# LAWS OF MOTION

**Force**

Force is the external form of push or pull which produces or tries to produce motion in a body at rest, stops or tries to stop a moving body, changes or tries to changes the direction of a body or produces a change in shape of the body.

Based on the nature of interaction between bodies, forces may be classified into

(i) Contact forces (ii) Noncontact forces. Contact forces act between bodies in contact.

eg :-Tension, Normal reaction friction etc.

Field forces act between two bodies separated by a distance without any actual contact.

**Newton's First law of motion**

According to this law, a body continues to be in its state of rest or uniform motion along a straight line unless it is acted upon by some external unbalanced force. A body on its own cannot change its state of rest or state of uniform motion along a straight line. This inability of a body is known as the inertia of the body.

**Inertia of rest**

It is the ability of a body to change by itself its state of rest.

eg:- A person standing in a bus is thrown backward when the bus start suddenly.

**Inertia of motion**

It is the inability of a body to change itself its state of uniform motion.

eg :- A person jumping out of a moving train may fall forward.

**Inertia of direction**

It is the ability of a body to change itself the direction of motion.

eg :- When a car goes round a curve suddenly, the person sitting inside is thrown outwards.

**Linear momentum**

Linear momentum is a vector quantity. It is the quantity of motion in the body. It is given by product of mass an velocity.

$\boxed{P = mV}$ or $\boxed{\vec{P} = m\vec{V}}$

**Newton's second law of motion**

According to this law, the rate of change of linear momentum of a body is directly proportional to the external force applied on the body and this change takes place always in the direction of force applied.

When an unbalanced force is applied on a body, the momentum of body changes.

i.e. $\vec{F}_{ext} = \frac{d\vec{p}}{dt}$

$$\vec{F}_{ext} = \frac{d}{dt}(m\vec{v}) = \frac{dm}{dt}\vec{v} + m\frac{d\vec{v}}{dt}$$

$$\frac{dm}{dt} = 0$$

$$\vec{F}_{ext} = m\frac{d\vec{v}}{dt}$$

$$\boxed{\vec{F}_{ext} = m\vec{a}}$$

**Note**

$\Sigma F_x = ma_x$ $\Sigma F_y = ma_y$ $\Sigma F_z = ma_z$

**Newton's third law of motion**

According to this law, to every action, there is always equal and opposite reaction

**Impulse**

From Newton's second law, we can write

$$\vec{F} = \frac{d\vec{p}}{dt} \qquad d\vec{p} = \vec{F}\,dt$$

$$\int_{p_1}^{p_2} d\vec{p} = \int_{t_1}^{t_2} dt$$

Right side term is a vector quantity and known as impulse of force in the interval $t_1$ to $t_2$

$$\boxed{\bar{J} = \int_{t_1}^{t_2} \vec{F}\,dt}$$

If F is constant then

$$\boxed{J = F(t_2 - t_1)}$$

$$\boxed{J = \text{area of force time graph}}$$

$$\int_{p_1}^{p_2} d\vec{p} = \int_{t_1}^{t_2} F dt$$

$$J = \int_{t_1}^{t_2} \vec{F} dt = \int_{p_1}^{p_2} d\vec{p} = \vec{p}_2 - \vec{p}_1$$

$$\therefore \boxed{\text{Impulse} = \text{Change in momentum}}$$

**Example 1**

A force 5 N gives a mass $M_1$ an acceleration equal to 8 m/s$^2$ and $M_2$ and acceleration is equal to 24 m/s$^2$. What is the acceleration of both the masses are tied together?

**Solution**

F = Ma

$5 = M_1 \times 8$ ...(1)

$5 = M_2 \times 24$ ...(2)

$5 = (M_1 + M_2)a$

$$5=\left(\frac{5}{8}+\frac{5}{24}\right)a$$

$$5=\left(\frac{15}{24}+\frac{5}{24}\right)a$$

$$5=\frac{20}{24}a$$

a = 6 m/s$^2$

**Example 2**

A 50 kg mass is sitting on a frictionless surface. An unknown constant force pushes the mass for 2 sec until the mass reaches a velocity of 3 m/s. Then the impulse acting on the ball is

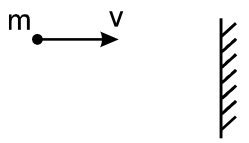
m = 50 kg u = 0 m/s v = 3 m/s

I = mv – mu = m (v – u)

= 50 (3 – 0) = 50 × 3

= 150 kg m/s

**Example 3**



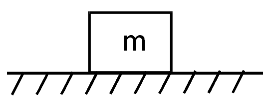

In the above figure ball is reflected back without any loss in speed.

Find the magnitude and direction of impulse.

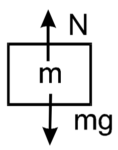
**Solution**

**Free Body Diagram**

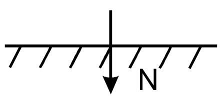
It is the diagram of a body, it represents all the forces acting on that body.




**Free Body diagram of m**




**Free body diagram of surface**

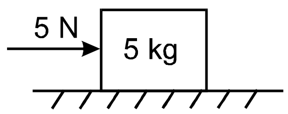



body is at rest N = mg

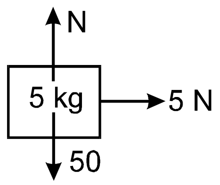
**Example**

i. Acceleration of body

ii. Normal reaction




**Solution**



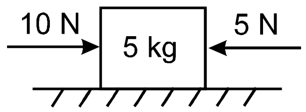

Net F = mass × acc

5 = 5 × a

a = 1 right

**N = 50**

**Example**

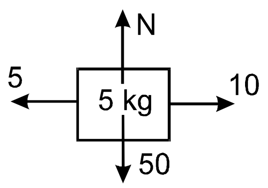



**Find**

i. acceleration of body

ii. Normal reaction

**Solution**



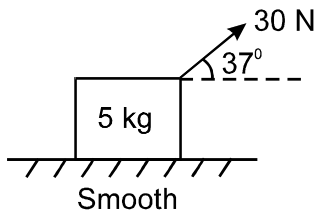

Net F = mass × acc

10 – 5 = 5 × a

a = 1 m/s$^2$ right

**N = 50 Newton**

**Example**

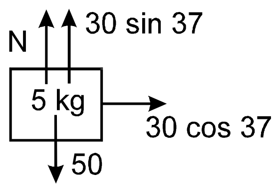



**Find**

i. acceleration of body

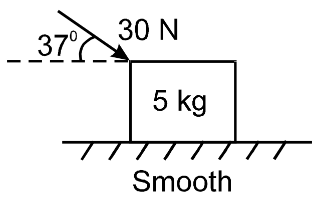
ii. Normal reaction

**Solution**




| | |
|---|---|
| N + 30 sin 37 = 50 | Net F = mass × acc |
| N + 30 × 3/5 = 50 | 30 cos 37 = 5 × a |
| N + 18 = 50 | 30 × 4/5 = 5a |
| **N = 32 Newton** | 24 = 5a |
| | **a = 24/5 m/s² right** |

