CHAPTER - 11

MATRICES AND DETERMINANTS

A rectangular array of mn numbers in the form of horizontal lines (rows) and n vertical lines (columns) is called a matrix of order mby n $m \times n$ such an array in enclosed by [ ] or ( ) or || || or { }. An $m \times n$ matrix is usually written as $A = \begin{bmatrix} a_{11} & a_{12} & a_{1n} \\ a_{21} & a_{22} & a_{2n} \\ a_{m_1} & a_{m_2} & a_{mn} \end{bmatrix}$ or $A = \left[a_{ij}\right]_{m\times n}$. A matrix $A = \left[a_{ij}\right]_{m\times n}$ over the field of complex numbers is said to be

1) a rectangular matrix if $m \neq n$

2) a square matrix if m = n

3) a row matrix if m = 1

4) a column matrix if n = 1

5) a null (zero) matrix if $a_{ij} = 0$, for all i and j

6) a diagonal matrix if $a_{ij} = 0$ for $i \neq j$, m = n

7) a scalar matrix if m = n, $a_{ij} = 0$ for all $i \neq j$ and $a_{11} = a_{22} = a_{33} = \ldots\ldots\ldots = a_{nn}$

8) Unit (identity) matrix if m = n, $a_{ij} = 0$ for all $i \neq j$ and $a_{ii} = 1$

9) Comparable matrix means same order

10) Equal matrices $\Rightarrow$ same order and all the corresponding elements are equal

Addition: Let A and B be two matrices of same order then A + B is defined $A + B = [a_{ij} + b_{ij}]_{m \times n}$ where $A = \left[a_{ij}\right]_{m\times n}$, $B = \left[b_{ij}\right]_{m\times n}$

Scalar multiplication

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $KA = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$

Properties of addition

1) $A+B=B+A$ (commutative)

2) $(A+B)+C=A+(B+C)$ (Associative)

3) $A+0=0+A=0$ (Zero matrix is the additive identity)

Subtraction of matrices

A - B = A+(-B)

Multiplication of matrices

Let A and B be two matrices such that the number of columns of A is same as the number of rows of B ie,

$A=[aij]_{m\times n}$, $B=[bij]_{n\times p}$. Then $[AB]=[Cij]_{m\times p}$, where $Cij=\sum_{k=1}^{n} aik\, bkj$

Properties

$$(AB)C=A(BC)$$

$$A(B+C)=AB+AC$$

$$(A+B)C=AC+BC$$

$$(A+B)^2=A^2+AB+BA+B^2$$

$$(A-B)^2=A^2-AB-BA+B^2$$

$$(A+B)(A-B)=A^2-AB+BA-B^2$$

AI = IA=A where I is the identity matrix A is square matrix.

$$A^2=A\times A, A^3=A^2.A$$

If $A(\theta)=\begin{bmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{bmatrix}$, then $A^n(\theta)=\begin{bmatrix}\cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta\end{bmatrix}$ and $A(\theta)\times A(\phi)=A(\theta+\phi)$

If $A(\alpha)=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1\end{bmatrix}$ then $A(\alpha)\times A(\beta)=A(\alpha+\beta)$.

Idempotent matrix (A): If $A^2 = A$ where A is a square matrix.

Involuntory matrix if $A^2 = I$

Nilpotent matrix : $A^m = 0$, m is called the index of the nilpotent matrix

If AB= A and BA= B then both A and B are idempotent.

If $A=\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix}$ then $A^n=2^{n-1}A$

If $A=\begin{bmatrix}1 & k\\0 & 1\end{bmatrix}$, then $A^n=\begin{bmatrix}1 & kn\\0 & 1\end{bmatrix}$

Properties is transpose

1) $A^{T^T}=A$. Let $A=\begin{bmatrix}a & b\\c & d\end{bmatrix}$, $A^T=A'=\begin{bmatrix}a & c\\b & d\end{bmatrix}$

2) $(A\pm B)^T=A^T\pm B^T$

3) $(KA)^T=KA^T$

4) $(AB)^T=B^TA^T$

Orthogonal matrix A.

