DIFFERENTIAL EQUATIONS

Definitions

- An equation involving derivative (derivatives) of the dependent variable with respect to independent variable (variables) is called a differential equation (DE)
- A differential equation involving derivative of the dependent variable with respect to only ore dependent variable is called an ordinary differential equation

  eg: $2\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3=0$ is an ordinary DE

- A differential equation involving derivatives with respect to more than ore independent variables called partial differential equations

Note: We are only discussing about the ordinary differential equation in this chapter

Notations we used here

$$\frac{dy}{dx}=y',\frac{d^2y}{dx^2}=y'',....\frac{d^ny}{dx^n}=y_n$$

Order of a DE

Order of a DE is defined as the order of the highest order derivative of the dependent variable with respect to the independent variable involved in the given DE

Degree of a DE

Degree of a DE is the highest power (positive integral index) of the highest order derivative involved in the given DE when it is a polynomial equation in derivatives

Note: (1) Order and degree (it defined) of a DE are always positive integer

(2) For a DE degree may or may not occur

Solution of a DE

- The solution which contains arbitrary constants is called the general solution (primitive) of the DE
- The solution obtained from the general solution by giving particular values to the arbitrary constants is called a particular solution of the DE

Note: The order of a DE representing a family of curves is same as the number of arbitrary constant present in the equation corresponding to the family of curves

Methods of solving first order, first degree DEs

1) DE with variables seperable

A I[st] order I[st] degree DE is of the form $\frac{dy}{dx}=f(x,y)$ or $\frac{dy}{dx}=h(y).g(x)$ can be rewritten as $\frac{dy}{h(y)}=g(x)dx$ where $h(y)\neq 0$. Then by integrating both sides we get the solution as $H(y)=G(x)+c$, where C is the constant of integration (which is the arbitary constant)

2) Homogenous DE

A function $f(x,y)$ is said to be homogenous function of degree n if $f(\lambda x,\lambda y)=\lambda^n F(x,y)$ for any nonzero constant λ

$\therefore$ A function $f(x.y)$ is a homogenous function of degree n if $F(x,y)=x^n g\left(\frac{y}{x}\right)$ or $y^n h\left(\frac{x}{y}\right)$

Method I

A differential equaiton of the form $\frac{dy}{dx}=f(x,y)$ is said to be homogeneous if $f(x,y)$ is a homogenous function of degree zero

ie $\frac{dy}{dx}=F(x,y)$ or $=f\left(\frac{y}{x}\right)$

Here we make the substitution $y=vx$

$\therefore \frac{dy}{dx}=v+x\frac{dv}{dx}$

substituting and make it in variable seperable form and obtain the solution in v and x. Finally replace $v=\frac{y}{x}$ and obtain its general solution

Method I

A DE is of the form $\frac{dx}{dy}=F(x,y)$ or $=g\left(\frac{x}{y}\right)$ we substitute $x=vy \Rightarrow \frac{dx}{dy}=v+y\frac{dv}{dy}$

Sub and make it in variable seperable form and proceed like in method I

3. LINEAR DIFFERENTIAL EQUATION (LDE)

A DE of the form $\frac{dy}{dx}+py=Q$ where P & Q are constants or functions of x only is known as a first order linear differential equation

eg 1) $\frac{dy}{dx}+y=\sin x$

2) $\frac{dy}{dx}+\frac{y}{x}=e^{x}$

3) $\frac{dy}{dx}+\frac{y}{x\log x}=\frac{1}{x}$

Another Form

$$\frac{dx}{dy}+px=Q$$

where P & Q are constants or function of y only

Eg. 1) $\frac{dx}{dy}+x=\cos y$

2) $\frac{dy}{dx}-\frac{2x}{y}=y^{3}e^{-y}$

Steps: $\frac{dy}{dx}+py=Q$ for

1) make it is the form $\frac{dy}{dx}+py=Q$

2) Find $IF=e^{\int pdx}$

3) Write the solution of the given DE as $y(F)=\int Q(IF)dx+C$

For $\frac{dx}{dy}+px=Q$

steps $1:\text{Find } IF=e^{\int pdy}$

$2\ .x(IF)=\int Q.(IF)dy+c$

$$\frac{\text{Integrating factor}}{\text{let LDE be}}$$

$$\frac{dy}{dx}+py=Q$$

Multiplying a function g(x) on both sides

$$g(x)\frac{dy}{dx}+Pg(x)y=Q.g(x)$$

Choose g(x) in such a way that RHS becomes a derivative of y.g(x)

