Design and Testing of Parallel RLC Frequency Filters

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***Abstract*—The design and implementation of RLC (resistor, inductor, and capacitor) circuits is critical to the understanding of time dependent components and their applications in modern electrical systems. In particular, parallel combinations of these components can be used to design a variety of different frequency based filters for use in signal processing and information transmission. This report covers the experimental implementation of two different frequency filters in order to test the theoretical equations for natural frequencies and quality factors.**

I. INTRODUCTION

Sinusoidal signals form the basis for most modern electrical communication. Information can be encoded in the amplitudes and frequencies of different sinusoids, and the ability to filter out certain frequencies is vital to operations in signal processing, such as noise reduction.

These types of circuits can be created using only passive components: resistors, capacitors, and inductors in parallel. The parallel RLC forms of these filters are sometimes called **2nd order**, there are four main filters that are designed in this fashion, two of which will be explored in this report.

A. Low-Pass Filtering

The first circuit is called a **low-pass (LP)** filter, and as the name suggests, it is designed to allow lower frequencies and block high frequencies. The second order low-pass filter is designed using a resistor and capacitor in parallel, attached to an inductor, with the output taken across the capacitor.

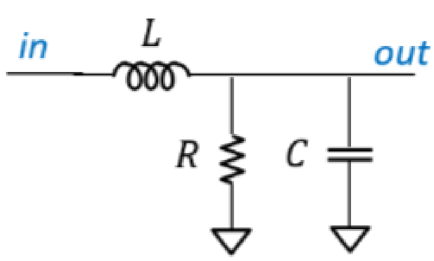

Fig. 1. a circuit diagram of a 2nd order passive low-pass filter

Figure 1 depicts the conventional design for a 2nd order low-pass filter using a resistor, inductor, and a capacitor. Low frequencies allow the capacitor to charge and discharge, passing the signal, while high frequencies are not registered across the capacitor. RLC circuits are best described by a laplace transfer function which can be used to find the poles and zeros, and certain properties of the topology. The transfer function is given in the following equation [1].

$$\mathcal{H} = \frac{1}{s^2LC + s\frac{L}{R} + 1} \tag{1}$$

This transfer function shows no zeroes and two poles. The transfer function can be converted into a complex representation like so:

$$\mathcal{H}(i\omega) = \frac{\omega_n^2}{(\omega_n^2 - \omega^2) + i\frac{\omega_n}{Q}\omega} \tag{2}$$

where ω_n is a the natural frequency of the circuit, and Q is the quality factor. The magnitude and frequency response of this circuit provide method of determining the quality factor and natural frequency.

At certain values of the RLC components this topology exhibits what is known as **overshoot** in response to a step input. In other words, the voltage output shoots above the step input and peaks. The time to first peak is related to the damping frequency:

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \tag{3}$$

ζ is the **damping ratio**, which is a different representation of the quality factor: $\zeta = \frac{1}{2Q}$. The time to first peak is easily measured on an oscilloscope, allowing for an experimental determination of the natural frequency. The quality factor can be calculated using the RLC component values [1]:

$$Q = \frac{R}{\sqrt{L/C}} \tag{4}$$

A step response is not sufficient to experimentally measure the quality factor of this type of circuit. The frequency response, or the response to a sinusoidal input provides insight into the quality factor and natural frequency of the circuit. It is known [**?**] that the natural frequency is the frequency at which the output experiences a ninety degree phase shift compared to the input. The quality factor is the magnitude response at the natural frequency.

For comparison, the natural frequency of a parallel RLC circuit can be expressed as [2]

$$\omega_n = \frac{1}{\sqrt{LC}} \tag{5}$$

This can be calculated from the circuit components and will be used to compare with the experimentally determined value.

The natural frequency is also sometimes called the resonant frequency, as it is the condition for what is known as **resonance**. If the driving (input) frequency is the same as the natural frequency, a maximum response is observed, a phenomenon called resonance.

B. Band-Pass Filtering

The second kind of filter constructed in this lab was a **band-pass (BP)** filter. The second order representation also includes parallel RLC components, which can be described in a similar manner.

