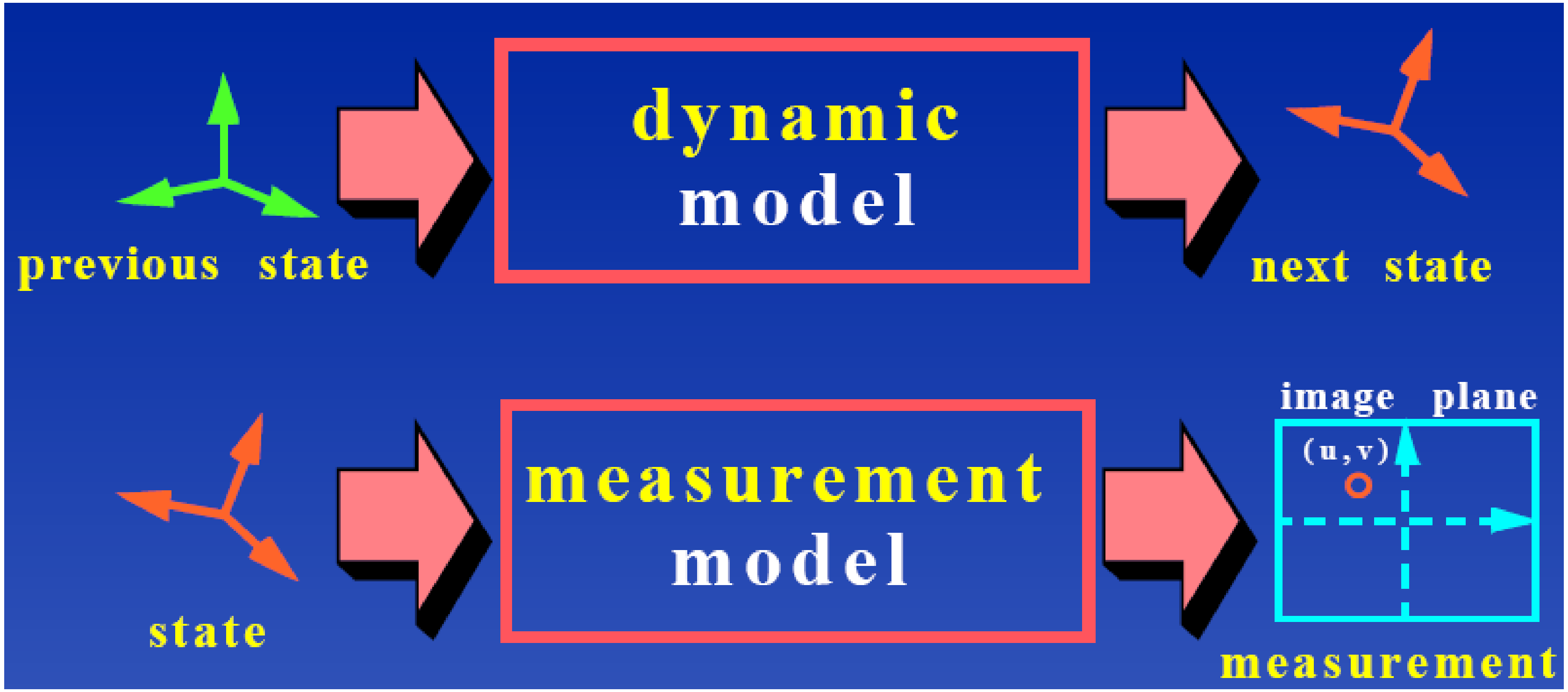
Discrete Kalman Filter - components

- The ingredients:
  - A discrete *process* model
    - Change in state over time (**Φ**)
    - Linear (or linearized) difference equation $(\delta \mathbf{\Phi}/dt)=\mathbf{F\Phi}$
  - A discrete *measurement* model
    - Relationship between state and measurements
    - Linear (or linearized) function (**H**)
  - Associated *stochastic* parameters (per model)
    - Process noise characteristics (**w**)
    - Measurement noise characteristics (**R**)

Discrete Kalman Filter

- Necessary models



Discrete Kalman Filter

- **Dynamic** model

$$x_{k+1} = \Phi_k x_k + "\Gamma w_k"$$

x_k – state vector; contains the *n*-states of the process

Φ – state transition matrix; (n x n) relates states at time step *k* to time step *k+1*

