KDCR Project 2

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Variables:

```
syms theta1 theta2 theta3 phi;
syms r R L x y;
```

Question 1:

```
x=1.5;
y=0;
phi=deg2rad(10); %% in rad
r = 1.5;
L = 3;
R = 3.5;

A=[x,y];
B=[x+r*cos(phi),y+r*sin(phi)];
C=[x+r*cos(phi+pi/3),y+r*sin(phi+pi/3)];

D=[R*cos(theta2), R*sin(theta2)];
E=[R*cos(theta3), R*sin(theta3)];
F=[R*cos(theta1), R*sin(theta1)];

r1=A-D;
r2=B-E;
r3=C-F;

eqn1=(sum(r1.^2)-L^2==0);
theta2=real(vpa(solve(eqn1,theta2),4))
eqn2=(sum(r2.^2)-L^2==0);
theta3=real(vpa(solve(eqn2,theta3),4))
eqn3=(sum(r3.^2)-L^2==0);
theta1=real(vpa(solve(eqn3,theta1),4))

q=zeros(3,8);
q=sym(q);
n=1;
for i=1:1:2
    for j=1:1:2
```

```
        for k=1:1:2
            q(:,n)=[theta1(i);theta2(j);theta3(k)];
            n=n+1;

        end
    end
end
```

Question 1 graph

```
for i=1:8
    figure
    patch([x  x+r*cos(phi)  ...
        x+r*cos(pi/3+phi)], ...
        [y y+r*sin(phi) ...
        y+r*sin(pi/3+phi)], 'black', 'edgecolor', 'red');
    hold on
    plot([x R*cos(q(2,i)) ], [y R*sin(q(2,i)) ], 'blue', 'linewidth', 2);
    plot([R*cos(q(3,i)) x+r*cos(phi)], ...
        [R*sin(q(3,i)) y+r*sin(phi)], 'blue', 'linewidth', 2);
    plot([R*cos(q(1,i)) x+r*cos(pi/3+phi)], ...
        [R*sin(q(1,i)) y+r*sin(pi/3+phi)], 'blue', 'linewidth', 2);
    viscircles([0,0],3.5);
    axis equal
    hold off
%     title(['Case',num2str(i)])
    print(['case',num2str(i),'.png'],'-dpng')
end
```

Question 2

```
clear all
syms r R L x y t;
theta1=1.5708;
theta2=2.0944;
theta3=0.698132;
r = 1.5;
L = 3;
R = 3.5;
% cos(phi)=(1-t^2)/(1+t^2);
% sin(phi)=2*t/(1+t^2);
% A=[x,y];
% B=[x+r*cos(phi),y+r*sin(phi)];
% C=[x+r*cos(phi+pi/3),y+r*sin(phi+pi/3)];
```

```
cp=(1-t^2)/(1+t^2);
sp=(2*t)/(1+t^2);
A=[x,y];
B=[x+r*cp,y+r*sp];
C=[x+r*(cp*cos(pi/3)-sp*sin(pi/3)),y+r*(cp*sin(pi/3)+sp*cos(pi/3))];

D=[R*cos(theta2), R*sin(theta2)];
E=[R*cos(theta3), R*sin(theta3)];
F=[R*cos(theta1), R*sin(theta1)];

r1=A-D;
r2=B-E;
r3=C-F;

f=[sum(r1.^2)-L^2;
  sum(r2.^2)-L^2;
   sum(r3.^2)-L^2 ];
expand(simplify(f))

[A1,b1] = equationsToMatrix(f(1)-f(2),[x,y]);
[A2,b2] = equationsToMatrix(f(1)-f(3),[x,y]);
Coeff=[A1;A2];
b=[b1;b2];
xy=inv(Coeff)*b;
simplify(xy)

x=xy(1);
y=xy(2);
A=[x,y];
D=[R*cos(theta2), R*sin(theta2)];
r1=A-D;
eqn4=sum(r1.^2)-L^2==0;
t_result=vpa(solve(eqn4,t),4)

solu=zeros(3,6);
x1=vpa(subs(x,t,t_result(1)),4);
