CHAPTER - 01

SET, RELATIONS AND REAL FUNCTIONS

I Number System

1. Natural numbers

N = {1, 2, 3, 4.................}

2. Whole numbers

W = {0, 1, 2, 3................}

3. Integers

I (z) = {...........-3, -2, -1, 0, 1, 2, 3.............}

4. Rational numbers

$$Q = \left\{\frac{p}{q}, p, q \in I, q \neq 0\right\}$$

5. Irrational numbers $= \left\{\sqrt{2}, \sqrt{3}, ...e,\ \pi........\right\}$

6. Real numbers

R = The union of rational and irrational numbers. Note : $N \subset W \subset Z \subset Q \subset R \subset C$

II. Types of Sets

1. Finite set	-	Contains finite number of elements
2. Infinite set	-	Contains infinite number of elements
3. Empty set *(void set), (Null set)*	-	Contains no element
4. Singleton set	-	Contains one element
5. Equivalent set	-	If $n(A) = n(B)$ then A and B are equivalent
6. Equal set	-	$A \subseteq B$ and $B \subseteq A \Leftrightarrow A = B$
7. Subset and super set	-	every element of A is an element of B then $A \subseteq B$ and $B \supseteq A$
8. Proper subset	-	A is a subset of B and $A \neq B$ then A is a proper sub set of B and is denoted as $A \subset B$
9. Power set	-	The set of all subsets of A is the power set of A and is denoted as $P(A)$, let $A = \{1,2,3\}$

$$P(A)=\{\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\phi\}$$

10. Universal set - In any discussion in set theory we consider a set which is the superset of all the sets under consideration is called the universal set

III. Set operations

1. Union - $A \cup B = \{x / x \in A \text{ or } x \in B\}$

2. Intersection - $A \cap B = \{x / x \in A \text{ and } x \in B\}$, If $A \cap B = \phi$ then A and B are disjoint.

3. Complement - $A^C = \{x / x \notin A \text{ and } x \in U\}$, U is the universal set

4. Difference - $A - B = \{x / x \in A \text{ and } x \notin B\} = \left\{x / x \in A \text{ and} x \in B^C\right\} = A \cap B^C$

5. Symmetric difference - $A\Delta B = (A \setminus B) \cup (B \setminus A) = (A \cup B) - (A \cap B) = \left(A \cap B^C\right) \cup \left(B \cap A^C\right)$

6. Compairable sets

If A and B are compairable if either $A \subseteq B$ or $B \subseteq A$

Examples :-

$A = \{1,2,3\}$ $B = \{2,3,4,5\}, U = \{1,2,3,4,5,6\}$

1. $A \cup B = \{1,2,3,4,5\}$
2. $A \cap B = \{2,3\}$
3. $A' = U - A = \{4,5,6\}$
4. $A - B = \{1\}$
5. $B - A = \{4,5\}$
6. $A\Delta B = (A \setminus B) \cup (B \setminus A) = \{1, 4, 5\}$
7. $A\Delta B = (A \cup B) - (A \cap B) = \{1, 2, 3, 4, 5\} - \{2, 3\} = \{1, 4, 5\}$

IV. Importants Laws

1. Demorgans Laws

a) $(A \cup B)' = A' \cap B'$

b) $(A \cap B)' = A' \cup B'$

c) $A - (B \cup C) = (A - B) \cap (A - C)$

e) $A - (B \cap C) = (A - B) \cup (A - C)$

2. Involution Laws

$(A')' = A$

3. Absorption Laws

a) $A \cup (A \cap B) = A$

b) $A \cap (A \cup B) = A$

4. Identity Laws

a) $A \cup \phi = A$

b) $A \cap U = A$

5. Complement Laws

a) $A \cup A' = U$

b) $A \cap A' = \phi$

V. Number of elements in a set (cardinality)

1. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

2. n(A-B) = $n(A \cap B^C) = n(A)\text{only} = n(A) - n(A \cap B)$

3. $n(A \Delta B) = n(A) + n(B) - 2n(A \cap B)$

4. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

VI. Properties of set operations

1. $A \cap B \subseteq A \subseteq A \cup B$ and $A \cap B \subseteq B \subseteq A \cup B$

2. $A \subseteq B \Leftrightarrow A \cup B = B$

3. $A \subset B \Leftrightarrow B^C \subset A^C$

4. $(A - B) = A \Leftrightarrow A \cap B = \phi$

5. $(A - B) \cup B = A \cup B$

6. $(A - B) \cap B = \phi$

7. $A \cap (B - C) = (A \cap B) - (A \cap C)$

8. $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$

VI. Cartesian product

1. $A \times B = \{(x, y) / x \in A \text{ and } y \in B\}$

   $(a, b) \neq (c, d)$

   $(a, b) = (c, d) \Leftrightarrow a = c \text{ and } b = d$

   If $A \times B = \phi$ then either one of A or B is null set
2. If n (A) = m and n (B) = n then n(A×B) = mn
3. If $n(A \cap B) = m$ then $n\left((A \times B) \cap (B \times A)\right) = m^2$
4. $A \times B \neq B \times A$ (in general) but $A \times B = B \times A \Leftrightarrow A = B$ where $A \neq \phi$ and $B \neq \phi$
5. $A \times (B \cup C) = (A \times B) \cup (A \times C)$ and $A \times (B \cap C) = (A \times B) \cap (A \times C)$ and

   $A \times (B - C) = (A \times B) - (A \times C)$
6. $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

