar{a},\bar{b},\bar{c}$ are collinear then $|\bar{b}\times\bar{c}+\bar{c}\times\bar{a}+\bar{a}\times\bar{b}|=0$

40. $(\bar{a}\times\bar{b})^2+(\bar{a}.\bar{b})^2=|a|^2|b|^2$

41. Area of the parallelogram whose diagonals are $\bar{d_1}$ and $\bar{d_2}$ is $\frac{1}{2}|\bar{d_1}\times\bar{d_2}|$

42. Area of the quadrilateral whose diagonals are $\bar{d_1}$ and $\bar{d_2}$ is $\frac{1}{2}|\bar{d_1}\times\bar{d_2}|$

43. $(\bar{a}\times\bar{b}).\bar{c}$ or $\bar{a}.(\bar{b}\times\bar{c})$ is called the scalar tripple product of $\bar{a},\bar{b},\bar{c}$ and is denoted by $(\bar{a},\bar{b},\bar{c})$ or $(\bar{a}\ \bar{b}\ \bar{c})$

If $\bar{a}.\bar{b}=\bar{b}.\bar{c}=\bar{c}.\bar{a}=0$ then $|(\bar{a},\bar{b},\bar{c})|=|\bar{a}||\bar{b}||\bar{c}|$

44. $(\bar{a},\bar{b},\bar{c})=\begin{vmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}$ where $\bar{a}=a_1\bar{i}+a_2\bar{j}+a_3\bar{k},\ \bar{b}=b_1\bar{i}+b_2\bar{j}+b_3\bar{k},\ \bar{c}=c_1\bar{i}+c_2\bar{j}+c_3\bar{k}$

45. In a scalar tripple product the dot and the cross can be interchanged ie., $\bar{a}\times\bar{b}.\bar{c}=\bar{a}.\bar{b}\times\bar{c}$

46. $(\bar{a},\bar{b},\bar{c})=(\bar{b},\bar{c},\bar{a})=(\bar{c},\bar{a},\bar{b})$

47. $(\bar{a},\bar{b},\bar{c})=-(\bar{a},\bar{c},\bar{b})=-(\bar{b},\bar{a},\bar{c})$

48. $(\bar{a},\bar{a},\bar{b})=(\bar{a},\bar{b},\bar{b})=(\bar{a},\bar{c},\bar{c})=0$

49. $(\bar{a},\bar{b},\bar{c})=$ Volume of the parallelopiped whose coterminus edges are $\bar{a},\bar{b},\bar{c}$

50. If $\bar{A}=l_1\bar{a}+l_2\bar{b}+l_3\bar{c},\ \bar{B}=m_1\bar{a}+m_2\bar{b}+m_3\bar{c},\ \bar{C}=n_1\bar{a}+n_2\bar{b}+n_3\bar{c}$ then

$$(\bar{A},\bar{B},\bar{C})=\begin{vmatrix}l_1 & l_2 & l_3\\ m_1 & m_2 & m_3\\ n_1 & n_2 & n_3\end{vmatrix}(\bar{a},\bar{b},\bar{c})$$

51. If $\bar{a},\bar{b},\bar{c}$ are non zero, non parallel vectors then $\bar{a},\bar{b},\bar{c}$ are coplanar if $(\bar{a},\bar{b},\bar{c}) = 0$

52. If $\bar{a},\bar{b},\bar{c}$ are coplanar then $\bar{a}\times\bar{b},\ \bar{b}\times\bar{c},\ \bar{c}\times\bar{a},$ also are coplanar, again

$\bar{a}+\bar{b},\ \bar{b}+\bar{c},\ \bar{c}+\bar{a};\ \bar{a}-\bar{b},\ \bar{b}-\bar{c},\ \bar{c}-\bar{a}$ also are coplanar vectors

53. $\bar{i}\times\bar{j}.\bar{k}+\bar{j}\times\bar{k}.\bar{i}+\bar{k}\times\bar{i}.\bar{j}=3$

54. $\left[\bar{a}-\bar{b},\bar{b}-\bar{c},\bar{c}-\bar{a}\right]=0$

55. $\left[\bar{a}+\bar{b},\bar{b}+\bar{c},\bar{c}+\bar{a}\right]=2\left[\bar{a},\bar{b},\bar{c}\right]$