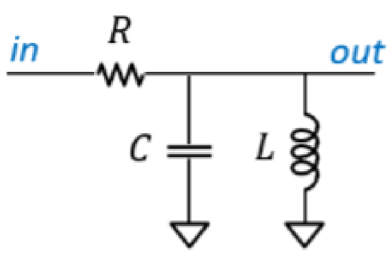

Fig. 2. circuit topology of a second order band-pass filter

Figure 2 depicts the design of a passive band-pass filter. The capacitor blocks high frequencies and the inductor blocks low frequencies, creating a small 'band' of frequencies that both will pass. The transfer function for this topology is given to be [1]:

$$\mathcal{H}(s) = \frac{s\frac{L}{R}}{s^2LC + s\frac{L}{R} + 1} \tag{6}$$

This function contains two poles and one zero. As before, this function can be transformed into the complex domain to garner insight about the frequencies.

$$\mathcal{H}(i\omega) = \frac{i\frac{\omega_n}{Q}\omega}{(\omega_n^2 - \omega^2) + i\frac{\omega_n}{Q}\omega} \tag{7}$$

The parameters of the circuit can be determined in a similar fashion to the low-pass filter, with slightly different specifications. The frequency response of the second order band-pass topology can provide a method of determining both the quality factor and the natural frequency. The natural frequency is the frequency at which the output phase shift is zero [1]. In addition, a band pass filter has a 'low frequency' ω_L and a 'high frequency' ω_H, which can be used to calculate the quality factor like so: [1]

$$Q = \frac{\sqrt{\omega_L\omega_H}}{\omega_H - \omega_L} \tag{8}$$

These characteristic frequencies are defined as the frequencies at which the phase responses of the circuit are -45 and 45 degrees, which are measurable.

For theoretical predictions of these values, equations (4) and (5) apply for this circuit as well, since it is also an example of a parallel RLC circuit. The theoretical quality factor and natural (resonant) frequency will be calculated from measured values of the components.

II. Experimental Procedure

To validate theoretical predictions, the circuits in figures (1) and (2) were constructed on a breadboard in order to observe the step and frequency responses.

A. Experimental Parameters

The circuits were built and tested with three different resistance values, of 1.5k, 510, and 390 ohms. The actual values were measured to be 1.468 kΩ, 501.7 Ω, and 384.8 Ω, as seen on the digital multimeter [3]. In addition, the values of the capacitor and inductor were determined to be 10 nF and 10 mH. These values were used to calculate the theoretical quality factors and natural frequencies discussed in the introductory section.

B. Low-Pass Filter

The circuit depicted in Figure (1) was constructed and tested with different inputs to understand the properties of the topology. First, a step input of 0 V to 1 V was applied at the input, and the voltage was taken across the capacitor, and displayed on an oscilloscope [4].

Fig. 3. step response of a low-pass filter exhibiting overshoot and damped oscillation, helping to determine the natural frequency

The time to the first peak can be used to find the natural frequency, as per equation (3). The oscilloscope was used to measure the time to first peak as 32 μs. The percent overshoot was measured to be 28.9%. Using the time to first peak, the natural frequency was calculated to be 16.603 kHz, or 104 krad/s. The ζ in equation (3) was calculated using the values in section 2.A, and equation (4). This measurement was repeated for the 510 Ω and 390 Ω resistors, but no overshoot was observed due to overdamping. The equation (3) only applies to a response with overshoot, so the natural frequency could not be calculated that way.

After exploring the step response, the frequency response was observed by inputting sinusoids of varying frequencies. As stated before, the magnitude response at the natural frequency is equal to the quality factor. By varying the input frequency, the natural frequency was found by observing a ninety degree phase shift in the output, and the peak to peak value was measured to find the magnitude response.

