PERMUTATION AND COMBINATION

I Fundamental Principles of Counting: Multiplication Principle (Multiplication Rule)

If an event can occur in ‘m’ different ways following which another event can occur in ‘n’ different ways, then the total number of occurrence of the events in given order is m×n.

Multiplication principle can be extended to any number of events

Addition Principle (Addition Rule)

If an event can occur in ‘m’ different ways and an another event can occur in ‘n’ different ways, then the total number of occurence of event one or event two can be in m+n difference ways

Addition principle can also be extended to any number of events

Note: ‘and’ represents multiplication and ‘or’ represents addition

II Factorial

Factorial of a natural number n in represented by n! or $\underline{|n}$ and in equal to the product of first ‘n’ natural numbers

ie, n! = n(n–1) (n–2) (n–3)..3.2.1

Note:

1) 0! is defined as 1

2) n! = n(n–1)!=n(n–1)(n–2)! so on

III Permutation (Arrangement)

Result: The number of ways of arranging n distinct objects taken ‘r’ at a time without repetition is denoted by nP_r or P(n,r) and in equal to $n.(n-1).(n-2).(n-3)...(n-r+1)$

$$nP_r=\frac{n!}{(n-r)!},0\le r\le n$$

Note:

1) $nP_0=1$

2) $nP_n=n!$

3) $nP_n=nP_{n-1}=n!$

Result: Number of arrangements of n distinct objects taken ‘r’ at a time if repetition is allowed is n^r

Result: Number of arrangements of n objects in which p items are alike of type 1, q objects are alike of type 2 and r items are alike of type 3 is $\frac{n!}{p!q!r!}$

Result: Number of arrangements of n distinct objects in which ‘r’ particular objects should comes in an order (not necessarily together) is $\frac{n!}{r!}$

Combinations (Selections)

Result: Relation between permutation and combination is $nP_r=r!.nC_r$

Result: Number of selections of ‘n’ distinct objects taken ‘r’ at a time in denoted as nC_r or C(n, r) or $\binom{n}{r}$ and $nC_r=\frac{nP_r}{r!}$

ie, $nC_r=\frac{n!}{r!(n-r)!}$

Note:

1) $nC_0=1$

2) $nC_1=n$

3) $nC_n=1$

Result: $nC_r=nC_{n-r}$

Result: $nC_r=nC_s \Rightarrow$ either r=s or r+s=n

Result: **Pascal’s rule**

$nC_r+nC_{r-1}={}^{n+1}C_r$

Result: $nC_r=\frac{n}{r}.{}^{n-1}C_r$ or $\frac{nC_r}{{}^{n-1}C_{r-1}}=\frac{n}{r}$

Circular Permutation

Result: Number of circular arrangements of ‘n’ distinct objects taken all at a time is $\frac{nP_n}{n}=\frac{n!}{n}=(n-1)!$

Result: Number of circular arrangement of ‘n’ distinct objects taken ‘r’ at a time is $\frac{nP_r}{r}$

Restricted combination

Number of selections of ‘n’ distinct objects taken ‘r’ at a time so that,

1) ‘m’ particular objects are always included is $n-mC_{r-m}$

2) ‘m’ particular objects are always excluded is $n-mC_r$

Restricted Permutation

Number of arrangements of 'n' distinct objects taken 'r' at a time so that,

1) 'm' particular objects are always excluded in $n{-}mP_r$

2) 'm' particular objects are always included in $n{-}mC_{r-m}\times r!$ or $n{-}mP_{r-m}\times rP_m$

Result: $nP_r + r.nP_{r-1} = {}^{n+1}P_r$

Combinatories in Geometry

1) Maximum intersections of 'n' non-parallel lines is a plane in nC_2

2) Maximum intersections of 'n' non-concentric circles in a plane is nP_2

3) Maximum number of straight lines formed using 'n' non collinear points in a plane is nC_2

