

# The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science

## Series 7

ISSN: 1941-5982 (Print) 1941-5990 (Online) Journal homepage: <http://www.tandfonline.com/loi/tphm18>

## CXVII. A theory of work-hardening of metal crystals

N.F. Mott F.R.S.

**To cite this article:** N.F. Mott F.R.S. (1952) CXVII. A theory of work-hardening of metal crystals, The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, 43:346, 1151-1178, DOI: [10.1080/14786441108521024](https://doi.org/10.1080/14786441108521024)

**To link to this article:** <http://dx.doi.org/10.1080/14786441108521024>



Published online: 29 Jul 2010.



Submit your article to this journal [↗](#)



Article views: 156



View related articles [↗](#)



Citing articles: 42 View citing articles [↗](#)

CXVII. *A Theory of Work-hardening of Metal Crystals*

By N. F. MOTT, F.R.S.

H. H. Wills Physical Laboratory, University of Bristol\*

[Received August 8, 1952]

## SUMMARY

A theory of work-hardening is put forward; the salient points are as follows. There are two types of hardening; the normal rapid hardening of cubic metals and the slower hardening characteristic of hexagonal crystals and cubic metals under conditions of easy glide. A quantitative theory of the rapid hardening is given. This does not occur unless dislocations are retained in the slip planes. The retained dislocations are supposed to occur in groups of about 1000 at the ends of the slip-bands. The strains round these groups are responsible for hardening as in Taylor's (1934) theory. The obstacles which stop slip are sessile dislocations, and the formation of sessile dislocations stabilizes the groups of dislocations. It is shown that the formation of slip lines must be a dynamic process, and on this assumption a parabolic stress-strain curve is deduced. Some discussion is given of the formation of vacancies by moving dislocations, and it is shown that this can account for the clustering of slip lines observed under the electron microscope. The theory agrees qualitatively with observations on the amount of retained energy in a cold-worked metal.

## §1. INTRODUCTION

THE aim of this paper is to give a theory of work-hardening of metal crystals, based on the properties of dislocations. The basic assumption made is that a crystal contains a network of dislocations,† and that a dislocation in an otherwise perfect crystal moves when the crystal is subjected to a small stress. We shall attempt from these assumptions to build up a quantitative theory for close-packed cubic crystals, and shall make a few qualitative remarks about hexagonal crystals.

The first theory of work-hardening based on dislocations is that due to Taylor (1934). Qualitatively this may be expressed as follows. The stress round an edge dislocation is of the form

$$\sigma = \pm (Gb/r)f(\theta), \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

---

\* Communicated by the Author.

† It is not suggested that it is inherently impossible to prepare crystals without dislocations; the dislocations in real crystals are a result of the method of growth. Some of the factors involved have been summarized by Frank (1952). A recent note by Herring and Galt (1952) shows that very thin fibres formed on tin crystals have yield strengths approaching the values expected for a material without dislocations.

where  $G$  is the shear modulus,  $b$  the Burgers vector,  $r$  the distance from the dislocation line and  $f(\theta)$  a function of angle. If, then, a cold-worked solid contains a random array of dislocations of both signs at a mean distance  $l$  from each other, there will be in the material a stress  $\sigma_i$ , varying with position in some roughly sinusoidal way, of which the magnitude is of order

$$\sigma_i = Gb/l. \quad (2)$$

If each dislocation of the array is in some way 'stuck', any additional dislocation introduced into the lattice cannot move through distances large compared with  $l$  unless the applied stress  $\sigma$  exceeds  $\sigma_i$ .  $\sigma_i$  is then identified with the flow stress of the crystal at any point of the stress-strain curve. It is supposed in Taylor's model that dislocations are constantly being created while the substance is being deformed, and that they move a distance  $L$  before they are stuck. The plastic strain  $\epsilon$  is thus

$$\epsilon = Lb/l^2. \quad (3)$$

Eliminating  $l$  between (2) and (3), we find

$$\frac{\sigma_i}{G} = \sqrt{\frac{b}{L}} \epsilon^{1/2}. \quad (4)$$

To obtain agreement with experiment for the stress-strain curve of aluminium, Taylor had to assume that  $L$  is about  $10^{-4}$  cm, and he deduced that, after a strain of order unity,  $l \sim 3 \times 10^{-6}$  cm. The concept of an oscillating internal field  $\sigma_i$  has been adapted by Mott and Nabarro (1948) and Nabarro (1952) to the explanation of age-hardening and the high yield-stress of alloys; while an explanation of yield points in iron and similar materials has been given by Cottrell (1948; see also Cottrell and Bilby 1949) in terms of the segregation of impurities actually in or very near the dislocations.

The explanation of strain hardening which we shall give is an extension of Taylor's; we suppose that the hardness is due to an internal field due to dislocations generated in the process of plastic flow. The actual distribution, however, is very different from that of Taylor, dislocations of like sign being grouped together, some hundreds being in each group. It shares with Taylor's theory, however, the feature that the hardening is, to a first approximation,\* independent of the direction in which slip has occurred; in a cubic crystal dislocations generated by slip along one set of (111) planes harden the crystal for slip along another set of (111) planes.

To complete Taylor's theory, we have to ask:

(a) How are the dislocations generated during slip? For this a satisfactory mechanism has been proposed by Frank and Read (1950), and will be discussed in the next section.

---

\* This point has been examined further by Stroh (1952) who in a paper to be published shortly finds a small dependence.

(b) How do the dislocations become 'stuck' after moving a certain distance, what limits slip on a given plane, and why do not the dislocations move back when the stress is reversed? Here we propose different mechanisms for the edge and screw components of a dislocation loop. The edge components in a cubic close-packed crystal become stuck by combining with edge dislocations on other octahedral planes, as proposed by Lomer (1951) and Cottrell (1952). A join of two such dislocations produces what we call a sessile dislocation. This cannot happen in hexagonal crystals, which is why they harden so much more slowly. The screw dislocations are probably stopped by a process connected with the generation of vacancies by moving dislocations (Seitz 1952). These vacancies also play a large role in the temperature-dependence of strain hardening and in the increase in electrical resistance due to strain hardening.

Some of the experimental facts which have to be explained by any theory are:

- (a) The occurrence of slip lamellae.
- (b) The fact that at low temperatures, in aluminium at any rate, a slip lamella consists of slip *on one plane* through distances of the order 1200–1800 Å. As the temperature, however, is raised, the visible slip bands are found to consist of clusters of these elementary steps, about 200 Å apart and with roughly the same slip on each (Heidenreich and Shockley 1948, Brown 1949, 1951, 1952).
- (c) The very considerable temperature-dependence of hardening.
- (d) The form of the stress-strain curve.
- (e) The formation of deformation bands and asterisms in cold-worked cubic metals, and their absence in hexagonal crystals.
- (f) The phenomenon of easy glide (Andrade and Henderson 1951, Lücke and Lange 1952).

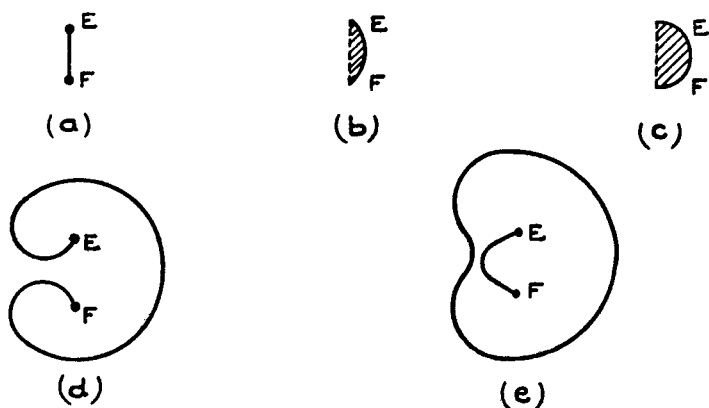
## §2. THE FRANK-READ SOURCE AND THE ORIGIN OF SLIP BANDS

Several accounts have already appeared of the explanation given by Frank and Read of the origin of slip bands (Frank and Read 1950, Frank 1951, Mott 1951). The model is illustrated in fig. 1. It is based on the concept of a dislocation line **EF** lying in a slip plane, which at the points **E**, **F** moves on to other planes lying at an angle to it. We consider movement in which **EF**, rather than its continuation in the other planes, is displaced.

Consider then what will happen to the line **EF**, when the material is stressed in such a way as to cause slip. If the line moves from the position shown in fig. 1 (a) to that shown in fig. 1 (b), the material over the area shown shaded in fig. 1 (b) slips relative to that underneath it. The line will move in such a way as to extend this area. On the other hand the points **E**, **F** are locked. Successive forms of the line are shown in figs. 1 (a) to (e). It will be seen that ultimately a complete loop is generated, and that at the same time the line **EF** is formed again.

