RELATION & FUNCTION

Ordered pairs

(a,b) is called ordered pair

- $(a, b) \neq (b, a)$
- $(a, b) = (c, d) \Leftrightarrow a = c \ \& \ b = d$

Cartesian product of sets (cross product)

If A and B are two sets then the cartesian product is given

$$A \times B = \{(x, y) : x \in A, y \in B\}$$

$$B \times A = \{(x, y) : x \in B, y \in A\}$$

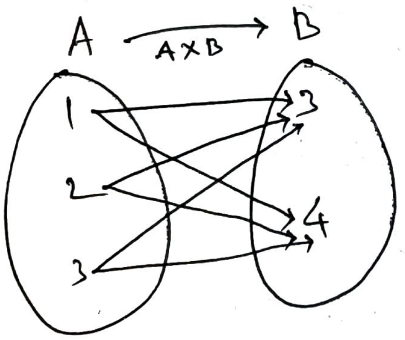
eg: $A = \{1, 23\}$, $B = \{3, 4\}$

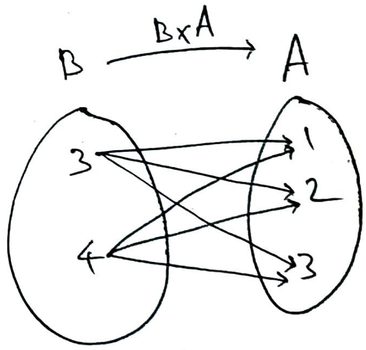
$$A \times B = \{(1,3), (1,4), (2,3), (2,4), (3,3), (3,4)\}$$

$$B \times A = \{(3,1), (3,2), (3,3), (4,1), (4,2), (4,3)\}$$

Thus $A \times B \neq B \times A$

Arrow Diagram representation





Now $A\times A=\{(x,y)\ \ x\in A,y\in A\}$

Here $A\times A=\{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)\}$

Important Results

If $n(A)=m\ \ \&\ n(B)=n$

Then $n(A\times B)=n(B\times A)=mn$

$\&\ n(A\times A)=m^2$

NCERT Ex.1: If $(x+1,y-2)=(3,1)$ find $x\ \&\ y$

$x+1=3, y-2=1$

$x=2, y=3$

NCERT Ex.4: If $A=\{1,2\}$ find $A\times A\times A$

$A\times A\times A=\{(1,1,1),(1,1,2),(1,2,1),(1,2,2),(2,1,1),(2,1,2),(2,2,1),(2,2,2)\}$

NCERT Ex.6:If $A\times B=\{(p,q),(p,r),(m,q),(m,r)\}$

Then find sets A & B

$A=\{p,m\};B\{q,r\}$

NCERT Ex.2.1 Qn.7. If $A=\{1,2\}, B=\{1,2,3,4\}, C=\{4,5\}$

Then find $(i) A\times(B\cap C)$

$(ii)(A\times B)\cap(A\times C)$

$(iii) A\times(B\cup C)$

$(iv)(A\times B)\cup(A\times C)$

Important Results

(1) $A\times(B\cap C)=(A\times B)\cap(A\times C)$

(2) $A\times(B\cup C)=(A\times B)\cup(A\times C)$

(3) $A\times B=\phi$ then either A or B is ϕ

(4) $n[(A\times B)\cap(B\times A)]=[n(A\cap B)]^2$

Relations: A Relation R from a non empty set A to a non empty set B is a subset of A×B or Any subset of A×B is a relation from set A to set B. If R is a relation from set A to set B we denote it as $R:A\to B$. Any subset of A×A is called a relation from $A\to A$ or relation on A.

<u>Domain , Range & codomain of a relation.</u>

Let $R:A\to B$ be a relation from $A\to B$. Then set of all first elements of ordered pairs of R is called

<u>Domain of R</u>

The set of all second elements of ordered pairs of R is called

<u>Range of R</u>

Set B is called co-domain of ordered pairs of R is called

ex: Let $A=\{1,2,3,4,5\}$ $B=\{3,5,7,9\}$

Let $R:A\to B$ be a relation by $R=\{(x,y):x\in A, y\in B, y=x+2\}$

Now R in Roster form is given by

$R=\{(1,3),(3,5),(5,7)\}$

Domain = $\{1,3,5\}$

Range = $\{3,5,7\}$

Codomain = $\{3,5,7,9\}$

Result (1) $\boxed{\text{If } R : A \to B \text{ is a relation then Domain} \subseteq A \text{ and Range} \subseteq B}$

(2) $\boxed{n(A) = m \;\&\; n(B) = n \text{ then total no.of Relations from } A \to B = 2^{mn}}$

(3) $\boxed{\text{If } n(A) = n \text{ then total number of relations from } A \to A \text{ (or total relations on } A) \text{ is } 2^{(n^2)}}$