56. $\left[\bar{a}\times\bar{b},\bar{b}\times\bar{c},\bar{c}\times\bar{a}\right]=\left(\bar{a},\bar{b},\bar{c}\right)^2=\begin{vmatrix}\bar{a}.\bar{a} & \bar{a}.\bar{b} & \bar{a}.\bar{c}\\ \bar{b}.\bar{a} & \bar{b}.\bar{b} & \bar{b}.\bar{c}\\ \bar{c}.\bar{a} & \bar{c}.\bar{b} & \bar{c}.\bar{c}\end{vmatrix}$

57. Vector tripple product $\bar{a}\times\left(\bar{b}\times\bar{c}\right)=\left(\bar{a}.\bar{c}\right)\bar{b}-\left(\bar{a}.\bar{b}\right)\bar{c}$

$\bar{a}=$ Outer vector, $\bar{c}=$ remote vector, $\bar{b}=$ adjacent vector

$\bar{a}\times\left(\bar{b}\times\bar{c}\right)=$ (outer . remote) adjacent - (outer . adjacent) remote

Note : $\bar{a}\times(\bar{b}\times\bar{c})$ is coplanar with $\bar{b}$ and $\bar{c}$, A unit vector. Unit vector perpendicular to $\bar{a}$ and coplanar with $\bar{b}$ and $\bar{c}$ is given by $\bar{n}=\dfrac{\bar{a}\times\left(\bar{b}\times\bar{c}\right)}{|\bar{a}\times\left(\bar{b}\times\bar{c}\right)|}$

Again $\left(\bar{a}\times\bar{b}\right)\times\bar{c}=\left(\bar{a}.\bar{c}\right)\bar{b}-\left(\bar{b}.\bar{c}\right)\bar{a}$. In general $\bar{a}\times\left(\bar{b}\times\bar{c}\right)\neq\left(\bar{a}\times\bar{b}\right)\times\bar{c}$

If $\bar{a}\times(\bar{b}\times\bar{c})=(\bar{a}\times\bar{b})\times\bar{c}$ then $(\bar{c}\times\bar{a})\times\bar{b}=\bar{0}$ and $\bar{a}$ is parallel to $\bar{c}$

58. $\bar{i}\times\left(\bar{a}\times\bar{i}\right)+\bar{j}\times\left(\bar{a}\times\bar{j}\right)+\bar{k}\times\left(\bar{a}\times\bar{k}\right)=2\bar{a}$

59. $\bar{i}\times\left(\bar{j}\times\bar{k}\right)=\bar{0}$

60. $\bar{a}\times\left(\bar{b}\times\bar{c}\right)+\bar{b}\times\left(\bar{c}\times\bar{a}\right)+\bar{c}\times\left(\bar{a}\times\bar{b}\right)=\bar{0}$

61. $\bar{a}\times\left(\bar{b}\times\bar{c}\right),\bar{b}\times\left(\bar{c}\times\bar{a}\right),\bar{c}\times\left(\bar{a}\times\bar{b}\right)$ are coplanar

62. A unit vector perpendicular to $\bar{a}$ and coplanar with $\bar{a}$ and $\bar{b}$ is given by $\bar{n}=\dfrac{\bar{a}\times\left(\bar{a}\times\bar{b}\right)}{|\bar{a}\times\left(\bar{a}\times\bar{b}\right)|}$

63. $\left(\bar{a}\times\bar{b}\right)\bullet\left(\bar{c}\times\bar{d}\right)=\begin{vmatrix}\bar{a}.\bar{c} & \bar{a}.\bar{d}\\ \bar{b}.\bar{c} & \bar{b}.\bar{d}\end{vmatrix}$

64. $\left(\bar{a}\times\bar{b}\right)\times\left(\bar{c}\times\bar{d}\right)=\left(\bar{a},\bar{c},\bar{d}\right)\bar{b}-\left(\bar{b},\bar{c},\bar{d}\right)\bar{a}$ and $\left(\bar{a}\times\bar{b}\right)\times\left(\bar{c}\times\bar{d}\right)=\left(\bar{a},\bar{b},\bar{d}\right)\bar{c}-\left(\bar{a},\bar{b},\bar{c}\right)\bar{d}$

$$\Rightarrow \bar{d} = \frac{(\bar{d},\bar{b},\bar{c})\bar{a} + (\bar{d},\bar{c},\bar{a})\bar{b} + (\bar{d},\bar{a},\bar{b})\bar{c}}{(\bar{a},\bar{b},\bar{c})}$$