$$\therefore g(x)\frac{dy}{dx}+Pg(x)y=\frac{d}{dx}\left(y.g(x)\right)$$

$$g(x)\frac{dy}{dx}+Pg(x)y=y.g'(x)+g(x)\frac{dy}{dx}$$

$$\Rightarrow\therefore P=\frac{g'(x)}{g(x)}$$

$$\int Pdx=\int\frac{g'(x)}{g(x)}dx$$

$$=\log\left(g(x)\right)$$

$\Rightarrow g(x)=e^{\int pdx\rightarrow}\rightarrow$ which is the integrations factor

on multiplying IF, the LHS becomes the derivative of same function of x & y

LEVEL - I

1. A

$$\cos x\,dy = y(\sin x - y)dx$$

$$\Rightarrow \cos x dy - y\sin x\,dx = -y^2 dx$$

$$\Rightarrow d(y\cos x) = -y^2 dx$$

$$\int \frac{d(y\cos x)}{y^2\cos^2 x} = -\int \frac{1}{\cos^2 x}dx$$

$$\Rightarrow \frac{-1}{y\cos x} = -\tan x - c$$

$$\Rightarrow \sec x = (\tan x + c)y$$

2. C

$$\frac{dy}{dx} = y + 3$$

$$\Rightarrow \frac{dy}{y+3} = dx$$

$$\Rightarrow \log(y+3) = x + c$$

$$y(0) = 2$$

$$\Rightarrow \log 3 = c$$

$$\therefore\ y + 3 = x + \log 5$$

$$\log(y+3) = \log 2 + \log 5$$

$$= \log 10$$

$$y = 7$$

3. B x intercept = $x - \frac{y}{\frac{dy}{dx}}$

A is $\left(x - \frac{y}{\left(\frac{dy}{dx}\right)}, 0 \right)$

B is $\left(0, y - x\frac{dy}{dx} \right)$

$$\frac{dx}{x} + \frac{dy}{y} = 0$$

Integrating $\log x + \log y = \log c$

since the curve passes through (2,3)

C= 6

4. A,D The given D.E. is $y' - y\tan x = 2x\sec x$

I.F: cos x

$\therefore$ solution is given by

$$y\cos x = \int 2x \sec x \cos x \, dx$$

$$\Rightarrow y\cos x = \int 2x \, dx + c$$

$$\Rightarrow y\cos x = x^2 + c$$

$$y(0) = 0 \Rightarrow c = 0$$

$$y = x^2 \sec x$$

$$\therefore y(\pi/4) = \pi^2 / 8\sqrt{2}$$

$$y' = 2x\sec x + x^2 \sec x \tan x$$

$$y'\left(\frac{\pi}{3}\right) = 2\left(\frac{\pi}{3}\right)(2) + \frac{\pi^2}{9} 2\sqrt{3}$$

$$= \frac{4\pi}{3} + \frac{2\pi^2}{3\sqrt{3}}$$

5. B $\frac{dx}{dy}+\frac{x}{y}=-x^2$

$\Rightarrow x^{-2}\frac{dx}{dy}+x^{-1}\frac{1}{g}=-1$

Find integrating factor

Solution is $\frac{-1}{xy}+\log y=c$

6. C Slope of tangent = $\frac{dy}{dx}$

$\therefore$ slope of normal = $\frac{-dx}{dy}$

$\therefore$ The equation of normal is

$Y-y=\frac{-dx}{dy}(X-x)$

This meets x - axis (y = 0) where

$-y=\frac{-dx}{dy}(X-x)\Rightarrow yX=x+y\frac{dy}{dx}$

$\therefore G\text{ is}\left(x+y\frac{dy}{dx},0\right)$

$\therefore OG=2x\Rightarrow x+y\frac{dy}{dx}=2x$

$\Rightarrow y\frac{dy}{dx}=x$

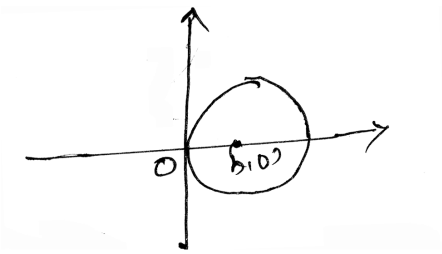
$\Rightarrow y\,dy=x\,dx \;\Rightarrow x^2-y^2=c$, which is a hyper bola

