Experimental Determination of the Mass-Charge Ratio for Electrons

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Abstract

We can affect the motion of electrons by the use of a magnetic field, allowing us to measure physical properties of each particle. In this experiment, an electron beam is deflected by a perpendicular magnetic field in order to determine the mass to charge ratio of the electron. The electrons are accelerated through an electron gun and then curve through a magnetic field. We measure the curvature, and use that to calculate the physical properties of the electrons.

1 Introduction

The electron is one of the most important fundamental particles in physics. It is the defining particle of electricity and magnetism, and is one of the building blocks of all matter. This experiment makes use of the theories of electricity and magnetism to model an electron beam passing through a magnetic field. The curvature of such a beam would be affected by the electron mass-charge ratio and thus we can calculate the latter by measuring the former. To start, the magnetic force on an electron from an external field is given to be:

$$\vec{F} = q\vec{v} \times \vec{B} \tag{1}$$

The speed of an electron after being accelerated through a known voltage V_a can be found by setting its initial potential energy eV_a equal to its final kinetic energy $\frac{1}{2}mv^2$. The key in this experiment is to use the Hemholtz rings to create a magnetic field perpendicular to the motion of the electrons. This simplifies the cross product in equation (2) to

$$\vec{F} = qvB\sin\theta = evB \tag{2}$$

$$m\frac{v^2}{r} = evB \tag{3}$$

We can then combine equation (3) with the energy conservation equation discussed previously, giving us the mass-charge ratio.

$$\frac{e}{m} = \frac{2V_a}{B^2r^2} \tag{4}$$

Now it becomes important to find what the magnetic field is. The Hemholtz rings are two loops of conducting wire both with a radius R at a distance R apart from each other. Each ring has N turns (loops) of the wire. We want to find the magnetic field at an axis through the center of each ring, an equal distance between them, meaning at a distance $R/2$ along the axis through the center of each ring. Thankfully, the magnetic field at a distance z along such an axis is known for a single loop of wire.

$$B = \frac{\mu_0 I R^2}{2(R^2 + z^2)^{\frac{3}{2}}} \tag{5}$$

We can substitute $R/2$ for z in this equation:

$$B = \frac{\mu_0 I R^2}{2(\frac{5R^2}{4})^{\frac{3}{2}}} = \frac{\mu_0 I R^2}{2R^3\frac{\sqrt{125}}{8}} = \frac{8\mu_0 I}{D\sqrt{125}}$$

Where D is the diameter of one of the Hemholtz rings. If each ring has N turns, then we effectively have $2N$ loops of current, so we can multiply the previous result by that amount to get the total magnetic field.

$$B = \frac{16\mu_0 NI}{D\sqrt{125}} \tag{6}$$

We can substitute equation (6) into equation (4) to get a nice result:

$$\frac{e}{m} = \frac{125D^2V_a}{128(\mu_0 NIr)^2} \tag{7}$$

Equation (7) is fully in terms of measurable quantities, so we can use our experimental results to find the desired mass-charge ratio.

2 Procedures and Data

The experimental setup consisted a of mica screen placed in between two Hemholtz coils. The mica screen has a grid drawn on it for measurement purposes. In a vacuum tube electrons are released from a heated filament (called an 'electron gun') and accelerated through a high potential. Once the electrons enter the magnetic field generated by the coils, they are deflected and curves either upwards or downwards. The beam hits the screen, and the path is drawn along the grid for measurement.

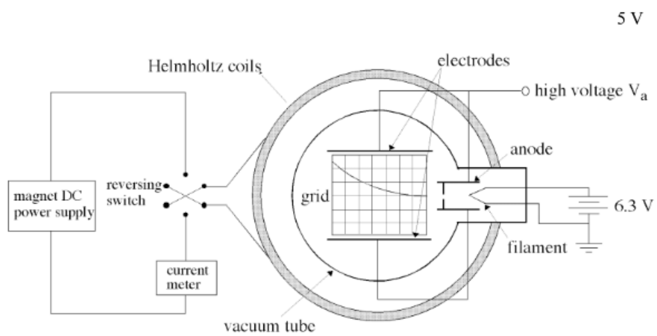

Figure 1: Experimental apparatus including Hemholtz coils, mica screen, electron gun, and circuitry to provide current and voltage.

