Generate the Dynamics of the Three-Link 2D Biped

This function calculates the dynamics of the three-link biped, that is, it generates the mass matrix, M, the Coriolis Matrix C, the gravity Matrix G as well as the control matrix B:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = Bu$$

Note that you first would need to complete and run the generate_kinematics.mlx.

```
syms m1 m2 m3 g;
syms ddq1 ddq2 ddq3;

% T1, T2, T3: kinetic energies of m1, m2, m3
T1 = 0
T2 = 0
T3 = 0

% V1, V2, V3: potential energies of m1, m2, m3
V1 = 0
V2 = 0
V3 = 0

T = 0 % total kinetic energy
V = 0 % total potential energy

T = simplify(T, 'steps', 50);
V = % simplify V

L = 0; % Lagrangian
```

We use dLdqi for $\frac{\partial L}{\partial q_i}$ and dLddqi for $\frac{\partial L}{\partial \dot{q}_i}$.

```
dLdq1 = 9
dLdq2 = 0
dLdq3 = 0

dLddq1 = 0
dLddq2 = 0
dLddq3 = 0

dLddq1_dt = 0
dLddq2_dt = 0
dLddq3_dt = 0
```

Lagrange equations of motion

Recall:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$$

```
Eq1 = dLddq1_dt - dLdq1;
Eq2 = 0
Eq3 = 0
```

Calculate the matrices M, C, G in the equations of motion:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = 0$$

Recall how you did this in assignment 1.

```
G(1, 1) = 0 % use subs as in assignment 1
G(2, 1) = 0
G(3, 1) = 0

M(1, 1) = 0
M(1, 2) = 0
M(1, 3) = 0
M(2, 1) = 0
M(2, 2) = 0
M(2, 3) = 0
M(3, 1) = 0
M(3, 2) = 0
M(3, 3) = 0

C(1, 1) = 0
C(1, 2) = 0
C(1, 3) = 0
C(2, 1) = 0
C(2, 2) = 0
C(2, 3) = 0
C(3, 1) = 0
C(3, 2) = 0
C(3, 3) = 0
```

```
G = simplify(G, 'steps', 50)
M = 0 % simplify M
C = 0 % simplify C
```

To check if the extraction of M, C, G is correct. Note that error being zero does not mean that all your calculations of M, C, and G are correct.

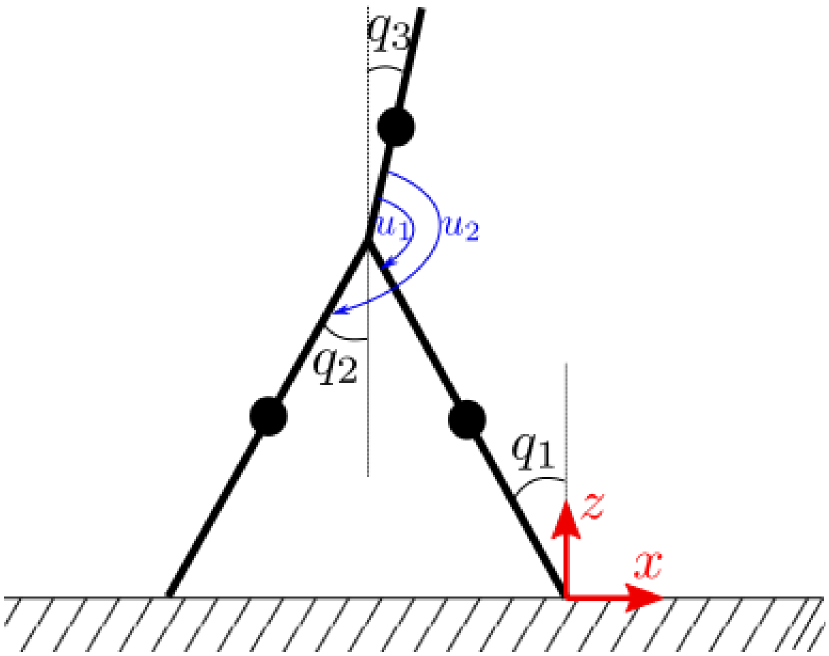
```
ddq = [ddq1;ddq2;ddq3];
dq = [dq1;dq2;dq3];
Eq = [Eq1;Eq2;Eq3];
error = simplify(M * ddq + C * dq + G - Eq)
```

Calculate the B matrix:

As shown in the figure below we have two controllers u_1 and u_2, which drive the angles between the stance leg and torso and between the swing leg and torso. What are the virtual work δW_1 and δW_2? From the expression of $\delta W = \delta W_1 + \delta W_2$, calculate the B matrix in the equations of motion:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = Bu$$

where $q = [q_1; q_2; q_3]$ and $u = [u_1; u_2]$. Note that the control matrix B is a 3×2 matrix.




```
syms u1 u2 delq1 delq2 delq3
%th1 = ; % angle between link 1 and link 3
%th2 = ; % angle between link 2 and link 3
%del_th1 = ; % virtual angle variation for th1
%del_th2 = ; % virtual angle variation for th2
%del_W1 = ; % virtual work done by u1
%del_W2 = ; % virtual work done by u2
%del_W = ; % total virtual work
%del_W = collect(del_W, [delq1, delq2, delq3])
```

Extract the B matrix:

First calculate the right hand side of the equations of motion from the expression of del_W:

Eq = R_Eq1

Eq2 = R_Eq2

Eq3 = R_Eq3

Calculate R_Eq1, R_Eq2, R_q3

```
%R_Eq1 = % use subs function
```

```
%R_Eq2 =
%R_Eq3 =
```

From the equations above write the B matrix:

```
%B(1, 1) = ;
%B(1, 2) = ;
%B(2, 1) = ;
%B(2, 2) = ;
%B(3, 1) = ;
%B(3, 2) = ;
B = sym(B) % why do we need this line of code?
```

Write the symbolic functions to a MATLAB *.m function.

Note: After running this section, you must modify the functions eval_M, eval_C, eval_G to have the following signature:

function M = eval_M(q)

function C = eval_C(q, dq)

function G = eval_G(q)

function B = eval_B()

```
matlabFunction(M, 'File', '../dynamics/eval_M_tmp');
matlabFunction(C, 'File', '../dynamics/eval_C_tmp');
matlabFunction(G, 'File', '../dynamics/eval_G_tmp');
matlabFunction(B, 'File', '../dynamics/eval_B_tmp');
```

Finally, remove the temporary functions eval_M_tmp, eval_C_tmp, eval_G_tmp, eval_B_tmp. We later, will be using the functions eval_M.m, eval_C.m, eval_G.m, eval_B.m.

Test your functions:

To run the test function (in the 'test' folder) you should add some folders to the path. Make sure that your are in the generate_model folder and then run the following code:

```
addpath('../set_parameters', '../dynamics', '../test');
```