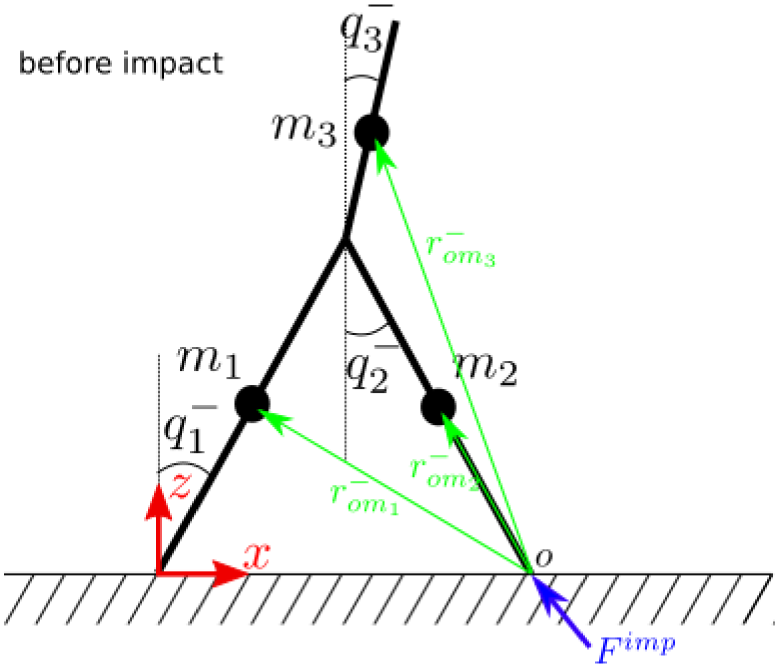
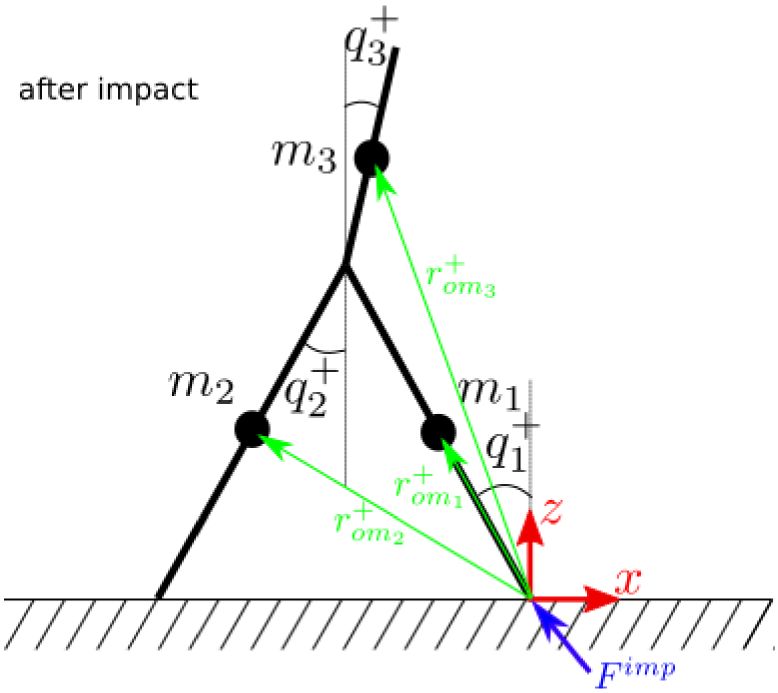
On the Impact Map Calculation

You will be using the conservation of angular momentum for calculation of the impact map.

Before impact:



After impact:



Note that $m_1 = m_2$ but m_1 is used to denote the mass on the stance leg and m_2 to denote the mass on the swing leg, so after the impact I have switched the indices.

Angles

The above two figures show the robot right before and after impact. Comparing the two configurations, we have:

$$\begin{bmatrix} q_1^+ \\ q_2^+ \\ q_3^+ \end{bmatrix} = \begin{bmatrix} q_2^- \\ q_1^- \\ q_3^- \end{bmatrix}$$

This is part of the transition map $(q^+, \dot{q}^+) = \Delta(q^-, \dot{q}^-)$.

Angular velocities

Calculation of the angular velocities after impact (i.e., $\dot{q}^+$) involves some physics. As discussed in class we use the method of conservation of angular momentum (as in McGeer 1998) to calculate the angular velocities after impact. Here I explain how to calculate H_a^- , that is, the angular momentum of the **whole system** about the point of impulse (denoted by o) before impact and after impact, H_a^+.