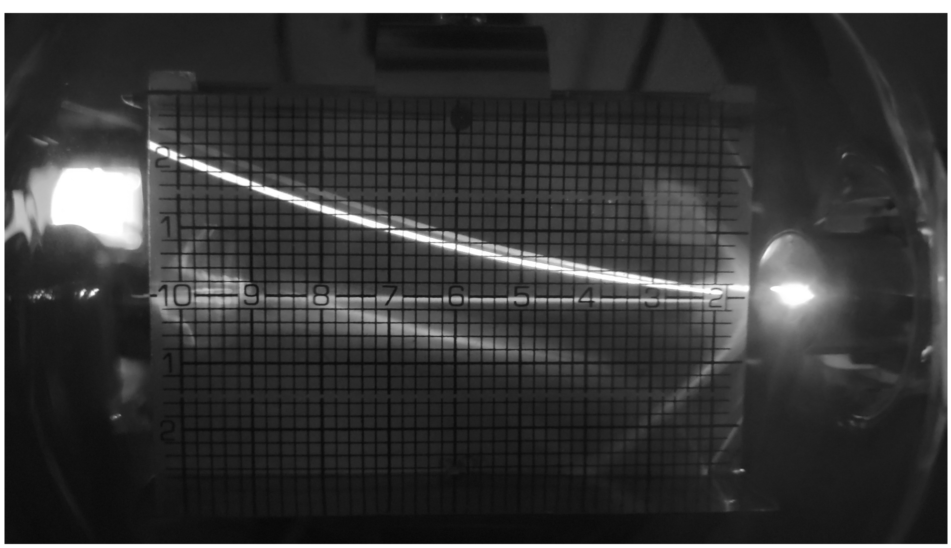

Figure 2: Mica grid with an electron beam deflected upwards across it

We measured the diameter of the coils to be 21.51 ± 0.01 cm, and each coil had 131 turns . Five trials were run, each at a different high voltage. In order to provide consistency between trials, the point (10,2) on the grid was chosen as a reference point. Before recording any data, the magnetic field was adjusted such that the curve intersected the reference point. For each voltage we collected data for deflections both upwards and downwards, so the reference point for downwards curves was (10,-2). The position of the curve was adjusted by changing the DC voltage supplied to the coils. The resulting current was measured for each trial. The up and down deflections were averaged for each voltage to get one curve for each trial. In addition, we measured the electron path when there was no voltage to get an offset. This offset was added to all data to account for the presence of any other fields (like the Earth's magnetic field). As can be seen in the table, the radii of curvature are all similar, and they

Voltage ± 1 (V)	Current ± 0.001 (A)	Fitted Radius of Curvature (cm)	Calculated e/m ratio (C/kg)
1000	0.345	28.0 ± 0.6	$1.8 \pm 0.4 \quad \times 10^{11}$
1500	0.419	27.6 ± 0.3	$1.9 \pm 0.6 \quad \times 10^{11}$
2000	0.480	26 ± 1	$2.1 \pm 0.8 \quad \times 10^{11}$
2500	0.537	28 ± 1	$1.9 \pm 0.6 \quad \times 10^{11}$
3000	0.588	29 ± 1	$1.7 \pm 0.5 \quad \times 10^{11}$

x position ± 0.01 cm	2	4	6	8	10
y-offset ± 0.01 cm	-0.05	-0.06	-0.08	-0.09	-0.15

Table 1: Measured and calculated values for this experiment. Uncertainty for the radius was calculated from MatLab's 68% confidence interval, and the uncertainty for the mass-charge ratio was found using error propagation. More discussion of this will follow in the Analysis section

were found by fitting five points from the electron trajectory to a circle in MatLab. The points were found with high accuracy by taking a photo of each trajectory, then using ImageJ software to measure distances.

3 Analysis and Discussion

The measurements of the previous section can be used to calculate the average mass-charge ratio, but first some uncertainty propagation must be performed. From chapter 3 of [1], we can use formulas to determine the uncertainty from equation (7). For simplicity, we assume N (the number of turns) has no uncertainty, and we assume the variables are independently uncertain. The uncertainy in a square variable, for example I, is given to be:

$$\delta(I^2) = 2 * I * \delta I \tag{8}$$

The same goes for other variables D and r. Equation (7) is of the form

$$\frac{D^2 \times V_a}{I^2 \times r^2}$$

And thus from error propagation techniques, the uncertainty is

$$\frac{\delta(e/m)}{e/m} = \sqrt{\left(\frac{\delta V_a}{V_a}\right)^2 + \left(\frac{\delta I^2}{I^2}\right)^2 + \left(\frac{\delta r^2}{r^2}\right)^2 + \left(\frac{\delta D^2}{D^2}\right)^2}$$

We can then substitute equation (8) into this previous one for variables I, D, and r, and get our final equation:

$$\frac{\delta(e/m)}{e/m} = \sqrt{\left(\frac{\delta V_a}{V_a}\right)^2 + 4\left(\frac{\delta I}{I}\right)^2 + 4\left(\frac{\delta r}{r}\right)^2 + 4\left(\frac{\delta D}{D}\right)^2} \tag{9}$$

Equation (9) is used to calculate the uncertainty for the charge-mass ratio in table (1). Now I can present the averaged values of the ratio and its uncertainty, to give the final experimental determination of this ratio.

$$\frac{e}{m} = 1.9 \pm 0.6 \quad \times 10^{11} (C/kg) \tag{10}$$

The accepted value for the charge-to-mass ratio of the electron is 1.75882×10^{11} [2], meaning my experimental value has a 7.2% difference.

Another way to calculate the desired value is to attempt a linear fit using equation (6). If we plot the high voltage versus the current squared.