y1=vpa(subs(y,t,t_result(1)),4);
x2=vpa(subs(x,t,t_result(2)),4);
y2=vpa(subs(y,t,t_result(2)),4);
x3=vpa(subs(x,t,t_result(3)),4);
y3=vpa(subs(y,t,t_result(3)),4);
x6=vpa(subs(x,t,t_result(6)),4);
y6=vpa(subs(y,t,t_result(6)),4);
solu(1,1)=(x1);
solu(2,1)=(y1);
```

```
solu(1,2)=(x2);
solu(2,2)=(y2);
solu(1,3)=(x3);
solu(2,3)=(y3);
solu(1,6)=(x6);
solu(2,6)=(y6);
for i=1:1:6
    if imag(t_result(i))==0
t=t_result(i);
cp=(1-t^2)/(1+t^2);
sp=(2*t)/(1+t^2);
phi=vpa(atan2(sp,cp),4);
 solu(3,i)=[phi];
    end
end

solu

for j=1:1:6
    if solu(1,j)~= 0
        solu(1,j)
        solu(2,j)
        solu(3,j)
inverse_kin(solu(1,j),solu(2,j),solu(3,j))
    end
end
```

```
theta1=1.5708;
```

```
theta2=2.0944;
theta3=0.698132;
% phi=rad2deg(phi)
```

Question 2 graph

```
for i=1:6
    if solu(1,i)~=0
    x=solu(1,i);
    y=solu(2,i);
    phi=solu(3,i);
    figure
    patch([x  x+r*cos(phi)  ...
        x+r*cos(pi/3+phi)], ...
```

```
        [y y+r*sin(phi) ...
        y+r*sin(pi/3+phi)], 'black', 'edgecolor', 'red');
    hold on
    plot([x R*cos(theta2) ], [y R*sin(theta2) ], 'blue', 'linewidth', 2);
    plot([R*cos(theta3) x+r*cos(phi)], ...
        [R*sin(theta3) y+r*sin(phi)], 'blue', 'linewidth', 2);
    plot([R*cos(theta1) x+r*cos(pi/3+phi)], ...
        [R*sin(theta1) y+r*sin(pi/3+phi)], 'blue', 'linewidth', 2);
    viscircles([0,0],3.5);
    axis equal
    hold off
    title(['Case',num2str(i)])
    print(['case',num2str(i),'.png'],'-dpng')
    end
end
```

Question 3

```
%robot parameters
r = 1.5;
L=3;
R=3.5;
params = [r, L, R];
%variables
x0 = [1, -0.1, 0];
xf = [0, 0.75, 90];
t = linspace(0,2,50)
n = length(t);
T = t(end);
%xplan
x = x_plan(x0,xf,t)
%inv kin
q = inv_kin(x, t, params);
%drawing
x1 = x0(1); x2 = x0(1)+r*cos(x0(3)); x3 = x0(1)+r*cos(x0(3)+deg2rad(60));
y1 = x0(2); y2 = x0(2)+r*sin(x0(3)); y3 = x0(2)+r*sin(x0(3)+deg2rad(60));


figure
hold on;
patch([x1 x2 x3], [y1 y2 y3], 'green', 'edgecolor', 'red')
plot([x1 R*cos(q(3,1)) ], [y1 R*sin(q(3,1)) ], 'blue', 'linewidth', 2);
plot([R*cos(q(5,1)) x1+r*cos(x0(3))],[R*sin(q(5,1)) y1+r*sin(x0(3))], 'blue',
'linewidth', 2);
```

```
 plot([R*cos(q(1,1)) x1+r*cos(pi/3+x0(3))],[R*sin(q(1,1)) y1+r*sin(pi/3+x0(3))],
'blue', 'linewidth', 2);

 viscircles([0,0],R);
 grid on
 axis equal

 xlim([-4.67 4.54])
 ylim([-3.65 3.61])

 figure
 for i=1:n
