Information and Policing

Shota Ichihashi*

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Abstract

Agents decide whether to commit crimes based on their heterogeneous returns from crimes, or their types. The police have information about these types. The police search agents, without commitment, to detect crimes subject to a search capacity constraint. The deterrent effect of policing is lost when the police have full information about types. The information structure that minimizes crime prevents the police from identifying high types while enabling search intensity to be tailored to low types. The result extends to the case in which the police endogenously choose search capacity at a cost.

*Queen's University, Department of Economics. Email: shotaichihashi@gmail.com. I thank the editor, Marco Ottaviani, the associate editor, and anonymous referees for their comments and suggestions, which greatly improved the paper. I also thank Moritz Meyer-ter-Vehn for providing valuable feedback on an early draft. For valuable suggestions and comments, I thank Yeon-Koo Che, Christopher Cotton, Navin Kartik, Qingmin Liu, Ruslan Momot, Alex Smolin, and the participants of seminars and talks at Osaka University, Columbia Business School, Toulouse School of Economics, University of Michigan (Ross School of Business), Queen's University, Michigan State University, University of Bristol, University of Warwick, and University of Oxford.

1 Introduction

Law enforcement agencies are increasingly relying on data and algorithms to predict crimes (Perry, 2013; Brayne, 2020). They use a variety of data sources, such as criminal records, social media posts, financial records, and local environmental information. Private vendors, such as Palantir and PredPol, also offer predictive algorithms to police departments. This trend, which is driven by the pursuit of more effective law enforcement, has raised a number of concerns. As a case in point, the EU's regulation "Artificial Intelligence Act" classifies a certain use of artificial intelligence to predict crime as a prohibited practice.[1]

Motivated by recent discussions, I study how information available to law enforcement affects its ability to deter crimes, when the law enforcer cannot commit to how to use this information. I examine this question from the perspective of information design.

The model consists of a unit mass of agents and a law enforcement, called the "police." The agents face heterogeneous returns from crimes, or their types. At the outset, the police observe information about the type of each agent according to an exogenous signal structure. Each agent decides whether to commit a crime, and simultaneously, the police search agents subject to a search capacity constraint.

The simultaneous-move assumption implies that the police cannot commit to a search strategy. It captures a situation in which agents decide whether to commit crimes—such as illegal parking, tax fraud, or drug trafficking—and then individual officers or auditors search agents to detect these crimes without directly observing their actions. With this assumption, although the police may be concerned about the crime rate (i.e., the mass of agents committing crimes), in equilibrium, the police focus solely on uncovering crimes instead of deterring them, taking the crime rate as given.

I begin with a preliminary observation that information restriction is necessary for crime deterrence: If the police have full information about the agents' types, then in any equilibrium, every agent commits a crime with probability 1. The fully informed police will allocate search resources only to the agents who are most likely to commit crimes. This search strategy fails to deter any crime: Each agent is either not searched at all, and thus commits a

[1] See https://www.europarl.europa.eu/news/en/press-room/20230609IPR96212/meps-ready-to-negotiate-first-ever-rules-for-safe-and-transparent-ai.

crime, or is searched with a positive probability but still commits a crime as predicted by the police.

The main result is the characterization of the signal structure that minimizes a crime rate. To do so, I first solve a relaxed problem in which I choose a joint signal structure for the police and the agents to minimize a crime rate, under the assumption that the agents do not observe their types. The relaxed problem has a solution in which the police receive no information and randomly search agents with a constant probability, and each agent learns only whether their type exceeds some cutoff. The relaxed problem identifies a lower bound of possible crime rates that can arise in the original problem. I then turn to the original problem—where the agents observe their types—and construct a signal structure for the police that attains the same outcome as the solution to the relaxed problem.

The crime-minimizing signal structure pools high types (i.e., types above some cutoff and committing crimes in equilibrium) with low types (i.e., types below the cutoff and not committing crimes), which prevents the police from identifying agents who face high returns from crime. At the same time, it never pools low types together, which enables the police to tailor search rates to the types of agents who face low returns from crime. The first property ensures that the police do not concentrate search resources on those who face high returns from crimes, which would lower the deterrent effect of searches. The second property enables the police to allocate search resources more effectively. The result implies that simply predicting who or where is most prone to crime could be counterproductive when law enforcement lacks the commitment power to act based on the prediction.

In practice, a law enforcement agency has some level of commitment power in choosing a search strategy. In Section 3.3, I show that the crime-minimizing signal remains relevant in such a situation. Specifically, suppose that a designer (e.g., a police organization) possesses a noisy signal of agents' types and can commit to a search strategy that is measurable with respect to the signal. The designer can also provide additional information to the police (e.g., individual officers), who, without commitment, use this information to further allocate the search capacity set by the designer. I show that the designer's optimal information policy incorporates the crime-minimizing signal structures of the baseline model.

The above results assume that the police face an exogenous search capacity. In Section

4, the police endogenously choose the total search capacity at a cost. For example, an individual officer might exert different levels of search effort depending on the available information. Under a certain condition, the crime-minimizing signal structure of the baseline model continues to minimize the crime rate. I also provide an example in which this result fails, and a crime-minimizing signal structure enables the police to identify a fraction of the agents who do not commit crimes in equilibrium.

The contribution of the paper is to offer a new application of information design—the issue of what information law enforcement should be endowed with. By characterizing the information structure that minimizes crime, I show how attempts to predict crimes could backfire due to law enforcement's lack of commitment to using the information. Theoretically, the paper studies an information design problem with a rich space of players and strategies, as well as a constraint on feasible information structures. It illustrates how a solution technique based on a relaxed problem facilitates the characterization of the optimal information structure.

Related work. The paper relates to the literature on Bayesian persuasion and information design (see Kamenica (2019) and Bergemann and Morris (2019) for surveys). Several papers, such as Lazear (2006), Eeckhout, Persico, and Todd (2010), and Hernández and Neeman (2022), study the intersection of Bayesian persuasion and law enforcement. These papers study how to disclose information to criminals or players to deter their socially undesirable actions. In contrast, this paper studies what information a law enforcer should have about such players. Methodologically, it addresses an information design problem with a continuum of players, states, and actions (for the police), which seems to be less understood than a single-player Bayesian persuasion problem (see Smolin and Yamashita (2022) for a discussion). The information design literature provides conditions under which an optimal signal takes a tractable form, such as monotone partitional signals, censorship policies, and nested intervals (e.g., Guo and Shmaya 2019; Dworczak and Martini 2019; Kolotilin, Mylovanov, and Zapechelnyuk 2022). The crime-minimizing signal structure does not belong to these classes of signals. Finally, the relaxed problems examined in this paper relate to Bayesian persuasion with moral hazard, in which the state distribution is endogenously determined by a player's action (Rodina, 2017; Boleslavsky and Kim, 2018; Zapechelnyuk, 2020; Hörner and Lambert, 2021).

This paper also relates to the economic literature on crime and policing, which starts from Becker (1968). The question of what information about agents should or should not be used for policing is discussed in the context of racial profiling (Knowles, Persico, and Todd 2001; Persico and Todd 2005; Bjerk 2007; Persico 2009). In terms of the timing and payoffs of the game, my paper builds closely on Persico (2002), who studies whether requiring the police to adopt a fairer search strategy reduces crime. Instead of constraining police behavior, I restrict information available to police and study what information renders policing effective. A model of predictive enforcement is also studied by Che, Kim, and Mierendorff (2023), who consider a bandit model that captures the endogenous generation and use of information for law enforcement. To focus on the role of information in policing, the model abstracts away from other important considerations, such as the design of judicial systems, richer responses by potential criminals and victims, as well as the fairness of predictive algorithms (e.g., Curry and Klumpp 2009; Cotton and Li 2015; Jung, Kannan, Lee, Pai, Roth, and Vohra 2020; Vasquez 2022; Liang, Lu, and Mu 2022).

The rest of the paper is organized as follows. Section 2 presents the baseline model as well as the benchmark in which the police have full information. Section 3 characterizes the crime-minimizing signal structure for the baseline model and for a model in which a designer has partial control over searches. Section 4 considers the police that can increase search capacity at a cost. In Section 5, I consider several extensions that relax various assumptions about the timing and payoffs in the baseline model.

2 Model

The model consists of police and a unit mass of agents. Each agent $i \in [0, 1]$ has some underlying returns on crime, or *type*, $x_i \in [0, 1]$. Each agent observes their type. We may interpret an agent's type as reflecting the individual characteristics that affect their returns on crime (e.g., legal earning opportunities) or crime opportunities specific to certain locations or times. Types are independently and identically drawn from distribution function

$F \in \Delta[0,1]$, which has a positive density f and is commonly known.[2] I use $\mathbb{E}_F[\cdot]$ for the expectation operator under F. I also use $F(\cdot|\tilde{x} \leq c)$ for the conditional distribution of F on $[0,c]$ and $\mathbb{E}_F[\cdot|\tilde{x} \leq c]$ for the corresponding expectation operator.

The police learn about each agent's type according to a *signal structure* (S, π), which consists of a set S of signals and a collection $\pi = \{\pi(\cdot|x)\}_{x\in[0,1]}$ of conditional distributions $\pi(\cdot|x) \in \Delta S$ over signals for each type x. For each agent $i \in [0,1]$, the police observe a signal $s_i \in S$ drawn from distribution $\pi(\cdot|x_i)$. Conditional on types, signals are independent across agents. The signal structure is exogenous and commonly known, but only the police observe realized signals.[3]

Given the signal structure, the police and the agents play the following simultaneous-move game: Each agent decides whether or not to commit a crime, and simultaneously, the police allocate search resources across agents. Specifically, the police choose a *search strategy* $p : S \to [0,1]$, where $p(s)$ is the probability of searching agents with signal $s \in S$. The police have a measure $\overline{P} \in (0,1)$ of searches to allocate; thus, the police can choose a search strategy $p(\cdot)$ if and only if the total mass of searches does not exceed $\overline{P}$, i.e.,

$$\int_0^1 \int_S p(s)\, \mathrm{d}\pi(s|x)\, \mathrm{d}F(x) \leq \overline{P}. \tag{1}$$

The payoff of an agent from committing a crime is $x - \rho$, where x is the agent's type and ρ is the search probability the agent faces. The payoff of not committing a crime is 0.[4] For a given signal structure and a search strategy, an agent's expected payoff of committing a

[2] I write ΔX for the set of all probability distributions on a set X. See, e.g., Sun (2006) for a formal treatment of a continuum of independent random variables and the corresponding law of large numbers.

[3] Because the signal structure is exogenous, we should view the results as comparative statics that examine how information affects the deterrent effect of policing. A related but separate question is which signal structure would be chosen if information were endogenous; the answer would depend on the objective of the player who chooses the signal structure and whether this choice is observable to the agents.

[4] This payoff specification is equivalent to the following richer setup: The payoff of committing a crime is $U(y, \rho)$, which strictly increases in an agent's type y; strictly decreases in search probability ρ; and has a threshold search probability $\hat{p}(y)$ that solves $U(y, \hat{p}(y)) = 0$ for each type y (without loss, the payoff of not committing a crime is normalized to 0). An agent will commit a crime if $\hat{p}(y) > \rho$. But once I redefine the agent's type as $x = \hat{p}(y)$, this richer setup and our original setup lead to the same set of best responses by agents under any search strategy. This setup subsumes $U(y, \rho) = (1 - \rho)y - L\rho$, i.e., an agent enjoys the returns on crime if they are not searched, but incurs a loss of L if they are searched.

crime is written as

$$x - \int_S p(s)\,\mathrm{d}\pi(s|x).$$

I now describe the police's payoff. Fix any strategy profile, and let $a(x)$ denote the probability with which the agents with type x commit crimes. Define the *mass of successful searches* as the mass of agents who commit crimes and are searched by the police, i.e.,

$$\sigma \triangleq \int_0^1 \int_S p(s)\,\mathrm{d}\pi(s|x)a(x)\,\mathrm{d}F(x). \tag{2}$$

Define the *crime rate* as the mass of agents who commit crimes, i.e.,

$$r \triangleq \int_0^1 a(x)\,\mathrm{d}F(x). \tag{3}$$

The police's payoff is given by

$$\sigma - \lambda r, \tag{4}$$

where parameter $\lambda \in \mathbb{R}_+$ captures the degree to which the police prioritize deterring crimes over uncovering crimes. I do not constrain λ, so the police may place a large weight on crime deterrence. However, under the simultaneous-move assumption, the police take the crime rate r as exogenous because it does not directly depend on a search strategy (see (3)). Therefore, for any given λ, in equilibrium, the police act as if their payoffs depend only on the mass of successful searches, i.e., $\lambda = 0$.[5]

The solution concept is Bayesian Nash Equilibrium (BNE). For expositional simplicity, I focus on equilibria in which the agents of the same type x commit crimes with the same probability. Hereafter, an *equilibrium* refers to a BNE that satisfies this condition.

To minimize possible case classifications, I assume that the primitives—i.e., the distribution of returns from crime and the police's search capacity—satisfy the following:

[5]This observation does not imply that the police are better off when the crime rate is high: If λ is high but the crime rate is also high, the police in this model could have been better off—in the sense of obtaining a greater payoff—had they publicly committed to a different search strategy or faced a different signal structure. The model is not suitable for analyzing police welfare because we can derive arbitrary welfare implications depending on λ.

Assumption 1. The primitives, F and $\overline{P}$, satisfy

$$\overline{P} < \int_0^1 x \, \mathrm{d}F(x). \tag{5}$$

The assumption means that the police do not have enough resources to fully deter crime: To attain a crime rate of 0, the police would have to search each type x with a probability of at least x, but inequality (5) implies that such a search strategy violates the capacity constraint. As a result, a positive mass of agents, facing search probabilities strictly below their types, commit crimes.

