CHAPTER - 07

STRAIGHT LINE

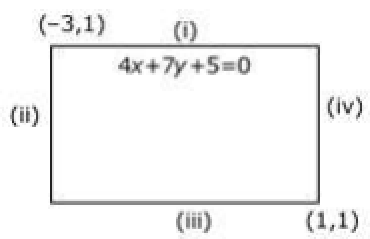
JEE MAIN - SECTION I

1. 2 Let the coordinates of axes are A (a, 0) and $B(0,b)$, but the point (–5,4) divides the line AB in the ratio of 1 : 2. Therefore, the coordinates of axes are $\left(\frac{-15}{2},0\right)$ and (0, 12).
Therefore, the equation of line passing through these coordinate axes is given by
$8x-5y+60=0$.

2. 1 The intersection point of $x-3y+1=0$ and $2x+5y-9=0$ is (2,1) and $m=\frac{1}{0}$.
So the required line is $y-1=\frac{1}{0}(x-2)\Rightarrow x=2$.

3. 2
Putting $k=1,2$, we get
$3x+2y=12$ .....(i)
$4x+3y=19$ .....(ii)
Obviously, the given lines are not parallel. Hence on solving them,
We get $x=-2$, $y=9$.
Therefore the lines pass through $(-2,9)$.

4. 1



(i) $4x+7y=4(-3)+7=-5$
(ii) $7x-4y=7(-3)-4=-25$
(iii) $4x+7y=4(1)+7(1)=11$
(iv) $7x-4y=7(1)-4(1)=3$

5. 4 Dividing both sides of relation $3a+2b+4c=0$ by 4,
We get $\frac{3}{4}a+\frac{1}{2}b+c=0$, which shows that for all values of a, b and c each member of the set of lines $ax+by+c=0$ passes through the point $\left(\frac{3}{4},\frac{1}{2}\right)$.

6. 3

$(x_1, y_1) \to \left(\frac{y_1 - 1}{x_1 - 4}\right) = -1$ and $\frac{x_1+4}{2} = \frac{y_1+1}{2}$

$\Rightarrow x_1 + y_1 = 5$ and $x_1 - y_1 = -3 \Rightarrow x_1 = 1, y_1 = 4$

2^{nd} operation $\Rightarrow (3,4)$

3^{rd} operation $\Rightarrow \left(\frac{3}{\sqrt{2}} - \frac{4}{\sqrt{2}}, \frac{3}{\sqrt{2}} + \frac{4}{\sqrt{2}}\right) = \left(\frac{-1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$.

7 1 Let the coordinates of A be (a, 0).

Then the slope of the reflected ray is $\frac{3-0}{5-a} = \tan\theta, (\text{say})$.

The slope of the incident ray $= \frac{2-0}{1-a} = \tan(\pi - \theta)$

Since $\tan\theta + \tan(\pi - \theta) = 0 \Rightarrow \frac{3}{5-a} + \frac{2}{1-a} = 0$

$\Rightarrow 13 - 5a = 0 \Rightarrow a = \frac{13}{5}$

Thus the coordinates of A are $\left(\frac{13}{5}, 0\right)$.

8. 4

$y = \cos(x+1-1)\cos(x+1+1) - \cos^2(x+1)$

$= \cos^2(x+1) - \sin^2 1 - \cos^2(x+1) = -\sin^2 1$,

which represents a straight line parallel to x-axis with

$y = -\sin^2 1$ for all x and so also for $x = \pi/2$.

9. 1 Taking co-ordinates as $\left(\frac{x}{r}, \frac{y}{r}\right)$; (x,y) and (xr, yr)

Above co-ordinates satisfy the relation $y = mx$,

So lie on a straight line.