65. If $\bar{a},\bar{b},\bar{c}$ and $\bar{a}',\bar{b}',\bar{c}'$ are reciprocal system of vectors where $(\bar{a},\bar{b},\bar{c}) \neq 0$ then

$$\bar{a}' = \frac{\bar{b}\times\bar{c}}{(\bar{a},\bar{b},\bar{c})}, \bar{b}' = \frac{\bar{c}\times\bar{a}}{(\bar{a},\bar{b},\bar{c})}, \bar{c}' = \frac{\bar{a}\times\bar{b}}{(\bar{a},\bar{b},\bar{c})}$$

66 Any vector $\bar{r} = (\bar{r}.\bar{a})\bar{a}' + (\bar{r}.\bar{b})\bar{b}' + (\bar{r}.\bar{c})\bar{c}'$ or $\bar{r} = (\bar{r}.\bar{a}')\bar{a} + (\bar{r}.\bar{b}')\bar{b} + (\bar{r}.\bar{c}')\bar{c}$

67. $\bar{a}.\bar{a}' + \bar{b}.\bar{b}' + \bar{c}.\bar{c}' = 3$; $\bar{a}\times\bar{a}' + \bar{b}\times\bar{b}' + \bar{c}\times\bar{c}' = \bar{0}$; $\bar{a}.\bar{b}' = \bar{a}.\bar{c}' = \bar{b}.\bar{a}' = \bar{b}.\bar{c}' = \bar{c}.\bar{a}' = \bar{c}.\bar{b}' = 0$

$\bar{a}'\times\bar{b}' + \bar{b}'\times\bar{c}' + \bar{c}'\times\bar{a}' = \frac{\bar{a}+\bar{b}+\bar{c}}{(\bar{a},\bar{b},\bar{c})}$; $(\bar{a},\bar{b},\bar{c})(\bar{a}',\bar{b}',\bar{c}') = 1$, $(\bar{a}+\bar{b}+\bar{c}).(\bar{a}'+\bar{b}'+\bar{c}') = 3$

68. If $\bar{F}$ is the force causing displacement $\bar{S}$ then work done $= \bar{F}.\bar{S}$

69. Moment of the force $\bar{F}$ about a point P is $\overline{PQ}\times\bar{F}$ where Q is a point on the line of action of $\bar{F}$.

70. Moment of a couple = $(\bar{r}_1 - \bar{r}_2)\times\bar{F}$ where $\bar{r}_1$ and $\bar{r}_2$ are the position vectors of the points of application of F and -$\bar{F}$ respectively.

71. $|\bar{a}+\bar{b}| = |\bar{a}-\bar{b}| \Leftrightarrow \bar{a}$ is perpendicular to $\bar{b}$; $|\bar{a}+\bar{b}| = |\bar{a}| + |\bar{b}| \Leftrightarrow \bar{a}$ is parallel to $\bar{b}$

$|\bar{a}+\bar{b}|^2 = |\bar{a}|^2 + |\bar{b}|^2 \Leftrightarrow \bar{a}$ is perpendicular to $\bar{b}$, $|\bar{a}+\bar{b}| = |\bar{a}| - |\bar{b}| \Rightarrow$ angle between $\bar{a}$ and $\bar{b}$ is π

72. If $\bar{u}$ is any vector then $(\bar{u}.\bar{i})\bar{i} + (\bar{u}.\bar{j})\bar{j} + (\bar{u}.\bar{k})\bar{k} = \bar{u}$

73. In triangle ABC, if D, E, F are the midpoints of the sides BC, CA, AB respectively then

$\overline{AD} + \overline{BE} + \overline{CF} = \bar{0}$ and $\overline{AB} + \overline{AC} = 2\overline{AD}$

74. If A, B, C, D are the vertices of a tetrahedron ABCD, then its volume = $\frac{1}{6}[\overline{AB}, \overline{AC}, \overline{AD}]$

75. If $\bar{a},\bar{b},\bar{c}$ form the sides BC, CA, AB of a triangle ABC then $\bar{a}\times\bar{b} = \bar{b}\times\bar{c} = \bar{c}\times\bar{a}$ and if $\bar{a}\times\bar{b} = \bar{b}\times\bar{c} = \bar{c}\times\bar{a}$ then $\bar{a}+\bar{b}+\bar{c} = 0$

76. Position vector of the incentre of triangle ABC is $\frac{BC\,\bar{a} + CA\,\bar{b} + AB\bar{c}}{BC+CA+AB}$ where $\bar{a},\bar{b},\bar{c}$ are the position vectors of A,B,C.