Γw – processing noise; expressing uncertainty of the dynamic model – ONLY in covariance propagation!

w – white noise

Γ – noise transition matrix

KF-dynamic model

- Formulated by 1st order differential equation

$$\dot{x} = Fx + "Gw"$$

x – state vector; contains the *n*-states of the process

F – dynamic matrix; (n x n) expresses the derivative of states with respect to time

Gw – time differential equation of the processing noise; expressing uncertainty of the dynamic model

w – white noise (sometimes '*u*' if it is uniform white noise)

G – noise shaping matrix

Discrete Kalman Filter

- **Measurement** model

$$z_k = Hx_k + v_k$$

z_k — measurement vector; size *m*

x_k — state vector; size *n*

H — measurement “design” matrix (m x n)

v_k — measurement noise

Discrete Kalman Filter

- **State** estimates

  - *A priori* state estimate (prediction)

$$\tilde{x}_k$$

  - Note: mean of *system noise* (prediction)

$$E\left[\Gamma w_k\right] = 0$$

  - *A posteriori* state estimate (after measurement)

$$\hat{x}_k$$

Discrete Kalman Filter

- **Covariance** estimates

  - *A priori* estimate of state covariance (prediction)

$$\tilde{P}_k = E\left[\left(x_k - \tilde{x}_k\right)\left(x_k - \tilde{x}_k\right)^T\right]$$

  - *A posteriori* state estimate (after measurement)

$$\hat{P}_k = E\left[\left(x_k - \hat{x}_k\right)\left(x_k - \hat{x}_k\right)^T\right]$$

Discrete Kalman Filter

- Covariance matrix of the **system noise**:

$$Q_k = E\left[\left(\Gamma w_k\right)\left(\Gamma w_k\right)^T\right] = \int_{k-1}^{k} \Phi_{k-1} G(\tau) Q(\tau) G^T(\tau) \Phi_{k-1}^T d\tau$$

- The covariance matrix of the **measurement noise**:

$$R_k = E\left[\left(v_k\right)\left(v_k\right)^T\right] \qquad E\left[v_k\right] = 0$$

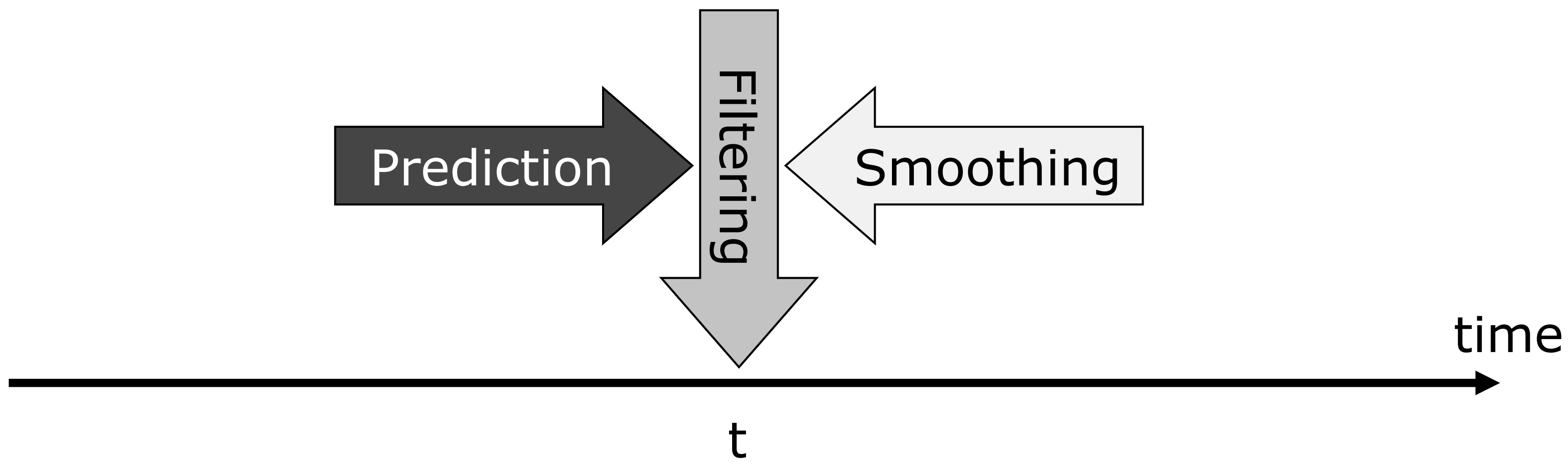
- *Basic assumption*: system and measurement noise are **NOT correlated!**

