CHAPTER - 00
MAGNETISM AND MATTER

Magnetic field line

It is an imaginary curve, the tangent to which at any point give us the direction of magnetic field at that point.

Properties

✶ The magnetic field lines of a magnet (or a solenoid) from a continuous closed loops.

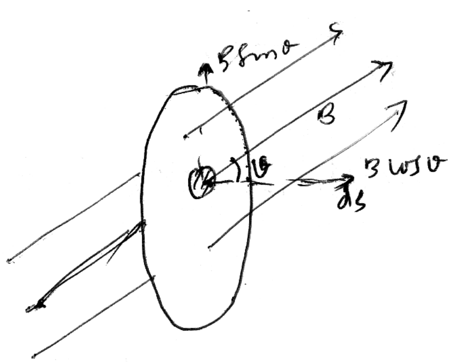
✶ The tangent to the field line at a given point represents the direction of the net magnetic field at that point.

✶ The larger the number of field lines crossing per unit normal area, the larger is the magnitude of magnetic field.

✶ The magnetic field lines do not intersect. This is so since the direction of the magnetic field would not be unique at the point of intersection.

Magnetic flux

It is the total no of magnetic field lines passing normally through unit area.



flux through the small area 'ds' is given by

$$d\phi = B\cos\theta.ds = \overline{B}.\overline{ds}$$

$\therefore$ Total flux through the entire surface is $\phi = \int_s \overline{B}.d\overline{s}$

Gauss's Theorem for Magnetism

The net magnetic flux through any closed surface is always zero.

$$\boxed{\phi_B = \oint_s \bar{B}.d\bar{s} = 0}$$

ie magnetic monopoles does not exist.

Magnetic dipole

A magnetic dipole consists of two unlike poles of equal strength and seperated by a small distance.

eg: Bar magnet, compass needle

Magnetic dipole moment of Bar magnet

It is the product of strength of either pole (m) and the magnetic length (2ℓ) of the magnet.

$$\boxed{\vec{M} = m \times 2\ell}$$

Magnetic dipole moment is a vector quantity directed from South to North pole of the magnet.

S.I unit $\rightarrow$ Am^2 or J/T

S.I unit of pole strength $\rightarrow$ Am.

Force of attraction or repulsion between two magnetic poles.

Force of attraction or repulsion between two magnetic poles of strength m_1 and m_2 separated by a distance r is directly proportional to the product of the strength and inversely proportional to square of the distance between them.

$$F \,\alpha \frac{m_1 m_2}{r^2}$$

$$F = \frac{k m_1 m_2}{r^2}$$

$$k = \frac{\mu_0}{4\pi}$$

$$\boxed{F = \frac{\mu_0}{4\pi}\frac{m_1 m_2}{r^2}}$$

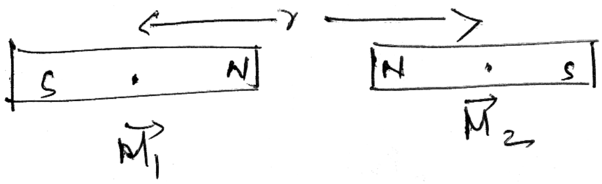
$\mu_0 \rightarrow$ magnetic permeability of free space or air.

$$\mu_0 = 4\pi \times 10^{-7}\,wb\,A^{-1}m^{-1}$$

Force between short Bar magnets

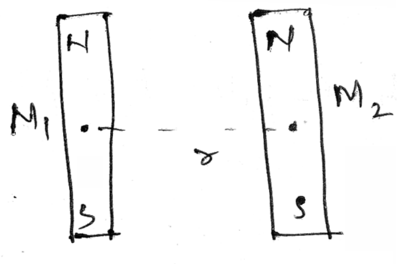
Consider two short bar magnets of magnate moments of M_1 and M_2 where centres are separated by a small distance 'r'.