**Example**

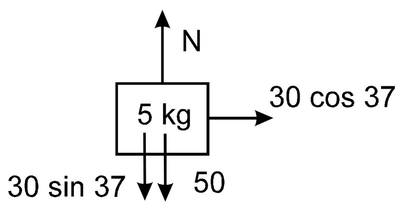



**Find**

i. acceleration of body

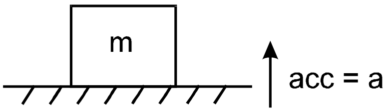
ii. Normal reaction

**Solution**



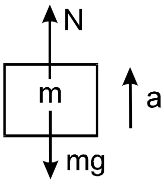

| | |
|---|---|
| N = 50 + 30 sin 37 | Net F = mass × acc |
| N = 50 + 30 × 3/5 | 30 cos 37 = 5 × a |
| N = 50 + 18 | 30 × 4/5 = 5 a |
| $\boxed{N = 68}$ | 24 = 5 a |
| | $\boxed{a = 24/5\,m/s^2}$ right |

**Example II**




Find normal reaction

**Solution**



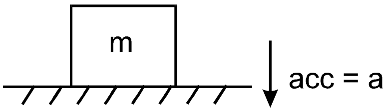

Net F = mass × acc

N – mg = ma

N = mg + ma

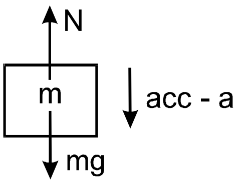
$\boxed{N = m(g + a)}$

**Example**




Find normal reaction

**Solution**



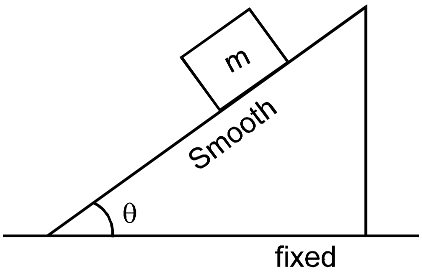

Net F = mass × acc

mg – N = ma

mg – ma = N

$\boxed{N = m(g - a)}$

**Example**

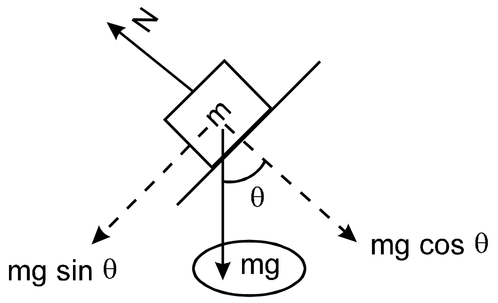


**Find**

i. acceleration of body

ii. Normal reaction

**Solution**

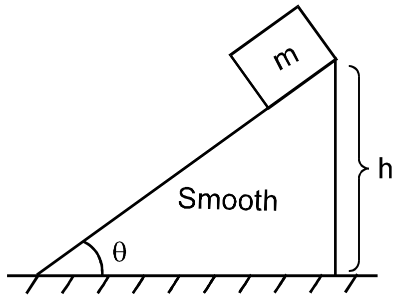



Net F = mass × acc

$mg\sin\theta = ma$

$a = g\sin\theta \qquad N = mg\cos\theta$

**Example**




If the block is released from the top of the inclined as shown, then time taken by block to reach bottom of the inclined plane.

**Solution**

dist trav by block = s

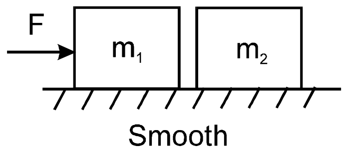
$S = ut + \frac{1}{2}at^2$

$\frac{h}{\sin\theta} = 0xt + \frac{1}{2}g\sin\theta\, t^2 ; \quad \boxed{\begin{array}{l}\sin\theta = h/s \\ s = \frac{h}{\sin\theta}\end{array}}$

$\frac{2h}{g\sin^2\theta} = t^2$

$\boxed{t = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}}$

**Example**

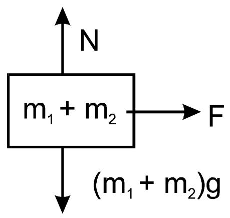



**Find**

i. acceleration of body
ii. Normal reaction

**Solution**

**fBD – Total System**

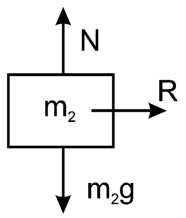



Net F = mass × a

F = $(m_1 + m_2)$ a

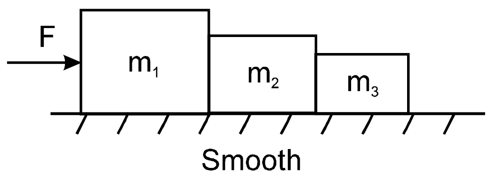
a = F /$(m_1 + m_2)$ right

**fBD – $m_2$**




Net F = mass × acc R = $m_2$ × a R = $m_2F/(m_1 + m_2)$

**Example**

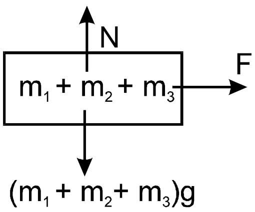



**Find**

i. acceleration of blocks
ii. reaction between $m_1$ and $m_2$
iii. reaction between $m_2$ and $m_3$

