Experimental Determination of Planck's Constant

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Abstract

The goal of this experiment is to determine the proportionality constant between the energy of a light wave and its frequency, known as Planck's Constant. We utilize the photoelectric effect and electrical engineering techniques to measure these properties and find the value of Planck's constant.

1 Introduction

In late nineteenth and early twentieth century, Max Planck discovered an unusual phenomenon when working with black-body radiation. When trying to fit his experimental data it seemed that the energy of light waves could only take on certain values, or only change by specific increments. This was the first evidence of what we now call quantization, the idea that the physical properties of very small objects can only take on certain values. In the case of light waves, Planck realized that the energies of these waves were proportional to their frequencies, defining the following equation:

$$E = hf \tag{1}$$

The value h then became known as **Planck's Constant** and would become one of the most important values in quantum mechanics, even appearing in the famous Schrodinger equation.

The original way Planck measured his constant was through experimental data in the form of a black-body curve, but there are other methods of using light waves to measure it. In the late ninteenth century, Hertz demonstrated that electrons in a metal could be liberated using beams of light, and these electrons could be detected in the form of an electrical current [**randy**]. This phenomenon was named the **Photoelectric Effect**. It was known that a certain amount of energy was required to liberate the electrons, and in the early twentieth century, Albert Einstein demonstrated that this energy was quantized. When the electrons are liberated, a certain amount of energy is used to excite the electron to the freed state, and the surplus energy becomes kinetic energy in the electron (hence the electrical current). Einstein demonstrated that light waves and electrons had quantized energy states by showing that the kinetic energy of the electrons would only take on certain values.

$$KE = hf - \phi \tag{2}$$

Equation (2) shows the energy conservation equation that governs the photoelectric effect. As in the black-body curve, hf represents the energy of the incoming light wave, and ϕ represents the amount of energy required to liberate the electron, called the **work function**. Using the photoelectric effect, and equation (2), we can measure the work function and the frequency of the light wave, allowing us to determine Planck's constant.

2 Procedures and Data

This experiment was based on an electric circuit designed to measure the physical properties of freed electrons. Firstly, to observe the photoelectric effect, a small capacitor containing a cathode and an anode was set up opposite from an LED torch (see fig. 1). This LED torch could be configured to shine a light beam of chosen wavelength onto the cathode. This would excite electrons, hence the photoelectric effect.

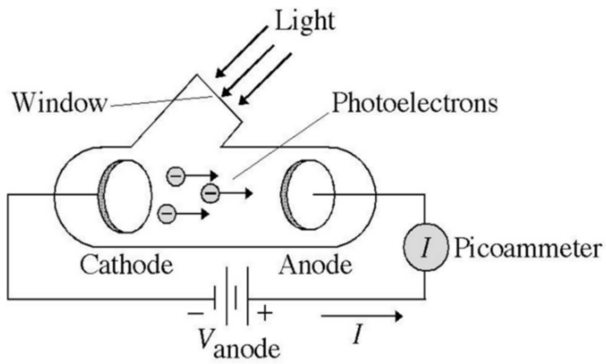

Figure 1: Experimental setup for observing the photoelectric effect (taken from the lab manual).

In addition, a complex measurement circuit was set up (see fig. 1), with a voltage supply, voltmeter, and ammeter. The voltage supply was set up in parallel with the cathode and anode, in order to supply voltage across the capacitor, and a voltmeter was set up to measure the voltage supplied. The ammeter was a special device that could measure with an accuracy of picoamps, which is the scale at which the photoelectric effect is detectable. This ammeter was set up in series with the capacitor in order to find the current from the cathode to the anode.

The procedure of the experiment was based on finding the kinetic energy of the electrons for use in equation (2). When the light beam hits the cathode, electrons are liberated from the metal and flow towards the anode with a kinetic energy described by equation (2). If we apply a voltage (V_s) across the anode and cathode, the electrons will be repelled and slow down. At the point when the voltage causes the electrons to completely stop, we have applied a potential that is equal and opposite to the kinetic energy. This potential can be measured, and thus we can find the kinetic energy.

$$KE = U = e \times V_s \tag{3}$$

In the above equation, e represents the charge of an electron, in coulombs. Thus during the experiment, different wavelengths of light were directed at the cathode, and then the energy of the photoelectrons was determined by adjusting the voltage supply across the capacitor, to find the voltage at which the current was zero.