1) When they are in Axial position



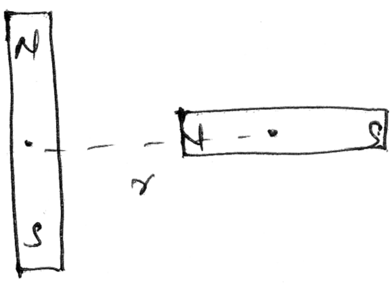
$$\boxed{F = \frac{\mu_0}{4\pi}\frac{6M_1M_2}{r^4}}$$

2) When they are in equatorial position



$$\boxed{F = \frac{\mu_0}{4\pi}\frac{3M_1M_2}{r^4}}$$

3) When the two magnets are $\perp$ to each other



$$\boxed{F = \frac{\mu_0}{4\pi}\frac{3M_1M_2}{r^4}}$$

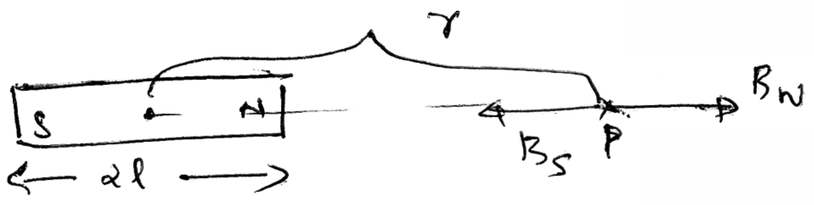
<u>Magnetic field intensity due to Bar magnet</u>

The magnetic field at any point is defined as the force experienced by a hypothetical unit, north pole placed at that point.

If a magnetic north pole of pole strength 'm' is placed in field of intensity $\vec{B}$, it experiences a force $\vec{F} = m\vec{B}$ directed along the direction of field.

If a magnetic South pole of pole strength ‘m’ placed in field of intensity $\vec{B}$, it experiences the force of same magnitude, but in opposite direction of the field.

Axial field



Field at point P due to N-pole $B_N = \dfrac{\mu_0}{4\pi}\dfrac{m}{(r-\ell)^2}$ (directed away from pole)

Field at point P due to R-pole $B_S = \dfrac{\mu_0}{4\pi}\dfrac{m}{(r+\ell)^2}$ (directed towards the pole)

Net field intensity, $B_{axial} = B_N - B_S$

$$= \frac{\mu_0}{4\pi} m\left[\frac{1}{(r-\ell)^2} - \frac{1}{(r+\ell)^2}\right]$$

$$= \frac{\mu_0}{4\pi} m \times \frac{4r\ell}{(r^2-\ell^2)^2}$$

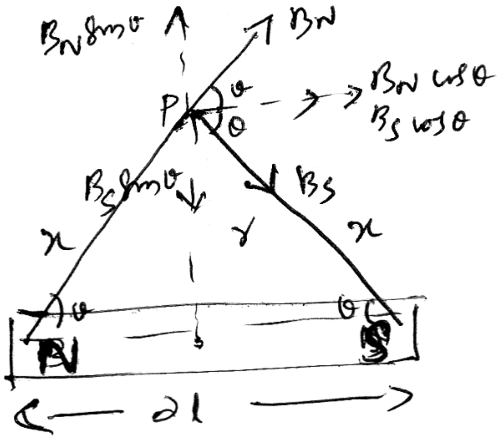
$$\boxed{B_{axial} = \frac{\mu_0}{4\pi}\frac{2Mr}{(r^2-\ell^2)^2}}$$

For short magnet $r^2 >> \ell^2 \therefore\ \ell^2$ can be neglected

$$\boxed{B_{axial} = \frac{\mu_0}{4\pi}\frac{2M}{r^3}}$$

direction is in the direction of dipole moment vector.

