CHAPTER - 18
THREE DIMENSIONAL GEOMETRY

JEE MAIN - SECTION I

1. 2 Check option (3), $\frac{4-(-2)}{-3-4} \neq \frac{-3-4}{-2-(-3)}$

Therefore, this set of points is non-collinear.

2. 2 Co-ordinates of P are $(\ell r, mr, nr)$

Here $\ell = \frac{-1}{\sqrt{1^2+2^2+2^2}} = \frac{-1}{3}, m = \frac{2}{3}, n = \frac{-2}{3}$ and r = 3, (given)

$\therefore$ Co-ordinates of P are (−1, 2, −2).

3. 2 $\frac{-2}{\ell} = \frac{-2}{m} = \frac{2}{n}$; $\therefore (\ell, m, n)$ are $(1,1,-1)$.

4. 1 D.r's of AB $\equiv (1,2,-2)$, D.r's of $CD \equiv (2,3,4)$

$\therefore a_1a_2 + b_1b_2 + c_1c_2 = 0$; $\therefore \cos\theta = 0 \Rightarrow \theta = \pi/2$.

5. 3

6. $\frac{4}{2} = \frac{-y}{y+3} = \frac{10-z}{z-4} \Rightarrow z = 6$ and $y = -2$

$\Rightarrow R(4,-2,6)$ distance from origin = $\sqrt{16+4+36} = 2\sqrt{14}$

7. 2 $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, $A \equiv (a,0,0)$, $B \equiv (0,b,0)$, $C \equiv (0,0,c)$

Centroid $\equiv \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right) = (1,1,2)$, a = 3, b= 3, c = 6

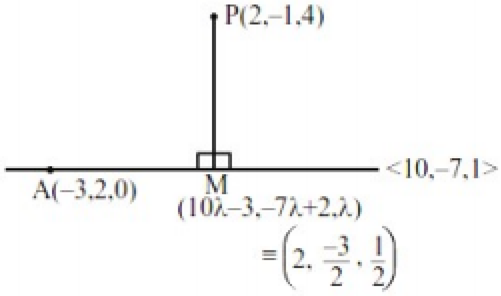
Plane: $\frac{x}{3} + \frac{y}{3} + \frac{z}{6} = 1$, $2x + 2y + z = 6$

Line perpendicular to the plane (DR of line = $2\hat{i} + 2\hat{j} + \hat{k}$)

$\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$.

8. 2 Shortest distance = $\dfrac{\left\|\begin{vmatrix} 6 & 15 & -3 \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix}\right\|}{\sqrt{11\times 29-49}}=\dfrac{270}{\sqrt{270}}$ = $3\sqrt{30}$

9. 2 Now, $\overline{MP}(10\hat{i}-7\hat{j}+\hat{k})=0$



$\Rightarrow \lambda=\dfrac{1}{2}$

∴ Length of perpendicular (=PM)

= $\sqrt{0+\dfrac{1}{4}+\dfrac{49}{4}}$ = $\sqrt{\dfrac{50}{4}}=\sqrt{\dfrac{25}{2}}=\dfrac{5}{\sqrt{2}}$

Which is greater than 3 but less than 4.

10. 3

Point on $L_1(\lambda+3, 3\lambda-1, -\lambda+6)$

Point on $L_2(7\mu-5, -6\mu+2, 4\mu+3)$

$\Rightarrow \lambda+3=7\mu-5$ ..... (1)

$\Rightarrow 3\lambda-1=-6\mu+2$ ..... (2)

$\Rightarrow \lambda=-1$, $\mu=1$

Point $R(2,-4,7)$

Reflection is $(2,-4,-7)$

11. 2 Line $x=ay+b$, $z=cy+d \Rightarrow \dfrac{x-b}{a}=\dfrac{y}{1}=\dfrac{z-d}{c}$

Line $x=a'z+b'$, $y=c'z+d' \Rightarrow \dfrac{x-b'}{a'}=\dfrac{y-d'}{c'}=\dfrac{z}{1}$

Given both the lines are perpendicular $aa'+c'+c=0$

12. 2 s.d.=$\dfrac{\left|\left(\bar{a}_2-\bar{a}_1\right)\times\bar{b}\right|}{\left|\bar{b}\right|} \to$ parallel lines

13. 4 $L_1 \equiv \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$

$L_2 \equiv \frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1}$

Point $A(-1,2,1)$, $B(-2,-1,-1)$

$\because$ L_1 and L_2 are coplanar

$\Rightarrow \begin{vmatrix} 2 & -1 & 1 \\ \alpha & 5-\alpha & 1 \\ 1 & 3 & 2 \end{vmatrix} = 0$, $\alpha = -4$

$L_2 \equiv \frac{x+2}{-4} = \frac{y+1}{9} = \frac{z+1}{1}$

Check options $(2,-10,-2)$ lies on L_2.