     x1 = x(1,i); x2 = x(1,i)+r*cos(x(3,i)); x3 =
x(1,i)+r*cos(x(3,i)+deg2rad(60));
     y1 = x(2,i); y2 = x(2,i)+r*sin(x(3,i)); y3 =
x(2,i)+r*sin(x(3,i)+deg2rad(60));
     hold on;
     p1 = patch([x1 x2 x3], [y1 y2 y3], 'green', 'edgecolor', 'red');
     p2 = plot([x1 R*cos(q(3,i)) ], [y1 R*sin(q(3,i)) ], 'blue', 'linewidth',
2);
     p3 = plot([R*cos(q(5,i)) x1+r*cos(x(3,i))],[R*sin(q(5,i))
y1+r*sin(x(3,i))], 'blue', 'linewidth', 2);
     p4 = plot([R*cos(q(1,i)) x1+r*cos(pi/3+x(3,i))],[R*sin(q(1,i))
y1+r*sin(pi/3+x(3,i))], 'blue', 'linewidth', 2);
     hold off;
     viscircles([0,0],R);
     grid on
     axis equal
     pause(T/n);
     xlim([-4.67 4.54])
     ylim([-3.65 3.61])
     grid on
     xlabel('x')
     ylabel('y')
     title = ['visualization of trajectory'];
     a(i) = norm([x1  y1] - [R*cos(q(3,i)) R*sin(q(3,i)) ]);
     b(i) = norm([R*cos(q(5,i)) R*sin(q(5,i))] - [x1+r*cos(x(3,i))
y1+r*sin(x(3,i))]);
     c(i) = norm([R*cos(q(1,i)) R*sin(q(1,i))] - [x1+r*cos(pi/3+x(3,i))
y1+r*sin(pi/3+x(3,i))]);
     delete(p1);delete(p2);delete(p3);delete(p4);
 end
 hold off;
```

Question 4:

```
clear all
syms r R L x y t theta1 theta2 theta3 phi;

A=[x,y];
B=[x+r*cos(phi),y+r*sin(phi)];
C=[x+r*cos(phi+pi/3),y+r*sin(phi+pi/3)];

D=[R*cos(theta2), R*sin(theta2)];
E=[R*cos(theta3), R*sin(theta3)];
F=[R*cos(theta1), R*sin(theta1)];

theta=[theta1, theta2, theta3];
X=[x, y, phi];

r1=A-D;
r2=B-E;
r3=C-F;

f=[sum(r1.^2)-L^2;
  sum(r2.^2)-L^2;
   sum(r3.^2)-L^2 ]
Jq=zeros(3,3);
Jq=sym(Jq);
for r=1:1:3
for i=1:1:3
   Jq(i,r)=diff(f(i),theta(r));
end
end
Jq=-Jq

Jx=zeros(3,3);
Jx=sym(Jx);
for r=1:1:3
for i=1:1:3
   Jx(i,r)=diff(f(i),X(r));
end
end
```

Question 5

```
%constants
clear all
syms x y t theta1 theta2 theta3 phi;
```

```
r = 1.5;
L=3;
R=3.5;
%initial stuff
A=[x,y];
B=[x+r*cos(phi),y+r*sin(phi)];
C=[x+r*cos(phi+pi/3),y+r*sin(phi+pi/3)];

D=[R*cos(theta2), R*sin(theta2)];
E=[R*cos(theta3), R*sin(theta3)];
F=[R*cos(theta1), R*sin(theta1)];

theta=[theta1, theta2, theta3];
X=[x, y, phi];

r1=A-D;
r2=B-E;
r3=C-F;

f=[sum(r1.^2)-L^2;
  sum(r2.^2)-L^2;
   sum(r3.^2)-L^2 ]
Jx=zeros(3,3);
Jx=sym(Jx);
for r=1:1:3
    for i=1:1:3
       Jx(i,r)=diff(f(i),X(r));
    end
end
Jx
Jq=zeros(3,3);
Jq=sym(Jq);
for r=1:1:3
for i=1:1:3
   Jq(i,r)=diff(f(i),theta(r));
end
end
Jq=-Jq

%inv kin
% x=1.5;
y=0;
% phi=0.174533; %% in rad
phi = deg2rad(10)
r = 1.5;