VII. Relation

1. ***A relation from A to B is a subset of A ×B,*** If (x, y) $\in R$ means x related to y. ie., if xRy then (x, y) $\in R$.

   ie R = $\{(x, y) / x \in A, y \in B\}$

   If n(A) = m and n(B) = n then number relations from A to B = 2^{mn}
2. ***Inverse relation***

   If $R : A \to B$ then $R^{-1} : B \to A$

   $R^{-1} = \{(y, x) / (x, y) \in R\}$

   Note : Dom (R) = Ran (R^{-1}) and Ran (R) = Dom(R^{-1})
3. ***Relation on a set***

   $R : A \to A$ is the relation on a set **A**

   $\therefore$ No. of relation on a set have n elements = 2^{n^2}

VIII. Types of relations

1. ***Reflexive***

   If a R a $\forall a \in A$
2. ***Symmetric***

   If a R b $\Rightarrow$ b R a $\forall a, b \in A$
3. ***Transitive***

   If a R b and b R c $\Rightarrow aRc \ \forall a, b, c \in A$

4. *Equivalence relation*

If R is equivalence then it is reflexive, symmetric and transitive.

5. *Identity relation - $I_A : A \to A$*

Let A be a set then the relation $I_A = \{(x, y) / x \in A, y \in A \text{ and } x = y\}$ is called identity relation

6. *Inverse relation*

If $R : A \to B$ then $R^{-1} : B \to A$

$R^{-1} = \{(y, x) / (x, y) \in R\}$

7. *Void relation :-* Let A be any set $\phi \subset A \times A$, ϕ is called the void relation

8. *Universal relation*

Let A be any set then $A \times A \subseteq A \times A$, $A \times A$ is called the universal relation

IX. Properties on relations

Let R_1 and R_2 be two relations on a set A

1. If $R_1 \subset R_2$ and R_1 is reflexive then R_2 is reflexive.
2. If R_1 or R_2 is reflexive then $R_1 \cup R_2$ is reflexive
3. If R_1 and R_2 is reflexive then $R_1 \cap R_2$ is reflexive
4. If R_1 and R_2 are symmetric then $R_1^{-1}, R_2^{-1}, R_1 \cup R_2, R_1 \cap R_2, R_1 - R_2$ and $R_2 - R_1$ are symmetric
5. If R_1 and R_2 are transitive then $R_1 \cap R_2$ is transitive but $R_1 \cup R_2$ is need not be transitive
6. If R_1 and R_2 are two equivalence relation then $R_1 \cup R_2$ is need not be an equivalence relation. But $R_1 \cap R_2$ is equivalence relation.
7. No. of possible relation from $A \to B = 2^{O(A).O(B)}$
8. **Congruent modulo m**

   Let m be any fixed intiger then two intiger a and b are said to be congruence modulo m if a - b is divisible by m and is written as $a \equiv b \pmod{m}$.

X. Function (mapping)

A function from set A to set B is a relation R from A to B having the following properties.

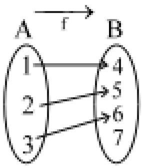
1. *Domain of R is A*

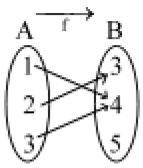
2. *If xRy and xRz then y = z* ie., If R is a function from A to B then for each $x \in A$ there exists one and only $y \in B$ such that x R y

Note : If f is a function from A to B is denoted as f : $A \to B$ and read as f from A to B

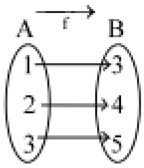
XI. Types of function

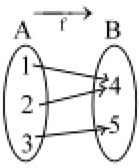
1. ***In to function :*** If f : $A \to B$ is an in to function then Ran (f) $\subset B$ or Ran (f) $\neq$ B

Eg. 1) 

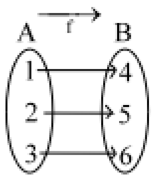
2) 

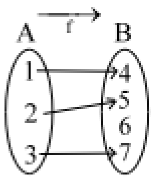
2. ***on to function (surjection) :*** If f : $A \to B$ is an on to function then Ran (f) = B

Eg. 1) 

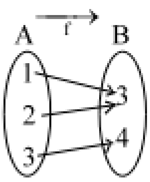
2) 

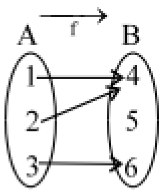
3. ***one one function (injection) :-*** If f : $A \to B$ is a one one function then different elements in A have different images in B

Eg. 1) 

2) 

4. ***many one function :*** If f : $A \to B$ is a many one function then different elements in A have same images in B

Eg. 1) 

2) 

5. ***Bijection :*** A function which is both one one and on to

6. Inverse function : If f : $A \to B$ is a bijection then f^{-1}exists and it denoted as $f^{-1} : B \to A$

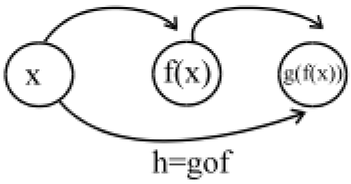
7. ***Identity function :*** $I_A : A \to A$ and $I_A(a) = a \quad \forall a \in A$

8. ***Constant function :*** f : $A \to B$ is constant function then f(a) = k $\forall a \in A$ where k is a constant and $k \in B$

XII. Composition of functions

If f : $A \to B$ and g : $B \to C$ then, g of : $A \to C$ is defined g of (x) = g(f(x)) $\forall x \in A$

- $h(x) = g(f(x)) = (gof)(x)$



gof $\neq$ fog

gof exists, iff the range of $F \subseteq$ domain of g

- If fo(goh) & (fog) oh are defined, then fo(goh) = (fog) oh
- Composite of two bijection is a bijection