14. 2 The equation of the line through A is $\frac{x-3}{2} = \frac{y-1}{-1} = \frac{z+1}{2} = \lambda$

Take P as arbitary point a the line make AP = 5

15. 4 $\frac{x-1}{1} = \frac{y+\frac{3}{2}}{\frac{3}{2}} = \frac{z+5}{2}, \frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-2}{0}$

$1.3 + \frac{3}{2} \times -2 + 2 \times 0 = 0$

16. 4 Solve first two lines and the point substitute in third line

17. 1 The drst line is $\vec{r} = (7+2s, 10+3s, 13+4s) .....(2)$

The second line is $\vec{r} = (3+t, 5+2t, 7+3t)$

Equate (1) and (2) and solve and find s or t

18. 4 $l+m+n=0....(1)$

$l^2+m^2+n^2=0...(2)$

$l+m=-n \;\; \text{sub in}(2) i+m^2-(1+m)^2=0$

$-2(m=0)$

$l=0 \;\; \text{or}\; m=0$

$\text{case I}\; L=0, m=-n$

$l^2+m^2+n^2=1 \Rightarrow 2n^2=1$

$n = \frac{1}{\sqrt{2}}$

$\left(0, \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

case II m = 0, l = –n

$l^2 + m^2 + n^2 = 1 \Rightarrow n^2 + n^2 = 1$

$r = \frac{1}{\sqrt{2}}$

$\left(\frac{-1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$

$\cos\theta \; 0 + 0 + \frac{1}{2} \Rightarrow \theta = 60^6$

19. 2 The equation of the line through A is $\frac{x-3}{2} = \frac{y-1}{-1} = \frac{z+1}{2} = \lambda$

Take P as arbitary point a the line make AP = 5

20. 1 Since $\bar{a}$ and $\vec{c}$ are non-collinear. Equating the coefficients of $\bar{a}$ and $\vec{c}$ in the two values of $\vec{r}$, we get $6 - \lambda = 1 + \mu$, $2\lambda - 1 = 3\mu - 1 \Rightarrow \lambda = 3, \mu = 2$

So, there exist values for λ and μ such that the two values of $\vec{r}$ are same showing that the lines intersect and hence they are coplanar. Thus, statement-I and statement-II are true and the first follows from the second.

SECTION II (NUMERICAL)

21. 1 One of the point on line is $P(0,1,-1)$ and given point is $Q(\beta,0,\beta)$

So, $\overline{PQ} = \beta\hat{i} - \hat{j} + (\beta+1)\hat{k}$

Hence, $\beta^2 + 1 + (\beta+1)^2 - \frac{(\beta-\beta-1)^2}{2} = \frac{3}{2}$

$\Rightarrow 2\beta^2 + 2\beta = 0 \Rightarrow \beta = 0, -1$

$\Rightarrow \beta = -1$ (as $\beta \neq 0$)

22. 2

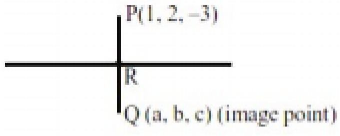
Line is $\dfrac{x+1}{2}=\dfrac{y-3}{-2}=\dfrac{z}{-1}=\lambda$

Let point R is $(2\lambda-1,-2\lambda+3,-\lambda)$

Direction ratio of $PQ\equiv(2\lambda-2,-2\lambda+1,3-\lambda)$

PQ is perpendicular to line

$\Rightarrow\ 2(2\lambda-2)-2(-2\lambda+1)-1(3-\lambda)=0$



$4\lambda-4+4\lambda-2-3+\lambda=0$

$9\lambda=9\ \Rightarrow\ \lambda=1.$

$\Rightarrow$ Point R is $(1,1,-1)$

$\dfrac{a+1}{2}=1 \quad \Big| \quad \dfrac{b+2}{2}=1 \quad \Big| \quad \dfrac{c-3}{2}=-1$

$a=1 \quad \Big| \quad b=0 \quad \Big| \quad c=1$

$\Rightarrow$ a + b + c = 2.

