$$l(\theta) = \sum_{i=1}^{m}\sum_{j=1}^{n} \log\left(\binom{p^{(i)}}{x_j^{(i)}} \alpha_{j|y=y^{(i)}}^{x_j^{(i)}} \cdot (1-\alpha_{j|y=y^{(i)}})^{p^{(i)}-x_j^{(i)}}\right) + \sum_{i=1}^{m} \log p(y^{(i)})$$

$$\frac{\partial l}{\partial \theta} = 0 \implies \frac{\partial l}{\partial \alpha_{j|y=l}} = 0 \quad \frac{\partial l}{\partial \alpha_l} = 0$$

$$\frac{\partial l}{\partial \alpha_{j|y=l}} \implies \sum_{i=1}^{m} \frac{\partial}{\partial \alpha_{j|y=l}}\left(\log\binom{p^{(i)}}{x_j^{(i)}} \alpha_{j|y=l}^{x_j^{(i)}} (1-\alpha_{j|y=l})^{p^{(i)}-x_j^{(i)}} + \sum_{i=1}^{m} \log p(y^{(i)})\right)$$

$$= \sum_{i=1}^{m}\left(\cancel{\frac{1}{\binom{p^{(i)}}{x_j^{(i)}}}} \times (0) + x_j \times \frac{1}{\alpha_{j|y=l}} + \frac{(p^{(i)}-x_j^{(i)}) \times 1}{(1-\alpha_{j|y=l})}(-1) + 0\right)$$

$$\implies \sum_{i=1}^{m}\left(\frac{x_j^{(i)}}{\alpha_{j|y=l}} - \frac{p^{(i)}-x_j^{(i)}}{(1-\alpha_{j|y=l})}\right) = 0$$

$$\sum_{i=1}^{m} \frac{x_j^{(i)}}{\alpha_{j|y=l}} = \sum_{i=1}^{m} \frac{p^{(i)}-x_j^{(i)}}{(1-\alpha_{j|y=l})}$$

$$\implies \frac{a}{\alpha_{j|y=l}} = \frac{b}{(1-\alpha_{j|y=l})}$$

$$\implies \alpha_{j|y=l} = \frac{a}{a+b} = \frac{\sum_{i=1}^{m} x_j^{(i)}}{\cancel{\sum_{i=1}^{m} x_j^{(i)}} + \sum_{i=1}^{m} (p^{(i)} - \cancel{x_j^{(i)}})}$$

$$\longrightarrow \boxed{\alpha_{j|y=l} = \frac{\sum_{i=1}^{m} x_j^{(i)}}{\sum_{i=1}^{m} p^{(i)}}} \longrightarrow \boxed{\alpha_{j|y=l} = \frac{\sum_{i=1}^{m} 1(y^{(i)}=l)\, x_j^{(i)}}{\sum_{i=1}^{m} 1(y^{(i)}=l)\, p^{(i)}}}$$

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$$l(\theta) = \sum_{i=1}^{M} \sum_{j=1}^{n} \log \binom{p^{(i)}}{x_j^{(i)}} \alpha_{j|y=y^{(i)}}^{x_j^{(i)}} \left(1-\alpha_{j|y=y^{(i)}}\right)^{p^{(i)}-x_j^{(i)}} + \sum_{i=1}^{m} \log p(y^{(i)})$$

$$\frac{\partial l}{\partial \alpha_p} = 0$$

$$\frac{\partial l}{\partial \alpha_p} = \sum_{i=1}^{M} \frac{\partial}{\partial \alpha_p} \left(\log \binom{p^{(i)}}{x_j^{(i)}} \alpha_{j|y=y^{(i)}}^{x_j^{(i)}} \left(1-\alpha_{j|y=y^{(i)}}\right)^{p^{(i)}-x_j^{(i)}} + \sum_{i=1}^{m} \log p(y^{(i)}) \right)$$

$$\Rightarrow \sum_{i=1}^{M} \left(\cancel{0} + \frac{\cancel{x_j^{(i)} 0}}{\cancel{\alpha_{j|y=y^{(i)}}}} + \frac{(p^{(i)} \cancel{- x_j^{(i)}}) \times \cancel{0}}{1-\cancel{\alpha_{j|y=y^{(i)}}}} \right) + \sum_{i=1}^{M} \frac{\partial}{\partial \alpha_p} \log p(y^{(i)})$$

$$\Rightarrow \sum_{i=1}^{M} \frac{\partial}{\partial \alpha_p} \left(\log p(y^{(i)}) \right) = 0$$

$$\boxed{\alpha_p = \frac{1}{M} \sum_{i=1}^{M} 1(y^{(i)} = p)}$$