Addictive Platforms*

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Abstract

We study competition for consumer attention in which platforms choose the addictiveness of their services. A more addictive platform yields a lower service utility, but a higher marginal utility for consumers of allocating attention. Consumer surplus is maximized by either a monopoly or a market with many small platforms, but never by a oligopoly. Platforms decrease addictiveness when they can charge for their services, but consumers may be better off when platforms provide free services that monetize attention. We also study the impact of platform mergers and a digital curfew. We extend our analysis to naive consumers who underestimate true addictiveness.

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1 Introduction

Online platforms, such as Facebook, Google, and Twitter, monetize consumer attention. Competition for attention may encourage them to offer better services that attract consumers. However, there is a growing concern for consumers and policymakers—that competition for attention could also incentivize firms to design their services to increase attention at the expense of consumer welfare. For example, a platform may adopt algorithmic news feeds that display low quality content users are likely to click; a platform may also design user interface, such as notification systems or infinite scrolling (Scott Morton et al., 2019).[1]

We study competition for consumer attention in the following game: First, platforms choose the addictiveness of their services. Second, a consumer decides which platforms to join, then allocates her attention. A more addictive platform yields a lower service utility but a higher marginal utility for the consumer of allocating attention (Section 2.1 motivates this formulation). As a result, the consumer prefers to join less addictive platforms, but after joining, she allocates more attention to more addictive platforms. The consumer incurs an increasing cost of allocating attention, such as the opportunity costs of using services. She also faces an attention constraint that caps the maximum attention she can allocate. The attention cost and constraint capture the elastic and inelastic components of supplying attention.

We study three questions. First, we ask which market structure minimizes addictiveness and maximizes consumer welfare. On the one hand, competition incentivizes platforms to decrease addictiveness: A consumer who faces many platforms loses less by refusing to join a single platform, because she can use other services. As a result, platforms need to reduce addictiveness and offer high service quality to encourage participation. On the other hand, competition introduces business stealing incentives—i.e., a platform can increase its addictiveness to capture attention the consumer would allocate to its rivals.

The countervailing incentives lead to the following result. If the consumer faces a tight attention constraint, higher addictiveness does not expand the total attention, but only changes how the

[1]For example, Scott Morton and Dinielli (2020) argue that "another reduction in quality that Facebook's market power allows is the serving of addictive and exploitative content to consumers. Facebook deploys various methods to maintain user attention—so that it can serve more ads—using techniques that the medical literature has begun to demonstrate are potentially addictive."

consumer divides her attention across platforms. In such a case, monopoly, which has no business stealing incentive, leads to the lowest addictiveness and highest consumer welfare. In contrast, if the consumer does not face a tight attention constraint, the monopolist chooses high addictiveness to increase attention without discouraging participation. In this case, the limit economy with many small platforms maximizes consumer welfare. An oligopoly—perhaps, prevailing state for many consumers—never maximizes consumer welfare, because platforms have market power and business stealing incentives, leading to high equilibrium addictiveness.

Second, we study how the business models of platforms affect the equilibrium welfare. We compare two models: In one model, platforms earn revenue from attention; in the other model, they earn revenue by charging monetary prices, such as subscription fees. We find price competition leads to lower addictiveness. The finding supports the idea that ad-financed services tend to adopt more addictive features than those that do not monetize attention.[2] However, consumer welfare can be higher under attention competition: The consumer faces high marginal utilities from addictive services, so she can earn a high incremental gain by refusing to join a platform and continuing to use other services. The better outside option encourages platforms to offer higher net utilities to the consumer under attention competition.

Third, we examine the impact of a digital curfew, which restricts the total amount of attention the consumer can allocate. For example, the Social Media Addiction Reduction Technology Act (the "SMART" Act) proposed in the US aims at curbing social media addiction by requesting that companies limit the time a user may spend on their services.[3] In our model, a digital curfew restricts the consumer's choice, but induces platforms to decrease their addictiveness. A digital curfew benefits the consumer more in a less competitive market, and a monopoly with a digital curfew attains the consumer's first best.

In our baseline model, the consumer correctly perceives the addictiveness of platforms. In

[2]For example, Scott Morton et al. (2019) state that "a business that depends on users staying online to watch ads and have their preferences harvested will focus its resources on keeping users online—for example, with intelligent and flexible algorithms."

[3]See https://www.congress.gov/bill/116th-congress/senate-bill/2314 (accessed on November 24, 2020). Several other countries have implemented some restrictions to protect young students from addictive games. In 2003 Thailand implemented a shutdown law that banned young people from playing online games between 22:00 and 06:00. In 2011, South Korea passed a similar legislature, known as the Youth Protection Revision Act. In 2007, China introduced the so-called "fatigue" system under which game developers need to reduce or stop giving out rewards (e.g., game items, experience value) in games after a player reached certain hours of play.

practice, consumers may systematically underestimate the addictive features of platforms. As an extension, we study such a consumer and show that her naivete increases equilibrium addictiveness and decreases her welfare. In contrast, consumer welfare is independent of her naivete under price competition, because platforms set positive prices but zero addictiveness. As a result, sufficiently naive consumers are better off under price competition than under attention competition. Finally, the naivete possibly eliminates the beneficial impact of a digital curfew in reducing the equilibrium addictiveness. Naive consumers have a wrong expectation that they would not spend much time on platforms. Anticipating that the digital curfew does not affect consumer behavior, platforms choose not to reduce the addictiveness of their services.

The paper provides several policy implications. First, a merger between free-to-use platforms into a concentrated oligopoly can increase the number of low-quality services capable of capturing attention. Such a merger shifts consumer attention and profits away from small competitors. At the same time, a merger to monopoly is not necessarily more harmful for consumers than other types of mergers. Second, we highlight the welfare cost and benefit of business models adopted by attention intermediaries: Free services come with a non-monetary cost of being addictive, but the impact of different business models on consumer welfare depends on the market structure and consumer sophistication. Third, on the regulatory actions against digital addiction, we show that the efficacy of a digital curfew depends on the market structure, platforms' business model, and consumers' behavioral biases. Our findings help policymakers understand the interaction between competition for consumer attention and the addictiveness of digital services.

Related literature First, this paper contributes to the nascent literature on possible negative impacts of digital services on consumers (Allcott and Gentzkow, 2017; Allcott et al., 2020; Mosquera et al., 2020). A recent discussion points out that technology companies may have an incentive to adopt features (e.g., user interfaces) that increase user engagement at the expense of their welfare (Alter, 2017; Scott Morton et al., 2019; Newport, 2019; Rosenquist et al., 2020). We contribute to this literature by examining interactions between competition for attention and the addictiveness of digital services. Although we later motivate our model based on habit formation with a time-inconsistent agent, we largely abstract away from dynamics and behavioral biases relevant to addiction (Becker and Murphy, 1988; Gruber and Köszegi, 2001; Orphanides and Zervos, 1995).

This simplification enables us to study various regulations and business models.

Second, our paper relates to the emerging literature on regulation against dominant platforms that monetize attention (Crémer et al., 2019; U.K. Digital Competition Expert Panel, 2019; Scott Morton and Dinielli, 2020). More broadly, our work relates to the literature on platform competition for attention.[4] Relative to this literature, our model has two new features. First, a consumer in our model decides whether to join a platform *and* how much attention to allocate. This feature renders both a platform's service utility and a marginal utility (of allocating attention) relevant. Second, a platform can choose higher addictiveness to decrease the service utility and increase marginal utility. The choice of addictiveness has a different welfare implication compared to the choice of price or quality.[5]

2 Model

There are $K \in \mathbb{N}$ platforms and a single consumer. We write K for the number and the set of the platforms. Suppose the consumer joins a set $K' \subset K$ of platforms, and allocates attention $a_k \geq 0$ to each platform $k \in K'$. If $K' = \emptyset$, she receives a payoff of zero. Otherwise, her payoff is

$$\sum_{k \in K'} u(a_k, d_k) - C\left(\sum_{k \in K'} a_k\right). \tag{1}$$

In the first term, $u(a_k, d_k)$ is the utility from platform k's service. The utility $u(a_k, d_k)$ depends on the *addictiveness* $d_k \in \mathbb{R}_+$ of platform k. We impose the following assumption (see also Figure 1).

Assumption 1. The function $u(\cdot, \cdot) : \mathbb{R}_+^2 \to \mathbb{R}$ is continuously differentiable and satisfies the following:

(a) For every $d \geq 0$, utility function $u(a, d)$ is strictly increasing and concave in a.

[4]See the following references, among many: Rochet and Tirole (2003); Anderson and Coate (2005); Armstrong (2006); Anderson and De Palma (2012); Bordalo et al. (2016); Wu (2017); Evans (2017, 2019); Prat and Valletti (2019); Galperti and Trevino (2020); Anderson and Peitz (2020).

[5]Relatedly, de Corniere and Taylor (2020) study the role of data in competition in utilities framework. There, data affect the marginal incentives of firms to offer utilities to consumers. Choi and Jeon (2020) study the incentive of platforms to adopt a technology that shifts surplus from one side to the other.

(b) For every $a \geq 0$, utility function $u(a, d)$ is strictly decreasing in d, and $\max_{a \geq 0} u(a, d) - C(a) < 0$ for some d.

(c) For every $a \geq 0$, the marginal utility for attention $\frac{\partial u}{\partial a}(a, d)$ is strictly increasing in d.

(d) $u(0, 0) \geq 0$.

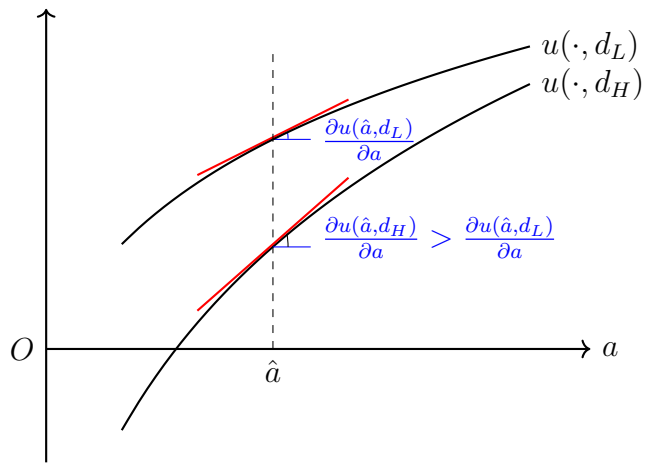


Figure 1: Utilities under d_L and $d_H > d_L$.

Points (b) and (c) imply that higher addictiveness decreases the consumer's utility of joining a platform but increases her marginal utility of allocating attention. Assumption 1 holds if, for example, $u(a, d) = (1 + d)(1 - e^{-\rho a}) - cd$ with $\rho > 0$ and $c > 1$, or $u(a, d) = \hat{u}(a - d)$ with an increasing and concave $\hat{u}(\cdot)$. Section 2.1 provides discussion and applications.

The second term $C\left(\sum_{k \in K'} a_k\right)$ of the consumer's payoff (1) is the *attention cost*—e.g., the opportunity and cognitive costs of using services. We impose the following assumption.

Assumption 2. The function $C(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+$ is increasing, convex, and continuously differentiable.

The consumer also faces the *attention constraint*: She can allocate the total attention of at most $\overline{A} \in (0, \infty]$ across platforms. Introducing $\overline{A} < \infty$ is equivalent to relaxing Assumption 2 to accommodate a discontinuous cost function such that $C(a) = +\infty$ for any $a > \overline{A}$. The bound $\overline{A}$ comes from, for example, the consumer's preferences, physical constraints, and an exogenous restriction such as a digital curfew.

If the consumer allocates a to platform k, it earns a payoff of ra, where $r > 0$ is an exogenous value of attention to a platform (if the consumer does not join platform k, it receives a payoff of zero). For example, r is the unit price of attention in the (unmodeled) advertising market. Total surplus refers to the sum of the payoffs of the consumer and all platforms.[6]

The timing of the game is as follows: First, each platform $k \in K$ simultaneously chooses its addictiveness, $d_k \geq 0$. Second, the consumer chooses which platforms to join and how much attention to allocate. In equilibrium the consumer solves

$$\max_{\hat{K} \subset K, (a_k)_{k \in \hat{K}}} \sum_{k \in \hat{K}} u(a_k, d_k) - C\left(\sum_{k \in \hat{K}} a_k\right) \tag{2}$$
$$\text{s.t.} \quad \sum_{k \in \hat{K}} a_k \leq \overline{A} \quad \text{and} \quad a_k \geq 0, \forall k \in \hat{K}.$$

Because $u(0, d) < 0$ for $d > 0$, the consumer's payoff of not joining platform k can differ from the payoff of joining but setting $a_k = 0$. Our solution concept is pure-strategy subgame perfect equilibrium, which we call *equilibrium*. Under monopoly, we focus on equilibrium in which the platform breaks ties in favor of the consumer.