## Solution

**fBD – Total System**

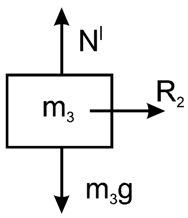



Net F = mass × acc

$F = (m_1 + m_2 + m_3)\,a$

$$\boxed{a = \frac{F}{m_1 + m_2 + m_3}}$$ μght

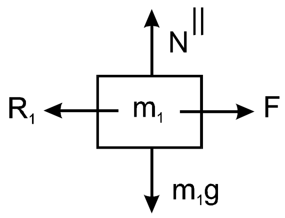
**fBD – $m_3$**




Net F = mass × acc

$$\boxed{R_2 = m_3 \frac{F}{m_1 + m_2 + m_3}}$$

**fBD – $m_1$**




Net F = mass × acc

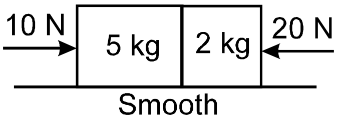
$F - R_1 = m_1 a$

$$F - R_1 = m_1 \frac{F}{m_1 + m_2 + m_3}$$

$$F - \frac{m_1 F}{m_1 + m_2 + m_3} = R_1$$

$$\boxed{R_1 = \frac{(m_2 + m_3)F}{m_1 + m_2 + m_3}}$$

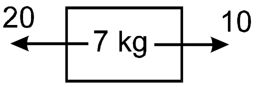
**Example**




Find acc of system
also find reaction between blocks

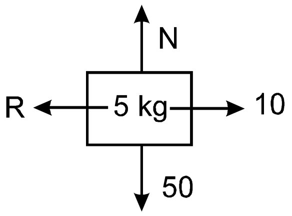
**Solution**

FBD – Total System



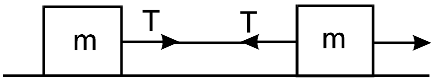

Net F = mass × acc
20 – 10 = 7 a
10 = 7 a
a = 10/7 m/s² left

**fBD – 5 kg**



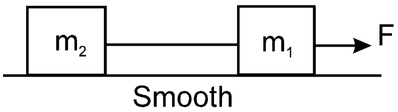

Net F = mass × acc
R – 10 = 5 × 10/7
R = 50/7 + 10 = 120/7 Newton

**Tension (T)**




**Note** (i) If string is massless, tension at every point on the string will be the same.
(ii) If string has mass, then tension will be different of different points.

**Example**

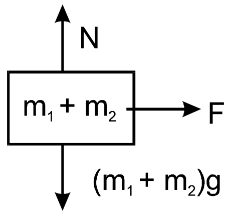



**Find**

i. acceleration of the system
ii. Tension in the string

**Solution**

FBD – Total System

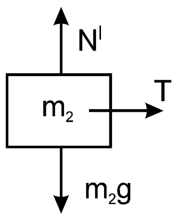



Net F = mass × a

$F = (m_1 + m_2)\,a$

$a = F/(m_1 + m_2)$ right

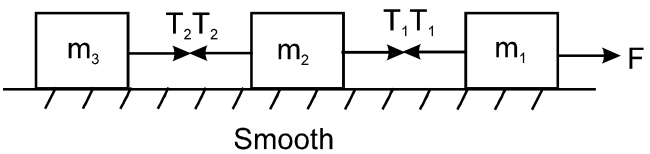
**fBD – $m_2$**




Net F = mass × acc

$T = m_2 \times a$ $\quad T = m_2F/(m_1 + m_2)$

**Example**




**Find**

i. acc of system

ii. Tension in the string

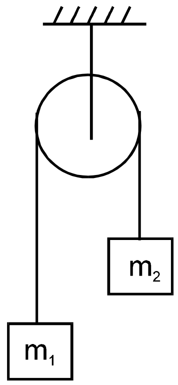
**Solution**

$F = (m_1 + m_2 + m_3)\,a \quad T_1\,(m_2 + m_3)\,a$

$$\boxed{a = \frac{F}{m_1 + m_2 + m_3}} \qquad \boxed{T_1 = \frac{(m_2 + m_3)F}{m_1 + m_2 + m_3}}$$

$T_2 = m_3a$ $\quad \boxed{T_2 = m_3\,\frac{F}{m_1 + m_2 + m_3}}$

**Example**



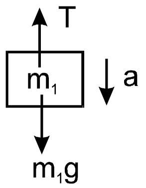

If $m_1 > m_2$

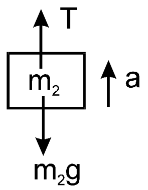
**Find**

i. acc of system

ii. Tension in the string

**Solution**




Net F = mass × acc

$m_1g - T = m_1 a$ ...(1)

Net F = mass × acc

$T - m_2g = m_2 a$ ...(2)

(1) + (2) $\Rightarrow m_1g - T + T - m_2g = m_1a + m_2a$

$(m_1 - m_2)g = (m_1 + m_2) a$

$$\boxed{a = \frac{(m_1 - m_2)g}{(m_1 + m_2)}}$$

(1) $\div$(2)

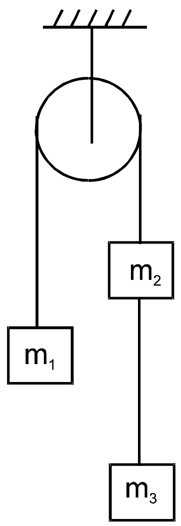
$$\frac{m_1 g - T}{T - m_2 g} = \frac{m_1 a}{m_2 a}$$

$m_1 m_2\, g - m_2 T = m_1 T - m_1 m_2 g$

$2m_1 m_2 g = m_1 T + m_2 T$

$$\boxed{T = \frac{2m_1 m_2 g}{m_1 + m_2}}$$

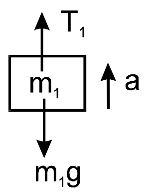
**Example**

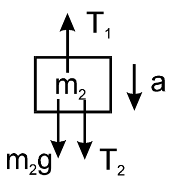



if $(m_2 + m_3) > m_1$

find acc of system

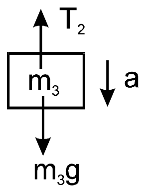
**Solution**




$T_1 - m_1 g = m_1 a$ ...(1)

$T_2 + m_2 g - T_1 = m_2 a$ ...(2)

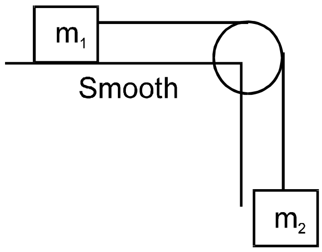



$m_3g - T_2 = m_3a$ ...(3)

(1) + (2) + (3)

$T_1 - m_1g + T_2 + m_2g - T_1 + m_3g - T_2 = m_1a + m_2a + m_3a \quad (m_3 + m_2 - m_1)g = (m_3 + m_2 + m_1)a$

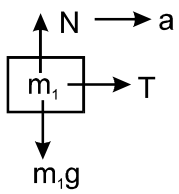
$$\boxed{a = \frac{(m_3 + m_2 - m_1)g}{(m_3 + m_2 + m_1)}}$$

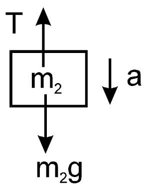



**Find**

i. acc of system

ii. Tension in the string




$T = m_1a$ ...(1)

$m_2g - T = m_2a$ ...(2)

(1) + (2)