Based on the definition of angular momentum and the figures above:

$$H_a^- = mr_{om_1}^- \times \dot{r}_1^- + mr_{om_2}^- \times \dot{r}_2^- + m_3 r_{om_3}^- \times \dot{r}_3^-$$

and

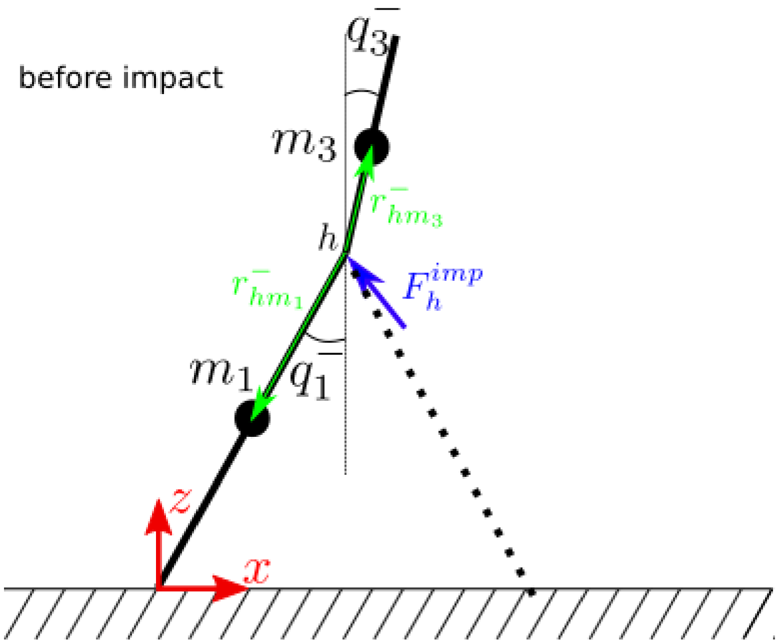
$$H_a^+ = mr_{om_1}^+ \times \dot{r}_1^+ + mr_{om_2}^+ \times \dot{r}_2^+ + m_3 r_{om_3}^+ \times \dot{r}_3^+$$

where $\dot{r}_i$ is the the velocity of the mass m_i in the inertial frame $x - z$. Note that I have substituted m for $m_1 = m_2$.

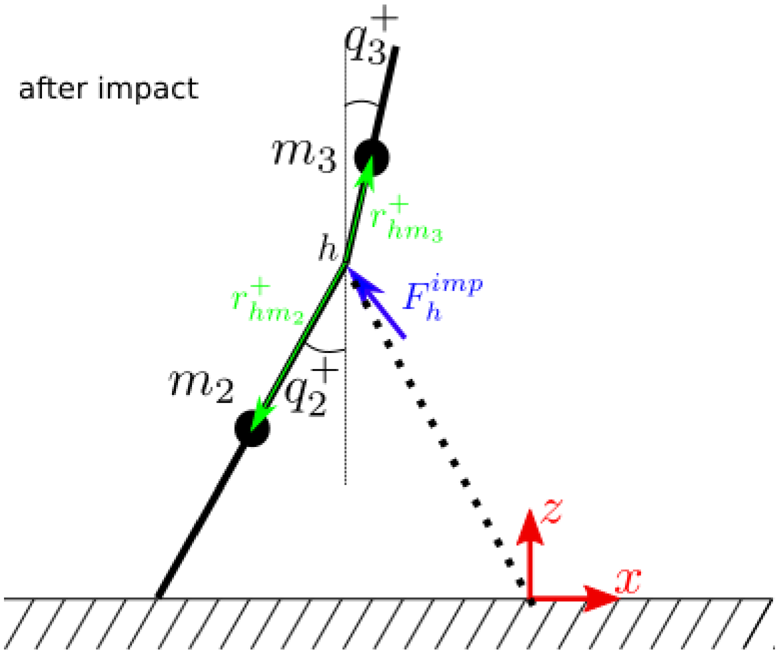
Note: You can calculate $r_{om_i}^-$ as a function of q^- and $r_{om_i}^+$ as a function of q^+ based on the `generate_kinematics.mlx` results.

Other than H_a^+ and H_a^- you need to calculate H_b, H_c before and after impact as discussed in class. Recall that H_b^- is the angular momentum of the **stance** leg before the impact *about the hip* joint and H_b^+ is its angular momentum *about the hip* joint after the impact. Similarly, H_c^- is the angular momentum of the **torso** *about the hip* joint before the impact and H_c^+ is its angular momentum *about the hip* joint after the impact. Look at the following two figures and write down the equations for H_b and H_c before and after impact.

Before impact:



After impact:



You can then calculate the impact map from the conservation of angular momentum. Letting $H^- = [H_a^-; H_b^-; H_c^-]$ and $H^+ = [H_a^+; H_b^+; H_c^+]$, we have:

$$H^+ = H^-$$

You can write the left side as $A^+ \dot{q}^+$ and the right hand side as $A^- \dot{q}^-$, thus:

$$\dot{q}^+ = (A^+)^{-1} A^- \dot{q}^-$$