As a benchmark, consider the police who have no information, i.e., the distribution $\pi(\cdot|x)$ over signals is independent of type x. In this case, every agent faces search probability $\int_S p(s) \, \mathrm{d}\pi(s|x)$, which is independent of x and thus equal to $\overline{P}$.[6] In equilibrium, every type $x > \overline{P}$ commits a crime, resulting in crime rate $1 - F(\overline{P})$.

The main focus is on a signal structure that minimizes the crime rate (3) in equilibrium.

Definition 1. A *crime-minimizing signal structure* is a signal structure that has an equilibrium with the lowest crime rate across all signal structures and equilibria. The corresponding equilibrium is called a *crime-minimizing equilibrium*.

The equilibrium search strategy is different from the crime-minimizing strategy because, even though the police might care about reducing crime, they act to maximize the mass of successful searches under the simultaneous-move assumption. As a result, providing the police with more information does not necessarily reduce crime. The crime-minimizing signal structure restricts the police's information such that the search strategy exhibits maximum deterrence.

2.1 Discussion of Timing and Payoffs

The results of this paper hinge on two assumptions: First, the police and agents move simultaneously, which implies that the police cannot pre-commit to a search strategy. Second,

[6] To see this, note that $\int_S p(s) \, \mathrm{d}\pi(s|x) > \overline{P}$ violates the search capacity constraint. The other case $\int_S p(s) \, \mathrm{d}\pi(s|x) < \overline{P}$ also leads to a contradiction, because the police could increase search rates and thus the mass of successful searches.

the police are rewarded for uncovering crimes. Together, these assumptions imply that the police act as if they care only about successful searches. This section motivates these assumptions (see Sections 5.1 and 5.2 for the consequences of relaxing them).

First, the literature has considered police pre-committing to their strategy as well as police moving simultaneously with potential criminals (see, e.g., Eeckhout et al. (2010) for the former). The validity of each assumption depends on the context. For example, the literature on decentralized law enforcement—such as Persico (2002) and Porto et al. (2013)—adopts the simultaneous-move assumption by viewing the "police" as a collection of individual officers or auditors. This assumption arises from the idea that the action of each individual officer does not directly influence the decisions of potential criminals (Appendix D formalizes this idea). The police's lack of commitment might also stem from the notion that a signal structure captures a predictive algorithm used by a law enforcement agency. Predictions generated by an algorithm (i.e., signals) would be invisible to the public and too complex to describe in advance, making it difficult for the police to commit to a predetermined search strategy.

The validity of the second assumption—that the police are (partly) rewarded for uncovering crimes—also depends on the context. For example, papers such as Eeckhout et al. (2010) present evidence favoring pure crime minimization. In contrast, papers such as Baicker and Jacobson (2007), Makowsky and Stratmann (2011), Nagin et al. (2015), and Owens and Ba (2021) argue that the incentives of police are tied to detecting and resolving crimes.[7] Stashko (2023) analyzes police stop-and-search data in the U.S. and concludes that "empirical evidence is consistent with a model of arrest maximization and inconsistent with a model of crime minimization."

[7]Nagin et al. (2015) note that rewards from making arrests "are institutionalized in organizational performance metrics that emphasize arrest and clearance," while "generally no tangible reward is given for preventing crime in the first place, precisely because it is a nonevent and therefore is difficult to measure." Similarly, Owens and Ba (2021) observe that CompStat-style reviews "create strong incentives for officers to provide a high level of engagement—in particular, to make arrests, issue citations, and lower crime."

2.2 Preliminary Analysis: The Fully Informed Police

To illustrate that restricting the police's information is necessary for crime deterrence, I begin with the analysis of the fully informed police. Specifically, suppose that the signal structure is such that for every type $x \in [0,1]$, $\pi(\cdot|x)$ places probability 1 on $s = x$.

Theorem 0. *Suppose that the police have full information. An equilibrium exists, and in any equilibrium, almost every agent commits a crime with probability* 1.

Proof. First, I construct an equilibrium. Suppose that the police adopt a search strategy $p^*(x) = \frac{x\overline{P}}{\mathbb{E}_F[\tilde{x}]}$ for all $x \in [0,1]$. This search strategy induces the total search capacity of $\int_0^1 p^*(x)\,\mathrm{d}F(x) = \overline{P}$ and thus is feasible. Assumption 1 implies that $\overline{P} < \mathbb{E}_F[\tilde{x}]$, which means that $p^*(x) < x$ for all $x \in (0,1]$. As a result, every agent (except possibly $x = 0$) commits a crime with probability 1. The police then find it optimal to choose any search strategy that exhausts search capacity $\overline{P}$, including p^*. Thus we obtain an equilibrium.

Second, take any equilibrium. The police cannot choose $p(x) \geq x$ for (almost) every x, because by Assumption 1, it violates the search capacity constraint. Thus, the set $X \triangleq \{x \in [0,1] : x > p(x)\}$ has a positive mass, and any type in X commits a crime with probability 1. Let Y be the set of types that commit crimes with probability strictly below 1. If Y has a positive mass, the police must be allocating a positive mass of searches to types in Y, because otherwise the types in Y would commit crimes for sure. We then obtain a contradiction, because the police could increase the mass of successful searches by shifting search probabilities from types in Y to X.[8] Thus, the set Y must have measure zero, i.e., almost every agent commits a crime with probability 1. □

The intuition for Theorem 0, which appears in the existing work such as Persico (2002), is that the police's ability to predict crimes—combined with their incentive to uncover crimes and a lack of commitment in search strategy—eliminates the deterrent effect of policing (see

[8]The police's deviation would be profitable even if the police's payoffs depend on a crime rate, because the deviation does not change the agents' actions (and thus the crime rate) under the simultaneous-move assumption.

also Section 5.1 for the role of the simultaneous-move assumption).[9]

In contrast to the crime rate, the police's equilibrium strategy is not unique. For example, suppose that $F = U[0,1]$ (i.e., the uniform distribution on $[0,1]$) and $\overline{P} = \frac{1}{4}$. In the proof of Theorem 0, I constructed an equilibrium in which the police set $p(x) = \frac{x}{2}$ for every $x \in [0,1]$. The following is another equilibrium: The police search any type $x \leq \frac{1}{\sqrt{2}}$ with probability x and never search any type $x > \frac{1}{\sqrt{2}}$. In this equilibrium, each type $x \leq \frac{1}{\sqrt{2}}$ is indifferent between committing a crime and not, yet breaks ties for committing a crime. Indeed, if types below $\frac{1}{\sqrt{2}}$ did not commit crimes, the police would profitably deviate and search types above $\frac{1}{\sqrt{2}}$ instead. In general, under full information, a strategy profile is an equilibrium if and only if (i) the search capacity $\overline{P}$ is allocated across agents in a way that almost every agent weakly prefers to commit a crime, and (ii) almost every agent commits a crime with probability 1.

Theorem 0 relies on the assumption that the police's payoff from successful searches is independent of the underlying types of agents. To see this, suppose that, in the previous example, the police's payoff from searching an agent who committed a crime is equal to their type, x (in the baseline model, this payoff is 1 for any type x). Then in equilibrium, the police search any type $x \geq \frac{1}{\sqrt{2}}$ with probability x and never search any type $x < \frac{1}{\sqrt{2}}$. Any type $x \geq \frac{1}{\sqrt{2}}$ commits a crime with probability $\frac{1}{\sqrt{2}x}$, and any type $x < \frac{1}{\sqrt{2}}$ commits a crime with probability 1.[10] The crime rate is strictly below 1, because the police with type-dependent payoffs can credibly search agents even when they commit crimes with low probability. In Appendix A.4, I construct an equilibrium when the police have full information and earn arbitrary type-dependent payoffs from searching criminals. I show that as the degree of type-dependency goes to 0, the equilibrium crime rate converges to 1.

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[9]See also Goldman and Pearl (1976) for a related result. Theorem 0 also echoes the recent findings by Adda and Ottaviani (2024), who, among other things, show that symmetric information eliminates incentives for costly activities in the non-market environment (see, e.g., their Proposition 2E). Here, the costly activity for the police—which does not contribute to their payoffs—is searching agents who do not commit crimes. Searching innocents is necessary for crime deterrence in my model but never occurs under full information.

[10]The police's search strategy is optimal. Indeed, searching any type $x < \frac{1}{\sqrt{2}}$ leads to a payoff of at most $\frac{1}{\sqrt{2}}$, whereas searching any type $x \geq \frac{1}{\sqrt{2}}$ leads to a payoff equal to $x \cdot \frac{1}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$. Thus, a search strategy is optimal if and only if it searches only types above $\frac{1}{\sqrt{2}}$ and exhausts the search capacity $\overline{P}$. This condition holds for the search strategy presented here.

3 Crime-Minimizing Signal Structure

I now turn to the crime-minimizing signal structure. The analysis consists of two steps. First, I study a relaxed problem in which the agents do not directly observe their types and I choose a joint information structure for the police and the agents in order to minimize crime. Because the incentive constraints of the agents are relaxed, the resulting crime rate becomes a lower bound of the possible crime rates in the original crime-minimization problem. Second, in the original setup, I construct a signal structure for the police that attains this lower bound in an equilibrium.

3.1 Relaxed Problem

The relaxed problem is defined as follows: The agents know the type distribution F but do not directly observe their types. Instead, information is determined by a *joint signal structure*, (S_P, S_A, π). Here, S_P and S_A are the sets of signals for the police and agents, respectively, and $\pi = \{\pi(\cdot|x)\}_{x\in[0,1]}$ is the collection of conditional probability distributions on $S_P \times S_A$ for each type. If agent i has type x_i, the police observe s_i^P and agent i observes s_i^A, where $(s_i^P, s_i^A) \sim \pi(\cdot|x_i)$. The rest of the game remains the same: Each agent i observes s_i^A and decides whether to commit a crime, and simultaneously, the police choose a search strategy $p : S_P \to [0,1]$ to maximize the mass of successful searches. The following result characterizes a crime-minimizing joint signal structure.

Lemma 1. *In the relaxed problem, the following joint signal structure and equilibrium minimize a crime rate: The police learn no information, e.g., $S_P = \{\phi\}$, and each agent learns whether their type exceeds cutoff $\hat{c} \in (0,1)$ that uniquely solves*

$$\mathbb{E}_F[\tilde{x}|\tilde{x} \leq \hat{c}] = \overline{P}. \tag{6}$$

In equilibrium, the police search every agent with probability $\overline{P}$, and each agent commits a crime if and only if their type exceeds $\hat{c}$.

Proof. Take any joint signal structure (S_P, S_A, π) and equilibrium. Let $p : S_P \to [0,1]$ be the equilibrium search strategy and $r \in (0,1)$ be the crime rate.[11] The proof consists of three steps. First, following the revelation principle of information design (e.g., Bergemann and Morris 2019), replace the agents' signals with action recommendations that replicate the equilibrium behavior of each type. The signal space for agents is now $S_A^* = \{crime, not\}$ and the obedience constraints hold:

$$\mathbb{E}[\tilde{x} - p(\tilde{s}^P)|crime] \geq 0 \quad \text{and} \quad \mathbb{E}[\tilde{x} - p(\tilde{s}^P)|not] \leq 0, \tag{7}$$

where the expectations in the first and the second inequalities are with respect to the agent's type $\tilde{x}$ and the police's signal $\tilde{s}^P$ conditional on action recommendations *crime* and *not*, respectively. The obedience constraints ensure that each agent commits a crime after observing signal *crime* and not after signal *not*. It holds that

$$\mathbb{E}[p(\tilde{s}^P)|crime] \geq \overline{P} \geq \mathbb{E}[p(\tilde{s}^P)|not]. \tag{8}$$

Indeed, if one inequality is violated, the other inequality must also be violated because of the binding search capacity constraint. However, if $\mathbb{E}[p(\tilde{s}^P)|crime] < \overline{P} < \mathbb{E}[p(\tilde{s}^P)|not]$, the mass of successful searches is $r\mathbb{E}[p(\tilde{s}^P)|crime] < r\overline{P}$. The police would then deviate and randomly search every agent with probability $\overline{P}$, securing a higher mass of successful searches, $r\overline{P}$. This is a contradiction.

Second, replace the police's signal with an uninformative signal, $S_P^* = \{\phi\}$. Under the resulting signal structure (S_P^*, S_A^*, π^*), the police can only search every agent with probability $\overline{P}$. Inequalities (7) and (8) then imply

$$\mathbb{E}[\tilde{x}|crime] - \overline{P} \geq 0 \quad \text{and} \quad \mathbb{E}[\tilde{x}|not] - \overline{P} \leq 0,$$

i.e., the obedience constraints continue to hold.

Finally, the above two steps imply that, to derive the crime-minimizing joint signal structure, we can without loss of generality assume that the police receive no information, the

[11] Assuming $r \in (0,1)$ is without loss of generality: By Assumption 1, there is no equilibrium with $r = 0$. Also, to solve the relaxed problem, we do not need to consider an equilibrium with $r = 1$, because we can ensure $r < 1$ by providing full information to the agents and no information to the police.

agents receive action recommendations, and the obedience constraints hold. The problem is equivalent to the following Bayesian persuasion problem: The receiver (i.e., an agent) obtains a payoff of $x - \overline{P}$ from committing a crime and 0 from not. The sender discloses information about type $x \sim F$ in order to minimize the probability of the receiver committing a crime. As shown in Gentzkow and Kamenica (2016) (Section IV.A), the solution is to disclose whether type x exceeds a cutoff $\hat{c}$ solving (6), which deters the types below $\hat{c}$ from committing a crime. The type distribution has a density and satisfies Assumption 1, so the cutoff $\hat{c} \in (0, 1)$ exists and is unique. □

Lemma 1 implies that, in the original setup, the minimum crime rate is attained if all types below $\hat{c}$ abstain from committing crimes. However, this outcome cannot arise if the agents observe their types and the police have no information, because the resulting random search induces types between $\overline{P}$ and $\hat{c}$ to commit crimes. Nevertheless, the next section shows that providing the police with partial information achieves the same crime rate.

