Observation of the Zeeman Effect in Cadmium Atoms

Alex Short

February 20, 2024

Abstract

1 Introduction

Atoms consist of a nucleus orbited by an electron, both of which interact magnetically. Electrons orbiting a nucleus have a magnetic moment given by

$$\boldsymbol{\mu} = \frac{2\pi\mu_B\mathbf{J}}{h} \tag{1}$$

where μ_B is the **Bohr Magneton** [1]. The vector $\mathbf{J}$ is the angular momentum of the electron, for which the direction is quantized depending on the energy state. If an electron is placed into an external magnetic field $\mathbf{B}_0$, then the energy state of the electron splits into multiple, as a magnetic moment in a field has energy given by [2]

$$U = -\boldsymbol{\mu} \cdot \mathbf{B}_0 \tag{2}$$

The energy depends on the direction of the magnetic moment, which is the same direction of the angular momentum $\mathbf{J}$. For the lowest energy state, the angular momentum can only point in two directions, usually denoted as the positive and negative z direction. Therefore the lowest energy state splits into two magnetic states [1], which are denoted by the magnetic quantum number m. This phenomenon is called the **Zeeman Effect**.

Transitions between these states are accompanied by the emission or absorption of photons with energies equal to the energy split ΔE. These photons can be observed, and thus the Zeeman effect can be seen through optical methods.

It's important to note that while the two new states are created, the original ground state is still possible, as the magnetic quantum number can be $-1, 0, 1$ in first energy state. Thus photons will still be emitted with energies corresponding to the original state. These photons can be differentiated using a polarizer. If the magnetic quantum number is -1 or 1, the light waves will be polarized in the same plane, although one will be the opposite of the other. However, if the magnetic quantum number is 0, then the light wave will be polarized ninety degrees relative to that plane. Therefore the unwanted photons can be filtered out by choosing the correct polarizer setting.

1.1 The Optics

The optical setup (as will be described in section 2) contains what is called a **Fabry-Perot inferometeter** which refracts the incoming light beams a number of times to create a very visible pattern. The measurement apparatus captures the angles of incoming light beams have the energies that represent the state splitting. From basic quantum mechanics, the energy of a light wave is [1]

$$E = \frac{hc}{\lambda} \tag{3}$$

We can approximate for ΔE by differentiating that equation to get

$$\Delta E \approx -\frac{hc\Delta\lambda}{\lambda^2} = -\frac{hc}{\lambda}\frac{\Delta\lambda}{\lambda} \tag{4}$$

The second term in this equation can be found through measurement. Fabry-Perot theory states that an inferometer will show a maximum at a wavelength of

$$\lambda = \frac{2nt}{m} \cos\beta \tag{5}$$

The change in wavelength $\Delta\lambda$ is simply the difference between the wavelength emitted from the lowest energy state and the wavelength emitted when the states are split. *beta* in equation (5) represents the angle of the light beam through the inferometer, which is refracted at the border into a new angle, measured by the camera. The measured angle, called α, can be related to β using the indices of refraction and **Snell's Law**.

Suppose we measure an angle when there is no magnetic field. Let's call this α_1. This represents our base state, with no splitting. After applying an external field B_0 the states split and we measure a photon with a new energy, at a different angle, α_2. From Snell's Law, we can relate the angle α which is measured through the air to the angle β which is through the inferometer:

$$n_{\text{air}} \sin\alpha = n_{\text{FP}} \sin\beta \tag{6}$$

Therefore,

$$\beta = \sin^{-1}\left(\frac{n_{\text{air}}}{n_{\text{FP}}} \sin\alpha\right) \tag{7}$$

Now we can find the wavelength *lambda* in each situation.

$$\Delta\lambda = \lambda_2 - \lambda_1 = \frac{2nt}{m}(\cos\beta_2 - \cos\beta_1) \tag{8}$$

Dividing by λ removes the outside constant and gives us the desired expression:

$$\frac{\Delta\lambda}{\lambda} = \frac{\cos\beta_2 - \cos\beta_1}{\cos\beta_1} \tag{9}$$

Thus we can use equation (4) to find the energy splitting, which we can verify by determining the experimental value of the Bohr Magneton.

2 Experimental Setup and Procedure

In order to observe the Zeeman effect, a large electromagnet was used to generate a strong magnetic field inside of a cadmium lamp. This lamo continuously emitted photons through a series of lenses to reach the camera, where the measured angles were recorded. The emitted light beams traveled through a condenser lens (focal length 150 mm), then the inferometer, then another lens, this time for imaging, then through an interference filter, and finally to the camera [3].