- **Properties of composite function**

1.	**f**	**g**	**fog**
	even	even	even
	odd	odd	odd
	even	odd	even
	odd	even	even

Note :

1. If f : $A \to B$ is a function and $I_A : A \to A$ and $I_B : B \to B$ are identity functions on A and B then $I_B$of = f = fo I_A
2. If f: $A \to A$ is a map then foI_A = I_A of = f
3. If f is a bijection from A to B and I_A and I_B are identity function on A and B then f of^{-1} = I_B and f^{-1} of = I_A
4. If f and g are bijections then g of is also a bijection
5. If A is a non empty set then f, g : $A \to A$ such that fog and gof = I_A then f and g are bijections and g = f^{-1}
6. If f : $A \to B$ and $g : B \to C$ are two bijections then gof : $A \to C$ is a bijection and (gof)$^{-1}$ = f^{-1} og^{-1}

Inverse of a function: $y : B \to A \Rightarrow f(x) = y \Leftrightarrow g(y) = x \; \forall x \in A$ and $y \in B$. Then g is inverse of f (one to one onto function)

Properties

1) Inverse of a bijection is unique

2) If $f : A \to B$ is a bijection and $g : B \to A$ is the inverse of f, then $fog = I_B$ & $gof = I_A$ where I_A & I_B are identity function

3) The inverse of a bijection is also a bijection

4) $(gof)^{-1} = f^{-1}og^{-1}$

XIII. Functional Relations

- i) $f(xy) = f(x) + f(y) \Rightarrow f(x) = k\log x$ or $f(x) = 0$

ii) $f(xy) = f(x).f(y) \Rightarrow f(x) = x^n . n \in R$

iii) $f(x+y) = f(x).f(y) \Rightarrow f(x) = a^{kx}$

iv) $f(x+y)=f(x)=f(y)\Rightarrow f(x)=k$

v) $f(x).f\left(\frac{1}{x}\right)=f(x)+f\left(\frac{1}{x}\right)\Rightarrow f(x)=\pm x^n+1$

XIV. Number of functions

1. Number of functions from $A \to B$ is $[O(B)]^{O(A)}$

2. Number of one one function from A to B = $\begin{bmatrix} {}^np_m & \text{If } n \geq m \\ 0 & \text{If } n<m \end{bmatrix}$ Where O(A) = m and O(B) = n

3. If n(A) = m = n (B) then number of bijections from A to B = m !

4. If n(A) = m and n (B) = n where $1 \leq n \leq m$ then number of on to functions from A to B is = $\sum_{r=1}^{n}(-1)^{n-r}\,{}^nc_r r^m$

5. The number of relation from A to B which are not functions = $2^{mn} - n^m$ where n(A) = m and n(B) = n

XV Various Types of Functions :

(i) Polynomial Function :

If a function f is defined by $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n$, where n is a **non negative integer** and $a_0, a_1, a_2, \ldots\ldots, a_n$ are real numbers and $a_0 \neq 0$, then f is called a polynomial function of degree n.

Note : ☞ There are only two polynomial functions, satisfying the relation; f(x).f(1/x) = f(x) + f(1/x), which are $f(x) = 1 \pm x^n$

Proof : Let $f(x) = a_0x^n + a_1x^{n-1} + \ldots\ldots + a_n$, then $f\left(\frac{1}{x}\right) = \frac{a_0}{x^n} + \frac{a_1}{x^{n-1}} + \ldots\ldots + a_n$.

Since the relation holds for many values of x,

$\therefore$ Comparing the coefficients of x^n, we get $a_0 a_n = a_0$ $\Rightarrow$ $a_n = 1$

Similary comparing the coefficients of x^{n-1}, we get $a_0 a_{n-1} + a_1 a_n = a_1$ $\Rightarrow$ $a_{n-1} = 0$, like wise $a_{n-2}, \ldots\ldots, a_1$ are all zero.

Comparing the constant terms, we get $a_0^2 + a_1^2 + \ldots\ldots + a_n^2 = 2a_n^2$ $\Rightarrow$ $a_0 = \pm 1$

(ii) Algebraic Function :

y is an algebraic function of x, if it is a function that satisfies an algebraic equation of the form, $P_0(x)\,y^n + P_1(x)\,y^{n-1} + \ldots\ldots + P_{n-1}(x)\,y + P_n(x) = 0$, where n is a positive integer and $P_0(x), P_1(x)\ldots\ldots$ are polynomials in x. e.g. $y = |x|$ is an algebraic function, since it satisfies the equation $y^2 - x^2 = 0$.

Note : ☞ All polynomial functions are algebraic but not the converse.

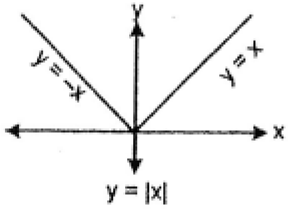
☞ A function that is not algebraic is called **Transcendental Function.**

(iii) Rational Function :

A rational function is a function of the form, $y = f(x) = \frac{g(x)}{h(x)}$, where g (x) and h (x), $h(x) \neq 0$ are polynomials.