Equatorial field



$$B_S = B_N = \frac{\mu_0}{4\pi}\frac{m}{r^2}$$

But they are inclined at angle 2θ

$\therefore$ Net field intensity

$$B_{equitorial} = B_N \cos\theta + B_S \cos\theta = 2B_N \cos\theta$$

$$= 2 \times \frac{\mu_0}{4\pi}\frac{m}{r^2} \times \frac{\ell}{r} = \frac{\mu_0}{4\pi}\frac{M}{r^3}$$

$$x = \left(r^2 + \ell^2\right)^{1/2}$$

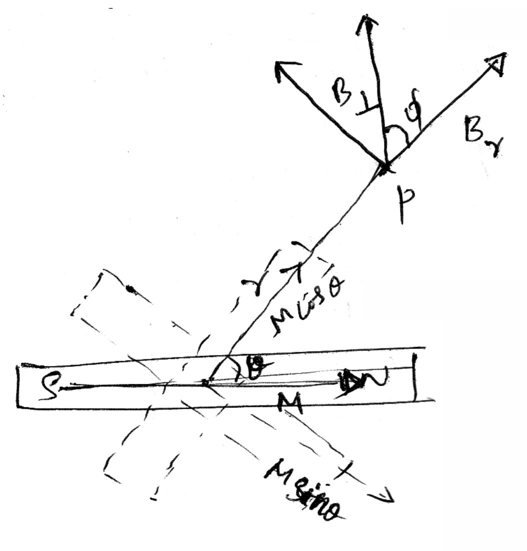
$$\therefore \boxed{B_{equitorial} = \frac{\mu_0}{4\pi}\frac{M}{\left(r^2+\ell^2\right)^{3/2}}}$$

For short Bar magnet $r^2 >> \ell^2, \therefore \ell^2$ can be neglected.

hence $\boxed{B_{equitorial} = \frac{\mu_0}{4\pi}\frac{M}{r^3}}$

Direction is opposite to the direction of dipole moment.

3) At any point



$$B_r = \frac{\mu_0}{4\pi}\frac{2M\cos\theta}{r^3}$$

$$B_\perp = \frac{\mu_0}{4\pi}\frac{M\sin\theta}{r^3}$$

$\therefore$ Net field $B = \sqrt{B_r^2 + B_\perp^2}$

$$= \frac{\mu_0}{4\pi}\frac{M}{r^3}\sqrt{\qquad\theta + \sin^2\theta}$$

$$\boxed{B = \frac{\mu_0}{4\pi}\frac{M}{r^3}\sqrt{\qquad\theta + 1}}$$

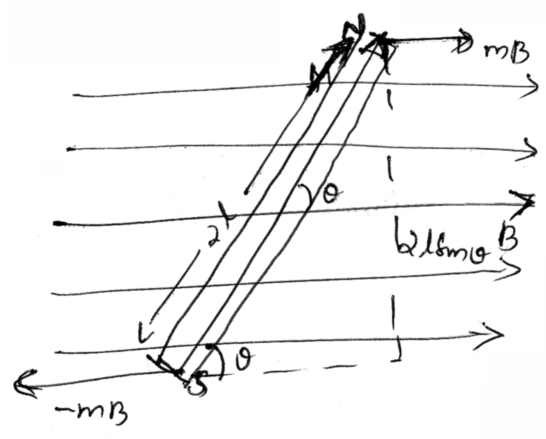
$$\tan\phi = \frac{B_\perp}{B_r} = \frac{\sin\theta}{2\cos\theta}$$

$$\therefore \boxed{\tan\phi = \frac{1}{2}\tan\theta}$$

ϕ is the angle between net field and the radius vector joining the point P and centre of dipole.

θ is the angle between dipole moment vector and the radius vector to the point P.

Torque on a bar magnet in a magnetic field



A bar magnet is held at an angle θ with uniform magnetic field B, then $F_{net} = -mB + mB = \theta$

ie magnet is in translational equilibrium.

But the two equal and opposite forces constitute a couple.