7. A Given equation is

$$\left(\frac{d^2y}{dx^2}\right)^2\frac{d^3y}{dx^3}+4\left(\frac{d^2y}{dx^2}\right)^3+\left(\frac{d^3y}{dx^5}\right)^2=\left(x^2-1\right)\frac{d^3y}{dx^3}$$

$m=3; n=2$

8. B



$y^2=x^2+2xy\left(\frac{dy}{dx}\right)$

LEVEL - II

9. B It is given that

$$8\sqrt{x}\left(\sqrt{9+\sqrt{x}}\right)dy = \left(\sqrt{4+\sqrt{9+\sqrt{x}}}\right)^{-1} dx, x > 0$$

$$\Rightarrow dy = \frac{\left(\sqrt{4+\sqrt{9+\sqrt{x}}}\right)}{8\sqrt{x}\left(\sqrt{9+\sqrt{x}}\right)} dx$$

$$\Rightarrow y = \frac{1}{8}\int \frac{\left(\sqrt{4+\sqrt{9+\sqrt{x}}}\right)}{\sqrt{x}\left(\sqrt{9+\sqrt{x}}\right)} dx$$

$$= \frac{1}{8}\int \frac{1}{\left[\sqrt{x}\left(\sqrt{9+\sqrt{x}}\right)\right]\left[\sqrt{4+\sqrt{9+\sqrt{x}}}\right]} dx$$

Put $\sqrt{9+\sqrt{x}} = t \Rightarrow \dfrac{dx}{\sqrt{x}\sqrt{9+\sqrt{x}}} = 4dt$

$$\therefore y = \frac{4}{8}\int \frac{dt}{\sqrt{4+t}}$$

$$\Rightarrow y = \sqrt{4+t} + c$$

$$\Rightarrow y(x) = \sqrt{4+\sqrt{9+\sqrt{x}}}$$

$$\Rightarrow y(256) = 3$$

10. A $\dfrac{dy}{dx} = \dfrac{y}{x} + \sec\dfrac{y}{x}$

Let y = vx

$$v + x\frac{dy}{dx} = v + \sec v$$

$$\cos v\, dv = \frac{dx}{x}$$

$$\sin v = \log x + c$$

$$\sin\left(\frac{y}{x}\right) = \log x + c$$

$$\sin\frac{\pi}{6} = c \Rightarrow c = \frac{1}{2}$$

11. B I.F = $\sqrt{1-x^2}$

$\therefore$ solution is $y\sqrt{1-x^2} = \int\left(x^4+2x\right)dx = \frac{x^5}{5}+x^2+c$

$f(0)=0 \Rightarrow c=0$

$\Rightarrow f(x)\sqrt{1-x^2} = \frac{x^5}{5}+x^2$

$\therefore \int_{-\sqrt{3}/2}^{\sqrt{3}/2} f(x)dx = 2\int_0^{\sqrt{3}/2} \frac{x^2}{\sqrt{1-x^2}}dx$

$= \left(\theta - \frac{\sin 2\theta}{2}\right)_0^{\pi/3}$

$= \pi/3 - \sqrt{3}/4$

12. D $f'(x)-2f(x)<0$

$e^{-2x}f'(x)-2e^{-2x}f(x)<0$

$\Rightarrow \frac{d}{dx}\left[e^{-2x}f(x)\right]<0$

$g(x)=e^{-2x}f(x)$

$\Rightarrow g(x)$ is a decreasing function

$x>\frac{1}{2}$

$g(x)<g\left(\frac{1}{2}\right)$

$\Rightarrow c^{-2x}f(x)<\frac{1}{e}$

$\Rightarrow f(x)<e^{2x-1}$

$\Rightarrow \int_{\frac{1}{2}}^{1} f(x)dx < \frac{1}{e}\int_{\frac{1}{2}}^{1} e^{2x}dx$

$\Rightarrow \int_{\frac{1}{2}}^{1} f(x)dx < \frac{e-1}{2}$

$\therefore \int_{\frac{1}{2}}^{1} f(x)dx > 0$

13. C $\frac{dy}{dx} = \frac{1 - y^2}{y}$

$\Rightarrow \frac{y}{\sqrt{1.y^2}} dy = dx$

Integrating both sides, we get

$\int \frac{ydy}{\sqrt{1 - y^2}} = \int dx$

$\Rightarrow (x - c)^2 + y^2 = 1$

14. A The given differential equation can be written as

$g^2 \left(\frac{dy}{dx}\right)^2 + 4x^2 + 4xy\frac{dy}{dx} = \left(y^2 + 2x^2\right)\left[1 + \left(\frac{dy}{dx}\right)^2\right]$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x} + \sqrt{\frac{1}{2}\left(\frac{y}{x}\right)^2 + 1}$.....(1)