$T + m_2g - T = m_1a + m_2a$

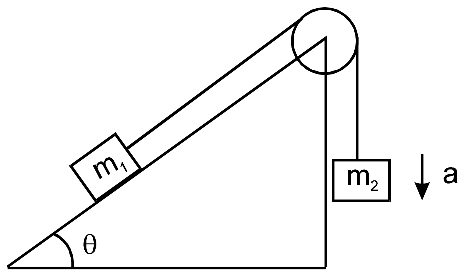
$m_2g = (m_1 + m_2)a$

$$\boxed{a = \frac{m_2 g}{m_1 + m_2}}$$

$T = m_1a$

$$\boxed{T = m_1 \frac{m_2 g}{m_1 + m_2}}$$

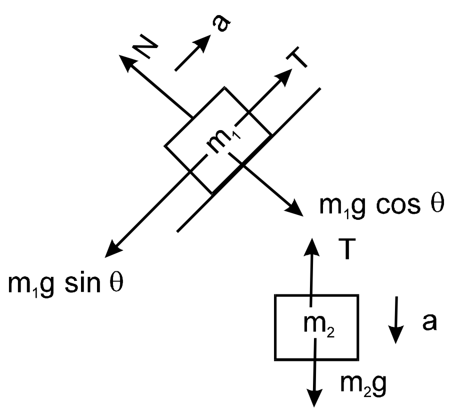
**<u>Example</u>**




**Find**

acc of the system

**<u>Solution</u>**




$T = m_1 g \sin\theta = m_1 a$ ...(1)

$m_2g - T = m_2a$ ...(2)

(1) + (2)

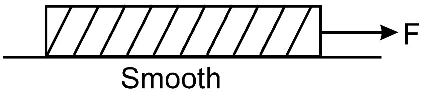
$m_2 g - m_1 g \sin\theta = (m_2 + m_1)a$ $\boxed{a = \frac{(m_2 - m_1 \sin\theta)g}{m_2 + m_1}}$

**String with mass**

If string is massless, tension will be same on every point of the string.

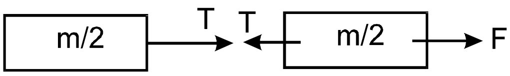
If string having mass, then tension will be different at different points.

**Example**




What is the tension at the mid point of the string?

**Solution**

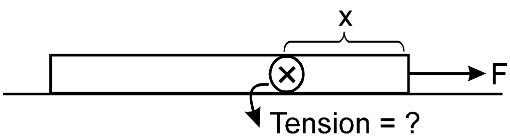



$F = ma$ $\qquad$ $T = m/2 \times a$

$\boxed{a = F/m}$ $\qquad$ $T = m/2 \times F/m$

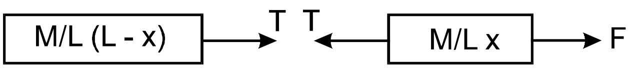
$T = F/2$

**Example**




Mass of string = M $\qquad$ length = L

**Solution**

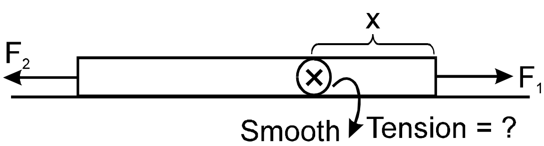



$F = Ma$ $\qquad$ $T = M/L\,(L - x) \times a$

$a = F/M$ $\qquad$ $T = M/L\,(L - x) \times F/M$

$\boxed{T = F/L(L - x)}$

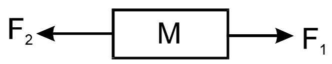
**Example**

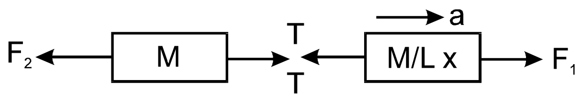



Mass of string = M

Length of string = L

**Solution**




$F_1 - F_2 = Ma$ Net F = mass × acc

$$\boxed{a=\frac{F_1-F_2}{M}} \qquad F_1 - T = (M/L)\ x \times a$$

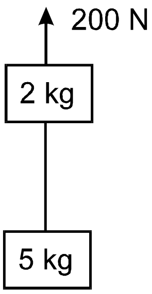
$$F_1 - T = (M/L) \times x \times \frac{F_1 - F_2}{M}$$

$$F_1 - T = (F_1 - F_2)x/L$$

$$T = F_1 - F_1x/L + F_2x/L$$

$$T = F_1 - F_1x/L + F_2x/L$$

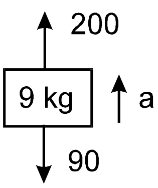
**Example**




Mass of the string is 2 kg. Find tension at the midpoint of the string.

**Solution**

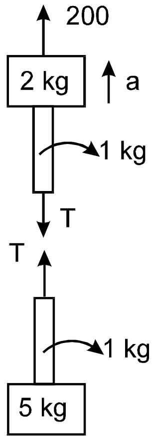
Fbd – Total




$200 - 90 = 9\ a$

$110 = 9\ a$

$a = 110/9$




Net F = mass × acc

T – 60 = 6 × 110/9

T = 660/9 + 60

## Frame of Reference

It is a platform used to observe a body.

There are two types of frame of reference.

(i) Inertial frame

(ii) Non inertial frame

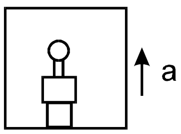
Non accelerating frames are inertial and accelerating frames are non inertial. When we draw non inertial free body diagram of a body, an extra force must included in that diagram, which is known as pseudoforce.

(i) Direction of pseudoforce is opposite to the direction of acceleration of the frame.

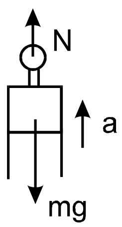
(ii) Magnitude of pseudo force is the product of mass of the body and acceleration of the frame.

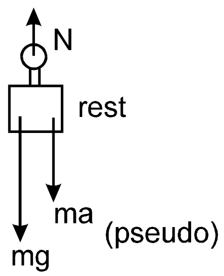
## Apparent weight of a man in a lift

i. **Lift moving upward with an acceleration 'a'**




**Inertial FBD** **Non inertial FBD**



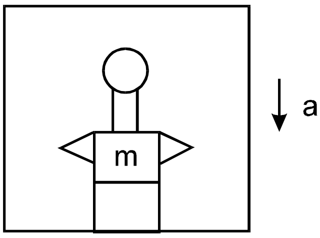
Net F = mass × acc     N = mg + ma

N – mg = ma

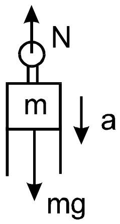
N = mg + ma

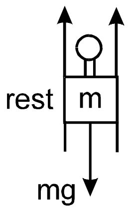
$\boxed{N = m(g + a)}$     $\boxed{N = m(g + a)}$

ii. **Lift moving downward with an acceleration 'a'**




**Inertial FBD**     **Non inertial FBD**




ma (pseudo)

N + ma = mg

N = mg - ma

Net F = mass × acc     $\boxed{N = m(g - a)}$

mg – N = ma

mg – ma = N

$\boxed{N = m(g - a)}$

Note :-

i. If lift is freely falling then a = g,

N = m (g – g) = 0

ii. If the lift is moving with constant velocity then a = 0

N = m (g – 0)     N = mg

**Frictional force**

It is an opposing force which opposes the relative motion between two surfaces in contact.