The wavelengths were measured using emission spectroscopy, and the uncertainty was determined by the FWHM of the peak. A spectrum like Fig. 2 was taken for each wavelength tested, and then the

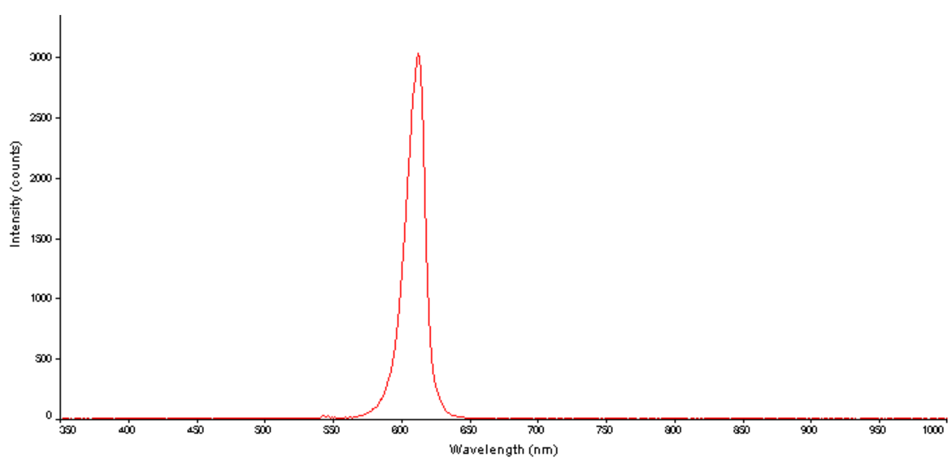

Figure 2: Emission spectrum of an LED light valued at 604nm. The peak was measured to be at 612nm with a FWHM of 16nm.

stopping voltage was measured. Table 1 contains the results of those measurements, along with the uncertainties. In addition to this, we measured the relationship between voltage and current across

Peak [nm]	**FWHM [nm]**	**Wavelength [nm]**	**Stopping Voltage [V]** ±0.001
402	19	402 ± 9.5	1.350
468	31	468 ± 15.5	0.996
507	29	507 ± 14.5	0.835
520	32	520 ± 16	0.787
569	28	569 ± 14	0.665
593	16	593 ± 8	0.615
612	16	612 ± 8	0.558

Table 1: Experimentlly measured values for the wavelength and stopping voltage.

the cathode and anode for a wavelength of 569 nanometers. Table 2 contains that data collected from around -3 volts to 3 volts in steps of about 0.5 volts. The uncertainties come from the measuring

Voltage [V] ±0.001	**Current [nA]** ±0.001	**Voltage [V]** ±0.001	**Current [nA]** ±0.001
-2.882	0.705	0.512	0.309
2.505	0.606	1.005	0.373
2.007	0.430	1.500	0.439
1.506	0.225	1.999	0.554
1.005	0.085	2.502	0.669
0.503	0.065	2.961	0.781
0	0.215		

Table 2: Measured values of the current for different supplied voltages. The table is organized in two columns.

devices. The values in tables 1 and 2 are used in the next section in SDOM analysis and linear fit analysis, in order to derive Planck's constant.

One final element to note is the factor of light intensity. The intensity of the light can affect the number of photoelectrons being freed per unit time, and therefore the current. Thus each experiment needed to have the same light intensity in order to obtain accurate results. The test for a measure of intensity was to lower the supplied voltage to zero, and measure the current. We chose 30 nA as our baseline current, and placed a iris in between the LED torch and the cathode. The iris allowed us to adjust the amount of light getting through, and thus the intensity. For each wavelength, we would set the applied voltage to zero, and adjust the iris so the measured current was 30 nA. This would 'normalize' the values so that there were no large discrepancies.

3 Analysis

To characterize the experiment in a more quantitative way, we tested the relationship between the current and voltage in the case of a single wavelength of light. The wavelength chosen was 570 nm (measured 569 nm), and produced the curve shown in Figure 3. The graph has this shape due to the direction of voltage and current. In our circuit, the voltage is applied from the anode to the cathode, and this was the direction of the current. At low voltages, the current would be in the opposite direction, since nothing was stopping photoelectrons from moving towards the anode.