Let y = vx

$\Rightarrow v + x\frac{dv}{dx} = \frac{dy}{dx}$

(1) becomes

$v + x\frac{dv}{dx} = v \pm \sqrt{\frac{1}{2}v^2 + 1}$

Integrating $\sqrt{2}\log\left|\frac{y + \sqrt{y^2 + 2x^2}}{x}\right| = \log|xc|$

Put $x = 1; y = 0 \Rightarrow c = \left(\sqrt{2}\right)$

$\therefore$ curves are given by $\frac{y + \sqrt{y^2 + 2x^2}}{x} = \sqrt{2x}^{\pm\frac{1}{\sqrt{2}}}$

15. A

LEVEL - III

16. D $\left(\sqrt{x}\frac{dy}{dx} + \sqrt{y}\right)^2 = 0$

$\sqrt{x}\frac{dy}{dx} + \sqrt{y} = 0$

$\frac{dy}{\sqrt{y}} = \frac{-dx}{\sqrt{x}}$

$2\sqrt{y} = -2\sqrt{x} + 2\sqrt{c}$

$\sqrt{x} + \sqrt{y} = \sqrt{c}$

17. C Given D.E can be written as

$$(x+2)^2 + y(x+2) = y^2 \frac{dx}{dy}$$

$$\Rightarrow \frac{dx}{dy} = \frac{(x+2)^2}{y^2} + \frac{(x+2)}{y}$$

$$\Rightarrow \frac{1}{(x+2)^2}\frac{dx}{dy} = \frac{1}{y^2} + \frac{1}{y(x+2)}$$

$$\Rightarrow \frac{1}{(x+2)^2}\frac{dx}{dy} - \frac{1}{y(x+2)} = \frac{1}{y^2}$$

Put $\frac{1}{x+2} = t \Rightarrow \frac{dt}{dy} = \frac{-1}{(x+2)^2}\frac{dx}{dy}$

On substituting we get

$$\frac{-dt}{dy} - \frac{t}{y} = \frac{1}{y^2} \Rightarrow \frac{dt}{dy} + \frac{t}{y} = \frac{-1}{y^2}$$

Then $I.F. = e^{\int \frac{1}{y} dy = y}$

$\therefore$ solution is $ty = C - \log y$

Given that the solution curve passes through the point (1,3)

$\Rightarrow 1 = C - \log^3 \Rightarrow C = 1 + \log^3$

Substituting the value of C

we get $\frac{y}{x+2} = 1 + \log{}^3 - \log y$

for $y = x+2$

$\log y = \log^3$

$\Rightarrow$ it intersects y = x+ 2 at exactly one point

$\Rightarrow$ (A) is correct

for y = (x+2)^2 $\Rightarrow \frac{(x+2)^2}{x+2} = 1 + \log\left(\frac{3}{y}\right)$

$\Rightarrow y = 3e^{-(x+1)}$

$\Rightarrow$ intersects y = $(x+2)^2$

$\Rightarrow$ c is correct

18. A,C The given D.E is $\left(1+e^{x}\right)y' + ye^{x} = 1$

$$\Rightarrow y' + \frac{ye^{x}}{1+e^{x}} = \frac{1}{1+e^{x}}$$

This is a linear D.E

$\therefore$ I.F $= 1+e^{x}$

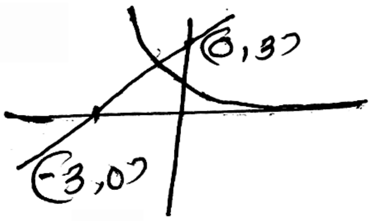
$\therefore$ soln.is $y\left(1+e^{x}\right) = x + c$

$y(0) = 2 \Rightarrow c = 4$

$y = \dfrac{4+x}{1+e^{x}} \therefore y(-4) = 0$

$y(-2) = \dfrac{2}{x^{-2}+1}$

For critical points, $\dfrac{dy}{dx} = 0 \Rightarrow 3 + x = e^{-x}$



From the graph the critical point lies in the interval (–1,0)