Torque, $\tau = \text{Force} \times \perp$ distance between the forces $= mB \times 2\ell \sin\theta$

$\tau = MB\sin\theta$

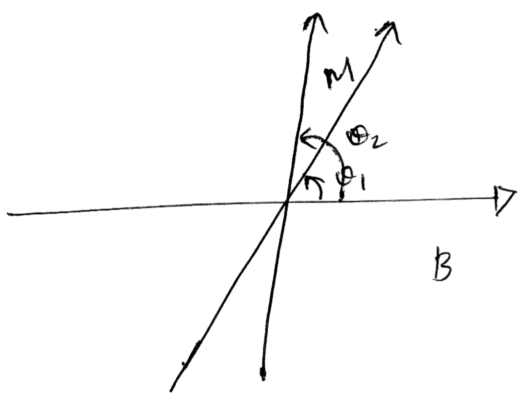
Invector form, $\boxed{\vec{\tau} = \vec{M} \times \vec{B}}$

when $\theta = 0^o, \tau = 0$ stable equilibrium

when $\theta = 180^o, \tau = 0$ unstable equilibrium

when $\theta = 90^o, \tau = MB$ maximum torque

Work done in rotating a dipole



The small amount of work done in rotating the dipole through a small angle $d\theta$ against the restoring is

$$dW = \tau d\theta = MB\sin\theta\, d\theta$$

Total work done in rotating the dipole from θ_1 to θ_2 is

$$W = \int_{\theta_1}^{\theta_2} MB\sin\theta\, d\theta = MB\left(-\cos\theta\right]_{\theta_1}^{\theta_2} = -MB\cos\theta_2 -^{-} MB\cos\theta_1$$

$$\boxed{W = MB\left(\cos\theta_1 - \cos\theta_2\right]}$$

According to work - Energy theorem

$$W = \Delta U = U_f - U_i$$

$$U_f = -MB\cos\theta_2 \qquad U_i = -MB\cos\theta_1$$

$\therefore$ In general $\boxed{U = -MB\cos\theta = -\bar{M}.\vec{B}}$

When $\theta = 0$, U = -MB P.E is minimum. This is the position of stable equilibrium.

When $\theta = 180^o, U = +MB$ P.E is maximum. This is the position of unstable equilibrium.

Vibration Magnetometer

It is used for

1) The comparison of dipole moments of 2-bar magnets.

2) For the comparison of earth's magnetic fields at two places.

Principle

When a bar magnet is suspended freely in a magnetic field it alligns itself in the direction of the field. It is deflected through an angle θ from the direction field, then if experiences a torque.

$$\tau = -MB\sin\theta$$

(-ve sign shows that torque is restoring in nature)

This restoring torque tend to allign the dipole in the direction of field. But due to rotational inertia dipole overshoots and set into oscillations.

For small angle oscillations $\sin\theta = \theta$

$$\therefore \tau = -MB\theta$$

$$I\frac{d^2\theta}{dt^2} + \frac{MB}{I}\theta = 0$$

$$\therefore \omega^2 = \frac{MB}{I} \Rightarrow \omega = \sqrt{\frac{MB}{I}}$$

$$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{MB}}}$$

Special cases

✶ When the bar magnet is divided into n-identical parts $\perp$ to its length, then for each part

pole strength, $\boxed{m' = m}$

dipole moment $\boxed{M' = \frac{M}{n}}$

M.I, $I' = \frac{I}{n^3}$

$\therefore$ time period of oscillation $\boxed{T' = \frac{T}{n}}$

✶ When the bar magnet is divided into n-identical parts parallel to its length, then for each part.

pole strength, $m' = \frac{m}{n}$

dipole moment, $M' = \frac{M}{n}$

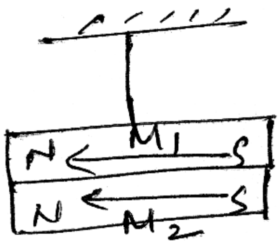
M.I, $I' = \frac{I}{n}$

$\therefore$ time period of oscillation, $\boxed{T' = T}$

Comparison of dipole moments of two bar magnets

Sum position

Like poles of the magnets are held together



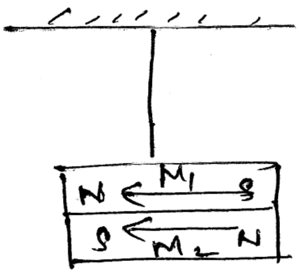
M = M_1 + M_2

I = I_1 + I_2

$$\therefore T_1 = 2\pi\sqrt{\frac{I + I_2}{(M_1 + M_2)B}}$$

Difference position

Unlike poles of the magnets are held together.