Types of Relations

(1) Inverse Relation: If $R : A \to B$ is a relation defined by $R = \{(x, y) : x \in A \;\&\; y \in B\}$ then inverse relation of R is given by $R^{-1} : B \to A$ by $R^{-1} = \{(y, x) : x \in A, y \in B\}$

Let $A = \{1,2,3,4,5\}, B = \{3,5,7,9\}$

Let $R : A \to B$ given by $R = \{(1,3),(3,5),(5,7)\}$ Then $R^{-1} = \{(3,1),(5,3),(7,5)\}$

We can see that Domain of $R^{-1} = \text{Range of } R$ & Range of $R^{-1} = \text{Domain of } R$

(2) Identity relation

A relation $R : A \to A$ given by

$R = \{(x, y) : x, y \in A, y = x\}$ is called identity relation

Ex.Let $A = \{1,2,3,4\}$

Then relation R on A by

$R = \{(1,1),(2,2),(3,3),(4,4)\}$ is called identity relation

(3) Void Relation (Null relation or empty relation)

We know a relation on A is a subset of A×A and ϕ is a subset of A×A $\therefore \phi$ is also a relation from $A \to A$. This is known as void relation

(4) Universal Relation

$A \times A$ itself is a subset of A×A and hence A×A is a relaiton on A. This is called universal Relation

Exercise 2.2 NCERT

1. $A = \{1, 2, 3.....14\}$. Define a relation R on A by $R = \{(x, y) : x, y \in A, y = 3x\}$. Write this on Roster form. Also write its domain and Range.

$R = \{(1,3),(2,6),(3,9),(4,12)\}$

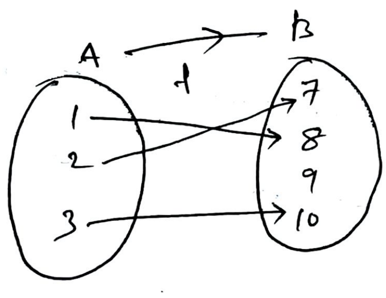
Domain $= \{1, 2, 3, 4\}$

Range $= \{3, 6, 9, 12\}$

Functions: A relation from set A to set B is said to be function from $A \to B$, if every element of set A has one and only one image in set B.

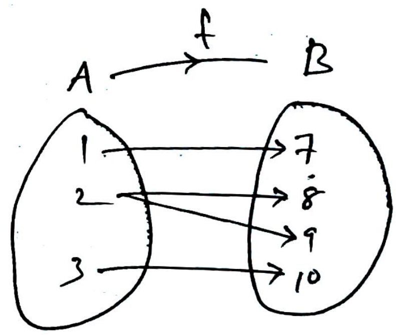
Examples:

(1)



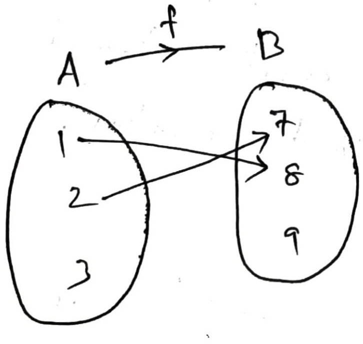
Function from $A \to B$

(2)



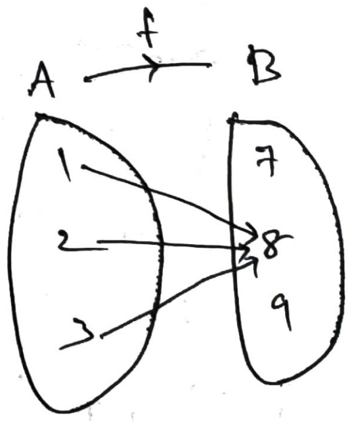
Not a function from $A \to B$

(3)



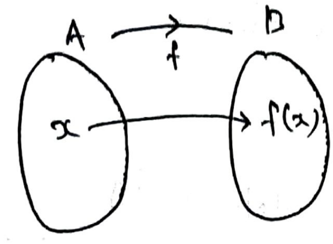
Not a function

(4)



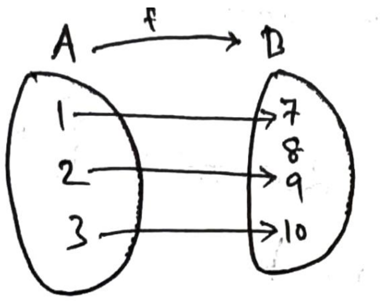
Function from $A \to B$

$f: A \to B$ is a function and if $(x, y) \in f$, then y is called the image of x with respect to the function f. We write it as $y = f(x)$. And x is called the pre image of y under f.



Then for $x \in A, y = f(x) \in B$

ex.