A line such as **EF** is thus capable of generating an infinite series of loops, when a large enough stress is applied. Moreover, it will be shown later in this section that the stress is of the order of that for which yield of actual metal crystals occurs.

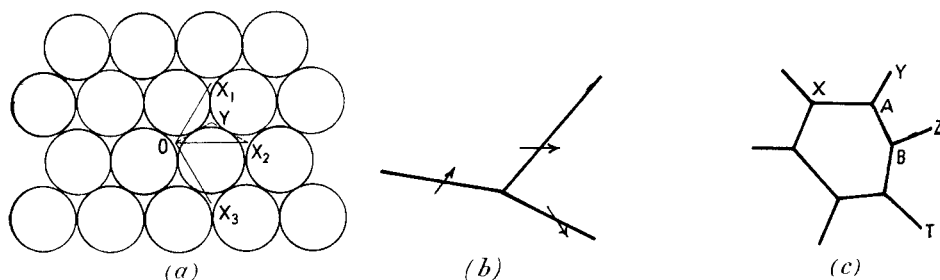
Fig. 1



Successive stages in the formation of a dislocation ring from a Frank-Read source.

For close packed cubic crystals—to which much of this article will be devoted—the model becomes more plausible when we consider the Burgers vectors possible for slip in the (111) planes. Such a plane is shown in fig. 2 (a). The possible values of the Burgers vector are  $OX_1$ ,  $OX_2$  and  $OX_3$ . It is clear that the displacement when two dislocations with vectors  $OX_1$ ,  $OX_3$  move across the plane is the same as if a dislocation with vector  $OX_2$  moves across it. From this it follows that it is geometrically possible for three dislocations to meet in a point, as in fig. 2 (b), the Burgers vectors of the three being respectively  $OX_1$ ,  $OX_2$ ,  $OX_3$ . Further, it is obviously not necessary that the three lines should all lie in the same slip plane.

Fig. 2



(a) A close-packed plane, showing the Burgers vector for complete and half dislocations. (b) Join of three dislocations. (c) Network of dislocations (from Mott 1951).

A possible arrangement of dislocations in a cubic close-packed solid, and probably the most stable in view of the line energy of dislocations, is therefore a network such as that shown in fig. 2 (c). This is the model that we shall adopt throughout this report for the arrangement of dislocations in an annealed metal of close-packed structure; it will probably replace the old model of a 'mosaic crystal' so far as the interpretation of intensity measurements in the reflection of x-rays is concerned. With this model, any element such as **AB** can act as a Frank-Read source.

There are certain further points that we may note about this model. These are :

*Extended Dislocations.* As first pointed out by Heidenreich and Shockley (1948), a dislocation in a close-packed plane with Burgers vector say  $OX_2$  in fig. 2 (a), can and will split into 'half-dislocations' with Burgers vectors  $OY$ ,  $YX_2$ . These two half dislocations repel each other; it is easily seen that the elastic energy, proportional to the square of the Burgers vector, is diminished when they separate in the ratio

$2OY^2/OX,^2$

and hence in the ratio  $\frac{2}{3}:1$ . On the other hand, when they separate they create a stacking fault between them, the energy of which increases with the distance between them. According to Heidenreich and Shockley, they will be in equilibrium when separated by about  $2 \times 10^{-7}$  cm.

As a result of this dissociation, the dislocations in close-packed planes will have low energy, and the elements such as **AB** (fig. 2 (c)) in cubic close-packed structures will mainly be in close-packed planes. This is believed to be the reason why glide in close-packed structures shows a marked preference for close-packed planes, while for materials with some other structures (e.g. body-centred metals, silver halides), various planes show glide.

**Yield Points.** In a pure single crystal flow will begin when the applied stress is great enough to initiate the process illustrated in fig. 1, namely the generation of loops at the most favourably situated Frank-Read source. The force resisting the generation of loops is due to the energy per unit length, or tension of a dislocation. If  $\sigma$  is the resolved part of the stress about a line lying in the slip plane parallel to the Burgers vector, the force exerted on unit length of the dislocation is  $\sigma b$ . If  $W$  is the energy of the line per unit length, the resultant force in the opposite direction is  $W/\rho$ , where  $\rho$  is the radius of curvature. The maximum value which  $1/\rho$  can reach as the loop expands is  $1/2l$ , where  $l$  is the length EF of the source. Thus the source will generate loops if

$$\sigma \geq W/2lb.$$

Putting in estimated numerical values for  $W$  for isotropic materials, Frank (1950) finds that in practical cases  $W \sim 2Gb^2$ , so that the condition for yield is

$$\sigma \geq Gb/l. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

For the best single crystal of, for example, silver (Andrade and Henderson 1951), yield points  $\sigma$  of the order  $50 \text{ g/mm}^2$  are observed, giving values of  $l$  of about  $3 \times 10^{-3} \text{ cm}$ ; for aluminium Rosi and Mathewson (1951) observed yield points of the order  $200 \text{ g/mm}^2$  give  $l \sim 3 \times 10^{-4} \text{ cm}$ . We emphasize that the values of  $l$  deduced are for the largest values of the length AB in the dislocation network discussed; the average value may be smaller. No theory has, as yet, been given which enables us to estimate the mean value of  $l$  that will result from a given method of preparing a crystal.

There are two further effects to be considered. Firstly, there will exist a random internal stress due to other dislocations or impurities which changes sign in a distance  $\lambda$  smaller than the estimated values of  $l$ ; this effectively introduces a frictional force, say  $\sigma_0 b$  per unit length resisting the motion of a dislocation; a dislocation initially at rest will not move unless the applied stress exceeds  $\sigma_0$ , and a Frank-Read source will not generate dislocations unless

$$\sigma > \sigma_0 + Gb/l. \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Secondly we have to consider the locking effect of impurities which have by diffusion moved into the immediate neighbourhood of a dislocation. This effect and its relation to yield points have been considered in a number of papers by Cottrell and his co-workers (Cottrell 1948, Cottrell and Bilby 1949). For our purpose we need to emphasize the following effects of such locking:

(a) The stress ( $\sigma_{\text{Cot}}$ ) required to initiate a slip band is now simply that required to make a dislocation break away from its impurities, provided that  $\sigma_{\text{Cot}}$  is greater than  $\sigma_0 + Gb/l$ . Once the dislocation has broken away, however, the resistance to further slip is still given by (6).

(b) Since  $\sigma_{\text{Cot}}$  will be the same for all loops and is thus independent of  $l$ , the conditions are favourable for sharp yield points.

(c)  $\sigma_{\text{Cot}}$  is very dependent on temperature (Cottrell and Bilby 1949), while  $\sigma_0$  is very little dependent on temperature (Mott and Nabarro 1948).

It is of some interest to consider the effect of a network such as that considered here on the apparent elastic modulus. If the dislocations are locked in the Cottrell sense, there should be no movement for stresses less than  $\sigma_{\text{Cot}}$ . Up to this stress the material should be perfectly elastic with the theoretical shear modulus. In the absence of Cottrell locking, however, a stress greater than  $\sigma_0$  but less than  $\sigma_0 + Gb/l$  will lead to a reversible movement of any dislocations present in the material. If the material is highly purified so that  $\sigma_0$  is very small, this will lead to an apparent change in the elastic modulus. We shall now estimate the magnitude of this for the network illustrated in fig. 2 (c). If the stress applied is  $\sigma$  in the plane considered, the dislocation takes up the form of a circle of radius  $\rho$  given by

$$\sigma b = W/\rho.$$

Substituting for  $W$  we find

$$\rho = 2bG/\sigma.$$

The area swept out by the dislocation (the shaded area in fig. 1 (b)) is thus  $\rho^2(\theta - \frac{1}{2} \sin 2\theta)$ , where  $\sin \theta = \frac{1}{2}l/\rho$ . For small  $\theta$  this reduces to

$$l^3/12\rho=l^3\sigma/24bG.$$

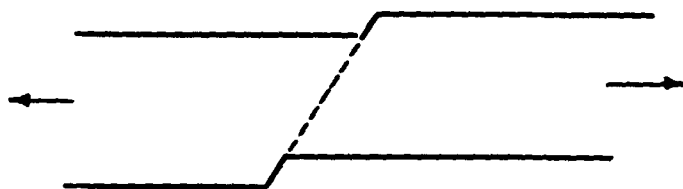
If there are  $N$  Frank–Read sources per unit volume, it follows that the shear produced by a stress  $\sigma$  is

$$Nl^3\sigma/24G. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Since  $N^3$  is probably slightly less than unity, it follows that the correction to the elastic modulus is of the order of at most a few per cent, unless  $\sigma$  approaches the stress  $Gb/l$ .

