— Expected Time complexity of Randomized Binary Search —

Let $T(n)$ be the expected time required for n elements. After we choose one pivot the array size reduced to say k. Since pivot is choosen with equal probability ie $p = 1/n$.

Hence,

$$T(n) = p * T(1) + p * T(2) + \cdots + p * T(n) + 1$$

$$= \frac{T(1) + T(2) + \cdots + T(n)}{n} + 1$$

$$n * T(n) = T(1) + T(2) + \cdots + T(n) + n \quad \text{— (1)}$$

Similarly

$$(n-1) * T(n-1) = T(1) + T(2) + \cdots + T(n-1) + n-1 \quad \text{— (2)}$$

Subtracting (1) & (2)

$$\Rightarrow n * T(n) - (n-1) * T(n-1) = T(n) + 1$$

$$\Rightarrow (n-1) * T(n) - (n-1) * T(n-1) = 1$$

$$\Rightarrow T(n) = \frac{1}{(n-1)} + T(n-1)$$

$$\Rightarrow T(n) = \frac{1}{(n-1)} + \frac{1}{(n-2)} + T(n-2)$$

$$\Rightarrow T(n) = \frac{1}{(n-1)} + \frac{1}{(n-2)} + \frac{1}{(n-3)} + T(n-3)$$

Similtarly

$$\Rightarrow T(n) = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{(n-1)}$$

Hence $T(n)$ is $\frac{1}{(n-1)}^{th}$ harmonic no.

$$\boxed{T(n) \simeq O(\log N)}$$

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