We have f(1)=7, f(2) = 9, f(3) =10

$$\boxed{\text{Result: If } n(A) = m \,\&\, n(B) = n \text{ then total no.of functions from } A \to B = n^m}$$

Domain & Range of a Function

Let $f: A \to B$ be a function . Then A is called Domain of f. Or in other words if $f: A \to B$ is a function. Then the set of all values of x in which f is defined is called Domain of f.

The set of all images of elements of A in set B is called. Range of f. ie. The set of values of f(x) in B is the Range of f. The whole set B is called codomain of f.

Some Important Functions

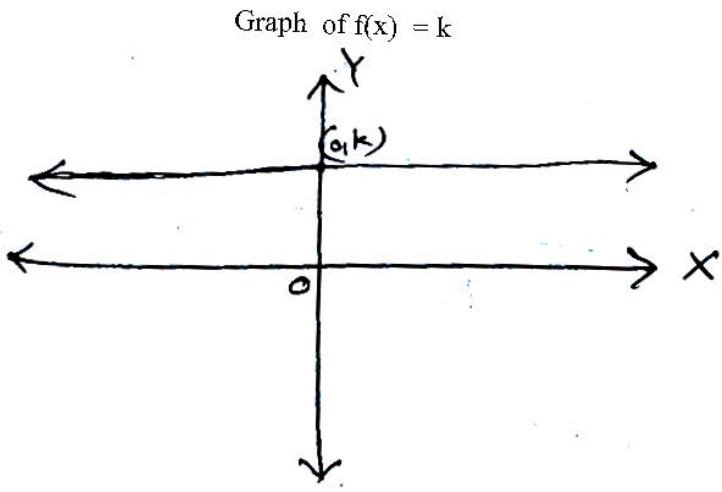
(1) Real function

A function $f: A \to B$ such that A and B are either R or subsets of R is called real function

(2) Constant function

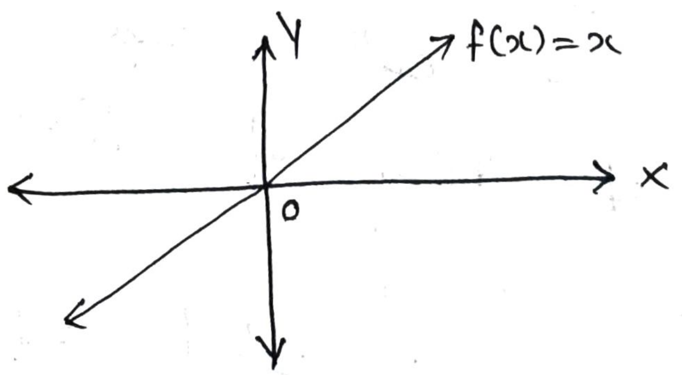
A function $f:R \to R$ defined by $f(x) = k$ for any $x \in \text{Domain}\, R$ is called constant function

Here Domain = R, Range = $\{k\}$



(2) Identity function

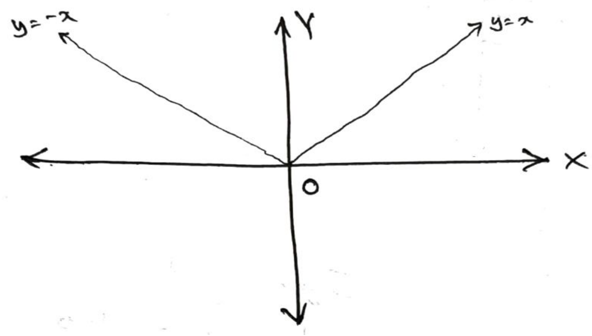
A real function $f:R \to R$ by f(x) = x for all $x \in R$ is called identity function. Here Domain =Range = R



Modulus function (Absolute value function)

A function $f:R \to R$ by $f(x)=|x|$ is called modulus function

Here Domain = R Range= $R^{+} \cup \lfloor 0 \rfloor \text{or} [0,\infty)$



Thus we have $|x| = \begin{cases} x \text{ if} & x \geq 0 \\ -x \text{ if} & x < 0 \end{cases}$

Greatest Integer value funciton (GIV function)