Two types

i. Static friction

ii. Kinetic friction

If there is no relative motion between contact surface, then friction will be static friction. If there is any relative motion between contact surfaces, then friction will be kinetic.

Maximum value of friction is known as limiting friction ($f_{max}$)

$$\boxed{f_{max} = \mu_s N}$$

Where $\mu_s$ is the coefficient of static friction. Kinetic friction is given by

$$\boxed{f_k = \mu_k N}$$

**Important points**

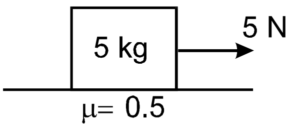
i. Check whether body is moving or not

ii. If net force greater than limiting friction then body will move, otherwise body is at rest.

iii. If the body is at rest, then friction is static friction and it is equal to net force and direction opposite to net force.

iv. If there is any relative motion, then friction will be kinetic friction, and its direction is opposite to relative motion.

**Example 1**

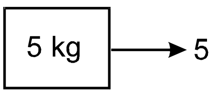



**Find**

i. acc of body

ii. frictional force

**Solution**




$f_{max} = \mu_s N$

$= 0.5 \times 50 = 25$

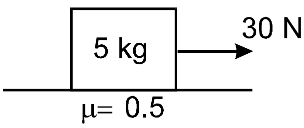
$5 < f_{max}$

body will not move

acc = 0

f = 5

**Example 2**

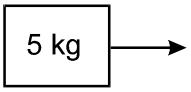



**Find**

i. acc of body

ii. frictional force

## Solution




$f_{max} = \mu_s N$

$= 0.5 \times 50 = 25$

$30 > f_{max}$

body will move

$f = f_k = \mu_k N = 0.5 \times 50 = 25$

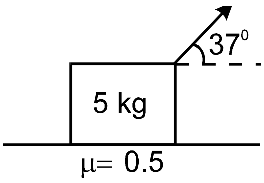
Net F = mass × acc

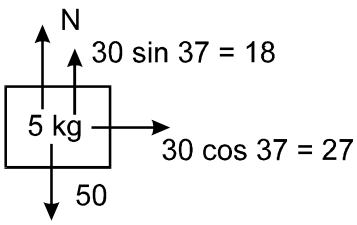
$30 - 25 = 5 \times a$

$5 = 5a$

$a = 1\ m/s^2$; $f = f_k = 25$ N

## Example 3




**Find**

i. acc of body

ii. frictional force

## Solution

$N + 18 = 50$

$N = 32$

$f_{max} = \mu_s N$

$= 0.5 \times 32 = 16$

$30 \cos 37 = 30 \times 4/5 = 24$ $\quad 24 > f_{max}$

$30 \sin 37 = 30 \times 3/5 = 18$ $\quad$ So body will move

then $f = f_k = \mu_k N$

$= 0.5 \times 32 = 16$

Net F = mass × acc

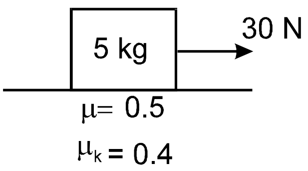
24 – 16 = 5 × a

8 = 5a

a = 8/5 m/s$^2$

f = $f_k$ = 16 N

**Example 4**

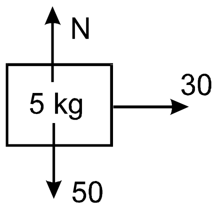



**Find**

i. acc of body

ii. frictional force

**Solution**




$f_{max} = \mu_s N$

= 0.5 × 50 = 25

30 > 25

So body will move.

Friction will be kinetic friction f = $f_k$

$f = \mu_k N = 0.4 \times 50 = 20$

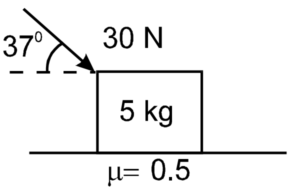
Net F = mass × acc

30 – 20 = 5 × a

10 = 5a

a = 2 m/s$^2$

**Example**

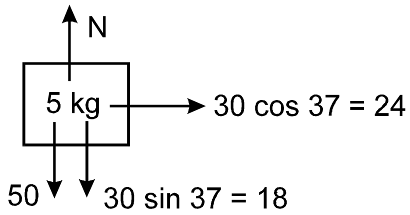



**Find**

i. acc of body

ii. frictional force

**Solution**




N = 50 + 18        N = 68

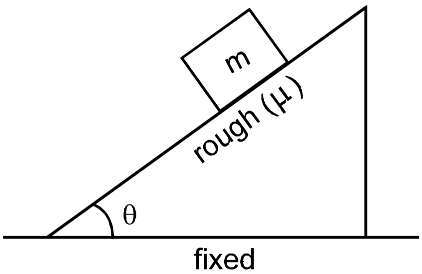
$f_{max} = \mu_s N$

$f_{max} = 0.5 \times 68 = 34$

$24 \leq 34$

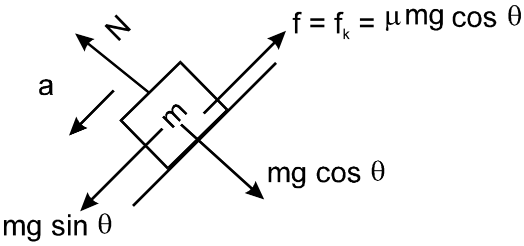
So body will not move

So     acc = 0     f = 24



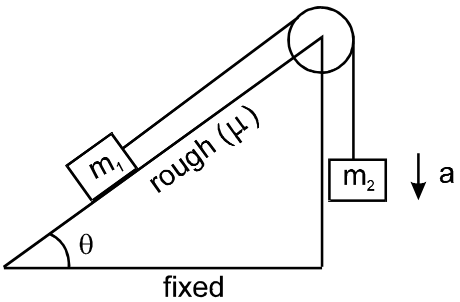

If block k sliding down with an acceleration 'a', then value of a is ______

**Solution**

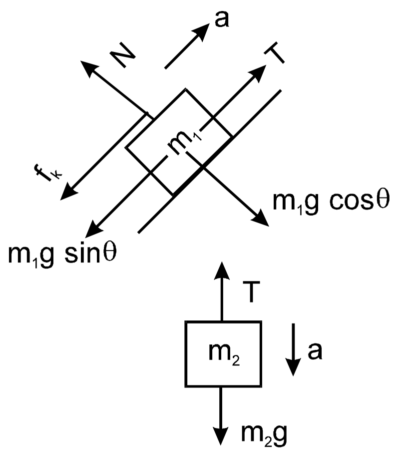



Net F = mass × acc

$mg\sin\theta - \mu mg\cos\theta = ma$          $\boxed{a = g\sin\theta - \mu g\cos\theta}$