If $A.A^T=A^T.A=I$, then A is orthogonal , example, $\begin{bmatrix}\cos\theta & \sin\theta\\-\sin\theta & \cos\theta\end{bmatrix}$, $\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$

Symmetric matrix A: if $A^T=A\Rightarrow aij=aji$

skew symmetric A : if $A^T=-A\Rightarrow aij=-aji$

If A is symmetric then $A+A^T$ is symmetrix, A^n, A^T, A and **AA**^{T} are also symmetric . $A-A^T$ is skew symmetric.

If A and B are symmetric matrixes of same order than AB + BA is symmetric and AB-BA is skew symmetric. If A is skew symmetric matrix than A^n is skew symmetrix when n is odd and symmetric when n is even.

Determinant A $=|A|=\det A=\begin{vmatrix}a & b\\c & d\end{vmatrix}=ad-bc$

Determinant of a matrix otherthan squarematrix does not exist

Properties of Determinant

1) $|A|=|A'|$, $\begin{vmatrix}a_1 & b_1 & c_1\\a_2 & b_2 & c_2\\a_3 & b_3 & c_3\end{vmatrix}=\begin{vmatrix}a_1 & a_2 & a_3\\b_1 & b_2 & b_3\\c_1 & c_2 & c_3\end{vmatrix}$

2) $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = -\begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$

3) $\begin{vmatrix} \lambda a_1 & \lambda a_2 & \lambda a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \lambda \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

4) $|KA| = K^n |A|$ where n is the order of **A**

5) A skew symmetric matrix of odd order has determinant value zero and that even odder is a perfect square

6) $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ ka_1 & kb_1 & kc_1 \end{vmatrix} = 0 \because R_1 \propto R_3$

7) $\begin{vmatrix} a_1+b_1 & a_2+b_2 & a_3+b_3 \\ c_1 & c_2 & c_3 \\ d_1 & d_2 & d_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \\ d_1 & d_2 & d_3 \end{vmatrix} + \left[\begin{matrix} b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \\ d_1 & d_2 & d_3 \end{matrix}\right)$

8) $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1+\lambda a_2 & b_1+\lambda b_2 & c_1+\lambda c_2 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} R_1 \to R_1 + \lambda R_2$

9) $|AB| = |A||B|$ where A and B are square matrices of the same order.

10) $\frac{d}{dx}\begin{vmatrix} f(x) & g(x) \\ h(x) & f(x) \end{vmatrix} = \begin{vmatrix} f'(x) & g'(x) \\ h(x) & \phi(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) \\ h'(x) & \phi'(x) \end{vmatrix}$

11) $\sum_{r=1}^{n} \Delta(r) = \begin{vmatrix} \sum_{r=1}^{n} f(r) & \sum_{r=1}^{n} g(r) & \sum_{r=1}^{n} h(r) \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\Delta(r) = \begin{vmatrix} f(r) & g(r) & h(r) \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

Minors, Cofactors and adjoint of a square matrix

Minor of a_{11} in $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ is $= \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = M_{11}$

cofactor of aij = $(-1)^{i+j} m^{ij}$ = Aij or Cij

Adjoint is the transpose of cofactor matrix , $\text{adj}A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$

$A(\text{adj}A) = |A|I = (\text{adj}A)A$

$\text{adj}(AB) = (\text{adj}B)(\text{adj}A)$

$\text{adj}(\text{adj}A) = |A|^{n-2}A$

$|\text{adj adj } A| = \left||A|^{n-2}A\right| = |A|^{(n-1)^2}$

Inverse of A = $A^{-1} = \dfrac{\text{adj}A}{|A|} \because |A| \neq 0$

$(A^{-1})^{-1} = A$

$(AB)^{-1} = B^{-1}A^{-1}$

$\text{adj}(A^{-1}) = (\text{adj}A)^{-1}$

If A is an orthogonal matrix and B is any square matrix of the same order of A then $(ABA^T)^n = AB^nA^T$

and $(ABA^{-1})^n = AB^nA^{-1}$

$|\text{adj } A| = |A|^{n-1}$ where n is the order of A .