77. If $\bar{a},\bar{b},\bar{c}$ form a right handed system then $\bar{c} = \bar{a}\times\bar{b}$. If they form a left handed system then $\bar{c} = -\bar{a}\times\bar{b}$

78. If $\bar{a}$, $\bar{b},\bar{c}$ are mutually perpendicular then $|\bar{a}+\bar{b}+\bar{c}| = \sqrt{a^2+b^2+c^2}$

79. If $\bar{a}$ perpendicular to $\bar{b}+\bar{c}$, $\bar{b}$, perpendicular to $\bar{c}+\bar{a}$, $\bar{c}$ perpendicular $\bar{a}+\bar{b}$ then $|\bar{a}+\bar{b}+\bar{c}| = \sqrt{a^2+b^2+c^2}$

80. $\left(\overline{a}.\overline{i}\right)^2+\left(\overline{a}.\overline{j}\right)^2+\left(\overline{a}.\overline{k}\right)^2=a^2$

81. $\left(\overline{a}\times\overline{i}\right)^2+\left(\overline{a}\times\overline{j}\right)^2+\left(\overline{a}\times\overline{k}\right)^2=2a^2$

PART I - (JEEMAIN LEVEL)

PART 1

SECTION - I - Straight objective

1. If vectors $\overline{AB}=-3\hat{i}+4\hat{k}$ and $\overline{AC}=5\hat{i}-2\hat{j}+4\hat{k}$ are the sides of triangle ABC, then the length of the median through A is

(1) $\sqrt{14}$ (2) $\sqrt{18}$ (3) $\sqrt{29}$ (4) 5

2. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=4\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{c}=\hat{i}+\alpha\hat{j}+\beta\hat{k}$ are linearly dependent vectors and $|\vec{c}|=\sqrt{3}$ then the values of α & β are

(1) $\alpha=1,\beta=-1$ (2) $\alpha=1,\beta=\pm1$ (3) $\alpha=-1,\beta=\pm1$ (4) $\alpha=\pm1,\beta=1$

3. If $\hat{i}-3\hat{j}+5\hat{k}$ bisects the angle between $\hat{a}$ and $-\hat{i}+2\hat{j}+2\hat{k}$, where $\hat{a}$ is a unit vector, then

(1) $\hat{a}=\frac{1}{105}(41\hat{i}+88\hat{j}-40\hat{k})$ (2) $\hat{a}=\frac{1}{105}(41\hat{i}+88\hat{j}+40\hat{k})$

(3) $\hat{a}=\frac{1}{105}(-41\hat{i}+88\hat{j}-40\hat{k})$ (4) $\hat{a}=\frac{1}{105}(41\hat{i}-88\hat{j}-40\hat{k})$

4. If $\vec{a},\vec{b}$ and $\vec{c}$ are vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=7$, $|\vec{b}|=5$ and $|\vec{c}|=3$, then the angle between $\vec{b}$ and $\vec{c}$ is

(1) 60^0 (2) 30^0 (3) 45^0 (4) 90^0

5. If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are three unit vectors inclined to each other at an angle θ, then the maximum value of θ is

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{2}$ (3) $\frac{2\pi}{3}$ (4) $\frac{5\pi}{6}$

6. If $\vec{r}=x_1(\vec{a}\times\vec{b})+x_2(\vec{b}\times\vec{a})+x_3(\vec{c}\times\vec{d})$ and $4[\vec{a}\ \vec{b}\ \vec{c}]=1$, then $x_1+x_2+x_3$ is equal to

(1) $\frac{1}{2}\vec{r}\cdot(\vec{a}+\vec{b}+\vec{c})$ (2) $\frac{1}{4}\vec{r}\cdot(\vec{a}+\vec{b}+\vec{c})$

(3) $2\vec{r}\cdot(\vec{a}+\vec{b}+\vec{c})$ (4) $4\vec{r}\cdot(\vec{a}+\vec{b}+\vec{c})$

7. Let O be the origin of the coordinate system in the Cartesian plane, $\overline{OP}$ and $\overline{OR}$ be vectors making angle 45^0 and 135^0 respectively with the positive directions of the x-axis (i.e., in the counter clock wise). Rectangle OPQR is completed and M is the midpoint of PQ. If the line $\overline{OM}$ meets the diagonal PR at T, and $|\overline{OP}|=3$, $|\overline{OR}|=4$, then $\overline{OT}$ is