The first test was to observe the quantum states splitting with the eye. An image was taken without the magnetic field, as seen in Figure (1). The distinct bulls-eye pattern represents the emitted photons having quantized energy states, reflected a number of times by the inferometer. The camera captures the bulls-eye and converts the radii of the rings to angular distance, allowing us to calculate the energy.

When the magnetic field is turned on, the energy states split into three distinct states, the original one and higher and lower states due to the magnetic field. These can also be observed through the lens, as a more interesting bulls-eye pattern, seen in Figure (2).

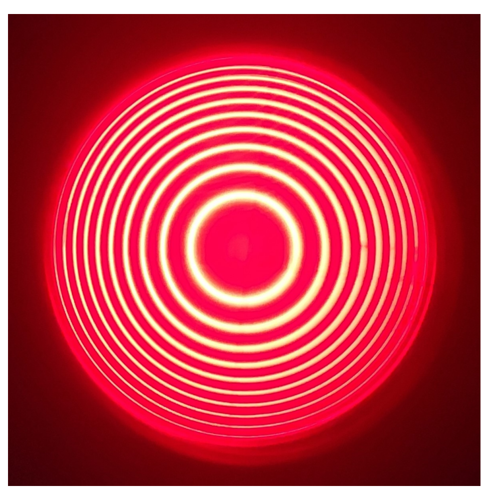
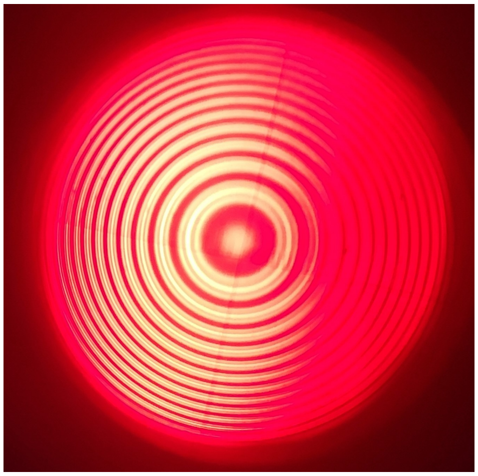


Figure 1: bulls-eye pattern of emitted light with no magnetic field present



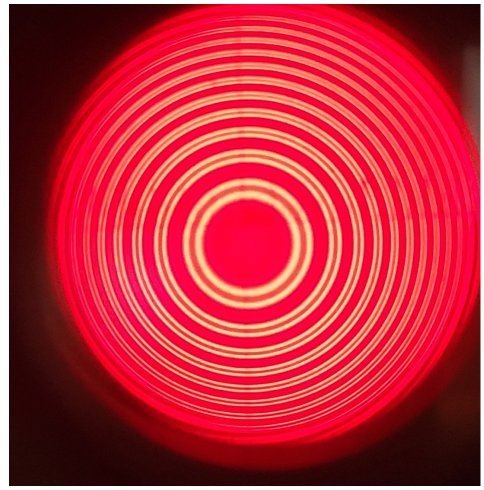


Figure 2: bulls-eye pattern of photons emitted from the cadmium lamp. The left image shows three distinct states, as it is without the polarizer. The right image shows the rings splitting into two distict states as the polarizer is placed into the setup at an angle of around 45 degrees

To observe the splitting more accurately, we would like to filter out the center line to observe the two shifted lines. This can be done by inserting a polarizer into the apparatus. The polarizer only allows light oscillating in a specific plane to pass through, and once calibrated to the plane of the shifted beams filters out the center beam. The polarizer was inserted between the cadmium lamp and the rest of the lenses in order to better see the shifted light.

The first step in conducting this experiment was to measure the magnetic field for different currents. The controlled variable is the current supplied to the electromagnet, but the desired relationship is between the magnetic field and the energy shifting, so we need to know the magnetic field at different supplied currents.