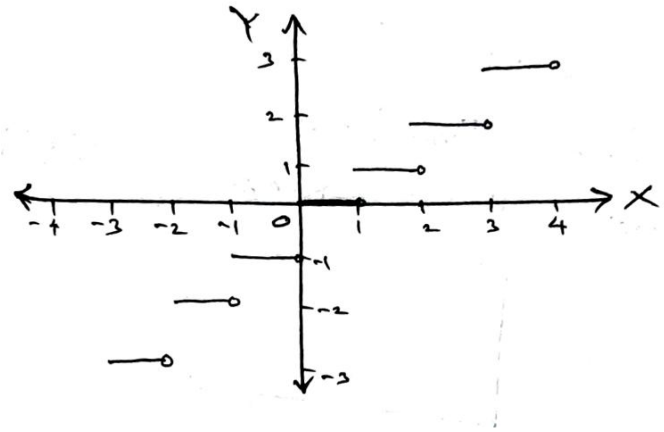
If x is any real number then the greatest integer less than or equal to x is called GIV of x denote as [x]

ex: $[3.72]=3$ $[0.214]=0$ $[4]=4$ $[-7]=-7$ $[\sqrt{3}]=1$ $[\pi]=3$ $[-2.1]=-3$ $[-5.92]=-6$ $[-10.001]=-11$

Note: [x] is always an integer for any $x \in R$

A real function $f:R \to R$ by $f(x)=[x]$ for every $x \in R$ is called GIV function

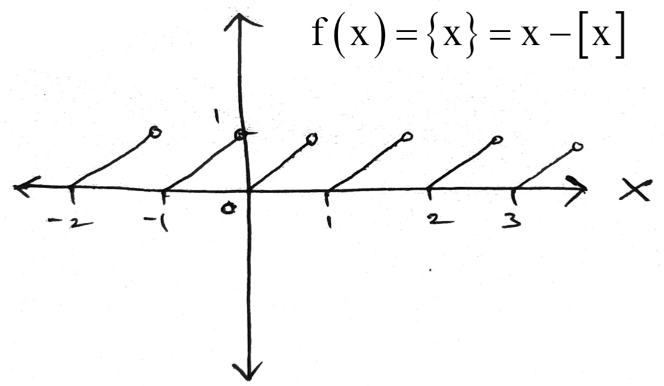
Here Domain = R, Range = Z



Fractional Part function

A real function $f : R \to R$ by $f(x) = \{x\} = x - [x]$ is called fractional part of x

Domain = R, Range = $[0,1)$



Signum function

For any $x \in R$ signum of x is defined as $\text{sig}(x) = \begin{cases} \dfrac{|x|}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

OR

$$\operatorname{sig}(x)=\begin{cases}1 & \text{if} \quad x>0\\ 0 & \text{if} \quad x=0\\ -1 & \text{if} \quad x<0\end{cases}$$

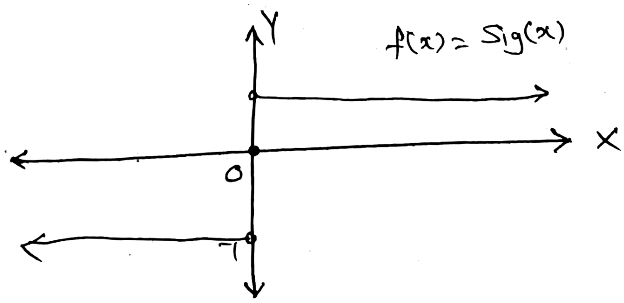
$\operatorname{Sig}(100)=1$ $\quad$ $\operatorname{Sig}(0)=0$

$\operatorname{Sig}(2.3)$ $\quad$ $\operatorname{Sig}(-7)=-1$

$\operatorname{Sig}(\sqrt{3})=1$ $\quad$ $\operatorname{Sig}(-\pi)=-1$

The function $f:R\to R$ by $f(x)=\operatorname{Sig}(x)$ is called signum function

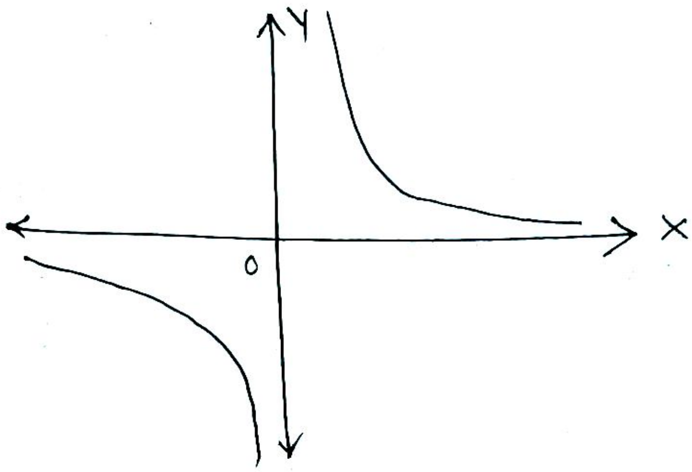
Domain = R, Range = $\{-1,0,1\}$



Reciprocal function

A function $f:R-\{0\}\to R$ by $f(x)=\frac{1}{x}$, $x\neq 0$ is called reciprocal function

Domain $=R-\{0\},\text{Range}=R-\{0\}$



Rational function

A function $f:R \to R$ by $f(x)=\frac{p(x)}{q(x)}$ where P(x) and q(x) are polynomials in x is called rational function