(1) $\frac{1}{2}(\hat{i}+\hat{j})$ (2) $\frac{2}{3}(\hat{i}+5\hat{j})$ (3) $\frac{\sqrt{2}}{3}(\hat{i}-5\hat{j})$ (4) $\frac{\sqrt{2}}{3}(\hat{i}+5\hat{j})$

8. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be vectors of equal magnitude such that the angle between $\vec{a}$ and $\vec{b}$ is $\alpha, \vec{b}$ and $\vec{c}$ is β and $\vec{c}$ and $\vec{a}$ is γ. Then the minimum value of $\cos\alpha+\cos\beta+\cos\gamma$ is

(1) $\frac{1}{2}$ (2) $-\frac{1}{2}$ (3) $\frac{3}{2}$ (4) $-\frac{3}{2}$

9. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be unit vectors such that $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}=0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$. If $\vec{a}=n(\vec{b}\times\vec{c})$, then value of n is

(1) ± 1 (2) ± 2 (3) $\pm\sqrt{3}$ (4) 0

10. $\vec{a}=2\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}$ if $\vec{c}$ is a vector such that $\vec{a}\cdot\vec{c}=|\vec{c}|$, $|\vec{a}-\vec{c}|=2\sqrt{2}$ and the angle between $\vec{a}\times\vec{b}$ and $\vec{c}$ is 30^0, then $|(\vec{a}\times\vec{b})\times\vec{c}|=$

(1) 2/3 (2) 3/2 (3) 2 (4) 3

11. If D, E, F are midpoints of the sides BC, CA, AB respectively of a triangle ABC and O is any point then

$\overline{AD}+\frac{2}{3}\overline{BE}+\frac{1}{3}\overline{CF}=$

A) $\overline{AC}$ B) $2\overline{AC}$ C) $\frac{1}{2}\overline{AC}$ D) $\vec{0}$

12. Let $\vec{a}=3\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$ be two vectors. If a vector perpendicular to both the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ has the magnitude 12 then one such vector is

(1) $4(2\hat{i}+2\hat{j}-\hat{k})$ (2) $4(-2\hat{i}-2\hat{j}+\hat{k})$ (3) $4(2\hat{i}-2\hat{j}-\hat{k})$ (4) $4(2\hat{i}+2\hat{j}+\hat{k})$

13. Let $\vec{a}=2\hat{i}+\lambda_1\hat{j}+3\hat{k}$, $\vec{b}=4\hat{i}+(3-\lambda_2)\hat{j}+6\hat{k}$ and $\vec{c}=3\hat{i}+6\hat{j}+(\lambda_3-1)\hat{k}$ be three vectors such that $\vec{b}=2\vec{a}$ and $\vec{a}$ is perpendicular to $\vec{c}$. Then a possible value of $(\lambda_1,\lambda_2,\lambda_3)$ is:

(1) $\left(\frac{1}{2},4,-2\right)$ (2) $\left(-\frac{1}{2},4,0\right)$ (3) (1, 3, 1) (4) (1, 5, 1)

14. Let $\vec{a}=\hat{i}+\hat{j}+\sqrt{2}\hat{k}, \vec{b}=b_1\hat{i}+b_2\hat{j}+\sqrt{2}\hat{k}$ and $\vec{c}=5\hat{i}+\hat{j}+\sqrt{2}\hat{k}$ be three vectors such that the projection of $\vec{b}$ on $\vec{a}$ is $|\vec{a}|$. If $\vec{a}+\vec{b}$ is perpendicular to $\vec{c}$, then $|\vec{b}|$ is equal to

(1) $\sqrt{22}$ (2) 4 (3) $\sqrt{32}$ (4) 6

15. Let $\vec{a}=\hat{i}-2\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$ be two vectors. If $\vec{c}$ is a vector such that $\vec{b}\times\vec{c}=\vec{b}\times\vec{a}$ and $\vec{c}\cdot\vec{a}=0$, then $\vec{c}\cdot\vec{b}$ is equal to

(1) $\frac{1}{2}$ (2) -1 (3) $-\frac{1}{2}$ (4) $-\frac{3}{2}$

16. The magnitude of the projection of the vector $2\hat{i}+3\hat{j}+\hat{k}$ on the vector perpendicular to the plane containing the vectors $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+3\hat{k}$, is:

(1) $\frac{\sqrt{3}}{2}$ (2) $\sqrt{\frac{3}{2}}$ (3) $\sqrt{6}$ (4) $3\sqrt{6}$

17. Let $\vec{a}=3\hat{i}+2\hat{j}+x\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$ for some real x. Then $|\vec{a}\times\vec{b}|=r$ is possible if:

(1) $3\sqrt{\frac{3}{2}}<r<5\sqrt{\frac{3}{2}}$ (2) $0<r\le\sqrt{\frac{3}{2}}$ (3) $\sqrt{\frac{3}{2}}<r\le3\sqrt{\frac{3}{2}}$ (4) $r\ge5\sqrt{\frac{3}{2}}$

18. Let $\sqrt{3}\hat{i}+\hat{j}$, $\hat{i}+\sqrt{3}\hat{j}$ and $\beta\hat{i}+(1-\beta)\hat{j}$ respectively be the position vectors of the points A, B and C with respect to the origin O. If the distance of C from the bisector of the acute angle between OA and OB is $\frac{3}{\sqrt{2}}$, then the sum of all possible values of β is:

(1) 2 (2) 1 (3) 3 (4) 4

Assertion & Reasoning

(a) If both Statement-I and Statement-II are true and the reason is the correct explanation of the statement-I.

(b) If both Statement-I and Statement-II are true but reason is not the correct explanation of the statement-I.

(c) If Statement-I is true but Statement-II is false.

(d) If Statement-I is false but Statement-II is true.

19. **Statement-I:** If $\vec{u}$ and $\vec{v}$ are unit vectors inclined at an angle α and $\vec{x}$ is a unit vector bisecting the angle between them, then $\vec{x} = \dfrac{\vec{u}+\vec{v}}{2\sin\dfrac{\alpha}{2}}$.

Statement-II: If ABC is an isosceles triangles with AB = AC = 1, then vectors representing bisector of angle A is given by $\overline{AB} = \dfrac{\overline{AB}+\overline{AC}}{2}$.

20. $\vec{a}$ and $\vec{c}$ are unit vectors and $|\vec{b}| = 4$. The angle between $\vec{a}$ and $\vec{c}$ is $\cos^{-1}\left(\frac{1}{4}\right)$. If $\vec{b} - 2\vec{c} = \lambda\vec{a}$ then value of λ is

A) $3, -4$ B) $\frac{1}{4}, \frac{3}{4}$ C) $-3, 4$ D) $\frac{-1}{4}, \frac{3}{4}$

SECTION - II

Numerical type Questions

21. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors satisfying $|\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2 = 9$, then $|2\vec{a}+5\vec{b}+5\vec{c}|$ is ___________

22. $\vec{a}$, $\vec{b}$, $\vec{c}$ are 3 unit vectors such that each is inclined at an angle θ with the other. A unit vector $\vec{d}$ is equally inclined with these vectors at an angle α, then $4\cos\theta - 3\cos 2\alpha$ is ___________

23. Let $\overline{V}_1 = \hat{i} + \hat{j} - 2\hat{k}$, $\overline{V}_2 = \hat{i} - 2\hat{j} + \hat{k}$, $\overline{V}_3 = -2\hat{i} + 2\hat{j} + \hat{k}$ are three vectors. Let $\overline{V}$ be a vector such that it can be expressed as a linear combination of $\overline{V}_1$ and $\overline{V}_2$ also $\overline{V} \cdot \overline{V}_3 = 0$ and the projection of vector $\overline{V}$ on $\hat{i} - \hat{j} + \hat{k}$ is $6\sqrt{3}$. If $\overline{V} = t(\hat{i} + 3\hat{j} - 4\hat{k})$ then the absolute value of 't' is ___________

24. If $\vec{a}$ and $\vec{b}$ are vectors in space given by $\vec{a} = \dfrac{\hat{i} - 2\hat{j}}{\sqrt{5}}$ and $\vec{b} = \dfrac{2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{14}}$, then the value of $(2\vec{a}+\vec{b}) \cdot [(\vec{a}\times\vec{b}) \times (\vec{a} - 2\vec{b})]$ is ___________

25. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be coplanar unit vectors such that $\vec{b}\cdot\vec{c} = \cos\alpha$, $\vec{c}\cdot\vec{a} = \cos\beta$, $\vec{a}\cdot\vec{b} = \cos\gamma$ then the value of $\cos^2\alpha + \cos^2\beta + \cos^2\gamma - 2\cos\alpha\cos\beta\cos\gamma$ is ___________

PART - II (JEE ADVANCED)

SECTION - III (Only one option correct type)