(vi) Absolute value function / modulus function

The symbol of modulus functions is $f(x)=|x|$ and is defined as : $y=|x|=\begin{cases} x & \text{if } x\geq 0 \\ -x & \text{if } x<0 \end{cases}$



Properties of Modulus functions

1. $|x|^2 = x^2$

2. $\sqrt{x^2} = |x|$

3. $||x|| = |-x| = |x|$

4. $|x| = \max\{-x, x\}$

5. $-|x| = \min\{-x, x\}$

6. $\max(a,b) = \dfrac{a+b}{2} + \left|\dfrac{a-b}{2}\right|$

7. $\min(a,b) = \dfrac{a+b}{2} - \left|\dfrac{a-b}{2}\right|$

8. $|x+y| \leq |x| + |y|$

9. $|x+y| = |x| + |y|$ iff $xy > 0$

10. $|x-y| = |x| + |y|$ iff $xy \leq 0$

11. $|x| \leq a\,(a \text{ is} +\text{ve}) -a \leq x \leq a$

12. $|x| \geq a\,(a \text{ is} +\text{ve})\, x \leq -a$ or $x \geq a$

13. $|x| \leq a\,(a \text{ is} -\text{ve})$ no solution

14. $|x| \geq a\,(a \text{ is} -\text{ve})\, x \in R$

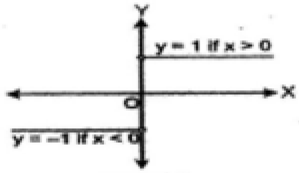
15. $a \leq |x| \leq b\,(a, b + \text{ve}) \Rightarrow -b \leq x \leq -a$ or $a \leq x \leq b$

$x \in [-b,-a] \cup [a,b]$

(vii) **Signum Function :** (Also known as sgn(x))

A function f (x) = sgn (x) is defined as follows :

$$f(x) = \text{sgn}(x) = \begin{cases} 1 & \text{for} \quad x > 0 \\ 0 & \text{for} \quad x = 0 \\ -1 & \text{for} \quad x < 0 \end{cases}$$



y = sgn x

It is also written as $\text{sgn}\, x = \begin{cases} \dfrac{|x|}{x}; & x \neq 0 \\ 0; & x = 0 \end{cases}$

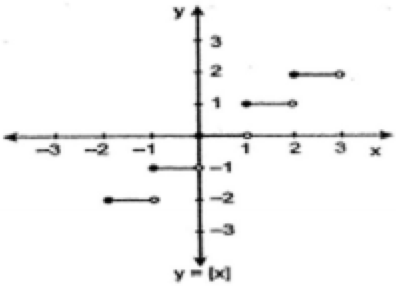
Note : $\text{sgn}\, f(x) = \begin{cases} \dfrac{|f(x)|}{f(x)}; & f(x) \neq 0 \\ 0; & f(x) = 0 \end{cases}$

(viii) **Greatest Integer Function or Step Function :**

The function y = f (x) = [x] is called the greatest integer function, where [x] equals to the greatest integer less than or equal to x. For example :

for $-1 \le x < 0$; $[x] = -1$; for $0 \le x < 1$; $[x] = 0$

for $1 \le x < 2$; $[x] = 1$; for $2 \le x < 3$; $[x] = 2$ and so on.



y = [x]

Properties of greatest integer function :

(a) $x - 1 < [x] \le x$

(b) If m is an integer, then $[x \pm m] = [x] \pm m$.

(c) $[x] + [y] \le [x + y] \le [x] + [y] + 1$

(d) $[x] + [-x] = \begin{cases} 0 & ; \text{ if } x \text{ is an integer} \\ -1 & ; \text{ if } x \text{ is not an integer} \end{cases}$

Properties of Greatest Integer Function

1. $[x] \le x < [x] + 1$

2. $x - 1 < [x] < x$

3. $I \le x < I + 1 \Rightarrow [x] = I$

4. $[[x]] = [x]$

5. $[x] + [-x] = \begin{cases} 0, x \in I \\ -1, x \in I \end{cases}$

6. $[x] - [-x] = \begin{cases} 2x, \; x \in I \\ 2x + 1, x \notin I \end{cases}$

7. $[x \pm n] = [x] \pm n, n \in I$

8. $[x] \geq n \Leftrightarrow x \geq n, n \in I$

9. $[x] > n \Leftrightarrow x \geq n+1, n \in I$

10. $[x] \leq n \Leftrightarrow x < n+1, n \in I$

11. $[x] < n \Leftrightarrow x < n$

12. $[x] = \left[\frac{x}{2}\right] + \left[\frac{x+1}{2}\right]$

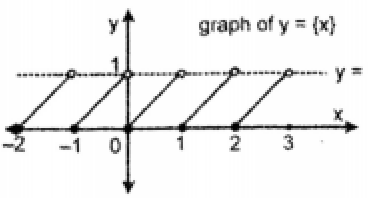
$[x_1] + [x_2] + ... + [x_n] \leq [x_1 + x_2 + ... + x_n]$

13. $\left[\frac{n+1}{2}\right] + \left[\frac{n+2}{4}\right] + \left[\frac{n+4}{8}\right] + ... = n$

14. $[x] + [y] \leq [x+y] \leq [x] + [y] + 1$

15. $[x] + \left[x + \frac{1}{n}\right] + \left[x + \frac{2}{n}\right] + ... + \left[x + \frac{n-1}{n}\right] = [nx]$

Fractional Part Function:
It is defined as, $y = \{x\} = x - [x]$, where [.] denotes greatest integer function.
e.g. the fractional part of the number 2.1 is $2.1 - 2 = 0.1$ and $\{-3.7\} = 0.3$.
The period of this function is 1 and graph of this function is as shown.