$\therefore$ correct options are A and C

19. A,D $f'(x) = 2 - \dfrac{f(x)}{x}$

$$\Rightarrow \frac{dy}{dx} = 2 - \frac{y}{x} \Rightarrow \frac{dy}{dx} + \frac{y}{x} = 2$$

I.F = x

$\therefore$ Required solution is $y = x + \dfrac{c}{x}$

$$\lim_{x\to 0^{+}} f'\left(\frac{1}{x}\right) = \lim_{x\to 0^{+}} = \lim_{x\to 0^{+}}\left(1-\left(x^{2}\right)\right) = 1$$

$\therefore$ (A) is correct

$f(x) = x + \dfrac{c}{x}, C \neq 0$

for $C > 0, \lim\limits_{x\to 0^{+}} f(x) = \infty$

$\therefore$ function is not bounded in (0,2)
D is correct

20. B,C Let the family of circles be $\left(x-h\right)^2\left(y-h\right)^2 = r^2$....(1)

$$\Rightarrow x^2+y^2-2xh-2hy+2h^2-r^2=0$$

Differentiating (1), w.r.to x

$$2x+2gy'-2h-2hy'=0$$

$$\Rightarrow x+yy'-h-hy'=0....(2)$$

Differential again w.r.t x

$$\Rightarrow \left(y-h\right)y''+\left(y'\right)^2+1=0....(3)$$

from (2) $b=\dfrac{x+yy'}{1+y'}$.....(4)

using (4) in (3), we

$$p=y-x;Q=1+y'+\left(y'\right)^2$$

$$\Rightarrow P+Q=1-x+y+y'+\left(y'\right)^2$$

21. A,B,C $\dfrac{dy}{dx}-y\cot x=\dfrac{\sin x}{x^2}$

$$I.F=\frac{1}{\sin x}$$

$\therefore$ solution is $y.\dfrac{1}{\sin x}=\displaystyle\int\frac{-\sin x}{x^2}\frac{1}{\sin x}dx+c$

$$\Rightarrow \frac{y}{\sin x}=\frac{1}{x}+c$$

As $x \to \infty, \Rightarrow c = 0$

$$\therefore y=\frac{\sin x}{x}=\lim_{x\to 0}f\left(x\right)=1$$

$$\therefore I=\int_0^{\pi/2}\frac{\sin x}{x}dx$$

since $\dfrac{\sin x}{x}$ is decreasing, when $x > 0$

$$\Rightarrow f\left(x\right)<f\left(0\right)\Rightarrow \int_0^{\pi/2}f\left(x\right)<\frac{\pi}{2}\text{and } x<\pi/2$$

$$\Rightarrow f\left(x\right)>\int\left(\frac{\pi}{2}\right)=\int_0^{\pi/2}f\left(x\right)dx>1$$

22. A,B,C A) $f(x,tx)=e^t+\tan^{-1}(t)$, independent of $x \Rightarrow$ Homogeneous differential equation

B) $\log\left(\frac{y}{x}\right)dx+\frac{y^2}{x^2}\sin^{-1}\left(\frac{y}{x}\right)dy=0$

$$\Rightarrow \frac{dy}{dx}=\frac{\log\left(\frac{dy}{dx}\right)}{\frac{y^2}{x^2}\sin^{-1}\left(\frac{y}{x}\right)}$$

$$f(x,y)=\frac{\log(y/x)}{\frac{y^2}{x^2}\sin^{-1}(y/x)}$$

$$\therefore f(x,tx)=\frac{\log t}{t^2\sin(t)}\text{independent of } x$$

$\Rightarrow$ Homogeneous DE

C) $f(x,y)=x^2+\sin x.\cos y$

$\Rightarrow$ not homogenous

D) $f(x,y)=\frac{x^2+y^2}{xy^2-y^3}\Rightarrow f(x,tx)$ is not independent of x

23. A,D The given differential equatio can be written as $\left(\frac{dy}{dx}-e^{-x}\right)\left(\frac{dy}{dx}-e^{x}\right)=0 \Rightarrow dy-e^{-x}dx=0$ or