It follows that the stress-strain curve of a pure substance having a number of Frank-Read sources shows a (reversible) strain slightly in excess of that given by the ideal elastic modulus. Under alternating stresses there will be a slight dissipation of energy, of amount proportional to the temperature, due to a mechanism first put forward by Eshelby (1949). If, however, there is any frictional resistance to the motion of dislocations, one will obtain a hysteresis loop (cf. Nabarro 1951). Such a resistance will certainly exist in alloys for the reasons already stated.

Fig. 3



Glide as the result of stress.

We may now sum up the elementary consequences which follow from the assumption that the crystal contains a number of Frank-Read sources, or a network of the type shown in fig. 2 (c). If the crystal is submitted to a uniform shear stress, then as the stress is increased a value will be reached at which one of the sources, that for which  $l$  is largest, begins to generate loops. The process will then continue indefinitely; nothing introduced into the model so far will stop it. Glide as shown in fig. 3 is thus to be expected, until owing to the change of shape of the crystal the stress is no longer uniform, when other sources would be brought into play. If by some mechanism slip on the first plane was stopped, the source with the next largest value of  $l$  would be brought into play. The next task of a theory is to explain why slip ever stops on a given plane. In particular we have to explain the distance of  $c. 2000 \text{ \AA}$  shown by the work of Heidenreich and Shockley and of Brown to be characteristic of slip on each active slip band.





The stress concentration is thus large, the dislocations producing the same stress concentration as does a freely slipping crack of the same length.

(e) The total slip produced by the sequence is the same as would be produced if a single dislocation of strength  $nb$  moved a distance  $3L/4$ .

Some numerical values will be of interest here. First of a well, estimate the magnitude of the distance  $d_0$ , which is equal to the equilibrium distance between two dislocations of the same sign, one of which is held up by a barrier and the other forced towards it by a stress  $\sigma$ . For a metal such as Cd or Al, we may take  $G \sim 2 \times 10^{11}$  dynes/cm<sup>2</sup>; for the yield point of a pure metal,  $\sigma \sim 100$  g/mm<sup>2</sup>  $\sim 10^7$  dynes/cm; this gives  $d_0 \sim 3 \times 10^3 b \sim 5 \times 10^{-5}$  cm. For a crystal that has been heavily deformed the flow stress will be from 10 to 100 times greater and the magnitude of  $d_0$  is  $10^{-5}$  or  $10^{-6}$  cm. Any theory which seeks to ascribe work-hardening to the interaction between dislocations fairly uniformly distributed (e.g. Taylor 1934) must assume that the distance between them is of this order, and thus considerably less than the distance between observed slip bands. One has to assume, *either* that in a cold-worked metal the space between the slip bands is filled up with dislocations, *or* that in the slip bands there are clusters of piled-up dislocations, formed as described above. Each of these clusters will behave at large distances like a dislocation of large Burgers vector. In the latter sections of this report we shall develop particularly the second of these hypotheses, though we believe that there will be some formation of dislocations between the slip bands.

A second point of interest is the high stress concentration that exists near the limit of a row of piled up dislocations. Frank (1951) has suggested that, when this occurs at a grain boundary, the stress may be great enough to generate loops. This point we believe to be important, and it will be considered in further detail in § 6. Here we wish to emphasize that such a process is likely to be temperature-dependent; temperature can assist the formation of a loop in a region of very high stress.

When the surfaces of aluminium crystals which have been deformed at low temperatures are examined with the electron microscope, *single* steps of the order of 1000–2000 Å in height are observed. At higher temperatures the steps are clustered; a hypothesis to account for this clustering will be given in § 8. At low temperatures, however, this suggests that about 1000 dislocations are produced by each Frank–Read source. The edge dislocations of one sign producing the visible slip band have come out of the crystal; those of the opposite sign, at the opposite side of the ring, must in general be piled up against some barrier. We shall now estimate the distance from the surface that this barrier must be. Let us assume for the moment, then, that when each ring expands the screw elements go to the boundary of the crystal or at any rate go a large distance compared with the edge components, but that the latter are stopped at a barrier after moving a distance  $L$ . We have to ask how big this distance  $L$  must be in order to account for a value of  $n$  of *c.*1000.

We shall take the case of aluminium single crystals subjected to a shear stress of  $500 \text{ g/mm}^2$  or  $5 \times 10^7 \text{ dynes/cm}^2$ . If  $G \sim 3 \times 10^{11}$ , and  $n \sim 1000$ , then formula (8) gives  $L = 0.05 \text{ cm}$ . The distance of the first barrier from the surface may be as much as twice this. Thus, for slip of  $2000 \text{ \AA}$  to take place for the lowest stresses at which yield occurs, a slip distance  $L$  of nearly  $1 \text{ mm}$  is necessary.

We shall now discuss the *dynamics* of the formation of a slip line. From the theoretical point of view, it is not yet certain how much a moving dislocation in a perfect crystal is damped by the thermal vibration of the lattice (Leibfried 1949, 1950, Nabarro 1951). We believe, however, that the facts about the formation of slip bands show conclusively that dislocations must acquire a velocity not less than about half the speed of sound. For all the observations of slip bands (Brown 1951) show that the *elementary* slip band through *c.*  $2000 \text{ \AA}$  is formed completely or not at all, though the formation of other members of a *cluster* may occur after some time. The evidence suggests, then, that the elementary slip band is formed quickly, and not gradually as the deformation proceeds. While a crystal is being strained, the stress rising slowly, slip from a given source begins then presumably when  $\sigma$  reaches the value of  $Gb/l$  for that source, and the band is formed before  $\sigma$  has risen appreciably above this value. If the dislocations moved slowly, say with one tenth of the velocity of sound, this would be impossible; as soon as, say,  $\Delta n$  dislocations were piled up at the end of the slip distance  $L$ , they would exert a stress  $\Delta n G / 2\pi L$  at the source, *preventing* the generation of new dislocations. This would stop the source from acting long before the final value  $n$  was reached. If the motion of dislocations is heavily damped, we should expect  $n$  to increase gradually with  $\sigma$ ; it could not reach its full value until  $\sigma$  reached about twice the yield stress.

If on the other hand dislocations can move with nearly the speed of sound, once a Frank-Read source is set into action, it will go on generating dislocations under its own momentum, the momentum generated in moving from the configuration of fig. 1 (*e*) carrying the line EF to the configuration of fig. 1 (*c*). Generation of loops will then go on until the stress of the piled-up dislocation stops it, the whole process being over in a few microseconds.\*

Moreover, if the process is rapid as described here, we can, assuming the presence of a barrier at a distance  $L$  from the source, account for the fact that the dislocations do not move back when the stress is taken off. This can be explained in terms of the formation of *sessile* dislocations, which is likely to occur as soon as a piled-up group of more than about

---

\* A somewhat similar suggestion has been put forward by J. C. Fisher, E. W. Hart and R. H. Pry (to be published). These authors consider that the stress due to the still moving dislocations generated by the source will stop the source from acting after some hundred are formed, without their coming up against any barrier. I am indebted to Dr. Fisher for allowing me to see his manuscript prior to publication.

six dislocations is brought to rest, but will not occur while they are in motion. We have already stated that when two dislocations on two octahedral planes join, they form a sessile dislocation. A piled up group of  $n$  dislocations has a stress field round it of order  $nGb/2\pi r$ . There are other sources at a mean distance from it of order  $l$ , and a stress of order  $Gb/l$  is required to make any one of them operate. Thus a piled-up group of six is sufficient to cause plastic flow round it, and to produce one or more edge dislocations on a different plane which will form a sessile element with one of the six. We suppose, then, that this is what happens, and a proportion of about one in six of the dislocations piled up at the end of a slip plane become sessile, thus preventing any appreciable recovery of strain when the stress is removed.

We emphasize that this phenomenon cannot occur for hexagonal crystals, where there is only one set of slip planes.

It is clear that in ordinary plastic flow, only a small proportion of the available sources are used. For supposing that the slip distance for edge and screw dislocations is  $L$ , then the strain  $\epsilon$  produced by the operation of  $N$  sources per  $\text{cm}^3$  is

$$\epsilon = NLL'bn.$$

With  $L \sim L' \sim 10^{-2}$  cm,  $n \sim 1000$  and  $\epsilon \sim 1$ , this gives  $N \sim 10^8$ , considerably less than the value  $10^{10}$ – $10^{12}$  which we believe to represent the total number.