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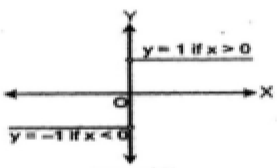


y = sgn x

It is also written as $\text{sgn}\, x = \begin{cases} \frac{|x|}{x}; & x \neq 0 \\ 0; & x = 0 \end{cases}$

Note : $\text{sgn}\, f(x) = \begin{cases} \frac{|f(x)|}{f(x)}; & f(x) \neq 0 \\ 0; & f(x) = 0 \end{cases}$

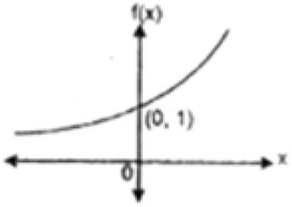
polynomials.

(iv) **Exponential Function :**

A function $f(x) = a^x = e^{x \ln a}$ $(a > 0,\ a \neq 1,\ x \in R)$ is called an exponential function. Graph of exponential function can be as follows :

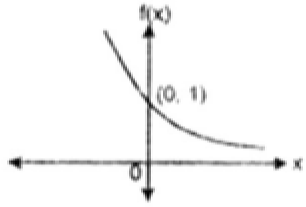
Case - I

For a > 1



Case - II

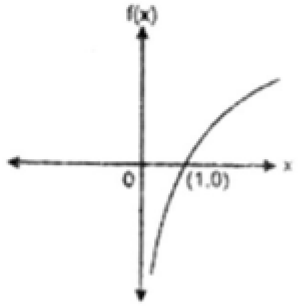
For 0 < a < 1



(v) **Logarithmic Function :** $f(x) = \log_a x$ is called logarithmic function, where $a > 0$ and $a \neq 1$ and $x > 0$. Its graph can be as follows

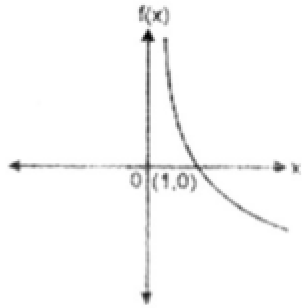
Case- I

For a > 1



Case- II

For 0 < a < 1



Properties of Logarithmic functions

1. $\log_e(ab) = \log_e a + \log_e b$

2. $\log_e\left(\dfrac{a}{b}\right) = \log_e a - \log_e b$

3. $\log_e a^m = m \log_e a$

4. $\log_a a = 1$

5. $\log_{b^m} a = \dfrac{1}{m}\log_b a$

6. $\log_b a = \dfrac{1}{\log_a b}$

7. $\log_b a = \dfrac{\log_m a}{\log_m b}$

8. $a^{\log_a m} = m$

9. $a^{\log_c b} = b^{\log_c a}$

10. $\log_m x > \log_m y$

$\Rightarrow x > y, m > 1$

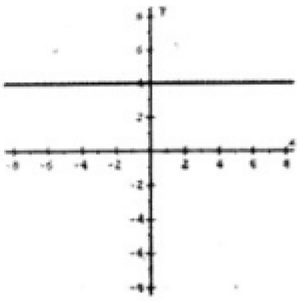
$x < y, 0 < m < 1$

11. $\log_m a = b \Rightarrow a = m^b$

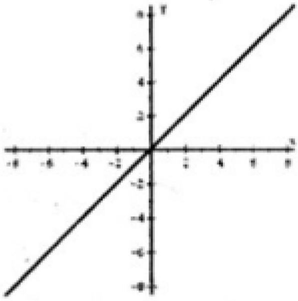
12. $\log_m a > b \Rightarrow \begin{cases} a > m^b, m > 1 \\ a < m^b, 0 < m < 1 \end{cases}$

13. $\log_m a < b \Rightarrow \begin{cases} a < m^b, m > 1 \\ a > m^b, 0 < m < 1 \end{cases}$

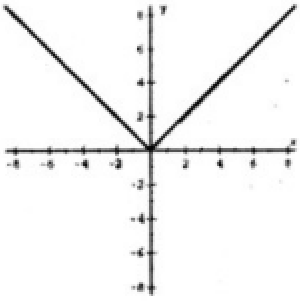
PARENT FUNCTIONS



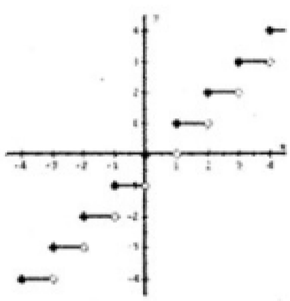
$f(x) = a$
Constant



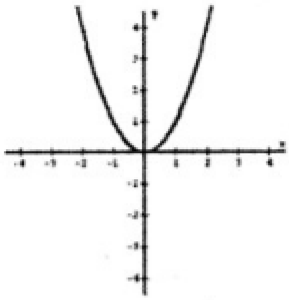
$f(x) = x$
Linear



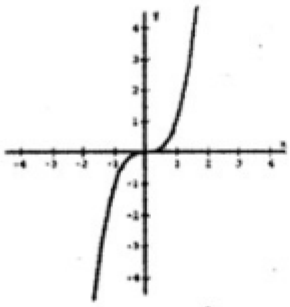
$f(x) = |x|$
Absolute Value



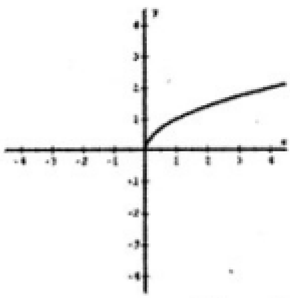
$f(x) = \text{int}(x) = [x]$
Greatest Integer