Domain of a function & Finding domain

It is the set of all values of x in which f(x) is defined

Important steps to find domain

- If a function is in the form $\frac{Nr}{Dr}$ then to be defined, $Dr \neq 0$ eg: $f(x)=\frac{1}{x}$; Domain $R-\{0\}$
- If the function is in the form $\sqrt{\text{Expression}}$ the expression ≥ 0
- If the function is in the form $\frac{1}{\sqrt{\text{Expression}}}$ then expression >0

Examples

Find domain of (i) $f(x)=x^3-7; \text{Domain}=R$

(2) $f(x)=\frac{1}{x+2}$

$f(x)$ is defined only when $x+2\neq 0, x\neq -2$

$\text{Domain}=R-\{-2\}$

(3) $f(x)=\frac{x^2+x+1}{x^2-5x+6}$

f(x) is defined only when $x^2-5x+6\neq 0$

$(x-2)(x-3)\neq 0$

$x\neq 2, x\neq 3$

$\therefore$ Domain $=R-\{2,3\}$

(4) $f(x)=\sqrt{2-x}$

$f(x)$ is defined only when $2-x\geq 0$

$2\geq x; x\leq 2$

Domain $=(-\infty,2]$

(5) $f(x)=\dfrac{1}{\sqrt{x-5}}$

f(x) is defined only when $x-5>0; x>5$

Domain = $(5,\infty)$

6) $f(x)=\sqrt{\dfrac{x+1}{3-x}}$

$f(x)$ is defined only when $3-x\neq 0 \,\&\, x\neq 3$

$\dfrac{x+1}{3-x}\geq 0$

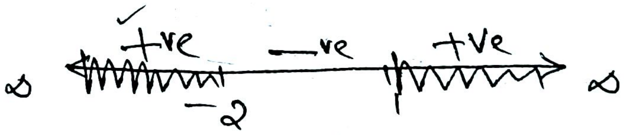
Real line method (Wavi curve method)

$x\in[-1,3)$

Do main $=[-1,3)$

7) $f(x)=\sqrt{(x-1)(2+x)}$

f(x) is defined only when $(x-1)(2+x)\geq 0$



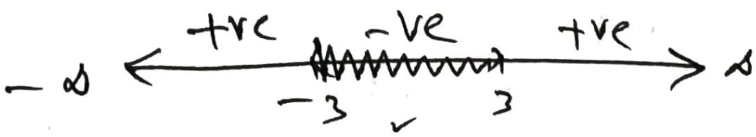
$x\in(-\infty,-2]\cup[1,\infty)$

Domain = $(-\infty,-2]\cup[1,\infty)$

8) $f(x)=\sqrt{9-x^2}$

f(x) to be defined $9-x^2 \geq 0; x^2-9 \leq 0$

$(x+3)(x-3) \leq 0$

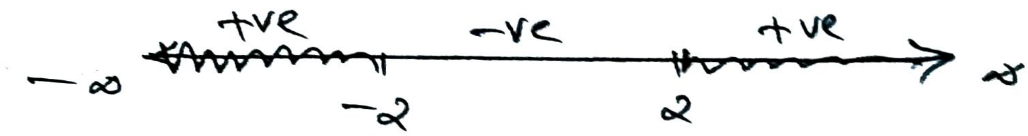


Domain = $[-3,3]$

9) $f(x)=\dfrac{1}{\sqrt{x^2-4}}$

$f(x)$ to be defined $x^2-4>0$

$(x+2)(x-2)>0$



Domain = $(-\infty,-2)\cup(2,\infty)$

Range of a function and method of finding Range

If $y=f(x)$ is a function then the set of all values of f(x) or set of all values of y is called Range.

Methods for find the Range of a function

1) Put $y=f(x)$
2) Express x as a function of y
3) Find possible values of y(just like domain)
4) Eliminate the values of y by looking at the definition to write the exact range

Example

1) $f(x)=x-1$

Put $y=x-1$

$x=y+1; y \in R$

Range = R

2) $f(x)=\dfrac{x-2}{3-x}$ Domain = $R-\{3\}$

$y=\dfrac{x-2}{3-x}$

$3y-xy=x-2$; $x+xy=3y+2$

$x(1+y)=3y+2; x=\dfrac{3y+2}{1+y}$

x is defined only when $1+y\neq 0$

$y\neq -1; y\in R-\{-1\}$

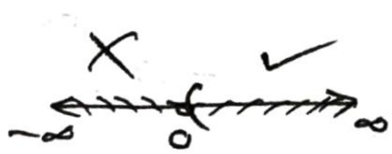
Range = $R-\{-1\}$

3) $f(x)=\dfrac{1}{\sqrt{x-5}}$; $\dfrac{1}{\sqrt{x-5}}=y$

$\dfrac{1}{x-5}=y^2 \Rightarrow 1=xy^2-5y^2$

$xy^2=1+5y^2 \Rightarrow x=\dfrac{1+5y^2}{y^2}$

$y\neq 0$



$y\in R-\{0\}$

Result

$$\boxed{\begin{array}{c} x^2=25\Rightarrow x=\pm 5 \\ \text{But} \\ x=\sqrt{25}=5 \end{array}}$$