Evidence in favour of this conclusion is provided by the work of Brown and Honeycombe (1951), who have shown that the slip markings distant  $c.1$  micron from each other are characteristic only of surfaces which have been mechanically polished, and that they do not appear on surfaces that have been electrolytically polished and etched. We believe, however, that the former markings are characteristic of the interior, and for the following reason. As first pointed out by Holloman (1952), dislocations which terminate on the surface of an otherwise perfect crystal will behave as though locked between the real locking point and its reflection in the surface. They will have, therefore, much longer effective values of  $l$ . If

$$l \sim 10^{-4} \text{ cm},$$

there will be per unit area of surface  $c. 10^8$  sources which terminate in the way shown; and if a stress is applied which will operate any of the sources in the interior, it will operate also most of the surface ones. If the dislocations generated move a distance  $10^{-2}$  cm, we see that a density of slip lines  $10^6$  per cm is to be expected on the surface. Under such conditions it is natural that each line can only have a small displacement.

If the surface has been cold-worked or hardened in any other way, the situation is quite different; sources on the surface do not move more easily than those on the interior, and if the hardened layer is thin compared with the slip distance (say  $10^{-2}$  cm), the markings shown should be characteristic of the interior.

## §4. STRAIN-HARDENING OF CUBIC CLOSE-PACKED CRYSTALS

We give in this section a theory, allied to that of G. I. Taylor, of the work-hardening of cubic close-packed crystals at low temperatures. The theory will be limited to one rather rapid hardening characteristic of impure crystals; the 'easy glide' sometimes obtained with pure crystals will be discussed in the next section. The assumptions that we shall make are:

(a) There exist on all active slip planes barriers at a mean distance  $2L$  from each other; the nature of the barriers will be discussed below.

(b) The screw components of the dislocation loops move a distance  $L'$  comparable with or greater than  $L$ ; the screws will be discussed in § 6.

The hardening will be ascribed to the stress round the piled-up groups of  $n$  edge dislocations already described. If the surrounding material were elastic, the stress at a distance  $r$  from such a group would be

$$nGb/2\pi r \quad (n \sim 1000).$$

However, at a distance greater than  $l$  ( $l \sim 10^{-4}$  cm) the material is plastic, as it contains Frank-Read sources. As, however, the stress is extremely unhomogeneous, we do not believe that the dislocations from any source are likely to move a distance much greater than  $l$  before meeting a dislocation on another plane and becoming sessile. We therefore have to relieve a local strain  $\epsilon(r)$  ( $=nb/2\pi r$ ) by allowing dislocations to move from sources distant  $l$  apart through a distance  $l$ . It is clear that, at a distance  $r$  from the pile-up, each source must contribute  $n'(r)$  dislocations, where  $n'(r)=l\epsilon(r)/b$ , to do this. Round each primary group of  $n$  which have moved a large distance  $L$ , we have other secondary groups of  $n'$  which have moved a small distance  $l$ . Moreover, these produce a stress field  $\sigma_i$  at distance  $r$  from the primary group having 'wave-length'  $l$  and magnitude

$$\sigma_i = Gbn'/l = G\epsilon(r) = Gbn/2\pi r.$$

The magnitude is thus the same as if secondary slip had not occurred.

If, then, a material has slipped so that slip bands are at a distance  $x$  apart, the mean distance between the primary groups of piled-up dislocations is  $(xL)^{1/2}$ .

The mean strain  $\sigma_i$  in the material is then of wave-length  $l$ , the dimensions of the network, and of magnitude

$$\sigma_i = nGb/2\pi (xL)^{1/2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The strain, on the other hand, is given by

$$\epsilon \sim nb/x. \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Eliminating  $x$ , we find

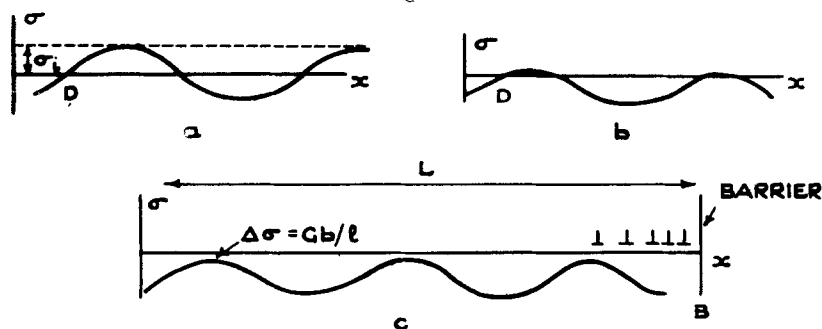
$$\frac{\sigma_i}{G} = \frac{1}{2\pi} \sqrt{\frac{nb}{L}} \epsilon^{1/2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

This formula is to be compared with Taylor's formula (4).

This stress  $\sigma_i$  cannot, as in the original theory of Taylor, be identified with the flow stress at a strain  $\epsilon$ . The situation is to be pictured as in fig. 5, in which the stress acting on a dislocation is plotted against position. Before an external stress is applied, the situation will be as in fig. 5 (a), and the loop of a Frank-Read source will be at or near a point such as D, where the stress vanishes. As the applied stress is increased, the stress within the material will change to that shown in fig. 5 (b), the dislocation remaining at D. But a slip band will not be formed until the configuration of fig. 5 (c) is reached, the stress at the maximum reaching the value  $\Delta\sigma = Gb/l$  required to operate the source. As soon as this is reached, the (dynamic) generation of a slip band occurs; if there is a barrier at B, the dislocations generated will *all* pile up in its immediate neighbourhood, the mean stress acting on them being  $\sigma_i + \Delta\sigma$ . But the *number*  $n$  of dislocations generated depends on  $\Delta\sigma$  only; for  $n$  dislocations near B give at the source a stress field

$$nGb/2\pi L,$$

Fig. 5



Showing internal stress on a material; dislocations in fig. 5 (c) are shown as  $\perp$ .

so that the source will stop generating when

$$nGb/2\pi L = \Delta\sigma;$$

when this is so,

$$L = nGb/2\pi\Delta\sigma,$$

a formula differing by a factor 2 from (8).

Substituting  $Gb/l$  for  $\Delta\sigma$ , we have

$$nb/L = 2\pi b/l,$$

whence formula (11) for  $\sigma_i$  takes the form

$$\frac{\sigma_i}{G} = \sqrt{\left(\frac{b}{2\pi l}\right) \epsilon^{1/2}}. \quad \dots \dots \dots (12)$$

We may take the flow stress to be

$$\sigma_i + Gb/l; \quad \dots \dots \dots (13)$$

except for small strains the second term is small compared with the first.

Several points of interest may be noted about formula (12). The first is that *the stress-strain curve is independent of the slip distance  $L$* . This is a most important result, because the discussion that we shall give below suggests that the slip distance may depend rather sensitively on the physical conditions. The analysis above shows how the stress-strain curve may not be so dependent.

A second point is that, if we assume that  $l$  remains constant throughout the hardening process, formula (12) gives a parabolic stress-strain curve, and if  $l \sim 10^{-4}$  cm, it is of the form

$$\sigma/G \sim 100 \epsilon^{1/2};$$

the constant has about the value required for agreement with experiment. In this case, too, it is instructive to write the formula

$$\sigma = \sigma_0 + \sigma_i = \sigma_0 + G \sqrt{\frac{\sigma_0}{G}} \epsilon^{1/2}, \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where  $\sigma_0$  is the yield stress. It is, however, likely that  $l$  will decrease during work-hardening, owing to the processes already described; in that case the stress will increase as  $\epsilon^m$ , where  $m$  is slightly greater than  $\frac{1}{2}$ .

One criticism may perhaps be made of this theory. We have to ask why, when the stress is raised, further slip does not occur on planes that have already been active. All the dislocations in the active slip plane, when  $\sigma \gg Gb/l$ , will have gone to the end; there are none left in the main part of the slip plane. We suggest that further slip *may* happen on this plane; but as the random internal stress is constantly changing, there is no reason why the particular centre that has slipped should be more favoured than any other; and there are always far more sources than are used. In the same way, if the stress is reversed, it will normally be other sources that are active, and not those that have already slipped.

We now turn to a discussion of the nature of the barriers. We remark first that various investigators (Rohm and Kochendörfer 1950, Andrade and Henderson 1951, Lücke and Lange 1952) have observed easy glide in crystals of close-packed metals. The conditions seem to be

- (a) Very pure materials.
- (b) Orientation for slip on one plane only.
- (c) In some cases absence of oxide layer.

By easy glide we mean either the absence of hardening, or slight hardening compared with that normally observed, of magnitude comparable with that in hexagonal metal crystals. Easy glide is not accompanied by the formation of asterisms. It appears that in crystals showing easy glide—as perhaps in hexagonal crystals, the dislocations pass out of the crystals. We believe that when rapid hardening takes place the barriers which stop the dislocations, apart from grain boundaries and oxide layers, are likely to be sessile dislocations, either those already existing or those formed by slip on two planes. If this is so, easy glide on one slip system will occur most easily in very carefully annealed crystals of small total diameter—as in Andrade's work.