23. 108

$\dfrac{x-2}{1}=\dfrac{y}{-1}=\dfrac{z-7}{8}=\lambda$

$\dfrac{x+3}{4}=\dfrac{y+2}{3}=\dfrac{z+2}{1}=k$

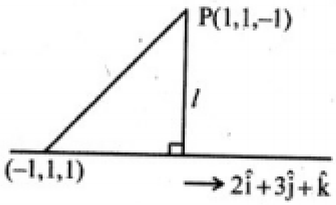
$\Rightarrow \lambda+2=4k-3$

$-\lambda=3k-2$

$\Rightarrow k=1, \lambda=-1$

$8\lambda+7=k-2$

$\therefore P=(1,1,-1)$



Projection of $2\hat{i}-2\hat{k}$ on $2\hat{i}+3\hat{j}+\hat{k}$ is

$=\dfrac{4-2}{\sqrt{4+9+1}}=\dfrac{2}{\sqrt{14}}$

$\therefore l^2=8-\dfrac{4}{14}=\dfrac{108}{14}$

$\Rightarrow 14l^2=108$

24. 9

$L_1 : \frac{x-5}{4} = \frac{y-4}{1} = \frac{z-5}{3} = \lambda$ drs $(4,1,3) = b_1$

$M(4\lambda+5, \lambda+4, 3\lambda+5)$

$L_2 : \frac{x+8}{12} = \frac{y+2}{5} = \frac{z+11}{9} = \mu$

$N(12\mu-8, 5\mu-2, 9\mu-11)$

$\overrightarrow{MN} = (4\lambda - 12\mu + 13, \lambda - 5\mu + 6, 3\lambda - 9\mu + 16)$ ..(1)

Now

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 1 & 3 \\ 12 & 5 & 9 \end{vmatrix} = -6\hat{i} + 8\hat{k} \quad ...(2)$$

Equation (1) and (2)

$\therefore \frac{4\lambda - 12\mu + 13}{-6} = \frac{\lambda - 5\mu + 6}{0} = \frac{3\lambda - 9\mu + 16}{8}$

I and II

$\lambda - 5\mu + 6 = 0$ ....(3)

I and III

$\lambda - 3\mu + 4 = 0$ ....(4)

Solve (3) and (4) we get

$\lambda = -1, \mu = 1$

$\therefore$ M (1, 3, 2)

N (4, 3, –2)

$\therefore \overrightarrow{OM} \cdot \overrightarrow{ON} = 4 + 9 - 4 = 9$

25. 2

Given equation of line $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-1}{-1}$ ..... (1)

Since, line(i) intersects the curve $xy = c^2$ ..... (2)

In XY-plane, put z = 0 in Eq.(i),we get x = 5, y = 1

Now, substituting the values of x and y in Eq. (ii), we get

$5 \times 1 = c^2 \quad \Rightarrow c^2 = 5$

$\Rightarrow c = \pm\sqrt{5}$

$\Rightarrow |c| = \sqrt{5}$

$\therefore [|c|] = \left[\sqrt{5}\right] = 2$

JEE ADVANCED LEVEL
SECTION III

26. C

Let a, b and c be the direction ratios of the required line. Since the required line lies in both the given planes, we must have

a − b + 2c = 0

and

3a + b + c = 0

In order to find a point on the required line, we put z = 0 in the two given equations to obtain x − y = 5 and 3x + y = 6.

Solving these two equations, we obtain x = 11/4, y = -9/4

Therefore, coordinates of a point on the required line are (11/4, −9/4, 0).

Hence, the equation of the required line is

$$\frac{x-\frac{11}{4}}{-3}=\frac{y-\left(-\frac{9}{4}\right)}{5}=\frac{z-0}{4}$$

or

$$\frac{4x-11}{-12}=\frac{4y+9}{20}=\frac{z-0}{4}$$

or

$$\frac{4x-11}{-3}=\frac{4y+9}{5}=\frac{z-0}{1}.$$

27. D

Any point on the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ is

$(3\lambda + 2, 4\lambda - 1, 12\lambda + 2)$

If this is the point of intersection with plane

$x - y + z = 5$, then

$3\lambda + 2 - (4\lambda - 1) + 12\lambda + 2 = 5$

or $\lambda = 0$

$\therefore$ Point of intersection is (2, − 1, 2).