$\therefore f(x)$ should be +ve

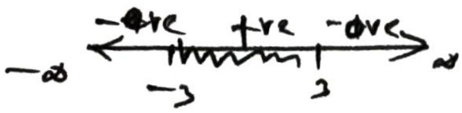
Range = $(0,\infty)$

4) $f(x)=\sqrt{9-x^2}$

$y=\sqrt{9-x^2}$; $y^2=9-x^2$

$x^2 = 9 - y^2$; $x = \sqrt{9 - y^2}$; $9 - y^2 \geq 0$

$(3+y)(3-y) \geq 0$



$\Rightarrow y \in [-3, 3]$

But $y = f(x) \geq 0$; $\therefore$ Range $= [0, 3]$

5) $y = \sqrt{x^2 - 9}$

$y^2 = x^2 - 9$; $x^2 = y^2 + 9$

$x = \sqrt{y^2 + 9}$; $\therefore y^2 + 9 \geq 0$

$\therefore y \in R$

But y > 0 Range = $[0, \infty)$

6) $f(x) = \dfrac{1}{\sqrt{9 - x^2}} \Rightarrow y^2 = \dfrac{1}{9 - x^2}$

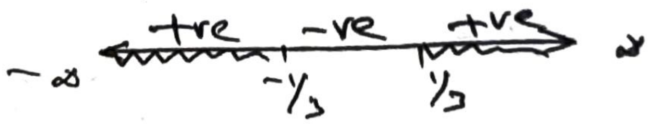
$y = \dfrac{1}{\sqrt{9 - x^2}}$ $\qquad$ $9 - x^2 = \dfrac{1}{y^2}$

$$x^2 = 9 - \frac{1}{y^2} = \frac{9y^2 - 1}{y^2}$$

$$x = \sqrt{\frac{9y^2 - 1}{y^2}}; y \neq 0$$

$$\frac{9y^2 - 1}{y^2} \geq 0 \text{ ; } \therefore 9y^2 - 1 \geq 0 \left[\because y^2 > 0\right]$$

$$(3y+1)(3y-1) \geq 0$$



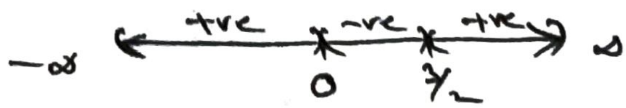
$$y \in \left(-\infty, \frac{-1}{3}\right] \cup \left[\frac{1}{3}, \infty\right)$$

