$$Z = e^{\frac{1}{2}x}(x+y^2)$$

$$\{f(x)\cdot g(x)\}' = f'(x)g(x) + f(x)g'(x)$$

$$\boxed{\begin{array}{ll} \frac{d}{dx}\cdot e^{\frac{1}{2}x} = \frac{1}{2}e^{\frac{1}{2}x}, & \frac{d}{dx}(x+y^2) = 1 \\ \frac{d}{dy}\cdot e^{\frac{1}{2}x} = 0, & \frac{d}{dy}(x+y^2) = 2y \end{array}}$$

$$Z_x = \frac{1}{2}e^{\frac{1}{2}x}(x+y^2) + e^{\frac{1}{2}x} = e^{\frac{1}{2}x}\left(\frac{1}{2}x+\frac{1}{2}y^2+1\right) = 0$$

$$Z_y = (x+y^2) + e^{\frac{1}{2}x}\cdot 2y = 0$$

$$Z_x = \frac{1}{2}x+\frac{1}{2}y^2+1 = 0 \qquad Z_y = -2 + 2ye^{\frac{1}{2}x} = 0$$

$$\Leftrightarrow x+y^2 = -2, \qquad ye^{\frac{1}{2}x} = 1$$

$$\Rightarrow (-2,0) \qquad \Rightarrow (0,1)$$

$$Z_{xx} = 1, \quad Z_{yy} = e^{\frac{1}{2}x}, \quad Z_{xy} = 2y$$

$$H = \begin{vmatrix} 1 & 2y \\ 2y & e^{\frac{1}{2}x} \end{vmatrix} = e^{\frac{1}{2}x} - 4y^2$$

$(-2,0)$ で $H = \frac{1}{e} > 0$

$(0,1)$ で $H = 1-4 = -3 < 0$

$(-2,0)$ で極値をとり、Z_{xx} が 1 なので極小値をとる。

$$f(-2,0) = e^{-1}(-2) = 2e^{-1} = \frac{2}{e}$$