$|A^{-1}| = \dfrac{1}{|A|}, (kA)^{-1} = \dfrac{1}{k}A^{-1}$

Solution of linear equations by matrix method

Let $a_1x_1 + b_1y + c_1z = d_1$, $a_2x + b_2y + c_2z = d_2$ and $a_3x + b_3y + c_3z = d_3$

Let $|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, $B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$

$AX = B$ where $B = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Test for consistency

Find $|A|$

- If $|A| \neq 0$ consistent and unique solution $X = A^{-1}B$
- $|A| = 0$
  - $(\text{adj } A)\, B = 0$ consistent and infinite solutions
  - $(\text{adj } A)\, B \neq 0$ inconsistent, No solution

Cramer's Rule (Solution of Linear equation by determinant)

Let $a_1x + b_1y + c_1z = d_1$, $a_2x + b_2y + c_2z = d_2$ and $a_3x + b_3y + c_3z = d_3$ and

$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ $D_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$; $D_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$ and $D_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$

Test for consistency by Cramer's Rule (Non Homogeneous)

Find D

- $D \neq 0$ The system is consistent and unique solution
- $D = 0$
  - All $D_i = 0$ consistant and infinite number of solutions
  - At least one $D_i = 0$, then the system is inconsistent (no solution)

Homogeneous ($d_1 = d_2 = d_3 = 0$)

If $D \neq 0$ then the system is consistent and Trivial solution only. If D = 0, then the system is consistent and infinite number of solutions. Singular matrix A : if $|A| = 0$; for non singular $|A| \neq 0$. Minor of a_{11}

Factor theorem

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$

when a = b R_1 and R_2 are equal .

$\therefore$ a- b is a factor LHS degree 3 = RHS degree 3

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

LHS degree 4 = RHS degree 4.

Special Determinants

(i). $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$ = (a-b) (b-c) (c-a)

(ii) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$ = (a-b) (b-c) (c-a) (a + b + c)

(iii) $\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$ = (a-b) (b-c) (c-a) (ab+bc+ca)

(iv). $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$ = abc $\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

(v) $\begin{vmatrix} 1^2 & 2^2 & 3^2 & 4^2 \\ 2^2 & 3^2 & 4^2 & 5^2 \\ 3^2 & 4^2 & 5^2 & 6^2 \\ 4^2 & 5^2 & 6^2 & 7^2 \end{vmatrix} = 0$

(vi). $\begin{vmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{vmatrix} = -8$

(vii). $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a^3+b^3+c^3-3abc)$

$= -(a+b+c)(a^2+b^2+c^2-ab-bc-ac)$

$= \frac{-1}{2}(a+b+c)\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$

$= -(a+b+c)(a+bw+cw^2)(a+bw^2+cw)$

(viii) $\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = 4a^2b^2c^2$

(ix) $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$

(x) $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$

(xi) $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

(xii) $\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

PART I - (JEEMAIN)

SECTION - I - Straight objective type questions

1. If $\begin{bmatrix} \lambda^2-2\lambda+1 & \lambda-2 \\ 1-\lambda^2+3\lambda & 1-\lambda^2 \end{bmatrix} = A\lambda^2+B\lambda+C,$ where A, B, C are matrices then B+ C=

1) $\begin{bmatrix} -1 & -1 \\ 4 & 1 \end{bmatrix}$ 2) $\begin{bmatrix} 1 & -1 \\ 4 & 1 \end{bmatrix}$ 3) $\begin{bmatrix} 1 & 1 \\ -4 & 1 \end{bmatrix}$ 4) $\begin{bmatrix} -1 & -1 \\ -4 & 1 \end{bmatrix}$

2. For the matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, then values of ‘a’ and ‘b’ such that $A^2+aA+bI=O$ are

1) 2,3 2) 1,4 3) –4,1 4) –2,3

3. If $A = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$ then $A^3-35A=$

1) A 2) 2A 3) 3A 4) 4A

4. If $A = \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix}$ and $f(t)=t^2-3t+7$ then $f(A)+\begin{bmatrix} 3 & 6 \\ -12 & -9 \end{bmatrix}$ is equal to

1) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ 2) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ 3) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ 4) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

5. If $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$ and $A^n = O$, then the minimum value of n is

1) 2 2) 3 3) 4 4) 5

6. Matrix A is such that $A^2 = 2A - I$, where I is the unit matrix. Then for $n \geq 2, A^n =$

1) nA-(n-1) I 2) nA– I 3) $2^{n+1}A-(n-1)I$ 4) $2^{n+1}A-I$

7. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}, a, b \in N$ then

1) there exists exactly one B such that AB = BA
2) there exists infinitely many B's such that AB = BA
3) there cannot exist may B such that AB = BA
4) there exists more than one but finite number of B's such that AB = BA

8. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}$ and I be the identity matrix of order 3. If $Q = \left[q_{ij}\right]$ is a matrix such that $P^{50} - Q = I$,

then $\dfrac{q_{31}+q_{32}}{q_{21}}$ equals

1) 52 2) 103 3) 201 4) 205

9. The values of λ and μ for which the system of linear equations

$x+y+z=2$

$x+2y+3z=5$

$x+3y+\lambda z=\mu$

has infinitely many solutions are, respectively

(1) 5 and 7 (2) 6 and 8 (3) 4 and 9 (4) 5 and 8

10. If the system of equations $x-ky-z=0$, $kx-y-z=0$, $x+y-z=0$, has a non-zero solution, then the possible values of k are

(1) – 1, 2 (2) 1, 2 (3) 0, 1 (4) – 1, 1

Assertion & Reasoning

1) If both Statement-I and Statement II are true and the reason is the correct explanaiton of the statement I
2) If both Statement -I and Statement -II are true but reason is not the correct explanaiton of the statement -I
3) If Statement-I is true but Statement -II is false
4) If Statement-I is fals ebut Statement-II is true

11. Statement - I: A is singular matrix of order n×n then adj A is singular

Statement - II: $|\text{adj}\,A| = |A|^{n-1}$

12. Statement-I : If $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$ then $A^3 + A^2 + A = I$

Statement -II: If $\det(A - \lambda I) = C_0\lambda^3 + C_1\lambda^2 + C_2\lambda + C_3 = 0$ then $C_0A^3 + C_1A^2 + C_2A + C_3I = O$

13. Matrix A is given by $A = \begin{bmatrix} 6 & 11 \\ 2 & 4 \end{bmatrix}$ then the determinant of $A^{2015} - 6A^{2014}$ is

1) 2^{2016} 2) $(-11).2^{2015}$ 3) $-2^{2015} \times 7$ 4) $(-9)2^{2014}$

14. The determinant $\begin{vmatrix} xp+y & x & y \\ yp+z & y & z \\ 0 & xp+y & yp+z \end{vmatrix} = 0$ if

1) x,y,z are in A.P. 2) x,y,z are in G.P 3) x,y,z are in H.P 4) xy, yz, zx are in A.P

15. If $A = \begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{bmatrix}$, then $\left[A(\text{adj}\,A)A^{-1}\right]A =$

1) $2\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ 2) $\begin{bmatrix} -6 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & -6 \end{bmatrix}$ 3) $\begin{bmatrix} 0 & 1/6 & -1/6 \\ 1/3 & 1/6 & 1/2 \\ 1/2 & 1/3 & -61/6 \end{bmatrix}$ 4) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

16. If $a_1, a_2, a_3, \ldots a_n, \ldots$ are in G.P, then the value of the determinant $\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$ is

1) 0 2) –2 3) 2 4) 1

17. The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ is

1) 0 2) 2 3) 1 4) 3

18. Let $A=\begin{bmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{bmatrix}$, where b>0. Then the minimum value of $\frac{\det(A)}{b}$ is

1) $\sqrt{3}$ 2) $-\sqrt{3}$ 3) $-2\sqrt{3}$ 4) $2\sqrt{3}$

19. If $a^2+b^2+c^2=-2$ and $f(x)=\begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2x) & (1+b^2)x & 1+c^2x \end{vmatrix}$, then f(x) is a polynomial of degree