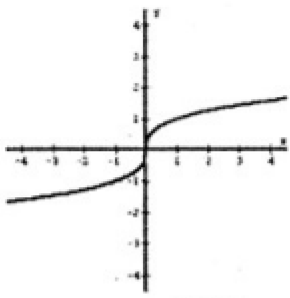
$f(x) = x^2$
Quadratic



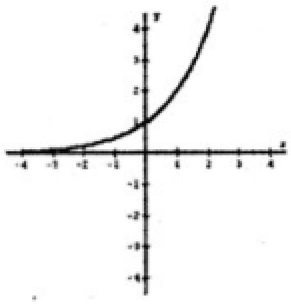
$f(x) = x^3$
Cubic



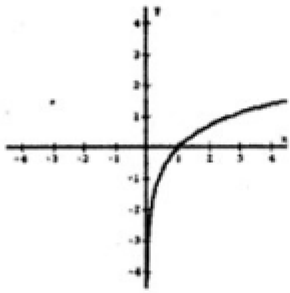
$f(x) = \sqrt{x}$
Square Root



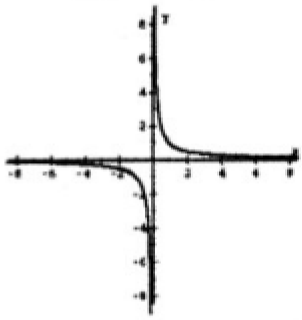
$f(x) = \sqrt[3]{x}$
Cube Root



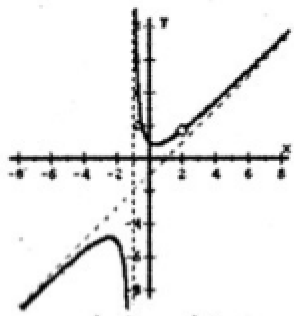
$f(x) = a^x$
Exponential



$f(x) = \log_a x$
Logarithmic



$f(x) = \dfrac{1}{x}$
Reciprocal



$f(x) = \dfrac{(x^2+1)(x-2)}{(x+1)(x-2)}$
Rational

XVI. BINARY OPERATIONS

I. Definition

Let S be a non empty set. A function f from S × S $\to S$ is a binary operation on S.

Binary operations denoted by symbols like $*, \oplus, \otimes$ etc.

II. Properties of binary operations

Let $*$ be a binary operation on S

1. $*$ is said to be commutative if $a * b = b * a \ \forall ab \in S$
2. $*$ is said to be associative if $(a * b) * c = a * (b * c) \ \forall a, b, c \in S$
3. $*$ is said to be Binary operation with identity element. If $\exists$ an element $e \in S$ such that $a * e = a = e * a$ $\forall a \in S$
4. Let $*$ be a binary operation with an element e then an an element $a \in S$ such that $a * a^1 = e = a^1 * a$ then a^1 is called an inverse of a.

III. Number of Elements in Binary Operations

1. Let A be a set having 'n' elements. Then the number of Binary operations on A is n^{n^2}
2. Let n(A) = n, Then total number of commutative Binary operations on A is $n^{\left[\frac{n(n+1)}{2}\right]}$
3. Let n(A) = n. Then total number of Non-commutative Binary operations is $n^{n^2} - n^{\left[\frac{n(n+1)}{2}\right]}$

PART I - (JEEMAIN)

SECTION - I - Straight objective type questions

1. If the sets A and B are defined as $A = \{(x, y) : y = e^x, x \in R\}$; $B = \{(x, y) : y = x, x \in R\}$, then

(1) $B \subseteq A$ (2) $A \subseteq B$ (3) $A \cap B = \phi$ (4) $A \cup B = A$

2. If A and B are two sets, then $A \cap (A \cup B)'$ is equal to

(1) A (2) B (3) ϕ (4) None of these

3. In a certain town 25% families own a phone and 15% own a car, 65% families own neither a phone nor a car. 2000 families own both a car and a phone. Consider the following statements in this regard:

(a) 10% families own both a car and a phone
(b) 35% families own either a car or a phone
(c) 40,000 families live in the town

Which of the above statements are correct

(1) (a) and (b) (2) (a) and (c) (3) (b) and (c) (4) (a), (b) and (c)

4. Let N denote the set of all natural numbers and R be the relation on $N \times N$ defined by (a, b) R (c, d) if $ad(b + c) = bc(a + d)$, then R is

(1) Symmetric only
(2) Reflexive only
(3) Transitive only
(4) An equivalence relation

5. If $A=\{(x,y):x^2+y^2=25\}$ and $B=\{(x,y):x^2+9y^2=144\}$, then $A\cap B$ contains

1) One point 2) Three points C) Two points D) Four points

6. Let R and S be two non-void relations on a set A. Which of the following statements is false

(1) R and S are transitive $\Rightarrow$ R $\cup$ S is transitive

(2) R and S are transitive $\Rightarrow$ R $\cap$ S is transitive

(3) R and S are symmetric $\Rightarrow$ R $\cup$ S is symmetric

(4) R and S are reflexive $\Rightarrow$ R $\cap$ S is reflexive

7. Let $S=\{1,2,3,...,100\}$. The number of non-empty subsets A of S such that the product of elements in A is even is:

(1) $2^{50}(2^{50}-1)$ (2) $2^{100}-1$ (3) $2^{50}-1$ (4) $2^{50}+1$

8. Let Z be the set of integers. If $A=\left\{x\in Z:2^{(x+2)(x^2-5x+6)}=1\right\}$ and $B=\{x\in Z:-3<2x-1<9\}$, then the number of subsets of the set A×B, is