26. Let $\hat{a}$ and $\hat{b}$ be mutually perpendicular unit vectors. If $\vec{r}$ is any arbitrary vector then

A) $\vec{r}=(\vec{r}.\hat{a})\hat{a}+(\vec{r}.\hat{b})\hat{b}+(\vec{r}.(\hat{a}\times\hat{b}))(\hat{a}\times\hat{b})$

B) $\vec{r}=(\vec{r}.\hat{a})\hat{a}-(\vec{r}.\hat{b})\hat{b}-(\vec{r}.(\hat{a}\times\hat{b}))(\hat{a}\times\hat{b})$

C) $\vec{r}=(\vec{r}.\hat{a})\hat{a}-(\vec{r}.\hat{b})\hat{b}+(\vec{r}.(\hat{a}\times\hat{b}))(\hat{a}\times\hat{b})$

D) $\vec{r}=(\vec{r}.\vec{a})\vec{a}+(\vec{r}.\vec{b})\vec{b}$

27. If $\vec{a}$ satisfies $\vec{a}\times(\hat{i}+2\hat{j}+\hat{k})=\hat{i}-\hat{k}$ then $\vec{a}$ is equal to

A) $\lambda\hat{i}+(2\lambda-1)\vec{j}+\lambda\hat{k},\lambda\in R$

B) $\lambda\hat{i}+(1-2\lambda)\hat{j}+\lambda\hat{k},\lambda\in R$

C) $\lambda\hat{i}+(2\lambda+1)\hat{j}+\lambda\hat{k},\lambda\in R$

D) $\lambda\hat{i}-(1+2\lambda)\hat{j}+\lambda\hat{k},\lambda\in R$

28. If $\vec{a},\vec{b},\vec{c}$ are unit vectors such that $\vec{a}.\vec{b}=0=\vec{a}.\vec{c}$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$ then the value of $|\vec{a}\times\vec{b}-\vec{a}\times\vec{c}|$ is

A) $\frac{1}{2}$ B) 1 C) 2 D) $\frac{1}{3}$

29. If $\vec{a}$ and $\vec{b}$ are any two vectors of magnitudes 1 and 2 respectively, and $(1-3\vec{a}.\vec{b})^2+|2\vec{a}+\vec{b}+3(\vec{a}\times\vec{b})|^2=47$ then the angle between $\vec{a}$ and $\vec{b}$ is

A) $\frac{\pi}{3}$ B) $\pi-\cos^{-1}\left(\frac{1}{4}\right)$ C) $\frac{2\pi}{3}$ D) $\cos^{-1}\left(\frac{1}{4}\right)$

30. If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}+\vec{b}|=\sqrt{29}$ and $\vec{a}\times(2\hat{i}+3\hat{j}+4\hat{k})$ $=(2\hat{i}+3\hat{j}+4\hat{k})\times\vec{b}$ then a possible value of $(\vec{a}+\vec{b}).(-7\hat{i}+2\hat{j}+3\hat{k})$ is

A) 0 B) 3 C) 4 D) 8

31. $\vec{a}$ and $\vec{b}$ are two mutually perpendicular unit vectors and $\vec{c}$ is a unit vector inclined at an angle θ to both $\vec{a}$ and $\vec{b}$ if $\vec{c}=x\vec{a}+x\vec{b}+y(\vec{a}\times\vec{b})$, where $x,y\in R$, exhaustive range of θ is

A) $\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$ B) $\left[\frac{\pi}{4},\frac{3\pi}{4}\right]$ C) $\left[\pi,\frac{\pi}{2}\right]$ D) $\left[0,\frac{\pi}{2}\right]$

32. $\vec{b}$ and $\vec{c}$ are unit vectors. Then for any arbitrary vector $\vec{a}$ the value of $(((\vec{a}\times\vec{b})+(\vec{a}\times\vec{c}))\times(\vec{b}\times\vec{c})).(\vec{b}-\vec{c})$ is always equal to

A) $|\vec{a}|$ B) $\frac{1}{2}|\vec{a}|$ C) $\frac{1}{3}|\vec{a}|$ D) 0

33. If $\vec{a}=2\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}+2\hat{k}$, $\vec{c}=\hat{i}+\hat{j}+2\hat{k}$ and $(1+\alpha)\hat{i}+\beta(1+\alpha)\hat{j}+\gamma(1+\alpha)(1+\beta)\hat{k}=\vec{a}\times(\vec{b}\times\vec{c})$ then α,β and γ are

A) $-2,-4,-\frac{2}{3}$ B) $2,-4,\frac{2}{3}$ C) $-2,4,\frac{2}{3}$ D) $2,4,-\frac{2}{3}$

