$$z = e^{\frac{1}{2}x}(x+y^2)$$

$$\{f(x)\cdot g(x)\}' = f'(x)g(x) + f(x)g'(x)$$

$$\boxed{\begin{array}{ll} \dfrac{d}{dx}\cdot e^{\frac{1}{2}x} = \dfrac{1}{2}e^{\frac{1}{2}x}, & \dfrac{d}{dx}(x+y^2) = 1 \\ \dfrac{d}{dy}\cdot e^{\frac{1}{2}x} = 0, & \dfrac{d}{dy}(x+y^2) = 2y \end{array}}$$

$$z_x = \frac{1}{2}e^{\frac{1}{2}x}(x+y^2) + e^{\frac{1}{2}x} = e^{\frac{1}{2}x}\left(\frac{1}{2}x + \frac{1}{2}y^2 + 1\right) = 0$$

$$z_y = + e^{\frac{1}{2}x}\cdot 2y = 0$$

$z_x = \frac{1}{2}x + \frac{1}{2}y^2 + 1 = 0$　　$z_y = +2ye^{\frac{1}{2}x} = 0$

$\Leftrightarrow x + y^2 = -2$,　$\cancel{ye^{\frac{1}{2}x} = 0}$

$\Rightarrow (-2, 0)$　　$\Rightarrow \cancel{(0, 0)}$　　$\Rightarrow$ $y=0$ なら どこでも

$z_{xx} = 1$, $z_{yy} = e^{\frac{1}{2}x}$, $z_{xy} = 2y$

$$H = \begin{vmatrix} 1 & 2y \\ 2y & e^{\frac{1}{2}x} \end{vmatrix} = e^{\frac{1}{2}x} - 4y^2$$

$(-2,0)$で $H = \frac{1}{e} > 0$

~~$(0,1)$で $H = 1-4 = -3 < 0$~~

$(-2,0)$で極値をとり、z_{xx} が 1 なので極小値をとる.

$$f(-2,0) = e^{-1}(-2) = -2e^{-1} = \underline{\underline{\frac{-2}{e}}}$$