1) 1 2) 0 3) 3 4) 2

20. Statement-I : The value of determinant $\begin{vmatrix} \sin\pi & \cos\left(x+\frac{\pi}{4}\right) & \tan\left(x-\frac{\pi}{4}\right) \\ \sin\left(x-\frac{\pi}{4}\right) & -\cos\left(\frac{\pi}{2}\right) & \log\left(\frac{x}{y}\right) \\ \cot\left(\frac{\pi}{4}+x\right) & \log\left(\frac{y}{x}\right) & \tan\pi \end{vmatrix}$ is zero

Statement II: The value of skew-symmetric determinant of odd order equals to zero

1) If both Statement -I and statement II are true and the reason is the correct explanaiton of the statement I

2) If both statement-I and statement II are true but reason is not the correct explanaiton of the statement I

3) If statement-I is true but statement-II is false

4) If statement -I is false but statement-II is true

SECTION - II

Numerical Type Questions

21. Let $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B=\begin{bmatrix} \alpha \\ \beta \end{bmatrix}\neq\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ such that AB = B and a+d= 2021, then the value of ad-bc is

22. Let I be an identity matrix of order 2×2 and $P=\begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$. Then the value of $n\in N$ for which $P^n=5I-8P$ is

23. If $\Delta=\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix}=Ax^3+Bx^2+Cx+D$, then $B\times C$ is

24. The total number of distinct $x\in R$ for which $\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix}=10$ is

25. $A=\begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$ and f(x) is defined as $f(x)=\det.(A^TA^{-1})$ then the value of $\underbrace{f(f(f(f.....f(x))))}_{n\,\text{times}}$ is $(n\geq 2)$

PART - II (JEE ADVANCED)

SECTION - III (Only one option correct type)

26. If $P=\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A=\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $Q=PAP^T$ and if $P^TQ^{2012}P=\begin{bmatrix} \alpha & \gamma \\ \beta & \alpha \end{bmatrix}$,then the value of $(\alpha+\beta)$ is

A) 1 B) -1 C) 2 D) -2

27. Let $A=\begin{bmatrix} 0 & \alpha \\ 0 & 0 \end{bmatrix}$ and $(A+I)^{50}-50A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Then the value of $a+b+c+d$ is

(a) 2 (b) 1 (c) 4 (d) 3

28. If $D_k=\begin{vmatrix} 3^k & \frac{1}{(k+1)(k+2)} & \cos(k+1)\theta \\ \frac{3^n-1}{2} & \frac{n}{n+1} & \frac{\sin\frac{n\theta}{2}\cos\frac{(n-1)\theta}{2}}{\sin\frac{\theta}{2}} \\ a & b & c \end{vmatrix}$ then $\sum_{k=0}^{n-1} D_k$ is

A) independent of n

B) independent of a,b,c

C) a + b + c

D) a + 2b + c

29. If x, y, z are all positive and are the p^{th}, q^{th} and r^{th} terms of a geometric progression respectively, then the value of the determinant $\begin{vmatrix} \log x & p & 1 \\ \log y & q & 1 \\ \log z & r & 1 \end{vmatrix} =$

A) log xyz B) (p-1)(q-1)(r-1) C) pqr D) 0

30. If $c<1$ and the system of equations $x+y-1=0$, $2x-y-c=0$ and $-bx+3by-c=0$ is consistent, then the possible real values of b are

(a) $b \in \left(-3, \frac{3}{4}\right)$ (b) $b \in \left(\frac{-3}{2}, 4\right)$ (c) $b \in \left(\frac{-3}{4}, 3\right)$ (d) $b \in \left(\frac{-3}{2}, \frac{3}{4}\right)$

SECTION - IV (More than one correct answer)

31. If the adjoint of a 3×3 matrix P is $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$ then the possible values of the determinant of P is (are)

A) -2 B) -1 C) 1 D) 2

32. If $A=\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ and $A^{2012}=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then which of the following is(are) correct