$$M = M_1 - M_2$$

$$I = I_1 + I_2$$

$$I_2 = 2\pi\sqrt{\frac{I + I_2}{(M_1 - M_2)B}}$$

$$\boxed{\frac{T_1}{T_2} = \sqrt{\frac{M_1 - M_2}{M_1 + M_2}}}$$

$$\frac{T_1^2}{T_2^2} = \frac{M_1 - M_2}{M_1 + M_2}$$

$$\therefore \boxed{\frac{M_1}{M_2} = \frac{T_1^2 + T_2^2}{T_2^2 - T_1^2}}$$

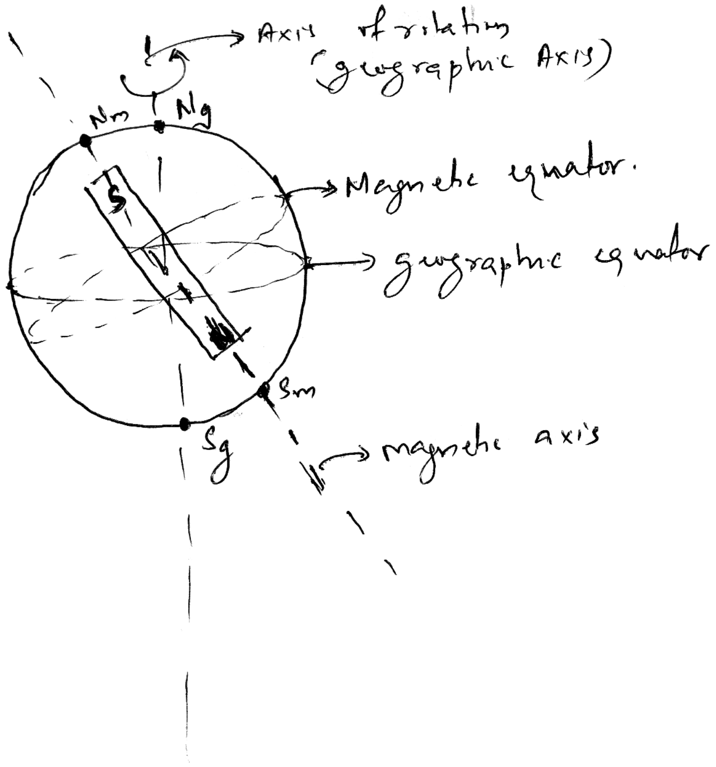
Similarly $\boxed{\frac{\nu_1}{\nu_2} = \sqrt{\frac{M_1 + M_2}{M_1 - M_2}}}$

$$\frac{\nu_1^2}{\nu_2^2} = \frac{M_1 + m_2}{M_1 - m_2}$$

$$\boxed{\frac{M_1}{M_2} = \frac{\nu_1^2 + \nu_2^2}{\nu_1^2 - \nu_2^2}}$$

Magnetic field of earth

The study of magnetism of earth is called terrestrial magnetism and geomagnetism.



Geographic Meridian

The vertical which includes geographic axis of earth and a given point on the surface of earth.