This assumption about the nature of the barriers enables us to estimate the slip distance in the absence of other boundaries. If a reasonable proportion of the dislocations in a material are sessile, there should be  $b/l^3$  lines, each of length  $l$ , per unit area in any plane. If any one of these is capable of stopping a dislocation in the plane, and if the slip distance  $L$  is the same for the screw as for the edge component, we have

$$bL^2/l^3 \sim 1,$$

whence

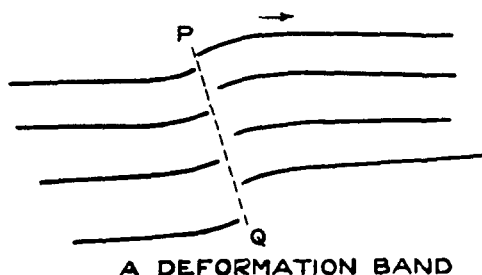
$$L \sim l\sqrt{(l/b)} \sim 10^{-2} \text{ cm.}$$

This gives a value of  $L$  of the right order of magnitude, perhaps greater than  $10^{-2}$  cm, since certainly the distance between sessile dislocations will be greater than between the normal ones. Moreover, it gives for  $n$

$$n = \Delta\sigma L/bG = L/l = \sqrt{(l/b)},$$

a value which varies with  $l$  only as the square root, and may perhaps explain the comparatively constant value of  $n$ . On the other hand, conditions which promote flow on two or more sets of planes are bound to

Fig. 6



increase the number of sessile dislocations, and so decrease the slip distance and hence  $n$ ; we should certainly expect some decrease in  $n$  as the strain is increased. Further experimental work on this point would be of interest.

### §5. DEFORMATION BANDS

Up to the present we have assumed that all slip bands, except those that reach the surface, terminate in *isolated* groups of piled-up dislocations. In fact, however, there are frequently observed slip lines which terminate in deformation bands; a schematic representation of a deformation band is shown in fig. 6. Deformation bands are an origin of asterisms, and it has frequently been suggested that they are the origin of hardening.\*

---

\* e.g. Mott (1951). Asterisms are not caused by isolated groups of piled-up dislocations. It should be possible in principle to harden a material without forming asterisms.



In our view, however, a group of piled-up dislocations in a deformation band cannot contribute to the hardening at all, because a grid of dislocations or of groups of dislocations of this type has no long-range stress field.\* Groups of dislocations which terminate in a deformation band are, from the point of view of work-hardening, wasted.

A deformation band grows in our view in the following manner. The regions P and Q form, owing to the stress field there, very effective traps for dislocations of sign such as to lengthen the band. In a material in which the density of barriers is otherwise low, deformation bands will tend to grow. But unless the density of barriers is very low indeed, there is no reason why a high proportion of slip lines should terminate there.

It is known (Chen and Mathewson 1951, Honeycombe 1951, Laloeuf and Crussard 1951) that deformation bands do not appear when a crystal is stressed under conditions such that slip occurs simultaneously on two sets of planes. On our model, this will be because, with slip on two sets of planes, a large number of sessile dislocations will be formed, leading to a low value of  $L$  and inhibiting the growth of deformation bands.

Cottrell (1952) has mentioned another factor which may contribute to strain-hardening when slip on two or more planes occurs. Under such conditions, a dislocation cannot move far without cutting other dislocations and so forming jogs. Energy is required for this, and so an increasingly large stress is required to drive a dislocation through the material. We believe that this cannot be the main cause of hardening; if it were, then, for impure materials for which there is no question of easy glide, the hardening would depend markedly on the orientation. This is not the case. As Cottrell points out, however, the activation energy required to form a jog is of the order of a only few electron volts even in the absence of a stress. Thus after plastic flow such dislocations as are held up by other dislocations will be easily freed by heat. Cottrell (1953) has given an analysis of logarithmic creep in pure aluminium which suggests that it is due to this process. We believe that Cottrell's mechanism introduces a small temperature-dependent frictional term in the resistance to the motion of dislocations.

## §6. MOVEMENT OF SCREW DISLOCATIONS AND THE GENERATION OF VACANCIES

The theory of hardening given in the last sections is based on the hypothesis that groups of piled-up edge dislocations can be formed through the agency of the sessile dislocations which results when edge dislocations on intersecting planes coalesce. We have made no mention of the screws, except to assume that their slip distance  $L'$  is at least as great as  $L$ . We shall in this section discuss the behaviour of the screws; they are important, because

---

\* cf. Nabarro 1952.

(a) We believe that moving screw dislocations generate vacancies, and that these are important for the explanation of the dependence of work-hardening on temperature, clustering of slip lines, electric resistance due to cold-work and other effects.

(b) In hexagonal crystals or others in which only one slip plane is operative, the blocking of edge dislocations through the formation of sessiles is impossible. We believe that in hexagonal crystals the slip distance  $L$  for the edge components must be the same as linear dimensions of the crystal; apart from oxide layers there is nothing to prevent their passing right out of the crystal. The (much slower) hardening which occurs in hexagonal crystals may be associated with piled up groups of screws.

(c) A discussion of the behaviour of screws is necessary in order to understand cross-slip.

All these phenomena depend in our opinion on the properties of vacant lattice sites or vacancies, and we shall give a few preliminary facts about these. According to the vacancy theory of diffusion (Johnson 1939), a vacancy is characterized by the energy  $W$  required to form it and the activation energy  $U$  to move it. In terms of these quantities the coefficient of self-diffusion  $D$  is given, apart from a numerical factor, by

$$D = \nu a^2 \exp \{-(W+U)/kT\}.$$

According to the calculations of Huntington and Seitz (1942),  $W$  and  $U$  should be about equal for a close-packed metal. Thus, if vacancies are formed artificially, e.g. by quenching from a high temperature or by neutron bombardment, self-diffusion is possible at about half the temperature at which it normally occurs. Recent work by Nowick (1951) has confirmed that the exchange of atoms in an alloy (AgZn) is more rapid after quenching, the effect being explained along these lines; Nowick finds for this alloy that, if this hypothesis is correct,  $U$  must be somewhat greater than  $W$ .

At temperatures at which self-diffusion is possible, the absorption or generation of vacancies enables an edge dislocation to 'climb', i.e. to move in a direction perpendicular to its slip direction. As described by the present author (Mott 1951), such a process is almost certainly responsible for the polygonization of a deformation band.

There is much evidence that *moving* dislocations generate vacancies at temperatures too low for them to be formed thermally. The evidence has been fully reviewed by Seitz (1952). The two main facts quoted by Seitz are:

(a) Cold-work increases the conductivity of alkali-halides, a conductivity thought to be ionic and to be due to vacancies. After cold-work the resistance gradually increases at a rate constant with this hypothesis.

(b) The increased electrical resistance of metals due to cold work can be partly or mainly removed by heating to a temperature much below that at which the mechanical 'recovery', or drop in the flow stress, occurs. It is thought that a large part of the electrical resistance is due to vacancies

formed by the cold-work; this will disappear if the metal is raised to a temperature at which they can move; while to get rid of mechanical hardness one has to allow edge dislocations to 'climb' out of the piled-up groups (Mott 1951), which will occur only if the specimen is heated to near to the temperature of self-diffusion.

Recent work at Harwell (Dugdale 1952) has given striking confirmation of the correctness of this hypothesis about the origin of the residual resistance due to cold-work. Platinum wires have been exposed to fast neutron bombardment in the pile, and an increase in the electrical resistance of the order of  $\frac{1}{2}\%$  observed. It is found that this resistance begins to diminish on annealing at about  $100^{\circ}\text{C}$ , and the activation energy for this process was determined, and found to be about 1.1 ev. Specimens with a residual resistance due to work-hardening were annealed in the same way and for these specimens too the activation energy for resistance decrease was found to be 1.1 ev. Since in the first case the resistance was almost certainly due to vacancies and interstitial atoms and the annealing involved the movement of the former, it seems highly probable that this is true also for the cold-worked specimens.

We turn now to the mechanism by which vacancies are formed by moving dislocations. Seitz (1952) gives a review of the various possibilities; of these we consider the following the most likely.

Consider the movement of dislocations in a given slip plane. Suppose that a certain number of screw dislocations cut this plane; in this context by a screw dislocation we mean one with Burgers vector *not* in the slip plane considered.\* We shall call such dislocations 'crossing dislocations' and the points where they cut the slip-plane under consideration 'crossing points'. Now, whenever a moving dislocation passes a 'crossing point', a jog is generated in that dislocation; and although a jog once formed is no impediment to the motion of an edge dislocation, a screw dislocation with a jog in it cannot move without leaving behind it a row of vacant lattice sites or interstitial atoms, according to the sign. This geometrical result of the dislocation concept was first put forward by Thornton Read, and is fully discussed by Seitz (1952) and by Mott (1951).

