Dynamic Privacy Choices

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Abstract

I study a dynamic model of consumer privacy and platform data collection. In each period, consumers choose their level of platform activity. Greater activity generates more precise information about the consumer, thereby increasing platform profits. Although consumers value privacy, a platform is able to collect much information by gradually lowering the level of privacy protection. In the long-run, consumers become "addicted" to the platform, whereby they lose privacy and receive low payoffs, but continue to choose high activity levels. Competition is unhelpful because consumers have a higher incentive to use a platform to which they have lower privacy.

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1 Introduction

Online platforms, such as Amazon, Facebook, Google, and Uber, analyze user activities and collect a large amount of data. This data collection may improve their services and benefit consumers. However, it also raises important concerns among consumers and policymakers (Crémer et al., 2019; Furman et al., 2019; Morton et al., 2019).

Consider a consumer (she) and a social media platform (it). The consumer writes posts, shares photos, and reads news on the platform. The platform analyzes her activity and collects data such as her race, location, and political preferences. The platform can then generate revenue, e.g., via improved targeted advertising. The consumer faces a trade-off: On the one hand, she enjoys the services provided by the platform. On the other hand, the consumer may value her privacy, or be concerned about data collection leading to the risk of data leakage, identity theft, and price or non-price discrimination.[1] The latter is a "privacy cost" of using the platform. If the consumer anticipates a high privacy cost, she may use the platform less actively, or may not join it in the first place. The platform can influence this trade-off through its privacy policy. For example, Facebook committed to not use cookies to track users.[2]

I formalize this story in a dynamic model: In each period, a consumer chooses her level of platform activity. Based on the level of activity, the platform observes a signal about her time-invariant type. The precision of the signal is increasing in the activity level, but decreasing in the platform's privacy level, which specifies the amount of noise added to the signal. The platform's per-period profit is increasing but the consumer's payoff is decreasing in the amount of information the platform has collected. Thus, the consumer chooses activity levels balancing the benefits from the service and the privacy costs. Anticipating the consumer's behavior, the platform sets privacy levels to maximize profits.

The key idea is that, when the consumer has less privacy, she faces a lower marginal privacy cost of using the same platform, repeatedly. For example, if Google already knows a lot about a consumer, she might not care about letting Google maps track her location. In an extreme case, if the platform knows everything, then the marginal privacy cost is zero. This is because the

[1] Such concerns are highlighted by, for example, the *Cambridge Analytica* scandal.

[2] In 2004, Facebook's privacy policy stated that "we do not and will not use cookies to collect private information from any user." https://web.archive.org/web/20050107221705/http://www.thefacebook.com/policy.php (accessed on March 12, 2020)

consumer's activity on a platform no longer affects what it knows about her.

The paper examines the dynamic implications of this idea. First, in equilibrium, the consumer chooses higher activity levels but receives lower payoffs over time. In the long-run, the consumer loses privacy and incurs a high privacy cost, but behaves as if there is no privacy cost. This long-run outcome arises even when the consumer values privacy highly. This is because the platform can set high privacy levels in early periods to incentivize the consumer to use the platform. However, the platform later reduces the privacy level to accelerate data collection.

The decreasing marginal privacy cost implies that the consumer is more willing to use a platform to which she has less privacy. This makes platform competition less effective. To see this, suppose that an incumbent (e.g., Google) has a lot of data on a consumer. Then, even if the entrant (e.g., DuckDuckGo) offers a better privacy protection, the consumer may stick to the incumbent since her marginal privacy cost of using it is negligible. As a result, the incumbent can keep collecting data without losing the consumer to the entrant.

I consider various privacy regulations. For example, mandating the platform to adopt a stricter privacy policy may lower consumer welfare in the long-run. Enforcing the long-term commitment of the platform to its privacy policy may not change the equilibrium. In contrast, the "right to be forgotten," which enables consumers to erase past information, may enhance consumer welfare and induce competition.

The paper has implications for consumer privacy. First, the consumer's long-run behavior seems consistent with the so-called privacy paradox: Consumers express concerns about data collection but actively share data with third parties (Acquisti et al., 2016). Second, the platform's equilibrium strategy rationalizes how online platforms, such as Facebook, have relaxed its privacy policy over time. Third, the analysis of competition helps us understand why competition using better privacy policies or more privacy controls has not been successful (Marthews and Tucker, 2019). The result also poses a challenge to firms that offer privacy-friendly alternatives to dominant platforms. Finally, the paper offers novel policy implications. For example, restricting data collection can lower the welfare of myopic consumers in the long-run.

This paper contributes to the literature on markets for data and the economics of privacy. First, the paper is related to Acemoglu et al. (2019), Bergemann et al. (2019), and Choi et al. (2019). They consider static models in which a platform collects data in exchange for monetary compensation. If some consumers sell their data, the platform can also learn about other consumers. This "data externality" lowers the incentive of each consumer to protect privacy, leading to low compensation and excessive data sharing.[3] The key economic force of this paper is similar to theirs: A consumer's data provision today lowers her incentive to protect privacy in the future. However, the dynamic model enables me to study the new implications of this effect. Specifically, I consider market entry, commitment to privacy policies, the right to be forgotten, data retention, consumer myopia, and the evolution of a platform's privacy policy. In terms of how data is generated, the previous works assume that consumers hold data at the outset, and platforms purchase data in exchange for monetary compensation. In my model, data arise as a byproduct of activity from which a consumer enjoys benefits. This formulation directly applies to online platforms that do not pay consumers for data.

This paper also relates to the recent work on competition and data. Hagiu and Wright (2020) study "data-enabled learning" whereby firms can improve their products and services through learning from the data they obtain from their customers. Prufer and Schottmüller (2017) assume that the cost of investing in quality is decreasing in the firm's past sales, and greater investment in quality leads to higher demand in the current period. In contrast to this literature, I assume that data collection lowers consumer welfare. This assumption enables me to study the value of consumer privacy. Another important difference is whether a consumer regards collected data as sunk. For example, in Hagiu and Wright (2020), regardless of how much data a firm has, a consumer's payoff from an outside option (i.e., not buying a product) is zero. In my paper, once a platform collects data, the consumer incurs the associated privacy cost even when she does not use the platform. This assumption is important for the key mechanism. Hagiu and Wright (2020) allow price competition and consider a rich learning dynamics incorporating "within-user" and "across-user" learning. In contrast, I abstract from pricing, and focus on within-user learning.

The way that the consumer's incentive changes over time is similar to the one in career concern models originated with Holmström (1999). In career concern models, a young worker, whose ability has not yet revealed to the market, works hard to influence the market's belief about her ability. In my model, a consumer who has not yet lost privacy uses the platform less actively to generate less information. Over time, the private information of the consumer and the worker are

[3] Relatedly, Easley et al. (2018) consider data externalities in a model where market transaction generates data.

revealed, and they have lower incentives to engage in signal-jamming. Despite this connection, the two signal-jamming activities are quite different. In career concern models, the market wants the worker to engage in signal-jamming, which corresponds to higher effort. Thus, there is a trade-off between learning the worker's ability and motivating high effort (i.e., Hörner and Lambert 2018). In my model, the platform wants the consumer to engage less in signal jamming. Thus, the platform prefers to collect information for not only increasing profit today but motivating the consumer to raise activity levels in the future. Many of my results come from this complementarity between data collection and higher activity levels.

Bonatti and Cisternas (2020) study consumer privacy in a continuous-time dynamic model. They consider a long-lived consumer with short-lived sellers. Sellers can learn about consumer preferences based on scores that aggregate purchase histories, and sellers use information for price discrimination. In contrast, I consider long-lived platforms and abstract away from how platforms use consumer information. Fainmesser et al. (2019) study the optimal design of a platform to store data and invest in information security. They consider a platform that cares about both the activity levels of consumers and the amount of data it can extract. They study how different objectives lead to different platform designs. I adopt simpler preferences for consumers and platforms, but consider a dynamic environment. Hörner and Skrzypacz (2016) considers a dynamic game of selling information. They find that the agent's optimal selling strategy is to transmit information gradually. In my model, the benefit of gradualism appears in the platform's optimal privacy policy. Casadesus-Masanell and Hervas-Drane (2015) considers a static vertical differentiation framework in which firms compete in price and the level of privacy protection.

The rest of the paper is as follows. Section 2 presents a model of a monopoly platform. Section 3 presents the long-run equilibrium outcome and characterizes the equilibrium privacy policy of the platform. Section 4 considers platform competition. Section 5 studies the incentive of the consumer or platforms to erase past information. Section 6 considers extensions, including a forward-looking consumer and general payoff functions.

2 Model

Time is discrete and infinite, indexed by $t \in \mathbb{N}$. There are a consumer (she) and a platform (it). The consumer's type X is drawn from a normal distribution $\mathcal{N}(0, \sigma_0^2)$. X is realized before $t = 1$ and fixed over time. The consumer does not observe X.[4] The platform does not observe X but receives signals about it.

In each period $t \in \mathbb{N}$, the consumer publicly chooses an *activity level* $a_t \geq 0$. After the consumer chooses a_t, the platform privately observes a signal $s_t = X + \varepsilon_t + z_t$, where $\varepsilon_t \sim \mathcal{N}\left(0, \frac{1}{a_t}\right)$ and $z_t \sim \mathcal{N}(0, \gamma_t)$. A greater a_t reduces the variance of ε_t and makes s_t more informative about X, whereas a greater γ_t increases the variance of z_t and makes s_t less informative about X.[5] $\gamma_t \geq 0$ is the *privacy level* of the platform in period t. All the random variables, $X, \varepsilon_1, z_1, \varepsilon_2, z_2, \dots,$ are mutually independent.

The payoffs are as follows. Suppose that the consumer has chosen activity levels $\boldsymbol{a}_t = (a_1, \dots, a_t)$ and the platform has chosen privacy levels $\boldsymbol{\gamma}_t = (\gamma_1, \dots, \gamma_t)$ up to period t. Then, at the end of period t, the platform receives a payoff of $\sigma_0^2 - \sigma_t^2(\boldsymbol{a}_t, \boldsymbol{\gamma}_t) \geq 0$. $\sigma_t^2(\boldsymbol{a}_t, \boldsymbol{\gamma}_t)$ is the variance of the conditional distribution of X given $(\boldsymbol{a}_t, \boldsymbol{\gamma}_t)$, derived from Bayes' rule.[6] I take $(\sigma_t^2(\cdot, \cdot))_{t \in \mathbb{N}}$ as a primitive. A lower $\sigma_t^2(\boldsymbol{a}_t, \boldsymbol{\gamma}_t)$ means that the platform has a more accurate estimate of X, which also means that the consumer has less privacy. For any $\tau \leq t$, $\sigma_t^2(\boldsymbol{a}_t, \boldsymbol{\gamma}_t)$ is decreasing in a_τ, increasing in γ_τ, and independent of a realized signal s_τ. Where it does not cause confusion, I write $\sigma_t^2(\boldsymbol{a}_t, \boldsymbol{\gamma}_t)$ as σ_t^2. The platform discounts future payoffs with a discount factor $\delta_P \in (0, 1)$.

The consumer's payoff in period t is $U(\boldsymbol{a}_t, \boldsymbol{\gamma}_t) := u(a_t) - v\left[\sigma_0^2 - \sigma_t^2(\boldsymbol{a}_t, \boldsymbol{\gamma}_t)\right]$. The first term $u(a_t)$ is the gross benefit of using the platform. $u : \mathbb{R}_+ \to \mathbb{R}$ is strictly concave, continuously differentiable, maximized at $a^* \in (0, \infty)$, and satisfies $u(0) = 0$. The second term $v\left[\sigma_0^2 - \sigma_t^2(\boldsymbol{a}_t, \boldsymbol{\gamma}_t)\right]$ is a *privacy cost*, which captures the negative impact of data collection on the consumer. $v \geq 0$ is the exogenous parameter that captures the value of privacy. The consumer is myopic and chooses a_t to maximize $U(\boldsymbol{a}_t, \boldsymbol{\gamma}_t)$ in each period t (Section 6 considers a forward-looking consumer).

Payoffs are normalized so that if $a_t = 0$ for all t, then the platform and the consumer obtain

[4] This is to simplify exposition. If the consumer observes X, then her activity level may become a signal of X. However, all the results of this paper hold with respect to a pooling equilibrium in which the consumer of all types chooses the same activity level after any history.

[5] If $a_t = 0$, then I define s_t as a pure noise that is independent of X.

[6] The equivalent formulation is that the platform observes (a_t, s_t), chooses $b_t \in \mathbb{R}$, and obtains an ex post payoff of $-(X - b_t)^2$, which the platform does not observe. Writing the payoffs in terms of σ_t^2 simplifies exposition.

zero payoffs in all periods. The primitives, σ_0^2, $u(\cdot)$, and v, are commonly known to the consumer and the platform (Section 6 considers the consumer who is privately informed of v).

The timing of the game is as follows. Before $t = 1$, the platform commits to a *privacy policy* $\boldsymbol{\gamma} = (\gamma_1, \gamma_2, \dots) \in \mathbb{R}_+^\infty$. After observing $\boldsymbol{\gamma}$, the consumer chooses an activity level in each period. An *equilibrium* is a strategy profile such that (i) the consumer myopically chooses a_t to maximize $U(\boldsymbol{a}_t, \boldsymbol{\gamma}_t)$ following every history, breaking ties in favor of the highest activity level, and (ii) the platform, anticipating (i), optimally chooses a privacy policy $\boldsymbol{\gamma}$ before $t = 1$.

I do not explicitly model the consumer's decision to join the platform. However, we may think that the consumer joins the platform in t if t is the earliest period in which $a_t > 0$. We can extend the model so that the consumer incurs a fixed cost $\kappa > 0$ to join, but the results continue to hold if κ is not too high.

2.1 Discussion of Modeling Assumptions

The model captures the situation in which a consumer uses a platform, which analyzes her activity and collects data. The platform uses data to generate revenue, but data collection reduces the consumer's welfare. This subsection provides further discussion on modeling assumptions.

Data Generation In practice, consumer data are generated along with their activity on a platform, such as browsing news and liking posts. The model captures such a situation by assuming that the precision of a signal is increasing in the activity level. The current formulation distinguishes this paper from the recent literature on markets for data, which assumes that consumers are exogenously endowed with data (Acemoglu et al., 2019; Bergemann et al., 2019; Choi et al., 2019; Ichihashi, 2019). To focus cleanly on the consumer's incentives to protect privacy, I do not focus on belief manipulation, such as a consumer strategically manipulating (say) browsing history to influence the platform's inference.

Privacy Cost The privacy cost $v(\sigma_0^2 - \sigma_t^2)$ captures monetary or non-monetary reasons for which a consumer wants a platform to have less information. For instance, a consumer may have intrinsic preferences for privacy (Kummer and Schulte, 2019; Lin, 2019; Tang, 2019).[7] For another instance, a consumer may consider the risk of data breach, identity theft, and price or non-price discrimination by platforms and third parties who may access data.

One important observation is that the consumer cannot erase data collected in the past, thereby perceiving privacy cost as sunk. One way to see this is that, even if the consumer "quits" the platform by choosing $a_s = 0$ for all $s \geq t$, σ_s^2 remains σ_t^2 and does not go back to σ_0^2. For example, suppose that a platform collects sensitive personal information, and a consumer faces a risk of data leakage. Upon quitting the platform, the consumer faces a technical difficulty of erasing information she provided, a phenomenon referred to as "data persistence" (Tucker, 2018). Alternatively, the consumer may (correctly or incorrectly) believe that the platform keeps collected data even after users quit. If so, the consumer will incur a privacy cost (i.e., the expected loss from data leakage) even after she stops using the services. Since the consumer regards the privacy cost as sunk, she chooses activity levels based on the marginal privacy cost rather than the level of privacy cost. If there are multiple platforms as in Section 4, the consumer decides which platform to use based on the marginal privacy cost associated with each platform.

Arguably, consumers' preferences on data collection should be more complicated than what is assumed. For example, a consumer may prefer data collection if it leads to better services. Also, the benefit $u(a)$ of using a platform may depend on the amount of data collected. While Section 6 considers more general payoffs, I mainly focus on $u(a_t) - v[\sigma_0^2 - \sigma_t^2]$, because it is a tractable benchmark where I can characterize the equilibrium. Also, the above generalization seems to push the equilibrium toward more data collection. However, my results show that even if the preferences are such that the consumer wants the platform to have no information, the equilibrium involves full privacy loss (i.e. Proposition 2). Thus, I conjecture that the main insight is robust to those extensions.

Platform I comment on the platform's strategy and payoffs. First, this paper considers a platform that either chooses a privacy policy at the outset (as in the baseline model) or sets a privacy level at the beginning of each period. They are not fully general. For example, I could consider a mechanism that maps each history of activity levels $(a_s)_{s=1}^t$ to a privacy level γ_t in period t. However,

[7] Lin (2019) considers an experiment that empirically distinguishes intrinsic and instrumental preferences for privacy.

such a mechanism seems impractical and prohibitively costly to communicate to consumers.

Second, the platform cannot use monetary transfer. This is consistent with online platforms such as Facebook and Google. If the platform could commit to a mechanism mapping $\boldsymbol{a}_t$ to monetary transfer in each period t, then the platform may commit to compensate the consumer in exchange for a higher activity level. However, I conjecture that positive compensation eventually disappears. This is because, even without transfer, the consumer chooses the highest activity level and the platform learns full information in the long-run.

As to payoffs, the platform's payoff can be any decreasing function of $(\sigma_t^2)_{t\in\mathbb{N}}$. All the results and proofs continue to hold without modification (see Section 6 for details).

3 Monopoly Platform

I begin with studying the consumer's behavior, taking the platform's strategy as given. Then, I present the long-run equilibrium outcome. After that, I characterize the platform's equilibrium privacy policy.

3.1 Consumer Behavior

Bayes' rule implies[8]

$$\sigma_t^2(\boldsymbol{a}_t, \boldsymbol{\gamma}_t) = \frac{1}{\frac{1}{\sigma_{t-1}^2(\boldsymbol{a}_{t-1}, \boldsymbol{\gamma}_{t-1})} + \frac{1}{\frac{1}{a_t}+\gamma_t}}. \tag{1}$$

Thus, in each period t, the consumer chooses a_t to maximize

$$\begin{aligned} &u(a_t) - v\left[\sigma_0^2 - \sigma_t^2(\boldsymbol{a}_t, \boldsymbol{\gamma}_t)\right] \\ =&u(a_t) - v\left[\sigma_0^2 - \frac{1}{\frac{1}{\sigma_{t-1}^2(\boldsymbol{a}_{t-1}, \boldsymbol{\gamma}_{t-1})} + \frac{1}{\frac{1}{a_t}+\gamma_t}}\right], \end{aligned}$$

taking $\sigma_{t-1}^2(\boldsymbol{a}_{t-1}, \boldsymbol{\gamma}_{t-1})$ and γ_t as given. Define the privacy cost function as $C(a, \gamma, \sigma^2) := v\left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma^2}+\frac{1}{\frac{1}{a}+\gamma}}\right)$. The following lemma summarizes the key properties of the privacy cost C and the marginal privacy

[8]This follows from the fact that if $x|\mu \sim N(\mu, \sigma^2)$ and $\mu \sim N(\mu_0, \sigma_0^2)$, then $\mu|x \sim N\left(\frac{\sigma_0^2}{\sigma^2+\sigma_0^2}x + \frac{\sigma^2}{\sigma^2+\sigma_0^2}\mu_0, \left(\frac{1}{\sigma_0^2}+\frac{1}{\sigma^2}\right)^{-1}\right)$.

cost $\frac{\partial C}{\partial a}$.

Lemma 1. *The following holds.*

1. $C(a, \gamma, \sigma^2)$ *is decreasing in* γ *and* σ^2*, and increasing in* a*.*
2. $\dfrac{\partial C}{\partial a}(a, \gamma, \sigma^2)$ *is decreasing in* γ *and increasing in* σ^2*.*

Proof. Point 1 follows from equation (1). Point 2 follows from

$$\frac{\partial C}{\partial a} = v \cdot \frac{\frac{\frac{1}{a^2}}{\left(\frac{1}{a}+\gamma\right)^2}}{\left(\frac{1}{\sigma^2} + \frac{1}{\frac{1}{a}+\gamma}\right)^2} = \frac{v}{\left(\frac{1}{\sigma^2}\left(1+\gamma a\right)+a\right)^2}. \tag{2}$$

□

The privacy cost C is decreasing but the marginal cost $\frac{\partial C}{\partial a}$ of using the platform is increasing in σ^2. Thus, if the consumer has less privacy, then she obtains a lower payoff but has a higher incentive to use the platform. Intuitively, if the platform has more information about the consumer, then information generated today affects the platform's learning less, which implies a lower marginal privacy cost. In particular, if the consumer has no privacy ($\sigma^2 \to 0$), then the marginal cost is zero, because a change in the activity level has no impact on the platform's information.

The rest of Lemma 1 is intuitive. The privacy cost C is increasing in the activity level a and the amount of information the platform has collected, $\frac{1}{\sigma^2}$. A higher privacy level γ makes the signal less informative and lowers the privacy cost. A higher γ also reduces the marginal cost, because adding noise makes the platform's learning less sensitive to the consumer activity.

Let $a^*(\gamma, \sigma^2)$ denote the optimal activity level given a privacy level γ in the current period and the conditional variance σ^2 from the previous period:

$$a^*(\gamma, \sigma^2) := \max \left\{ \arg\max_{a\geq 0} u(a) - v\left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma^2} + \frac{1}{\frac{1}{a}+\gamma}}\right)\right\}. \tag{3}$$

Lemma 1 implies that the consumer chooses a higher activity level when the platform sets a higher privacy level or she has less privacy (see Appendix A for the proof). Recall $a^* = \arg\max_{a\geq 0} u(a)$.

Lemma 2. *$a^*(\gamma,\sigma^2)$ is increasing in γ and decreasing in σ^2. For any (γ,σ^2), $\lim_{\hat{\gamma}\to\infty} a^*(\hat{\gamma},\sigma^2) = \lim_{\hat{\sigma}^2\to 0} a^*(\gamma,\hat{\sigma}^2) = a^*$.*

The next result presents the long-run outcome when the platform adopts a stationary privacy policy (see Appendix B for the proof).

Proposition 1. *Suppose that the platform chooses a privacy level $\gamma_t = \gamma$ for all $t \in \mathbb{N}$. Let $(a_t^*)_{t\in\mathbb{N}}$ denote the equilibrium activity levels of this subgame. There is a cutoff value $v^*(\gamma) \in (0,\infty)$ such that:*

1. *If $v < v^*(\gamma)$, then a_t^* increases in t, $\lim_{t\to\infty} a_t^* = a^*$, and $\lim_{t\to\infty} \sigma_t^2 = 0$. The consumer's per-period payoff decreases over time.*

2. *If $v > v^*(\gamma)$, then $a_t^* = 0$ and $\sigma_t^2 = \sigma_0^2$ for all $t \in \mathbb{N}$.*

Moreover, $v^*(\gamma)$ is increasing in γ.

The intuition is as follows. If the value of privacy is low, then it is optimal for the consumer to choose a positive activity level $a_1^* > 0$ in $t = 1$. The consumer activity generates some information, which reduces her payoff but increases the marginal net benefit of using the platform. Thus, in $t = 2$, the consumer chooses $a_2^* \geq a_1^*$. Repeating this argument, we can conclude that a_t^* increases over time. This also implies that the platform can perfectly learn the consumer's type X as $t \to \infty$. This is associated with $a_t^* \to a^*$, because the platform's perfectly learning X implies that the marginal privacy cost goes to zero. Thus, in the long-run, the consumer loses her privacy but behaves as if there is no privacy cost. In contrast, the consumer with a high v does not use the platform in $t = 1$, which implies $a_t = 0$ in all $t \geq 2$.

Proposition 1 has a policy implication: Suppose that a regulator, who cares about consumer privacy, mandates a stricter privacy policy ($\gamma_t = \gamma$ becomes $\gamma_t = \bar{\gamma} > \gamma$ for all $t \in \mathbb{N}$). Since $v^*(\cdot)$ is increasing, the regulation increases the cutoff and expands the range of v's under which the consumer experiences the long-run privacy loss (Point 1). To see the welfare implication, suppose that v satisfies $u(a^*) - v\sigma_0^2 < 0$. For a small γ, the consumer might choose $a_t^* = 0$ and obtain a payoff of zero in all periods. If the regulator enforces a large γ, then the consumer chooses $a_1^* > 0$ in $t = 1$ because it is less costly to use the platform. However, this leads to $(a_t^*, \sigma_t^2) \to (a^*, 0)$, and thus the consumer's per-period payoff converges to $u(a^*) - v\sigma_0^2 < 0$. Thus, the regulation can increase the consumer's per-period payoffs in the short-run but decrease them in the long-run. If the regulator puts a large weight on the long-run welfare, it might consider the regulation as detrimental.[9]

3.2 Equilibrium

I now turn to the equilibrium of the entire game (see Appendix C for the proof, which uses the characterization result in Proposition 3).

Proposition 2. *Take any v, and let $(a_t^*, \gamma_t^*, \sigma_t^2)_{t\in\mathbb{N}}$ denote the outcome of any equilibrium. Then,*

$$\lim_{t\to\infty} a_t^* = a^*, \lim_{t\to\infty} \gamma_t^* = 0, \textit{ and } \lim_{t\to\infty} \sigma_t^2 = 0. \tag{4}$$

Moreover, for any $T \in \mathbb{N}$, there is a $\underline{v}$ such that, for any $v \geq \underline{v}$, any equilibrium privacy policy satisfies $\gamma_t^* > 0$ for all $t \leq T$.

In equilibrium, even if the value v of privacy is arbitrarily high, in the long-run, the consumer becomes an active user with no privacy. This contrasts with Proposition 1, where the consumer with a high v does not use the platform. The result also shows that if v is high, the equilibrium privacy policy is nonstationary: In early periods, the platform sets positive privacy levels. In the long-run, the platform offers a vanishing level of privacy. The long-run payoff of the consumer is $u(a^*) - v\sigma_0^2$, which can be arbitrarily low for a large v.

The intuition is as follows. In early periods, the platform does not know much about the consumer's type. Then, consumer activity has a large impact on what the platform learns about her type. Thus, the consumer faces a high marginal privacy cost, which discourages her from raising the activity level. To reduce the marginal cost, the platform sets a high privacy level. Thus, in early periods, the platform offers high privacy levels and slowly learns the consumer type. After a long period of interaction, the platform accurately knows the consumer type. Then, the consumer faces a low marginal cost. Thus, the platform can lower a privacy level to accelerate learning.

[9]The caveat "if the regulator puts a large weight on the long-run welfare" is important. This is because a higher γ increases the consumer's ex ante sum of discounted payoffs from her optimal policy. This is true even if the consumer is forward-looking. Thus, we can argue that a higher γ can be undesirable, only if we evaluate the consumer's behavior under an alternative objective that exhibits more patience than the consumer.

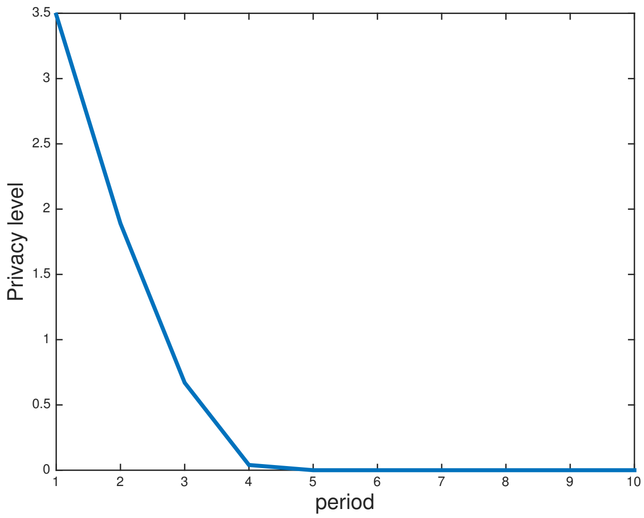

Figure 1(a): Privacy level γ_t

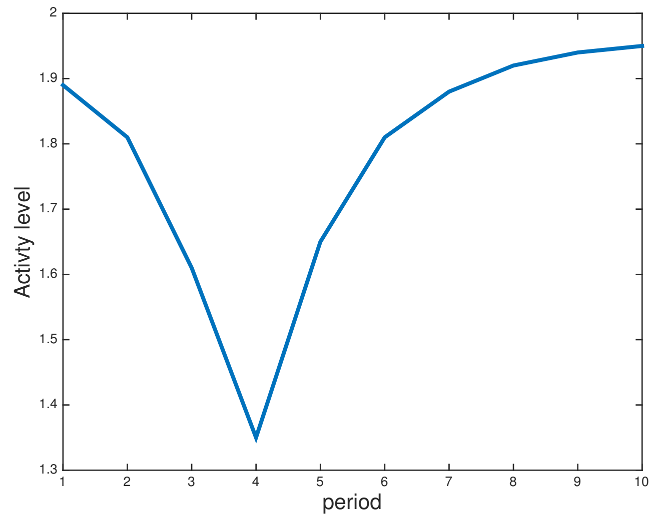

Figure 1(b): Activity level a_t

Figure 1: Equilibrium under $u(a) = 2a - \frac{1}{2}a^2$, $v = 10$, and $\sigma_0^2 = 1$.