**Find 'a'**




$T - m_1g\sin\theta - \mu m_1g\cos\theta = m_1a$ ...(1)

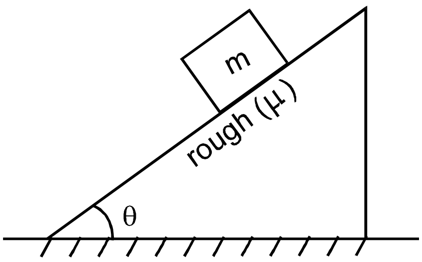
$m_2g - T = m_2a$ ...(2)

(1) + (2)

$$\frac{(m_2 - m_1\sin\theta - \mu m_1\cos\theta)g}{(m_1 + m_2)} = a$$

**<u>Angle of Repose</u>**

It is the angle of the incline plane at which block just starts to slide down.

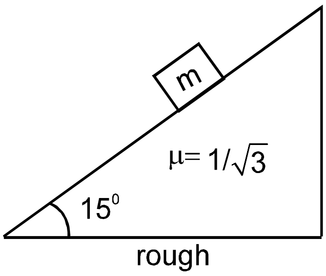



$\boxed{\text{Angle of repose} = \tan^{-1}(\mu)}$.

If angle of inclined plane is grater than angle of repose, then block will slide down.

If angle of inclined plane is less than the angle of repose, then the block will not move.

**<u>Example 1</u>**



Find acc of block?

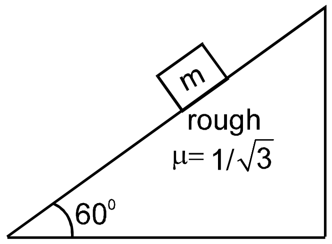
angle f repose = $\tan^{-1}(\mu)$

$$\tan^{-1}(1/\sqrt{3}) = 30^\circ$$

angle of inclined plane < angle of repose

So block will not move

**Example**




Find acc of block?

**Solution**

$\text{angle of repose} = \tan^{-1}(1/\sqrt{3}) = 30^\circ$

angle of inclined plane = $60^\circ$

angle of inclined plane > angle of repose

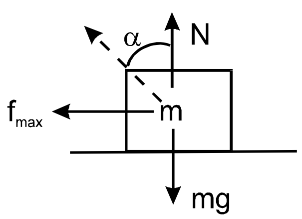
So block will move

$a = g\sin\theta - \mu g\cos\theta$

$a = 10 \times \sin 60 - \frac{1}{\sqrt{3}} \times 10 \times \cos 60$

**Angle of friction**

Angle made by resultant of normal reaction and limiting friction with normal reaction is called angle of friction.




$\tan\alpha = \frac{f_{max}}{N}$ $\tan\alpha = \frac{\mu_s N}{N}$

$\boxed{\alpha = \tan^{-1}(\mu)}$

## Principle of conservation of linear momentum

It states that if no external force acts on a system, then total linear momentum of the system remains constant.

## Proof

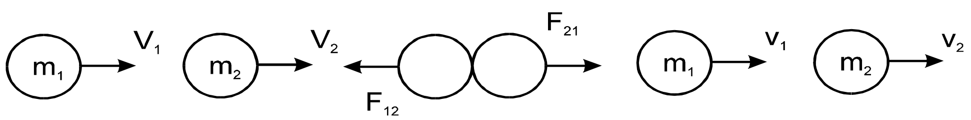
Acc to Newton's $2^{nd}$ law

$$\vec{f}_{ext} = \frac{d\vec{p}}{dt}$$

$$\text{If } \vec{f}_{ext} = 0 \text{ then} \frac{d\vec{p}}{dt} = 0$$

$$\boxed{\vec{p} = \text{const}}$$

## Alternative method




$m_1$ and $m_2$ together considered as a system.

$\therefore$ External force = 0

$\vec{F}_{21} \Rightarrow$ force acting on $2^{nd}$ body due to $1^{st}$ body.

$\vec{F}_{12} \Rightarrow$ force acting on $1^{st}$ body due to $2^{nd}$ body.

According to Newton's third law

$$\vec{F}_{21} \Rightarrow -\vec{F}_{12}$$

$$m_2\vec{a}_2 = -m_1\vec{a}_1$$

$$m_2\left(\frac{\vec{v}_2 - \vec{u}_2}{t}\right) = -m_1\left(\frac{\vec{v}_1 - \vec{u}_1}{t}\right)$$

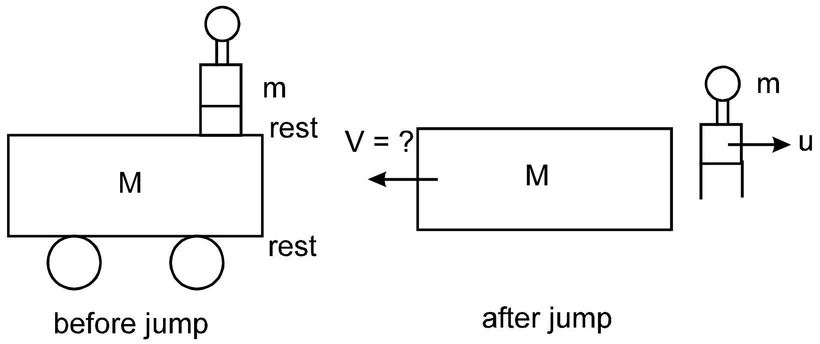
$$m_2\vec{v}_2 - m_2\vec{u}_2 = -m_1\vec{v}_1 + m_1\vec{v}_1$$

$$m_2\vec{v}_2 + m_1\vec{v}_1 = m_1\vec{u}_1 + m_2\vec{u}_2$$

$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$ i.e. Total linear momentum before = Total linear momentum after

## Example

A flat car of mass M at rest on a frictionless floor with a child of mass m, standing at its edge. If the child jumps off from the car towards right with an initial velocity V, find velocity car after it jumps.