```

```
L = 3;
R = 3.5;

A=[x,y];
B=[x+r*cos(phi),y+r*sin(phi)];
C=[x+r*cos(phi+pi/3),y+r*sin(phi+pi/3)];

D=[R*cos(theta2), R*sin(theta2)];
E=[R*cos(theta3), R*sin(theta3)];
F=[R*cos(theta1), R*sin(theta1)];

r1=A-D
r2=B-E
r3=C-F

'check this out'
norm(r1)^2
eqn1=(sum(r1.^2)-L^2==0);
t2=vpa(solve(eqn1,theta2),4)
t21=t2(1)
t22=t2(2)
eqn2=(sum(r2.^2)-L^2==0);
t3=vpa(solve(eqn2,theta3),4)
t31 = t3(1)
t32 = t3(2)
eqn3=(sum(r3.^2)-L^2==0);
t1=vpa(solve(eqn3,theta1),4)
t11 = t1(1)
t12 = t1(2)

Jx_num = zeros(3,3,8);
Jx_num = sym(Jx_num);
Jx_num(:,:,1) = vpa(subs(Jx, {sym('phi'),sym('y'), sym('theta1'),
sym('theta2'), sym('theta3')}, {deg2rad(10),0, t11, t21, t31}), 2);
Jx_num(:,:,2) = vpa(subs(Jx, {sym('phi'),sym('y'), sym('theta1'),
sym('theta2'), sym('theta3')}, {deg2rad(10),0, t11, t21, t32}), 2);
Jx_num(:,:,3) = vpa(subs(Jx, {sym('phi'),sym('y'), sym('theta1'),
sym('theta2'), sym('theta3')}, {deg2rad(10),0, t11, t22, t31}), 2);
Jx_num(:,:,4) = vpa(subs(Jx, {sym('phi'),sym('y'), sym('theta1'),
sym('theta2'), sym('theta3')}, {deg2rad(10),0, t11, t22, t32}), 2);
Jx_num(:,:,5) = vpa(subs(Jx, {sym('phi'),sym('y'), sym('theta1'),
sym('theta2'), sym('theta3')}, {deg2rad(10),0, t12, t21, t31}), 2);
Jx_num(:,:,6) = vpa(subs(Jx, {sym('phi'),sym('y'), sym('theta1'),
sym('theta2'), sym('theta3')}, {deg2rad(10),0, t12, t21, t32}), 2);
```

```
 Jx_num(:,:,7) = vpa(subs(Jx, {sym('phi'),sym('y'), sym('theta1'),
sym('theta2'), sym('theta3')}, {deg2rad(10),0, t12, t22, t31}), 2);
 Jx_num(:,:,8) = vpa(subs(Jx, {sym('phi'),sym('y'), sym('theta1'),
sym('theta2'), sym('theta3')}, {deg2rad(10),0, t12, t22, t32}), 2);
 Jq_num = zeros(3,3,8);
 Jq_num = sym(Jx_num);
 Jq_num(:,:,1) = vpa(subs(Jq, {sym('phi'),sym('y'), sym('theta1'),
sym('theta2'), sym('theta3')}, {deg2rad(10),0, t11, t21, t31}), 2);
 Jq_num(:,:,2) = vpa(subs(Jq, {sym('phi'),sym('y'), sym('theta1'),
sym('theta2'), sym('theta3')}, {deg2rad(10),0, t11, t21, t32}), 2);
 Jq_num(:,:,3) = vpa(subs(Jq, {sym('phi'),sym('y'), sym('theta1'),
sym('theta2'), sym('theta3')}, {deg2rad(10),0, t11, t22, t31}), 2);
 Jq_num(:,:,4) = vpa(subs(Jq, {sym('phi'),sym('y'), sym('theta1'),
sym('theta2'), sym('theta3')}, {deg2rad(10),0, t11, t22, t32}), 2);
 Jq_num(:,:,5) = vpa(subs(Jq, {sym('phi'),sym('y'), sym('theta1'),
sym('theta2'), sym('theta3')}, {deg2rad(10),0, t12, t21, t31}), 2);