Figure 1 depicts the equilibrium dynamics in a numerical example.[10] Figure 1(a) shows that the platform offers a decreasing privacy level, hitting zero in $t = 5$. Figure 1(b) shows that the equilibrium activity level first decreases but eventually approaches $a^* = 2$. This non-monotonicity contrasts with the case of a stationary privacy policy in Proposition 1.[11]

Proposition 2 has several implications. First, it gives an economic explanation of the so-called privacy paradox: Consumers seem to casually share their data with online platforms, despite their demand for privacy and concerns about data collection.[12] One may view this puzzle as the long-run equilibrium outcome, where the consumer faces a high privacy cost and zero marginal cost. In contrast to the explanation based on information externalities among consumers, the long-run outcome in Proposition 2 occurs without a priori market imperfection. Moreover, the long-run outcome $(a_t^*, \sigma_t^2) \to (a^*, 0)$ arises even if the consumer is forward-looking and arbitrarily patient (Proposition 9).

Second, observing consumers' intertemporal privacy choices may be useful for applying the

[10] I compute the equilibrium strategy profile using Proposition 3.

[11] While I have not manged to prove that the non-monotonicity always occurs, a numerical exercise suggests that, for a wide range of parameters such that the equilibrium privacy level is strictly decreasing in early periods, the activity level also decreases.

[12] Acquisti et al. (2016) contains an insightful review of research on the privacy paradox. Recent empirical work, for example, includes Athey et al. (2017).

revealed preference argument to infer their values of privacy. Indeed, a consumer's privacy choice in a single period may not tell much about his or her preferences for privacy (v), if the choice is made after the consumer revealed much information in the past.

Third, the consumer's payoff is decreasing but her incentive to use the platform is increasing in past activity levels. This complementarity between past and present consumption is what Becker and Murphy (1988) call rational addiction. One difference from a typical model of rational addiction is that the platform can dynamically adjust the degree of addiction through its privacy policy. As a result, even if the consumer values privacy arbitrarily highly ex ante, she becomes "addicted" to the platform.

At an anecdotal level, the equilibrium strategy of the platform seems consistent with how the privacy policies of online platforms have evolved. In 2004, Facebook's privacy policy stated that it would not use cookies to collect consumer information. However, in 2020, the privacy policy states that it uses cookies to track consumers on and possibly off the website.[13] Srinivasan (2019) describes how Facebook has acquired dominance in the social media market as follows:

> "When Facebook entered the market, the consumer's privacy was paramount. The company prioritized privacy, as did its users—many of whom chose the platform over others due to Facebook's avowed commitment to preserving their privacy. Today, however, accepting Facebook's policies in order to use its service means accepting broad-scale commercial surveillance."

Relatedly, Fainmesser et al. (2019) describe how the business models of online platforms have changed from the initial phase where they expand a user base to the mature phase where they monetize collected information. The equilibrium dynamics Proposition 2 rationalize the described pattern.

3.3 Characterizing Equilibrium Privacy Policy

The following result characterizes the platform's optimal privacy policy. Recall that $a^*(\gamma, \sigma^2)$ is the activity level a_t chosen by the consumer given $\gamma_t = \gamma$ and $\sigma^2_{t-1} = \sigma^2$.

[13] In 2020, Facebook's privacy policy states that "we use cookies if you have a Facebook account, use the Facebook Products, including our website and apps, or visit other websites and apps that use the Facebook Products (including the Like button or other Facebook Technologies)." https://www.facebook.com/policies/cookies

Proposition 3. *The equilibrium privacy policy* $(\gamma_1^*, \gamma_2^*, \dots)$ *is recursively defined as follows.*

$$\gamma_t^* \in \arg\min_{\gamma \geq 0} \frac{1}{a^*(\gamma, \hat{\sigma}_{t-1}^2)} + \gamma, \forall t \in \mathbb{N}, \tag{5}$$

$$\hat{\sigma}_0^2 = \sigma_0^2, \tag{6}$$

$$\hat{\sigma}_t^2 = \frac{1}{\frac{1}{\hat{\sigma}_{t-1}^2} + \frac{1}{\frac{1}{a^*(\gamma_t^*, \hat{\sigma}_{t-1}^2)} + \gamma_t^*}}, \forall t \in \mathbb{N}. \tag{7}$$

For any privacy policy $\boldsymbol{\gamma}$*, the conditional variances* $(\sigma_t^2)_{t \in \mathbb{N}}$ *induced by* $\boldsymbol{\gamma}$ *and the consumer's best responses satisfy* $\hat{\sigma}_t^2 \leq \sigma_t^2$ *for all* $t \in \mathbb{N}$.

Proof. Take any privacy policy $(\gamma_t)_{t \in \mathbb{N}}$, and let $(\sigma_t^2)_{t \in \mathbb{N}}$ denote the sequence of the conditional variances induced by $a^*(\cdot, \cdot)$. I show $\hat{\sigma}_t^2 \leq \sigma_t^2$ for all $t \in \mathbb{N}$. The inequality holds with equality for $t = 0$. Take any $\tau \in \mathbb{N}$. Suppose that $\hat{\sigma}_t^2 \leq \sigma_t^2$ for $t = 0, \dots, \tau - 1$. Then, it holds

$$\sigma_\tau^2 = \frac{1}{\frac{1}{\sigma_{\tau-1}^2} + \frac{1}{\frac{1}{a^*(\gamma_\tau, \sigma_{\tau-1}^2)} + \gamma_\tau}} \geq \frac{1}{\frac{1}{\hat{\sigma}_{\tau-1}^2} + \frac{1}{\frac{1}{a^*(\gamma_\tau, \hat{\sigma}_{\tau-1}^2)} + \gamma_\tau}} \geq \frac{1}{\frac{1}{\hat{\sigma}_{\tau-1}^2} + \frac{1}{\frac{1}{a^*(\gamma_\tau^*, \hat{\sigma}_{\tau-1}^2)} + \gamma_\tau^*}} = \hat{\sigma}_\tau^2.$$

The first inequality follows from the inductive hypothesis and decreasing $a^*(\gamma, \cdot)$. The second inequality follows from (5). $\forall t \in \mathbb{N}, \hat{\sigma}_t^2 \leq \sigma_t^2$ implies that the strategy described by (5), (6), and (7) is optimal, because it gives the platform a weakly greater profit than any other privacy policy in all periods. □

The objective of the minimization problem (5), $\frac{1}{a^*(\gamma, \hat{\sigma}_{t-1}^2)} + \gamma$, is the variance of the noise term $\varepsilon_t + z_t$ in the signal $s_t = X + \varepsilon_t + z_t$ given the consumer's best response. The minimization problem captures the platform's trade-off. On the one hand, a higher privacy level γ leads to a higher activity level, which leads to a lower variance $\frac{1}{a^*(\gamma, \hat{\sigma}_{t-1}^2)}$ of ε_t. On the other hand, given any activity level, a higher γ lowers the informativeness of the signal. This cost is captured by the second term γ, a variance of z_t. The platform chooses γ_t^* by resolving this trade-off. As the platform solves (5) in each period, the conditional variance evolves according to (7) with the initial condition (6).

The platform chooses its strategy to maximize the sum of discounted payoffs. However, the equilibrium policy is as if it chooses each γ_t to *statically* maximize the informativeness of the signal. The reason is follows. In principle, the platform chooses (say) γ_1 to maximize the sum of period-1 profit and the continuation value. The period-1 profit is increasing in the precision of the signal in $t = 1$ by construction. As more information is generated in $t = 1$, the consumer faces lower marginal costs and chooses higher activity levels in the future. Thus, the continuation value is also increasing in the precision of the signal in $t = 1$. As a result, the platform can maximize the sum of discounted profits by maximizing the informativeness of signal in $t = 1$. A similar argument implies that the equilibrium privacy level in any period statically maximizes the informativeness of the signal in that period.

Proposition 3 implies that the platform does not require as strong a commitment power as currently assumed.[14] To state the result formally, I say that the platform has the *short-run commitment power* if it chooses a privacy level γ_t at the beginning of each period t (before the consumer chooses a_t) without committing to future privacy levels.

Corollary 1. *Let* $(\boldsymbol{a}^*, \boldsymbol{\gamma}^*)$ *denote the equilibrium outcome of the baseline model, in which the platform can commit to any privacy policy. The same outcome* $(\boldsymbol{a}^*, \boldsymbol{\gamma}^*)$ *can arise in an equilibrium when the platform has the short-run commitment power.*

Proof. Consider the short-run commitment power. Consider the following strategy profile: Following every history with the conditional variance σ^2, the platform sets $\gamma \in \arg\min_{\gamma \geq 0} \frac{1}{a^*(\gamma, \sigma^2)} + \gamma$. The consumer always acts according to $a^*(\cdot, \cdot)$. By construction, $(\boldsymbol{a}^*, \boldsymbol{\gamma}^*)$ arises on the path of play. If the platform deviates, it weakly increases the conditional variances in all periods (Proposition 3). Thus, the platform has no profitable deviation. $\square$

The result shows that the long-run commitment has no value relative to the short-run commitment. In contrast, the platform could be strictly worse off if it has no commitment power: If the platform sets γ_t after observing a_t, then in any equilibrium, the platform sets $\gamma_t = 0$ whenever $a_t > 0$. Anticipating this, the consumer chooses a weakly lower activity level than under the short-run or long-run commitment. In practice, the short-run commitment seems a reasonable assumption, because a platform could be sanctioned for the outright violation of its privacy policy.

[14] The proof of the following result also reveals that the equilibrium outcome is independent of the platform's discount factor or time horizon.

4 Platform Competition

I now explore the effect of competition among platforms. There are two platforms, an incumbent (I) and an entrant (E). I is in the market from the beginning of $t = 1$. In period $t^* \geq 2$, E enters the market. t^* is exogenous and known to I at the outset. Thus, there is no issue of strategic entry.[15]

Before the entry ($t < t^*$), the consumer chooses her activity level $a_t^I \geq 0$ for I. After the entry ($t \geq t^*$), the consumer chooses $(a_t^I, a_t^E) \in \mathbb{R}_+^2$, where a_t^E is the activity level for E. I assume single-homing: The consumer can choose (a_t^I, a_t^E) if and only if $\min(a_t^I, a_t^E) = 0$. This is natural if two platforms offer substitutable services such as search engines.

I consider two games that differ in the timing of moves. One is *competition with long-run commitment:* I publicly commits to $(\gamma_1^I, \gamma_2^I, \dots)$ at the beginning of $t = 1$, and E publicly commits to $(\gamma_{t^*}^E, \gamma_{t^*+1}^E, \dots)$ at the beginning of period t^*. The other is *competition with short-run commitment:* In each period, each platform chooses a privacy level, after which the consumer chooses an activity level. In particular, I and E set privacy levels γ_t^I and γ_t^E simultaneously in each period $t \geq t^*$.

As in the baseline model, activity on platform $k \in \{I, E\}$ generates a signal $s_t^k = X + \varepsilon_t^k + z_t^k$ with $\varepsilon_t^k \sim \mathcal{N}\left(0, \frac{1}{a_t^k}\right)$ and $z_t^k \sim \mathcal{N}(0, \gamma_t^k)$. Each platform k privately observes s_t^k. Thus, there is no information spillover between platforms. All the noise terms (ε_t^k, z_t^k) are independent across (t, k).

The payoff of platform $k \in \{I, E\}$ in period t is $\sigma_0^2 - \sigma_{t,k}^2$, where $\sigma_{t,k}^2$ is the conditional variance of X given activity levels $a_1^k, \dots, a_t^k$ on platform k. The consumer's payoff in period t is given by

$$u(a_t^I) - v\left(\sigma_0^2 - \sigma_{t,I}^2\right) + \mathbf{1}_{\{t \geq t^*\}} \cdot \left[u(a_t^E) - v\left(\sigma_0^2 - \sigma_{t,E}^2\right)\right], \tag{8}$$

where $\mathbf{1}_{\{t \geq t^*\}}$ is the indicator function that equals 1 or 0 if $t \geq t^*$ or $t < t^*$, respectively. Payoff (8) implies that even if the consumer switches to E and never uses I from some period on, she continues to incur a privacy cost based on the information collected by I in the past (and vice versa).

To ensure the existence of an equilibrium, I impose an upper bound on the feasible privacy levels. In practice, the bound might reflect the minimum amount of data that a platform needs to collect for offering services, or the maximum privacy level that a platform can credibly enforce.

[15] Exogenous entry is to simplify the description of equilibrium. If E can choose to enter in any period $t \geq 2$ at a positive entry cost, then we obtain a similar result where E does not enter in equilibrium.

Assumption 1. There is a $\bar{\gamma} \in (0, +\infty)$ satisfying $a^*(\bar{\gamma}, \sigma_0^2) > 0$ such that each platform can choose a privacy level of at most $\bar{\gamma}$.

$a^*(\bar{\gamma}, \sigma_0^2) > 0$ implies that if a platform chooses $\bar{\gamma}$, then the consumer chooses $a_t^I > 0$ or $a_t^E > 0$. This restriction is necessary for a non-trivial equilibrium in which the consumer uses a platform.

4.1 Equilibrium under Competition

I present an equilibrium in which the consumer never switches to the entrant and the long-run outcome equals the one under monopoly.[16] If the entry is sufficiently late, then the equilibrium outcome exactly equals the monopoly outcome. Moreover, there is no equilibrium in which the consumer permanently switches to the entrant.

Proposition 4. *Regardless of the commitment assumption:*

1. *There is an equilibrium in which $a_t^E = 0$ for all $t \in \mathbb{N}$, $\lim_{t\to\infty} a_t^I = a^*$, $\lim_{t\to\infty} \sigma_{I,t}^2 = 0$, and $\lim_{t\to\infty} \gamma_t^I = 0$.*

2. *There is a $\underline{t} \geq 2$ such that, if the entry time t^* is after $\underline{t}$, then I's privacy levels $(\gamma_t^I)_{t\in\mathbb{N}}$ in the above equilibrium equal the monopoly strategy.*

3. *Switching never occurs: There is no equilibrium in which for some $\hat{t} \in \mathbb{N}$, $a_t^E > 0$ and $a_t^I = 0$ for all $t \geq \hat{t}$.*

The intuition is as follows. Before the entry, I chooses privacy levels that make it optimal for the consumer to choose positive activity levels. Suppose that, upon entry, E chooses the highest privacy level $\bar{\gamma}$. Since the privacy cost from collected data is sunk, the consumer chooses which platform to use based on the marginal (or more precisely, incremental) cost. Because the incumbent has collected data, the consumer faces a lower marginal cost of using I. Thus, if I also chooses $\bar{\gamma}$, then the consumer strictly prefers to use I. This ensures that the consumer only uses I even after the entry. However, the equilibrium choice of I may not be $\bar{\gamma}$: I chooses a privacy level to maximize the amount of information generated subject to the constraint that the consumer prefers

[16] I do not consider other equilibria, which are likely to exist because there are two long-lived players in the game.

I. As time goes by, the constraint is relaxed, because the consumer's marginal cost of using I goes to zero. This ensures that I can offer a vanishing privacy level over time.

The threat of future entry has no impact on I's strategy: Before the entry, I chooses the same privacy levels as monopoly, regardless of commitment assumption. This is because maximizing the amount of information makes consumer switching least likely.

Point 3 implies that, for switching to occur, E needs some advantage in terms of the quality of service or the privacy level. For the next result, I say that E can *successfully enter the market* if there exists an equilibrium in which the consumer switches to E upon entry, i.e. $a_t^E > 0$ and $a_t^I = 0$ for all $t \geq t^*$.[17]

Proposition 5. *Suppose that the gross benefit of E's service is given by $u^E(\cdot) = u(\cdot) + \Delta$. There is a $\Delta^* > 0$ such that for any $\Delta \geq \Delta^*$, E can successfully enter the market. The minimum Δ^* satisfying this property is increasing in t^*.*

4.2 Antitrust Implication of Propositions 4 and 5

The results imply that data held by incumbents can be a barrier to entry. To see this, suppose that a platform has collected much data from consumers. This lowers the welfare of consumers who value their privacy. However, consumers also face lower marginal privacy costs, because there is less for them to lose on the margin. Suppose that consumers regard collected data as sunk for the reasons discussed in Section 2. If so, consumers decide whether to switch to an entrant based on their marginal privacy costs. Then, the low marginal privacy cost favors the incumbent, and makes switching and market entry less likely to occur.

In the model, low marginal costs are associated with high privacy costs. Thus, switching and market entry are less likely when consumers suffer from a lack of privacy and receive low payoffs from the incumbent. This welfare implication contrasts with the existing idea of data being an entry barrier, where dominant platforms use data to improve their services. For example, Furman et al. (2019) states that:

> "Data can act as a barrier to entry in digital markets. A data-rich incumbent is able to cement its position by *improving its service and making it more targeted for users*,

[17] The result considers the entrant's advantage in terms of service quality. We obtain a similar result by considering the entrant that can choose a higher maximum privacy level $\bar{\gamma} + \Delta$.

as well as making more money by better targeting its advertising" (italicized by the author).

As an application, consider the market for search engines: The incumbent is Google, and the entrant is a privacy-preserving alternative of Google, such as DuckDuckGo. My results suggest that, even if DuckDuckGo is as good a search engine as Google, DuckDuckGo may not be able to poach consumers. If consumers have no privacy to Google, then their marginal privacy cost of using Google is nearly zero. Thus, DuckDuckGo may have to offer much better a search engine than Google without collecting data, which seems unrealistic.

The results also suggest a potential remedy. If consumers could costlessly erase collected data upon switching, then competition is more likely to occur. This is because erasing past information eliminates the incumbency advantage. The next section considers such an extension.

5 Erasing Past Information

So far, the amount of information a platform has on the consumer is weakly increasing over time. I now consider the incentive of the consumer or a platform to erase past information.

5.1 The Right to be Forgotten

I consider the right to be forgotten, whereby the consumer can request a platform to erase past information. Below, I describe the model of competition, but a similar description applies to monopoly.

In each period, the consumer makes two decisions. First, the consumer chooses whether to erase past information of each platform in the market. Second, the consumer chooses a_t^I or (a_t^I, a_t^E), depending on whether t is before or after the entry. If she erases information of platform $k \in \{I, E\}$ in period t, then the conditional variance for platform k at the beginning of t becomes σ_0^2. At the end of the period, the consumer still incurs a privacy cost based on information generated in period t. It is costless for the consumer to erase information.

For example, if the consumer erases information of both platforms in period t and uses platform

E, then her payoff is

$$u(a_t^E) - v\left[\sigma_0^2 - \sigma_{1,E}^2(a_t^E, \gamma_t^E)\right], \tag{9}$$

where $\sigma_{1,E}^2(a_t^E, \gamma_t^E)$ is the conditional variance for E given one signal based on (a_t^E, γ_t^E). Thus, the privacy cost from E is only based on the signal of period t. Since the consumer has erased information for I and does not use it, she does not incur a privacy cost from I in period t.

In contrast, suppose that the consumer has never erased information. If she uses platform E in period t, then her payoff in period t is

$$u(a_t^E) - v\left[\sigma_0^2 - \sigma_{t,E}^2(\boldsymbol{a}_t^E, \boldsymbol{\gamma}_t^E)\right] - v\left[\sigma_0^2 - \sigma_{t,I}^2(\boldsymbol{a}_t^I, \boldsymbol{\gamma}_t^I)\right], \tag{10}$$

where $\boldsymbol{a}_t^k = (a_1^k, \ldots, a_t^k)$ and $\boldsymbol{\gamma}_t^k = (\gamma_1^k, \ldots, \gamma_t^k)$ for each $k \in \{I, E\}$. Thus, the consumer incurs a privacy cost from both platforms based on past data collection. The following result summarizes the impact of the right to be forgotten.

Proposition 6. *If the consumer can costlessly erase past information, then regardless of the commitment assumption, there is the following equilibrium:*

1. *Under monopoly, in all periods, the consumer erases information and the platform sets a privacy level γ_1^* defined in (5).*
2. *Under competition, the consumer erases information in all periods, and both platforms set the highest privacy level $\bar{\gamma}$ in any period after the entry (i.e., in period $t \geq t^*$).*
3. *Suppose that the gross benefit of E's service is given by $u^E(\cdot) = u(\cdot) + \Delta$. For any $\Delta > 0$, E can successfully enter the market.*

The right to be forgotten benefits the consumer in three ways. First, it reduces privacy cost. Second, it incentivizes platforms to choose higher privacy levels: Once the consumer erases information, then she incurs a high marginal privacy cost. This ensures that a monopoly platform always offers a period-1 privacy level in any period (Point 1). Under competition, erasing information makes competing platforms homogeneous. This intensifies competition and incentivizes platforms offer the highest privacy level (Point 2). Finally, erasing past information eliminates the incumbency advantage and makes market entry more likely (Point 3).

5.2 Data Retention Policies

This section considers a platform's choice of data retention policies, which have recently been paid attention by economists and legal scholars (Chiou and Tucker, 2017). For simplicity, I focus on the short-run commitment.[18] If there is one platform in the market, then the platform chooses whether to erase past information and sets a privacy level. Then, the consumer chooses an activity level. If there are two platforms, they simultaneously choose whether to erase information. After observing this, they simultaneously set privacy levels. Finally, the consumer chooses an activity level for each platform.

Erasing information affects the conditional variances and payoffs in the same way as the consumer erasing information (see the previous subsection). The following result shows that a platform's optimal data retention policy is to not erase information at all.

Proposition 7. *A platform never erases information:*

1. *A monopoly platform never erases information in any period in any equilibrium. The equilibrium outcome equals Proposition 2.*

2. *Under competition, there is an equilibrium in which platforms never erase information in any period. Among the equilibria with this property, Proposition 4 holds.*

If consumer behavior is exogenous, then a platform has no incentive to raise a privacy level or erase past information, because it lowers the profit by reducing the amount of consumer information. However, if consumer behavior is endogenous, then a platform has different incentives for these choices. A platform may have an incentive to increase a privacy level (at least in early periods) because it reduces the consumer's marginal cost and increases her activity level. In contrast, a platform has no incentive to erase information because it increases the consumer's marginal privacy cost and decreases her activity level.

[18] The same result holds for other commitment assumptions. For example, if a platform can commit to a privacy policy and a data retention policy at the outset, then it chooses a privacy policy $(\gamma_1, \gamma_2, \dots)$ and $T \in \mathbb{N}$ before $t = 1$. T means that the platform erases information in every T periods.

6 Extensions

This section shows that the main insight of long-run privacy loss holds in a more general environment. Also, I examine how the equilibrium dynamics interact with the platform's ex ante incentive to invest in quality. Throughout the section, I consider a monopoly platform.

6.1 Consumers with Heterogeneous v

Proposition 2 holds when consumers have heterogeneous v's. In other words, the main insight does not depend on the platform knowing v at the outset. To see this, extend the model as follows: There is a unit mass of consumers. Each consumer $i \in [0,1]$ has v_i, which is distributed independently and identically according to some distribution with support $V = \{v_1, \ldots, v_N\} \subset \mathbb{R}$. Let $\alpha_v \in [0,1]$ denote the mass of consumers who have $v \in V$. The platform knows V and $(\alpha_v)_{v \in V}$. The result does not depend on whether the platform knows a realized v_i for each i.

The game is a natural extension of the baseline model. First, the monopoly platform chooses a privacy policy $(\gamma_t)_{t \in \mathbb{N}}$, which is common across all consumers. Second, each consumer i myopically chooses activity levels $(a_t(i))_{t \in \mathbb{N}}$. The types and signals are independent across consumers. Thus, the platform learns about i's type only based on her past activity levels and privacy levels.

For each $i \in [0,1]$, let $\sigma_t^2(i)$ denote the conditional variance for consumer i at the end of period t. Then, i's payoff is $u(a_t(i)) - v_i[\sigma_0^2 - \sigma_t^2(i)]$, and the platform's payoff is $\int_{i \in [0,1]} \sigma_0^2 - \sigma_t^2(i) di$. If (almost) all consumers who have the same v choose the same activity level, then we can write the platform's profit as $\sum_{v \in V} \alpha_v \left[\sigma_0^2 - \sigma_t^2(v)\right]$, where $\sigma_t^2(v)$ is the conditional variance of consumers with v.

The platform faces a new trade-off: A high privacy level encourages consumers with high v to choose positive activity levels. However, the platform obtains less information from consumers with low v, who would choose high activity levels even without privacy protection.

However, there is no trade-off for the platform in the long-run: All consumers eventually lose privacy and choose the highest activity levels (see Appendix E.3 for the proof).

Proposition 8. *Let $(a_t^*(v), \sigma_t^2(v), \gamma_t^*)_{t \in \mathbb{N}, v \in V}$ denote the outcome of any equilibrium. Then,*

$$\forall v \in V, \lim_{t \to \infty} (a_t^*(v), \sigma_t^2(v)) = (a^*, 0), \textit{and} \lim_{t \to \infty} \gamma_t^* = 0. \tag{11}$$

To see the intuition, suppose that v is either $L = 0$ or $H > 0$, and the platform sets $\gamma_t = 0$ in early stages to obtain information only from L-consumers. During this period, only $\sigma_t^2(L)$ decreases over time. However, once $\sigma_t^2(L)$ gets close to zero, the platform finds it more profitable to increase a privacy level to encourage H-consumers to use the platform. Thus, the platform eventually obtains information from all consumers.

6.2 Forward-looking Consumer

Suppose that the consumer is patient with one-period ahead discount factor $\delta_C \in (0, 1)$. As in the baseline model, assume that a monopoly platform can commit to any privacy policy at the outset. The following result shows that the long-run equilibrium outcome exhibits a high activity level and no privacy, as in the case of a myopic consumer (see Appendix F for the proof, which also proves the existence of an equilibrium).

Proposition 9. *For any v and $\delta_C, \delta_P \in (0, 1)$, in any subgame perfect equilibrium:*

$$\lim_{t\to\infty} a_t^* = a^* \text{ and } \lim_{t\to\infty} \sigma_t^2 = 0. \tag{12}$$

If the platform adopts a nonstationary privacy policy, then the consumer's problem is a non-stationary dynamic programming, which is difficult to solve. Thus, I first show that there is a stationary privacy policy that induces (12). That is, for any discount factor δ_C of the consumer and her value v of privacy, there is a stationary privacy policy such that she loses privacy in the long-run. I then show that the equilibrium policy, which may be nonstationary, also induces the same long-run outcome. The key lemma is that the consumer's sum of discounted payoffs is supermodular in activity levels and the informativeness of signals. This implies that the optimal activity levels are increasing in the amount of information collected in the past and the amount of information the platform will collect in the future.

6.3 Vanishing Incentive to Invest in Quality

So far, the benefit $u(\cdot)$ from the platform's service has been exogenous. Suppose now that, before $t = 1$, the platform chooses a quality $q \geq 0$. Then, in each period, the consumer receives a gross benefit of $u_q(a) = qa - \frac{1}{2}a^2$, and the platform receives a payoff of $\sigma_0^2 - \sigma_t^2 - c(q)$ for some strictly increasing $c(\cdot)$. The platform chooses q once, but incurs $c(q)$ in every period.

Proposition 10. *A patient platform does not invest in quality: For any $\delta_P \in (0,1)$, let $q(\delta_P)$ denote the quality in an (arbitrarily chosen) equilibrium. Then, $\lim_{\delta_P \to 1} q(\delta_P) = 0$. Thus, regardless of the consumer's discount factor, as $\delta_P \to 1$, her ex ante sum of discounted payoffs converges to zero, and her long-run equilibrium payoff converges to $-v\sigma_0^2 < 0$.*

Proof. Given (δ_P, q), Let $\Pi(\delta_P, q)$ denote the platform's ex ante sum of discounted profits. For any $q > 0$, the platform's per-period payoff is at most $\sigma_0^2 - c(q)$. Thus, $(1-\delta_P)\Pi(\delta_P, q) \leq \sigma_0^2 - c(q)$. Suppose to the contrary that there is a sequence $\delta_n \to 1$ such that for some $q' > 0$, $q(\delta_n) \geq q'$ for infinitely many n's (for some selection of equilibria). Without loss of generality, assume $(1-\delta_n)\Pi(\delta_n, q(\delta_n)) \in [0, \sigma_0^2]$ has a limit. Then, $\lim_{n\to\infty}(1-\delta_n)\Pi(\delta_n, q(\delta_n)) \leq \sigma_0^2 - c(q') < \sigma_0^2 - c(q'/2)$. Proposition 9 implies that there is a $\boldsymbol{\gamma}$ under which $(a_t^*, \sigma_t^2) \to (a^*, 0)$ given quality $q'/2$. If the platform chooses $q'/2$ and $\boldsymbol{\gamma}$, then as $\delta_P \to 1$, its average payoff converges to $\sigma_0^2 - c(q'/2)$. Thus, the platform with a large δ_n strictly prefers $q'/2$ to $q(\delta_n)$, which is a contradiction. This implies $\lim_{\delta_P \to 1} u_{q(\delta_P)}(a^*) - v\sigma_0^2 = -v\sigma_0^2$. Also, as the consumer's ex ante payoff is nonnegative but lower than $\frac{u_{q(\delta_P)}(a^*)}{1-\delta_C}$, it converges to 0 as $\delta_P \to 1$. $\square$

6.4 General Privacy Cost Function

Suppose that the consumer's per period payoff is $u(a_t) - C\left(\sigma_t^2\right)$. Assume $C(\cdot) : \mathbb{R}_+ \to \mathbb{R}$ is continuously differentiable. This implies that the cost and the marginal cost at no privacy (i.e., $C(0)$ and $C'(0)$) are bounded. $C(\cdot)$ can be nonmonotone: For example, it can be convex and minimized at some σ^*, which means that the consumer benefits from data collection that reduces σ_0^2 to σ^*, but the additional data collection is harmful. The following result shows that the long-run outcome remains the same (see Appendix G).

Proposition 11. *If the consumer is myopic, in any equilibrium:*

$$\lim_{t\to\infty} a_t^* = a^* \textit{ and } \lim_{t\to\infty} \sigma_t^2 = 0. \tag{13}$$

6.5 General Payoffs for the Platform

All the results of this paper continue to hold if the platform's payoff from any induced sequence $(\sigma_t^2)_{t\in\mathbb{N}}$ of variances is $\Pi((\sigma_t^2)_{t\in\mathbb{N}})$, where $\Pi : \mathbb{R}_+^\infty \to \mathbb{R}$ is bounded and coordinate-wise strictly decreasing. This is because, in the equilibrium for each result, any deviation by a platform weakly increases σ_t^2 for all $t \in \mathbb{N}$. For example, suppose that the platform sells information to a sequence of short-lived data buyers. Any data sold in period t is freely replicable in the future and thus have a price of zero in any period $s \geq t+1$. Then, the platform's payoff in period t equals the value of information newly generated in period t. Thus, the ex ante payoff is $\sum_{t=1}^\infty \delta_P^{t-1}(\sigma_{t-1}^2 - \sigma_t^2)$. This objective is strictly decreasing in each σ_t^2, because the coefficient of each σ_t^2 equals $-\delta_P^{t-1}+\delta_P^t < 0$.

7 Conclusion

I consider a dynamic model in which a consumer chooses how actively to use a platform in each period. The activity generates information about the consumer's type, which benefits the platform but hurts the consumer. Thus, the consumer balances the benefit of using the service and the cost of losing privacy. The model captures the idea that the consumer's incentive to protect privacy decreases as she reveals more information. The paper examines the implications of this insight on the equilibrium dynamics and long-run outcomes. Notably, the platform designs its privacy policy so that the long-run equilibrium outcome exhibits no privacy and a high activity level. This occurs even if the consumer is forward-looking and values privacy arbitrarily highly. When there are multiple platforms, the consumer's decreasing marginal privacy cost creates an endogenous switching cost. The switching cost leads to the situation in which consumers are more likely to use a platform to which they have surrendered privacy. Enabling the consumer to erase past information mitigates the problem, although platforms have no incentive to do so.

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Appendix

A Properties of the Consumer's Best Response: Proof of Lemma 2

Proof. Define $U(a,\gamma,\sigma^2) := u(a) - v\left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma^2}+\frac{1}{\frac{1}{a}+\gamma}}\right)$. Lemma 1 implies that $\frac{\partial U}{\partial a}$ is increasing in γ and decreasing in σ^2. The standard argument of monotone comparative statics implies that $a^*(\gamma,\sigma^2)$ is increasing in γ and decreasing in σ^2 (e.g., Milgrom et al. 1994).

Suppose to the contrary that $\lim_{\sigma^2\to 0} a^*(\gamma,\sigma^2) = a^*$ fails. As $a^*(\gamma,\sigma^2) \leq a^*$ for all (γ,σ^2), there are $\varepsilon > 0$ and $(\sigma_n^2)_{n\in\mathbb{N}}$ such that $\lim_{n\to\infty} \sigma_n^2 = 0$, $a^*(\gamma,\sigma_n^2) \leq a^* - \varepsilon$ for all $n \in \mathbb{N}$, and $\lim_{n\to\infty} a^*(\gamma,\sigma_n^2) = a^* - \varepsilon$. Suppose that the consumer chooses a^* instead of $a < a^*$. Then, the payoff difference is

$$\Delta(a,\sigma^2) := u(a^*) - u(a) - v\left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma^2}+\frac{1}{\frac{1}{a^*}+\gamma}}\right) + v\left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma^2}+\frac{1}{\frac{1}{a}+\gamma}}\right). \tag{14}$$

Note that $\lim_{n\to\infty} \Delta(a^*(\gamma,\sigma_n^2),\sigma_n^2) = u(a^*) - u(a^*-\varepsilon) > 0$. This implies that $\Delta(a^*(\gamma,\sigma_n^2),\sigma_n^2) > 0$ for a large n. Thus, for a large n, the consumer strictly prefers a^* to $a^*(\gamma,\sigma_n^2)$, which is a contradiction. A similar argument implies that $\lim_{\hat{\gamma}\to\infty} a^*(\hat{\gamma},\sigma^2) = a^*$. □

B The Long-run Outcome Under a Stationary Privacy Policy: Proof of Proposition 1

Proof. To emphasize that the optimal activity level depends on v, I write $a^*(\gamma, \sigma^2)$ as $a^*(v, \gamma, \sigma^2)$. Define $v^*(\gamma)$ as follows:

$$v^*(\gamma) = \sup\left\{v \in \mathbb{R} : a^*\left(v, \gamma, \sigma_0^2\right) > 0\right\}. \tag{15}$$

Note that $\frac{\partial U}{\partial a} = u'(a) - v\frac{1}{\left(\frac{1}{\sigma_0^2}(1+\gamma a)+a\right)^2}$. This implies that

$$\begin{aligned}
\left.\frac{\partial U}{\partial a}\right|_{a=0} &= u'(0) - v \cdot (\sigma_0^2)^2,\\
\left.\frac{\partial U}{\partial a}\right|_{a=a'} &\leq u'(0) - v \cdot \frac{1}{\left(\frac{1}{\sigma_0^2}\left(1+\gamma a^*\right)+a^*\right)^2}, \forall a' \in [0, a^*].
\end{aligned}$$

The second inequality holds because $u(\cdot)$ and the privacy cost function are increasing and concave. For a sufficiently small v, the right hand side of the first equality is positive. Thus, $v^*(\gamma)$ is well-defined and positive. For a sufficiently large v, the right hand side of the second inequality is negative. Thus, $v^*(\gamma)$ is finite.

Suppose $v < v^*(\gamma)$. By the construction of $v^*(\gamma)$, $a^*(v, \gamma, \sigma_0^2) > 0$. σ_t^2 decreases in t for any sequence of activity levels. Thus, if $\gamma_t = \gamma$ for any $t \in \mathbb{N}$, then $a^*(v, \gamma, \sigma_t^2)$ is increasing in t and greater than $a_1^* > 0$ for all t. This implies that $\lim_{t\to\infty} \sigma_t^2 = 0$, because

$$0 \leq \sigma_t^2 \leq \frac{1}{\frac{1}{\sigma_0^2} + \frac{t}{\left(\frac{1}{a_1^*}+\gamma\right)}} \to 0 \quad \text{as} \quad t \to \infty.$$

Lemma 2 implies $\lim_{t\to\infty} a_t^* \to a^*$. For $v > v^*(\gamma)$, note that $a^*(v, \gamma, \sigma_0^2) = 0$, which implies that $a_t^* = 0$ for all t. Finally, $v^*(\gamma)$ is increasing in γ, because $a^*\left(v, \gamma, \sigma_0^2\right)$ is increasing in γ. □

C Properties of Equilibrium: Proof of Proposition 2

Proof. Let $(a_t^*, \hat{\sigma}_t^2)_{t\in\mathbb{N}}$ denote the equilibrium activity levels and conditional variances. First, I prove $\lim_{t\to\infty} \hat{\sigma}_t^2 = 0$. By Lemma 2 and Proposition 1, there is a stationary privacy policy $\gamma_t \equiv \gamma$

such that $\lim_{t\to\infty} \sigma_t^2 = 0$. By Proposition 3, $\hat{\sigma}_t^2 \leq \sigma_t^2$ for all $t \in \mathbb{N}$, which implies $\lim_{t\to\infty} \hat{\sigma}_t^2 = 0$.

To show $\lim_{t\to\infty} a_t^* = a^*$, suppose to the contrary that there is an $\varepsilon > 0$ such that $a_t^* \leq a^* - \varepsilon$ for infinitely many t's. Without loss of generality, suppose $a_t^* \leq a^* - \varepsilon$ for all t. Following the proof of Lemma 2, we can conclude that, for a large t, the consumer strictly prefers a^* to a_t^*. Indeed, if the consumer chooses a^* instead of a_t^*, $u(\cdot)$ increases by at least $u(a^*) - u(a^* - \varepsilon) > 0$ whereas the increment of privacy cost goes to zero. This is a contradiction.

Next, suppose to the contrary that there is a $\underline{\gamma} > 0$ such that $\gamma_t^* \geq \underline{\gamma}$ for infinitely many t's. To simplify exposition, suppose $\gamma_t^* \geq \underline{\gamma}$ for all $t \in \mathbb{N}$. Take any $\varepsilon \in (0, \underline{\gamma})$. Then, as $\lim_{t\to\infty} a_t^* = a^* > 0$, for a sufficiently large t, the minimized value in (5) is weakly greater than $\frac{1}{a^*} - \varepsilon + \underline{\gamma}$. To show a contradiction, let T denote the first period such that $a^*(0, \hat{\sigma}_{T-1}^2) > 0$. Then, $a^*(0, \hat{\sigma}_{t-1}^2) > 0$ for any $t \geq T$. If the platform chooses $\gamma_t = 0$ instead of γ_t^* in period $t \geq T$, then the minimand in (5) equals $\frac{1}{a^*(0,\hat{\sigma}_t^2)}$, which converges to $\frac{1}{a^*} < \frac{1}{a^*} - \varepsilon + \underline{\gamma}$ for a sufficiently large t. This implies that for a sufficiently large t, the platform can strictly increase its payoff in period t by setting $\gamma_t = 0$, which is a contradiction.

To show the final part, I write $\gamma_t^*(v)$ to emphasize the dependence of the equilibrium privacy level on v. Suppose to the contrary that there is a T such that, for any $\underline{v}$, there is some $v \geq \underline{v}$ such that $\gamma_t^*(v) = 0$ for some $t \leq T$. Then, we can find $v_n \to \infty$ and $t^* \leq T$ such that $\gamma_{t^*}^*(v_n) = 0$ for all n. However, for a sufficiently large v_n, $a_{t^*}^* = 0$ if $\gamma_{t^*}^*(v_n) = 0$. This follows from the proof of Proposition 1, where I show that the consumer with a sufficiently large v chooses $a_1^* = 0$ for a fixed γ_1. This contradicts the optimality of $\boldsymbol{\gamma}^*$ because if the platform sets a sufficiently large γ_{t^*}, then the consumer chooses a positive activity level and the minimand in (5) becomes finite. $\square$

D Equilibrium under Competition: Omitted Proofs in Section 4

D.1 Proof of Proposition 4

Proof. For each $k \in \{I, E\}$, I use $-k$ to mean E or I if $k = I$ and $k = E$, respectively. Suppose that, at the beginning of period $t \geq t^*$, the conditional variance for platform k is $\sigma_{t-1,k}^2$. Let γ_t^k denote the privacy level of platform k in period t. The consumer weakly prefers to use platform k

(i.e. $a_t^{-k} = 0$ maximizes her period-t payoff) if

$$\begin{aligned}
&\arg\max_{a\geq 0} u(a) - v[\sigma_0^2 - \sigma_{t,k}^2(\gamma_t^k, a|\sigma_{t-1,k}^2)] - v[\sigma_0^2 - \sigma_{t-1,-k}^2] \\
\geq &\arg\max_{a\geq 0} u(a) - v[\sigma_0^2 - \sigma_{t,-k}^2(\gamma_t^{-k}, a|\sigma_{t-1,-k}^2)] - v[\sigma_0^2 - \sigma_{t-1,k}^2],
\end{aligned}$$

where $\sigma_{t,k}^2(\gamma, a|\sigma_{t-1,k}^2)$ is the conditional variance at the end of period t when platform k chooses γ, the consumer chooses a, and the conditional variance from the previous period is $\sigma_{t-1,k}^2$. Arranging this inequality, we obtain

$$\arg\max_{a\geq 0} u(a) - v[\sigma_{t-1,k}^2 - \sigma_{t,k}^2(\gamma_t^k, a|\sigma_{t-1,k}^2)] \geq \arg\max_{a\geq 0}\ u(a) - v[\sigma_{t-1,-k}^2 - \sigma_{t,-k}^2(\gamma_t^{-k}, a|\sigma_{t-1,-k}^2)]. \tag{16}$$

The above inequality implies that the consumer prefers to use k if and only if the gross benefit from the service net of the incremental privacy cost is greater for k than $-k$.

First, assume competition without commitment (a similar proof applies to competition with commitment). Consider the following strategy profile. For each period $t < t^*$, I chooses a monopoly privacy level γ_t^*. Take any period $t \geq t^*$. Let $k^* \in \arg\min_{k=I,E} \sigma_{t-1,k}^2$ denote the platform that has the lower conditional variance (if k^* is not unique, then set $k^* = I$). Then, platform $-k^*$ chooses the highest privacy level $\bar{\gamma}$. Platform k^* chooses a privacy level $\gamma_t^{k^*}$ that solves

$$\begin{aligned}
&\min_{\gamma\in[0,\bar{\gamma}]} \frac{1}{a^*(\gamma, \sigma_{t-1,k^*}^2)} + \gamma \\
\text{s.t.}\quad &\arg\max_{a\geq 0} u(a) - v[\sigma_{t-1,k^*}^2 - \sigma_{t,k^*}^2(\gamma, a|\sigma_{t-1,k^*}^2)] \\
&\geq \arg\max_{a\geq 0} u(a) - v[\sigma_{t-1,-k^*}^2 - \sigma_{t,-k^*}^2(\bar{\gamma}, a|\sigma_{t-1,-k^*}^2)].
\end{aligned} \tag{17}$$

In each period, the consumer myopically chooses a_t^I or (a_t^I, a_t^E) to maximize her per-period payoff. If indifferent, then the consumer uses the platform for which she chose a positive activity level in the most recent period. (If she chose zero activity levels up to period $t-1$, then she sets $a_t^k = 0$ for one of $k \in \{I, E\}$ with equal probability, and chooses a_t^{-k} to maximize her period-t payoff.)