External force = 0

Linear momentum before = Linear momentum after

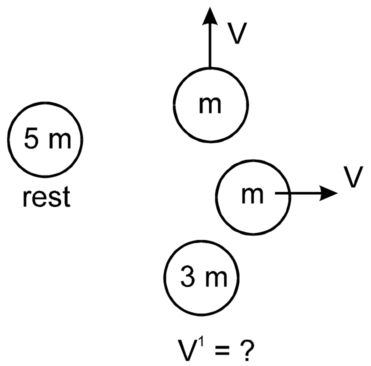
$M \times 0 + m \times 0 = mu + M(-v)$ $\quad\quad$ $0 = mu - Mv$

$Mv = mu$ $\quad\quad$ $\boxed{V = \frac{mu}{M}}$

**Example**

A bomb of mass 5m initially at rest explodes and breaks into three pieces of masses in the ratio 1 : 1 : 3. The two pieces of equal masses fly off perpendicular to each other with speed v. Then the velocity of the heavier piece is _______

**Solution**




Ext f = 0

Linear momentum before = Linear momentum after

$$5m \times 0 = m_1\vec{V}_1 + m_2\vec{V}_2 + m_3\vec{V}_3$$

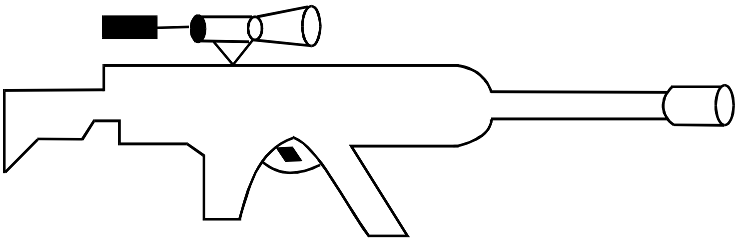
$$0 = mV\hat{j} + mV\hat{i} + 3m\vec{V}_3$$

$$-3m\vec{V}_3 = mV\hat{j} + mV\hat{i}$$

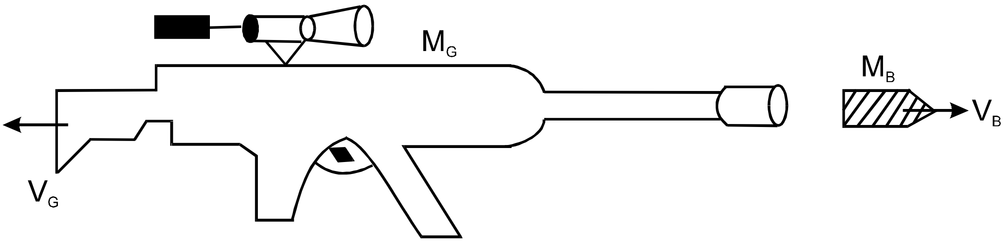
$$\vec{V}_3 = -V/3\,\hat{i} + -V/3\,\hat{j}$$

$$V_3 = |\vec{V}_3| = \sqrt{(V/3)^2 + (V/3)^2} = V\sqrt{2}/3$$

## Applications
## Recoil velocity of gun



Before firing




After firing

$V_B \rightarrow$ Velocity of bullet $M_B \rightarrow$ mass of bullet $M_G \rightarrow$ mass of gun
System $\Rightarrow$ gun + bullet
Ext F = 0
Linear momentum before = Linear momentum after

$$M_G \times 0 + M_B \times 0 = M_B V_B + M_G(-V_G)$$

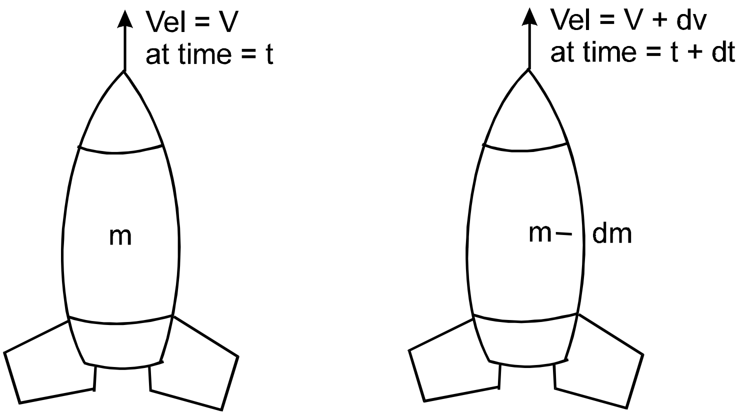
$$0 = M_B V_B - M_G V_G$$

$$M_G V_G = M_B V_B$$

$$\boxed{V_G = \frac{M_B V_B}{M_G}}$$

"$V_G$ is the recoil velocity of gun"

## Variable mass system
## Rocket propulsion



System $\rightarrow$ rocket + fuel

Ext F = 0

Total linear momentum before = Total linear momentum after

$$mv\hat{j} = (m - dm)(v + dv)\hat{j} + dm\,\vec{v}_f$$

Velocity of fuel w.r.t rocket = $\vec{V}_e$

$$\vec{V}_f - V_R = V_e(-\hat{j})$$

$$\vec{V}_f = V_e(-\hat{j}) + \vec{V}_R$$

$$\vec{V}_f = V_e(-\hat{j}) + (V + dv)\hat{j}$$

$$mv\hat{j} = (m - dm)(v + dv)\hat{j} + dm(v_e(-\hat{j}) + (v + dv)\hat{j})$$

$$mv\,\hat{j} = mv\,\hat{j} + mdv\,\hat{j} - dmv\,\hat{j} - dmdv\,\hat{j} - dmv_e\,\hat{j} + dmv\,\hat{j} + dmdv\,\hat{j}$$

$$0 = mdv\,\hat{j} - dmv_e\hat{j}$$

$mdv = dmv_e$

$$m\frac{dv}{dt} = V_e\frac{dm}{dt}$$

$$ma = V_e\frac{dm}{dt}$$

$$\boxed{F = V_e\frac{dm}{dt}}$$

Net force acting on the rocket in the upward direction is given by

$\boxed{F = V_e\frac{dm}{dt}}$ $\quad$ $\boxed{F = V_e\frac{dm}{dt} - mg}$ if gravity present

**Velocity of rocket at any time**

$$F = V_e\frac{dm}{dt}$$

$$m\frac{dv}{dt} = V_e\frac{dm}{dt}$$

$$mdv = -V_e dm$$

$$dV = -V_e\frac{dm}{m}$$

$$\int_{V_0}^{V} dV = -V_e \int_{m_0}^{m} \frac{dm}{m}$$

$$V - V_0 = -V_e \log_e (m)_{m_0}^{m}$$

$$V - V_0 = V_e \log m_0 / m \qquad \boxed{V = V_0 + V_e \log\left(\frac{m_0}{m}\right)}$$

**Example**

A 500 kg rocket is set for vertical firing. The exhaust speed is 800 m/s. To give an initial upward acceleration of 20 m/s$^2$, the amount of gas ejected per second to supply the needed thrust will be _____ (g = 10 m/s$^2$)