1) 2^{18} 2) 2^{10} 3) 2^{15} 4) 2^{12}

9. Let R_1 and R_2 be two relations defined as follows:

$R_1=\{(a,b)\in R^2:a^2+b^2\in Q\}$ and $R_2=\{(a,b)\in R^2:a^2+b^2\notin Q\}$,

where Q is the set of all rational numbers. Then:

(1) R_2 is transitive but R_1 is not transitive

(2) R_1 is transitive but R_2 is not transitive

(3) R_1 and R_2 are both transitive

(4) Neither R_1 nor R_2 is transitive

10. Let $A=\{x_1,x_2,....,x_7\}$ and $B=\{y_1,y_2,y_3\}$ be two sets containing seven and three distinct elements respectively. Then the total number of functions $f:A\to B$ that are onto, if there exist exactly three elements 'x' in A such that $f(x)=y_2$, is equal to

(1) $14\cdot{}^7C_3$ (2) $16\cdot{}^7C_3$ (3) $14\cdot{}^7C_2$ (4) $12\cdot{}^7C_2$

11. If P(S) denotes the set of all subsets of a given set S, then the number of one-to-one functions from the set S = {1, 2, 3} to the set P(S) is

(1) 24 (2) 8 (3) 336 (4) 320

Assertion & Reasoning
(1) If both Statement-I and Statement-II are true and the reason is the correct explanation of the assertion.
(2) If both Statement-I and Statement-II are true but reason is not the correct explanation of the assertion.
(3) If Statement-I is true but Statement-II is false.
(4) If Statement-I is false but Statement-II is true.

12. **Statement-I:** If $A = \{x \mid g(x) = 0\}$ and $B = \{x \mid f(x) = 0\}$, then $A \cap B$ be a root of $\{f(x)\}^2 + \{g(x)\}^2 = 0$.
Statement-II: $x \in A \cap B \Rightarrow x \in A$ or $x \in B$.

13. Let R be the set of real numbers.
Statement-I: A = $\{(x,y) \in R \times R :\ y - x$ is an integer$\}$ is an equivalence relation on R.
Statement-II: B = $\{(x,y) \in R \times R :\ x = \alpha y$ for some rational number $\alpha\}$ is an Equivalece relation on R.

14. The function $f(x) = \log(x + \sqrt{x^2 + 1})$, is
(1) An even function
(2) An odd function
(3) A Periodic function
(4) Neither an even nor odd function

15. If x is real, then value of the expression $\dfrac{x^2 + 14x + 9}{x^2 + 2x + 3}$ lies between
1) 5 and 4
2) 5 and –4
3) –5 and 4
4) none of these

16. Let $f : (2,3) \to (0,1)$ be defined by $f(x) = x - [x]$ then $f^{-1}(x)$ equals
1) $x - 2$
2) $x + 1$
3) $x - 1$
4) $x + 2$

17. The domain of the definition of the function $f(x) = \dfrac{1}{4 - x^2} + \log_{10}(x^3 - x)$ is:
1) $(1,2) \cup (2,\infty)$
2) $(-1,0) \cup (1,2) \cup (3,\infty)$
3) $(-1,0) \cup (1,2) \cup (2,\infty)$
4) $(-2,-1) \cup (-1,0) \cup (2,\infty)$

18. The inverse function of

$$f(x)=\frac{8^{2x}-8^{-2x}}{8^{2x}+8^{-2x}}, x\in(-1,1), \text{ is}$$

1) $\frac{1}{4}(\log_8 e)\log_e\left(\frac{1-x}{1+x}\right)$ 2) $\frac{1}{4}\log_e\left(\frac{1-x}{1+x}\right)$

3) $\frac{1}{4}(\log_8 e)\log_e\left(\frac{1+x}{1-x}\right)$ 4) $\frac{1}{4}\log_e\left(\frac{1+x}{1-x}\right)$

19. The domain of the function $f(x)=\log(\sqrt{x-4}+\sqrt{6-x})$ is

(1) $[4,\infty)$ (2) $(-\infty,6]$ (3) $[4, 6]$ (4) None of these

20. The inverse of $y=5^{\log x}$ is

1) $x=5^{\log y}$ 2) $x=y^{\log 5}$ 3) $x=y^{\frac{1}{\log 5}}$ 4) $x=5^{\frac{1}{\log y}}$

SECTION - II

Numerical Type Questions

21. Out of 800 boys in a school, 224 played cricket, 240 played hockey and 336 played basketball. Of the total, 64 played both basketball and hockey; 80 played cricket and basketball and 40 played cricket and hockey; 24 played all the three games. The number of boys who did not play any game is

22. If the number of reflexive relations of a set with four elements is 2^k, then the value of k is ?

23. Let $\bigcup_{i=1}^{50} X_i=\bigcup_{i=1}^{n} Y_i=T$, where each X_i contains 10 elements and each Y_i contains 5 elements. If each element of the set T is an element of exactly 20 of sets X_i's and exactly 6 of sets Y_i's, then 'n' is equal to:

24. Let f(x) satisfy f(x) + f(x+10)= f(x+5)+f(x+15), $\forall x\in R$, then f(x) is periodic with period

25. Let $X=\{n\in N:1\le n\le 50\}$. If A = $\{n\in X:\ n$ is a multiple of $2\}$ and B = $\{n\in X:\ n$ is a multiple of $7\}$, then the number of elements in the smallest subset of X containing both A and B is_____

PART - II (JEE ADVANCED)