But $y > 0 \therefore \text{Range} = \left[\frac{1}{3}, \infty\right)$

7) $f(x) = \dfrac{3}{2-x^2} \Rightarrow y = \dfrac{3}{2-x^2}$

$$2 - x^2 = \frac{3}{y} \Rightarrow x^2 = 2 - \frac{3}{y} = \frac{2y-3}{y}$$

$$y \neq 0, \frac{2y-3}{y} \geq 0$$



$$\therefore y \in (-\infty, 0) \cup \left(\frac{3}{2}, \infty\right)$$

$$f(x) = \frac{x^2+x+2}{x^2+x+1}, x \in R$$

$$y = \frac{x^2+x+2}{x^2+x+1}$$

$$yx^2 + yx + y = x^2 + x + 2$$

$$(y-1)x^2 + (y-1)x + (y-2) = 0, x \in R$$

$$\because b^2 - 4ac \geq 0 [a = y-1, b = y-1, c = y-2$$

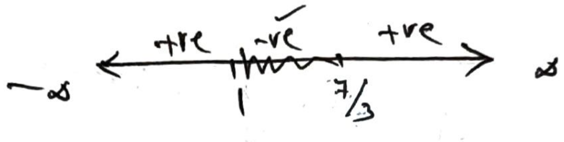
$$(y-1)^2 - 4(y-1)(y-2) \geq 0$$

$$y^2-2y+1-4y^2+12y-8\geq 0$$

$$3y^2-10y+7\leq 0$$

$$3y^2-3y-7y+7\leq 0$$

$$3y(y-1)-7(y-1)\leq 0\Rightarrow(y-1)(3y-7)\leq 0$$



$$y\in\left[1,\frac{7}{3}\right]$$

when $y=1\Rightarrow 1=\dfrac{x^2+x+2}{x^2+x+1}\Rightarrow x^2+x+1=x^2+x+2$

$1=2,\text{impossible}$; $\therefore y\neq 1$

$$\therefore \text{Range}=\left(1,\frac{7}{3}\right]$$

Algebra of functions

1) Addition If $f:R\to R\,\&\,g:R\to R$ be two function

Then $(f+g)(x)=f(x)+g(x)$

2) Subtraction: $(f-g)(x)=f(x)-g(x)$

3) Multiplication by scalar: $(kf)(x)=k.f(x)$

4) Multiplication of two functions: $(f.g)(x)=f(x).g(x)$

5) Division : $\left(\dfrac{f}{g}\right)(x)=\dfrac{f(x)}{g(x)},g(x)\neq 0$

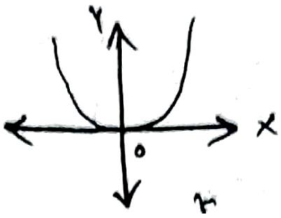
example of f(x) = x^2, $g(x)=2x+1$, find (i)

(2) $(f-g)(x),(3)(f.g)(x)(4)\left(\dfrac{f}{g}\right)(x)$

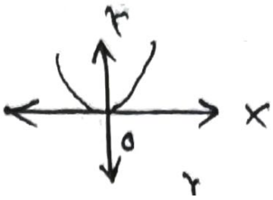
H.W. NCERT EXERCISE 2.3 Solve miscellaneous

Graph & Graph Transformations

Let $y = f(x) = x^2$

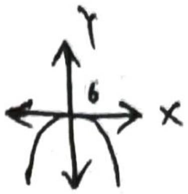


$y = f(x) = (-x)^2 = x^2$



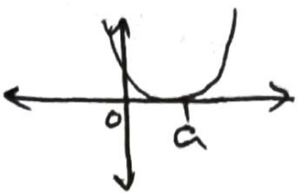
Draw reflection about Y-axis

$y = -f(x) = -x^2$



Draw reflection about X-axis

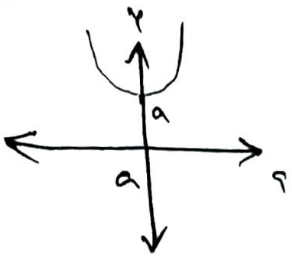
$y = f(x-a) = (x-a)^2$

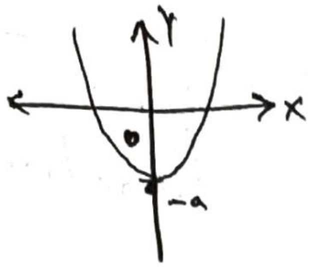


Shifting on +ve X-axis at a distance 'a'

$y = f(x+a) = (x+a)^2$

Shifting on –ve X-axis at a distance 'a'

$y = f(x) + a = x^2 + a$  Shifting on Y-axis above at a distance 'a'

$y = f(x) - a = x^2 - a$  Shifting on Y-axis below at a distance 'a'