3.2 Characterizing the Crime-Minimizing Signal Structure

I now turn to the original problem, in which the agents observe their types. First, I define a class of signal structures:

Definition 2. For each $c \in [0, 1]$, the *truth-or-noise signal structure with cutoff* c, denoted by (S_c, π_c), is the following signal structure: The signal space S_c is $[0, c]$; for each $x \leq c$, distribution $\pi_c(\cdot|x)$ draws $s = x$ with probability 1; and for any $x > c$, distribution $\pi_c(\cdot|x)$ is independent of x and equals $F(\cdot|\tilde{x} \leq c)$.

To understand the truth-or-noise signal structures, consider the police's posterior belief on an agent's type induced by a signal drawn from (S_c, π_c) (see Figure 1). The signal can be the "truth" (i.e., an agent's type is below c and the signal is equal to their true type) or a "noise" (i.e., an agent's type is above c and the signal is a realization of a random draw from $F(\cdot|\tilde{x} \leq c)$). As a result, the posterior belief places positive probabilities both on types below and above c. Specifically, the posterior induced by signal $s \in [0, c]$ contains a point mass $F(c)$ on type s and a mass $1-F(c)$ of types distributed according to $F(\cdot|\tilde{x}>c)$.[12] The posterior beliefs (indexed by their point masses) are distributed according to $F(\cdot|\tilde{x}\leq c)$ and average to prior distribution F.

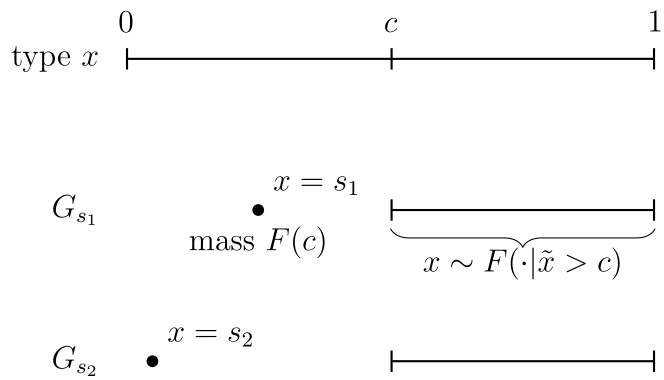

Figure 1: Posterior distribution G_s of types conditional on signal $s = s_1, s_2$ under (S_c, π_c).

Theorem 1. *Let $\hat{c}\in(0,1)$ denote the cutoff defined by equation (6), i.e., $\overline{P}=\mathbb{E}_F[\tilde{x}|\tilde{x}\leq\hat{c}]$. The truth-or-noise signal structure with cutoff $\hat{c}$ is a crime-minimizing signal structure.*

Proof. It suffices to show that signal structure $(S_{\hat{c}},\pi_{\hat{c}})$ has an equilibrium in which types below $\hat{c}$ do not commit crimes. Consider the following strategy profile: The police adopt search strategy $p^*(s)=s$ for every $s\in[0,\hat{c}]$, and each agent commits a crime if and only if their type exceeds $\hat{c}$. This is an equilibrium: First, from the police's perspective, each agent is committing a crime with probability $1-F(\hat{c})$ conditional on any signal. Thus any search strategy that exhausts search capacity $\overline{P}$ is optimal for the police. Search strategy p^* indeed exhausts the search capacity because of equation (6). The strategy of each agent is also optimal: Any agent with type $x\leq\hat{c}$ knows that the police will observe signal $s=x$ and search them with probability x, so the agent is indifferent and willing to abstain from committing a crime. Types above $\hat{c}$ will be searched with probability at most $\hat{c}$, so they commit crimes. Hence the strategy profile described above is an equilibrium and attains the same crime rate as the solution to the relaxed problem described in Lemma 1. $\square$

[12] To see this, let $f(s|\tilde{x}\leq c)$ denote the conditional density associated with $F(s|\tilde{x}\leq c)$ evaluated at $x=s$. Roughly, the "probability" with which signal s is realized is $f(s|\tilde{x}\leq c)$. The "probability" of the joint event—in which the realized signal and the true type are both equal to s—is equal to the probability of $x\leq c$, which is $F(c)$, multiplied by the "probability" of the true type s conditional on $x\leq c$, which is $f(s|\tilde{x}\leq c)$. Therefore, the posterior probability of type s conditional on signal s is $\frac{F(c)f(s|\tilde{x}\leq c)}{f(s|\tilde{x}\leq c)}=F(c)$.

The crime-minimizing signal structure prevents the police from identifying high-type agents while enabling the police to tailor search rates to low-type agents. The intuition is as follows. First, the crime-minimizing signal structure prevents the police from identifying types above $\hat{c}$ because the fraction of the types above $\hat{c}$ equals $1 - F(\hat{c})$ across all signals.[13] This property minimizes the distortion whereby the police concentrate search resources on agents who face high returns from crimes and are thus less responsive to searches, which lowers the deterrent effect of searches. Various signal structures have this first property, including one that discloses no information. Second, the crime-minimizing signal structure reduces wasteful searches by enabling the police to search each type $x < \hat{c}$ with probability x. The signals are differentiated according to the lowest possible types, and the equilibrium search rates are tailored to these types. Hence, the police will never search agents with a probability greater than what is minimally necessary to deter crimes.

Implementing the crime-minimizing signal structure requires full information about the types below $\hat{c}$, because no two types below $\hat{c}$ are pooled. However, a similar construction is possible with coarser information. For example, consider a monotone-partitional signal structure that partitions $[0, 1]$ into subintervals $0 = x_0 < x_1 < \cdots < x_{N-1} < x_N = 1$. This could represent a city divided into N areas, each assigned a "risk level" for crime. Instead of sharing this information directly with the police, it can be garbled similarly to the crime-minimizing signal structure: We create $K < N$ segments, where each segment $k = 1, \ldots, K$ contains all types in (x_{k-1}, x_k) and the fraction $\frac{F(x_k) - F(x_{k-1})}{F(x_K)}$ of types in $(x_K, 1]$. Every segment contains the same fraction $1 - F(x_K)$ of types above x_K, and no two subintervals below x_K are pooled together. As shown below, this garbling—which is a discrete version of the truth-or-noise signal structure—may achieve a lower crime rate than the original partitional signal structure.

Example 1. Suppose $F = U[0, 1]$ and $\overline{P} = 1/2$.[14] Consider a signal structure (S, π) that sends signal s_L if $x \in [0, 1/3]$, s_M if $x \in (1/3, 2/3]$, and s_H if $x \in (2/3, 1]$. In equilibrium

[13] A consequence of this property is that the crime rate is equal to $1 - F(\hat{c})$ across all signals, and thus the police are indifferent between any search strategies that exhaust $\overline{P}$. However, the police's indifference is not unique to this signal structure and holds for, e.g., any deterministic signal structure (see Appendix A.1).

[14] This violates Assumption 1 as $\int_0^1 x \, \mathrm{d}F(x) = \overline{P}$, but the same observation holds for any $\overline{P} < 1/2$.

under (S, π), the police adopt search strategy $p(s_L) = 1/6$, $p(s_M) = 1/2$, and $p(s_H) = 5/6$.[15] Within each signal, half of the agents abstain from committing crimes, so the equilibrium crime rate is $1/2$, the same as under no information.

Now, garble (S, π) by pooling all types in $[0, 1/3]$ with half of those in $(2/3, 1]$ to form signal s_L^*, and all types in $(1/3, 2/3]$ with the remaining half of $(2/3, 1]$ to form signal s_M^*. Let (S^*, π^*) denote the resulting signal structure. Under (S^*, π^*) and $F = U[0, 1]$, a fraction $2/3$ of agents within each signal have types below $2/3$. In equilibrium, the police search s_L^* and s_M^* with probabilities $1/3$ and $2/3$, respectively, deterring all types below $2/3$ from committing crimes. The crime rate is thus $1/3 < 1/2$.

Intuitively, the signal structure (S^*, π^*) prevents the police from identifying types above $2/3$. This reduces searches directed at high types and increases those directed at low types, which are more responsive to searches. At the same time, it allows the police to search types in $[0, 1/3]$ and $(1/3, 2/3]$ with distinct probabilities, leading to a more effective allocation of search resources across low types than under no information.

Remark 1 (Multiplicity of Crime-Minimizing Signals). The crime-minimizing signal structure is not unique in either the relaxed or the original problem. The relaxed problem has multiple solutions such as those in Lemma 1 and Theorem 1. The original problem also has multiple solutions: For example, let $F = U[0, 1]$ and $\overline{P} = \frac{1}{4}$. Solving equation (6) gives $\hat{c} = \frac{1}{2}$. Another crime-minimizing signal structure, different from the one in Theorem 1, is as follows: Signal $s \in [0, \frac{1}{2}]$ is realized with probability 1 if and only if $x = s$ or $x = 1 - s$. Conditional on signal s, the posterior belief assigns equal probability to $x = s$ and $x = 1 - s \geq \hat{c}$. Since $F(\hat{c}) = \frac{1}{2}$, the proof of Theorem 1 applies directly. Thus, this signal structure achieves the same crime rate as that in Theorem 1.

Remark 2 (Fragility of the Crime-Minimizing Signal Structure). The signal structure $(S_{\hat{c}}, \pi_{\hat{c}})$ in Theorem 1 exhibits two kinds of fragility. First, it achieves the minimal crime rate only if the agents who are indifferent break ties by not committing crimes. Indeed, under $(S_{\hat{c}}, \pi_{\hat{c}})$, there is another equilibrium with crime rate 1, in which the police use the same

[15] This search strategy deters all types in $[0, 1/6] \cup [1/3, 1/2] \cup [2/3, 5/6]$ from committing crimes, equalizing the crime rate of every signal to $1/2$. It also exhausts the search capacity, as $\frac{1}{3}\left(\frac{1}{6} + \frac{1}{2} + \frac{5}{6}\right) = \frac{1}{2} = \overline{P}$. Thus, the search strategy is optimal.

search strategy p^* as in the crime-minimizing equilibrium and all types below $\hat{c}$ break ties and commit crimes. Second, the signal structure fails to deter crimes if the police's search capacity is slightly misspecified: If the crime-minimizing signal structure $(S_{\overline{P}}, \pi_{\overline{P}})$ is designed and employed for a certain value of $\overline{P}$ but the actual search capacity is $\overline{P} - \epsilon$ with $\epsilon > 0$, then, according to logic similar to that in Theorem 0, all agents commit crimes in a unique equilibrium. At the same time, these kinds of fragility are not necessary for a signal structure to reduce crime (relative to providing the police with no information). For example, the binary signal structure (S^*, π^*) in Example 1 has a unique equilibrium—which is the one described in the example—and the equilibrium is robust to the slight misspecification of search capacity $\overline{P}$. Appendix C formalizes these ideas.

3.3 Partial Commitment

Theorem 1 applies even when a search strategy is partially determined by an entity with some commitment power. I clarify this point using the following variation of the model.

First, I introduce a player called the *designer*, whose goal is to minimize the crime rate. The designer could represent a law enforcement organization, with the police as individual enforcers. The designer faces an exogenous signal structure, $(\overline{S}, \overline{\pi})$. For simplicity, assume $\overline{S}$ is finite. Each $t \in \overline{S}$ is called a *pre-signal*, where $m(t) \triangleq \int_0^1 \overline{\pi}(t|x)\,\mathrm{d}F(x)$ denotes the mass of agents who receive t, and F_t denotes the posterior type distribution conditional on t. Assume that each F_t has a positive density on $[0, 1]$. The designer has search capacity $\overline{P}$ that satisfies Assumption 1.

The designer commits to the allocation of search capacity and the information available to the police: First, the designer chooses an *allocation policy* $\overline{a} : \overline{S} \to \mathbb{R}_+$, subject to the capacity constraint:

$$\sum_{t \in \overline{S}} \overline{a}(t) = \overline{P}, \tag{9}$$

where $\overline{a}(t)$ is the mass of searches allocated to pre-signal t.

Second, for each $t \in \overline{S}$, the designer chooses a signal structure (S_t, π_t), which captures the information that the police can use to search agents with each pre-signal t. As in the baseline model, the police cannot commit to how to use this information.

After the designer commits to an allocation policy $\overline{a}$ and signal structures $\{(S_t, \pi_t)\}_{t\in\overline{S}}$, the game unfolds as follows: The pre-signal of each agent is realized and observed by the agent and the police. In addition, each agent privately observes their type. Then, for each $t \in \overline{S}$, the agents with pre-signal t and the police play the game described in Section 2, where the mass of agents is $m(t)$ (instead of 1); the type distribution is F_t; and the search capacity is $\overline{a}(t)$. The police's payoff equals the total mass of successful searches aggregated across all pre-signals. The designer's objective is to minimize the total mass of agents who commit crimes.[16] I focus on the designer's preferred equilibrium, which minimizes the crime rate across all equilibria. If $|\overline{S}| = 1$, the model reduces to the baseline model.

As an example, imagine that a police organization (i.e., the designer) has some information about how different times of the day or locations in a city are associated with crime opportunities. This information is captured by pre-signals, $(\overline{S}, \overline{\pi})$, where each pre-signal represents a particular time or location.[17] The organization commits to the allocation of officers across different times or locations, which corresponds to an allocation policy $\overline{a}$. The organization can also provide the officers assigned to each pre-signal t with additional information (S_t, π_t), generated by a predictive algorithm.[18] However, neither the organization nor the officers can pre-commit to its use.

The designer can replicate the same outcome as when the police have no information in the baseline model. To do so, it allocates search capacity $\overline{a}(t) = m(t)\overline{P}$ and chooses an uninformative (S_t, π_t), such as $S_t = \{\phi\}$, for each t. The police can then only search each pre-signal t with probability $\overline{P} = \frac{m(t)\overline{P}}{m(t)}$, because they use search capacity $m(t)\overline{P}$ to randomly search a mass $m(t)$ of agents. Alternatively, the designer might provide the police with more information, but the resulting outcome could involve more crimes. How should the designer balance control over the allocation of search capacity with the use of more information?