SECTION - IV (More than one correct answer)

34. If $\vec{a},\vec{b}$ and $\vec{c}$ are non-coplanar vectors and λ is a real number , then the vectors $\vec{a}+2\vec{b}+3\vec{c}, \lambda\vec{b}+\mu\vec{c}$ and $(2\lambda-1)\vec{c}$ are coplanar for

A) all values of μ B) $\lambda=\frac{1}{2}$ C) $\lambda=0$ D) no value of λ

35. The vector $\hat{i}+x\hat{j}+3\hat{k}$ is rotated through an angle θ and doubled in magnitude and becomes $4\hat{i}+(4x-2)\hat{j}+2\hat{k}$. The values of x are

A) 1 B) -2/3 C) 2 D) 4/3

36. Let $\bar{x},\bar{y}$ *and* $\bar{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$. If $\bar{a}$ is a non-zero vector perpendicular to $\bar{x}$ and $\bar{y}\times\bar{z}$ and $\bar{b}$ is a non zero vector perpendicular to $\bar{y}$ and $\bar{z}\times\bar{x}$, then

A) $\bar{b}=(\bar{b}.\bar{z})(\bar{z}-\bar{x})$ B) $\bar{a}=(\bar{a}.\bar{y})(\bar{y}-\bar{z})$ C) $\bar{a}.\bar{b}=-(\bar{a}.\bar{y})(\bar{b}.\bar{z})$ D) $\bar{a}=(\bar{a}.\bar{y})(\bar{z}-\bar{y})$

SECTION - V (Numerical Type)

37. Let $|\bar{p}|=\frac{2}{3}\sqrt{2}, |\vec{q}|=1$ and the angle between $\bar{p}$ and $\vec{q}$ be $\frac{\pi}{4}$. If a parallelogram is formed with adjacent sides $\vec{a}=\bar{p}-3\vec{q}$ and $\vec{b}=5\bar{p}+2\vec{q}$, then the length of the shorter diagonal is

38. If $\vec{a}=x\hat{i}+(x-1)\hat{j}+\hat{k}$ and $\vec{b}=(x+1)\hat{i}+\hat{j}+a\hat{k}$ always make an acute angle for all $x\in R$, then the least integral value of a is

39. Given $|\vec{a}|=|\vec{b}|=1$ and $|\vec{a}+\vec{b}|=\sqrt{3}$. If $\vec{c}$ is a vector such that $\vec{c}-\vec{a}-2\vec{b}=3(\vec{a}\times\vec{b})$, then the value of $[\vec{c}.\vec{b}]$, where [.] represents the greatest integerfunction, is

SECTION VI - (Matrix match type)

40.

	Column I		ColumnII
A)	If $\lvert\vec{a}\rvert=\lvert\vec{b}\rvert=\lvert\vec{c}\rvert$, angle between each pair of vectors is $\frac{\pi}{3}$ and $\lvert\vec{a}+\vec{b}+\vec{c}\rvert=\sqrt{6}$, then $2\lvert\vec{a}\rvert$ is equal to	p	3
B)	If $\bar{a}$ is a perpendicular to $\vec{b}+\vec{c}$, $\vec{b}$ is perpendicular to $\vec{c}+\vec{a}$, $\vec{c}$ is perpendicular to $\vec{a}+\vec{b}$, $\lvert\vec{a}\rvert=2, \lvert\vec{b}\rvert=3$ and $\lvert\vec{c}\rvert=6$. Then $\lvert\vec{a}+\vec{b}+\vec{c}\rvert$ is equal to	q	2
C)	$\vec{a}=2\hat{i}+3\hat{j}-\hat{k}, \vec{b}=-\hat{i}+2\hat{j}-4\hat{k}, \vec{c}=\hat{i}+\hat{j}+\hat{k}$ and $d=3\hat{i}+2\hat{j}+\hat{k}$ then $\frac{1}{7}(\vec{a}\times\vec{b}).(\vec{c}\times\vec{d})$ is equal to	r	4
D)	If $\lvert\vec{a}\rvert=\lvert\vec{b}\rvert=\lvert\vec{c}\rvert=2$ and $\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}=2$, then $\left[\vec{a}\,\vec{b}\,\vec{c}\right]\cos45^0$ is equal to	s	5

A) A-Q,B-S,C-P,D-R
B) A-R,B-S,C-P,D-R
C) A-Q,B-S,C-P,D-Q
D) A-Q, R,B-S,C-P,D-R