A) a = d B) a + b + c + d = 4026

C) $a^2+b^2+d^2=2$ D) b = 2012

33. If A is 3×3 matrix whose $(i, j)^{th}$ element is given by $a_{ij}=\begin{cases} 2 & if\ i=j \\ -1 & if\ |i-j|=1 \\ 0 & else\ where \end{cases}$ then

A) A is symmetric B) Trace A = 6

C) det A is a perfect square D) A^{-1} is skew symmetric

34. If maximum and minimum values of $\begin{vmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ \sin^2 x & 1+\cos^2 x & \sin 2x \\ \sin^2 x & \cos^2 x & 1+\sin 2x \end{vmatrix}$ are α, β then which of the following is/are true?

A) $\alpha+\beta^{99}=4$ B) $\alpha^3-\beta^{17}=26$

C) $\alpha^{2n}-\beta^{2n}$ is always an even integer for $n \in N$

D) A triangle can be constructed having its sides as $\alpha, \beta, \alpha-\beta$

35. Let α, β be the roots of $ax^2+bx+c=0$ and let $S_n = \alpha^n + \beta^n$, for $n \geq 1$ and

$$\Delta = \begin{vmatrix} 3 & 1+S_1 & 1+S_2 \\ 1+S_1 & 1+S_2 & 1+S_3 \\ 1+S_2 & 1+S_3 & 1+S_4 \end{vmatrix} \ \& \ \Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix}$$ Then $\frac{\Delta}{\Delta_1^2}$ is equal to

A) 1 B) 2 C) 0 D) 3

SECTION - V (Numerical Type)

36. If the system of linear equations

$x+y+z=5$

$x+2y+2z=6$

$x+3y+\lambda z=\mu$ $(\lambda, \mu \in R)$, has infinitely many solution, then the value of $\lambda \times \mu$ is________

37. If $D = \begin{vmatrix} 10! & 11! & 12! \\ 11! & 12! & 13! \\ 12! & 13! & 14! \end{vmatrix}$ then k/3 , where k is the total number of divisors of $\frac{D}{(10!)^3} - 4$,is

38. Let k be a positive real number and let $A = \begin{vmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{vmatrix}$ and $B = \begin{vmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{vmatrix}$.

If det (adj A) + det (adj B) = 10^6 then [k] is equal to [Note : adj M denotes the adjoint of a square matrix M and [k] denotes the largest integer less than or equal to k]

39. If the system of equations $\lambda x+y+z=0$, $-x+\lambda y+z=0, -x-y+\lambda z=0$ will have a non-zero solution then the real values of λ is

SECTION VI - (Matrix match type)

40. If $\begin{bmatrix} 4a^2 & 4a & 1 \\ 4b^2 & 4b & 1 \\ 4c^2 & 4c & 1 \end{bmatrix}\begin{bmatrix} f(-1) \\ f(1) \\ f(2) \end{bmatrix} = \begin{bmatrix} 3a^2+3a \\ 3b^2+3b \\ 3c^2+3c \end{bmatrix}$ where f(x) is a quadratic function and $f(x)=ax^2+bx+c$ whose maximum value occurs at a point V say (α,β). Let A be the point of intersection of $y=f(x)$ with negative x-axis, say (p,o) and point B is such that the chord AB subtends a right angle at V. Let B be (r,s). Let Δ be the area enclosed by $y=f(x)$ and the chord AB. Then

Column-I	**Column-II**
A) $\alpha+\beta=$	P) 125/3
B) $p=$	Q) -7
C) $r+s=$	R) -2
D) $\Delta=$	S) 1

A) $A\rightarrow S; B\rightarrow R; C\rightarrow Q; D\rightarrow P$

B) $A\rightarrow Q; B\rightarrow R; C\rightarrow Q; D\rightarrow P$

C) $A\rightarrow S; B\rightarrow R; C\rightarrow Q; D\rightarrow Q$

D) $A\rightarrow S; B\rightarrow P; C\rightarrow Q; D\rightarrow P$