 Jq_num(:,:,6) = vpa(subs(Jq, {sym('phi'),sym('y'), sym('theta1'),
sym('theta2'), sym('theta3')}, {deg2rad(10),0, t12, t21, t32}), 2);
 Jq_num(:,:,7) = vpa(subs(Jq, {sym('phi'),sym('y'), sym('theta1'),
sym('theta2'), sym('theta3')}, {deg2rad(10),0, t12, t22, t31}), 2);
 Jq_num(:,:,8) = vpa(subs(Jq, {sym('phi'),sym('y'), sym('theta1'),
sym('theta2'), sym('theta3')}, {deg2rad(10),0, t12, t22, t32}), 2);
 determinantx = zeros(8,1);
 determinantx = sym(determinantx);

 for i=1:8
     determinantx(i) = det(Jx_num(:,:,i));
 end
 determinantq = zeros(8,1);
 determinantq = sym(determinantq);

 for i=1:8
     determinantq(i) = det(Jq_num(:,:,i));
 end
 solve(determinantx(1)==0,x)
 solve(determinantx(2)==0,x)
 solve(determinantx(4)==0,x)
 solve(determinantx(5)==0,x)
 solve(determinantx(7)==0,x)
 round(solve(determinantx(8)==0,x), 6)
```

```
ans =
0.556001
```

```
solve(determinantq(1)==0,x)
solve(determinantq(2)==0,x)
```

```
solve(determinantq(3)==0,x)
solve(determinantq(4)==0,x)
solve(determinantq(5)==0,x)
solve(determinantq(6)==0,x)
solve(determinantq(7)==0,x)
solve(determinantq(8)==0,x)
Jx_final = round(subs(Jx_num, x, 1.8146), 4)
for i=1:1:8
det(Jx_final(i))
end
syms v1 v2 v3 sol;
assume(sol ~= 0)
vars = [v1 v2 v3];

for i=1:1:8
 null(Jx_final(:,:,i))
end

Jx_final = round(subs(Jx_num, x, 0.556), 4)
for i=1:1:8
det(Jx_final(i))
end
syms v1 v2 v3 sol;
assume(sol ~= 0)
vars = [v1 v2 v3];

for i=1:1:8
 null(Jx_final(:,:,i))
end

%%figure
r = 1.5;
L = 3;
R = 3.5;

x=1.8146;
y=0;
phi=deg2rad(10);
solu=inverse_kin(x,y,phi)

theta2=solu(:,2)
theta3=solu(:,3)
theta1=solu(:,1)
q=zeros(3,8);
q=sym(q);
```

```
n=1;
for i=1:1:2
    for j=1:1:2
        for k=1:1:2
            q(:,n)=[theta1(i);theta2(j);theta3(k)];
            n=n+1;

        end
    end
end

for i=1:8
    figure
    patch([x  x+r*cos(phi)  ...
        x+r*cos(pi/3+phi)], ...
        [y y+r*sin(phi) ...
        y+r*sin(pi/3+phi)], 'black', 'edgecolor', 'red');
    hold on
    plot([x R*cos(q(2,i)) ], [y R*sin(q(2,i)) ], 'blue', 'linewidth', 2);
    plot([R*cos(q(3,i)) x+r*cos(phi)], ...
        [R*sin(q(3,i)) y+r*sin(phi)], 'blue', 'linewidth', 2);
    plot([R*cos(q(1,i)) x+r*cos(pi/3+phi)], ...
        [R*sin(q(1,i)) y+r*sin(pi/3+phi)], 'blue', 'linewidth', 2);
    viscircles([0,0],3.5);
    axis equal
    hold off
%     title(['Case',num2str(i)])
    print(['case',num2str(i),'.png'],'-dpng')
end
```