10. 1 It is given that the lines

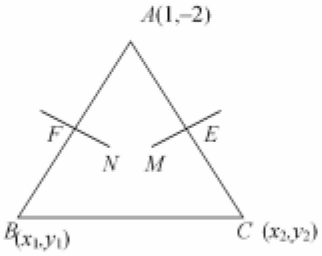
$ax + 2y + 1 = 0$, $bx + 3y + 1 = 0$ and $cx + 4y + 1 = 0$ are concurrent,

therefore $\begin{vmatrix} a & 2 & 1 \\ b & 3 & 1 \\ c & 4 & 1 \end{vmatrix} = 0$

$\Rightarrow -a + 2b - c = 0 \Rightarrow 2b = a + c$

$\Rightarrow a, b, c$ are in A. P.

11. 4 Let the equation of perpendicular bisector FN of AB is



$x - y + 5 = 0$ ......(i)

The middle point F of AB is

$\left(\frac{x_1+1}{2}, \frac{y_1-2}{2}\right)$ lies on line (i).

Therefore $x_1 - y_1 = -13$ .....(ii)

Also AB is perpendicular to FN. So the product of their slopes is –1.

i.e. $\frac{y_1+2}{x_1-1} \times 1 = -1$ or $x_1 + y_1 = -1$ ......(iii)

On solving (ii) and (iii), we get $B(-7, 6)$.

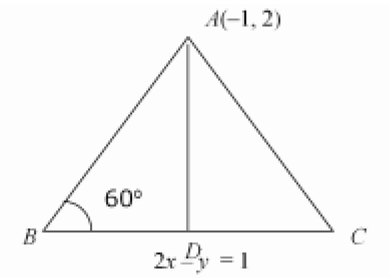
Similarly $C\left(\frac{11}{5}, \frac{2}{5}\right)$.

Hence the equation of BC is $14x + 23y - 40 = 0$.

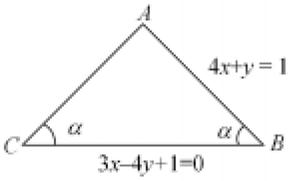
12. 1 $$AD = \left| \frac{-2-2-1}{\sqrt{(2)^2 + (-1)^2}} \right| = \left| \frac{-5}{\sqrt{5}} \right| = \sqrt{5}$$

$$\because \tan 60^o = \frac{AD}{BD} \Rightarrow \sqrt{3} = \frac{\sqrt{5}}{BD} \Rightarrow BD = \sqrt{\frac{5}{3}}$$

$$\therefore BC = 2BD = 2\sqrt{\frac{5}{3}} = \sqrt{\frac{20}{3}}.$$



13. 1 Slopes of AB and BC are – 4 and $\frac{3}{4}$ respectively.



If α be the angle between AB and BC, then

$$\tan\alpha = \frac{-4-\frac{3}{4}}{1-4\left(\frac{3}{4}\right)} = \frac{19}{8} \qquad .....(i)$$

Since $AB = AC$

$\Rightarrow \angle ABC = \angle ACB = \alpha$

Thus the line AC also makes an angle α with BC. If m be the slope of the line AC, then its equation is $y+7 = m(x-2)$ .....(ii)

Now $\tan\alpha = \pm\left[\frac{m-\frac{3}{4}}{1+m.\frac{3}{4}}\right] \Rightarrow \frac{19}{8} = \pm\frac{4m-3}{4+3m}$

$\Rightarrow m = -4$ or $-\frac{52}{89}$.

But slope of AB is – 4, so slope of AC is $-\frac{52}{89}$.

Therefore the equation of line AC given by (ii) is

$52x + 89y + 519 = 0$.

14. 4

The given lines are $\pm x \pm y = 1$

i.e. $x+y=1, x-y=1, x+y=-1$ and $x-y=-1$

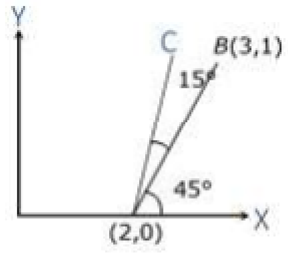
These lines form a quadrilateral whose vertices are $A(-1,0), B(0,-1), C(1,0)$ and $D(0,1)$

Obviously ABCD is a square.

Length of each side of this square is $\sqrt{1^2+1^2} = \sqrt{2}$

Hence area of square is $\sqrt{2}\times\sqrt{2} = 2sq.$ units

15. 1 Here slope of $AB = \frac{1}{1} \Rightarrow \tan\theta = m_1 = 1$ or $\theta = 45^o$.