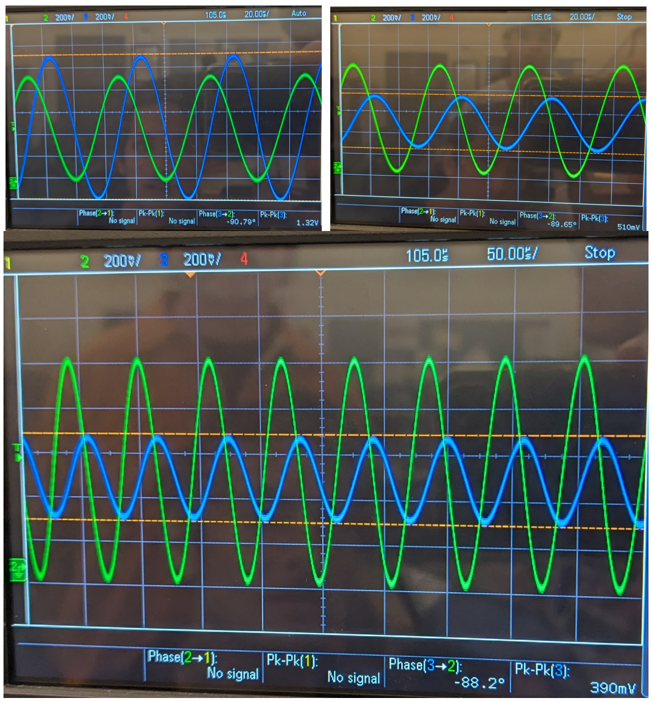

Fig. 4. frequency responses to sinusoidal inputs at the resonant frequency, exhibiting a 90 degree phase shift. The input is shown in green and the output is shown in blue. The responses are shown for three resistances, 1.5 kΩ (top left), 510 Ω (top right), 390 Ω (bottom).

TABLE I
MEASURED MAGNITUDE RESPONSES FOR THE RESONANT FREQUENCY OF THE LOW PASS FILTER

Resistance	Frequency (kHz)	Peak to Peak (V)	Quality Factor
1500 Ω	16.7	1.32	1.47
510 Ω	17	0.510	0.505
390 Ω	17	0.390	0.388

One can already see that the results make sense as the frequencies in the table are close to the calculated value from the overshoot. More will be discussed in section III.

C. Band-Pass Filter

For the band-pass filter, the step response was observed on the scope, to see the type of oscillations. For the higher resistor value (1.5 kΩ) the output response depicted over damped oscillations. The other resistances (510 Ω and 390 Ω) exhibited critical damping, meaning there was only one oscillation before the curve dipped back to equilibrium.

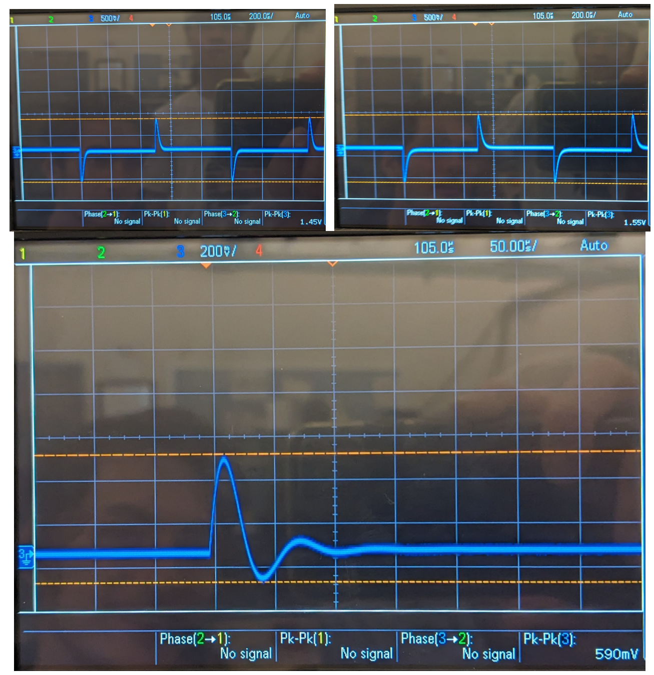

Fig. 5. step responses of the band pass filter to an input square wave. Scope captures taken for three different resistances: 1.5k (bottom), 510 ohm (top left), and 390 ohm (top right).

This behavior is shown in figure 5, but the step responses were not used in any calculations. To calculate the quality factors and the natural frequency, the frequency responses were analyzed for specific parameters. To find the desired parameters, responses were observed to input sinusoids of varying frequencies. The desired frequencies were those that gave 45, 0, and -45 degrees of phase shift in the output, corresponding to the ω_H, ω_n, and ω_L frequencies that can be used to calculate the quality factor.

TABLE II
FREQUENCY REPSPONSES OF THE BAND PASS FILTER

Resistance	-45° shift	0° shift	+45° shift	Quality Factor
1.5 $k\Omega$	23.3 kHz	16.7 kHz	500 Hz	1.47
510 Ω	40 kHz	16.8 kHz	6.7 kHz	0.505
390 Ω	50 kHz	16.6 kHz	4.8 kHz	0.388

Notice that the quality factors are the same as in table 1, an excellent verification of equation (4) as the circuits were built with the same components. The quality factors in table (2) were calculated using equation (8).

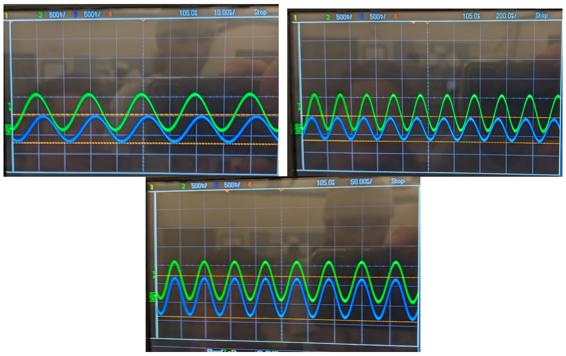

Fig. 6. frequency response (blue) of the BP filter to a sinusoidal input (green)

The frequency responses to different sinusoidal inputs are shown in figure (6) for the 390 Ohm resistor.

III. Results

The results of this experiment clearly support the theory laid out in section 1. The same inductor and capacitor were used for every circuit, and thus according to equation (5) the natural frequency should be the same for each experiment. The theoretical value for the natural frequency was calculated to be

$$\omega_n = 100 \text{ krad/s} = 15.92 \text{ kHz}$$

as can be seen in the experimental results (tables 1 & 2) the measured natural frequencies match up with the theoretical results, with the most inaccurate value having a percent difference of 6.8%.

The quality factor could be predicted using equation (4). Since this value depended on the resistance the quality factor was different for each circuit constructed. The following table contains measured values alongside theoretical values for the quality factor, to compare the experimental results to prediction.

TABLE III
Comparison of Experimental and Theoretical Quality Factors in Second Order Filters

Theoretical Value	Measured Value (LP)	Measured Value (BP)
1.5	1.47	1.47
0.51	0.505	0.505
0.39	0.388	0.388

All of the measured values are within a 2% difference from the theoretically predicted values.

IV. Conclusion

The field of electrical filters is a basis for many applications in the areas of signal processing, communication, and scientific instrumentation. There are many different types of filters, and this report focuses on frequency filtering, the blocking of specific ranges of frequencies from an AC input signal. Of the varied implementations, the passive designs are second order LTI circuits that utilize resistors, capacitors, and inductors.

Passive filters using time dependent components can be characterized by two main parameters, the natural frequency and the quality factor. In this experiment, different inputs were used to observe responses that could be used to determine these parameters. The natural frequency of a parallel RLC circuit can be determined based on the step and phase responses of the circuit. For the low-pass circuit, the magnitude response to a sinudoidal input was equal to the quality factor, while for the band-pass circuit the natural frequency occurred at a specific phase response.

The quality factor and natural frequency were determined for two different passive frequency filters by constructing the circuits and testing sinusoids and step function inputs while observing the responses. The circuits were tested with different resistances for a more comprehensive experiment. The quality factors for each circuit were experimentally determined within 2% difference from the theoretically predicted values, and the natural frequency was determined to within 7% of the predicted value.

The equations (1) through (8) were confirmed through the data and analysis of two second order frequency filters in this experiment, endorsing the circuit theory describing this scenario.

References

[1] N. Ellis and V. Prodanov, *Week #6: Natural Frequency and Quality Factor of a Parallel RLC Circuit*, 2024.

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