I show that the above strategy profile is an equilibrium. First, the consumer's behavior is optimal by construction. Second, I verify that platforms have no profitable deviation. Without loss of generality, consider a node in period t in which $I = k^*$ and $E = -k^*$. The strategy of E is optimal: By construction, even if E chooses $\bar{\gamma}$ in all periods $s \geq t$, the consumer uses I in any future periods as long as I and the consumer follow the above strategy.

Suppose now that I chooses a privacy level such that the consumer chooses E in period t. If $\sigma^2_{t,E} \leq \sigma^2_{t,I}$, then the consumer uses E in any period $s \geq t+1$. In this case, I's deviation is not profitable. Otherwise, $\sigma^2_{t,E} > \sigma^2_{t,I}$ hold. Note that I obtains a lower payoff in period t, because it is not maximizing the informativeness of the signal. Moreover, at any future period s, I faces an optimization problem

$$\begin{aligned} &\min_{\gamma} \frac{1}{a^*(\gamma, \sigma^2_{s-1,I})} + \gamma \\ \text{s.t.} \quad & \arg\max_{a \geq 0} u(a) - v[\sigma^2_{s-1,I} - \sigma^2_{s,I}(\gamma, a | \sigma^2_{s-1,I})] \\ & \geq \arg\max_{a \geq 0} u(a) - v[\sigma^2_{s-1,E} - \sigma^2_{s,E}(\bar{\gamma}, a | \sigma^2_{s-1,E})]. \end{aligned} \tag{18}$$

After deviation, I faces a strictly lower $\sigma^2_{s-1,E} - \sigma^2_{s,E}(\bar{\gamma}, a|\sigma^2_{s-1,E}) > 0$ because the consumer generated information on E in period t. This means that the set of γ that satisfies the constraint is smaller. Thus, the minimized value in (18) becomes greater for any period $s \geq t+1$ after deviation. This implies that I's payoff is weakly lower for any period $s \geq t$ after the deviation. A similar argument implies that it is not profitable for I to deviate from a monopoly strategy before entry. This is because the deviation lowers I's payoff before and after entry. In particular, the deviation shrinks the set of γ's satisfying the constraint in (18) by increasing $\sigma^2_{s-1,I} - \sigma^2_{s,I}(\gamma, a|\sigma^2_{s-1,I})$.

$\lim_{t\to\infty} \sigma^2_{I,t} = 0$ holds because it holds even if I adopts $\gamma_t = \bar{\gamma}$ for all t, and I chooses each γ^I_t to achieve even lower conditional variances. Given this result, $\lim_{t\to\infty} a^I_t = a^*$ follows the same proof as monopoly.

Suppose that γ^I_t does not converge to 0. Then, there is a convergent subsequence $\gamma^I_{t(n)}$ such that $\lim_{n\to\infty} \gamma^I_{t(n)} = \gamma' > 0$. For a sufficiently large n, both $\gamma = 0$ and $\gamma = \gamma^I_{t(n)}$ satisfy the constraint in (18), because $\sigma^2_{s-1,E} - \sigma^2_{s,E}(\bar{\gamma}, a|\sigma^2_{s-1,E}) = \sigma^2_0 - \sigma^2_{1,E}(\bar{\gamma}, a^*(\bar{\gamma}, \sigma^2_0)|\sigma^2_0) > 0$, but $\lim_{s\to\infty} \sigma^2_{s-1,I} - \sigma^2_{s,I}(0, a^*(0, \sigma^2_{s-1,I})|\sigma^2_{s-1,I}) \leq \lim_{s\to\infty} \sigma^2_{s-1,I} = 0$. As $n \to \infty$, the value of the objective converges to $\frac{1}{a^*}$ and $\frac{1}{a^*} + \gamma'$ for $\gamma = 0$ and $\gamma = \gamma'$, respectively. Thus, for a large n, $\gamma = 0$ achieves a strictly lower value in (18) than $\gamma = \gamma'$. This is a contradiction and thus $\lim_{t\to\infty} \gamma^I_t \to 0$

in the equilibrium.

For a sufficiently large t^*, $\sigma^2_{t^*-1,I} \leq \sigma^2_0 - \sigma^2_{t^*,E}(\bar{\gamma}, a^*(\sigma^2_0, \bar{\gamma})|\sigma^2_0)$. Then, for any period $t \geq t^*$, the constraint (18) holds for any γ. This implies that I's problem is equal to the monopolist's problem after the entry. Combined with the above result, I's choice equals the monopolist's.

Finally, there is no equilibrium in which $a^E_t > 0$ and $a^I_t = 0$ for all $t \geq t^*$. This is because I can then choose $\bar{\gamma}$ for all periods. Given this, the consumer strictly prefers to use I for any period $t \geq t^* \geq 2$, because the consumer has generated information on I in periods $t < t^*$, which leads to a strictly lower marginal privacy cost. $\square$

D.2 Successful Entry: Proof of Proposition 5

Proof. Consider the following strategy profile: In any period $t \leq t^*$, I chooses a monopoly strategy. In any period $t \geq t^*$, I chooses $\bar{\gamma}$, whereas E solves

$$\begin{aligned}
&\min_{\gamma\in[0,\bar{\gamma}]} \frac{1}{a^*(\gamma, \sigma^2_{t-1,E})} + \gamma \\
\text{s.t.} \quad & \arg\max_{a\geq 0} u(a) + \Delta - v[\sigma^2_{t-1,E} - \sigma^2_{t,E}(\gamma, a|\sigma^2_{t-1,E})] \\
&\geq \arg\max_{a\geq 0} u(a) - v[\sigma^2_{t-1,I} - \sigma^2_{t,I}(\bar{\gamma}, a|\sigma^2_{t-1,I})].
\end{aligned} \tag{19}$$

Let Δ^* denote the lowest Δ such that the set of γ's that satisfy (19) is nonempty given $t = t^*$, $\sigma^2_{t^*-1,E} = \sigma^2_0$, and the monopoly outcome $\sigma^2_{t-1,I}$. Δ^* is well-defined because the set of all γ's satisfying the constraint is upper hemicontinuous. The rest of the strategy profile is specified analogously to Proposition 4. The same argument as Proposition 4 confirms that this is an equilibrium. Δ^* is increasing in t^*, because a larger t^* decreases $\sigma^2_{t^*-1,I} - \sigma^2_{t^*,I}(\bar{\gamma}, a|\sigma^2_{t-1,I})$.

Finally, suppose $\Delta < \Delta^*$ but there is an equilibrium in which the consumer only uses E in any period $t \geq t^*$. If I adopts a monopoly strategy for any $t < t^*$ and chooses $\gamma^I_t = \bar{\gamma}$ in period t^*, then the consumer strictly prefers to use I in t^*. This weakly increases I's payoff for any period $t < t^*$ and strictly increases I's payoff in period t^*. This is a contradiction. $\square$

E Erasing Past Information: Omitted Proofs from Section 5

E.1 The Right to be Forgotten: Proof of Proposition 6

Proof. Consider monopoly with commitment. Since the consumer's action does not affect a privacy policy, it is optimal for the consumer to erase information in all periods. Anticipating this, the platform maximizes the amount of information generated in each period by solving the problem (5) with $t = 1$. If the platform has only a short-run commitment power, then the platform sets γ_t to maximize the amount of information in each period. Because $\frac{1}{a^*(\sigma^2,\gamma)} + \gamma$ is increasing in σ^2, it is optimal for the consumer to erase information, which leads to a weakly lower amount of information generated. In either case, the platform's problem leads to $\gamma_t = \gamma_1^*$ for all t.

For competition, consider the following strategy profile: Before entry, the consumer erases information in all periods, and chooses the activity level according to $a^*(\cdot,\cdot)$. After entry, the consumer erases information in all periods, and chooses the platform that offers $\gamma_t = \bar{\gamma}$ for all $t \geq t^*$ if there is such a platform (for a node in which both platform have deviated, I assign an equilibrium of that subgame arbitrarily). I sets $\gamma_t^I = \gamma_1^*$ for all $t < t^*$ and $\gamma_t^I = \bar{\gamma}$ for all $t \geq t^*$. E sets $\gamma_t^E = \bar{\gamma}$ for all $t \geq t^*$ upon entry. I can pick any equilibrium in any subgame in which the consumer deviates and chooses to not erase information, because the consumer is worse off relative to no deviation.

Finally, for any $\Delta > 0$, we can construct an equilibrium in which (i) the consumer erases in formation in all periods and sets $a_t^I = 0$ for any $t \geq t^*$, (ii) I sets $\bar{\gamma}$ in any period $t \geq t^*$, and (iii) E sets γ_t^E that makes the consumer indifferent between I and E. This is an equilibrium with successful entry. $\square$

E.2 Data Retention Policies: Proof of Proposition 7

Proof. A monopolists' problem is to solve (5) by choosing a privacy level and whether to erase information. Whenever $\sigma_{t-1}^2 < \sigma_0^2$, erasing information strictly increases the conditional variance, increases the consumer's marginal cost, and shifts $a^*(\cdot,\sigma^2)$ downward. Thus, erasing information strictly lowers the platform's payoff.

In the model of competition, consider the strategy profile in which platforms never erase information on the path of play, and all players behave in the same way as the strategy profile constructed for Proposition 4. The action of each player straightforwardly extends to nodes in which a platform has deleted information, because the relevant state variable in that strategy profile is $(\sigma^2_{I,t-1}, \sigma^2_{E,t-1})$. If a platform erases information, it lowers the payoff and increases the consumer's cost of using the platform. Thus, it is optimal for each platform to not erase information. $\square$

E.3 Heterogeneous Consumers: Proof of Proposition 8

Take any equilibrium with $(a_t^*(v), \sigma_t^2(v), \gamma_t^*)_{t\in\mathbb{N}, v\in V}$. Define $\sigma^2_\infty(v) := \lim_{t\to\infty} \sigma^2_t(v)$. First, suppose to the contrary that there is some $v^* \in V$ such that $\sigma^2_\infty(v^*) > 0$. Define

$$\Delta_t := \frac{1}{1-\delta_P}\sum_{v\in V}\alpha_v\left[\sigma_0^2 - \sigma^2_\infty(v)\right] - \frac{1}{1-\delta_P}\sum_{v\in V}\alpha_v\left[\sigma_0^2 - \sigma^2_t(v)\right]. \tag{20}$$

It holds $\lim_{t\to\infty}\Delta_t = 0$. Take any $\gamma_v^* \in \arg\max_\gamma \frac{1}{a^*(v^*,\gamma,\sigma_0^2)} + \gamma$. It holds that, for any $\sigma^2 \in [\sigma^2_\infty(v^*), \sigma_0^2]$,

$$\begin{aligned}
&\sigma^2 - \frac{1}{\frac{1}{\sigma^2} + \frac{1}{\frac{1}{a^*(v^*,\gamma_v^*,\sigma^2)}+\gamma_v^*}}\\
\geq&\sigma^2 - \frac{1}{\frac{1}{\sigma^2} + \frac{1}{\frac{1}{a^*(v^*,\gamma_v^*,\sigma_0^2)}+\gamma_v^*}}\\
\geq&B := \min_{\sigma^2\in[\sigma^2_\infty(v^*),\sigma_0^2]} \sigma^2 - \frac{1}{\frac{1}{\sigma^2} + \frac{1}{\frac{1}{a^*(v^*,\gamma_v^*,\sigma_0^2)}+\gamma_v^*}}\\
>&0.
\end{aligned}$$

The first inequality follows from $a^*(v^*, \gamma, \sigma_0^2) \leq a^*(v, \gamma, \sigma^2)$ for $\sigma^2 \leq \sigma_0^2$. The last inequality holds because the minimand is continuous and positive on $[\sigma^2_\infty(v^*), \sigma_0^2]$. For a sufficiently large t, we obtain $\frac{\alpha_v B}{1-\delta_P} > \Delta_t$, or equivalently,

$$\frac{\alpha_v B}{1-\delta_P} + \frac{1}{1-\delta_P}\sum_{v\in V}\alpha_v\left[\sigma_0^2 - \sigma_t^2(v)\right] > \frac{1}{1-\delta_P}\sum_{v\in V}\alpha_v\left[\sigma_0^2 - \sigma^2_\infty(v)\right].$$

The left hand side is the lower bound of the time-t continuation value that the platform can get by deviating to the privacy level γ_v^* from time t on. The right hand side is the upper bound of the time-t continuation value without deviation. Thus, the platform is strictly better off by committing to a privacy policy that sets γ_v^* from time t on. This is a contradiction. $\lim_{t\to\infty} a_t^*(v) = 0$ and $\lim_{\to\infty} \gamma_t^* = 0$ follow the proof of Proposition 2.

F Forward-looking Consumer: Proof of Proposition 9

This appendix consists of three steps. First, I prove the existence of an equilibrium in which the consumer breaks ties and chooses the "greatest" sequence of activity levels. Second, I prove useful properties of the consumer's value function in the dynamic programming. Finally, I use these results to prove Proposition 9.

I prepare notations. Let $\mathcal{A} := [0, a^*]^{\mathbb{N}}$ denote the set of all sequences of activity levels between 0 and a^*. It is without loss of generality to exclude an activity level strictly above a^*. Let $\boldsymbol{a}$ denote a generic element of $\mathcal{A}$, with the t-th coordinate denoted by a_t. Let $\boldsymbol{\Gamma}$ denote the set of all sequences of non-negative real numbers. Let $\boldsymbol{\gamma}$ denote a generic element of $\boldsymbol{\Gamma}$, with the t-th coordinate denoted by γ_t. I consider product topology for $\mathcal{A}$ and $\boldsymbol{\Gamma}$.

F.1 Existence of an Equilibrium

Take any privacy policy $\boldsymbol{\gamma} \in \boldsymbol{\Gamma}$. The consumer's problem is

$$\max_{\boldsymbol{a}\in\mathcal{A}} \sum_{t=1}^{\infty} \delta_C^{t-1} \left[u(a_t) - v \cdot \left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma_0^2} + \sum_{s=1}^{t} \frac{1}{\frac{1}{a_s}+\gamma_s}} \right) \right]. \tag{21}$$

For any $\boldsymbol{\gamma} \in \boldsymbol{\Gamma}$, let $\mathcal{A}^*(\boldsymbol{\gamma}) \subset \mathcal{A}$ denote the set of all maximizers in (21).