$$E\left[\left(\Gamma w_k\right)\left(v_k\right)^T\right] = 0$$

Estimation in time

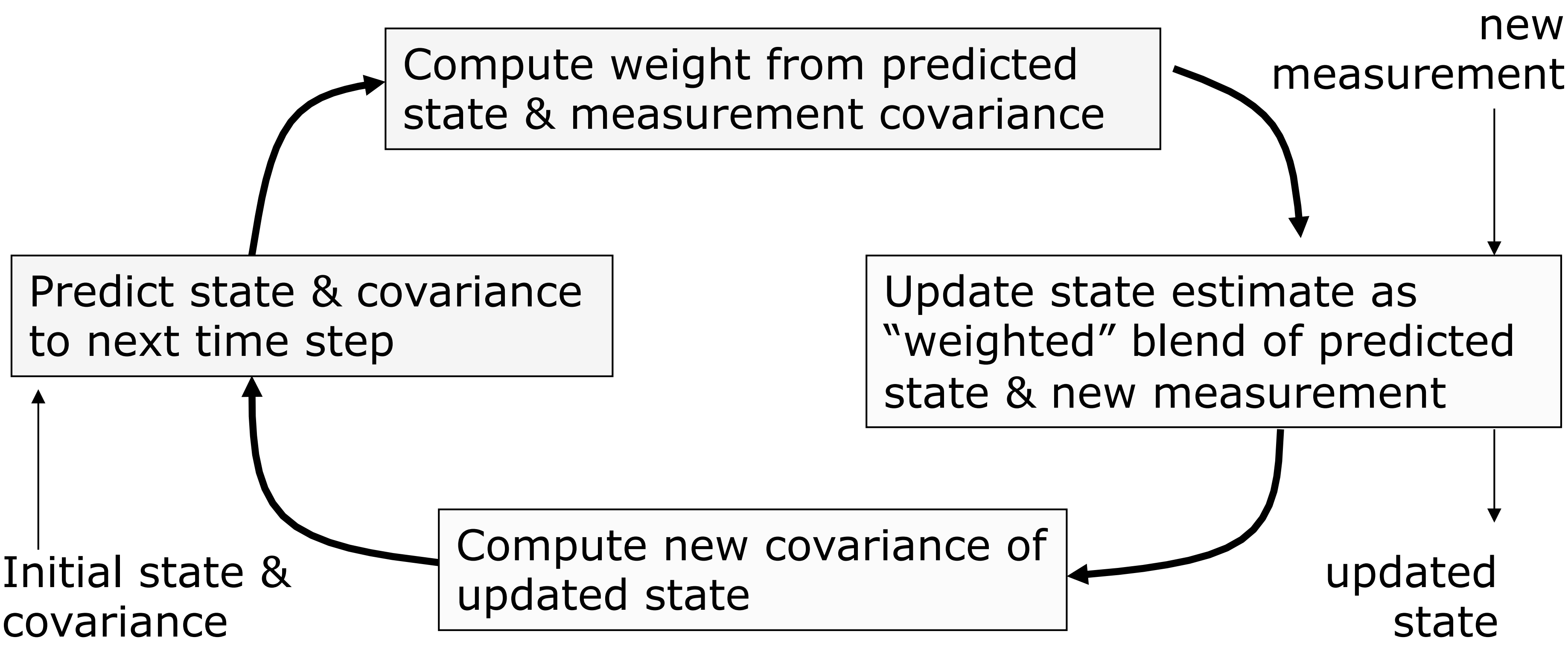
There are three types of estimation problems, based on the time (t) and the availability of measurements for which the estimate is required:

- Prediction: When (t) occurs after the last available measurement
- Filtering: When (t) coincides with the last measurements
- Smoothing: When (t) falls within the span of available measurements

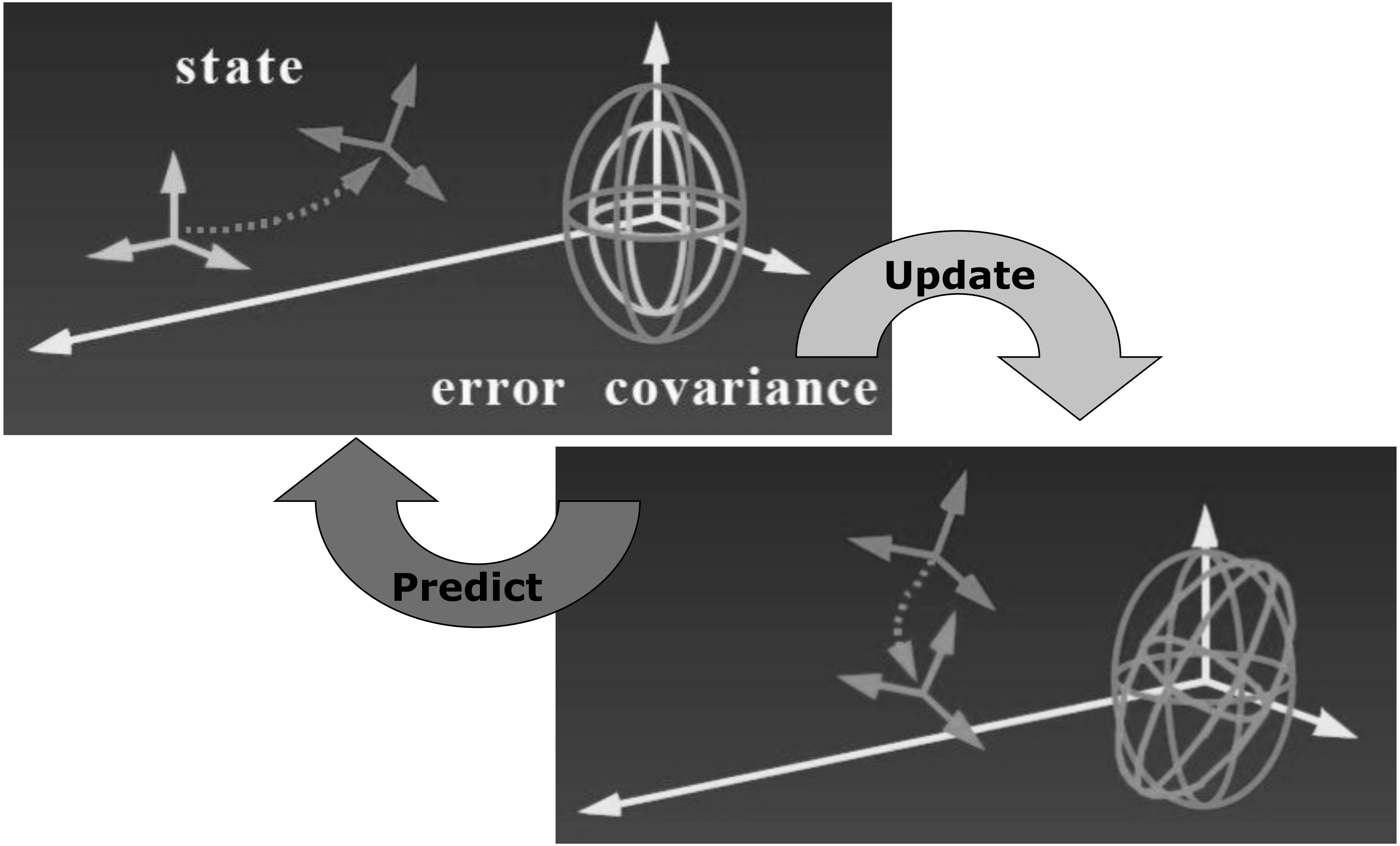


Kalman Filter operation

- The Kalman algorithm is a sequential recursive algorithm for an optimal least-mean square variance estimation of error states



Kalman Filter operation



Kalman Filter algorithm