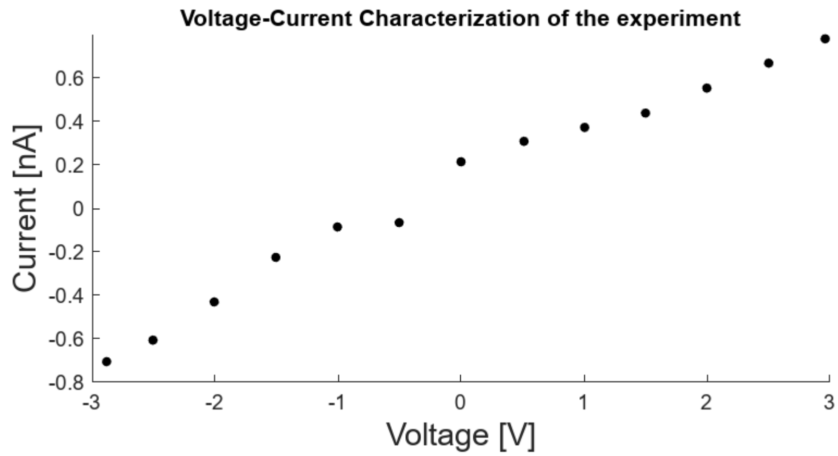

Figure 3: current-voltage characterization of the experimental setup to observe the photoelectric effect.

The main goal of this experiment was to use the photoelectric effect to measure the proportionality constant between frequency and energy of light waves. The first way this was done was through a linear fit process. The data from the previous section was used to calculate the frequency of each wavelength, and the energy for each frequency.

Wavelength [nm]	**Frequency [GHz]**	**Energy [J]**
402 ± 9.5	$7.5 \pm 0.2 \quad \times 10^5$	$2.163 \pm 0.002 \quad \times 10^{-19}$
468 ± 15.5	$6.4 \pm 0.2 \quad \times 10^5$	$1.596 \pm 0.002 \quad \times 10^{-19}$
507 ± 14.5	$5.9 \pm 0.2 \quad \times 10^5$	$1.338 \pm 0.002 \quad \times 10^{-19}$
520 ± 16	$5.8 \pm 0.2 \quad \times 10^5$	$1.261 \pm 0.002 \quad \times 10^{-19}$
569 ± 14	$5.3 \pm 0.1 \quad \times 10^5$	$1.065 \pm 0.002 \quad \times 10^{-19}$
593 ± 8	$5.06 \pm 0.07 \quad \times 10^5$	$0.985 \pm 0.002 \quad \times 10^{-19}$
612 ± 8	$4.90 \pm 0.06 \quad \times 10^5$	$0.894 \pm 0.002 \quad \times 10^{-19}$

Table 3: Calculated values for frequency and energy, in order to fit the data to equation (2)

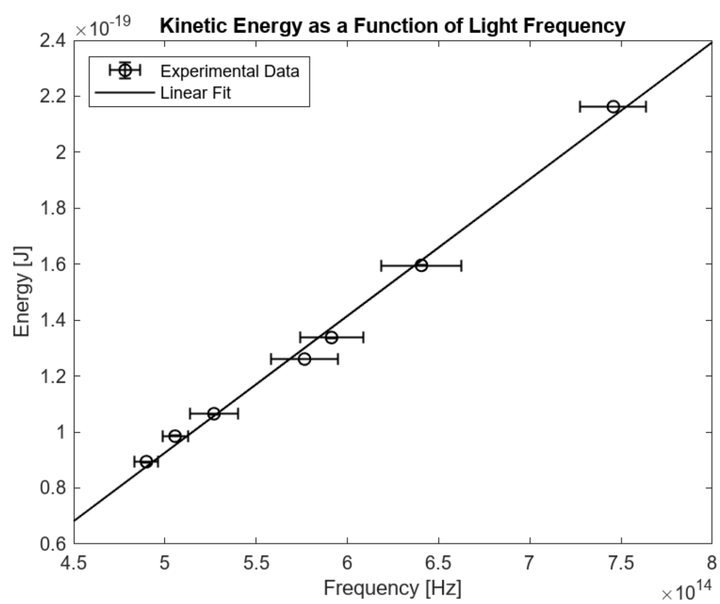

Figure 4: linear fit of energy versus frequency data, to use in finding Planck's constant. The linear fit has the equation: $4.892 \pm 0.398 \times 10^{-34} - 1.52 \times 10^{-19}$.

The graph in Fig. 4 has error bars in both the energy and frequency, but as can be seen in Tbl. 3, the error in the energy measurements is very small compared to the actual measurements. This is why they are not obvious on the graph. As stated in the caption, the linear fit has an error of aboout $\pm 0.4 \times 10^{-34}$, so the measured Planck's constant is $4.9 \pm 0.4 \quad \times 10^{-34}$ J $\cdot$ s . Interestingly, the accepted value of h, $6.62606957 \times 10^{-34}$ [1], is not within those error bounds, although it is close. The percent difference is about 26%, but with such small numbers that difference is actually quite small. The actual numerical difference is about 1.7×10^{-34} J $\cdot$ s which is incredibly small.