We may thus accept the hypothesis that moving screw dislocations do produce rows of vacancies and interstitial atoms, and that this process means that there is a resistance to the motion of screws which is not shared by edges. However, it is not immediately obvious that the passage of a *group* of dislocations along the slip plane, as in the formation of a slip band, will produce the vacancies and interstitials that we require. For, as pointed out by Mott (1951), provided the crossing screws do not move during the formation of a slip band, the rows of atoms added or subtracted by each succeeding dislocation will be side by side; all that will result from the motion of  $n$  screws a distance  $L'$  beyond a crossing

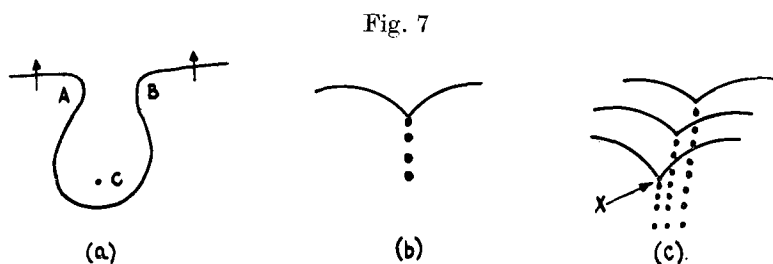
---

\* In the simple cubic structure such a dislocation *must* have a screw component, and this is also the case for hexagonal structures, but not for the close-packed cubic.

point will be the removal or addition of a slab of atoms of width  $nb$  and length  $L'$ . After the passage of the first few dislocations, the passage of further ones will meet with little resistance.

Seitz has suggested that, by a mechanism to be described below, enough energy is *released* in the formation of vacancies to disperse them. If this is true for the whole slip distance  $L'$  (say  $10^{-2}$  cm), far too many vacancies are formed and the resistance to the motion is much too large to be compatible with the observed facts. In order to explain the observed resistance, Seitz has to assume that a moving dislocation forms one vacancy for every  $10^4$  atoms that it crosses. If  $1/l^2$ , the density of crossing points, is such that  $l \sim 10^{-4}$  cm, this will be the case after a dislocation has moved a distance of the order  $l$ ; after it has moved a distance  $L'$  it will have jogs at intervals of  $l^2/L'$  along its length, and, if  $L'$  is  $10^{-2}$  cm, will be creating vacancies at 100 times the observed rate.

Moreover, if  $x$  is the distance between jogs, the resistance to the motion of a screw is equivalent to a stress of order  $Gb/x$ , and only if  $x$  is greater than  $l$  is this comparable with or less than the yield stress.



Motion of a screw dislocation cutting another screw.

We believe that the correct model is a compromise between the two points of view. Each screw dislocation, when it approaches a crossing point C, is repelled by the crossing dislocation, and curls round it, as in fig. 7 (a). When the two edge components, now on contiguous planes, join together and form a row of vacancies, enough energy is released to disperse them. Thereafter, however, and thus at a distance of order  $l$  from C, a continuous screw line is formed as in fig. 7 (b), which leaves behind it a row of vacancies which are not dispersed. The *first* process generated about  $l/b$  dispersed vacancies every time a screw passes a crossing point; the *second* generates rows of vacancies next to each other, and thus removes *layers* of atoms, presenting no effective resistance to the movement of dislocations after the first few.

Such a mechanism accounts for the generation of about the right number of vacancies; we have to ask further

(a) Can we give any explanation of the limited slip along one plane, similar to that provided by sessile dislocations in the case of edges?

(b) Can we find a mechanism which prevents the screws from moving back under a reversed stress?

(c) Can we account for the (slow) work-hardening of hexagonal single crystals?

As regards (a) we suggest the following: if  $n$  screw dislocations have moved a distance  $L$ , for each of the  $L/l^2$  crossing dislocations cut per unit length, a plane of atoms of width  $nb$  is either added or removed. The crossing dislocations are pushed sideways, therefore, a distance  $nb$ , for which energy is required to increase their length. We may suppose that the resultant force pushing the  $n$  screws back is equivalent to a stress

$$GnLb^2/l^3.$$

Which increases with increasing  $n$ .

The model suggests, then, that the slip on each band increases with increasing strain. That the slip on each plane increases with increasing strain is in accordance with the observations of Andrade and Roscoe (1937) on zinc.

The model will also lead us to expect that as the strain is increased the steps on the surface of a cubic metal may lengthen, though they do not increase in height (at low temperatures).

With regard to the sticking or anchoring of screw dislocations, we suppose that they will always have a certain number of 'free jogs' ~i.e. jogs which are not anchored to a point such as X in fig. 7 (c). These, and also any movement of the crossing screws since the slip bands were formed, will hinder movement in either direction.

Finally, we discuss the work-hardening of hexagonal crystals (or of cubic crystals under conditions of easy glide). This may be essentially of the same nature as in cubic crystals, the screws replacing the edges. We suppose that in hexagonal crystals the edges escape to the surface, so that only the screws are stuck, the worked crystal being full of screws. The slow rate of hardening of hexagonal crystals will be due primarily to the much greater tendency that one would expect, there being no possibility of sessile screws, for the groups of dislocations to form into grids, with the result that there is no *long-range* stress from any group. Some evidence that this occurs is presented in the next section (on cross-slip). We have not, however, been able to formulate any quantitative theory for these crystals.

An alternative suggestion is that in hexagonal crystals and in cubic crystals under conditions of easy glide the screws escape as well as the edges, and that the hardening is simply due to the strains round vacancies formed.

## § 7. CROSS-SLIP

Up to this point we have implicitly assumed that dislocation loops remain in the plane in which they start to move. If this were the case, slip bands observed on the surface of metals would be straight. Actually, in cubic metals, the slip bands observed frequently show intimate cross-slip. Examples of cross slip have been given by Cahn (1951). The points to note are:

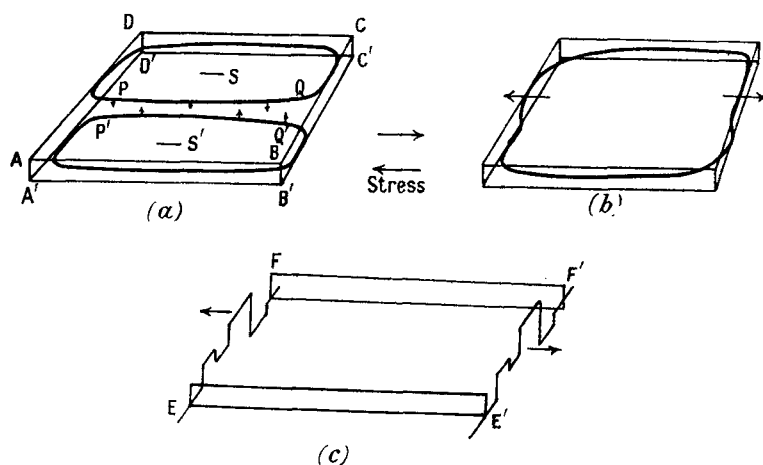
(i) Slip bands on the (111) planes on which the stress is largest are connected by short bands on the other sets of (111) planes.

(ii) Cross slip is more marked at high temperature.

(iii) The photographs at high magnification show that the step connecting the main bands is less sharp than the main steps.

An explanation of cross-slip (due to Frank) was given in the author's Guthrie lecture (Mott 1951). This explanation depends on the different kinematical properties of the edge and of the screw dislocation. The edge dislocation, if it is straight, can move only in the plane defined by its Burgers vector and itself. Moreover each element of an edge dislocation line which switches from one (111) plane to another has this property too. The same is true of dislocation lines which are partially edge, partially screw. A pure screw dislocation, on the other hand, has its Burgers vector parallel to itself, and so is not associated with any particular one

Fig. 8



Model for cross-slip.

of the (111) planes in which the Burgers vector lies. There is no kinematic reason, therefore, why a screw dislocation should not move for a certain distance in one of the (111) planes and then switch to another.

The explanation of cross-slip already referred to envisages, then, two loops spreading out from sources, marked S, S' in fig. 8, in parallel planes not too far from each other. PQ, P'Q' are the screw elements. When PQ reaches a position above P'Q' the two lines having opposite sign attract each other. They can then leave the planes in which they were moving originally and move towards each other until they join and annihilate each other. One is then left with a ring in which the two edge elements switch from one plane to another (fig. 8 (b)). This may happen for a number of sources, so that finally one is left with the two kinky edge

dislocations of fig. 8 (c), connected by screws. When these edge dislocations appear on the top surface of the crystal, and are followed by others produced in the same way, they will form kinky steps of the type observed.

This model shows why the steps joining the main slip bands are less distinct; when one loop after another starts out from the sources  $S, S'$ , there is no obvious reason why the screws should move from  $PQ$  to  $P'Q'$  always along the same plane.

We believe that this description is geometrically correct. It needs, however, some amplification. One may ask, for instance:

(a) Why should the sources  $SS'$  be generating loops at the same time? They will not in general have the same values of  $l$  and  $\sigma_i$  and one would expect one source to generate its full complement of loops before the next was brought into play (cf. Crussard 1951).

(b) In close-packed cubics—as opposed to the simple cubic, a very considerable activation energy will be required to make a screw dislocation moving in one plane switch to another. This is because, as already stated, all dislocations, screw or edge, split into half dislocations.

Fig. 9



Showing activation process for the motion of an extended screw dislocation out of its plane, the dotted lines being in new plane. Each line represents a half dislocation.

The half screw dislocation does not have its Burgers vector parallel to itself; it cannot move out of its plane. If a screw is to move out of its plane, a small length must be moved out first, and at the points where the two join, the two halves must be brought together, as in fig. 9. The necessary activation energy will be several electron volts, probably too large for the process to occur.

We believe that the clue to both criticisms is to be found in another property of the model which has already been emphasized. A screw dislocation, after it has cut a certain number of other screws, contains jogs, so that the resistance to the motion of a screw will increase as it moves along, and other screws will pile up behind it. The stress induced by this pile-up will readily induce some nearby source to generate loops, so that one is left with the configuration shown in fig. 10. Slip has occurred from  $A$  to  $B$  and  $A'$  to  $B'$ ; energy will be gained if the piled-up screws at  $B, B'$ , can move into the plane  $BB'$  (shown dotted) and so annihilate each other. The experimental evidence shows that this only occurs when the temperature is raised; we suggest that the clue is the

high stresses due to the pile-up ; these either allow spontaneous generation of loops in the plane  $BB'$ , or much reduce the activation energy for the process of fig. 9.

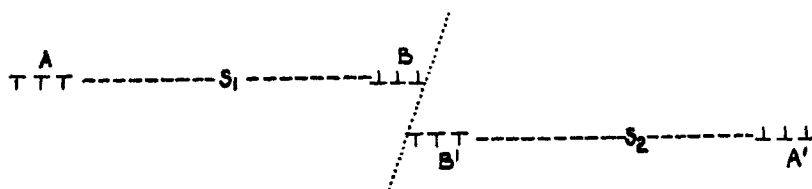
In a recent paper Smith (1952) has shown that pure copper bars extended by *c.* 10% show a drop in Young's modulus of about 10%, but that this is recovered on annealing at 100° c. This may well be due to the motion of screw dislocations bounding the slip bands (the edges being locked), and the process by which annealing takes place exactly the same as cross slip, namely the mutual annihilation of the screws.

#### §8. DEPENDENCE OF WORK-HARDENING ON TEMPERATURE

The theory of work-hardening of cubic crystals given in §§ 2, 3 and 4 is applicable as it stands to work-hardening at low temperatures only. In this section we shall suggest explanations of the following phenomena which depend on temperature :

(a) The 'clustering' of active slip planes at distances of the order of 200 Å from each other observed in aluminium at room temperature and above and in other metals, but absent according to Brown (1951) in aluminium at the temperature of liquid air.

Fig. 10



Showing groupings of screw dislocations.

(b) The dependence of the whole stress-strain curve on temperature.

(c) The behaviour observed when a crystal is strained at room temperature and then strained further at a lower temperature.

We remark first that the theoretical values of all the flow stresses derived in this article are linear functions of  $G$ , the shear modulus, and that  $G$  itself depends on temperature ; for aluminium the change is of the order 7% per 100° c (Ke 1947) above the characteristic temperature. Thus changes of from 10 to 20% between the absolute zero and room temperature may be expected from this cause alone. Apart from this, one expects very little temperature-dependence for a yield stress determined by a relation of the type

$$\sigma_0 = Gb/l.$$

The activation energy necessary to operate a Frank-Read source for stress  $\sigma$  less than  $\sigma_0$  is (Mott and Nabarro 1948)

$$k\sigma_0 b l^2 (1 - \sigma/\sigma_0)^{3/2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$



where  $k$  is a numerical factor of order 0.2. The term  $\sigma_0 b l^2$  is equal to  $G b^3 (l/b)$  and thus to  $c. 10^4$  ev. If, then, a yield point with a dependence on temperature greater than  $G$  is observed, as for instance in Rosi and Mathewson's (1950) work on aluminium, the most probable conclusion is that the yield point is determined by Cottrell locking.\*

Another process in which a large dependence on temperature may be expected is the spontaneous formation of dislocation loops in regions of very high stress; this has already been mentioned in connection with cross-slip. Another is Cottrell's mechanism of the cutting of one dislocation by another; this we believe responsible for transient creep at low temperatures.

Apart from these and the variation of  $G$  with temperature, it seems impossible to account for any temperature dependence of work-hardening in terms of an activated motion of dislocations; the term  $l$  in formula (15) should be at least  $10^{-4}$  cm or possibly even comparable with the slip distance. We consider, therefore, that the dependence on temperature is largely to be accounted for through the action of vacancies.

We assume that thermal recovery is associated with the 'climb' of dislocations out of their slip planes, a process possible at temperatures for which self-diffusion can occur; a theoretical account has been given by the author (Mott 1951). Such a process enables the piled-up arrays of dislocations, to which we have ascribed work-hardening, to disperse and deformation bands to polygonize. We suggest, however, that at considerably lower temperatures, namely those at which vacancies can move, the vacancies created by the moving screw dislocations will soften the material in the immediate neighbourhood of the planes which have already slipped.† This could occur through the diffusion of vacancies to the *secondary* groups of piled-up dislocations, the existence of which has already been deduced. The temperature above which this can occur will be  $c. -70^\circ \text{C}$  in aluminium, room temperature in copper,  $c. 120^\circ \text{C}$  in nickel.‡

If this occurs we have, following Brown, an immediate explanation of clustering. Once slip has occurred on a given plane there will occur a relatively slow reduction of  $\sigma_i$  in the immediate neighbourhood of that plane. Slip will then occur again, either on the plane which has slipped already or on some neighbouring one; but the source that has generated vacancies already is in no way preferred; in most cases it will be another one. The nearest of these should be at a distance from the first plane  $l^3/LL'$ ; with  $L \sim L' \sim 10^{-2}$  cm and  $1/l^3 \sim 10^{10}$  cm $^{-3}$ , this gives  $10^{-6}$  cm.

---

\* Note that Neurath and Koehler (1951) find almost the same yield point of pure single crystals of copper at liquid air and room temperature.

† Brown (1951) has suggested, without proposing a mechanism, that the material round a slip plane anneals, and has explained clustering and certain other phenomena in terms of this assumption. The discussion given here owes much to Brown's work.

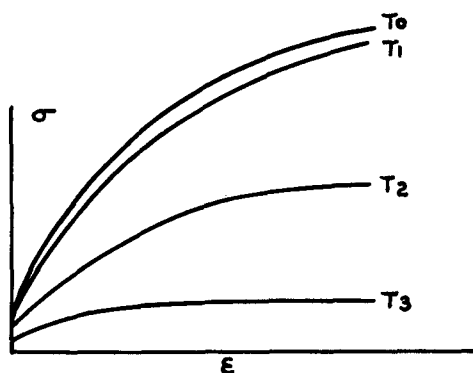
‡ The basis for this estimate is given below.

We thus expect sources at about this distance to become active. The lines of a cluster, on this hypothesis, will be similar in nature to those of the primary slip, and will, like the primary slip band, have groups of piled-up edge dislocations at the extremities. We suppose, moreover, that the limited supply of vacancies affects the secondary rather than the primary groups of piled-up dislocations.

If this hypothesis is correct, we may, following Brown, account for the temperature-dependence of the work-hardening curve. At temperatures  $T_0, T_1$  (fig. 11), too low for vacancies to move, we shall expect a *small* change in  $\sigma$  proportional to that for  $G$ . At a temperature  $T_2$ , at which vacancies can move, we shall expect :

- (a) A rather rapid drop of the whole curve, it being assumed that most new slip bands start near old ones.
- (b) The disappearance of the main part of the residual resistivity produced by cold-work.
- (c) The formation of clustered slip lines.

Fig. 11



Work-hardening at temperatures such that  $T_3 > T_2 > T_1 > T_0$ .