Thus slope of new line is $\tan(45^o + 15^o) = \tan 60^o = \sqrt{3}$

{$\because$ It is rotated anticlockwise so the angle will be $45^o + 15^o = 60^o$}

Hence the equation is $y = \sqrt{3}x + c$, but it still passes through (2,0), hence $c = -2\sqrt{3}$.

Thus required equation is $y = \sqrt{3}x - 2\sqrt{3}$.

16. 2 Centorid of $\Delta = (2,2)$ line passing through intersection of $x + 3y - 1 = 0$ and $3x - y + 1 = 0$, be given by $(x + 3y - 1) + \lambda(3x - y + 1) = 0$

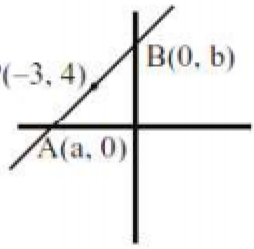
$\because$ It passes through (2, 2)

$\Rightarrow 7 + 5\lambda = 0 \Rightarrow \lambda = -\dfrac{7}{5}$

$\therefore$ Required line is $8x - 11y + 6 = 0$

$\because$ $(-9, -6)$ satisfies this equation.

17. 4

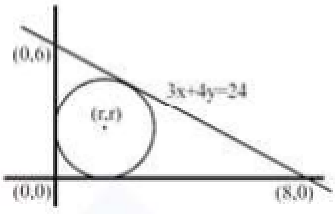


Let the line be $\dfrac{x}{a} + \dfrac{y}{b} = 1$

$(-3, 4) = \left(\dfrac{a}{2}, \dfrac{b}{2}\right)$

$a = -6, b = 8$ equation of line is $4x - 3y + 24 = 0$

18. 2 $\left|\dfrac{3r + 4r - 24}{5}\right| = r \Rightarrow 7r - 24 = \pm 5r$



$2r = 24$ or $12r = 24$

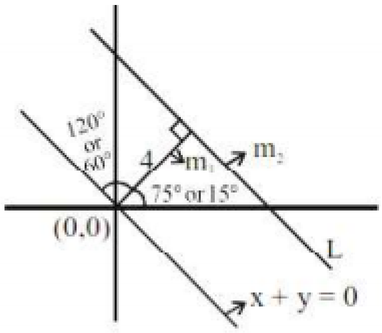
r= 2 or 12

19. 1

$$m_1 = \tan 75^0 = \frac{\sqrt{3}+1}{\sqrt{3}-1} \text{ or } m = \tan 15^0 = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$m_2 = \frac{-1}{m_1} = \frac{-(\sqrt{3}-1)}{\sqrt{3}+1} \text{ or } m_2 = \frac{-1}{m_1} = \frac{-(\sqrt{3}+1)}{\sqrt{3}-1}$$

$$\Rightarrow y = m_2 x + C \Rightarrow y = \frac{-(\sqrt{3}-1)x}{\sqrt{3}+1} + C \Rightarrow \text{L}$$



or

$$y = \frac{-(\sqrt{3}+1)x}{\sqrt{3}-1} + C \Rightarrow \text{L}$$

Distance from origin = 4

$$\therefore \left| \frac{C}{\sqrt{1+\frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)^2}}} \right| = 4 \text{ or } \left| \frac{C}{\sqrt{1+\frac{(\sqrt{3}+1)^2}{(\sqrt{3}-1)^2}}} \right| = 4$$

$$\Rightarrow C = \frac{8\sqrt{2}}{(\sqrt{3}+1)} \text{ or } C = \frac{8\sqrt{2}}{(\sqrt{3}-1)}$$

$$\Rightarrow (\sqrt{3}-1)y + (\sqrt{3}+1)x = 8\sqrt{2} \text{ or } (\sqrt{3}-1)x + (\sqrt{3}+1)y = 8\sqrt{2}$$

20. 1 Image of P(3, 5) on the line $x - y + 1 = 0$ is $\frac{x-3}{1} = \frac{y-5}{-1} = \frac{-2(3-5+1)}{2} = 1$

x = 4, y = 4

$\therefore$ Image is (4, 4)

Which lies on $(x-2)^2+(y-4)^2=4$.