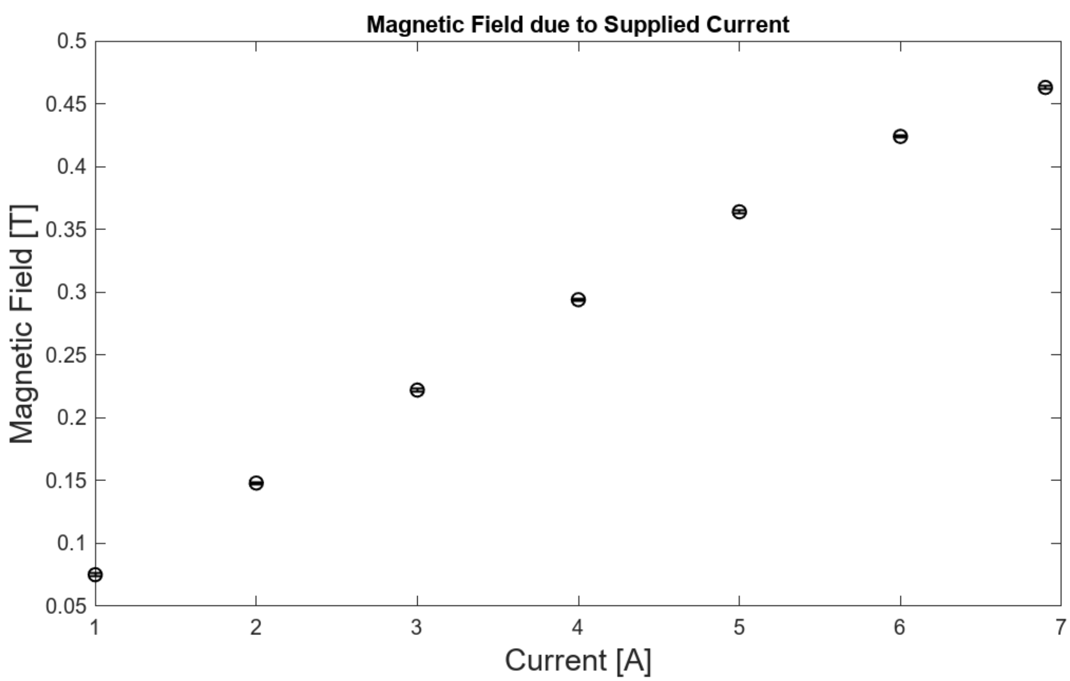


Figure 3: magnetic field as a function of current for future use in determining the Bohr Magneton

3 Results & Data

When the bulls-eye was interpreted by the camera, a plot of intensity as a function of angle was created, with each peak representing a ring, and therefore an energy state. Once the magnetic field was turned on, the peaks would split into two peaks representing the shifted energy states.

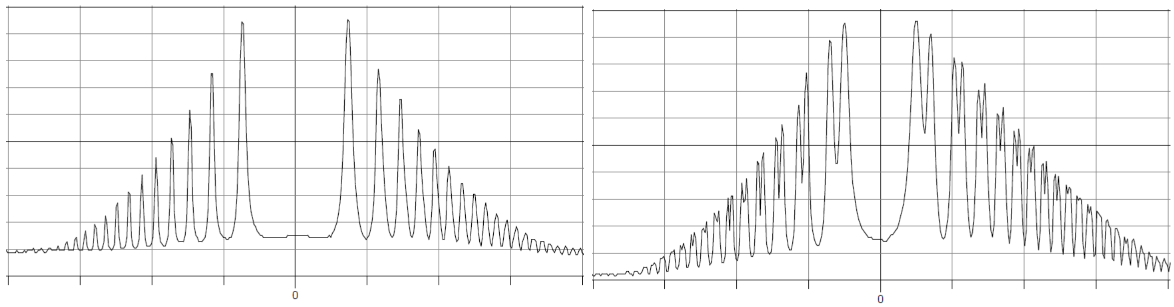


Figure 4: intensity plot of the cadmium lamp spectrum. The left image shows the output when there was no magnetic field, each state displays one distinct peak. The right image shows the light beams when the external magnetic field was present, displaying two distinct peaks for each state

The x-axis in these plots represents the angle α discussed in section 1.1, and therefore can be used to calculate the energy shift and the Bohr Magneton. Each peak in the left image has a corresponding up and down shift represented in the right image, and the angle shifts between those peaks each can be used to find our desired results. Multiple peaks were used to analyze the energy splitting in the Zeeman Effect, in order to get a more accurate calculation for the Bohr Magneton.