SECTION - III (Only one option correct type)

26. A real valued function $f(x)$ satisfies the function equation $f(x-y) = f(x)f(y) - f(a-x)f(a+y)$ where a is a given constant and $f(0) = 1$, $f(2a-x)$ is equal to

A) $f(a) + f(a-x)$ B) $f(-x)$ C) $-f(x)$ D) $f(x)$

27. The domain of the function $f(x) = \dfrac{\sin^{-1}(3-x)}{\ln(|x|-2)}$ is

A) [2, 4] B) $(2, 3) \cup (3, 4]$ C) $[2, \infty)$ D) $(-\infty, -3) \cup [2, \infty)$

28. Domain of the function $f(x) = \left[\log_{10}\left(\dfrac{5x-x^2}{4}\right)\right]^{1/2}$ is

A) $-\infty < x < \infty$ B) $1 \le x \le 4$ C) $4 \le x \le 16$ (4) $-1 \le x \le 1$

29. If $g:[-2,2] \to R$ where $g(x) = x^3 + \tan x + \left[\dfrac{x^2+1}{P}\right]$ is a odd function then the value of parametric P is

A) $-5 < P < 5$ B) $P < 5$ C) $P > 5$ D) None of these

30. Let $f:(1,3) \to R$ be a function defined by $f(x) = \dfrac{x[x]}{1+x^2}$, where $[x]$ denotes the greatest integer $\le x$. Then the range of f is

A) $\left(\dfrac{3}{5}, \dfrac{4}{5}\right)$ B) $\left(\dfrac{2}{5}, \dfrac{3}{5}\right] \cup \left(\dfrac{3}{4}, \dfrac{4}{5}\right)$ C) $\left(\dfrac{2}{5}, \dfrac{4}{5}\right]$ D) $\left(\dfrac{2}{5}, \dfrac{1}{2}\right) \cup \left(\dfrac{3}{4}, \dfrac{4}{5}\right]$

31. If $f(x) = \log_e\left(\dfrac{1-x}{1+x}\right)$, $|x|<1$, then $f\left(\dfrac{2x}{1+x^2}\right)$ is equal to:

A) $2f(x)$ B) $2f(x^2)$ C) $(f(x))^2$ D) $-2f(x)$

32. Solution of simultaneous inequations $(2\{x\}-1)(3\{x\}-2)\le 0$ and $(3[x]-4)(2[x]-8)\le 0$ (where [.] in GIF and $\{.\}$ denotes fractional part function) is $[a,b]\cup[c,d]\cup[e,f]$ then $|(a+c+e)-(b+d+f)|$

A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) $\frac{3}{2}$ D) $\frac{1}{2}$

SECTION - IV (More than one correct answer)

33. Consider the function $f(x)=|x-1|+|x-2|+|x-3|$ then

A) $f(x)=-3x+6 \text{ if } x<1$ B) $f(x)=-x+4 \text{ if}, 1\le x\le 2$

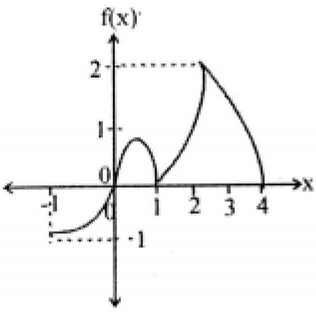
C) $f(x)=x \text{ if}, 2<x\le 3$ D) $f(x)=3x-6 \text{ if } x>3$

34. The domain and range of $f(x)=\dfrac{x^2+5x-6}{2x^2+7x-9}$ is

A) $D_f=R-\left\{\frac{-9}{2},1\right\}$ B) $D_f=R-\{0,1\}$

C) $R_f=R-\left\{\frac{1}{2},\frac{7}{11}\right\}$ D) $R_f=R-\left\{\frac{1}{2}\right\}$

35. The graph of a function f(x) which is defined in [–1,4] is shown in the adjacent figure. Identify the correct statement(s)



A) domain of $f(|x|-1)$ is [–5,5] B) range of $f(|x|+1)$ is [0,2]

C) range of $f(-|x|)$ is $[-1,0]$ D) domain of $f(|x|)$ is [–3,3]

36. Let $f(x)=\begin{cases} x^2-1, x<0 \\ 4-x^2, x\geq 0 \end{cases}$ & $g(x)=\begin{cases} x+1, x<0 \\ 2-x, x\geq 0 \end{cases}$ then g(f(x)) is

A) x^2, when $x \in (-1,0)$

B) $5-x^2$ wnen $x>2$

C) 3-x^2 when $x \leq -1$

D) x^2-2 when $x \in [0,2]$

SECTION - V (Numerical Type)

37. If $f(x)=ax^7+bx^5+cx-5$, a,b,c are real constants and f(-7)=7 then the maximum value of $|f(7)+17\cos x|$ is

38. If $f(x)=\cos(\log x)$, then the value of $f(x)\cdot f(4)-\frac{1}{2}\left[f\left(\frac{x}{4}\right)+f(4x)\right]$ is

39. If $f:R\rightarrow R, f(x)=x^3+3$ and $g:R\rightarrow R, g(x)=2x+1$ then $f^{-1}og^{-1}(23)$ equals

40. For a suitably chosen real constant 'a', let a function $f:R-\{-a\}\rightarrow R$ be defined by $f(x)=\frac{a-x}{a+x}$. Further suppose that for any real number $x\neq -a$ and $f(x)\neq -a$, $(fof)(x)=x$, then $f\left(-\frac{1}{2}\right)$ is equal to: