University of Tehran

College of Engineering

School of Electrical and Computer Engineering (ECE)

Robotics

Homework 1

Teaching Assistants:

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List of Problems

Problem 1: Snake Robot Locomotion - 10 points

The study of snake locomotion offers a rich source of inspiration for designing mobile robots capable of navigating complex, unstructured environments where traditional wheeled or legged robots might fail. Snakes utilize their slender, hyper-redundant bodies to generate propulsion through various gaits, enabling them to traverse tight spaces, climb obstacles, and move efficiently on diverse terrains. Understanding the underlying biomechanics is crucial for translating these capabilities into effective robotic systems.

(a) Biological Inspiration: Snake Locomotion

(b) Robotic Implementation: Modular Robot

Figure 1: Comparison of a biological snake and its robotic counterpart.

1. **Analyze Biomechanics:** Describe the biomechanical principles behind at least two distinct snake gaits (e.g., **Serpentine** and **Sidewinding**). Explain how each gait generates forward propulsion by creating and leveraging anisotropic friction and ground reaction forces, despite the absence of limbs.

2. **Propose a Robotic Design:** Propose a conceptual design for a modular snake robot capable of mimicking these gaits. Detail the necessary hardware components, including the structure of the individual modules, the type and arrangement of joints (e.g., providing both pitch and yaw motion), and the essential sensors for proprioception and environmental perception (e.g., contact sensors, IMUs).

Problem 2: Robotic Camera Systems (10 points)

Filming a stadium during a live game presents several challenges. Key issues include the necessity for unobstructed views of the action without disrupting the audience's experience, the ability to capture fast-paced movements from various angles, and maintaining stability in a crowded environment.

Figure 2: Camera systems for filming a stadium

1. Suggest a robot that can move a camera smoothly and precisely across a large stadium from various angles, including overhead shots, while minimizing obstruction to the audience.

2. Investigate the robot's abilities and describe other potential uses for this type of robot in different industries.

Problem 3: Yaw, Pitch and Roll (20 points)

Consider a disk that undergoes rotations around the y-axis, z-axis, and x-axis, respectively. The final orientation of the disk after these rotations is represented by a given rotation matrix.

$$Q = \begin{bmatrix} 0.354 & -0.612 & 0.707 \\ 0.927 & 0.127 & -0.354 \\ 0.127 & 0.780 & 0.612 \end{bmatrix}$$

1. Calculate the corresponding yaw, pitch, and roll angles using the final rotation matrix, assuming the rotations are performed about the local axis.

2. Calculate the yaw, pitch, and roll angles assuming the rotations are performed about the fixed axis.

3. Determine the natural invariant parameters (e, ϕ) and the natural Euler-Rodrigues parameters (r, r_0) from the given rotation matrix.

Problem 4: Rotation Matrix (20 points)

Assume we want to rotate an object by 90 degrees ($\theta = 90°$) around an axis defined by the vector $v = (1, 2, 2)$.

1. Construct the rotation matrix (R): Using the given axis and rotation angle, calculate the corresponding 3×3 rotation matrix (R).
2. Verify the validity of the constructed matrix: After obtaining the matrix R, prove that it is a valid rotation matrix.
3. Apply the rotation to a point: If we have a point in space with coordinates $P = (3, 0, 0)$, what will be the new coordinates of this point (P') after applying the above rotation?

Problem 5: Rotation Representation (15 points)

The final operator of a robot's initial orientation R_i has rotated to a final orientation R_f. Find the minimum relative rotation between the initial and final orientation using the following methods:

$$R_i = \begin{pmatrix} 0 & -0.5 & -\frac{\sqrt{3}}{2} \\ -1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -0.5 \end{pmatrix}, \quad R_f = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

1. Euler angles of type $YXY(\alpha_1, \alpha_2, \alpha_3)$ in the local coordinate system.
2. The values of e, ϕ.

Problem 6: Mathematical Background (10 points)

Considering the following 3 x 3 matrix: Considering the following 3×3 matrix:

$$\mathbf{X} = \begin{bmatrix} -2 & 0 & 1 \\ -5 & 3 & y \\ 4 & -2 & -1 \end{bmatrix}$$

Find all values of the variable 'y' that will demonstrate that matrix $\mathbf{X}$ has eigenvalues 0, 2, and -2.

Problem 7: (Coding Exercise) 15 points

Write three functions for the questions below, then test your functions with the given rotation matrix as their input.

1. Write a function for calculating natural invariant parameters $(\vec{e}, \phi)$ from the rotation matrix.
2. Write a function for calculating natural euler parameters (q_1, q_2, q_3, q_0) from the rotation matrix.
3. Write a function for calculating natural euler rodrigues parameters $(\vec{r}, r_0)$ from the rotation matrix.

Homework Guidelines and Instructions

- The deadlines are fixed and cannot be changed. If you need extra time, you can use your grace period (14 days) and upload your answers up to 5 days after the deadline.
- If you write your report of this homework in LaTeX, you will be rewarded 5% extra points.
- Programming problems can either be completed in Python or MATLAB and your codes must be executable and uploaded along with the report.
- This exercise is done by one person.
- If any similarity is observed in the work report or implementation codes, this will be considered as fraud for the parties.
- Using ready-made codes without mentioning the source and without changing them will constitute cheating and your practice score will be considered zero.
- If you do not follow the format of the work report, you will not be awarded the grade of the report.
- Handwritten exercise delivery is not acceptable. Only mathematical procedures and formulas are allowed to be handwritten.
- All pictures and tables used in the work report must have captions and numbers.
- A large part of your grade is related to the work report and problem solving process.
- Please upload the report, code file and other required attachments in the following format in the system: `HW1_[Lastname]_[StudentNumber].zip`
  For example: `HW1_Ahmadi_123456789.zip`
- You can ask your questions or doubts either via Telegram group or sending an email directly to the teaching assistants of this homework.
- The questions that are asked in Telegram PV of teaching assistants will not be answered as they are not traceable for the chief TA.
- If you wanted to ask your questions or doubts via email, use the following subject: "Robotics_HW1_[Lastname]_[StudentNumber]" and make sure to cc the chief TA at shayanahmadi9999@gmail.com:
  - Question#1,2,3,4,5,6,7: alizadeh.reza7997@gmail.com (Reza Alizadeh)
- Be happy and healthy