Before the data can be analyzed an uncertainty analysis must be performed for the energy shift equation (equation (4)). For the inferometer angle β, one can use equation 3.23 from [4] to calculate the uncertainty as

$$\delta\beta = \frac{n\cos\alpha}{\sqrt{1-n^2\sin^2\alpha}}\delta\alpha \tag{10}$$

Then, using the uncertainty for β we can calculate the uncertainty in the energy shift to be (using equation 3.47 in [4]):

$$\delta(\Delta E) = -\frac{hc}{\lambda}\sqrt{(\cos\beta_2\sec\beta_1\tan\beta_1)^2\delta\beta_1^2 + (-\sin\beta_2\sec\beta_1)^2\delta\beta_2^2} \tag{11}$$

We took data for four peaks on the spectrum in order to obtain the relationship between the magnetic field and the energy shift. From this point on all angle measurements are in degrees.

The first peak (we'll call this peak 1) was the first (tallest) peak on the left side of the spectrum. It had a measured base angle of $\alpha_1 = -0.5968 \pm 0.0367$. This resulted in a calculated $\beta_1 = -0.41 \pm 0.03$.

Current [A]	**Upshifted Angle**	**Downshifted Angle**	$\boldsymbol{\beta_2}$ **(up)**	$\boldsymbol{\beta_2}$ **(down)**
2	-0.5505 ± 0.0107	-0.6446 ± 0.0107	-0.38 ± 0.01	-0.44 ± 0.01
3	-0.5270 ± 0.0165	-0.6683 ± 0.0165	-0.36 ± 0.01	-0.46 ± 0.01
4	-0.4929 ± 0.0368	-0.6915 ± 0.0254	-0.34 ± 0.03	-0.47 ± 0.02
5	-0.4578 ± 0.0342	-0.7112 ± 0.0258	-0.31 ± 0.02	-0.49 ± 0.02
6	-0.4297 ± 0.0464	-0.7249 ± 0.0293	-0.29 ± 0.03	-0.50 ± 0.02
6.9	-0.4105 ± 0.0472	-0.7361 ± 0.0323	-0.28 ± 0.03	-0.50 ± 0.03

Table 1: Caption

The second peak (called peak 2) was directly to the left of peak 1, in other words it was the second peak from the center on the left side. The base angle was measured to be $\alpha_1 = -1.0840 \pm 0.0259$. The β_1 was calculated to be -0.74 ± 0.02

Current [A]	Upshifted Angle	Downshifted Angle	β_2 (up)	β_2 (down)
2	-1.0617 ± 0.0107	-1.1045 ± 0.0107	-0.73 ± 0.01	0.76 ± 0.01
3	-1.0395 ± 0.0107	-1.1152 ± 0.0107	-0.71 ± 0.01	0.76 ± 0.01
4	-1.0295 ± 0.0166	-1.1376 ± 0.0166	-0.71 ± 0.01	0.78 ± 0.01
5	-1.0173 ± 0.0204	-1.1505 ± 0.0205	-0.70 ± 0.01	0.79 ± 0.01
6	-1.0057 ± 0.0237	-1.1617 ± 0.0231	-0.69 ± 0.02	0.80 ± 0.02
6.9	-0.9980 ± 0.0227	-1.1653 ± 0.0227	-0.68 ± 0.02	0.80 ± 0.02

Table 2: Caption

The third peak (peak 3) was the first peak on the right side of the spectrum, the tallest peak on the right. The base angle was measured to be $\alpha_1 = 0.5894 \pm 0.0367$. The calculated β from that was $\beta_1 = 0.41 \pm 0.03$.

Current [A]	Upshifted Angle	Downshifted Angle	β_2 (up)	β_2 (down)
2	0.6837 ± 0.0107	0.5635 ± 0.0107	0.44 ± 0.01	0.39 ± 0.01
3	0.6632 ± 0.0263	0.5293 ± 0.0161	0.45 ± 0.02	0.36 ± 0.01
4	0.6943 ± 0.0351	0.4905 ± 0.0215	0.48 ± 0.02	0.34 ± 0.01
5	0.7178 ± 0.0426	0.4672 ± 0.0295	0.49 ± 0.03	0.32 ± 0.02
6	0.7358 ± 0.0472	0.4372 ± 0.0293	0.50 ± 0.03	0.30 ± 0.02
6.9	0.7424 ± 0.0509	0.4264 ± 0.0357	0.51 ± 0.03	0.29 ± 0.02

Table 3: Caption

The fourth and final peak was the second peak on the right (peak 4), the second tallest peak on the right side of the spectrum. The base angle was measured to be $\alpha_1 = 1.0887 \pm 0.0336$.

Current [A]	Upshifted Angle	Downshifted Angle	β_2 (up)	β_2 (down)
3	1.1207 ± 0.0106	1.0547 ± 0.0107	0.77 ± 0.01	0.72 ± 0.01
4	1.1361 ± 0.0150	1.0305 ± 0.0211	0.78 ± 0.01	0.71 ± 0.01
5	1.1523 ± 0.0165	1.0180 ± 0.0248	0.79 ± 0.01	0.70 ± 0.02
6	1.1613 ± 0.0205	1.0074 ± 0.0284	0.80 ± 0.01	0.69 ± 0.02
6.9	1.1709 ± 0.0241	1.0002 ± 0.0252	0.80 ± 0.02	0.69 ± 0.02

Table 4: Caption

4 Analysis & Discussion

The results in tables 1 through 4 were used to calculate the energy shift relations for the four peaks, resulting in concrete relationships between the external magnetic field and the energy shift of a split peak. The Bohr Magneton was extracted from these graphs and linear fits.

B [T]	ΔE Up [μeV]	ΔE Down [μeV]	ΔE Up [μeV]	ΔE Down [μeV]	ΔE Up [μeV]	ΔE Down [μeV]	ΔE Up [μeV]	ΔE Down [μeV]
0.148	7.3 ± 6	-8.2 ± 6	6.6 ± 8	-6.2 ± 8	6.7 ± 6	-5.6 ± 6	9.7 ± 10	-10.0 ± 10
0.222	10.8 ± 6	-12.4 ± 7	13.0 ± 8	-9.4 ± 8	11.3 ± 8	-10.7 ± 6	14.5 ± 11	-17.0 ± 12
0.294	15.6 ± 8	-16.8 ± 8	15.9 ± 9	-16.4 ± 9	17.1 ± 9	-16.2 ± 7	19.6 ± 11	-20.5 ± 12
0.364	20.2 ± 7	-20.6 ± 8	19.3 ± 10	-20.4 ± 10	21.6 ± 10	-19.2 ± 7	23.1 ± 12	-23.5 ± 13
0.424	23.6 ± 8	-23.3 ± 8	22.5 ± 10	-24.0 ± 11	25.2 ± 11	-23.0 ± 7	25.6 ± 13	-25.4 ± 12
0.463	25.8 ± 8	-25.6 ± 10	24.6 ± 10	-25.2 ± 11	26.6 ± 12	-24.3 ± 7		

Table 5: energy shifts as a function of the magnetic field for each peak, from one to four

Recall equation (1). If one realizes that the angular momentum is just the reduced planck's constant times the angular momentum quantum number, then one can make a series of substitutions to gain a simpler form of this equation. The angular momentum **J** is equal to $\hbar\sqrt{j(j+1)}$ [1]. In the lowest state, j can only be 0, meaning **J** is $\mathbf{J} = \hbar$. If this is the case, the magnetic moment of the electron reduced to a very simple form:

$$\boldsymbol{\mu} = \mu_B$$

. It is equal to the Bohr Magneton. Combining this with equation (2) yields an important equation:

$$\Delta E = -\mu_B B_0 \tag{12}$$

In other words, the slope of a ΔE vs. B_0 plot would be the Bohr Magneton, exactly the relationship we have in the previous tables. The above tables (table 5) display the energy shifting up or down, but we really want to measure the *split* of the states, and if there is a systematic shift in the optical equipment and light beams, then the angle data may be systematically off. Therefore it makes sense to average the slopes of the up and down shifted data, as the average energy shift will give a more accurately centered data set.

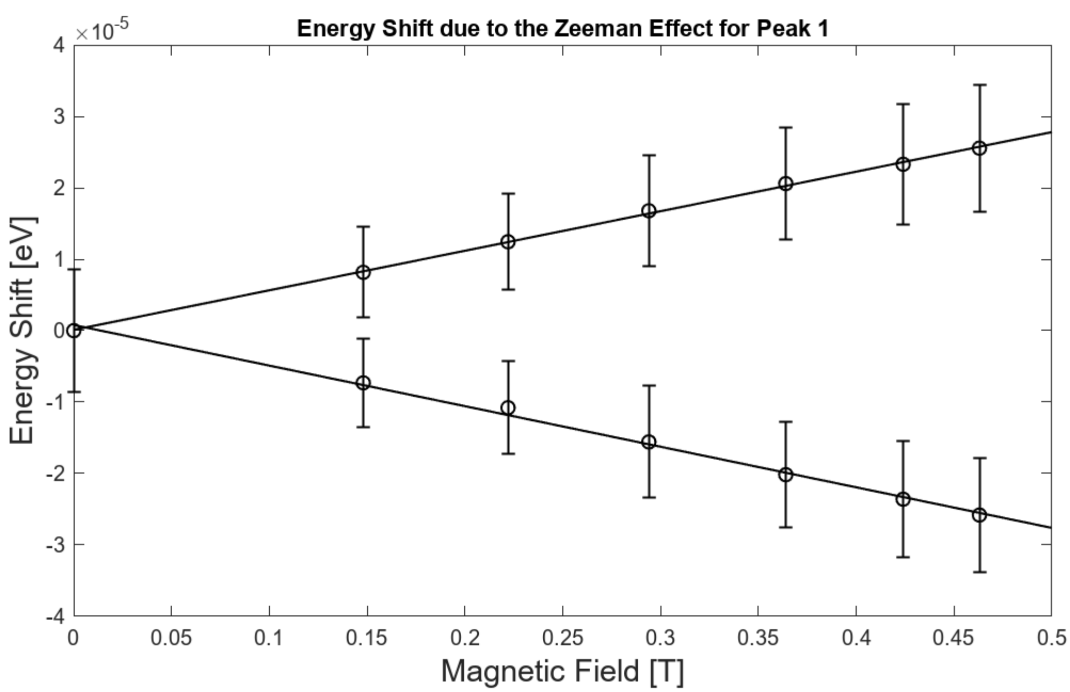


Figure 5: energy shifting as a function of the external magnetic field for peak 1

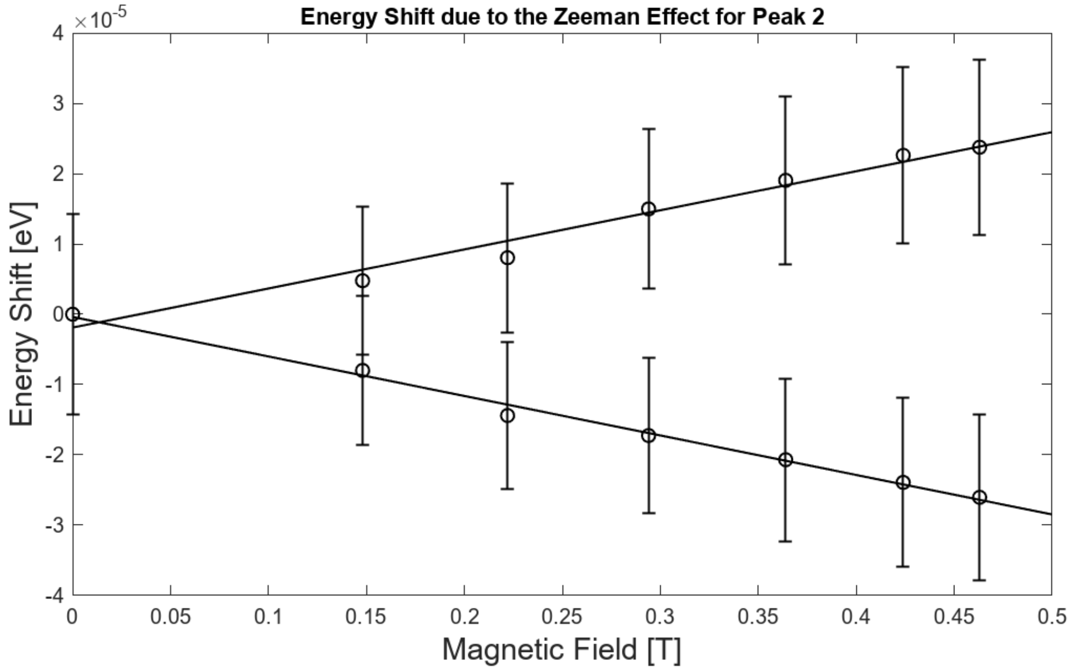


Figure 6: energy shifting as a function of the external magnetic field for peak 2