**Solution**

m = 5000 kg $\qquad F = V_e \frac{dm}{dt} - mg$

$V_e$ = 800 m/s $\qquad ma = V_e \frac{dm}{dt} - mg$

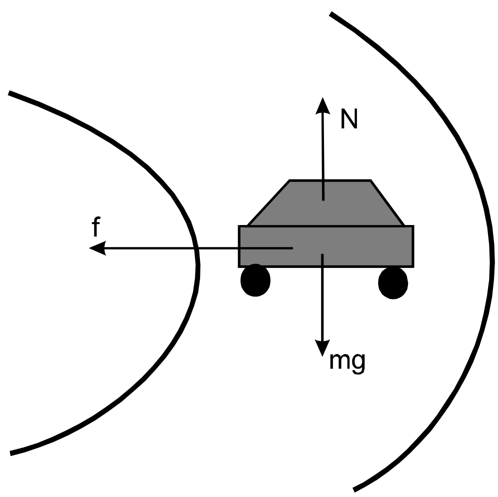
a = 20 m/s$^2$ $\qquad m(g + a) = V_e \frac{dm}{dt}$

$\frac{dm}{dt} = ?$ $\qquad 5000(10 + 20) = 800 \frac{dm}{dt}$

$$\frac{dm}{dt} = \frac{5000 \times 30}{800}$$

$$\boxed{\frac{dm}{dt} = \frac{1500}{8} Kg / s}$$

**Maximum speed for safe turning on circular level road**



friction will provide necessary centripetal force.

$$f = \frac{mv^2}{R} \qquad R \rightarrow \text{"radius of circular path".}$$

$$f \le f_{max}$$

$$\frac{mv^2}{R} \le \mu_s N$$

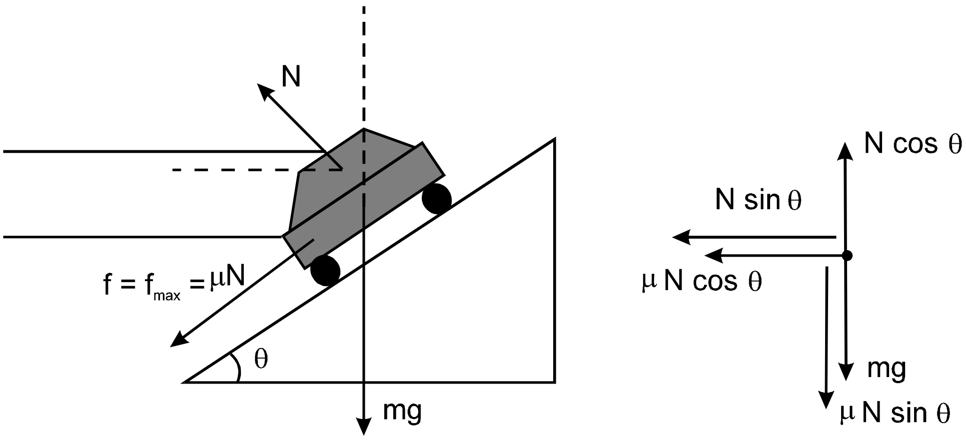
$$\frac{mv^2}{R} \le \mu mg$$

$$v^2 \le \mu Rg$$

$$v \le \mu Rg$$

$$\boxed{v_{max} = \sqrt{\mu Rg}}$$

**Banked road (with friction)**




Net F towards centre = Centripetal force

$$N\sin\theta + \mu N\cos\theta = \frac{mv^2}{R} ------(1)$$

$$mg + \mu N\sin\theta = N\cos\theta$$

$$N\cos\theta - \mu N\sin\theta = mg ----(2)$$

$$\frac{N\sin\theta + \mu N\cos\theta}{N\cos\theta - \mu N\sin\theta} = \frac{\frac{mv^2}{R}}{mg}$$

$$\frac{\frac{N\sin\theta}{N\cos\theta}+\frac{\mu N\cos\theta}{N\cos\theta}}{\frac{N\cos\theta}{N\cos\theta}-\frac{\mu N\sin\theta}{N\cos\theta}}=\frac{v^2}{Rg}$$

$$\frac{v^2}{Rg}=\frac{\tan\theta+\mu}{1-\mu+\tan\theta}$$

$$\boxed{v=\sqrt{\frac{Rg(\tan\theta+\mu)}{(1-\mu\tan\theta)}}}$$ Maximum speed for safe turning.

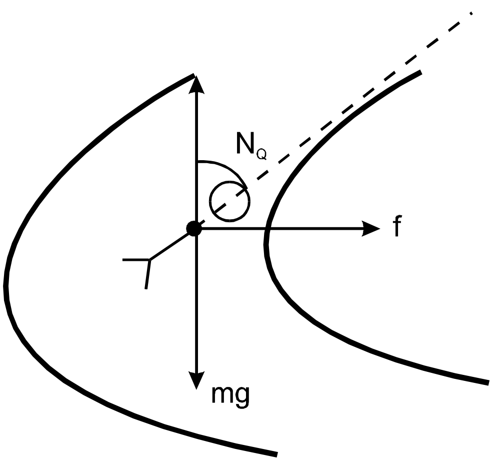
**Banked road (without friction)**

If there is no friction, then $\mu=0$

$$v=\sqrt{\frac{Rg\ (\tan\theta+0)}{(1-\text{ox}\tan\theta)}}$$

$$\boxed{v=\sqrt{Rg\tan\theta}}$$ Maximum speed for safe turning.

**Bending of Cyclist on circular level road**




**Angle made by cyclist with vertical =** $\theta$

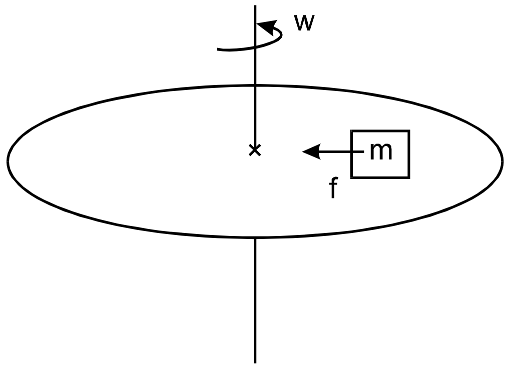
f $\rightarrow$ will provide centripetal force $f_{max}=\frac{mv^2}{R}$ $\mu_s N = mv^2/R$

$\mu mg = mv^2/R$; $\boxed{\mu=\frac{v^2}{Rg}}$

## Bending of cyclist with vertical

$$\tan\theta = \frac{\mu N}{N}; \qquad \tan\theta = \frac{v^2}{Rg}; \qquad \boxed{\theta = \tan^{-1}\left(\frac{v^2}{Rg}\right)}$$

## Note




When the table rotates, block is also rotating with it if that surface is rough and necessary centripetal force is provided by frictional force.