Lemma 3. $\mathcal{A}^*(\boldsymbol{\gamma})$ *is non-empty, compact, and upper hemicontinuous in product topology.*

Proof. First, $\mathcal{A}$ is compact with respect to product topology. Second, the objective function is continuous: To see this, take any sequence of the consumer's choices $(\boldsymbol{a}^n)_{n=1}^{\infty}$ such that $\boldsymbol{a}^n \to \boldsymbol{a}^*$. This implies that, for each $t \in \mathbb{N}$, $\lim_{n\to\infty} a_t^n \to a_t^*$. The consumer's period-$t$ payoff $U_t(\boldsymbol{a}, \boldsymbol{\gamma}) := u(a_t) - v \cdot \left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma_0^2}+\sum_{s=1}^{t} \frac{1}{\frac{1}{a_s}+\gamma_s}} \right)$ is bounded from above and below by $u(a^*) > 0$ and $-v\sigma_0^2 < 0$, respectively. Define $B := \max(u(a^*), v\sigma_0^2) > 0$. Take any $\varepsilon > 0$, and let T^* satisfy $\frac{\delta_C^{T^*}}{1-\delta_C} B < \frac{\varepsilon}{4}$. Take a sufficiently large n so that, for each $t \leq T^*$, $\delta_C^{t-1} |U_t(\boldsymbol{a}^n, \boldsymbol{\gamma}) - U_t(\boldsymbol{a}^*, \boldsymbol{\gamma})| < \frac{\varepsilon}{2T^*}$. These inequalities imply that

$$\left| \sum_{t=1}^{\infty} \delta_C^{t-1} U_t(\boldsymbol{a}^n, \boldsymbol{\gamma}) - \sum_{t=1}^{\infty} \delta_C^{t-1} U_t(\boldsymbol{a}^*, \boldsymbol{\gamma}) \right| < \varepsilon.$$

Thus, equation (21) is continuous in $\boldsymbol{a}$. For each privacy policy $\boldsymbol{\gamma}$, let $\mathcal{A}^*(\boldsymbol{\gamma}) \subset \mathcal{A}$ denote the set of all maximizers. Berge maximum theorem implies that $\mathcal{A}^*(\boldsymbol{\gamma})$ is non-empty, compact, and upper hemicontinuous. $\square$

Next, I prove some properties of the consumer's objective $U(\boldsymbol{a}, \boldsymbol{\gamma}) := \sum_{t=1}^{\infty} \delta_C^{t-1} U_t(\boldsymbol{a}, \boldsymbol{\gamma})$. Abusing notation, for any $\boldsymbol{a}, \boldsymbol{a}' \in \mathcal{A}$, write $\boldsymbol{a} \geq \boldsymbol{a}'$ if and only if $a_t \geq a'_t$ for all $t \in \mathbb{N}$. $\geq$ is a partial order on $\mathcal{A}$, and $(\mathcal{A}, \geq)$ is a lattice.

Lemma 4. *For any $\boldsymbol{\gamma}$, $U(\boldsymbol{a}, \boldsymbol{\gamma})$ is supermodular in $\boldsymbol{a}$.*

Proof. Take any $\boldsymbol{\gamma}$. Below, I omit $\boldsymbol{\gamma}$ and write $U(\cdot, \boldsymbol{\gamma})$ as $U(\cdot)$. Take any $\boldsymbol{a}, \boldsymbol{b} \in \mathcal{A}$. For each $n \in \mathbb{N}$, define $(\boldsymbol{a} \vee \boldsymbol{b})^n$ as

$$(\boldsymbol{a} \vee \boldsymbol{b})^n = \begin{cases} \max(a_t, b_t) & \text{if} \quad t \leq n, \\ a_t & \text{if} \quad t > n. \end{cases} \tag{22}$$

Similarly, define $(\boldsymbol{a} \wedge \boldsymbol{b})^n$ as

$$(\boldsymbol{a} \wedge \boldsymbol{b})^n = \begin{cases} \min(a_t, b_t) & \text{if} \quad t \leq n, \\ a_t & \text{if} \quad t > n. \end{cases} \tag{23}$$

Also, define $\boldsymbol{b}^n$ as

$$\boldsymbol{b}^n = \begin{cases} b_t & \text{if} \quad t \leq n, \\ a_t & \text{if} \quad t > n. \end{cases} \tag{24}$$

In product topology, $(\boldsymbol{a} \vee \boldsymbol{b})^n \to \boldsymbol{a} \vee \boldsymbol{b}$, $(\boldsymbol{a} \wedge \boldsymbol{b})^n \to \boldsymbol{a} \wedge \boldsymbol{b}$, and $\boldsymbol{b}^n \to \boldsymbol{b}$ as $n \to \infty$. For each $n \in \mathbb{N}$, $U(\boldsymbol{a})$ is supermodular in the first n activity levels, $(a_1, \ldots, a_n) \in \mathbb{R}^n_+$. Thus, $U((\boldsymbol{a} \vee \boldsymbol{b})^n) + U((\boldsymbol{a} \wedge \boldsymbol{b})^n) \geq U(\boldsymbol{a}) + U(\boldsymbol{b}^n)$. Since $U(\cdot)$ is continuous, we can take $n \to \infty$ and obtain

$U(\boldsymbol{a} \vee \boldsymbol{b})) + U(\boldsymbol{a} \wedge \boldsymbol{b}) \geq U(\boldsymbol{a}) + U(\boldsymbol{b})$. □

Lemma 5. *There is an $\bar{\boldsymbol{a}} \in \mathcal{A}^*(\boldsymbol{\gamma})$ such that, for any $\boldsymbol{a} \in \mathcal{A}^*(\boldsymbol{\gamma})$, $\bar{\boldsymbol{a}} \geq \boldsymbol{a}$.*

Proof. First, Corollary 2 of Milgrom et al. (1994) implies that $\mathcal{A}^*(\boldsymbol{\gamma})$ is a sublattice of $\mathcal{A}$. Since $\mathcal{A}^*(\boldsymbol{\gamma})$ is compact, for each $t \in \mathbb{N}$, the projection of $\mathcal{A}^*(\boldsymbol{\gamma})$ on the t-th coordinate, i.e.,

$$\mathcal{A}_t^*(\boldsymbol{\gamma}) := \left\{ a_t \in [0, a^*] : \exists \boldsymbol{a}_{-t} = (a_s)_{s \in \mathbb{N} \setminus \{t\}} \in \mathcal{A}^*(\boldsymbol{\gamma}) \text{ s.t. } (a_t, \boldsymbol{a}_{-t}) \in \mathcal{A}^*(\boldsymbol{\gamma}) \right\}, \tag{25}$$

is compact (here, $(a_t, \boldsymbol{a}_{-t})$ is a sequence of activity levels such that the consumer takes a_t in period t and acts according to $\boldsymbol{a}_{-t}$ in other periods). For each $k \in \mathbb{N}$, let $\boldsymbol{a}^k$ denote an optimal policy such that $\boldsymbol{a}^k = \max \mathcal{A}_k^*(\boldsymbol{\gamma})$. Define $\bar{\boldsymbol{a}}^k := \boldsymbol{a}^1 \vee \cdots \vee \boldsymbol{a}^k$. Since $\mathcal{A}^*(\boldsymbol{\gamma})$ is sublattice, for any each $k \in \mathbb{N}$, $\bar{\boldsymbol{a}}^k$ maximized (21). Also, $\bar{\boldsymbol{a}}^k \to \bar{\boldsymbol{a}}$, where $\bar{a}_t = \max \mathcal{A}_k^*(\boldsymbol{\gamma})$ for any $k \in \mathbb{N}$. Since $\mathcal{A}^*(\boldsymbol{\gamma})$ is compact, $\bar{\boldsymbol{a}} \in \mathcal{A}^*(\boldsymbol{\gamma})$. By construction, for any $\boldsymbol{a} \in \mathcal{A}^*(\boldsymbol{\gamma})$, $\bar{\boldsymbol{a}} \geq \boldsymbol{a}$. □

For each $\boldsymbol{\gamma} \in \boldsymbol{\Gamma}$, let $\bar{\boldsymbol{a}}(\boldsymbol{\gamma}) := (\bar{a}_t(\boldsymbol{\gamma}))_{t \in \mathbb{N}}$ denote the "greatest" strategy of the consumer defined in Lemma 5.

Lemma 6. *For each $t \in \mathbb{N}$, $\bar{\boldsymbol{a}}_t(\boldsymbol{\gamma})$ is upper semicontinuous in $\boldsymbol{\gamma}$.*

Proof. By Lemma 3, $\mathcal{A}^*(\boldsymbol{\gamma})$ is upper hemicontinuous. Thus, the set $\mathcal{A}_t^*(\boldsymbol{\gamma})$ of all activity levels for period t is upper hemicontinuous in $\boldsymbol{\gamma}$. Thus, it is enough to show that for any upper hemicontinuous and compact-valued correspondence $\phi : X \twoheadrightarrow \mathbb{R}$, $f(x) := \max \phi(x)$ is upper semicontinuous. To show this, take any $x_n \to x$. For each n, define $y_n = f(x_n)$. Because there is a subsequence $y_{n(k)}$ of y_n that converges to $\limsup y_n$, it holds that $\limsup y_n = \lim y_{n(k)} = \lim f(x_{n(k)}) \leq f(\lim x_{n(k)}) = f(x)$. The inequality holds because ϕ has a closed graph. Connecting the left and right sides, we establish that $f(\cdot)$ is upper semicontinuous. □

Lemma 7. *There exists an equilibrium.*

Proof. The platform's objective is

$$\sum_{t=1}^{\infty} \delta_P^{t-1} \left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma_0^2} + \sum_{s=1}^{t} \frac{1}{\frac{1}{\bar{a}_s(\boldsymbol{\gamma})} + \gamma_s}} \right). \tag{26}$$

To show it is upper semicontinuous, take $\boldsymbol{\gamma}^n \to \boldsymbol{\gamma}$. Then,

$$\begin{aligned}
&\limsup_{n\to\infty} \sum_{t=1}^{\infty} \delta_P^{t-1} \left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma_0^2} + \sum_{s=1}^{t} \frac{1}{\frac{1}{\bar{a}_s(\boldsymbol{\gamma}^n)} + \gamma_s^n}} \right) \\
&= \lim_{k\to\infty} \sup_{n\geq k} \sum_{t=1}^{\infty} \delta_P^{t-1} \left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma_0^2} + \sum_{s=1}^{t} \frac{1}{\frac{1}{\bar{a}_s(\boldsymbol{\gamma}^n)} + \gamma_s^n}} \right) \\
&\leq \lim_{k\to\infty} \sum_{t=1}^{\infty} \delta_P^{t-1} \left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma_0^2} + \sum_{s=1}^{t} \sup_{n\geq k} \frac{1}{\frac{1}{\bar{a}_s(\boldsymbol{\gamma}^n)} + \gamma_s^n}} \right) \\
&= \sum_{t=1}^{\infty} \delta_P^{t-1} \left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma_0^2} + \sum_{s=1}^{t} \lim_{k\to\infty} \sup_{n\geq k} \frac{1}{\frac{1}{\bar{a}_s(\boldsymbol{\gamma}^n)} + \gamma_s^n}} \right) \\
&= \sum_{t=1}^{\infty} \delta_P^{t-1} \left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma_0^2} + \sum_{s=1}^{t} \frac{1}{\liminf_{n\to\infty} \frac{1}{\bar{a}_s(\boldsymbol{\gamma}^n)} + \gamma_s^n}} \right) \\
&\leq \sum_{t=1}^{\infty} \delta_P^{t-1} \left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma_0^2} + \sum_{s=1}^{t} \frac{1}{\frac{1}{\limsup_{n\to\infty} \bar{a}_s(\boldsymbol{\gamma}^n)} + \lim_{k\to\infty} \inf_{n\geq k} \gamma_s^n}} \right) \\
&\leq \sum_{t=1}^{\infty} \delta_P^{t-1} \left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma_0^2} + \sum_{s=1}^{t} \frac{1}{\frac{1}{\bar{a}_s(\boldsymbol{\gamma})} + \gamma}} \right)
\end{aligned}$$

The second equality comes from the dominated convergence theorem, and the last inequality uses the upper semicontinuity of $\bar{a}_s(\boldsymbol{\gamma})$. Thus, given the consumer's optimal behavior, the platform's objective is upper semicontinuous. Since $\boldsymbol{\Gamma}$ is compact, there is a privacy policy $\boldsymbol{\gamma}^*$ that maximizes the platform's objective. $(\boldsymbol{\gamma}^*, \bar{\boldsymbol{a}}(\cdot))$ is an equilibrium. $\square$

F.2 Properties of Consumer Value Function

For each privacy policy $\boldsymbol{\gamma} \in \boldsymbol{\Gamma}$, define

$$V_{\boldsymbol{\gamma}}(\rho) := \sum_{t=1}^{\infty} \delta_C^{t-1} \left[u(\bar{a}_t(\boldsymbol{\gamma})) - v \cdot \left(\sigma_0^2 - \frac{1}{\rho + \sum_{s=1}^{t} \frac{1}{\frac{1}{\bar{a}_s(\boldsymbol{\gamma})} + \gamma_s}} \right) \right]. \tag{27}$$

$V_{\boldsymbol{\gamma}}(\rho)$ is the consumer's maximum value of the objective starting from the conditional variance $\sigma^2 = \frac{1}{\rho}$.

Lemma 8. *For any* $\boldsymbol{\gamma} \in \boldsymbol{\Gamma}$*,* $V_{\boldsymbol{\gamma}}(\cdot)$ *is decreasing and convex. For any* $\rho > 0$ *and* $\Delta > 0$*,* $\lim_{\rho\to\infty} V_{\boldsymbol{\gamma}}(\rho) - V_{\boldsymbol{\gamma}}(\rho+\Delta) = 0$.

Proof. Take any privacy policy $\boldsymbol{\gamma}$. Hereafter, I omit $\boldsymbol{\gamma}$ from subscripts (thus, the consumer value function is $V(\cdot)$). Consider the "T-period problem, " in which the consumer's payoff in any period $s \geq T$ is exogenously set to zero. For any $t \leq T$, let $V_t^T(\rho)$ denote the consumer's continuation value in the T-period problem starting from period t given $\frac{1}{\sigma_{t-1}^2} = \rho$:

$$V_t^T(\rho) = \max_{a_t,\ldots,a_T} \sum_{s=t}^{T} \delta_C^{s-t} \left(u(a_s) - v\left(\sigma_0^2 - \frac{1}{\rho_{s-1} + \frac{1}{\frac{1}{a_s}+\gamma_s}} \right) \right).$$

Here, $(\rho_t, \ldots, \rho_{T-1})$ are recursively defined by Bayes' rule given $(a_t, \ldots, a_{T-1})$ and $\rho_{t-1} = \rho$. The standard argument of dynamic programming implies that, for each $t = 1, \ldots, T$,

$$V_t^T(\rho) = \max_{a\geq 0} u(a) - v \cdot \left(\sigma_0^2 - \frac{1}{\rho + \frac{1}{\frac{1}{a}+\gamma_t}} \right) + \delta_C V_{t+1}^T\left(\rho + \frac{1}{\frac{1}{a}+\gamma_t} \right), \tag{28}$$

where $V_{T+1}^T(\cdot) \equiv 0$. I use induction to show that $V_1^T(\rho)$ is decreasing and convex. First, V_{T+1}^T is trivially (weakly) decreasing and convex. Suppose that V_{t+1}^T is decreasing and convex. Since $-v \cdot \left(\sigma_0^2 - \frac{1}{\rho + \frac{1}{\frac{1}{a}+\gamma_t}} \right)$ has the same property, $V_t^T(\cdot)$ is also decreasing and convex. Thus, $V_1^T(\cdot)$ is decreasing and convex.