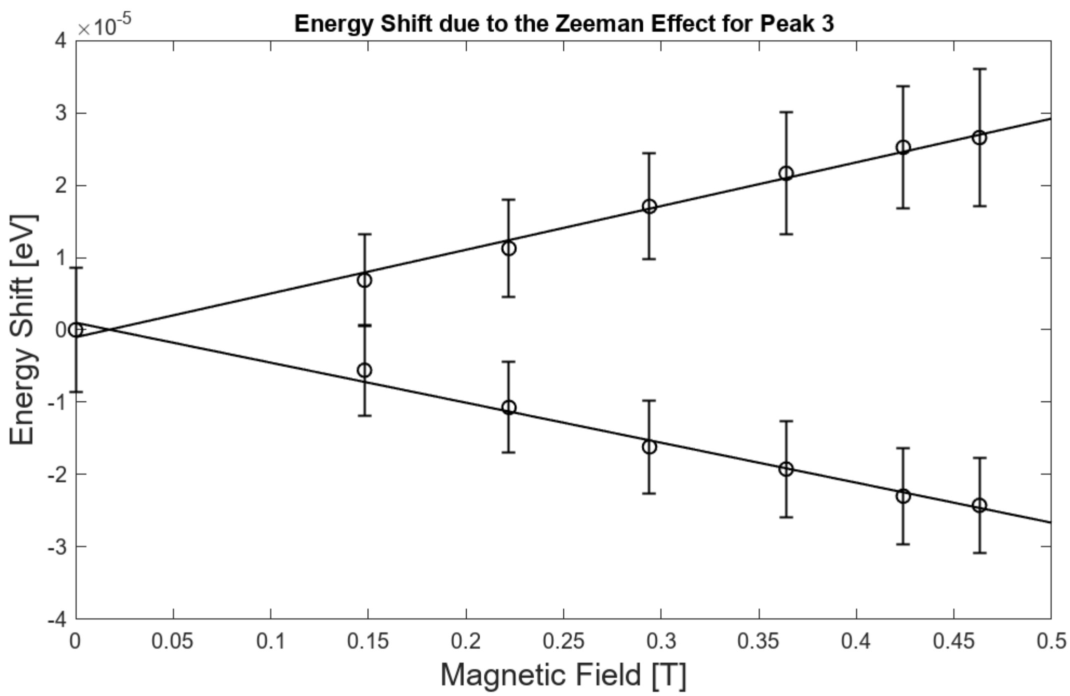


Figure 7: energy shifting as a function of the external magnetic field for peak 3

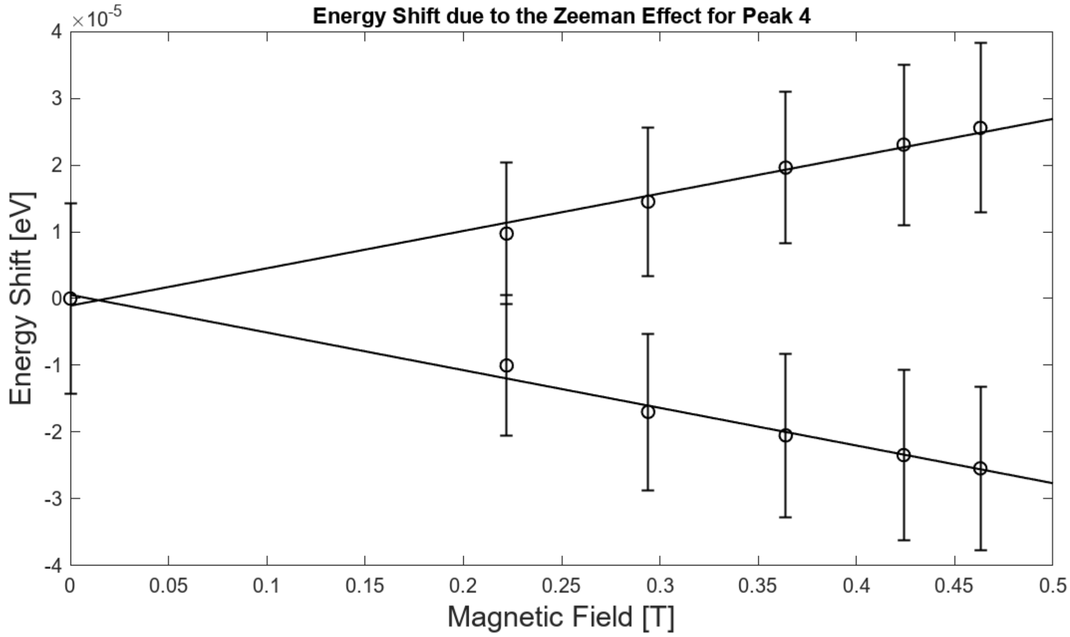


Figure 8: energy shifting as a function of the external magnetic field for peak 4

The currently accepted value of the Bohr Magneton is $\mu_B = 57.884$ μeV [5]. The measured values of the Bohr Magneton, extracted from figures 5-8 with 68% confidence intervals, are in Table (6). We expected the y-intercepts to be zero, but the linear fits are only near zero, indicating the presence of a background magnetic field in the vicinity.