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[16]Formally, let $r(t)$ denote the fraction of agents who commit crimes conditional on pre-signal t, and $d(t)$ denote the fraction of agents who commit crimes and are searched conditional on pre-signal t. The police's payoff is $\sum_{t\in\overline{S}} d(t)m(t)$, and the designer's payoff is $-\sum_{t\in\overline{S}} r(t)m(t)$.

[17]For example, people may face higher returns from illegal parking during certain times of the day. Another example is tax auditing for businesses: Some types of businesses, such as large corporations, may face higher returns from tax evasion. In such scenarios, the assumption that agents' types are exogenous implies that agents do not change the timing of their commutes or their business types based on opportunities for illegal parking or tax evasion.

[18]As shown in Appendix D, the police can be viewed as a population of individual officers.

Theorem 1 helps us solve the designer's problem. To see this, suppose that the designer has chosen an allocation policy $\overline{a}$. The game between the police and the agents with pre-signal t is equivalent to our baseline model in which the prior type distribution is $F = F_t$ and the search capacity is $\overline{P} = \frac{\overline{a}(t)}{m(t)}$. Here, I scale the mass of each pre-signal to 1 (instead of $m(t)$) and the search capacity to $\frac{\overline{a}(t)}{m(t)}$, so that Theorem 1 applies verbatim.

Theorem 1 implies that for each pre-signal t, the designer should provide the police with the truth-or-noise signal structure with cutoff $c_t = c_t\left(\frac{\overline{a}(t)}{m(t)}\right)$ that solves

$$\mathbb{E}_{F_t}\left[\tilde{x}|\tilde{x} \leq c_t\right] = \frac{\overline{a}(t)}{m(t)}. \tag{10}$$

The cutoff c_t does not exist if $\frac{\overline{a}(t)}{m(t)} > \mathbb{E}_{F_t}[\tilde{x}]$, in which case I set $c_t = 1$. The minimized crime rate conditional on pre-signal t is thus $1 - F_t\left(c_t\left(\frac{\overline{a}(t)}{m(t)}\right)\right)$. The designer's problem reduces to choosing an allocation policy to minimize the overall crime rate:

$$\min_{\overline{a}} \sum_{t\in\overline{S}} m(t)\left[1 - F_t\left(c_t\left(\frac{\overline{a}(t)}{m(t)}\right)\right)\right] \tag{11}$$

subject to the capacity constraint, (9). The following result formalizes the above observation (see Appendix A.2 for the full proof).

Proposition 1. *The designer chooses an allocation policy $\overline{a}$ that solves (11) and sets each (S_t, π_t) as the truth-or-noise signal structure with cutoff $c_t\left(\frac{\overline{a}(t)}{m(t)}\right)$. Under the designer's optimal allocation policy, $c_t\left(\frac{\overline{a}(t)}{m(t)}\right)$ solves (10) for each $t \in \overline{S}$.*

I consider an example with two possible pre-signals.

Example 2. There are two equally likely pre-signals, 1 and 2, such that $F_1(x) = F_2(x) = F(x) = x^\beta$ with $\beta > 0$, i.e., the pre-signals divide the population into two groups of equal size and type distribution. Since $F_1 = F_2$, I omit the dependence of F_t and c_t on t. For simplicity, assume that $0.5\mathbb{E}_F[x] < \overline{P} < \mathbb{E}_F[x]$. Because $m(1) = m(2) = 0.5$, the designer's problem (11) is

$$\min_{\overline{a}(1),\overline{a}(2)\geq 0} 0.5\left[1-F\left(c\left(2\overline{a}(1)\right)\right)\right]+0.5\left[1-F\left(c\left(2\overline{a}(2)\right)\right)\right]$$
$$\text{subject to}\quad \overline{a}(1)+\overline{a}(2)=\overline{P},$$

or equivalently,

$$\max_{\overline{a}(1),\overline{a}(2)\geq 0} F\left(c\left(2\overline{a}(1)\right)\right)+F\left(c\left(2\overline{a}(2)\right)\right) \tag{12}$$
$$\text{subject to}\quad \overline{a}(1)+\overline{a}(2)=\overline{P}.$$

Direct calculation reveals that for each $t \in \{1,2\}$,[19]

$$c\left(2\overline{a}(t)\right)=\frac{2(1+\beta)}{\beta}\overline{a}(t)\quad \text{and}\quad F\left(c\left(2\overline{a}(t)\right)\right)=\left(\frac{2(1+\beta)}{\beta}\right)^{\beta}\overline{a}(t)^{\beta}.$$

The solution to (12) then becomes as follows: If $\beta>1$, each term $F\left(c\left(2\overline{a}(t)\right)\right)$ of the objective in (12) is strictly convex in $\overline{a}(t)$, which implies that the designer should first allocate as much search capacity as possible to one pre-signal and, once that pre-signal has a crime rate of 0 (or $F\left(c\left(2\overline{a}(t)\right)\right)=1$), allocate the remaining search capacity to the other pre-signal. Specifically, for one pre-signal, say 1, the designer sets $\overline{a}(1)=0.5\mathbb{E}_F[x]$ and provides the police with full information, so that the police can search each type x with probability x and attain a crime rate of 0. For pre-signal 2, the designer sets $\overline{a}(2)=\overline{P}-0.5\mathbb{E}_F[x]$ and provides the police with the truth-or-noise signal structure with cutoff

$$c\left(2\overline{a}(2)\right)=\frac{2(1+\beta)}{\beta}\left(\overline{P}-0.5\mathbb{E}_F[x]\right).$$

In contrast, if $\beta\in(0,1)$, each term $F\left(c\left(2\overline{a}(t)\right)\right)$ of the objective in (12) is strictly concave in $\overline{a}(t)$, which implies that the designer should set $\overline{a}(1)=\overline{a}(2)=0.5\overline{P}$ and provide the police

[19] It holds that $\mathbb{E}[x|x\leq c]=\int_0^c \frac{x\cdot\beta x^{\beta-1}}{c^{\beta}}\,\mathrm{d}x=\frac{\beta}{1+\beta}c$, so $\mathbb{E}[x|x\leq c]=2\overline{a}(t)$ implies $c\left(2\overline{a}(t)\right)=\frac{2(1+\beta)}{\beta}\overline{a}(t)$.

with the truth-or-noise signal structure with cutoff

$$c\left(2\overline{a}(t)\right) = \frac{1+\beta}{\beta}\overline{P}$$

for both pre-signals.

Remark 3 (Partial Commitment with the Fully Informed Police). Considering the designer with partial commitment power also affects Theorem 0, which states that fully informing the police leads to maximal crime. Suppose that the designer is restricted to providing the police with full information, i.e., (S_t, π_t) is fully informative for each pre-signal $t \in \overline{S}$. If the designer allocates a search capacity weakly greater than $m(t)\mathbb{E}_{F_t}[x]$, the game between the police and the agents with pre-signal t has an equilibrium with a crime rate of 0, because the police can search each agent with a probability equal to their type, x. In contrast, if the search capacity is strictly below $m(t)\mathbb{E}_{F_t}[x]$, then, as in Theorem 0, the crime rate for pre-signal t is 1. Therefore, the designer's allocation problem is to choose a set of pre-signals to which the designer allocates search capacity $m(t)\mathbb{E}_{F_t}[x]$ subject to the capacity constraint (9). As shown in Appendix A.3, this is a knapsack problem.

4 Endogenous Search Capacity

In this section, I assume that the police can choose any search strategy p at cost $C(P)$, where P is the total mass of searches induced by p, i.e.,

$$P \triangleq \int_0^1 \int_S p(s)\ \mathrm{d}\pi(s|x)\,\mathrm{d}F(x).$$

The police's payoff is the mass of successful searches (2) minus cost $C(P)$. The cost function $C(\cdot)$ is strictly increasing, strictly convex, differentiable, and satisfies

$$C'(0) < 1 < C'\left(\int_0^1 x\,\mathrm{d}F(x)\right).$$

The first inequality implies that the police search a positive mass of agents under any signal structure and any equilibrium. The second inequality ensures that the equilibrium crime rate is positive, which plays the same role as Assumption 1. The rest of the model, such as the agents' payoffs and the timing, remains the same.

Theorem 0 holds in this setup with the same intuition. Therefore, information restriction is necessary for crime deterrence under endogenous search capacity (all omitted proofs of this section are in Appendix B).

Claim 1. *An equilibrium exists, and in any equilibrium, almost every agent commits a crime with probability* 1*.*

4.1 Relaxed Problem with Endogenous Search Capacity

I now consider a crime-minimizing signal structure with endogenous search capacity. First, I study the relaxed problem, in which the agents do not directly observe their types and the joint signal structure (S_P, S_A, π) is chosen to minimize the equilibrium crime rate.

Figure 2 describes a solution to the relaxed problem. Similar to the case of the exogenous search capacity (Lemma 1), the agents receive signals *crime* or *not* depending on whether their types exceed some cutoff c^*, and in equilibrium, they follow action recommendations. The police's signal is a garbling of the agents' signals and may be different from the uninformative signal. In particular, the police now privately identify—and therefore choose not to search—a fraction α^* of agents who receive signal *not*. For the remaining population, the police apply the same search rate $\rho^* > 0$. Parameter α^* can be 0, in which case the crime-minimizing joint signal structure takes qualitatively the same form as Lemma 1, i.e., the police receive no information and the agents receive a cutoff signal.

Lemma 2. *In the relaxed problem, the following joint signal structure (S_P^*, S_A^*, π^*), characterized by tuple $(\rho^*, c^*, \alpha^*) \in (0,1)^2 \times [0,1)$, minimizes a crime rate:*

1. *For every type $x > c^*$, the realized signal is $(s_i^P, s_i^A) = (\rho^*, crime)$ with probability* 1*. For every type $x < c^*$, the realized signal is $(0, not)$ or (ρ^*, not) with probability α^* or $1 - \alpha^*$, respectively. In equilibrium, the agents and the police follow the action recommendations.*

2. *Tuple (ρ^*, c^*, α^*) satisfies $\mathbb{E}_F[\tilde{x}|\tilde{x} \leq c^*] = (1-\alpha^*)\rho^*$, i.e., the agents who observe signal "not" are indifferent between committing a crime and not.*

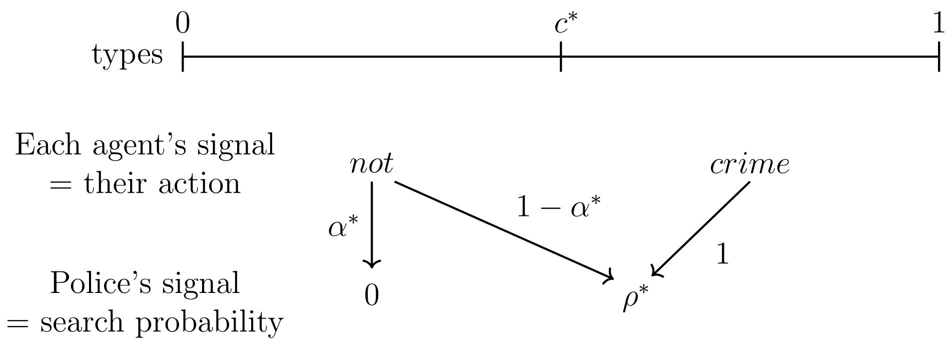

Figure 2: A solution to the relaxed problem with endogenous search capacity. The police identify a fraction α^* of the agents who do not commit crimes.

When $\alpha^* > 0$, the crime-minimizing signal structure of the relaxed problem enables the police to identify a fraction of agents who do not commit crimes in equilibrium. To see why such partial revelation can be optimal when the search capacity is endogenous, suppose that a mass $r \in (0,1)$ of agents are committing crimes. Compare the following two cases: In Case 0, the police have no information. In Case α, the police can privately identify a fraction α of the agents who do not commit crimes (or equivalently, a mass $\alpha(1-r)$ of such agents). Moving from Case 0 to Case α affects the costs of committing crimes in two ways. First, the agents who do not commit crimes are on average less likely to be exposed to search, because the police do not search a fraction α of them. Second, the police choose a higher total search capacity in Case α than Case 0, because the police can detect a crime with probability $\frac{r}{1-\alpha(1-r)} > r$ by searching a mass $1-\alpha(1-r)$ of unidentified agents in Case α, whereas the probability of detecting a crime is r in Case 0. The crime-minimizing signal structure involves partial revelation (i.e., $\alpha^* > 0$) when the second effect dominates, so that the overall costs for the agents of committing crimes increase as we move from Case 0 to Case α.

At the same time, consistent with the case of exogenous search capacity, the crime-minimizing signal structure does not enable the police to identify "criminals," the agents who commit crimes. To see why, consider Case β in which the police can identify a fraction β of criminals. In contrast to Case α, moving from Case 0 to Case β could reduce the police's effort, because the probability of detecting a crime in the unidentified population is $\frac{(1-\beta)r}{1-\beta r} < r$. Moreover, as in the baseline model, the information about criminals distorts the allocation of searches and reduces the deterrent effect of search. Therefore, allowing the police to detect who will commit crimes could increase the crime rate by both reducing search effort and distorting its allocation.

4.2 The Optimality of the Truth-or-Noise Signals

When $\alpha^* = 0$ in Lemma 2, the solution to the relaxed problem with endogenous search capacity is qualitatively the same as the case of exogenous search capacity. Correspondingly, the truth-or-noise signal structures continue to solve the original problem.

Lemma 3. *Suppose that the solution to the relaxed problem described in Lemma 2 has $\alpha^* = 0$, i.e., the police receive no information. Then, the crime-minimizing signal structure of the original problem is the truth-or-noise signal structure with cutoff c^*, where c^* and the equilibrium search capacity P^* jointly solve*

$$\mathbb{E}_F[\tilde{x}|\tilde{x} \le c^*] = P^*, \text{ and} \tag{13}$$

$$1 - F(c^*) = C'(P^*). \tag{14}$$

Proof. Equation (13) and the same argument as Theorem 1 imply that under the truth-or-noise signal structure with cutoff c^*, there exists a strategy profile such that: the agents optimally commit crimes if $x > c^*$ and not if $x < c^*$; the police adopt search strategy $p(s) = s$ for every $s \in [0, c^*]$; and the police cannot increase their payoffs by changing the search strategy while keeping the total search capacity P^* fixed. Thus, it remains to show that the police have no profitable deviation in terms of changing the total search capacity. The police's payoff from search capacity P is $(1 - F(c^*))P - C(P)$, because the posterior crime rate is equalized to be $1 - F(c^*)$ across all signals. Due to the convexity of $C(\cdot)$, equation (14) is sufficient for the optimality of P^*. □

Lemma 3 provides a sufficient condition under which Theorem 1 extends to the case of endogenous search capacity. However, it does not tell when $\alpha^* = 0$ holds. The following result provides a condition for $\alpha^* = 0$. It relies on the following restriction on the cost function.

Assumption 2. The police's search cost function takes the form of

$$C(P) = \frac{L}{1+\beta} P^{1+\beta}$$

for some $\beta > 0$ and $L > 1$.

Proposition 2. *Assume that (F, β, L) satisfies*

$$\mathbb{E}_F\left[\tilde{x} \,\middle|\, \tilde{x} < F^{-1}\left(\frac{\beta}{1+\beta}\right)\right] > ((1+\beta)L)^{-\frac{1}{\beta}}. \tag{15}$$

Then, $\alpha^* = 0$ holds in the relaxed problem. Therefore, the crime-minimizing signal structure of the original problem is a truth-or-noise signal structure, where the cutoff and the equilibrium search capacity jointly solve equations (13) and (14).

For a fixed β, inequality (15) is more likely to hold when the police face a greater search cost (i.e., L is high) or the agents face greater returns from crimes (i.e., $F^{-1}\left(\frac{\beta}{1+\beta}\right)$ is high). When these conditions hold, the police incur a high marginal cost of search in equilibrium. In such a case, revealing a fraction of agents with signal *not* has little effect on the police's search effort but decreases the probability $1-\alpha$ with which the agents with signal *not* are exposed to search. To relax the obedience constraint for signal *not* as much as possible and attain a lower crime rate, the crime-minimizing signal structure in the relaxed problem provides no information to the police. In this case, the crime-minimizing signal structure in the original problem becomes a truth-or-noise signal structure.

While the general analysis of the original problem is beyond the scope of the paper, the following example shows that even when $\alpha^* > 0$, a natural generalization of truth-or-noise signal structures may solve the original problem.

Claim 2. *Suppose that $F = U[0,1]$ and $C(P) = \frac{L}{2}P^2$ with $L \geq \frac{3+\sqrt{5}}{4} \approx 1.31$. The equilibrium crime rate is minimized by a signal structure that reveals a fraction $\alpha^* = \max(0, 2-\sqrt{2L})$ of the agents with types below c^* and discloses the truth-or-noise signal structure with cutoff c^* for the rest of the agents.*

In the above example, when $L \in [\frac{3+\sqrt{5}}{4}, 2)$, the crime-minimizing outcome is as follows: The police privately identify a fraction α^* of agents who have types below c^* as signal ϕ. As to the remaining population, the police observe the truth-or-noise signal with cutoff c^*, i.e., a signal coincides with an agent's true type if the type is below c^*, and otherwise the signal is a noise drawn from $F(\cdot|\tilde{x} \leq c^*)$. These signals are private to the police, so the agents with type $x < c^*$ do not know whether the police observe signal ϕ or x. In equilibrium, the police search signal $s \in [0, c^*]$ with probability $\frac{s}{1-\alpha^*}$ and signal ϕ with probability 0. Facing such a strategy, types below c^* are indeed willing to not commit crimes, because any agent with type $x < c^*$ believes that they will be searched with probability $(1-\alpha^*)\frac{x}{1-\alpha^*} = x$.[20]

5 Extension

I examine the consequences of modifying various assumptions in the baseline model. Throughout the section, the police have a fixed search capacity of $\overline{P} < \int_0^1 x \, \mathrm{d}F(x)$ as in Section 2.

5.1 Police with Commitment Power

Assume that the police have full information about the agents' types and can commit to any search strategy upfront. Formally, the police first choose a search strategy $p : S \to [0,1]$, which is observed by all agents, and then each agent takes an action. The police's payoff is given by

$$\hat{\lambda}\sigma - (1-\hat{\lambda})r,$$

where σ is the mass of successful searches (2), r is the crime rate (3), and $\hat{\lambda} \in [0,1]$. I focus on an equilibrium that maximizes the police's payoff.

The minimum search capacity the police need for attaining crime rate r is

$$\int_0^{F^{-1}(1-r)} t \, \mathrm{d}F(t), \tag{16}$$

which arises when the police search every type $t \leq F^{-1}(1-r)$ with probability t, who

[20]In general, the above signal structure and strategy profile may not solve the original problem. For example, the police's search strategy may be infeasible because we may have $\frac{s}{1-\alpha^*} > 1$. Also, some types above c^*, who anticipate search probability $\frac{\mathbb{E}[\tilde{x}|\tilde{x} \leq c^*]}{1-\alpha^*}$, may abstain from committing crimes, which leads to a different outcome from the solution to the relaxed problem.

abstains from committing a crime. The mass of successful searches consistent with crime rate r is then at most $\overline{P} - \int_0^{F^{-1}(1-r)} t \, \mathrm{d}F(t)$. The police can indeed attain this upper bound without changing the crime rate, by committing to search each type $t > F^{-1}(1-r)$ with probability equal to or less than t, so that (i) the police exhaust search capacity $\overline{P}$ and (ii) all types above $F^{-1}(1-r)$ commit crimes. The police's problem thus reduces to the choice of a crime rate r that maximizes their payoff

$$\hat{\lambda}\left(\overline{P} - \int_0^{F^{-1}(1-r)} t \mathrm{d}F(t)\right) - (1-\hat{\lambda})r$$

subject to $r \geq r^*$, where $r^* \in (0,1)$ is the minimum crime rate under commitment, which solves

$$\int_0^{F^{-1}(1-r^*)} t \, \mathrm{d}F(t) = \overline{P}. \tag{17}$$

Equivalently, I can state the police's problem as the choice of a cutoff type $x = F^{-1}(1-r)$, i.e.,

$$\max_{x \in [0,\overline{x}]} \hat{\lambda}\left(\overline{P} - \int_0^{x} t \mathrm{d}F(t)\right) - (1-\hat{\lambda})[1-F(x)],$$

where $\overline{x} \triangleq F^{-1}(1-r^*)$. The derivative of the police's objective with respect to x is

$$-\hat{\lambda}x + 1 - \hat{\lambda},$$

which crosses 0 at most once and always from above as x increases. Solving the first-order condition and taking into account the constraint $x \leq \overline{x}$, we can derive the optimal cutoff:

$$x^* = \min\left(\frac{1-\hat{\lambda}}{\hat{\lambda}}, \overline{x}\right).$$

Claim 3. *Suppose that the police can commit to their search strategy upfront and have full information about the agents' types. In the police-preferred equilibrium, the crime rate is* $1 - F\left(\min\left(\frac{1-\hat{\lambda}}{\hat{\lambda}}, \overline{x}\right)\right)$*, which increases from* $1 - F(\overline{x})$ *to* 1 *as the weight* $\hat{\lambda}$ *on successful searches increases from* 0 *to* 1.

If $\hat{\lambda} = 1$, the police maximize the mass of successful searches. In this case, Claim 3 implies that the equilibrium crime rate is 1: The police allocate search capacity so that every agent commits a crime, leading to the maximal crime rate and successful searches. This resembles Theorem 0 but is different: Theorem 0 holds for any $\hat{\lambda} \in (0, 1]$, whereas in the case of commitment, the crime rate of 1 arises only when $\hat{\lambda} = 1$.

If $\hat{\lambda} = 0$, the police minimize the crime rate. The police search the types below cutoff $\overline{x}$ with probabilities equal to their types. In equilibrium, all agents who are searched are indifferent and thus abstain from committing crimes. The police with commitment power can attain a lower crime rate than the police with no commitment power. Indeed, the cutoff type for the crime-minimizing outcome in Theorem 1 solves $\int_0^{\hat{c}} \frac{t}{F(\hat{c})} \, \mathrm{d}F(t) = \overline{P}$ (which is (6)), whereas the cutoff type in this commitment model with $\hat{\lambda} = 0$ solves $\int_0^{\overline{x}} t \, \mathrm{d}F(t) = \overline{P}$. Since $F(\hat{c}) < 1$, we must have $\hat{c} < \overline{x}$. Therefore, compared to the baseline model, the police with commitment power can additionally deter the types in $[\hat{c}, \overline{x}]$ from committing crimes.

5.2 Alternative Objectives of the Police

I consider two payoff specifications for the police under which the equilibrium entails the crime-minimizing outcome of the full commitment benchmark in Section 5.1 (i.e., Claim 3 with $\hat{\lambda} = 0$). The police are assumed to have full information about types and move simultaneously with the agents. Recall that in the crime-minimizing outcome under commitment, the agents commit crimes if and only if their types exceed $\overline{x}$.

Police that prefer to search innocents. Suppose that the police's payoff is equal to the mass of agents who did not commit crimes but are searched, i.e.,

$$\int_0^1 \int_S p(s) \, \mathrm{d}\pi(s|x)[1 - a(x)] \, \mathrm{d}F(x), \tag{18}$$

where $a(x)$ is the probability with which type x commits a crime. In equilibrium, the police search each type $x \leq \overline{x}$ with probability x and never search types $x > \overline{x}$. Correspondingly, the agents commit crimes if and only if $x > \overline{x}$. The police have no incentive to deviate, because they prefer to search agents who did not commit crimes rather than those who did.

Police that care only about crime rate. Suppose that the police's objective is to minimize crime rate (3), which does not directly depend on the police's strategy. Then, an arbitrary search strategy combined with the agents' best responses constitutes an equilibrium, including the crime-minimizing outcome under full commitment.

5.3 Balancing Crime Rate and Costs Imposed on Innocents

The cost of policing on innocent people has been an important consideration in the discussion of policing (see, e.g., Section VII of Persico (2002)). In this section, I characterize a signal structure that balances such costs and the crime rate. The timing and payoffs are the same as in the baseline model (Section 2).

Take any signal structure (S, π) and a strategy profile. The *search mass on innocents* is defined as (18), i.e., the mass of agents who did not commit crimes but are searched. I take this as the cost of policing imposed on innocent agents. The minimal search mass on innocents consistent with crime rate r is

$$\int_0^{F^{-1}(1-r)} x \, \mathrm{d}F(x), \tag{19}$$

which arises if all types below $F^{-1}(1-r)$ are searched with probabilities equal to their types and abstain from committing crimes. The crime-minimizing outcome in Theorem 1 has this property, because given the minimized crime rate $1 - F(\hat{c})$, the search mass on innocents equals $\int_0^{\hat{c}} x \, \mathrm{d}F(x)$.

Regardless of the signal structure, the equilibrium crime rate cannot be below $1 - F(\hat{c})$. Thus, I fix an arbitrary crime rate $r \in [1 - F(\hat{c}), 1]$ and characterize a signal structure and an equilibrium that minimize the search mass on innocents subject to achieving the crime rate r. Even though I do not explicitly formulate an information designer who cares about the crime rate and the costs imposed on innocents, the optimal choice of any such designer would belong to the class of signal structures characterized below.

Claim 4. *Let $\hat{r} = 1 - F(\hat{c})$ be the minimal crime rate of the baseline model. For any $r \in [\hat{r}, 1]$, there exists a signal structure and an equilibrium that attain the minimal search mass on innocents given by (19) and crime rate r.*

In Appendix E, I construct a signal structure and equilibrium in which any type $x > F^{-1}(1-r)$ commits a crime and any type $x < F^{-1}(1-r)$ faces search probability x and chooses not to commit a crime. This outcome attains the crime rate r and the minimal search mass on innocents (19). The signal structure resembles a truth-or-noise signal structure of Theorem 1 with one modification: For any type $x > F^{-1}(1-r)$, the signal reveals the type with some type-dependent probability. Whenever that happens, the police, knowing that the agent will commit a crime, set search probability equal to 1. Otherwise, each type $x > F^{-1}(1-r)$ is pooled with one of the types below $F^{-1}(1-r)$, as in the truth-or-noise signals. The probability of revelation is such that in expectation, any type above $F^{-1}(1-r)$ still commits a crime with probability 1. As the probability of revelation increases, the police can more easily find agents who commit crimes. Consequently, the police allocate more search resources to criminals, which results in a lower search mass on innocents and a higher crime rate.

6 Conclusion

From an applied perspective, the crime-minimizing signal structure in Theorem 1 provides a way to garble information so that a law enforcer cannot identify agents who face high returns from crime but can still learn about those with low returns from crime. The exact implementation of such an information structure is difficult in practice—e.g., the set of feasible information structures is limited by various constraints, such as legal restrictions. However, Example 1 suggests that a similar garbling procedure might help even if the underlying information is already coarse. From a theoretical perspective, the paper studies an information design problem in which the sets of players, actions, and states are infinite, and the feasible information structures are restricted (i.e., agents must know their types). The paper uses an approach based on a relaxed problem, which could be useful for other information design problems with similar constraints.

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Appendix A: Omitted Materials for Section 3

A.1. Proof of the Claim in Footnote 13

The argument in Footnote 13 rests on the following claim:

Claim 5. *Consider any deterministic signal structure (S, π), i.e., for each type $x \in [0, 1]$, the distribution $\pi(\cdot|x)$ over signals is degenerate at some $s \in S$. Then in any equilibrium, the crime rates are equal across almost all signals, and thus the police are indifferent between any search strategies that exhaust search capacity $\overline{P}$.*

Proof. If the signal structure is deterministic, each agent, who knows their type, also knows their signal. Take any equilibrium, and let $r(S')$ be the crime rate conditional on a set of signals $S' \subseteq S$. Suppose to the contrary that $r(S_2) > r(S_1)$ for some sets of signals, S_1 and S_2, that can arise with positive ex ante probabilities under (S, π). The inequalities imply $r(S_1) < 1$ and $r(S_2) > 0$. Inequality $r(S_1) < 1$ means that the police allocate positive search mass to signals in S_1, and $r(S_2) > 0$ implies that the search rate for signals in S_2 is strictly below 1.[21] Then the police would profitably deviate by shifting search mass from signal S_1 to S_2. This is a contradiction, so the crime rate must be equalized across almost all signals. □

A.2. Proof of Proposition 1

I solve the designer's problem in two steps. First, I fix an allocation policy $\overline{a}$ arbitrarily and then solve for the optimal signal structures. As discussed in the main text, if $\mathbb{E}_{F_t}[\tilde{x}] \geq \frac{\overline{a}(t)}{m(t)}$, the equation $\mathbb{E}_{F_t}[\tilde{x}|\tilde{x} \leq c] = \frac{\overline{a}(t)}{m(t)}$ has a unique solution $c_t\left(\frac{\overline{a}(t)}{m(t)}\right)$, and the designer can attain

[21] This argument could fail under a stochastic signal structure. For example, if some agents receive signal s_1 or s_2 with a positive probability, it is possible that, even though the police search agents with signal s_2 with probability 1, these agents still commit crimes because the search rate for s_1 may be low, and the agents may be uncertain about whether their signals will be s_1 or s_2. If we assume that the realized signals are publicly observable, Claim 5 holds under any signal structure.

the minimal crime rate by providing the police with the truth-or-noise signal structure with cutoff $c_t\left(\frac{\overline{a}(t)}{m(t)}\right)$. If $\mathbb{E}_{F_t}[\tilde{x}] < \frac{\overline{a}(t)}{m(t)}$, define $c_t\left(\frac{\overline{a}(t)}{m(t)}\right) = 1$. In this case, the minimal crime rate is indeed $1 - F_t\left(c_t\left(\frac{\overline{a}(t)}{m(t)}\right)\right) = 0$, because the designer can provide the police with full information, and the police can deter all crimes (by the agents with pre-signal t) with search strategy $p(x) = x$ for all $x \in [0,1]$. Overall, the crime-minimizing (S_t, π_t) leads to the equilibrium crime rate of $1 - F_t\left(c_t\left(\frac{\overline{a}(t)}{m(t)}\right)\right)$. The designer's optimal allocation policy then solves (11).

I now show that $\mathbb{E}_{F_t}[\tilde{x}] < \frac{\overline{a}(t)}{m(t)}$ never arises at the optimal allocation policy chosen by the designer. To see why, note that even in this extended model, the equilibrium crime rate is strictly positive for at least one pre-signal, say $t' \in \overline{S}$, because the designer faces a search capacity $\overline{P}$ that satisfies Assumption 1. Suppose to the contrary that $\mathbb{E}_{F_t}[\tilde{x}] < \frac{\overline{a}(t)}{m(t)}$ holds. Then the designer can improve its payoff in the following way. First, the designer slightly decreases the search capacity $\overline{a}(t)$ allocated to pre-signal t by $\epsilon > 0$ so that $\mathbb{E}_{F_t}[\tilde{x}] < \frac{\overline{a}(t)-\epsilon}{m(t)}$ holds. The resulting crime rate for pre-signal t remains 0. Then, the designer increases the search capacity allocated to a pre-signal (say t') that has a positive crime rate by ϵ. Originally, the signal structure for pre-signal t' was the truth-or-noise signal with cutoff c where $\mathbb{E}_{F_{t'}}[\tilde{x}|\tilde{x} \le c] = \frac{\overline{a}(t)}{m(t)}$. For a small $\epsilon > 0$, the cutoff now increases to $c_\epsilon > c$ that solves $\mathbb{E}_{F_{t'}}[\tilde{x}|\tilde{x} \le c_\epsilon] = \frac{\overline{a}(t)+\epsilon}{m(t)}$. The designer can then replace $(S_{t'}, \pi_{t'})$ with the truth-or-noise signal with cutoff c_ϵ. Because $c_\epsilon > c$, this will reduce the equilibrium crime rate for pre-signal t' as well as the overall crime rate. But then we obtain a contradiction. Therefore, under the designer's optimal search strategy, $\mathbb{E}_{F_t}[\tilde{x}] \ge \frac{\overline{a}(t)}{m(t)}$ holds, so $\mathbb{E}_{F_t}[\tilde{x}|\tilde{x} \le c] = \frac{\overline{a}(t)}{m(t)}$ has a unique solution c for every $t \in \overline{S}$. This proves the last part of Proposition 1. $\square$

A.3. Omitted Formalism for Remark 3

This appendix studies the case in which the designer can control an allocation policy based on pre-signals as in Section 3.3 but must provide the police with full information. We begin with a lemma that extends Theorem 0:

Lemma 4. *Suppose that signal structure (S_t, π_t) is fully informative for every $t \in \overline{S}$. Take any pre-signal t. If $\overline{a}(t) \ge m(t)\int_0^1 x\, \mathrm{d}F_t(x)$, then in the game between the police* *and the agents with pre-signal t, there exists an equilibrium with a crime rate of* 0. *If* $\overline{a}(t) < m(t)\int_0^1 x\,\mathrm{d}F_t(x)$*, any equilibrium has a crime rate of* 1.

Proof. To simplify notation, without loss of generality, we set $m(t) = 1$. Suppose that $\overline{a}(t) \geq \int_0^1 x\,\mathrm{d}F_t(x)$. First, we construct an equilibrium with a crime rate of 0. Suppose that the police adopt search strategy $p^*(x) = x, \forall x \in [0,1]$. This search strategy is feasible because it induces the total search capacity of at most $\int_0^1 p^*(x)\,\mathrm{d}F_t(x) \leq \overline{a}(t)$. Under p^*, it is optimal for every agent to not commit a crime . The police then find it optimal to choose any search strategy because the mass of successful searches is 0 regardless of the search strategy. Thus we obtain an equilibrium with crime rate 0. The second part follows from Theorem 0. $\square$

I now consider the designer's allocation problem when the police have full information. First, minimizing crime is equivalent to maximizing the mass of agents who do not commit crimes (which we call the mass of innocents) in equilibrium. Thus, we assume that the designer's payoff equals the mass of innocents. By Lemma 4, the designer's payoff from pre-signal t is $m(t)$ if $\overline{a}(t) \geq m(t)\int_0^1 x\,\mathrm{d}F_t(x)$ and 0 otherwise. It is then without loss of generality to assume that for each pre-signal t, the designer either allocates $m(t)\int_0^1 x\,\mathrm{d}F_t(x)$ or nothing. The designer's problem can then be restated as follows:

$$\max_{D\subseteq\overline{S}} \sum_{t\in D} m(t)$$

$$\text{subject to} \quad \sum_{t\in D} m(t)\int_0^1 x\,\mathrm{d}F_t(x) \leq \overline{P}.$$

This is a knapsack problem in which the set of items is $\overline{S}$, the weight of each item $t \in \overline{S}$ is $m(t)\int_0^1 x\,\mathrm{d}F_t(x)$, the value of item t is $m(t)$, and the maximum capacity is $\overline{P}$.

A.4. Type-Dependent Payoffs of the Police

I modify the baseline model so that the police's payoff of searching an agent who commits a crime is $1 + \eta g(x)$, where x is the agent's type, $\eta \geq 0$ is an exogenous parameter that captures the degree of type-dependency, and $g : [0,1] \to \mathbb{R}_+$ is an arbitrary nonnegative continuous function such that $g(x) = 0$ for some $x \in [0,1]$. Thus, if the police face signal structure (S, π) and adopt search strategy p and type x commits a crime with probability $a(x)$, the police's payoff—the mass of successful searches that reflects the type-dependent priority to search—is

$$\int_0^1 \int_S p(s)\, \mathrm{d}\pi(s|x) a(x)[1 + \eta g(x)]\, \mathrm{d}F(x).$$

To extend Theorem 0 in this general setup, define $\gamma > 0$ as follows:

$$\int_{\{x: g(x) \geq \gamma\}} x\, \mathrm{d}F(x) = \overline{P}. \tag{20}$$

For simplicity, we assume that γ exists.[22] Then we obtain the following result:

Claim 6. *Suppose that the police have the type-dependent payoffs and full information. There exists an equilibrium with the following property: The police search agents whose types satisfy $g(x) \geq \gamma$ with probability x. These agents are indifferent and commit crimes with probability*

$$a^*(x) = \frac{1 + \eta\gamma}{1 + \eta g(x)}. \tag{21}$$

The police search any type x with $g(x) < \gamma$ with probability 0, and these agents commit crimes with probability 1. As $\eta \to 0$, the equilibrium crime rate converges to 1.

Proof. Consider the strategy profile described in the statement. The agents' strategies are optimal. The police's payoff of searching any type x with $g(x) \geq \gamma$ is $a^*(x)(1 + \eta g(x)) = 1 + \eta\gamma$, whereas the payoff of searching any type $g(x) < \gamma$ is at most $1 + \eta\gamma$. As a result, the police's search strategy is optimal if it only searches types x with $g(x) \geq \gamma$ and exhausts search capacity $\overline{P}$. The candidate strategy satisfies the first property by construction and also the second property because of (20). Finally, in this equilibrium, the probability of any agent committing a crime is at least $\frac{1+\eta\gamma}{1+\eta \max_{x \in [0,1]} g(x)}$. Because γ is independent of η, as $\eta \to 0$, the crime rate converges to 1. $\square$

[22] Under Assumption 1, γ exists if for any $y \in g([0,1])$, its inverse image $g^{-1}(\{y\}) \subseteq [0,1]$ has measure zero.

Appendix B: Omitted Proofs for Section 4

Proof of Claim 1

First, I construct an equilibrium. Pick a unique $\bar{x}$ that solves $C'\left(\int_0^{\bar{x}} t \, \mathrm{d}F(t)\right) = 1$. Consider the strategy profile such that the police adopt a search strategy $p^*(x) = x$ for $x \leq \bar{x}$ and $p^*(x) = 0$ for $x > \bar{x}$, and all agents commit crimes with probability 1. The agents' strategies are optimal. The police's search strategy is also optimal, because the police's payoff from searching mass P of agents is $P - C(P)$, so the strategy is optimal if the police search mass P^* of agents where $C'(P^*) = 1$. Search strategy p^* has this property by construction.

Second, take any equilibrium. Let p be the equilibrium search strategy and $P = \int_0^1 p(x) \, \mathrm{d}F(x)$ be the total mass of searches under p. We have $C'(P) \leq 1$, because otherwise the police could profitably deviate by slightly lowering the mass of searches. Combining $C'(P) \leq 1$ with $1 < C'\left(\int_0^1 x \, \mathrm{d}F(x)\right)$, we obtain $P < \int_0^1 x \, \mathrm{d}F(x)$. We can then apply the same argument as the proof of Theorem 0: The set $X \triangleq \{x \in [0,1] : x > p(x)\}$ has a positive mass, and any type in X commits a crime with probability 1. Let Y be the set of types that commit crimes with probability strictly below 1. If Y has a positive mass, the police must be allocating a positive mass of searches to types in Y, because otherwise they would commit crimes. We then obtain a contradiction, because the police could increase the mass of successful searches without changing the search cost $C(P)$ by maintaining the total search capacity at P and shifting search probabilities from types in Y to X. Thus, the set Y has measure zero, i.e., almost every agent commits a crime with probability 1. $\square$

Proof of Lemma 2

In what follows, a joint signal structure is referred to as a signal structure. Take any signal structure (S_P, S_A, π) and any strategies chosen by agents. For each $s \in S_P$, the *posterior crime rate*, $r(s) \in [0,1]$, is defined as the probability that an agent commits a crime conditional on the police's signal s (but not conditional on the agent's signal).[23] To prove Lemma

[23] Let $a(t)$ denote the probability that an agent commits a crime after observing signal $t \in S_A$. The posterior crime rate is given by $r(s) = \mathbb{E}[a(\tilde{t})|s]$, where the expectation is with respect to an agent's signal $\tilde{t}$ conditional on the police's signal $s \in S_P$.

2, I first prove the following lemma to restrict the class of signal structures that need to be considered.

Lemma 5. *Consider the relaxed problem, and take any signal structure (S'_P, S'_A, π') and any equilibrium with crime rate $r \in (0,1)$. There is some $c \in (0,1)$ such that the same crime rate arises under a signal structure (S_P, S_A, π) and an equilibrium with the following properties:*

1. *Each agent receives signal "crime" or signal "not" if $x > c$ or $x < c$, respectively. In equilibrium, agents follow action recommendations.*

2. *The police's signal is a garbling of an agent's signal, i.e., there exist conditional distributions of the police's signal given an agent's signal, denoted by $\hat{\pi}_P(\cdot|crime), \hat{\pi}_P(\cdot|not) \in \Delta S_P$, such that for any $S \subset S_P$, $a \in \{crime, not\}$, and $x \in [0,1]$, we have $\pi(S \times \{a\}|x) = \hat{\pi}_P(S|a)\pi_A(a|x)$, where $\pi_A(a|x) \triangleq \pi(S_P \times \{a\}|x)$. In equilibrium, each signal of the police leads to a distinct posterior crime rate.*

Proof. Take any signal structure (S'_P, S'_A, π') and any equilibrium with crime rate $r \in (0,1)$. Let p' denote the police's equilibrium search strategy. First, as in Lemma 1, replace the signal space S'_A of agents with $S_A = \{crime, not\}$ and assume that each agent follows the action recommendation in equilibrium.

Second, replace each signal $s' \in S'_P$ of the police with its posterior crime rate $r(s')$ induced by the agents' strategies. Let $S_P \subset [0,1]$ be the resulting signal space for the police. This modification reduces the police's information, because different signals in S'_P may have the same posterior crime rate. I then assume that the police adopt search strategy

$$p(y) \triangleq \mathbb{E}[p'(s')|r(s') = y], \forall y \in S_P, \tag{22}$$

where the expectation is with respect to the police's original signal $s' \in S'_P$ conditional on that the posterior crime rate associated with the signal equals y. The police find it optimal to adopt p because they can ensure the same mass of successful searches as p' despite having less information under S_P than under S'_P. The agents' incentives remain the same (i.e., the obedience constraints continue to hold), because strategies p' and p induce the same expected search probability conditional on each action recommendation. Indeed, as explained below, it holds that for each $a \in \{crime, not\}$, $\mathbb{E}_{s'}[p'(s')|a] = \mathbb{E}_y[p(y)|a]$, or more specifically,

$$\mathbb{E}_{s'}[p'(s')|a] = \mathbb{E}_y[\mathbb{E}_{s'}[p'(s')|r(s') = y, a]|a] = \mathbb{E}_y[\mathbb{E}_{s'}[p'(s')|r(s') = y]|a] = \mathbb{E}_y[p(y)|a]. \tag{23}$$

Here, $\mathbb{E}_{s'}[\cdot|a]$ is the expectation with respect to the police's signal $s' \in S'_P$ conditional on action recommendation a; $\mathbb{E}_y[\cdot|a]$ is the expectation with respect to the police's signal $y \in S_P$ as a posterior crime rate conditional on a; $\mathbb{E}_{s'}[\cdot|r(s') = y, a]$ is the expectation with respect to the police's signal $s' \in S'_P$ conditional on posterior crime rate y and action recommendation a; and $\mathbb{E}_{s'}[\cdot|r(s') = y]$ is the expectation with respect to the police's signal $s' \in S'_P$ conditional only on posterior crime rate y.

I now explain why the equalities in (23) hold. The first equality is from the law of iterated expectation. The last equality is from the definition of search strategy p in (22). To show the second equality, suppose to the contrary that $\mathbb{E}_{s'}[p'(s')|r(s') = y, a] \neq \mathbb{E}_{s'}[p'(s')|r(s') = y]$ for some $y \in S_P$ and $a \in \{crime, not\}$, which is equivalent to

$$\mathbb{E}_{s'}[p'(s')|r(s') = y, crime] \neq \mathbb{E}_{s'}[p'(s')|r(s') = y, not]. \tag{24}$$

Hereafter, we fix such a y. Let $r^{-1}(y) \subseteq S'_P$ be the set of all signals $s' \in S'_P$ such that $r(s') = y$. The condition (24) means that there is some set $T \subsetneq r^{-1}(y)$ such that

$$\Pr(s' \in T|r(s') = y, crime) > \Pr(s' \in T|r(s') = y, not) \tag{25}$$

and thus

$$\Pr(s' \in T^c|r(s') = y, crime) < \Pr(s' \in T^c|r(s') = y, not) \tag{26}$$

where $T^c \triangleq r^{-1}(y) \setminus T$. For each $U \in \{T, T^c\}$, we obtain

$$\frac{\Pr(crime|s' \in U)}{\Pr(not|s' \in U)} = \frac{\Pr(crime|s' \in U, r(s') = y)}{\Pr(not|s' \in U, r(s') = y)} = \frac{\Pr(crime|r(s') = y)}{\Pr(not|r(s') = y)} \cdot \frac{\Pr(s' \in U|r(s') = y, crime)}{\Pr(s' \in U|r(s') = y, not)}, \tag{27}$$

where the first equality holds because $s' \in U$ implies $r(s') = y$, and the second equality follows from Bayes' rule. Combining (25), (26), and (27), we obtain

$$\frac{\Pr(crime|s' \in T)}{\Pr(not|s' \in T)} > \frac{\Pr(crime|s' \in T^c)}{\Pr(not|s' \in T^c)},$$

or equivalently, $\Pr(crime|s' \in T) > \Pr(crime|s' \in T^c)$. This contradicts the fact that signals in T and T^c have the same posterior crime rate y. Therefore, the second equality in (23) is valid.

Finally, let $\hat{\pi} \in \Delta(S_P \times S_A)$ denote the joint distribution of the police's signal (i.e., posterior crime rate) and an agent's signal (i.e., action recommendation). Let $\hat{\pi}_P(\cdot|a) \in \Delta S_P$ denote the associated conditional distribution of the police's signal given an agent's signal $a \in \{crime, not\}$. I then modify the signal structure as follows. First, given the equilibrium crime rate $r \in (0,1)$, assume that types above and below cutoff $c \triangleq F^{-1}(1-r)$ receive signals *crime* and *not*, respectively. Second, assume that conditional on each agent i's signal $a_i \in \{crime, not\}$, the police observe signal $y_i \sim \hat{\pi}_P(\cdot|a_i) \in \Delta S_P$ (regardless of i's type). These modifications (i) preserve the joint distribution of posterior crime rates and action recommendations across the population and (ii) relax the obedience constraints for the agents. As a result, the police optimally choose search strategy p defined in (22), and the agents continue to follow action recommendations. The resulting equilibrium has the properties stated in the lemma. □

I now prove Lemma 2, which characterizes the solution to the relaxed problem with endogenous search capacity.

Proof of Lemma 2. Take any signal structure (S_P, S_A, π) and any equilibrium that satisfy the properties described in Lemma 5 and have crime rate $r \in (0,1)$. Part 2 of the lemma ensures that the police's signal is a garbling of an agent's signal, generated by conditional distributions $\hat{\pi}_P(\cdot|crime), \hat{\pi}_P(\cdot|not) \in \Delta S_P$. Without loss, assume that the police's signals are recommended search probabilities that the police follow in equilibrium.

First, I show $S_P \subset \{0, \rho, 1\}$ for some $\rho \in (0,1)$. If S_P contains multiple interior search rates $\rho, \rho' \in (0,1)$, they must have the same posterior crime rate, i.e., $r(\rho) = r(\rho')$. For example, if $r(\rho) < r(\rho')$, the police would profitably deviate by shifting search probabilities from signal ρ to ρ' without changing the total search capacity. However, $r(\rho) = r(\rho')$ contradicts Part 2 of Lemma 5 that each signal leads to a distinct posterior crime rate.[24]

The unique interior search rate ρ (if exists) satisfies two properties. First, $\{\rho, 1\} \subset S_P$ implies $r(\rho) < r(1)$, because if $r(\rho) > r(1)$, the police would profitably deviate by shifting search masses from signal 1 to signal ρ (recall $r(\rho) \neq r(1)$ from Lemma 5). Second, the police equate the marginal cost of search with the marginal probability of detecting a crime. Hence, the equilibrium total search capacity P solves $C'(P) = r(\rho)$. Otherwise, the police would profitably deviate by slightly changing $p(\rho)$.

In the second step, we show that if $\{\rho, 1\} \subset S_P$, we can replace signals ρ and 1 with the same signal σ to increase the search probability allocated to signal *not*. Indeed, with the agents' strategies fixed, after we pool signals ρ and 1, the posterior crime rate $r(\sigma)$ for signal σ satisfies $r(\sigma) > r(\rho)$. Thus, if we let the police choose an optimal search strategy, the police will choose total search capacity $\tilde{P} > P$, because the marginal return on searches at P is now $r(\sigma) - C'(P) > r(\rho) - C'(P) = 0$. This pooling also reduces the police's information about the agents' signals (and their behavior) and thus increases the fraction of searches that go to signal *not*. As a result, if we fix the agents' strategies but let the police adopt an optimal search strategy, pooling signals ρ and 1 increases the expected search probability conditional on signal *not* and relaxes its obedience constraint.

The pooling procedure in the previous paragraph changes the police's signal space and the distribution of the police's signal conditional on the agent's action recommendation. However, to reduce notational burden, I continue using S_P for the police's signal space and $(\hat{\pi}_P(\cdot|crime), \hat{\pi}_P(\cdot|not))$ for the conditional distributions of the police's signal.

We now have a signal structure and a strategy profile such that $S_P = \{0, \sigma\}$ or $\{\sigma\}$ for some $\sigma > 0$, and the obedience constraint for signal *not* holds (possibly strictly).[25] If $S_P = \{0, \sigma\}$, we have $r(0) < r(\sigma)$, because otherwise the police would deviate by shifting search masses from signal σ to signal 0. ★ We then continuously increase cutoff type c defined in

[24] To be precise, this argument only implies that there exists some $\rho \in (0, 1)$ such that the ex ante probability of a signal belonging to $S_P \cap (0, 1) \setminus \{\rho\}$ is 0 (instead of this set being empty). However, we can replace all signals in $S_P \cap (0, 1) \setminus \{\rho\}$ with signal ρ without affecting the equilibrium crime rate.

[25] We cannot have $S_P = \{0\}$ because if the police follow signal 0 and do not search at all, then all agents commit crimes, which incentivizes the police to choose a positive search rate because of $C'(0) < 1$.

Lemma 5 while (i) letting the agents follow action recommendations (i.e., they receive signal *crime* and thus commit crimes if and only if $x > c$); (ii) maintaining conditional distributions $(\hat{\pi}_P(\cdot|crime), \hat{\pi}_P(\cdot|not))$ that garble the agents' signals to create the police's signals, and (iii) letting the police adopt the optimal search strategy at any given c. Increasing cutoff c continuously changes the expected search probabilities the agents face conditional on signals *crime* and *not*. At $c = 1$, all agents receive and follow signal *not*, the police choose search probability 0, and the obedience constraint for signal *not* is violated. Thus at some $c^* \in (0, 1)$, the agents become indifferent between committing crime and not after receiving signal *not*. The obedience constraint for signal *crime* also holds at the same c^*, because the assumption $C'\left(\int_0^1 x \, \mathrm{d}F(x)\right) > 1$ implies that the police's optimal searches never make all agents weakly prefer to abstain from committing crimes.

At this point, we have a signal structure and an equilibrium such that: each agent receives signal *crime* if $x > c^*$ or *not* if $x < c^*$, they follow action recommendations, and those who receive signal *not* are indifferent between the two actions; and the police's signal space S_P is $\{0, \sigma\}$ or $\{\sigma\}$. If $S_P = \{0, \sigma\}$, the inequality $r(0) < r(\sigma)$ continues to hold even though we increased cutoff c, because whether $r(0) < r(\sigma)$ holds depends only on $\frac{\hat{\pi}_P(0|crime)}{\hat{\pi}_P(0|not)}$ and $\frac{\hat{\pi}_P(\sigma|crime)}{\hat{\pi}_P(\sigma|not)}$, not on c.

In the last step, we consider two cases. If $S_P = \{\sigma\}$, we obtain the desired result where $\alpha^* = 0$ (note that $(s_i^P, s_i^A) = (0, not)$ is irrelevant if $\alpha^* = 0$).

Otherwise, the signal space is $\{0, \sigma\}$. In this case, we first change the police's signals back to the corresponding posterior crime rates, which results in a signal space $\{r_0, r_\sigma\}$ with $r_0 < r_\sigma$. We then split signal r_0 into signals t_0 and t_σ that have posterior crime rates 0 and $r_\sigma > r_0$, respectively.

We now need to consider two cases. One is when $p(\sigma) \in (0, 1)$, i.e., before splitting signal r_0 into t_0 and t_σ, the search rate for signal r_σ is interior. In this case, we must have $C'(P) = r_\sigma$ and the police do not search signal r_0 at all before splitting. Thus, after splitting, it continues to be optimal for the police to allocate search mass P to signal r_σ and not search signal t_σ or t_0. In particular, searching agents with signal t_σ (in addition to those with r_σ) will entail a higher marginal cost than $C'(P)$. Now, signals t_σ and r_σ have the same posterior crime rate, so we can pool them into the same signal, say u_σ. After this pooling, the police still find it optimal to apply search capacity P to signal u_σ and search capacity 0 to signal t_0, and the expected search probability conditional on each action recommendation remains the same.

The other case is when $p(\sigma) = 1$, i.e., before splitting signal r_0 into t_0 and t_σ, the police search signal r_σ with probability 1. In this case, we may have $C'(P) < r_\sigma$ before splitting. Thus, after splitting, the police continue to search agents with signal r_σ with probability 1, and may also prefer to search agents with signal t_σ with a positive probability. As a result, the obedience constraint for signal *not* may hold strictly—the agents used to face search mass P after signal r_σ and no search after signal r_0, but they now face search mass P after signal r_σ and possibly a positive search mass after signal t_σ. We then proceed as follows: First, we pool signals t_σ and r_σ into signal u_σ. As in the previous paragraph, signals t_σ and r_σ have the same posterior crime rate, and this pooling does not change the police's search capacity or the agents' incentives. If the obedience constraint for signal *not* holds strictly, we increase cutoff c from c^* as we did in the second step (the procedure ★ above) until the cutoff hits the point at which the obedience constraint for signal *not* binds and the one for signal *crime* holds. Redefine c^* as this new cutoff.

The police's signal is now binary, i.e., signal r_0 has posterior crime rate 0 and search rate 0, and signal u_σ has a positive crime rate and a positive search probability. In terms of action recommendations, the police's signal space becomes $S_P^* = \{0, \rho^*\}$ for some $\rho^* \in (0, 1]$.