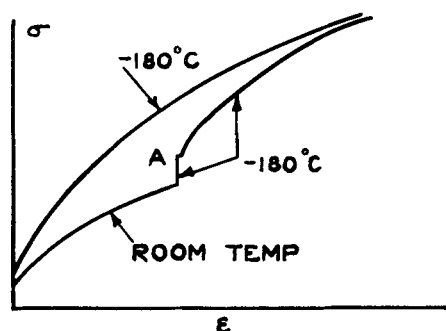
At a temperature  $T_3$  more than twice as high, true recovery may occur during straining.

Finally, following a suggestion of Brown, we can explain with this model the behaviour of the stress-strain curve of a crystal such as aluminium strained at room temperature and then lowered to liquid air temperature (fig. 12). On cooling to liquid air temperature there is a small jump in  $\sigma$ , doubtless due to the change in  $G$ ; then hardening is very rapid until the curve characteristic of liquid air temperature is approached. Now in our model the quantity  $n/L$ —i.e. the *strength* of the groups of piled-up dislocations divided by the distance between them, will not depend on the temperature of deformation; moreover it is this quantity, according to formula (11), which determines the flow stress. The crystal at room temperature flows more easily because the material round any slip plane softens, owing to a rearrangement of the secondary groups. Thus when slip

continues from the point A in fig. 12, the annealed region is quickly used up; thereafter the material differs from a crystal strained from the beginning at liquid air temperature only in that some of the slip bands are grouped in clusters, but not in the total number of dislocation groups or in the mean value of  $n/L$ .

A further consequence of the formation of vacancies is probably, as has been suggested by various authors (Hall and Williamson 1951), the fragmentation or substructure that is formed in a cold-worked material (Gough and Wood 1936, 1938). The elements of the substructure are, we suggest, simply polygonized crystallites, and polygonization is enabled to take place through the movement of vacancies formed by the cold work. This suggests that fragmentation would not take place at a low enough temperature.

Fig. 12



Stress strain curve initially at room temperature and then at liquid air temperature.

Finally, we give a very rough estimate of the critical temperatures  $T_1$  below which the vacancies cannot move. Unfortunately we have only one definite measurement of the activation energy  $U$  for movement of a vacancy, that of Dugdale (1952) for platinum, and this is less than one half the probable value of  $W$  for this element, instead of being equal to it. The following table gives  $U + W$ , the activation energy for self-diffusion for the elements for which it is known, and for Al, Pt, Ni deduces it on the assumption that these quantities are proportional to the melting points.  $U$  is deduced on the assumption that the ratio  $U$  to  $W$  is the same as for platinum. Finally  $T_1$  is taken, rather arbitrarily, as the temperature at which a vacancy makes one jump in ten seconds.

Commenting on these results, we may note that Seitz (1952, p. 45) reports unpublished experiments by Blewitt which record that the resistance of copper strained at liquid air temperature shows a partial recovery on annealing at  $-80^\circ\text{C}$ ; with the above value of  $U$ , we may estimate that at this temperature a vacancy would change places about once in 1 000 sec, so an anneal at this temperature is just possible. It is difficult on this basis

to understand the results of Bowen, Eggleston and Kropschott (1952) who after cold work at room temperature found an activation energy of 28.3 kcal for further recovery of the residual resistance of copper. Perhaps this is the activation energy for the motion of interstitial atoms.

Table 1

Metal	$W+U$	$U$	$T_1$ (degrees K)
Cu	<b>46.5</b>	16	230
Ag	<b>45.9</b>	16	230
Au	<b>53.0</b>	18	260
Pb	<b>27.5</b>	9.5	140
$\gamma$ -Fe	<b>67.9</b>	23	350
Al	39	13	200
Pt	75	<b>26</b>	430
Ni	64	23	390

$W$  and  $U$  are in kcal/g atom.

Experimentally determined quantities in bold-face type.

## REFERENCES

- ANDRADE, E. N. DA C., and HENDERSON, C., 1951, *Trans. Roy. Soc.*, **244**, 177.  
 ANDRADE, E. N. DA C., and ROSCOE, R., 1937, *Proc. Phys. Soc.*, **49**, 152.  
 BOWEN, D., EGGLESTON, R. R., and KROPSCHOT, R. H., 1952, *J. Applied Phys.*, **23**, 630.  
 BROWN, A. F., 1949, *Nature, Lond.*, **163**, 961 ; 1951, *J. Inst. Metals*, **80**, 115 ; 1952, *Advances in Physics*, **1** (in press).  
 BROWN, A. F., and HONEYCOMBE, R. W. K., 1951, *Phil. Mag.*, **42**, 1146.  
 CAHN, R. W., 1951, *J. Inst. Metals*, **79**, 129.  
 COTTRELL, A. H., 1948, *Bristol Conference on the Strength of Solids*, *Phys. Soc.*, p. 30 ; 1952, *Phil. Mag.*, **43** (in press) ; 1953, *Dislocations and Plastic Flow in Solids* (Oxford : Clarendon Press) (in press).  
 COTTRELL, A. H., and BILBY, B. A., 1949, *Proc. Phys. Soc. A*, **62**, 49.  
 CHEN, N. K., and MATHEWSON, C. H., 1951, *J. of Metals*, **3**, 653.  
 CHEN, N. K., and POND, R. B., 1952, *Phys. Rev.* (in press).  
 CRUSSARD, C., *Solvay Conference 1951*, discussion remarks p. 575.  
 DUGDALE, R. A., 1952, *Phil. Mag.*, **43**, 912.  
 ESHELBY, J. D., 1949, *Proc. Roy. Soc. A*, **197**, 396.  
 ESHELBY, J. D., FRANK, F. C., and NABARRO, F. R. N., 1951, *Phil. Mag.*, **42**, 351.  
 FISHER, J. C., HART, E. W., and PRY, R. H., 1952, *Phys. Rev.* (in press).  
 FRANK, F. C., 1951, *Pittsburgh Conference on Plastic Deformation of Crystalline Solids*, p. 89 ; 1952, *Advances in Physics*, **1**, 91.  
 FRANK, F. C., and READ, W. T., 1950, *Phys. Rev.*, **79**, 722.  
 GOUGH, H. J., and WOOD, W. A., 1936, *Proc. Roy. Soc. A*, **154**, 510 ; 1938, *Ibid.*, **165**, 358.  
 HALL, W. H., and WILLIAMSON, G. K., 1951, *Proc. Phys. Soc. B*, **64**, 937.  
 HEIDENREICH, R. J., and SHOCKLEY, W., 1948, *Bristol Conference on the Strength of Solids*, p. 57.  
 HERRING, C., and GALT, J., 1952, *Phys. Rev.*, **85**, 1060.  
 HONEYCOMBE, R. W. K., 1951, *J. Inst. of Metals*, **80**, 45.  
 HUNTINGTON, H. B., and SEITZ, F., 1942, *Phys. Rev.*, **61**, 315.  
 JOHNSON, R. P., 1939, *Phys. Rev.*, **56**, 814.  
 KE, T. S., 1947, *Phys. Rev.*, **71**, 533.

- LALOEUF, A., and CRUSSARD, C., 1951, *Revue de Metallurgie*, **48**, 461.  
LEIBFRIED, G., 1949, *Z. Physik*, **126**, 781 ; 1950, *Ibid.*, **127**, 344.  
LOMER, W. M., 1951, *Phil. Mag.*, **42**, 1327.  
LÜCKE, K., and LANGE, H., 1952, *Z. für Metallkunde*, **43**, 55.  
MASING, G., and RAFFELSPEISER, J., 1950, *Z. für Metallkunde*, **41**, 65.  
MOTT, N. F., 1951, *Proc. Phys. Soc. B*, **64**, 729.  
MOTT, N. F., and NABARRO, F. R. N., 1948, *Bristol Conference on the Strength of Solids*, p. 11.  
NABARRO, F. R. N., 1951, *Proc. Roy. Soc. A*, **209**, 278 ; 1952, *Advances in Physics*, **1**, 269.  
NEURATH, P. W., and KOEHLER, J. S., 1951, *J. Applied Phys.*, **22**, 621.  
NOWICK, A. S., 1951, *Phys. Rev.*, **82**, 551.  
RÖHM, F., and KOCHENDÖRFER, A., 1950, *Z. für Metallkunde*, **41**, 265.  
ROSI, F. D., and MATHEWSON, C. H., 1950, *J. of Metals*, **188**, 1159.  
SEITZ, F., 1952, *Advances in Physics*, **1**, 43.  
SMITH, A. D. N., 1951, *M.O.S. Report*, to be published.  
STROH, A. N., 1952, *Proc. Phys. Soc.* (in press).  
TAYLOR, G. I., 1934, *Proc. Roy. Soc. A*, **145**, 362.