Required distance

$= \sqrt{(2+1)^2+(-1+5)^2+(2+10)^2}$

$= 13$

28. A

Let A be any point on the plane x - y + z = 5 and B(1, -2, 3).

Then equation of the line AB whose direction ratios are 2, 3, – 6

$$\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{-6}=\lambda\left(\text{Let}\right)$$

$$\Rightarrow \quad x=1+2\lambda, y=-2+3\lambda, z=3-6\lambda$$

$$A(1+2\lambda,\ -2+3\lambda,\ 3-6\lambda)$$

A lies on plane.

Then, $1+2\lambda-(-2+3\lambda)+3-6\lambda=5$

$$\Rightarrow \quad 1+2\lambda+2-3\lambda+3-6\lambda=5 \Rightarrow \lambda=\frac{1}{7}$$

$$\therefore \quad A\left(\frac{9}{7},\frac{-11}{7},\frac{15}{7}\right)$$

$$\text{Distance } AB=\sqrt{\left(1-\frac{9}{7}\right)^2+\left(-2+\frac{11}{7}\right)^2+\left(3-\frac{15}{7}\right)^2}$$

$$=\sqrt{\frac{4}{49}+\frac{9}{49}+\frac{36}{49}}=1$$

29. D

Lines equation : $\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-2}{3}=\lambda$

General point (B) = $(2\lambda+1, 4\lambda+3, 3\lambda+2)$

$\overrightarrow{BP}=(2\lambda-2, 4\lambda-5, 3\lambda)$

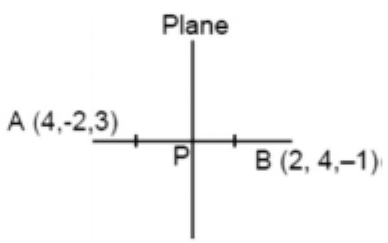
As $\overrightarrow{BP}$ is parallel to the plane, it is perpendicular to its normal,

$$3(2\lambda-2)+2(4\lambda-5)-2.3\lambda=0$$

$$\Rightarrow 6\lambda-6+8\lambda-10-6\lambda=0 \Rightarrow 8\lambda=16 \Rightarrow \lambda=2$$

$$\Rightarrow B=(5,11,8)=|\overrightarrow{BP}|=\sqrt{4+9+36}=\sqrt{49}=7$$

30. C



Sol. Mid-point $P=(3,1,1)$

Normal planes are along the line AB.

D.R,'s of normal = 4 –2, –2 –4, 3 –1 (–1) = 2, –6, 4,

= 1, –3, 2

Plane $\rightarrow 1(x-3)-3(y-1)+2(z-1)=0$

$\Rightarrow \quad x-3y+2z-2=0$

31. A

Equation of required plane is

$$P \equiv (x+2y+3z-2)+\lambda(x-y+z-3)=0$$

$$\Rightarrow (1+\lambda)x+(2-\lambda)y+(3+\lambda)z-(2+3\lambda)=0$$

Its distance from (3, 1, −1) is $\frac{2}{\sqrt{3}}$, therefore,

$$\frac{2}{\sqrt{3}}=\frac{|3(1+\lambda)+(2-\lambda)-(3+\lambda)-(2+3\lambda)|}{\sqrt{(\lambda+1)^2+(2-\lambda)^2+(3+\lambda)^2}}$$

$$\Rightarrow \frac{4}{3}=\frac{(-2\lambda)^2}{3\lambda^2+4\lambda+14} \Rightarrow 3\lambda^2+4\lambda+14=3\lambda^2$$

$$\Rightarrow \lambda=-\frac{7}{2} \Rightarrow -\frac{5}{2}x+\frac{11}{2}y-\frac{z}{2}+\frac{17}{2}=0$$

$$-5x+11y-z+17=0$$

32. B

Writing general points

x = 3λ + 1, y = λ + 2 and 2λ + 3

x = m + 3, y = 2m + 1 and 3m + 2

Lines intersect. Therefore

3λ + 1 = m + 3

and

λ + 2 = 2λ + 1

$$\begin{array}{lcccccc} 1\quad 2\quad 3 & 2 & & 3 & & 1 & 2 \\ 3\lambda-\mu\quad -2=0 & -1 & \times & -2 & \times\ 3 & \times & -1 \\ \lambda\quad -2\mu+1=0 & -2 & & 1 & 1 & & -2 \end{array}$$

$$\frac{\lambda}{-1-4}=\frac{\mu}{-2-3}=\frac{1}{-6+1} \Rightarrow \lambda=1 \text{ and } \mu=1$$

Therefore, point of intersection is (4, 3, 5).