Solution of Questions from study Material

Level I

1. D

2. B $R=\{(2,4),(4,3),(6,2),(8,1)\}$

$\therefore R^{-1}=\{(4,2),(3,4),(2,6),(1,8)\}$

3. D $\{(2,8),(3,27),(5,12,5),(7,343)\}$

Range = $\{8,27,125,343\}$

4. C $2^{(n^2)}=2^9$

5. D $R=\{(2,2),(3,5),(4,10),(5,17),(6,26)\}$

Domain = $\{2,3,4,5,6\}$

Range = $\{2,5,10,17,26\}$

6. C $n(A\times B)=n(A).n(B)$

7. D $n[(A\times B)\cap(B\times A)]=[n(A\cap B)]^2=3^2=9$

8. A $A-B=\{a\},(B\cap C)=\{c,d\}$

$(A-B)\times(B\cap C)=\{(a,c),(a,d)\}$

Level II

9. C

10. A

11. A $\{(11,10),(13,12)\}$

12. D $A=\{-2,-1,0,1,2\}$

$R=\{(-2,2),(-1,1),(0,0),(1,1),(2,2)\}$

13. C $f(x)=1+x^4$

$f(3)=1+3^n$

Given $f(3)=28 \Rightarrow 1+3^n=28; 3^n=27 \Rightarrow n=3$

$\therefore f(x)=1+x^3$

$f(4)=1+4^3=65$

14. B $\therefore f(x)=2(x-2)-(x+1)+x$

$=2x-4-x-1+x$

$=2x-5$

$x \geq 2; |x-2|=x-2; |x+1|=x+1$

15. B $g(1)=1 \Rightarrow g(1)=\alpha+\beta$

$g(2)=3 \quad g(2)=2\alpha+\beta$

$\therefore \alpha+\beta=1 \rightarrow (1)$

$2\alpha+\beta=3 \rightarrow (2)$

$(2)-(1) \Rightarrow \alpha=1$

$(1) \Rightarrow 2+\beta=1; \beta=-1$

16. D

Level III

17. D $A \times B \rightarrow (A \times B)$

No of relations on $A \times B = 2^{n(A\times B).n(A\times B)} = 2$

18. D $n(A)=P, n(B)=q$

$n(A\times B)=12, p, q\in W$

$pq=12$

$\Rightarrow p=12, q=1$ or $p=1, q=12 \Rightarrow p^2+q^2=145$

$p=2, q=6$ or $p=6, q=2 \Rightarrow p^2+q^2=40$

$p=3, q=4$ or $p=6, q=3 \Rightarrow p^2+q^2=25$

19. D $F(0)=2, F(1)=3$

$F(x+2)=2F(x)-F(x+1), x\geq 0$

$x=0 \Rightarrow F(2)=2F(0)-F(1)=4-3=1$

$x=1 \Rightarrow F(3)=2F(1)-F(2)=6-1=5$

$x=2 \Rightarrow F(4)=2F(2)-F(3)=2-5=-3$

$x=3 \Rightarrow F(5)=2F(3)-F(4)=10-3=13$

20. D $(n(B))^{n(A)}=3^4=81$

21. B $n[(A\times B)\cap(B\times A)]=[n(A\cap B)]^2=99^2$

22. C Universal relation = A×A

$\therefore n(A\times A)=10^2$

23. B $2^{100}-10^{10}$

24. B 10^{10}

Level IV

25. B Put n=1,2,3....

Generalising $f(n)=n\ f(1)$

26. A $x^2-8x+12\neq 0$

$(x-2)(x-6)\neq 0$

$x\neq 2, 6$

27. D $y = x^2 + 2x + 2$

$x^2 + 2x + (2 - y) = 0$

$x \in R \Rightarrow b^2 - 4ac \geq 0$

$4 - 4(2 - y) \geq 0$

$4 - 8 + 4y \geq 0$

$4y \geq 4; y \geq 1 \Rightarrow y \in [1, \infty)$

$y \geq 1 \Rightarrow y \in [1, \infty)$

Range $= [1, \infty)$

28. D Solved in theory

29. B Graph transformation $f(x) = |x - 2|$

30. Both I & II are true

31. C $R = (-\infty, \infty)$

32. C $f(x) = a^{nx} \quad f(2) = a^{2n} = 9 \Rightarrow (a^n)^2 = 3^2 \Rightarrow a^n = 3$

$\therefore f(x) = (a^x)^n = 3^x, \therefore f(5) = 3^5 = 243$

If $f(x).f(y) = f(x) + f(y)$

then $f(x) = a^x$

$f(2) = 9 \Rightarrow a^2 = 9; a = 3$

$f(x) = 3^x$

$f(5) = 3^5 = 243$

33. A

f(x) is defined when $\log_{10}\left(\frac{5x-x^2}{4}\right)\geq 0$

$\Rightarrow \frac{5x-x^2}{4}\geq 1$

$\Rightarrow x^2-5x+4\leq 0$

$\Rightarrow (x-1)(x-4)\leq 0$

$\Rightarrow x\in[1,4]$

34. D

f(x) is defined when $4-x^2\neq 0$ & $x^3-x>0$

$x\neq \pm 2$ & $x(x+1)(x-1)>0$

$x\in(-1,0)\cup(1,3)$

$\therefore$ Domain = $(-1,0)\cup(1,2)\cup(2,\infty)$

35. 4

$3f(x)+2f\left(\frac{x+59}{x-1}\right)=10x+30$

Put $x=7$, $3f(7)+2f(11)=100\rightarrow(1)$

Put $x=11$ $2f(7)+3f(11)=140\rightarrow(2)$

$(1)\times 3\Rightarrow 9f(7)+6f(11)=300$
$(2)\times 2\Rightarrow 4f(7)+6f(11)=280$

–ing $5f(7)=20$

$\therefore f(7)=4$

36. 66

$$\left[\frac{2}{3}\right]=0,\left[\frac{2}{3}+\frac{1}{99}\right]=0,\left[\frac{2}{3}+\frac{2}{99}\right]=0,\ldots..\left[\frac{2}{3}+\frac{32}{99}\right]=0$$

$$\left[\frac{2}{3}+\frac{33}{99}\right]=1,\left[\frac{2}{3}+\frac{34}{99}\right]=1,\ldots..\left[\frac{2}{3}+\frac{38}{99}\right]=1$$

$$\therefore \text{ Required value } = \therefore \underbrace{0+0+\ldots..+0}_{33\text{times}}+\underbrace{1+1+\ldots.+1}_{66\text{ times}}=66$$