SECTION II (NUMERICAL)

21. 2 Solving $3x+4y=9, y=mx+1$ we get $x=\frac{5}{3+4m}$

x is an integer if $3+4m=1,-1,5,-5$

$\therefore m=\frac{-2}{4},\frac{-4}{4},\frac{2}{4},\frac{-8}{4}$.

So, m has two integral values.

22. 5 $\frac{17-\beta}{-8}\times\frac{2}{3}=-1,\ \beta=5$

23. 4 Slope = $m=\frac{1}{1-k}$

Equation of perpendicular bisector is $y+4=(k-1)(x-0)$

$\Rightarrow y+4=x(k-1) \Rightarrow \frac{7}{2}+4=\frac{k+1}{2}(k-1)$

24. 6 Let the points be A(1, 1) and B(2, 4)

Let point C divides line AB in the ratio 3 : 2.

So, by section formula we have

$$C=\left(\frac{3\times2+2\times1}{3+2},\frac{3\times4+2\times1}{3+2}\right)=\left(\frac{8}{5},\frac{14}{5}\right)$$

Since line 2x + y = k passes through $C\left(\frac{8}{5},\frac{14}{5}\right)$

$\Rightarrow \frac{2\times8}{5}+\frac{14}{5}=k \Rightarrow$ k = 6.

25. 18 The equation of the line L, be $y-2=m(x-8), m<0$

Coordinates of P and Q are $P\left(8-\frac{2}{m},0\right)$ and $Q(0,2-8m)$

So, $OP+PQ=8-\frac{2}{m}+2-8m$

$= 10 + \frac{2}{(-m)} + 8(-m) \geq 10 + 2\sqrt{\frac{2}{(-m)} \times 8(-m)} \geq 18$

So, absolute minimum value of $OP + OQ = 18$.

JEE ADVANCED LEVEL

SECTION III

26. D $\quad a - 2\sqrt{bc} = b + c$

$\Rightarrow \left(\sqrt{b}+\sqrt{c}\right)^2 - \left(\sqrt{a}\right)^2 = 0$

or $\sqrt{b}+\sqrt{c}-\sqrt{a} = 0 \quad \left(\because \sqrt{b}+\sqrt{c}+\sqrt{a} \neq 0\right)$

$\therefore \sqrt{a}x + \sqrt{b}y + \sqrt{c}$ passes through the fixed point (-1, 1)

27. A $\quad$ Slope of CD is $\frac{1}{2} \Rightarrow C \equiv (-5,-1)$

Perpendicular distance from G to AB $= \frac{1}{3}$ (perpendicular distance from C to AB).

28. D $\quad$ P lies in the actue angle

$\Rightarrow \alpha^2 - 3\alpha > 0$ and $\alpha^2 - 5\alpha < 0$

$\Rightarrow \alpha \in (-\infty, 0) \cup (3, \infty)$ and $\alpha \in (0,5)$

$\therefore \alpha \in (3,5)$

29. B $\quad$ The equation of the line in the initial system is $\frac{x}{a} + \frac{y}{b} = 1.$

The equation of the same line after rotation of axes is $\frac{x}{p} + \frac{x}{q} = 1$

Since the origin remains the same, the perpendicular distance of the line from origin must be unchanged. So $\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}} = \frac{1}{\sqrt{\frac{1}{p^2}+\frac{1}{q^2}}} \Rightarrow \quad \frac{1}{a^2}+\frac{1}{b^2} = \frac{1}{p^2}+\frac{1}{q^2}$

30. A $\quad$ **Let the line be** $\frac{x-3}{\cos\theta} = \frac{y-4}{\sin\theta} = r$

Any point on this line be $(3 + r\cos\theta, 4 + r\sin\theta)$

Substituting this in y=8, we get PL$= \frac{4}{\sin\theta}$

Substituting this in x=6 we get PM$= \frac{3}{\cos\theta}$

$\frac{1}{PQ} = \frac{\sin\theta}{4} + \frac{\cos\theta}{3} \Rightarrow 12 = 3r\sin\theta + 4r\cos\theta \; = 3(y-4) + 4(x-3)$

$\therefore Locus$ is 4x+3y-36=0

SECTION IV (More than one correct)

31. B,C

For the two lines $24x + 7y - 20 = 0$ and $4x - 3y - 2 = 0$, the angle bisectors are given by

$$\frac{24x + 7y - 20}{25} = \pm \frac{4x - 3y - 2}{5}$$

Taking positive sign, we get

$$2x + 11y - 5 = 0$$

32. A,B,C

Given $A(-5, -2)$, $B(7, 6)$ and $C(5, -4)$

Slope of AB is $m_1 = \frac{8}{12} = \frac{2}{3}$; Slope of $BC = \frac{10}{2} = 5$; tan $B = \frac{5 - 2/3}{1 + 10/3} = \frac{13}{13}$

$\Rightarrow \angle B = 45°$

Equation of altitude through $C(5, -4)$ is $y + 4 = \left(\frac{-3}{2}\right)(x - 5)$

or $3x + 2y - 7 = 0$

33. A,B,C

From the given $A(2, 0)$, $B(0, -2)$

$\Rightarrow$ $AB = 2\sqrt{2}$

Distance of AB from the origin = $\sqrt{2}$

Maximum distance = $3\sqrt{2}$ units

Area of square = 8 square units.

Side through $A(2, 0)$ is $x + y = 2$

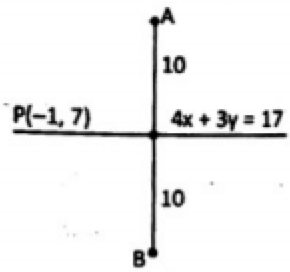
34. A,B

K must be the angular bisector

$$\frac{3x + 4y - 1}{5} = \pm \frac{5x - 12y + 2}{13}$$

$$\left.\begin{array}{l} 14x + 112y - 23 = 0 \\ 64x - 8y - 3 = 0 \end{array}\right\}$$

35. A



There are two points A & B which are at a distance of 10 units from P and farthest (10 unit distance) from the line $4x + 3y = 17$.

The slope of $AB = \frac{3}{4}$ {Because the slope of $4x + 3y = 17$ is $-\frac{4}{3}$ }

Let line AB makes an angle θ with x-axis then $\tan\theta = \frac{3}{4} \Rightarrow (\cos\theta, \sin\theta) = \left(\frac{4}{5}, \frac{3}{5}\right)$

$\Rightarrow A$ or $B = (10\cos\theta - 1, 10\sin\theta + 7)$ or $\Rightarrow A$ or $B = (-10\cos\theta - 1, -10\sin\theta + 7)$

$\Rightarrow A$ or $B = (7,13)$ or $(-9,1)$

36. A,B

37. B,C,D

38. B,C   A(1,1), B(k,k), C (2-k, k); Area $= 4n^2 \Rightarrow (k-1)^2 = 4h^2$; $k-1 = \pm 2h$

Locus : $2x + y - 1 = 0$ & $2x - y + 1 = 0$

SECTION V

39. A

SECTION VI - (Matrix match type)

40. A

A) $a + c = 2b \Rightarrow a + c - 2b = 0$

$\Rightarrow ax + by + c = 0$ Satisfies (1, -2)

B) Perpendicular distance of $P(\lambda, 4-\lambda)$ from $4x + 3y = 10$

$$\frac{|4\lambda + 3(4-\lambda) - 10|}{5} = 1$$

$\Rightarrow |\lambda + 2| = 5 \Rightarrow \lambda - 2 \pm 5 = 3, -7$

$P(3,1), (-7, +11)$

C) Point $B = (-1, 2)$

D) $\frac{y}{x} = \frac{7}{4}$

$\frac{4+1}{y-5} = \frac{2}{3}, \frac{21}{4}x = 2x - 13$; $x = -4, y = -7$