Now plane passing through (4, 3, 5) and at maximum distance from the origin must have directions of the normal as 4 − 0, 3 − 0 and 5 − 0.

Therefore, equation of required plane is

(x − 4)4 + (y − 3)3 + (z − 5)5 = 0

or

4x + 3y + 5z = 16 + 9 + 25 ⇒ 4x + 3y + 5z = 50

SECTION IV (More than one correct)

33. A,B,C Equation of plane in new position will be $ax+by+\lambda z=0$ $(\lambda \in R)$ and if its angle with $ax+by=0$ is θ then

$$\tan\theta=\frac{a^2+b^2}{\sqrt{a^2+b^2+\lambda^2}\sqrt{a^2+b^2}}$$

34. A,B,C Equation of plane in new position will be $ax+by+\lambda z=0$ $(\lambda \in R)$ and if its angle with $ax+by=0$ is θ then

$$\tan\theta=\frac{a^2+b^2}{\sqrt{a^2+b^2+\lambda^2}\sqrt{a^2+b^2}}$$

35. A,B L will be parallel to common line of intersection of planes P_1 and P_2.

36. A,D

Equation of $P_3 : x+\lambda y+z-1=0$

$$\left|\frac{\lambda-1}{\sqrt{2+\lambda^2}}\right|=1 \quad \lambda=-\frac{1}{2}$$

$$\left|\frac{\alpha+\lambda\beta+\gamma-1}{\sqrt{2+\lambda^2}}\right|=2$$

$$\frac{\alpha-\frac{1}{2}\beta+\gamma}{\frac{3}{2}}=\pm 2; \quad \alpha-\frac{1}{2}\beta+\gamma-1=\pm 3$$

$$2\alpha-\beta+2\gamma-2=\pm 6$$

37. B,D Equation of the line passing through P(1, 4, 3) is $\frac{x-1}{a}=\frac{y-4}{b}=\frac{z-3}{c}$ ....(1)

Since (1) is perpendicular to

$\frac{x-1}{2}=\frac{y+3}{1}=\frac{z-2}{4}$ and $\frac{x+2}{3}=\frac{y-4}{2}=\frac{z+1}{-2}$

Hence $2a+b+4c=0$ and $3a+2b-2c=0$

$\frac{a}{-2-8}=\frac{b}{12+4}=\frac{c}{4-3} \Rightarrow \frac{a}{-10}=\frac{b}{16}=\frac{c}{1}$

Hence the equation of the lines is

$\frac{x-1}{-10}=\frac{y-4}{16}=\frac{z-3}{1}$ ....(2)

Now any point Q on (2) can be taken as

$(1-10l, 16l+4, l+3)$

Distance of Q from P (1, 4, 3)

$= (10l)^2+(16l)^2+l^2 = 357$

$\Rightarrow (100+256+1)l^2 = 357 \Rightarrow l = 1$ or -1 Q is (–9, 20, 4) or (11, – 12, 2)

Hence $a_1+a_2+a_3 = 15$ or 1

SECTION V - (Numerical type)

38. 1 $\begin{vmatrix} 1 & -c & -b \\ -c & 1 & -a \\ -b & -a & -1 \end{vmatrix} = 0 \Rightarrow a^2+b^2+c^2+2abc = 1.$

39. 174 The line of intersection of plane $2x+3y+4z-1=0$ and $x+y+z-3=0$ is $\frac{x-8}{1}=\frac{y+5}{-2}=\frac{z-0}{1}$

and the line of intersection of plane $2x+3y+4z-1=0$ and $x+y+z+3=0$ is

$\frac{x+10}{1}=\frac{y-7}{-2}=\frac{z-0}{1}$

Shortest distance will be $\sqrt{174}$

SECTION VI - (Matrix match type)

40. A Use the concept of coplanarity of lines and planes in 3-dimensional space