Peak	**Averaged Bohr Magneton**
1	56.1 ± 1.3 μeV
2	55.9 ± 2.9 μeV
3	57.9 ± 2.6 μeV
4	56.2 ± 3.5 μeV

The slope of each graph was exactly the Bohr Magneton for that experiment, so each figure provided two values, which were averages to deal with systematic shift. Clearly, the closest value came from peak 3, which was 57.9 μeV, which had a 0.03% difference from the accepted value.

4.1 Free Spectral Range

One of the characteristics of the Fabry-Perot inferometer is calleed the **free spectral range**. It is defined as the spacing between maxima it produces, and is a function of the index of refraction and the thickness of the lens.

$$\text{FSR} = \frac{c}{2nt} \tag{13}$$

The index of refraction for the piece used in this experiment was 1.457, and the thickness of the lens was 4 mm [6]. Therefore, by the above equation, the free spectral range was 106.7 μeV. The actual equation gave a number in units of Hz, which is frequency, but multiplication with Planck's constant gave units of energy. Consider two peaks, side by side. When the states split, these peaks will split into two peaks each. Two of these new peaks, one from each of the original peaks, will shift towards each other, and when they overlap, the energy splitting is equal to the FSR.

For example, take the first and second peaks on the left. From equation (12), the magnetic field for 1 FSR of splitting would be $B_0 = 1.84$ T. For those two peaks, take for example the energy shifts of -23.3 and 25.5 μeV. They correspond to shifts of 0.22 FSR and 0.21 FSR, so based on the above calculation, the magnetic field should be 0.402 T and 0.388 T. The measured magnetic field was 0.424 T, which is not far off, demonstrating how the FSR can be used to determine certain properties. Another use for the FSR is to investigate systematic error. If one can measure the magnetic field at which the peaks overlap, and it is not 1 FSR, then there may be a systematic shift in the measurement somewhere, causing overlapping at the wrong points. It also can be used to investigate systematic error in B_0 as the peaks move through a fraction of the FST that can describe what the measured magnetic field should be.

5 Conclusion

We observed the Zeeman Effect, the quantum mechanical phenomenon in which an external magnetic field causes the energy states of particles (like electrons) to split into multiple. Electrons have intrinsic angular momentum and therefore magnetic moments that interact with external fields, giving them additional energy. This energy is related to a special value, the Bohr Magneton, and observing the Zeeman Effect allowed us to compute the value of μ_B to within a 0.1% difference.

The linear relationship between the magnetic field and energy splitting (equation 2) allows us to fit the experimental data and extract the slope as the Bohr Magneton. The closest measured value was $\mu_B = 57.9$ μeV, which is within 0.03% of the accepted value [5]. This operation was performed for four peaks on the spectrum generated by a Fabry-Perot inferometer. T

Overall this experiment worked with optics and quantum mechanics to show a clear picture of the quantum state splitting present in the Zeeman Effect, due to an external magnetic field.

References

[1] R. Harris, *Modern physics*, 2nd ed. (Pearson Education Inc., 2014).

[2] R. D. Knight, *Physics for scientists and engineers a strategic approach*, 4th ed. (Pearson Education Inc., 2017).

[3] *Measuring the zeeman splitting of the red cadmium line as a function of the magnetic field – spectoscopy using a fabry-perot etalon*, tech. rep. (Leybold).

[4] J. R. Taylor, *An introduction to error analysis :* 2nd ed. (University Science Books, Mill Valley, Calif : c1982.).

[5] P. J. Mohr, B. N. Taylor, D. B. Newell, and et al., *Crc handbook of chemistry and physics*, edited by W. Haynes, 95th ed. (CRC Press, 6000 Broken Sound Parkway NW, Suite 300, Boca Raton, FL, 2014).

[6] *Technical data for the fabry-perot etalon mirror*, tech. rep. (Leybold).