Magnetic Meridian

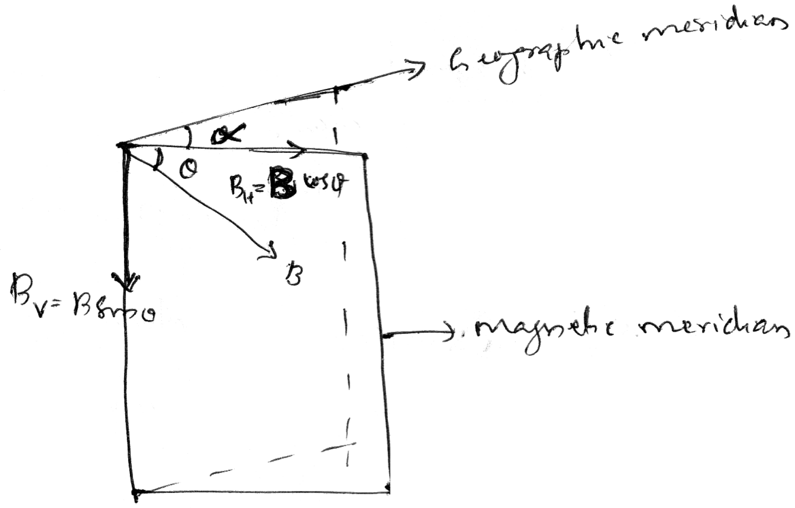
The vertical plane which includes magnetic axis of earth and a given point on the surface of earth.

Magnetic elements of earth

Declination(α)

The angle between magnetic axis and geographic axis of earth at a plane.

The angle between magnetic meridian and geographic meridian of earth at a place.



Inclination/Dip (θ)

Magnetic dip or inclination at a place is the angle which the direction of total strength of earth's field makes with horizontal line of magnetic meridian.

Horizontal component $B_H = B\cos\theta$

Vertical component $B_V = B\sin\theta$

$\boxed{\dfrac{B_v}{B_H} = \tan\theta}$

At poles $\theta = 90^o$

$\therefore B_H = 0$, $B_v = B$

$\boxed{B = \sqrt{B_H^2 + B_r^2}}$

At equator $\theta = 0^o$

$B_H = B$

$B_v = 0$

Apparent Dip

When a magnetic dip needle is placed in the magnetic meridian it shows the true dip at that place. If θ is the angle of dip, then

$$\tan\theta = \frac{B_v}{B_H}$$

If the magnetic dip needle makes an angle δ with magnetic meridian, then it measures an angle of dip, which is apparent, then

$$\boxed{\tan\theta' = \frac{B_v}{B_H\cos\delta} = \frac{\tan\theta}{\cos\delta}}$$

Neutral points

The points where net magnetic field due to the magnet and magnetic field (horizontal) due to earth is zero are called neutral points.

When a magnet is placed with its north pole towards north pole of earth, neutral point lies on equitorial line of magnet.

✶ When a magnet is placed with its north pole towards south pole of earth, neutral point lies on axial line of magnet.

Magnetic Materials

Properties of Magnetic Materials

1. **Magnetic permeability** (μ)

It is the ability of material to permit passage of magnetic lines of force through it.

Relative magnetic permeability is the ratio of number of magnetic field lines per unit area (then density B) in that material to the number of magnetic field lines per unit area that would be present, if the medium were replaced by vacuum (B_0).

$$\boxed{\mu_r = \frac{B}{B_0}}$$

It is also defined as the ratio of magnetic permeability of the material (μ) and magnetic permeability of free space (μ_0).

$$\boxed{\mu_r = \frac{\mu}{\mu_0}}$$

Magnetic force/magnetising Intensity $(\vec{H})$

Consider a solenoid with 'n' turns per unit length carrying a current $\overset{o}{\ell}$ would round a magnetic material. The magnetic induction of the field produced in the material.

$B_0 = \mu_0 ni$

The product nI is called magnetising force or magnetizing intensity H.

$\boxed{H = ni}$

If inside solenoid, there is free space, the magnetic induction $\boxed{B_0 = \mu_0 H}$

SI unit of $H \rightarrow$ A/m

Intensity of magnitisation (I)

It is the magnetic moment unit volume of the material.

$$\boxed{I=\frac{M}{V}}$$

unit $\rightarrow A/m$

Magnetic susceptibility (X)

It is the ratio of intensity of magnetisation (I) to the magnetising force (H) applied on it.

$\boxed{\chi=\frac{I}{H}}$ It has no units and no dimensions.