The linear fit actually provides more information than just Planck's constant. The y-intercept of the line indicates the **effective work function**, the energy required to liberate an electron. This is called the effective work function and not the actual work function because it is dependent on the intensity of light, and calculated based on current, which is an average over many moving photoelectrons.

In addition to the linear fit analysis, a standard deviation of the mean (SDOM) analysis was performed for a second perspective on acquiring Planck's constant. In this case, I calculated Planck's constant by equation (4) to each measurement in Table 3.

$$h = \frac{KE + \phi}{f} = \frac{eV_s + \phi}{f} \tag{4}$$

Of course, I am using the effective work function for ϕ, but since the wavelengths are the same, it will not matter too much.

Wavelength [nm]	**Planck's Constant [J $\cdot$ s]**
402 ± 9.5	$4.9 \pm 0.1 \quad \times 10^{-34}$
468 ± 15.5	$4.9 \pm 0.2 \quad \times 10^{-34}$
507 ± 14.5	$4.8 \pm 0.1 \quad \times 10^{-34}$
520 ± 16	$4.8 \pm 0.2 \quad \times 10^{-34}$
569 ± 14	$4.9 \pm 0.1 \quad \times 10^{-34}$
593 ± 8	$4.96 \pm 0.07 \quad \times 10^{-34}$
612 ± 8	$4.93 \pm 0.07 \quad \times 10^{-34}$

Mean:	4.89×10^{-34}
Standard Deviation:	0.02×10^{-34}
SDOM:	0.008×10^{-34}

Table 4: individually calculated values of Planck's constant alongside the mean, standard deviation, and SDOM

As can be seen, the calculated value for Planck's constant with this method is $4.893 \pm 0.008 \quad \times 10^{-34}$ J $\cdot$ s. This is still very close to the accepted value (although with a high percent difference), and incredibly close to the value calculated using a linear fit. It is interesting to note that the standard deviation is very small, and that is reflected in the following graph.

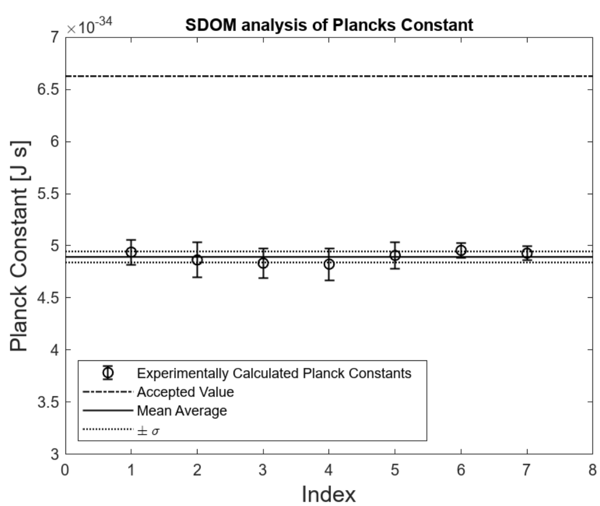

Figure 5: SDOM analysis for our measurements in order to calculate Planck's constant. The individually calculated values are plotted against the mean average (with one standard deviation of error) and the accepted value.

4 Conclusion

There are a few important points to note about this experiment. Firstly, the experimentally calculated value of Planck's constant is close to, but not within the error bounds of the accepted value. There is a high percent difference (26%) but at such small values any tiny difference is a high percent. Secondly, all the individually calculated constants (Table 4) seem practically the same, with only tiny differences. There was something potentially wrong with the experimental setup that caused all of our measurements to be slightly off. The very small standard deviation indicates we had high precision, but the percent difference may reflect a worse accuracy.

Lastly, it was briefly discussed that the light intensity plays an important role in the photoelectric effect. The number of electrons ejected is directly proportional to the light intensity, and therefore the intensity is a factor in the current. The photon model treats light as a beam of particles, and thus the higher the intensity, the more particles hitting the surface of the cathode. This would cause more electrons to eject, altering the current. This was a possible source of error in this experiment.

To conclude, Planck's constant is an extremely tiny value, and is therefore difficult to measure with a low percent difference. This experiment was no different, but we managed to get close to the accepted value.

References

[1] P. J. Mohr, B. N. Taylor, D. B. Newell, and et al., *Crc handbook of chemistry and physics*, edited by W. Haynes, 95th ed. (CRC Press, 6000 Broken Sound Parkway NW, Suite 300, Boca Raton, FL, 2014).