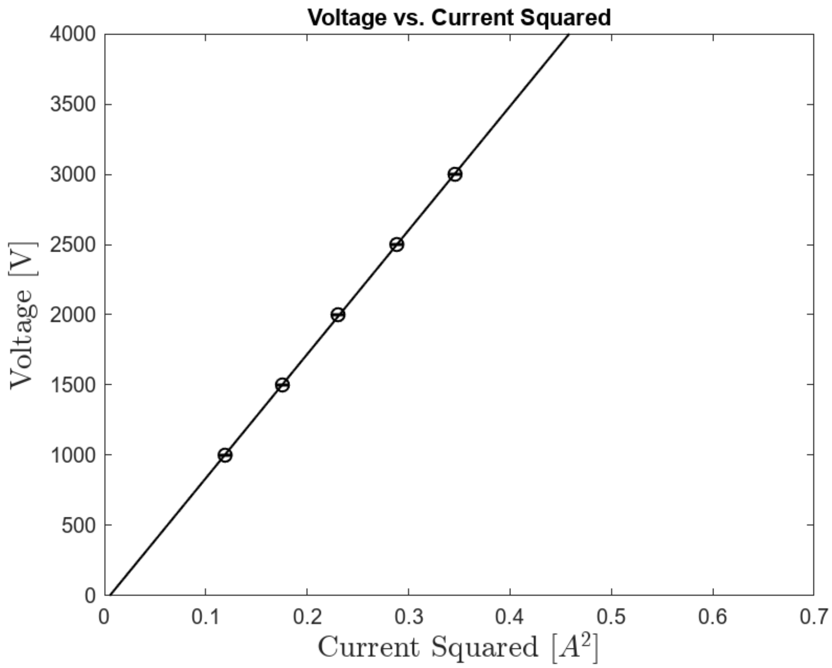

Figure 3: linear fit of z

The slope of this graph is 8829 V/A^2, and once we divide by all the constants in equation (6) in the correct way, we get that

$$\frac{e}{m} = 1.92 \times 10^{11}$$

which is of course very similar to the previously calculated value, which makes sense because the points on the graph are extremely linear.

One of the most prominent potential sources of error in this experiment was the presence of Earth's magnetic field. The speed of an electron accelerating through a 2000 volt potential can be determined by an energy conservation equation, relating potential and kinetic. Using accepted values from [2], I calculated that speed to be about 26.5 million meters per second. For this electron to have a radius of curvature of 35 cm, equation (3) can be rearranged to find the magnetic field should be 4.31×10^{-4} T.

In my experiment, the current used to generate the magnetic field when the voltage was 2000 V was 0.480 A. The magnetic field generated by the Helmholtz coils would then be found using equation (6) to be 5.26×10^{-4} which is decently similar to the theoretical value. In contrast, the Earth's magnetic field (the horizontal component) is about 2.5×10^{-5} T, an order of magnitude less. It is reasonable that this external field could affect the experiment, but only in a small way. We accounted for this by measuring an offset of the electron beam deflection with no current through the coils.

As I stated previously, the speed of these electrons is tens of millions of meters per second. This is around one tenth the speed of light, which may mean relativistic mechanics are at play. This is of course subjective, but as the speed gets faster with higher and higher voltages, the Lorentz factor becomes a significant term. Calculated using the speed at a 2000 V high voltage, the Lorentz factor comes out to be $\gamma = 1.00394$. At a speed of zero, the factor is exactly 1, so relativistic mechanics are only at play out to the third and fourth decimal places. In this experiment, with the uncertainty usually in the first decimal place, it may not be important to factor in relativity.

4 Conclusion

The charge-to-mass ratio for the electron is a quantity of great interest to all scientists, as the electron is a fundamental particle in a variety of applications. In this experiment, the charge-mass ratio of electrons was measured by using a strong magnetic field to deflect a beam of electrons. The curvature of the path is dependent on the charge-mass ratio, and thus can be used to experimentally determine it.

The value I measured was $1.9 \pm 0.6 \quad \times 10^{11}$ (C/kg), which has a percent difference of 7.2% from the accepted value. There are multiple possible sources of error discussed throughout this report. External fields are always a problem, especially the Earth's magnetic field. External fields would cause the beam to swing up or down slightly, so we measured the offset with zero current through the coils, and subtracted it off each data set to combat this. In addition, these electrons are moving at such high speeds that relativistic mechanics may be a concern, although it was calculated that this was only true for values accurate to the third decimal place, which was more accurate than the uncertainty.

References

[1] J. R. Taylor, *An introduction to error analysis :* 2nd ed. (University Science Books, Mill Valley, Calif : c1982.).

[2] P. J. Mohr, B. N. Taylor, D. B. Newell, and et al., *Crc handbook of chemistry and physics*, edited by W. Haynes, 95th ed. (CRC Press, 6000 Broken Sound Parkway NW, Suite 300, Boca Raton, FL, 2014).