Define $V^\infty(\rho) := \lim_{T\to\infty} V_1^T(\rho)$. $V^\infty(\rho)$ is decreasing and convex, because these properties are preserved under pointwise convergence. I show that $V^\infty(\rho)$ is the value function of the original problem, i.e., $V^\infty(\cdot) = V(\cdot)$. Take any ρ, and let $(\bar{a}_1, \bar{a}_2, \ldots) \in \mathcal{A}^*(\boldsymbol{\gamma})$ denote the optimal policy. For any finite T,

$$V_1^T(\rho) \geq \sum_{s=1}^{T} \delta_C^{s-1} \left(u(\bar{a}_s) - v\left(\sigma_0^2 - \frac{1}{\rho_{s-1} + \frac{1}{\frac{1}{\bar{a}_s}+\gamma_s}} \right) \right). \tag{29}$$

By taking $t \to \infty$, we obtain $V^\infty(\rho) \geq V(\rho)$. Suppose to the contrary that $V^\infty(\rho) > V(\rho)$. Then, there is a sufficiently large $T \in \mathbb{N}$ such that $V_1^T(\rho) - \frac{\delta_C^T}{1-\delta_C} v\sigma_0^2 > V(\rho)$. If the consumer in the original infinite horizon problem adopts the T-optimal policy that gives $V_1^T(\rho)$ up to period t, then she can obtain a strictly greater payoff than $V(\rho)$, which is a contradiction. Thus, $V^\infty(\rho) = V(\rho)$.

Suppose that the consumer starting from $\rho+\Delta$ chooses the policy $(\bar{a}_t^{\rho})_{t\in\mathbb{N}}$ that is optimal for ρ. Let $(\hat{\rho}_t)_{t=1}^{\infty}$ denote the induced sequence of the precisions after $\rho+\Delta$, i.e. $\hat{\rho}_t = \rho+\Delta+\sum_{s=1}^{t}\frac{1}{\frac{1}{\bar{a}_s^{\rho}}+\gamma_s}$. Note that $\hat{\rho}_t \geq \rho_t$ for all $t\in\mathbb{N}$. Then, it holds that $0 \leq V(\rho)-V(\rho+\Delta) \leq \sum_{t=1}^{\infty}\delta_C^{t-1}\left(\frac{1}{\rho}-\frac{1}{\rho+\Delta}\right) = \frac{1}{1-\delta_C}\left(\frac{1}{\rho}-\frac{1}{\rho+\Delta}\right)$. Thus, $\lim_{\rho\to\infty} V(\rho)-V(\rho+\Delta)=0$. □

F.3 Consequences of Previous Lemmas

The following result extends Proposition 1.

Proposition 12. *Take any $\boldsymbol{\gamma}\in\boldsymbol{\Gamma}$ such that $\gamma_t=\gamma$ for all $t\in\mathbb{N}$. Let $(\bar{a}_t)_{t\in\mathbb{N}}$ denote the equilibrium strategy in the subgame following $\boldsymbol{\gamma}$. There is a $v^*(\gamma)>0$ such that the following holds:*

1. *If $v<v^*(\gamma)$, then $\bar{a}_t$ is increasing in t, $\lim\limits_{t\to\infty}\bar{a}_t=a^*$, and $\lim\limits_{t\to\infty}\sigma_t^2=0$.*
2. *If $v>v^*(\gamma)$, then $\bar{a}_t=0$ for all $t\in\mathbb{N}$.*

Moreover, $v^*(\gamma)$ is increasing and $\lim_{\gamma\to\infty}v^*(\gamma)=\infty$.

Proof. Since $\gamma_t=\gamma$ for all t, the value function $V(\cdot)$ satisfies the Bellman equation

$$V(\rho)=\max_{a\geq 0} u(a)-v\left(\sigma_0^2-\frac{1}{\rho+\frac{1}{\frac{1}{a}+\gamma}}\right)+\delta_C V\left(\rho+\frac{1}{\frac{1}{a}+\gamma}\right). \tag{30}$$

Lemma 8 implies that $V(\cdot)$ is decreasing and convex. Thus, the maximand in (30) has the increasing differences in (a,ρ). Thus, $\bar{a}(v,\gamma,\rho)$, the greatest maximizer, is increasing in ρ.

Define

$$v^*(\gamma):=\sup\left\{v\in\mathbb{R}:\bar{a}_1(v,\gamma,\rho)>0\right\}. \tag{31}$$

$v^*(\gamma)$ is increasing because $\bar{a}_1(v,\gamma,\rho)$ is. Suppose to the contrary that there is a sequence $\gamma^n\to\infty$ such that $v^*(\gamma^n)\leq\bar{v}$ for some $\bar{v}<\infty$. Take the consumer with $v>\bar{v}$. Suppose that the consumer takes $a_t=a^*=\arg\max_{a\geq 0}u(a)$ for all $t\in\mathbb{N}$. As, $\gamma^n\to\infty$,the consumer's period-t payoff converges to $u(a^*)$ for each $t\in\mathbb{N}$. As the consumer's objective is continuous in per-period payoffs with product topology, the sum of discounted payoffs converges to $\frac{u(a^*)}{1-\delta_C}>0$. This contradicts that $v>\bar{v}$ should choose $a_t=0$ for all t. Thus, $\lim_{\gamma\to\infty}v^*(\gamma)=\infty$.

By the identical argument with the case of the myopic consumer, we can conclude that the consumer's activity level is positive and increasing in t if $v<v^*(\gamma)$. This implies $\lim_{t\to\infty}\sigma_t^2\to$

0, or equivalently, $\lim_{t\to\infty} \rho_t = \infty$ with $\rho := \frac{1}{\sigma_t^2}$. By Lemma 8, for any $\rho > 0$ and $\Delta > 0$, $\lim_{\rho\to\infty} V(\rho) - V(\rho+\Delta) = 0$. This, combined with $\lim_{t\to\infty} \rho_t = \infty$, implies $\lim_{t\to\infty} \bar{a}_t(v, \gamma, \rho) = a^*$. Finally, $v > v^*(\gamma)$ implies $\bar{a}_1 = 0$. This implies $\bar{a}_t = 0$ for all $t \in \mathbb{N}$ because the conditional variance does not change. $\square$

The following result is specific to the forward-looking consumer. It states that whenever the change in a privacy policy leads to more informative signals in some periods, the consumer also chooses greater activity levels in other periods.

Lemma 9. *Take $\boldsymbol{\gamma}, \boldsymbol{\gamma}' \in \boldsymbol{\Gamma}$. Define $\mathcal{T} = \{t \in \mathbb{N} : \gamma_t = \gamma'_t\}$. Suppose that $\frac{1}{\bar{a}_t(\boldsymbol{\gamma})} + \gamma_t \leq \frac{1}{\bar{a}_t(\boldsymbol{\gamma}')} + \gamma'_t$ for all $t \in \mathbb{N} \setminus \mathcal{T}$. Then, $\bar{a}_t(\boldsymbol{\gamma}) \geq \bar{a}_t(\boldsymbol{\gamma}')$ for all $t \in \mathcal{T}$.*

Proof. Let $\boldsymbol{\beta}$ be any one of $\boldsymbol{\gamma}$ and $\boldsymbol{\gamma}'$. I decompose the consumer's problem (21) into two steps. First, given any $(a_t)_{t\notin\mathcal{T}}$, the consumer chooses $(a_t)_{t\in\mathcal{T}}$ to maximize the following hypothetical objective function:

$$\sum_{t=1}^{\infty} \delta_C^{t-1} \left[\mathbf{1}_{\{t\in\mathcal{T}\}} u(a_t) - v \cdot \left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma_0^2} + \sum_{s=1}^{t} \frac{1}{\frac{1}{a_s} + \beta_s}} \right) \right]. \tag{32}$$

Note that the consumer does not receive a benefit of $u(a_t)$ in period $t \notin \mathcal{T}$. This leads to a mapping that maps any $(a_t)_{t\notin\mathcal{T}}$ to the (greatest) optimal choice of $(a_t)_{t\in\mathcal{T}}$. In the second step, the consumer chooses $(a_t)_{t\notin\mathcal{T}}$ to maximize her original objective, taking the mapping $(a_t)_{t\notin\mathcal{T}} \mapsto (a_t)_{t\in\mathcal{T}}$ as given.

For any $t \notin \mathcal{T}$, a_t affects (32) only through $\frac{1}{a_t} + \gamma_t$, because $\mathbf{1}_{\{t\in\mathcal{T}\}} = 0$. Moreover, the same argument as in the proof of Lemma 4 implies that (32) is supermodular in $\left((a_t)_{t\in\mathcal{T}}, \left\{ \left(\frac{1}{a_s} + \gamma_s \right)^{-1} \right\}_{s\notin\mathcal{T}} \right)$. This implies that if $\frac{1}{\bar{a}_t(\boldsymbol{\gamma})} + \gamma_t \leq \frac{1}{\bar{a}_t(\boldsymbol{\gamma}')} + \gamma'_t$ for all $t \in \mathbb{N} \setminus \mathcal{T}$, then $\bar{a}_t(\boldsymbol{\gamma}) \geq \bar{a}_t(\boldsymbol{\gamma}')$ for all $t \in \mathcal{T}$. $\square$

F.4 $\lim_{t\to\infty} a_t^* = a^*$ and $\lim_{t\to\infty} \sigma_t^2 = 0$: Proof of Proposition 9

Proof. Let $\boldsymbol{\gamma}^*$ denote the equilibrium privacy policy, and let $\boldsymbol{a}^*$ denote the equilibrium activity levels. First, I show $\lim_{t\to\infty} \sigma_t^2 = 0$. Suppose to the contrary that $\lim_{t\to\infty} \sigma_t^2 \neq 0$. As σ_t^2 is weakly decreasing, it holds that $\lim_{t\to\infty} \sigma_t^2$ exists. This implies that $\frac{1}{a_t^*} + \gamma_t^* \to \infty$, which I prove to be a contradiction.

By Proposition 12, there exists a $\hat{\gamma}$ such that $v^*(\hat{\gamma}) > v$. That is, if the platform chooses $\gamma_t = \hat{\gamma}$ for all t, then the consumer chooses $\bar{a} > 0$ in $t = 1$. Define $B := \frac{1}{\bar{a}} + \hat{\gamma}$. Consider T^* such that, for all $t \geq T^*$, $\frac{1}{a_t^*} + \gamma_t^* > B$. Suppose that the platform replaces γ_t^* with $\hat{\gamma}$ for all $t \geq T^*$, and commits to such a new policy ex ante. Take any period $t \geq T^*$. Since the consumer's activity levels after T^* solve the Bellman equation with the "initial state" of $\rho = \frac{1}{\sigma_{T^*}^2} \geq \frac{1}{\sigma_0^2}$, the consumer chooses an activity level weakly greater than $\bar{a} > 0$ after period T^*. Thus, the variance of the noise $\varepsilon_t + z_t$ in the signal s_t is at most $\frac{1}{\bar{a}} + \hat{\gamma} < \frac{1}{a_t^*} + \gamma_t^*$. Thus, this change strictly increases the platform's profit in any period $t \geq T^*$. By Lemma 9, this change also increases the consumer's activity level for any period $t < T^*$. Thus, as a result of the deviation, the platform's payoffs increase in all periods and strictly increase in some periods, which contradicts $\boldsymbol{\gamma}^*$ being optimal. Thus, in equilibrium, $\lim_{t\to\infty} \sigma_t^2 = 0$ holds. Finally, $\lim_{t\to\infty} \sigma_t^2 = 0$ implies $a_t^* \to a^*$: Otherwise, there is a convergent subsequence $a_{t(n)}^* \to a' < a^*$, however, the consumer could be strictly better off by choosing a^*, due to Lemma 8. $\square$

G General Privacy Cost: Proof of Proposition 11

Proof. Consider the consumer's problem in period t. Given the conditional variance σ^2 at the end of period $t-1$ and the privacy level γ in period t, the consumer chooses a to maximize $U(a, \gamma, \sigma^2) := u(a) - C\left(\frac{1}{\frac{1}{\sigma^2} + \frac{1}{\frac{1}{a}+\gamma}}\right)$. It holds that

$$\frac{\partial U}{\partial a} = u'(a) + C'\left(\frac{1}{\frac{1}{\sigma^2} + \frac{1}{\frac{1}{a}+\gamma}}\right) \cdot \frac{1}{\left(\frac{1}{\sigma^2}(1+\gamma a) + a\right)} \geq u'(a) - B \cdot \frac{1}{\left(\frac{1}{\sigma^2}(1+\gamma a) + a\right)}, \quad (33)$$

where $B := \sup_{x\in[0,\sigma_0^2]} |C'(x)| < \infty$. If $\lim\limits_{t\to\infty} \sigma_t^2 > 0$, then $\lim\limits_{t\to\infty} a_t^* = 0$. This implies that $\lim_{t\to\infty} \frac{1}{a_t^*} + \gamma_t^* = \infty$. Consider a hypothetical payoff function

$$U_B(a, \gamma, \sigma^2) = u(a) - B \cdot \left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma^2} + \frac{1}{\frac{1}{a}+\gamma}}\right).$$

(33) implies that $\frac{\partial U}{\partial a} \geq \frac{\partial U_B}{\partial a}$. Take any γ' such that $a_B^*(\gamma', \sigma^2) := \max\left\{\arg\max_{a\geq 0} U_B(a, \gamma', \sigma_0^2)\right\} > 0$. Then, for any $\sigma^2 \leq \sigma_0^2$, $a^*(\gamma', \sigma^2) \geq a_B^*(\gamma', \sigma^2) \geq a_B^*(\gamma', \sigma_0^2) > 0$. Take T such that for all $t \geq T$, $\frac{1}{a_t^*} + \gamma_t^* \geq \frac{1}{a_B^*(\gamma', \sigma_0^2)} + \gamma'$ Then, the platform can achieve a lower $\frac{1}{a_t} + \gamma_t$ for any $t \geq T$ by

replacing γ_t^* with γ', which is a contradiction. A similar argument implies that $\lim_{t\to\infty} a_t^* = a^*$. $\square$