2.1 Discussion on Modeling Addictive Platforms

Addictiveness d captures any choice of a platform that decreases consumer welfare but induces consumers to spend longer time on the service. We provide three examples.

2.1.1 Data Collection and Personalization

A platform requests consumers to provide their personal data upon participation. Let d denote the amount of data the platform requests. To provide data, consumers incur a privacy cost—e.g., the risk of data leakage, identity theft, and discrimination. Suppose consumers incur a linear privacy cost, $c \cdot d$. The platform can use their data to personalize offerings, which increases the value of the

[6]In some applications, we may consider only a part of the platforms' revenues as total surplus (e.g., wasteful advertising). In this case, we can redefine $r > 0$ as the social value of attention, and continue to define total surplus as the sum of the payoffs of all players. This interpretation does not affect the analysis, because the equilibrium is independent of r.

service from the base value $w(a)$ to $(1+d)w(a)$, where $w(\cdot)$ is increasing, concave, and bounded. A consumer's utility from joining the platform is $u(a,d) := (1+d)w(a) - cd$. If $c > \sup_{a\geq 0} w(a)$, then $u(a,d)$ satisfies Assumption 1. If consumers join platforms but do not use the services, they may receive a negative utility, because they experience the downside of data collection without enjoying the service.

2.1.2 Providing Tempting Content

Scott Morton et al. (2019) provide an example in which a video streaming platform recommends videos that lead viewers towards false or dangerous content.[7] A higher d can correspond to a platform that offers such low-quality but attention-grabbing content. In our model the consumer may receive a negative utility if she joins a platform with $d > 0$ but allocates no attention. We may understand such a loss as the cost of self-control, which the consumer incurs when they know tempting content is available, but do not consume it (e.g., Gul and Pesendorfer, 2001).

2.1.3 Habit Formation

The consumer faces a low utility and a high marginal utility on a platform with high addictiveness. This assumption resembles rational addiction, in which an addicted consumer faces a low utility and a high marginal utility of consumption (e.g., Becker and Murphy, 1988). While this literature models addiction as a dynamic process, we study addictive digital services in a static model. To reconcile this tension, we motivate the consumer's payoff $u(a,d)$ in a dual-self model with habit formation.[8] It helps us understand the implicit relation between the addictiveness of a platform and the consumer's addiction.

Given addictiveness $(d_1, \ldots, d_K)$, consider the following three-period problem (see Figure 2). In $t = 1$, the consumer chooses the set $K' \subset K$ of platforms to join. In $t = 2$, the consumer exogenously allocates attention $a_0 > 0$ and obtains utility $u_0 \geq 0$ on each platform in K'. This

[7] Scott Morton et al. (2019) describes the situation as follows: "More disturbing examples of low-quality content are YouTube recommended videos that lead the viewer towards false or dangerous content. Prior to having these patterns made public and criticized, a Google search about the earth's geology would lead to a chain of recommendations that resulted in 'flat earth' content; YouTube would offer teenage girls interested in diets videos about how to get anorexia, and so forth."

[8] The combination of time-inconsistent preferences and habit formation also appear, for example, in Gruber and Köszegi (2001) in a much more general way.

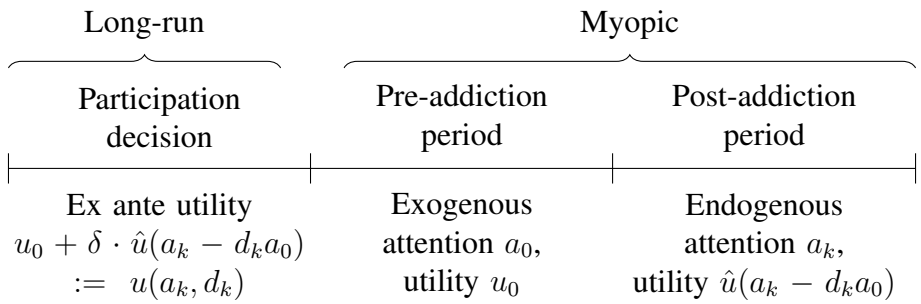


Figure 2: Three-period problem of the consumer

period is a "pre-addiction" stage—i.e., the consumer has yet to be addicted, and the service utilities and the optimal amount of attention do not depend on $(d_1, \ldots, d_K)$.[9] In $t = 3$, the consumer endogenously allocates her attention across platforms in K'. This period is a "post-addiction" stage: If the consumer allocates attention a to platform k, she receives $\hat{u}(a - a_0 d_k)$, where $\hat{u}(\cdot)$ is an increasing concave function with $\hat{u}(0) \geq 0$. This payoff $\hat{u}(a - a_0 d_k)$ captures linear habit formation (e.g., Rozen, 2010). Here, $a_0 d_k$ is the reference point against which the consumer evaluates service consumption of platform k in $t = 3$. We can interpret $\frac{1}{d_k}$ as the "rate of disappearance of the physical and mental effects of past consumption" (Becker and Murphy, 1988). A higher d_k imposes a greater harm on the consumer in $t = 3$, and she needs to increase her attention in $t = 3$ to ensure the same payoff as in $t = 2$.

Motivated by dual-self models, we assume that the long-run self makes the participation decision and the short-run selves allocate attention (e.g., Thaler and Shefrin, 1981; Fudenberg and Levine, 2006). Specifically, in $t = 1$ the long-run self decides which platforms to join, anticipating the behavior of future selves: In $t = 2$ the short-run self allocates attention a_0 to each platform, then in $t = 3$ she allocates attention $(a_k^*)_{k \in K'}$ to maximize $\sum_{k \in K'} \hat{u}(a_k - a_0 d_k) - C\left(\sum_{k \in K'} a_k\right)$. Assume the long-run self has discount factor δ. The consumer's participation decision is based on the service utility $u(a_k, d_k) := u_0 + \delta \hat{u}(a_k - a_0 d_k)$, which satisfies Assumption 1.

Our model is suitable when a consumer is susceptible to addictive features of digital services, but she recognizes it and may avoid joining platforms as a commitment device. The model is not suitable for a consumer who joins platforms but can use them cautiously to avoid addiction. Such

[9] Here, to derive our functional form, we assume that a_0 does not depend on the number of platforms the consumer has joined. One way to endogenize this behavior is to assume that the consumer's utility from each platform in $t = 2$ is $v(a)$, which is maximized at an interior optimum a_0 such that $K a_0 \leq A$.

a situation would correspond to the consumer who is forward-looking in periods 2 and 3.

2.2 Other Modeling Assumptions

Multi-homing. We allow the consumer to multi-home. For example, consumers may divide their time across social media, video streaming, and mobile applications, all of which monetize consumer attention. If the consumer can join only one platform, in equilibrium, all platforms set zero addictiveness and the consumer attains the first-best payoff.

Platform's revenue. A platform's payoff ra_k is proportional to the amount of attention a_k from the consumer. This formulation facilitates welfare analysis because the platforms' joint profit only depends on the total amount of attention the consumer allocates. However, most of the results continue to hold in the following setting: If the consumer allocates attention $(a_1, \ldots, a_K)$, platform k earns a payoff of $r_k(a_1, \ldots, a_K)$ that is strictly increasing in a_k and may depend arbitrarily on $(a_j)_{j \in K \setminus \{k\}}$. For example, a platform's payoff captures its revenue in the advertising market, in which platforms can sell consumer attention at a market price. Alternatively, more attention may lead to a larger revenue from ancillary services such as in-app purchases.

Addictiveness reduces welfare. We assume that higher addictiveness harms consumers, but platforms may also adopt features that increase consumer attention and their welfare. To incorporate such features, suppose the consumer's utility from a platform is $u(a, d, b)$, where $u(a, d, b)$ and $\frac{\partial u}{\partial a}(a, d, b)$ are increasing in $b \in [0, 1]$. Since a higher b encourages the consumer to join the platform and allocates more attention, we can redefine $u(a, d) = u(a, d, 1)$ and apply our model.

2.3 The First-Best Benchmark

As a benchmark, we characterize the level of addictiveness that maximizes total or consumer surplus, when the consumer joins platforms and allocates attention to maximize her payoff. For any $d \geq 0$, let $CS(d)$ denote the maximized value of (2) given $d_k = d$ for all k. It is the consumer's indirect utility when all platforms choose addictiveness d. Let $A(d)$ denote the total amount of attention she allocates to attain $CS(d)$. Note that for a high d, the consumer may not join some of the platforms (see Appendix A for the proof of the following result).

Proposition 1. *Total surplus is maximized by* $d^{TS} \in \arg\max_{d \geq 0} CS(d) + rA(d)$*. If* $A(0) < \overline{A}$*, for a sufficiently large* r*, we have* $d^{TS} > 0$*. Consumer surplus is maximized by* $d_k = 0$ *for all* $k \in K$*.*

The consumer-optimal outcome is $d_k = 0$ because higher addictiveness lowers consumer welfare. In contrast, given the consumer's optimal behavior, total surplus may be maximized by $d > 0$.[10] Because the consumer does not internalize the value of attention to platforms (or advertisers), for a large r, she chooses an inefficiently low level of attention. In such a case, positive addictiveness increases attention and total welfare; this result reflects the two-sidedness of the market.

3 Equilibrium

We derive a unique equilibrium for a given K, and compare it to the first-best.

3.1 Monopoly

A monopolist maximizes the consumer's attention subject to her participation constraint. To state the result, let d^0 denote the highest addictiveness that satisfies the consumer's participation constraint—i.e., $\max_{A \in [0, \overline{A}]} u(A, d^0) - C(A) = 0$. Also let $A(d) := \arg\max_{A \geq 0} u(A, d) - C(A)$ denote the consumer's unconstrained choice of attention given addictiveness d. Then, define $d^1 := \min\left\{d \in [0, \infty] : A(d) \geq \overline{A}\right\}$. If $d^1 < \infty$ the consumer's attention constraint binds at d^1, i.e., $A(d^1) = \overline{A}$.

Proposition 2. *In equilibrium, a monopolist sets addictiveness* $\min(d^0, d^1)$*. In particular the following holds.*

1. *If* $\overline{A} \leq A(0)$*, the monopolist sets zero addictiveness and the consumer chooses* $\overline{A}$*. The equilibrium maximizes consumer and total surplus.*

2. *If* $\overline{A} \geq A(d^0)$*, the consumer obtains a payoff of zero.*

[10] If the welfare-maximizing social planner could determine addictiveness and the consumer's behavior, the planner would choose zero addictiveness and allocate attention to maximize total surplus. Such a first-best outcome would be less relevant to a regulator who cannot influence consumer behavior.

Proof. Suppose the monopolist chooses $d^M := \min(d^0, d^1)$. Because $d^M \leq d^0$, it is optimal for the consumer to join the platform. If $d^M = d^0$ and the monopolist increases addictiveness, the consumer will not join it. If $d^M = d^1$ and the monopolist increases addictiveness, the consumer will continue to choose $\overline{A}$ because her marginal utility of allocating attention is increasing in d. In this case the monopolist continues to earn the same payoff, $r\overline{A}$. In either case, the monopolist does not strictly benefit from changing d^M. Point 1 holds because if $\overline{A} \leq A(0)$, we have $d^M = d^1 = 0$ and $A(d^M) = \overline{A}$. Point 2 holds because if $\overline{A} \geq A(d^0)$, we have $d^M = d^0$, so the consumer receives a payoff of zero. □

Proposition 2 connects the tightness of the attention constraint with the monopolist's incentive: If the constraint is tight, the consumer exhausts her attention capacity $\overline{A}$ even when the monopolist chooses zero addictiveness. In this case, the monopolist sets $d = 0$, and the equilibrium maximizes consumer and total surplus. If $\overline{A}$ is large, the monopolist raises addictiveness to increase consumer attention until she becomes indifferent between joining and not joining the platform.

3.2 Competing Platforms

We now study competing platforms (i.e., $K \geq 2$). To state the result, for each $K \geq 2$, define $A_K(d) := \arg\max_{A \in [0,\overline{A}]} Ku\left(\frac{A}{K}, d\right) - C(A)$. If the consumer joins K platforms with addictiveness d, she will choose total attention $A_K(d)$. The following result characterizes the equilibrium (see Appendix B for the proof).

Proposition 3. *In a unique equilibrium, all platforms choose addictiveness $d^* > 0$ that makes the consumer indifferent between joining and not joining each platform:*

$$K \cdot u\left(\frac{A_K(d^*)}{K}, d^*\right) - C\left(A_K(d^*)\right) = (K-1) \cdot u\left(\frac{A_{K-1}(d^*)}{K-1}, d^*\right) - C\left(A_{K-1}(d^*)\right). \tag{3}$$

The consumer allocates attention $\frac{A_K(d^*)}{K}$ to each platform.

The intuition is as follows. Upon choosing addictiveness, each platform faces a trade-off. On the one hand, higher addictiveness renders its service less attractive to consumers. On the other hand, conditional on joining, consumers allocate more attention to more addictive services because

of the complementarity between attention and addictiveness. Each platform then prefers to increase its addictiveness so long as the consumer joins it. As a result, the equilibrium addictiveness makes the consumer indifferent between joining and not joining each platform.

The equilibrium does not maximize consumer surplus because platforms choose positive addictiveness. In contrast, it is ambiguous whether the equilibrium addictiveness exceeds the welfare-maximizing level in Proposition 1. The equilibrium addictiveness is determined by the consumer's indifference condition and is independent of r, but the welfare-maximizing level may depend on r. Thus the equilibrium addictiveness can be higher or lower than the welfare-maximizing level. For example, if r is high but platforms decrease addictiveness, the consumer reduces total attention, which may decrease total surplus. Nonetheless, in some cases, reducing addictiveness can lead to Pareto improvement.

Corollary 1. *Suppose the supply of total attention is perfectly inelastic—i.e., $C(\cdot) \equiv 0$ and $\overline{A} < \infty$. If the platforms collectively reduce addictiveness relative to the equilibrium level and the consumer behaves optimally, consumer surplus strictly increases, and the platforms' payoffs remain the same.*

Proof. In equilibrium the consumer allocates $\frac{\overline{A}}{K}$ to each platform. If the platforms collectively reduce their addictiveness, the consumer's utility increases. The allocation of attention remains the same, and so are the platforms' profits. □

Does the consumer benefit from competition? To begin with, we compare monopoly and duopoly. Under duopoly the indifference condition implies that consumer welfare is equal to her payoff of joining a single platform. As a result, whenever the monopolist chooses lower addictiveness than duopoly platforms, the consumer is better off under monopoly.

Corollary 2. *If the attention constraint is tight so that $\overline{A} \leq A(0)$ holds, the consumer is strictly better off under monopoly than duopoly.*

Proof. Duopolists choose positive addictiveness, but if $\overline{A} \leq A(0)$ the monopolist chooses zero addictiveness (Proposition 2). In such a case the consumer is better off under monopoly because her payoff under duopoly is equal to the payoff from joining a single platform, which chooses higher addictiveness than the monopolist. □

Intuitively, when the attention constraint is tight, higher addictiveness does not induce greater attention. However, each platform under duopoly benefits from higher addictiveness because the consumer will allocate a greater fraction of her total attention $\overline{A}$ to more addictive services. As a result, competition for attention can lead to higher addictiveness and lower consumer surplus.

The condition in the corollary is not necessary for the monopoly to attain higher consumer surplus than duopoly. For example, for any $\overline{A} < \infty$, if $C'(\cdot)$ is small enough, the consumer chooses $\overline{A}$ on a monopoly platform that chooses positive but lower addictiveness than duopoly equilibrium. Then, the consumer is strictly better off under monopoly.

A natural question is how competition beyond duopoly affects the equilibrium addictiveness and welfare. To answer this question, we could compare the equilibrium in Proposition 3 across different K's. This approach has two problems. First, it mechanically favors competition because a higher K implies more services. Second, the limiting economy ($K \to \infty$) may not be well-defined—e.g., the equilibrium objects, such as the total attention allocated, may not converge along the sequence. To address these issues, in the next section we study a sequence of markets such that they have the same size and markets with larger indices are more competitive.

4 The Impact of Competition

4.1 Competition

We study the following sequence of markets, $(\mathcal{E}_K)_{K\in\mathbb{N}}$. In market $\mathcal{E}_1$ a monopoly platform provides service utility $u(a,d)$, as in Section 3.1. For each $K \geq 2$, the market $\mathcal{E}_K$ have K platforms, each of which provides service utility $\frac{1}{K}u(aK,d)$. The markets $(\mathcal{E}_K)_{K\in\mathbb{K}}$ have the same size, in that if the consumer allocates total attention A equally across K platforms, she obtains total utility $u(A,d)$. Conversely, in any market with this property, each platform provides utility $\frac{1}{K}u(aK,d)$. Indeed, if utility function $v(a,d)$ has the property that the consumer obtains $u(A,d)$ by allocating A/K to each of K platforms, we have $Kv(\frac{a}{K},d) = u(a,d)$, which implies $v(a,d) = \frac{1}{K}u(aK,d)$. In all of these markets $(\mathcal{E}_K)_{K\in\mathbb{K}}$, the consumer faces the same $(C(\cdot),\overline{A})$. As a result, the maximum total surplus and consumer surplus are constant across K. This way of capturing competition—i.e., increasing the number of firms while fixing the market size—appears in the classical work that

studies the relation between perfectly and imperfectly competitive equilibria in product markets (e.g., Novshek, 1980, 1985; Allen and Hellwig, 1986).

Proposition 3 implies each $\mathcal{E}_K$ has a unique equilibrium. The following result shows how the equilibrium changes as K grows, and characterizes the equilibrium addictiveness in the limiting market $K \to \infty$ (see Appendix C for the proof). We define the consumer's best response as $A(d) := \arg\max_{A \in [0,\overline{A}]} u(A,d) - C(A)$, and write $\frac{\partial u}{\partial a}(a,d)$ as $u_1(a,d)$.

Proposition 4. *For any $K \geq 2$, market $\mathcal{E}_K$ has a unique equilibrium, in which all platforms choose the same positive addictiveness. The equilibrium addictiveness is strictly decreasing in $K \geq 2$ and converges to $d^\infty > 0$ that solves*

$$u\left(A(d^\infty), d^\infty\right) = A\left(d^\infty\right) \cdot u_1\left(A(d^\infty), d^\infty\right). \tag{4}$$

Proposition 4 implies competition reduces addictiveness, except possibly when the market changes from monopoly to duopoly (i.e., Corollary 2). To see the intuition, consider the consumer's trade-off. The consumer's loss of not joining a platform (say k) is that she does not receive its service utility. The benefit is that she can allocate the saved attention to other platforms $j \neq k$. When there are fewer platforms, this benefit of reallocating the saved attention is smaller because the consumer will allocate a larger amount of attention to each platform $j \neq k$, but the marginal utility of allocating attention is decreasing. As a result, for a small K the consumer incurs a higher net loss of not joining a platform. Thus, platforms can set higher equilibrium addictiveness without deterring participation. Equation (4) captures this intuition in the limiting case. The left-hand side $u\left(A(d^\infty), d^\infty\right)$ is the consumer's loss of not joining a platform, and the right-hand side $A\left(d^\infty\right) \cdot u_1\left(A(d^\infty), d^\infty\right)$ is the incremental gain of reallocating the saved attention. The two terms are equal in equilibrium.

Proposition 4 implies that across all markets with $K \geq 2$ platforms, consumer surplus is maximized in the limit economy, in which platforms choose addictiveness d^∞. As a result, consumer surplus is maximized in either the limit economy or monopoly. The next result formalizes this idea (see Appendix C for the proof). For any $\overline{A} > 0$ and $K \in \mathbb{N}$, let $CS_K(\overline{A})$ denote the consumer surplus in the equilibrium of $\mathcal{E}_K$, and let $CS_\infty(\overline{A})$ denote the one in the limit economy, i.e., $CS_\infty(\overline{A}) = \max_{A \in \left[0,\overline{A}\right]} u\left(A, d^\infty\right) - C(A)$.

Corollary 3. *There is an $A^* \in \mathbb{R}_{++}$ that has the following properties.*

1. *If $\overline{A} \leq A^*$, a monopoly maximizes consumer surplus—i.e., $\max_{K \in \mathbb{N} \cup \{\infty\}} CS_K(\overline{A}) = CS_1(\overline{A})$. Also, a duopoly minimizes consumer surplus.*

2. *If $\overline{A} \geq A^*$, the limit economy maximizes consumer surplus—i.e., $\max_{K \in \mathbb{N} \cup \{\infty\}} CS_K(\overline{A}) = CS_\infty(\overline{A})$.*

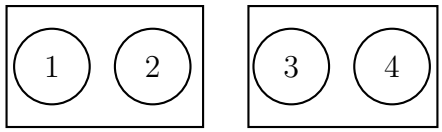
The intuition is as follows. When $\overline{A}$ is small, the consumer exhausts her attention $\overline{A}$ at low addictiveness. In this case, each platform increases its addictiveness to encourage the consumer to spend a greater fraction of $\overline{A}$ on the platform. Then, the monopolist, which has no business stealing incentive, chooses the lowest addictiveness and maximizes consumer surplus. When $\overline{A}$ is large, the monopolist prefers to increase addictiveness to encourage the consumer to spend a longer time on its service. The limit economy minimizes such an incentive and maximizes consumer surplus.

4.2 Platform Merger

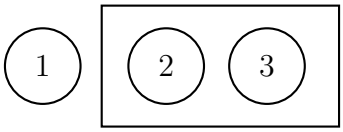
Another way to examine the effect of competition is to study the welfare impact of platform merger. We compare the original game to the *post-merger* game. It is characterized by a *market structure* $\mathcal{M}$, which is a partition of $\mathcal{K} := \{1, \ldots, K\}$. We write $\mathcal{M} = \{P_1, \ldots, P_M\}$, where each $P_m \subset \mathcal{K}$ is a merged platform that consists of platforms (or services) $k \in P_m$. The original game corresponds to $\mathcal{M} = \{\{k\}\}_{k \in \mathcal{K}}$. We write M for the set and the number of platforms in the post-merger game. Given any $\mathcal{M}$, the post-merger game works as follows. First, each platform $m \in M$ simultaneously chooses its addictiveness, d_m. The consumer observes $(d_m)_{m \in M}$, then chooses the set $M' \subset M$ of platforms to join and the allocation $(a_k)_{k \in \cup_{m \in M'} P_m}$ of attention, in order to maximize her payoff $\sum_{m \in M'} \sum_{k \in P_m} \frac{1}{K} u(K a_k, s d_m) - C(\sum_{m \in M'} \sum_{k \in P_m})$. The payoff of each platform m is $r \sum_{k \in P_m} a_k$ if $m \in M'$, and zero if $m \notin M'$.

A merger changes the game in two ways. First, each platform $m \in M$ chooses a single level of addictiveness for all services in P_m. Second, services operated by the same platform are tied—i.e., the consumer cannot join a nonempty strict subset of services in P_m.

We examine two types of mergers, which Figure 3 illustrates: Circles and rectangles are platforms before and after the merger, respectively.



(a) Symmetric merger



(b) All-but-one merger

Figure 3: Platform mergers

Definition 1. A *symmetric merger* refers to $\mathcal{M}$ such that $|P_m| = |P_\ell| \geq 2$ for all $m, \ell \in M$ and $M \geq 2$. An *all-but-one merger* refers to $\mathcal{M}$ such that $\mathcal{M} = \{\mathcal{K} \setminus \{k\}, \{k\}\}$ for some $k \in K$.

All-but-one merger is unique up to the identity of the non-merged platform. In contrast, there can be multiple symmetric mergers. The following result presents the welfare impacts of a merger (see Appendix C for the proof).

Proposition 5. *A symmetric merger increases the addictiveness of all of the K services. An all-but-one merger increases the addictiveness of $K - 1$ merged services and decreases that of the non-merged platform. In either case, a merger strictly decreases consumer surplus.*

To see the intuition, suppose two out of three firms merge to form a single platform M. After the merger if the consumer refuses to join platform M, she loses access to two services. Because the consumer faces a lower outside option, platform M can set higher addictiveness for its services than before the merger. The merger also encourages the non-merged platform to decrease its addictiveness; the consumer has a stronger incentive to stay with platform M, so the non-merged platform has to offer a higher service utility to ensure participation. On balance, the merger harms the consumer because her payoff is equal to the payoff from platform M alone.

