Primal Objective function

$$L_p = \frac{1}{2}\|w\|^2 + c\sum_{i=1}^{m} \xi_i - \sum_{i=1}^{m} \alpha_i \left(y^{(i)}(w^T x_i + w_0) - 1 + \xi_i\right) - \sum_{i=1}^{m} \beta_i \xi_i$$

$$\frac{\partial L_p}{\partial w_i} = \frac{1}{\cancel{2}} \times \cancel{2} w - \sum_{i=1}^{m} \alpha_i y^{(i)} x_i = 0$$

$$\therefore \quad w = \sum_{i=1}^{m} \alpha_i y^{(i)} x_i$$

$$\frac{\partial L_p}{\partial \xi_i} = 0 + c\sum_{i=1}^{m} 1 - \sum_{i=1}^{m} \alpha_i - \sum_{i=1}^{m} \beta_i = 0$$

$$\Rightarrow \quad c - \alpha_i - \beta_i = 0$$

$$\frac{\partial L_p}{\partial w_0} = 0 + 0 - \sum_{i=1}^{m} \alpha_i y^{(i)} = 0$$

$$\Rightarrow \quad \sum_{i=1}^{m} \alpha_i y^{(i)} = 0$$

$$\therefore \quad L_p = \frac{1}{2}\left(\sum_{i=1}^{m} \alpha_i y^{(i)} x_i^T\right)\left(\sum_{i=1}^{m} \alpha_i y^{(i)} x_i\right) + c\sum_{i=1}^{m} \xi_i - \sum_{i=1}^{m} \alpha_i y^{(i)} \left(\sum_{j=1}^{m} \alpha_j y^{(j)} x_j^T\right) x_i - \sum_{i=1}^{m} \alpha_i y^{(i)} w_0 + \sum_{i=1}^{m} \alpha_i - \sum_{i=1}^{m} \alpha_i \xi_i - \sum_{i=1}^{m} \beta_i \xi_i = 0$$

from above,

$$\sum_{i=1}^{m} \alpha_i y^{(i)} = 0 \quad \& \quad c - \alpha_i - \beta_i = 0$$

$\therefore$ for Dual,

$$L_D = \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} \alpha_i \alpha_j y_i^{(i)} y^{(j)} x^{(i)T} x^{(j)} + \sum_{i=1}^{m} \alpha_i$$

Maximize L_D

s.t $\alpha_i \geq 0$, $\sum_{i=1}^{m} \alpha_i y^{(i)} = 0$

$C - \alpha_i - \beta_i = 0 \quad \forall i$

$\beta_i \geq 0$

Maximize L_D

St $0 \leq \alpha_i \leq C$ $\sum_{i=1}^{m} \alpha_i y^{(i)} = 0$

$\because \alpha_i \geq 0$

$C - \alpha_i - \beta_i = 0 \Rightarrow \beta_i = C - \alpha_i \geq 0 \Rightarrow C > \alpha_i$

$\therefore 0 \leq \alpha_i \leq C$