$dy-e^{x}dx=0$

$\Rightarrow y+e^{-x}=c$ or $y-e^{x}=c$

24. A,D Put $y=vx$ so that

$$\frac{dy}{dx}=v+x=\frac{dv}{dx}$$

Then given differetial equation can be written as

$$\cot x=\frac{dv}{dx}$$

$$\Rightarrow \log|\sin v|=\log|x|+\log c$$

$$\Rightarrow \left|\sin\frac{y}{2}\right|=c|x|$$

Put $x=1;\left|\sin\frac{\pi}{2}\right|=c\Rightarrow c=1$

$$\therefore \left|\sin\frac{y}{x}\right|=|x|\Rightarrow \sin\frac{y}{x}=\pm x$$

$0<x<1, \sin\frac{y}{x}=x\Rightarrow y=x\sin^{-1}x$; $\therefore \int_0^1 y\,dx=\int_0^1 x\sin^{-1}x\,dx=\frac{\pi}{8}$

LEVEL - IV

25. 3 $\frac{dy}{dx}\left(\frac{2+\sin x}{1+y}\right)=\cos x, y(0)=1$

$\Rightarrow \frac{dy}{1+y}=\frac{-\cos x}{2+\sin x}dx$

Integrating both sides

$\log(1+y)=-\log(2+\sin x)+c$

Put x=0 and y = 1

$\log 2=-\log 2+c$

$\Rightarrow c=\log 4$

Put $x=\frac{\pi}{2};\log(1+y)=\log\frac{4}{3}$

$\therefore y=\frac{1}{3}$

26. 98 Let V(t) be the velocity of the object at time 't' $\frac{dv}{dt}=9.8-kv$

$\Rightarrow \frac{dv}{9.8kv}=dt$

integrating $\log(9.8-kv)=^{-}kt+c$

$\Rightarrow 9.8-kv=\text{cons}\tan t\, e^{-kt}$

$v(0)=0\Rightarrow \text{constant}=9.8$

$\therefore 9.8-kv=9.8e^{-kt}$

$\Rightarrow kv=9.8\left(1-e^{-kt}\right)$

$\Rightarrow v(t)=\frac{9.8}{k}\left(1-e^{-kt}\right)<\frac{9.8}{k}$

$\therefore v(t)\,\text{cannot exceed}\,\frac{9.8}{k}$; $\therefore 104k\,\text{is}\,98$

27. 0 The given differentiable equation is $y'(x)+y(x)g'(x)=g(x)g'(x)$ which is a linear differentiate equation whose integrating factor is

$I.F=e^{\int g'(x)dx}=e^{g(x)}$

$\therefore$ solution is given by

$g(x).I.F=\int g(x)g'(x)I.F.dx$

$\Rightarrow g(x)e^{g(x)}=\int g(x)g'(x)e^{g(x)}dx$

$\Rightarrow g(0)=0\,\text{and}\,g(0)=0\Rightarrow c=1$

$\Rightarrow g(x)=g(x)-1+e^{-g(x)}$

$\Rightarrow g(2)=\left[g(2)-1\right]+e^{-g(2)}=0$

28. 6 Given, $6\int_1^x f(t)\,dt = 3xf(x) - x^3$ and $f(1) = 2$

Differentiating w.r.t x, we for

$6f(x) = 3f(x) + 3xf'(x) - 3x^2$

$\Rightarrow f'(x) - \frac{f(x)}{x} = x$

This is an linear differential equaiton

$\therefore \text{I.F} = e^{-\int \frac{1}{x}dx} = e^{-\log x} = \frac{1}{x}$

$\therefore$ solution is

$y(\text{I.F}) = \int x.\frac{1}{x}dx$

$\Rightarrow \frac{y}{x} = x + c$

$\Rightarrow y = x^2 + cx$

$\therefore f(1) = 2 \Rightarrow c = 1$

$\therefore f(x) = x^2 + x$

$\therefore y(2) = 6$

29. 9 $y - y_1 = m(x - x_1)$

Put x = 0

$y_1 - mx_1 = x_1^3$

$y_1 - x_1\frac{dy}{dx} = x_1^3$

$x\frac{dy}{dx} - y = x^3$; $\frac{dy}{dx} - \frac{y}{x} = x^2$

$f(x) = \frac{-x^3}{2} + \frac{3x}{2}$ $\therefore f(-3) = 9$

30. 2 Let (u,v) be the vertex of parabola. The equation of required parabolas are of the form

$(x - u)^2 = 4a(y - v)$

So there are 2 unknown constants

31. 3

32. 3 The given equation contains one constant

Differentiating the equation we set

$2x - 2yy' = 2c(x^2 + y^2)(2x + 2yy')$

But $c = \frac{x^2 - y^2}{(x^2 + y^2)^2}$

Substituting for c are get

$y' = \frac{x(3y^2 - x^2)}{y(3x^2 - y^2)}$