Not all mergers harm the consumer. By Proposition 2, a merger to monopoly increases consumer surplus if she faces a tight attention constraint. In such a case, a merger eliminates the business stealing incentives, decreases the addictiveness of services, and increases consumer surplus. The observation contrasts with the idea that a merger to monopoly is more harmful than other

types of mergers.[11]

5 Price Competition and Attention Competition

We now examine how the business models of platforms affect the equilibrium addictiveness and welfare. We compare the model of competition for attention and a model of price competition, in which platforms earn revenue from monetary prices. As in Section 4, we describe the model of price competition by keeping the market size constant.

The game of price competition in market $\mathcal{E}_K$ is as follows. First, each platform $k \in K$ simultaneously chooses its addictiveness $d_k \geq 0$ and price $p_k \in \mathbb{R}$. The consumer observes $(d_k, p_k)_{k \in K}$, then chooses the set $\hat{K} \subset K$ of platforms to join and how much attention to allocate. The consumer has to pay price p_k to join platform k. Each platform $k \in \hat{K}$ receives a payoff of p_k, and any platform $k \notin \hat{K}$ obtains a payoff of zero. The consumer receives a payoff of $\sum_{k \in \hat{K}} \frac{1}{\hat{K}} \left[u(Ka_k, d_k) - p_k\right] - C\left(\sum_{k \in \hat{K}} a_k\right)$ if $\hat{K} \neq \emptyset$ and zero if $\hat{K} = \emptyset$ (recall that in $\mathcal{E}_K$, the service utility of each platform is $\frac{1}{K} u(Ka, d)$). In equilibrium, the consumer solves

$$\max_{\hat{K} \subset K, (a_k)_{k \in \hat{K}}} \sum_{k \in \hat{K}} \left[\frac{1}{K} u(Ka_k, d_k) - p_k\right] - C\left(\sum_{k \in \hat{K}} a_k\right) \tag{5}$$
$$\text{s.t.} \quad \sum_{k \in \hat{K}} a_k \leq \overline{A} \quad \text{and} \quad a_k \geq 0, \forall k \in \hat{K},$$

where the objective is zero if $\hat{K} = \emptyset$.

Under price competition, platforms can charge for their services, but they do not monetize attention. The model captures digital services not supported by advertising, such as Netflix and YouTube Premium.[12] The following result characterizes the equilibrium under price competition

[11] For example, Competition Policy Guidance of the Federal Trade Commission states that a unilateral anticompetitive effect of a merger is most obvious in the case of a merger to monopoly. See https://www.ftc.gov/tips-advice/competition-guidance/guide-antitrust-laws/mergers/competitive-effects (accessed on February 10, 2021). In our model, such a statement holds when platforms directly charge the consumer for their services.

[12] We do not consider the endogenous choice of business models, or a business model that offers both ad-supported and subscription plans. For recent studies on business models and price discrimination in two-sided markets by a monopoly platform, see, for instance, Gomes and Pavan (2016), Lin (2020), Carroni and Paolini (2020), and Jeon et al. (2021).

(see Appendix D for the proof).

Lemma 1. *The game of price competition has a unique equilibrium, in which all platforms choose zero addictiveness and set the same positive price that makes the consumer indifferent between joining K and $K-1$ platforms.*

Under price competition, the platforms' payoffs do not depend on attention. Thus each platform decreases addictiveness to increase service quality, and charges a higher price. In equilibrium all platforms set zero addictiveness, and the price p^* equals the incremental contribution of each platform to the consumer's total payoff.

The following result provides sufficient conditions under which the consumer is better off under attention competition (see Appendix D for the proof).

Proposition 6. *Consider the sequence of markets, $(\mathcal{E}_K)_{K\in\mathbb{N}}$, defined in Section 4. The consumer is strictly better off under attention competition than price competition in market $\mathcal{E}_K$ if any of the following holds:*

1. *The market is sufficiently competitive—i.e., K is greater than some cutoff $K^* \in \mathbb{N}$.*
2. *The supply of total attention is inelastic—i.e., $C(\cdot) \equiv 0$ and $\overline{A} < \infty$.*

Points 1 and 2 apply to distinct environments. For example, Point 1 holds for any $(C(\cdot), \overline{A})$, and Point 2 holds for any number of platforms. However, a similar intuition applies to both: Under attention competition, platforms set positive addictiveness, and thus the consumer faces higher marginal utilities of allocating attention. The consumer then faces a higher gain of refusing to join a platform and continuing to use other $K-1$ platforms. For example, under the inelastic supply of attention, the consumer can increase her attention to each platform $j \neq 1$ from $\frac{\overline{A}}{K}$ to $\frac{\overline{A}}{K-1}$ by not joining platform 1. The gain of doing so is higher when services are more addictive. To sum up, because the consumer faces a higher gain of not joining each platform under attention competition, platforms must provide high utilities to ensure consumer participation. As a result, consumer surplus is higher under attention competition.

The argument is more subtle when the supply of attention is elastic (i.e., Point 1). The consumer faces a steeper utility function under attention than price competition, but she evaluates these func-

tions at different levels of attention. In a sufficiently competitive market, higher marginal utilities ensure a higher consumer surplus under attention competition.

6 Digital Curfew

This section asks whether the consumer benefits from a digital curfew, which tightens the attention constraint (see Footnote 3 for real-world practices). To state the result, we prepare some notations. Recall that $(\mathcal{E}_K)_{K\in\mathbb{N}}$ is the sequence of markets, where $\mathcal{E}_K$ consists of K platforms, each of which provides service utility $\frac{1}{K}u(Ka, d)$. Let $\mathcal{E}_K(\overline{A})$ denote the market $\mathcal{E}_K$ in which the consumer faces attention constraint $\overline{A}$, and let $CS(\mathcal{E}_K(\overline{A}))$ denote the consumer's unique equilibrium payoff in market $\mathcal{E}_K(\overline{A})$. Finally, $A(0)$ is the consumer's optimal attention in a monopoly market $\mathcal{E}_1$ with zero addictiveness. See Appendix E for the proof of the following result.

Proposition 7. *A digital curfew is more effective in a less competitive market:*

1. *In a monopoly market, a digital curfew at $\overline{A} = A(0)$ attains the consumer-optimal outcome.*
2. *For any $K \geq 2$, some digital curfew benefits the consumer: $\exists \overline{A} < \infty$ s.t. $CS(\mathcal{E}_K(\overline{A})) > CS(\mathcal{E}_K(\infty))$. No digital curfew attains the consumer-optimal outcome.*
3. *The gain of digital curfew vanishes in a competitive market: $\lim_{K\to\infty} CS(\mathcal{E}_K(\overline{A}))$ is increasing in $\overline{A}$.*

Proposition 7 implies that under an optimally chosen digital curfew, the monopoly attains a higher consumer surplus than in any other market structure. To see the intuition, consider a digital curfew at $\overline{A} = A(0)$, which prevents the consumer from spending longer time on digital services than how much she would have spent if these services had zero addictiveness. Under monopoly, this digital curfew makes it optimal for the platform to set zero addictiveness, because the consumer cannot increase her attention beyond $\overline{A}$. However, a digital curfew does not eliminate business stealing incentives, because the consumer will allocate a greater fraction of her attention to more addictiveness platforms even if the total attention is fixed. As a result, a digital curfew can be more effective for a monopolist.

Remark 1 (Voluntary Reduction of A). In the above analysis, a digital curfew is exogenous to

the consumer. One question is whether consumers voluntarily adopt a digital curfew. To see this, suppose that there is a continuum of consumers, each of whom $i \in [0, 1]$ chooses the maximum amount of attention $A_i \in [0, A_{max}]$ she can spend on platforms ($A_{max} > 0$ is an exogenous cap on possible attention constraints). After consumers choose $(A_i)_{i\in[0,1]}$, the original game of attention competition is played.[13] In the unique equilibrium, all consumers choose the maximum attention A_{max}, because each consumer is atomless and her choice does not affect the behavior of platforms. Thus, consumers cannot voluntarily enforce a digital curfew, even though they could benefit from collectively reducing A_i's.

7 Extension: Naive Consumer

Thus far, we have assumed that the consumer observes the true addictiveness of the platforms. In practice, consumers may become overconfident about their future self-control or underestimate their lack of self-control; firms would design their contracts to exploit such traits (e.g., DellaVigna and Malmendier, 2004, 2006; Heidhues and Kőszegi, 2010). In this spirit, we now assume that the consumer systematically underestimates the addictiveness of platforms.[14]

We extend the model of Section 2 as follows (Appendix F provides details). In the first stage, all platforms simultaneously choose addictiveness, $(d_k)_{k\in K}$. In the second stage, the consumer decides which platforms to join, *based on the perceived addictiveness,* $(sd_k)_{k\in K}$. The parameter $s \in (0, 1]$ is exogenous and captures the degree of consumer sophistication. If (and only if) $s < 1$, the consumer falsely thinks that the addictiveness of each platform is lower than the true value. In the third stage, after joining platforms the consumer allocates attention to maximize her utility based on the true addictiveness. The consumer's attention allocation problem is the same as (2) except she has chosen the set of platforms to join based on the perceived addictiveness. Consumer surplus now refers to her payoff based on the true addictiveness.

Intuitively, a lower s renders it more attractive for a platform to increase its addictiveness, because the increase in d does not much affect the consumer's participation decision, but the plat-

[13] If platform k obtains attention a_k^i from each consumer i, then k's profit is $\int_{i\in[0,1]} a_k^i$.

[14] Our extension is different from a model in which the consumer does not observe addictiveness but correctly anticipates it in equilibrium. In this sense, our consumer naivete relates to "uninformed myopes" in Gabaix and Laibson (2006), whereby consumers do not recognize the full price of a product before making actual purchase decisions.

form with a high d can capture a greater fraction of attention. As a result, a lower s leads to higher addictiveness and lower consumer welfare.

Proposition 8. *Suppose $K \geq 2$. In a unique equilibrium, all platforms choose addictiveness $\frac{d^*}{s}$, where d^* is the equilibrium addictiveness under the sophisticated consumer in Proposition 3. A higher naivete (i.e., a lower s) leads to higher equilibrium addictiveness and a lower consumer surplus.*

We next turn to the comparison of business models. For any $s \in (0, 1]$, platforms choose zero addictiveness under price competition, because they prefer to decrease addictiveness and charge higher monetary prices. As a result, a lower s decreases consumer surplus under attention competition but not under price competition. This observations lead to the following result.

Proposition 9. *For any $K \geq 2$, there is an $s^* \in (0, 1]$ such that the following holds: The consumer is better off under price competition than attention competition if and only if $s \leq s^*$.*

It is ambiguous whether naive consumers benefit more from digital curfew than sophisticated consumers. For example, a sufficiently naive consumer, who has a small s, obtains a negative payoff in equilibrium. In such a case, a digital curfew at $\overline{A} = 0$ trivially increases consumer surplus. Appendix F presents the other possibility—i.e., some digital curfew can harm consumers only when they are naive. Indeed, the consumer's naivete can eliminate the beneficial impact of a digital curfew that reduces the equilibrium addictiveness. Naive consumers think that a digital curfew does not affect her payoff, because they believe that they will not spend much time on platforms. Such a wrong expectation reduces platforms' incentive to lower addictiveness to encourage participation.

8 Conclusion

In this paper, we study how firms compete for consumer attention by choosing the addictiveness of their services. We examine how competition affects the equilibrium addictiveness and consumer welfare through a trade-off associated with platforms' strategic choices of addictiveness: Ex ante, competition incentivizes each platform to lower its addictiveness because consumers face a higher

reservation utility with more platforms to choose. Ex post, platforms that face competitors have the attention stealing incentive. Consumer surplus is maximized by either a monopoly or a market with many small platforms, and possibly minimized by a duopoly. Platforms have incentives to lower their addictiveness when they compete on prices, but consumer can be better off when platforms monetize only attention. This result highlights a potential pitfall in the argument that the business model that monetize attention itself harms consumers. We also study the impact of a digital curfew, which restricts the consumer's choice, but incentivizes platforms to reduce their addictiveness. A digital curfew is more likely to be effective in a less competitive market. To our best knowledge, the paper is the first to theoretically study the interactions between competition and digital addiction, which have important policy implications.