4) Maximum number of straight lines formed using 'n' points in a plane so that 'm' are collinear is $nC_2 - mC_2 + 1$

5) Maximum number of triangles formed using 'n' non-collinear points in a plane in nC_3

6) Maximum number of triangles formed using 'n' points out of which 'm' are collinear is $nC_3 - mC_3$

7) Number of diagonals of an 'n' sided polygon is $nC_2 - n$ or $\frac{n(n-3)}{2}$

8) Maximum number of quadrilateral formed using 'n' non-collinear points in a plane is nC_4

9) Maximum number of quadrilaterals formed using 'n' points out of which 'm' are collinear is $n{-}mC_4 + n{-}mC_3.mC_1 + n{-}mC_2.mC_2$

10) A set of 'n' parallel lines in interested with another set of 'm' parallel lines, then the number of parallelogram thus formed in $nC_2\times mC_2$

Result: Number of selections of atleast one (one or more) object taken from n distinct objects is 2^n-1

Result: Number of ways of selecting 'r' items from p identical item is 1

Result: Number of ways of selecting items (zero or more)from p identical items is p+1

Result: Number of ways of selecting atleast one object form a collection which has 'p' identical items of type I, 'q' identical items of type q, r identical items of type is (p+1)(q+1)(r+1)–1

Grouping (Groups of unequal size)

1. Number of ways of dividing (m+n) distinct items into two groups each containing 'm' and 'n' items is $m{+}nC_m$ or $m{+}nC_n$ and its distribution among 2 is $m{+}nC_n\times 2!$
2. Number of ways of dividing m+n+p items into 3 groups each containing m,n and p items respectively in $m{+}n{+}pC_m \times n{+}pC_n$ or $\frac{(m+n+p)!}{m!n!p!}$ and its distribution among 3 is $\frac{(m+n+p)!}{m!n!p!}\times 3!$

Grouping (Groups of equal size)

1. Number of ways of division (parcels) of 2m distinct items into 2 equal groups each containing m items each in $\frac{2mCm}{2!}$ or $\frac{(2m)!}{m!m!2!}$ and its distribution among 2 is $\frac{(2m)!}{m!m!}$

2. Number of ways of divisions (parcels) of 3m distinct items into 3 equal groups each containing m items each in $\frac{3mC_m.2mC_m}{3!}$ or $\frac{(3m)!}{m!m!m!3!}$ and its distribution among 3 is $\frac{(3m)!}{m!m!m!}$

Grouping (Groups of equal and unequal size)

1. Number of ways of dividing (m+2n+3p) distinct items into groups of 'm' items 2 equal groups of 'n' items and 3 equal groups of 'p' items in $\frac{(m+2n+3p)!}{m!(n!)^2.(p!)^3.2!3!}$ and its distribution among 6 is

$$\frac{(m+2n+3p)!}{m!(n!)^2(p!)^3 2!3!} \times 6!$$

Derangement (De arrangement)

Number of derangement of n distinct items in denoted by D_n and is equal to

$$D_n = n!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + ... + \frac{(-1)^n}{n!}\right]$$

Note: $D_1=0$, $D_2=1$, $D_3=2$, $D_4=9$, $D_5=44$, $D_6=265$ etc

Result: Distribution of n identical objects among r persons/groups/boxes etc if blank groups are allowed or Number of non-negative integer solutions of the equation $x_1+x_2+x_3+...+x_r=n$ is ${}^{n+r-1}C_{r-1}$

Result: Distribution of n identical objects among r persons/ groups/boxes etc if blank groups not allowed or number of positive integer solutions of the equation $x_1+x_2+x_3+...+x_r=n$ is ${}^{n-1}C_{r-1}$

Result: Number of ways of distributing 'n' different items among 'r' persons. So that each receives atleast one item in $r^n - rC_1(r-1)^n + rC_2(r-2)^n + ... + (-1)^{r-1}\, rC_{r-1}$