Non-Linear Regression

As an example, the following model is chosen for fitting:

$$y = \ln(ax)$$

(a is unknown, and x and y data points are known)

The least squares error for the above function and any given data set $\{X_i, Y_i\}$ can be written as:

$$E = \sum_{i=1}^{N} (Y_i - \ln(aX_i)^2$$

where:
N is the number of data points available
E is the squared error
a is the unknown parameter
X_i and Y_i are the known data points

To minimize the squared error, we must differentiate the above expression with respect to a and equate it with zero:

$$\frac{dE}{da} = \sum_{i=1}^{N} 2\,(Y_i - \ln(aX_i)\left(-\frac{1}{a}\right) = 0$$

The above can be simplified to:

$$\sum_{i=1}^{N} (Y_i - \ln(aX_i)\left(\frac{1}{a}\right) = 0$$

To find a solution for the unknown value a using the root finding approach, let:

$$f(a) = \sum_{i=1}^{N} (Y_i - \ln(aX_i)(\frac{1}{a})$$

And

$$f'(a) = \frac{d\big(f(a)\big)}{da} = \sum_{i=1}^{N} \left(-\frac{1}{a^2}\right) - \frac{Y_i - \ln(aX_i)}{a^2}$$

Now, we can use the relation:

$$a_{n+1} = a_n - \frac{f(a_n)}{f'(a_n)}$$

This relation can be solved iteratively (after guessing an initial value of a_n) till the difference between successive iterations converges to a threshold difference value.

To obtain the desired fit for the given data, we consider a threshold difference value of 10^{-8}. (it can be chosen according to required accuracy)
Three different sets of data are used to demonstrate the fitting, and for each, the resulting plots are shown in the readme description of this repository.