We close this article by mentioning several directions for future research. First, it appears worth studying platforms' choices of business models and their implications on addictiveness and welfare. Second, allowing heterogeneous consumers and firms raises new issues. For example, a firm may offer both ad-supported and subscription plans to screen consumers who have different preferences. A regulator may prefer consumer- or content-specific digital curfews. Important questions also await on the empirical front. First, we model addictiveness as a parameter of a utility function, but it is important to identify which features of digital services satisfy our assumptions of addictiveness. Second, although we identified various factors—e.g., the market structure, business models, and consumer sophistication —that shape the addictiveness in equilibrium, it remains worthy to empirically examine the relative importance of each factor. Finally, little seems known about how consumers allocate their attention across multiple digital services and how their attention responds to various features of platforms. Building upon our work, we anticipate further studies on various intriguing questions on theoretical and empirical fronts.

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Appendix

A Proof of Proposition 1

Proof. Consumer surplus is maximized by $d_k = 0$ because service utilities decrease in addictiveness. To show $d^{TS} > 0$ for a large r, suppose all platforms choose $d = 0$. The consumer optimally joins all platforms. Denoting $TS(d) = CS(d) + rA(d)$, the envelope formula implies $TS'(0) = Ku_2\left(\frac{A(0)}{K}, 0\right) + rA'(0)$. Because $A'(0) > 0$, we have $TS'(0) > 0$ for a large r. For such an r, if all platforms choose a small but positive d, the consumer strictly increases her attention, and the total surplus strictly increases. □

B Proof of Proposition 3

To show the result, we first prove some lemmas.

Lemma 2. *Take any increasing, strictly concave, and differentiable function,* $u(\cdot) : \mathbb{R}_+ \to \mathbb{R}$. *Then,* $u(x) - xu'(x)$ *is strictly increasing in* x.

Proof. For any x and $y > x$, we have

$$\begin{aligned}
&\frac{u(y) - u(x)}{y - x} > u'(y) \\
\Rightarrow &u(y) - u(x) > u'(y)(y - x) \\
\Rightarrow &u(y) - u(x) > u'(y)y - u'(x)x \\
\Rightarrow &u(y) - yu'(y) > u(x) - xu'(x).
\end{aligned}$$

□

Lemma 3. *Consider the problem*

$$\max_{A\in[0,\overline{A}]} y\cdot u\left(\frac{A}{y},d\right)-C(A). \tag{A.1}$$

Let $A^*(y)$ and $V^*(y)$ denote the maximizer and the maximized value, respectively. Then, $A^*(y)$ is increasing in y, $\frac{A^*(y)}{y}$ is decreasing in y, $V^*(y)$ is strictly concave in y, and $\frac{\partial V^*}{\partial y}$ is decreasing in d.

Proof. Define $V(A,y):=y\cdot u\left(\frac{A}{y},d\right)-C(A)$. We have $\frac{\partial^2 V}{\partial A\partial y}=-\frac{A}{y^2}u_{11}\left(\frac{A}{y},d\right)>0$, and thus $A^*(y)$ is increasing in y. To show $\frac{A^*(y)}{y}$ is decreasing, we rewrite (A.1) as

$$\max_{a\in[0,\overline{A}/y]} y\cdot u\left(a,d\right)-C(ay). \tag{A.2}$$

The maximizer of (A.2) is $a^*(y):=\frac{A^*(y)}{y}$. If a^* is an interior solution (or in other words, when $A^*(y)<\overline{A}$), it satisfies the first order condition $u_1(a,d)-C'(ay)=0$, whose solution is decreasing in y. If y is so large that $A^*(y)=\overline{A}$, then for any such y, we have $a^*(y)=\frac{\overline{A}}{y}$, which is decreasing in y. Overall, $\frac{A^*(y)}{y}$ is decreasing in $y>0$.

We now show that $V^*(y)$ is concave. Let y^* denote the smallest value that satisfies $A^*(y)=\overline{A}$. First, we show $V^*(y)$ is concave on $[0,y^*)$. For any $y\in[0,y^*)$, $A^*(y)$ is an interior solution. The envelope theorem implies

$$\frac{dV^*}{dy}=u\left(\frac{A^*(y)}{y},d\right)-\frac{A^*(y)}{y}u_1\left(\frac{A^*(y)}{y},d\right).$$

This expression is decreasing in y, because $u(x,d)-xu'(x,d)$ is increasing in x (Lemma 2) and $\frac{A^*(y)}{y}$ is decreasing in y. Second, we show $V^*(y)$ is concave on $[y^*,\infty)$. After $A^*(y)$ hits $\overline{A}$, the maximized value is $V^*(y):=y\cdot u\left(\frac{\overline{A}}{y},d\right)-C(\overline{A})$. We have $\frac{dV^*}{dy}=u\left(\frac{\overline{A}}{y},d\right)-\frac{\overline{A}}{y}u_1\left(\frac{\overline{A}}{y},d\right)$. By the same argument as above, $\frac{dV^*}{dy}$ is decreasing in y. Finally, at $y=y^*$, the right and the left limits of $\frac{dV^*}{dy}$ coincides. Thus, $V^*(y)$ is globally concave.

Finally, $\frac{\partial^2 V^*}{\partial y\partial d}=u_2\left(\frac{A^*(y)}{y},d\right)+y\cdot u_{12}\left(\frac{A^*(y)}{y},d\right)\cdot\frac{\partial}{\partial y}\left(\frac{A^*(y)}{y}\right)<0$. The cross derivative $\frac{\partial^2 V^*}{\partial y\partial d}$ is well-defined (at least) for all $y\neq y^*$. Thus, $\frac{\partial V^*}{\partial y}=\int_0^d\frac{\partial^2 V^*}{\partial y\partial d}(y,t)dt+c$ (with some constant c) is decreasing in d. □

Lemma 4. *Fix any $d' \geq 0$, and consider the problem*

$$U(x, y, d) := \max_{A \in [0, \overline{A}], A_y \in [0, A]} x \cdot u\left(\frac{A - A_y}{x}, d\right) + y \cdot u\left(\frac{A_y}{y}, d'\right) - C(A). \tag{A.3}$$

Then, $U_2(x, y, d)$ is decreasing in d.

Proof. The envelope theorem implies $U_2(x, y, d) = u\left(\frac{A_y^*}{y}, d'\right) - \frac{A_y^*}{y} u_1\left(\frac{A_y^*}{y}, d'\right)$, where A_y^* is a part of the maximizer (A^*, A_y^*) of (A.3). Because the objective function in (A.3) is supermodular in $(A, -A_y^*, d)$, A_y^* is decreasing in d. Also, Lemma 2 implies $u(a, d') - a \cdot u_1(a, d')$ is increasing in a. Thus, $U_2(x, y, d)$ is decreasing in d. □

The following result shows that the consumer faces a decreasing incremental gain of joining platforms for any choices of addictiveness. We later use this lemma to establish the uniqueness of the equilibrium.

Lemma 5. *Take any $S, S' \subset K_{-1} := \{2, 3, \ldots, K\}$ such that $S' \subset S$. For any choice of addictiveness, the consumer's incremental gain of joining platform 1 is greater when she has already joined platforms S' than S. Formally, the following holds. Fix any $(d_1, \ldots, d_K) \in \mathbb{R}_+^K$. For any $y \in [0, 1]$ and $S \subset K_{-1}$, define*

$$V(y, S) := \max_{(a_k)_{k \in S \cup \{1\}}} \sum_{k \in S} u(a_k, d_k) + y \cdot u(a_1, d_1) - C\left(\sum_{k \in S \cup \{1\}} a_k\right) \tag{A.4}$$
$$\text{s.t.} \quad \sum_{k \in S \cup \{1\}} a_k \leq \overline{A} \quad \text{and} \quad a_k \geq 0, \forall k \in S \cup \{1\}.$$

Then for any $S', S \subset K_{-1}$ such that $S' \subsetneq S$,

$$\frac{\partial V}{\partial y}(y, S) \leq \frac{\partial V}{\partial y}(y, S'). \tag{A.5}$$

In particular, $V(1, S) - V(0, S) \leq V(1, S') - V(0, S')$. These inequalities are strict whenever the consumer allocates positive attention to every platform in S and S' upon solving (A.4).

Proof. Let $a_1(y, S)$ denote the optimal value of a_1 in (A.4). The envelope theorem implies

$$\frac{\partial V}{\partial y}(y, S) = u(a_1(y, S), d_1).$$

Thus, to show (A.5), we first show that $a_1(y, S) \leq a_1(y, S')$ for any S' and $S \supset S'$.

Suppose to the contrary that $a_1(y, S) > a_1(y, S')$. Note that in the problem (A.4), the marginal utilities from any two platforms in S are equal whenever the consumer allocates positive attention to them. Thus, for any $k \in S'$ such that $a_k(y, S') > 0$, we have $a_k(y, S) > a_k(y, S')$. This inequality leads to a contradiction if the attention constraint is binding under S', i.e., $\sum_{k \in S' \cup \{1\}} a_k(y, S') = \overline{A}$. Suppose the attention constraint is not binding under S'. Then for any $j \in S'$, we have

$$u_1(a_j(y, S), d_j) < u_1(a_j(y, S'), d_j) = C'\left(\sum_{k \in S' \cup \{1\}} a_k(y, S')\right) < C'\left(\sum_{k \in S \cup \{1\}} a_k(y, S)\right).$$

These inequalities imply that $(a_k(y, S))_{k \in S \cup \{1\}}$ does not solve (A.4), because the marginal cost exceeds the marginal utility from any platform $j \in S$. This is a contradiction. Thus, we obtain $a_1(y, S) \leq a_1(y, S')$. Integrating both sides of (A.5) from $y = 0$ to $y = 1$, we have $V(1, S) - V(0, S) \leq V(1, S') - V(0, S')$.

Now, suppose the consumer allocates positive attention to every platform in S and S' upon solving (A.4). Then, we can use the same argument to show that $a_1(y, S) \geq a_1(y, S')$ leads to a contradiction. Thus, we have $a_1(y, S) < a_1(y, S')$ and obtain (A.5) as a strict inequality. □

We are now ready to prove Proposition 3.

Proof of Proposition 3. STEP 1:*There is a unique d^* that satisfies (3).* To show this, define

$$f(K, d) := K \cdot u\left(\frac{A_K(d)}{K}, d\right) - C(A_K(d)) - \left[(K-1) \cdot u\left(\frac{A_{K-1}(d)}{K-1}, d\right) - C(A_{K-1}(d))\right].$$

The function $f(K, d)$ is the difference between payoffs when the consumer uses K platforms and when she uses $K - 1$ platforms, given optimally allocating attention. Hereafter, we use the notation $V^*(y, d)$ for $V^*(y)$ of Lemma 3 to make the dependence of $V^*(y)$ on d explicit. We can write $f(K, d) = V^*(K, d) - V^*(K - 1, d)$. Lemma 3 implies $V_1^*(y, d)$ is decreasing in d. Thus, $f(K, d) = \int_{K-1}^{K} V_1^*(y, d)dy$ is decreasing in d. Also, $f(K, 0) > 0$, and $f(K, d) < 0$ for a sufficiently large d. Thus, there is a unique d^* that solves (3) (i.e., $f(K, d^*) = 0$).

STEP 2: *There is an equilibrium in which each platform sets d^*.* Suppose all platforms choose d^*. First, we show that the consumer prefers to joins all the platforms. Given $d_k = d^*$ for all k,

the consumer's payoff from joining $J \leq K$ platforms is $V^*(J, d)$, which is strictly concave in J (Lemma 3). Also, we have $V^*(K, d^*) = V^*(K-1, d^*)$ by construction. As a result, $V^*(J, d^*)$ is strictly increasing in $y \leq K-1$. Thus, the consumer prefers to join all platforms.

Second, we show that no platform has a profitable deviation. Without loss of generality, we consider the incentive of platform 1. If it increases d_1, the consumer joins only platforms $2, \ldots, K$ to achieve the same payoff as without platform 1's deviation. Suppose platform 1 decreases d_1 from d^* to d. The consumer joins platform 1. If she additionally joins other y platforms, her payoff becomes $U(1, y, d)$ according to the notation of Lemma 4 (with $d' = d^*$). When $d_1 = d^* > d$, $U(1, y, d^*)$ is maximized at $y = K-1$ and $y = K$. Because $U_{23}(1, y, d) < 0$ by Lemma 4, the consumer's marginal gain from joining platforms increases after platform 1's deviation. As a result, $U(1, y, d)$ is uniquely maximized at $y = K$ across all $y \in \{1, \ldots, K\}$. However, the consumer will then allocate a smaller amount of attention to platform 1 compared to without deviation, because platform 1 now offers a lower marginal utility. Thus, platform 1 does not strictly benefit from the deviation to $d < d^*$.

STEP 3: *The above equilibrium is a unique one.* To show this, take any pure-strategy subgame perfect equilibrium. Because any platform can set $d_k = 0$ to ensure participation, the consumer joins all platforms in equilibrium. First, we show all platforms choose the same addictiveness. Suppose to the contrary that there is an equilibrium in which platforms choose $(d_k^*)_{k \in K}$ such that (without loss of generality) $d_2 = \max_k d_k^* > \min_k d_k^* = d_1$. Suppose now that platform 1 deviates and increases its addictiveness to $d_1 = d_1^* + \varepsilon < d_2^*$. We show that the consumer joins platform 1. Suppose to the contrary that she does not join platform 1. If she joins platform 2, it is a contradiction, because she could obtain a strictly higher payoff by replacing platform 2 with 1. Thus, the consumer does not join platform 2. Lemma 5 implies that the consumer's incremental gain of joining platform 1 is strictly higher when (i) she has joined some set of platforms $K' \subset \{1, \ldots, K\} \setminus \{1, 2\}$ than when (ii) she has joined platforms $2, \ldots, K$. Because the consumer weakly prefers to join platform 1 under (ii) at $d_1 = d_1^*$, she strictly prefers to join it under (i) at $d_1 = d_1^*$. As a result, the consumer strictly prefers to join platform 1 under (i) at $d_1 = d_1^* + \varepsilon$ for a small $\varepsilon > 0$. To sum up, if platform 1 deviates to $d_1^* + \varepsilon$ with small $\varepsilon > 0$, the consumer joins platform 1 and allocates strictly greater attention to it. This contradicts $(d_k^*)_{k \in K}$ being an

equilibrium.

Therefore, in any equilibrium, d_k^* is the same for all $k \in K$. Finally, take any equilibrium in which all platforms choose the same addictiveness. If the consumer's indifference condition (3) fails, then one of the following holds: (i) the left-hand side is strictly greater, in which case a platform prefers to deviate and increase its addictiveness, or (ii) the right-hand side is greater, in which case the consumer does not join at least one platform, which is a contradiction. □

C Proofs for Section 4: The Impact of Competition

Proof of Proposition 4. The first part of the result follows from Proposition 3. To show the second part, fix any $K \geq 2$ and let d^* denote the equilibrium addictiveness. Each platform provides a service utility of $\frac{1}{K}u\left(Ka, d\right)$. Let $A_x(d)$ denote the unique maximizer of the problem

$$V^*(x, d) := \max_{A \in [0, \overline{A}]} x \cdot u\left(\frac{A}{x}, d\right) - C(A). \tag{A.6}$$

If the consumer joins K platforms with addictiveness d, she allocates total attention $A_1(d)$. If the consumers joins $K-1$ platforms, she allocates total attention $A_{\frac{K-1}{K}}(d)$. In equilibrium, the consumer is indifferent between joining K and $K-1$ platforms. Thus, we have

$$u(A_1(d^*), d^*) - C(A_1(d^*)) = \frac{K-1}{K} u\left(\frac{K}{K-1} A_{\frac{K-1}{K}}(d^*), d^*\right) - C\left(A_{\frac{K-1}{K}}(d^*)\right). \tag{A.7}$$

Suppose to the contrary that for some K, the equilibrium addictiveness weakly increases from d^* to d^{**} as we move from K platforms to $K+1$ platforms. Equation (A.7) implies that $V^*(1, d^*) = V^*(\frac{K-1}{K}, d^*)$. Because $V^*(x, d)$ is strictly concave in x, this equation implies

$$u(A_1(d^*), d^*) - C(A_1(d^*)) < \frac{K}{K+1} u\left(\frac{K+1}{K} A_{\frac{K}{K+1}}(d^*), d^*\right) - C\left(A_{\frac{K}{K+1}}(d^*)\right). \tag{A.8}$$

If d^* increases, the left-hand side decreases more than the right-hand side. To see this, first, note that

$$\frac{\partial}{\partial d}V^*(x,d) = xu_2\left(\frac{A_x(d)}{x}, d\right), \tag{A.9}$$

$$\frac{\partial^2}{\partial x \partial d}V^*(x,d) = u_2\left(\frac{A_x(d)}{x}, d\right) + x \cdot \frac{\partial}{\partial x}\left(\frac{A_x(d)}{x}\right) \cdot u_{12}\left(\frac{A_x(d)}{x}, d\right) < 0. \tag{A.10}$$

The inequality uses $\frac{\partial}{\partial x}\left(\frac{A_x(d)}{x}\right) < 0$, which follows from Lemma 3. Now, we can write (A.8) as $V^*(1,d^*) < V^*\left(\frac{K}{K+1}, d^*\right)$, or equivalently, $\int_{\frac{K}{K+1}}^{1} \frac{\partial}{\partial x}V^*(x,d^*)dx < 0$. Because $\frac{\partial}{\partial x}V^*(x,d)$ is decreasing in d, we have $\int_{\frac{K}{K+1}}^{1} \frac{\partial}{\partial x}V^*(x,d^{**})dx < 0$, or equivalently, $V^*(1,d^{**}) < V^*\left(\frac{K}{K+1}, d^{**}\right)$. As a result, we have

$$u(A_1(d^{**}), d^{**}) - C(A_1(d^{**})) < \frac{K}{K+1}u\left(\frac{K+1}{K}A_{\frac{K}{K+1}}(d^{**}), d^{**}\right) - C\left(A_{\frac{K}{K+1}}(d^{**})\right),$$

which contradicts that the consumer joins all platforms in equilibrium even when there are $K+1$ platforms.

To show the last part, we write (A.7) as

$$\frac{u(A_1(d^*), d^*) - C(A_1(d^*)) - \left[xu\left(\frac{1}{x}A_x(d^*), d^*\right) - C\left(A_x(d^*)\right)\right]}{1-x} = 0, \quad \forall x \in \left\{\frac{K}{K+1}\right\}_{K\in\mathbb{N}} \tag{A.11}$$

By taking $x \to 1$ along the sequence $\left(\frac{K}{K+1}\right)_{K\in\mathbb{N}}$ and applying the envelope theorem, we obtain (4).

Finally, we show $d^\infty > 0$. If $d^\infty = 0$, we have $u(A,0) - Au_1(A,0) = 0$ for $A = A(0)$, which implies $\frac{u(A,0)-u(0,0)}{A-0} = u_1(A,0)$. This is a contradiction, because $u(x,0)$ is strictly concave and $A > 0$. □

Proof of Corollary 3. Excluding monopoly, we can use Proposition 4 to show that $\max_{K\geq 2} CS_K(\overline{A}) = CS_\infty(\overline{A})$ and $\min_{K\geq 2} CS_K(\overline{A}) = CS_2(\overline{A})$. Thus, it suffices to show that (i) the consumer is strictly better off under monopoly than in the limit economy for a small $\overline{A}$, (ii) the consumer is strictly better off in the limit economy than under monopoly for a large $\overline{A}$, and (iii) consumer surpluses in the limit economy are increasing in $\overline{A}$, but they are decreasing in $\overline{A}$.

Points (i) and (ii) follow from Proposition 2, because d^∞ is positive and gives the consumer a

positive payoff. As to (iii), in the limit economy, the consumer's payoff is $A_1(d^\infty)C'(A_1(d^\infty)) - C(A_1(d^\infty))$, because $u(A_1(d^\infty)) = A_1(d^\infty)C'(d^\infty)$. If $\overline{A}$ increases, d^∞ and $A_1(d^\infty)$ increase. Because $xC'(x) - C(x)$ is increasing in x, the consumer surplus increases. Under monopoly, Proposition 2 implies that the consumer obtains the first-best payoff for $\overline{A} \leq A(0)$, and a payoff of zero for $\overline{A} \geq A(d^0)$. On $[A(0), A(d^0)]$, the monopolist chooses d^1, which is the lowest addictiveness that makes it optimal for the consumer to choose $\overline{A}$. Because $\overline{A}$ globally maximizes the consumer's payoff given d^1, we have $u_1(\overline{A}, d^1) - C'(\overline{A}) = 0$. Because the left-hand side is strictly decreasing in $\overline{A}$ and strictly increasing in d^1, it follows that d^1 increases in $\overline{A}$. Now, for $\overline{A} \in [A(0), A(d^0)]$, the consumer's payoff is $u(\overline{A}, d^1) - C(\overline{A})$. Differentiating this expression in $\overline{A}$ and using the first-order condition, the change in consumer surplus with $\overline{A}$ is equal to $u_2(\overline{A}, d^1) \cdot \frac{d}{d\overline{A}} d^1 < 0$. As a result, the consumer surplus under monopoly is strictly decreasing in $\overline{A} \geq A(0)$. We can define A^* as the unique value at which the consumer is indifferent between monopoly and the limit economy. □

Proof for Proposition 5. The case of a symmetric merger follows Proposition 4: A symmetric merger that changes the number of platforms from K to L is equivalent to the change from $\mathcal{E}_K$ to $\mathcal{E}_L$.

We consider an all-but-one merger. To simplify notation, we use the original service utility function $u(a, d)$ instead of the normalized one. Suppose platforms $2, \ldots, K$ merge and become platform M. Let d_1 and d_M denote the addictiveness of platforms 1 and M. We denote their equilibrium values as d_1^* and d_M^*. In equilibrium the consumer joins all platforms, because any platform can choose $d_k = 0$ to obtain a positive amount of attention. Let $a_1(d_1, d_M)$ and $A_M(d_1, d_M)$ denote the attention allocated to platform 1 and M, respectively, when platforms 1 and M choose (d_1, d_M) and the consumer joins both in the post-merger market. Let $A_k(d)$ denote the total attention allocated when the consumer joins k platforms with addictiveness d in the pre-merger market. In equilibrium, we have

$$u\left(a_1(d_1^*, d_M^*), d_1^*\right) + (K-1)u\left(\frac{A_M(d_1^*, d_M^*)}{K-1}, d_M^*\right) - C\left(a_1(d_1^*, d_M^*) + A_M(d_1^*, d_M^*)\right) \quad \text{(A.12)}$$

$$= u\left(A_1(d_1^*), d_1^*\right) - C\left(A_1(d_1^*)\right) \quad \text{(A.13)}$$

$$= (K-1)u\left(\frac{A_{K-1}(d_M^*)}{K-1}, d_M^*\right) - C\left(A_{K-1}(d_M^*)\right). \quad \text{(A.14)}$$

The expression (A.12) is the consumer's payoff of joining platforms 1 and M: Platform M consists of $K-1$ symmetric services with decreasing marginal utilities, so the consumer allocates $\frac{A_M(d_1^*,d_M^*)}{K-1}$ to each of the $K-1$ services. The expressions (A.13) and (A.14) are the consumer's payoffs of joining only platform 1 and M, respectively. If any of the equalities fails, some platform will have a profitable deviation.