Relation between relative permeability and susceptibility

When a magnetic material is placed in a magnetising field H, the material gets magnetised. The total magnetic induction B in the material is the sum of magnetic induction B_0 in vacuum produced by magnetising intensity and magnetic B_m, due to magnetisation of material,

$$\therefore\ B=B_0+B_m$$

$$B_0=\mu_0H\ , \qquad B_m=\mu_0I$$

$$\boxed{B=\mu.(H+I)}$$

$$\chi=\frac{I}{H}$$

$$B=\mu_0H\left(1+\frac{I}{H}\right)=\mu_0H(1+X)$$

$$B=\mu H$$

$$\mu H=\mu_0H(1+X)$$

$$\frac{\mu}{\mu_0}=1+X$$

$$\boxed{\mu_r=1+X}$$

Diamagnetic Materials

- ✶ The diamagnetic substances are those in which individual atoms/molecules do not possess any net magnetic moment on their own.
- ✶ When they are placed in an external magnetic field they get fully magnetized is a direction opposite to the magnetising field.

- When placed in a non-uniform magnetic field, those substances have a tendency to move from stronger parts of the field to the weaker parts.
- When a specimen of diamagnetic material is placed in magnetising field, the magnetic field lines prefer not to pass through the specimen.

  i.e. $\mu_r < 1$

- Since $\mu_r < 1$, x is a small negative value.

  Susceptibility of diamagnetic materials does not change with temperature.

Paramagnetic Materials

- Paramagnetic substances are those is which each individual atoms/molecule has a not non-zero magnetic moment of its own.
- When placed in a non-uniform magnetic field, they tend to move from weaker parts of the field to the stronger parts.
- When a specimen of paramagnetic substance is placed in a magnetising field, the magnetic field lines prefer to pass through the specimen rather than through air.

  Relative permeability is always more than unity.

  $\mu_r < 1$

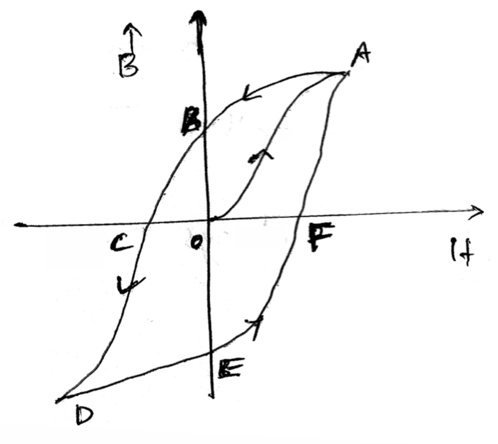
- X is a small +ve value.
- χ of paramagnetic subtances various inversity as the temperature of substance $\chi \; \alpha \frac{1}{T}$

  $\boxed{\chi = \frac{C}{T}}$ Curie's law

Ferromagnetic substances

- Ferromagnetic substances are those in which each individual atom/molecule has a non-zero magnetic moment as in a paramagnetic substances.
- When they are placed in an external magnetising field, they gets strongly magnetized in the direction of the field.
- They have a tendency to move from a region of weak magnetic field to the region of strong magnetic field
- μ_r is very large
- X is also a large +ve value
- With rise in temperature, susceptibility of ferromagnetic substances decreases. At a certain temperature, ferromagnetic change over to paramagnetic substances. This transition temperature is called curie temperature.

Hysteresis



OB, OE → Retentivity
OC, OF → coercivity

Hysteresis represents the relation between magnetic induction $\vec{B}$ of a ferromagnetic material with magnetising force $\vec{H}$.

The phenomenon of lagging of magnetic induction B behind magnetising field H is called hysteresis.

The area of B-H loop represents energy dissipated per unit volume of the material.

$\therefore$ Energy loss due to hysteresis is given by

$\boxed{E = VAft}$

V $\rightarrow$ volume of speciment

A $\rightarrow$ Area of B -H loop

f $\rightarrow$ frequency of magnetisation

t $\rightarrow$ time