First, we show that the merger of $K-1$ platforms increases the addictiveness of the merged services and decreases that of the non-merged platform. Let d_0 denote the equilibrium addictiveness before the merger. First, we show $d_M^* > d_0$. Suppose to the contrary that $d_M^* \leq d_0$. Lemma 3 implies that when all platforms set the same addictiveness in the pre-merger market, the consumer's optimal payoff (i.e., $V^*(y)$ in the lemma) is strictly concave in the number of platforms she joins. Also, given d_0 the consumer is indifferent between joining K and $K-1$ platforms. As a result, the consumer strictly prefers joining K platforms to a single platform, i.e.,

$$\begin{aligned}
&u\left(a_1(d_0,d_0),d_0\right)+(K-1)u\left(\frac{A_M(d_0,d_0)}{K-1},d_0\right)-C\left(a_1(d_0,d_0)+A_M(d_0,d_0)\right)\\
>&u\left(A_1(d_0),d_0\right)-C\left(A_1(d_0)\right).
\end{aligned}$$

Using $d_M^* \leq d_0$, we have

$$\begin{aligned}
f(d_0) :=& u\left(a_1(d_0,d_M^*),d_0\right)+(K-1)u\left(\frac{A_M(d_0,d_M^*)}{K-1},d_M^*\right)-C\left(a_1(d_0,d_M^*)+A_M(d_0,d_M^*)\right)\\
&-\left[u\left(A_1(d_0),d_0\right)-C\left(A_1(d_0)\right)\right]>0.
\end{aligned}$$

Because (A.12) equals (A.13), we need $f(d_1^*)=0$. Because $u_{12}>0$ and $a_1(d,d_M)<A_1(d)$, the envelope theorem implies

$$f'(d)=u_2\left(a_1(d,d_M),d\right)-u_2\left(A_1(d),d\right)<0.$$

As a result, we need $d_1^* > d_0$ to satisfy $f(d_1^*)=0$. However this is a contradiction. To see this, suppose $d_1^* > d_0$ and $d_M^* = d_0$. The consumer will join only platform M because it consists of $K-1$ services and the consumer is indifferent between joining K services and $K-1$ services when they all choose $d_0 < d_1^*$. If $d_M^* < d_0$, platform M can profitably deviate to d_0, because the

consumer will then join platform M alone. Therefore we obtain $d_M^* > d_0$.

Next, we show platform 1 reduces its addictiveness after the merger, i.e., $d_1^* < d_0$. If all services have d_0, the consumer is indifferent between joining K and $K-1$ platforms:

$$\begin{aligned}
&u\left(a_1(d_0,d_0),d_0\right)+(K-1)u\left(\frac{A_M(d_0,d_0)}{K-1},d_0\right)-C\left(a_1(d_0,d_0)+A_M(d_0,d_0)\right)\\
=&(K-1)u\left(\frac{A_{K-1}(d_0)}{K-1},d_0\right)-C\left(A_{K-1}(d_0)\right).
\end{aligned}$$

Using $d_M^* > d_0$ and $u_{12} > 0$, we obtain

$$\begin{aligned}
&u\left(a_1(d_0,d_M^*),d_0\right)+(K-1)u\left(\frac{A_M(d_0,d_M^*)}{K-1},d_M^*\right)-C\left(a_1(d_0,d_M^*)+A_M(d_0,d_M^*)\right) \quad \text{(A.15)}\\
<&(K-1)u\left(\frac{A_{K-1}(d_M^*)}{K-1},d_M^*\right)-C\left(A_{K-1}(d_M^*)\right).
\end{aligned}$$

If we replace d_0 in the left-hand side of (A.15) with d_1^*, it is equal to the right-hand side of (A.15), because (A.12) equals (A.14). Therefore, we have $d_1^* < d_0$.

Consumer surplus in the post-merger game is $(K-1)u\left(\frac{A_{K-1}(d_M^*)}{K-1},d_M^*\right)-C\left(A_{K-1}(d_M^*)\right)$, and the one in the pre-merger game is $(K-1)u\left(\frac{A_{K-1}(d_0)}{K-1},d_0\right)-C\left(A_{K-1}(d_0)\right)$. Because $d_M^* > d_0$, the merger decreases consumer surplus.

Finally, we show that there is a pure-strategy subgame perfect equilibrium in the post-merger game. First, let $\bar{d}$ be the unique value that satisfies $\max_{A\in[0,\overline{A}]} u\left(A,\bar{d}\right)-C(A)=0$. Consider the equation

$$\begin{aligned}
&u\left(a_1(d_1,d_M),d_1\right)+(K-1)u\left(\frac{A_M(d_1,d_M)}{K-1},d_M\right)-C\left(a_1(d_1,d_M)+A_M(d_1,d_M)\right) \quad \text{(A.16)}\\
=&\,u\left(A_1(d_1),d_1\right)-C\left(A_1(d_1)\right).
\end{aligned}$$

Fix $d_1 \in [0,\bar{d}]$. If $d_M = 0$, the left-hand side is weakly greater. As $d_M \to \infty$, the left-hand side goes to $-\infty$. Also, the left-hand side is continuous and strictly decreasing in d_M. As a result, there is a unique d_M that satisfies the above equation. Let $d_M(d_1)$ denote such a d_M. Note that $d_M(d_1)$

is continuous. To show $d_M(d_1)$ is decreasing, define

$$g(d_1, d_M) := u\left(A_1(d_1), d_1\right) - C\left(A_1(d_1)\right) \\ - \left[u\left(a_1(d_1, d_M), d_1\right) + (K-1)u\left(\frac{A_M(d_1, d_M)}{K-1}, d_M\right) - C\left(a_1(d_1, d_M) + A_M(d_1, d_M)\right)\right].$$

Note that $d_M(d_1)$ satisfies $g(d_1, d_M(d_1)) = 0$. By the envelope theorem,

$$g_1(d_1, d_M) = u_2\left(A_1(d_1), d_1\right) - u_2\left(a_1(d_1, d_M), d_1\right) \geq 0,$$

because $u_{12}(a, d) > 0$ and $A_1(d_1) > a_1(d_1, d_M)$. Because $g_2(d_1, d_M) > 0$, to satisfy equation $g(d_1, d_M) = 0$, d_M must decrease whenever d_1 increases. As a result, $d_M(d_1)$ is weakly decreasing.

Next, for each $d_1 \in [0, \bar{d}_1]$, let $\hat{d}_M(d_1)$ solve

$$(K-1)u\left(\frac{A_{K-1}(d_M)}{K-1}, d_M\right) - C\left(A_{K-1}(d_M)\right) = u\left(A_1(d_1), d_1\right) - C\left(A_1(d_1)\right). \tag{A.17}$$

By the similar argument as above, we can show that $\hat{d}_M(d_1)$ is unique, continuous, and strictly increasing. At $d_1 = 0$, the left-hand side of (A.16) is weakly greater than that of (A.17). Thus, $d_M(0) \geq \hat{d}_M(0)$. At $d_1 = \bar{d}$, the left-hand side of (A.16) is weakly smaller than that of (A.17), because

$$\begin{aligned} & u\left(a_1(d_1, d_M), d_1\right) + (K-1)u\left(\frac{A_M(d_1, d_M)}{K-1}, d_M\right) - C\left(a_1(d_1, d_M) + A_M(d_1, d_M)\right) \\ & \leq (K-1)u\left(\frac{A_M(d_1, d_M)}{K-1}, d_M\right) - C\left(A_M(d_1, d_M)\right) \\ & \leq (K-1)u\left(\frac{A_{k-1}(d_M)}{K-1}, d_M\right) - C\left(A_{K-1}(d_M)\right). \end{aligned}$$

Here, the first inequality holds because at $d_1 = \bar{d}$, the consumer's payoff decreases by joining platform 1. Thus, $d_M(\bar{d}) \leq \hat{d}_M(\bar{d})$. Because $d_M(d_1)$ is weakly decreasing and $\hat{d}_M(d_1)$ is strictly increasing, they have a unique crossing point, which corresponds to an equilibrium. □

D Proofs for Section 5: Price Competition and Attention Competition

Proof of Lemma 1. Throughout the proof, we fix K and use the notations and the results in Lemma 3 with $u(a,d)$ replaced by $\hat{u}(a,d) := \frac{1}{K}u(Ka,d)$. Define

$$p^* := K\hat{u}\left(\frac{A_K^*(0)}{K},0\right) - C(A_K^*(0)) - \left[(K-1)\hat{u}\left(\frac{A_{K-1}^*(0)}{K-1},0\right) - C(A_{K-1}^*(0))\right].$$

Then, we show that there is an equilibrium in which each platform k sets $d_k = 0$ and $p_k = p^*$.

Suppose each platform k sets $(d_k, p_k) = (0, p^*)$. The consumer chooses the number K' of platforms to join to maximize $V(K')$, where

$$V(K') := \max_{A\in[0,\overline{A}]} K'\hat{u}\left(\frac{A}{K'},0\right) - C(A) - K'p^*$$

Lemma 3 implies that $V(K')$ is concave on $[0,K]$. Because p^* makes the consumer indifferent between joining K and $K-1$ platforms, it is optimal for her to join all platforms.

Suppose platform k deviates and chooses (d_k', p_k'). If $d_k' > 0$, platform k has to set $p_k' < p^*$; otherwise, the consumer strictly prefers to join only platforms $2,\ldots,K$. In this case the deviation reduces k's payoff. Conditional on $d_k' = 0$, p^* is the maximum price that platform k can charge, because the consumer is indifferent between joining $K-1$ and K platforms at price p^*. Thus, platform k has no profitable deviation.

The above equilibrium is unique. To show this, take any equilibrium, and suppose each platform k chooses (d_k^*, p_k^*). First, we show that the consumer joins all platforms in equilibrium. Fix $\hat{k} \in K$, and suppose platform $\hat{k}$ sets $(d_{\hat{k}}, p_{\hat{k}}) = (0,0)$, which may or may not be a deviation. Let K_0 denote the set of platforms the consumer joins, following $(d_{\hat{k}}, p_{\hat{k}}) = (0,0)$. Take any $K' \subset K$ such that $\hat{k} \notin K'$. First, if $d_j^* > 0$ for some $j \in K'$, then the consumer strictly prefers joining $(K' \setminus \{j\}) \cup \left\{\hat{k}\right\}$ to joining K'. Second, if $d_j^* = 0$ for all $j \in K'$ or $K' = \emptyset$, then the consumer strictly prefers $K' \cup \left\{\hat{k}\right\}$ to K'. Thus, for any set K' of platforms such that $\hat{k} \notin K'$, we can find some set S of platforms such that $\hat{k} \in S$ and the consumer strictly prefers S to K'. As a result, for a sufficiently small $p_{\hat{k}} > 0$ and $d_{\hat{k}} = 0$, the consumer still joins platform $\hat{k}$. This argument implies that any platform earns a positive profit in any equilibrium. Therefore, the consumer joins all platforms.

Second, we show all platforms set zero addictiveness in any equilibrium. Suppose to the contrary that $d_k^* > 0$ for some k. Suppose platform k deviates and chooses $(d_k, p_k) = (0, p_k^*)$. Before the deviation, the consumer weakly prefers joining all platforms to joining any set K' of platforms that does not contain k. Thus, after the deviation to $(0, p_k^*)$, the consumer strictly prefers to joining platform k. As a result, platform k can slightly increase its price while retaining the consumer. This is a contradiction.

We have shown that in any equilibrium, the consumer joins all platforms, which set zero addictiveness. The price of each platform makes the consumer indifferent between joining and not joining the platform; otherwise, the platform can deviate by slightly increasing its price. Therefore, $(d_k^*, p_k^*) = (0, p^*)$ is a unique equilibrium. □

Proof of Proposition 6. First, we show Point 1. Under price competition, all platforms choose zero addictiveness. To simplify notation, we write $u(a)$ instead of $u(a, 0)$, and A_x instead of $A_x(0)$. In equilibrium, the consumer is indifferent between joining K and $K-1$ platforms that choose zero addictiveness. Thus, we have

$$u(A_1) - C(A_1) - Kp^* = \frac{K-1}{K} u\left(\frac{K}{K-1} A_{\frac{K-1}{K}}\right) - C\left(A_{\frac{K-1}{K}}\right) - (K-1)p^*. \tag{A.18}$$

The equation implies

$$Kp^* = K(1-x) \cdot \frac{u(A_1) - C(A_1) - \left[xu\left(\frac{A_x}{x}\right) - C\left(A_x\right)\right]}{1-x} \tag{A.19}$$

for any $x \in \left\{\frac{K-1}{K}\right\}_{K \in \mathbb{N}}$. Now, define $f(x) := xu\left(\frac{A_x}{x}\right) - C\left(A_x\right)$. Since $K(1-x) = 1$ for any $x \in \left\{\frac{K-1}{K}\right\}_{K \in \mathbb{N}}$, the right-hand side of (A.19), as $K \to \infty$, converges to $f'(1)$. Corollary 4 of Milgrom and Segal (2002) implies $f'(1) = u(A_1) - A_1 u'\left(A_1\right)$. Thus, by taking $K \to \infty$, we obtain $\lim_{K \to \infty} Kp^* = u(A_1) - A_1 u'\left(A_1\right)$.

Thus, the consumer's payoff converges to

$$u(A_1) - C(A_1) - [u(A_1) - A_1 u'(A_1)] = A_1 u'(A_1) - C(A_1).$$

In the limit $K \to \infty$, the consumer's payoffs under attention competition and price competition

are $A_1(d^*)u_1(A_1(d^*), d^*) - C(A_1(d^*))$ and $A_1(0)u'(A_1(0), 0) - C(A_1(0))$, respectively.

To show $A_1(d^*)u_1(A_1(d^*), d^*) - C(A_1(d^*)) > A_1(0)u'(A_1(0), 0) - C(A_1(0))$, we consider three cases. Note that we always have $A_1(0) \leq A_1(d^*)$. First, suppose $A_1(d^*) < \overline{A}$. Then by the first-order conditions, these payoffs are respectively equal to $A_1(d^*)C'(A_1(d^*)) - C(A_1(d^*))$ and $A_1(0)C'(A_1(0)) - C(A_1(0))$. The function $xC'(x) - C(x)$ is increasing because its first derivative is $xC''(x) > 0$. As a result,

$$A_1(d^*)C'(A_1(d^*)) - C(A_1(d^*)) > A_1(0)C'(A_1(0)) - C(A_1(0)).$$

Second, suppose $A_1(0) = A_1(d^*) = \overline{A}$. Then, the consumer's payoffs under attention competition and price competition are $\overline{A}u_1(\overline{A}, d^*) - C(\overline{A})$ and $\overline{A}u_1(\overline{A}, 0) - C(\overline{A})$, respectively. The former is strictly greater than the latter as $u_{12} > 0$.

Third, suppose $A_1(0) < A_1(d^*) = \overline{A}$. Then, the consumer's payoffs under attention competition is $\overline{A}u_1(\overline{A}, d^*) - C(\overline{A}) \geq \overline{A}C'(\overline{A}) - C(\overline{A}) > A_1(0)C'(A_1(0)) - C(A_1(0))$. Thus, the consumer is strictly better off under attention competition in the limit.

Second, we show Point 2. Suppose the supply of attention is inelastic at $\overline{A}$. Under monopoly, the consumer's payoffs under attention competition and price competition are $u(\overline{A}, 0)$ and 0, respectively. Indeed, the monopolist chooses $d = 0$ under attention competition, but charges a price of $u(\overline{A}, 0)$ under price competition. Thus, the consumer is strictly better of under attention competition. Next, consider market $\mathcal{E}_K$ for any $K \geq 2$, and let d^* denote the equilibrium addictiveness. We have

$$\begin{aligned}
& u\left(\overline{A}, 0\right) - K\left[u\left(\overline{A}, 0\right) - \frac{K-1}{K}u\left(\frac{K\overline{A}}{K-1}, 0\right)\right] && \text{(A.20)} \\
&= (K-1)\left[u\left(\frac{K\overline{A}}{K-1}, 0\right) - u\left(\overline{A}, 0\right)\right] \\
&< (K-1)\left[u\left(\frac{K\overline{A}}{K-1}, d^*\right) - u\left(\overline{A}, d^*\right)\right] \\
&= u\left(\overline{A}, d^*\right). && \text{(A.21)}
\end{aligned}$$

The inequality holds because $u_{12} > 0$. The last equality follows from the equilibrium condition

that the consumer is indifference between joining K and $K-1$ platforms in $\mathcal{E}_K$:

$$u\left(\overline{A}, d^*\right) = \frac{K-1}{K} u\left(\frac{K\overline{A}}{K-1}, d^*\right).$$

The first (A.20) and the last (A.21) expressions are the consumer's payoffs under price and attention competition, respectively. Therefore, she is strictly better off under attention competition. □

E Proof of Proposition 7: The Impact of Digital Curfew

Proof. First, we show Point 1. If $\overline{A} = A_1(0)$, the consumer chooses the maximum possible attention at zero addictiveness. Thus, the monopolist chooses $d = 0$. This outcome maximized the consumer's payoff across all outcomes, because she obtains $\max_{A \geq 0} u(A, 0) - C(A)$.

Second, we show Point 2. Let $d^*(A)$ denote the equilibrium addictiveness in the game where the consumer has a total attention of at most A. If $\overline{A} \geq A_K(d^*)$, the equilibrium addictiveness is the same as $\overline{A} = \infty$, i.e., the original game in which there is no cap on A. Note that the equilibrium addictiveness satisfies

$$K \cdot u\left(\frac{A_K(d^*)}{K}, d^*\right) - C\left(A_K(d^*)\right) = (K-1) \cdot u\left(\frac{A_{K-1}(d^*)}{K-1}, d^*\right) - C\left(A_{K-1}(d^*)\right). \quad \text{(A.22)}$$

If we cap the maximum attention at $X < A_K(d^*)$, we have

$$K \cdot u\left(\frac{X}{K}, d^*\right) - C\left(X\right) < (K-1) \cdot u\left(\frac{A^X_{K-1}(d^*)}{K-1}, d^*\right) - C\left(A^X_{K-1}(d^*)\right). \quad \text{(A.23)}$$

Generally, if the consumer joins y platforms with addictiveness d at cap X, her optimal payoff is $U(y, d) := \max_{A \in [0,X]} yu\left(\frac{A}{y}, d\right) - C(A)$. The envelope formula (Milgrom and Segal 2002) implies $U_2(y, d) = yu_2\left(\frac{A(y,d)}{y}, d\right)$. Now, $u_2\left(x, d\right)$ is negative and increasing in x. Also, $\frac{A(y,d)}{y}$ is decreasing in y. Thus, $U_2(y, d) = yu_2\left(\frac{A(y,d)}{y}, d\right)$ is decreasing in y.

The above observation implies that if platforms increased addictiveness after a cap of X, the consumer continues to join at most $K-1$ platforms, which contradicts the equilibrium condition. Thus, after a curfew, the platforms set a strictly lower addictiveness.

Consider a digital curfew with a cap $X \in [A^*(K-1), A^*(K))$. Before platforms adjust addictiveness, this digital curfew does not change the consumer's payoff, because she can join

$K-1$ platforms and allocates attention $A^*(K-1)$ optimally. After the cap, the platforms strictly decrease their addictiveness. Thus, the consumer is strictly better off than without the digital curfew.

Third, we show Point 3. In the limit economy, the consumer's payoff is $A_1(d^\infty)C'(A_1(d^\infty)) - C(A_1(d^\infty))$. If $\overline{A}$ increases, d^∞ and $A_1(d^\infty)$ increase. Because $xC'(x) - C(x)$ is increasing in x, the consumer surplus increases. □

F Appendix for Section 7: Naive Consumer

Let us formally describe the timing of the game and the optimization problems of the consumer. First, each platform $k \in K$ simultaneously chooses its addictiveness, $d_k \geq 0$. Second, given the perceived addictiveness $(sd_k)_{k\in K}$, the consumer chooses the set $\hat{K} \subset K$ of platforms to join. In equilibrium, $\hat{K}$ maximizes the perceived indirect utility $V(K')$ across all $K' \in 2^K$, where

$$V(K') := \max_{(a_k)_{k\in K'}\in\mathbb{R}_+^{K'}} \sum_{k\in K'} u(a_k, sd_k) - C\left(\sum_{k\in K'} a_k\right) \tag{A.24}$$
$$\text{s.t.} \quad \sum_{k\in\hat{K}} a_k \leq \overline{A} \quad \text{and} \quad a_k \geq 0, \forall k \in \hat{K}.$$

If $\hat{K} = \emptyset$, all players obtain a payoff of zero, and the game ends. After joining platforms $\hat{K} \neq \emptyset$, the consumer observes the true addictiveness of each platform, then allocates her attention. In equilibrium, the consumer solves

$$\max_{(a_k)_{k\in\hat{K}}\in\mathbb{R}_+^{\hat{K}}} \sum_{k\in\hat{K}} u(a_k, d_k) - C\left(\sum_{k\in\hat{K}} a_k\right) \tag{A.25}$$
$$\text{s.t.} \quad \sum_{k\in\hat{K}} a_k \leq \overline{A} \quad \text{and} \quad a_k \geq 0, \forall k \in \hat{K}.$$

The above two maximization problems coincide if $s = 1$. Our solution concept continues to be pure-strategy subgame perfect equilibrium. Even if $s < 1$, we can use SPE by treating the consumer who solves (A.24) and the consumer who solves (A.25) as different players who have different objectives.

First, we prove Proposition 8, which characterizes the equilibrium and conducts comparative

statics in s.

Proof of Proposition 8. Define $d_s^* := \frac{d^*}{s}$, where d^* is the equilibrium addictiveness of the original model (i.e., $s = 1$). Recall that d^* satisfies the sophisticated consumer's indifference condition, which we can rewrite as

$$K \cdot u\left(\frac{A_K(sd_s^*)}{K}, sd_s^*\right) - C\left(A_K(sd_s^*)\right) = (K-1) \cdot u\left(\frac{A_{K-1}(sd_s^*)}{K-1}, sd_s^*\right) - C\left(A_{K-1}(sd_s^*)\right). \tag{A.26}$$

The equation means that the consumer with s is indifferent between joining K and $K-1$ platforms that choose addictiveness d_s^*. Note that the participation decision uses the perceived addictiveness, sd_s^*. By the same argument as the proof of Proposition 3, we can use this indifference condition to show the following: (i) given addictiveness d_s^*, the consumer joins all platforms; (ii) if platform k deviates and increases its addictiveness, the consumer joins all platforms but k; and (iii) if platform k deviates and decreases its addictiveness, she joins all platforms. Points (i) and (ii) imply that any platform cannot profitably deviate by increasing its addictiveness. For Point (iii), although the consumer's attention allocation is based on d_s^* (not on sd_s^*), she still allocates less attention to less addictive platforms. Thus, Point (iii) implies that any platform cannot profitably deviate by decreasing its addictiveness.

The equilibrium addictiveness $\frac{d^*}{s}$ is decreasing in s, and the consumer joins all platforms for any s. Thus, the consumer surplus is increasing in s. □

Under price competition, the platforms first set addictiveness and prices. Then, the consumer decides which platforms to join by maximizing $V^P(K')$, where

$$\begin{aligned} V^P(K') := \max_{(a_k)_{k\in K'}\in\mathbb{R}_+^{K'}} & \sum_{k\in K'} \left[u(a_k, sd_k) - p_k\right] - C\left(\sum_{k\in K'} a_k\right) \\ \text{s.t.} \quad & \sum_{k\in\hat{K}} a_k \leq \overline{A} \quad \text{and} \quad a_k \geq 0, \forall k \in \hat{K}. \end{aligned} \tag{A.27}$$

Note that the consumer now pays p_k to join platform k. After joining platforms, the consumer allocates her attention by solving (A.25). As before, the payoff of platform k is p_k and 0 if the consumer does and does not join platform k, respectively.

Proof of Proposition 9. For any $s \in (0, 1]$ the same argument as Lemma 1 implies that all platforms set zero addictiveness in a unique equilibrium under price competition. Thus the consumer's payoff is independent of s under price competition, and it is increasing in s under attention competition. Also, for a small s the consumer's payoff under attention competition is negative because of Point (b) of Assumption 1. As a result, price competition gives the consumer a greater payoff if s is below some $s^* \in (0, 1]$. □

We now turn to the impact of digital curfew. Fix K, and let d_s^* denote the equilibrium addictiveness given $s \in (0, 1]$. For each $J \leq K$, let $A^*(d)$ denote the total amount of attention the consumer allocates to platforms, conditional on joining all platforms with the true addictiveness d.

Claim 1. *Take any $A \geq A^*(K, d_1^*)$. If $s = 1$, a digital curfew at $\overline{A} = A$ (as opposed to $\overline{A} = \infty$) does not affect consumer surplus. If $s = s^* \in (0, 1)$, the digital curfew weakly decreases consumer surplus. In particular, it strictly decreases consumer surplus if $\overline{A} \in [A^*(K, d_1^*), A^*(K, d_{s^*}^*))$.*

Proof. If $A \geq A^*(K, d_1^*)$, the cap at A has no impact on the equilibrium addictiveness or the consumer's participation decision, because the consumer's perceived addictiveness is $sd_s^* = d_1^*$ for any s and thus the maximum attention she believes she will allocate is $A^*(K, d_1^*)$. However, if $s < 1$ and $A \in [A^*(K, d_1^*), A^*(K, d_{s^*}^*))$, the cap strictly reduces the total attention she can allocate after joining the platforms. Such a digital curfew harms the consumer with $s < 1$. □