We now have signal structure (S_P^*, S_A^*, π^*) such that: $S_P^* = \{0, \rho^*\}$ and $S_A^* = \{crime, not\}$; the agents with types above and below cutoff c^* receive signals *crime* and *not*, respectively; the agents who receive signal *not* are indifferent; and the police's signal divides the population into two groups, one with zero posterior crime rate and the other with a positive posterior crime rate. This signal structure and equilibrium satisfy Part 1. Part 2 holds because I have constructed the outcome so that the agents with signal *not* are indifferent.

Finally, note that at the beginning of this proof, I chose a signal structure (S_P, S_A, π) whose corresponding equilibrium crime rate r lies in $(0, 1)$. This choice is without loss of generality in the following sense. We do not need to consider $r = 0$ because it never arises: If no agent commits a crime, the police with a strictly increasing cost function would never search, which, in turn, implies that everyone would commit a crime. Similarly, we do not need to consider $r = 1$ to derive the crime-minimizing joint signal structure, because we can provide the agents with full information and the police with no information. The resulting equilibrium crime rate is strictly less than 1, because $C'(0) < 1$. $\square$

Proof of Proposition 2

Proof. I first use Lemma 2 to solve the relaxed problem. I then turn to the original problem and show that inequality (15) ensures $\alpha^* = 0$.

To solve the relaxed problem, I focus on signal structures that take the form described in Figure 2. Instead of parameters (ρ^*, c^*, α^*), we use (ρ, c, α) to indicate that they may not be the crime-minimizing signal structure. Recall that α is the probability with which the police observe signal 0 conditional on that an agent observes signal *not*. When types above some cutoff commit crimes, a crime rate r pins down the cutoff type through $c = F^{-1}(1-r)$.

I fix $\alpha \in [0, 1]$ arbitrarily and then determine the cutoff type c and the unique positive search probability ρ from the mutual best responses of the agents and the police. By Part 2 of Lemma 2, the equilibrium crime rate $r(\alpha)$ is determined by the condition that the police's optimal search strategy given crime rate $r(\alpha)$ makes the agents who observe signal *not* indifferent between committing a crime and not.

I derive the expected search probability given signal *not*. As in Figure 2, if the crime rate is r, the posterior crime rate for signal ρ is $\frac{r}{r+(1-r)(1-\alpha)}$. The police's mass of searches P then solves the first-order condition $C'(P) = LP^{\beta} = \frac{r}{r+(1-r)(1-\alpha)}$, or

$$P = \frac{1}{L^{\frac{1}{\beta}}} \cdot \left(\frac{r}{r + (1-r)(1-\alpha)} \right)^{\frac{1}{\beta}}.$$

For the moment, I ignore the constraint $\rho \leq 1$ and verify it later. The expected search probability conditional on signal *not* is $(1-\alpha)\frac{P}{r+(1-r)(1-\alpha)}$, or

$$I(r, \alpha) \triangleq \frac{1}{L^{\frac{1}{\beta}}} \frac{(1-\alpha) r^{\frac{1}{\beta}}}{[r + (1-r)(1-\alpha)]^{\frac{1+\beta}{\beta}}}.$$

The binding obedience constraint for signal *not* is written as

$$I(r,\alpha) = \mathbb{E}_F[\tilde{x}|\tilde{x} \le F^{-1}(1-r)]. \tag{28}$$

At $r = 0$, we have $I(0,\alpha) = 0 < \mathbb{E}_{x\sim F}[x]$. At $r = 1$, we have $I(1,\alpha) = \frac{1-\alpha}{L^{\frac{1}{\beta}}} \ge 0$. Also, $I(r,\alpha)$ and $\mathbb{E}_{\tilde{x}\sim F}[\tilde{x}|\tilde{x} \le F^{-1}(1-r)]$ are continuous in r. Thus equation (28) has a solution. Let $r(\alpha)$ denote the smallest solution. The minimal crime rate in the relaxed problem is given by $r^* \triangleq \min_{\alpha\in[0,1]} r(\alpha)$. However, instead of solving this minimization problem, I first derive $\alpha(r) \triangleq \arg\max_{\alpha\in[0,1]} I(r,\alpha)$ and then determine the minimal crime rate r^* through $I(r,\alpha(r)) = \mathbb{E}_F[\tilde{x}|\tilde{x} \le F^{-1}(1-r)]$. Hereafter, I restrict attention to $\alpha \in [0,1)$ and $r \in (0,1)$, because $\alpha = 1$ or $r \in \{0,1\}$ cannot be a part of a crime-minimizing equilibrium.

We have

$$\frac{\partial}{\partial\alpha}\log I(r,\alpha) = -\frac{1}{1-\alpha} + \frac{1+\beta}{\beta}\cdot\frac{1-r}{1-\alpha(1-r)}.$$

Note that $(1-\alpha)(1-\alpha(1-r)) > 0$, but the expression

$$(1-\alpha)(1-\alpha(1-r))\frac{\partial}{\partial\alpha}\log I(r,\alpha) = -r + \frac{1}{\beta}(1-r)(1-\alpha)$$

changes its sign at most once from positive to negative as α increases, and $\frac{\partial}{\partial\alpha}\log I(r,\alpha) < 0$ for α close to 1. As a result, $\frac{\partial}{\partial\alpha}\log I(r,\alpha)$ is either always negative or changes its sign exactly once from positive to negative. Examining the first-order condition $\frac{\partial}{\partial\alpha}\log I(r,\alpha) = 0$, we obtain the following solution:

$$\alpha(r) = \begin{cases} 1 - \beta\frac{r}{1-r} & \text{if} \quad r \le \frac{1}{1+\beta}, \\ 0 & \text{if} \quad r \ge \frac{1}{1+\beta}. \end{cases}$$

and

$$I(r,\alpha(r)) = \begin{cases} \frac{1}{L^{\frac{1}{\beta}}}\cdot\frac{\beta}{(1+\beta)^{\frac{1+\beta}{\beta}}}\frac{1}{1-r} & \text{if} \quad r \le \frac{1}{1+\beta}, \\ \left(\frac{r}{L}\right)^{\frac{1}{\beta}} & \text{if} \quad r \ge \frac{1}{1+\beta}. \end{cases}$$

Now, go back to the equation

$$I(r, \alpha(r)) = \mathbb{E}_{\tilde{x} \sim F}[\tilde{x} | \tilde{x} \leq F^{-1}(1-r)].$$

The left-hand side is strictly increasing, and the right-hand side is strictly decreasing in r. Hence the equilibrium crime rate r^* is unique. Moreover, we have $r^* \geq \frac{1}{1+\beta}$ and thus $\alpha(r^*) = 0$ when

$$I\left(\frac{1}{1+\beta}, 0\right) < \mathbb{E}_{\tilde{x} \sim F}\left[\tilde{x} \mid \tilde{x} \leq F^{-1}\left(\frac{\beta}{1+\beta}\right)\right],$$

which reduces to inequality (15). Finally, recall that when I derived the police's best response, I temporarily ignored the condition that the search probability ρ^* can be at most 1. This condition is satisfied at equilibrium because ρ^* satisfies $\rho^* \leq \mathbb{E}_F[\tilde{x} | \tilde{x} \geq F^{-1}(1-r^*)] \leq 1$. □

Proof of Claim 2

Proof. Substituting $F = U[0, 1]$ and $\beta = 1$ into the solutions to the relaxed problem for Proposition 2, I obtain the following:

$$\alpha(r) = \begin{cases} \frac{1-2r}{1-r} & \text{if} \quad r \leq \frac{1}{2}, \\ 0 & \text{if} \quad r \geq \frac{1}{2}. \end{cases}$$

and

$$I(r, \alpha(r)) = \begin{cases} \frac{1}{4L(1-r)} & \text{if} \quad r \leq \frac{1}{2}, \\ \frac{r}{L} & \text{if} \quad r \geq \frac{1}{2}. \end{cases}$$

Solving $I(r, \alpha(r)) = \frac{1-r}{2}$, I obtain the minimized crime rate under the relaxed problem:

$$r^* = \begin{cases} 1 - \frac{1}{\sqrt{2L}} & \text{if} \quad L \leq 2, \\ \frac{L}{2+L} & \text{if} \quad L \geq 2. \end{cases}$$

Because the type distribution is uniform, a crime rate of r^* means that an agent commits a crime if and only if their type exceeds $c^* = 1 - r^*$.

I now show that if $L \geq L^* = \frac{3+\sqrt{5}}{4}$, crime rate r^* can be implemented in the original problem. To do so, I modify the signal structure in Theorem 1 as follows: If $x < 1 - r^*$, with probability $\alpha^* = \alpha(r^*)$, the police observe signal 0. Other parts of the signal structure follow the truth-or-noise signal structure with cutoff c^*: The police observe signal x with probability $1 - \alpha^*$ if $x \leq c^*$ and observe signal $s \sim F(\cdot|\tilde{x} \geq c^*)$ whenever $x \geq c^*$.

If $L \geq L^*$, this signal structure has an equilibrium in which each agent commits a crime if and only if $x > c^*$, and the police search agents with signal s with probability $\frac{s}{1-\alpha^*}$. The agents' strategies are optimal: Any agent with a type below c^* is indifferent between committing a crime and not because they anticipate search probability $(1-\alpha^*)\frac{x}{1-\alpha^*} = x$ in expectation. Any type $x \geq c^*$ will face a search probability of $\frac{1-r^*}{2(1-\alpha(r^*))} < 1 - r^* \leq x$, where the first inequality uses $\alpha^* < 1/2$, which follows from $L \geq L^*$. The police's strategy is also optimal: The police never search signal 0 and are indifferent regarding how to allocate a given mass of searches across the positive signals. The choice of a total search capacity is also optimal: Indeed, the posterior crime rate for any positive signal is $\frac{r^*}{r^*+(1-r^*)(1-\alpha^*)}$, so the total search capacity induced by the above search strategy equates the marginal cost with the marginal crime rate because of the police's first-order condition in the relaxed problem. Also, $L \geq L^*$ ensures that the highest search probability $\frac{c^*}{1-\alpha^*}$ is below 1, so the police's strategy is feasible. Finally, if $L \in [L^*, 2)$, then we have $r^* = 1 - \frac{1}{\sqrt{2L}}$ and $\alpha(r^*) = \frac{1-2r^*}{1-r^*} = 2 - \sqrt{2L}$. If $L \geq 2$, we have $\alpha^* = 0$, so the police's signal reduces to the truth-or-noise signal structure. $\square$

Appendix C: Omitted Formalism for Remark 2

As I discussed in Remark 2, the crime-minimizing signal structure in Theorem 1 is fragile in two ways. In this appendix, I elaborate on those points and then show that, as claimed in the remark, signal structure (S^*, π^*) in Example 1 does not have the same kinds of fragility.

First, under the crime-minimizing signal structure of Theorem 1, for any crime rate $r \in [1 - F(\hat{c}), 1]$ greater than the minimized level, there exists an equilibrium in which the crime rate is r and the police continue to adopt the same search strategy as in the crime-minimizing equilibrium. In this equilibrium, any type above $\hat{c}$ commits a crime with probability 1, and any type below $\hat{c}$, who is indifferent, commits a crime with probability $1 - \frac{1-r}{F(\hat{c})}$. The mass of agents who abstain from crime is $F(\hat{c}) \cdot \left(1 - \left[1 - \frac{1-r}{F(\hat{c})}\right]\right) = 1 - r$, so the crime rate is r.

Second, the crime-minimizing signal structure fails to deter crimes at all if the police's search capacity is misspecified. To see this, suppose that we construct the crime-minimizing signal structure, denoted by $(S_{\overline{P}}, \pi_{\overline{P}})$, for a certain value of $\overline{P}$, but the actual search capacity turns out to be $\overline{P} - \epsilon$ with $\epsilon > 0$. The signal structure $(S_{\overline{P}}, \pi_{\overline{P}})$ generates signals between 0 and $\hat{c}$, and the search capacity $\overline{P}$ is just enough for the police to make all types below $\hat{c}$ indifferent. Thus, if the search capacity is $\overline{P} - \epsilon$, for any search strategy, there is always a positive measure of signals associated with crime rate 1. The same argument as in the proof of Theorem 0 implies that every signal generated by the signal structure $(S_{\overline{P}}, \pi_{\overline{P}})$ must have crime rate 1. Therefore, if the search capacity is slightly overestimated, the equilibrium crime rate jumps from the minimized level to 1.

The binary signal structure (S^*, π^*) in Example 1 reduces the crime rate relative to no information, but it does not exhibit the same kind of fragility. First, I show that the equilibrium described in Example 1, where $p(s_L^*) = 1/3$ and $p(s_M^*) = 2/3$, is unique. For instance, if $p(s_L^*) > 1/3$ in equilibrium, the binding search capacity constraint implies $p(s_M^*) < 2/3$. Then, the crime rate for signal L becomes strictly lower than that for signal M. The police would then profitably deviate by shifting search probabilities from signal L to signal M, which leads to a contradiction. Symmetrically, $p(s_L^*) < 1/3$ leads to a contradiction. Therefore, a search strategy can be part of an equilibrium only if $p(s_L^*) = 1/3$ and $p(s_M^*) = 2/3$, which implies that the equilibrium described in Example 1 is unique.

Moreover, the equilibrium crime rate is continuous with respect to $\overline{P}$ at $\overline{P} = 1/2$. To see this, suppose we construct the signal structure (S^*, π^*) for $\overline{P} = 1/2$, but the actual search capacity is $\overline{P}_\epsilon = \overline{P} - \epsilon$ for some ϵ. I allow $\epsilon < 0$, in which case the actual search capacity is underestimated. Let $\hat{p}(s_L^*)$ and $\hat{p}(s_M^*)$ denote the equilibrium search probabilities for signals L and M, respectively. Here, the binding search capacity constraint $0.5\hat{p}(s_L^*) + 0.5\hat{p}(s_M^*) = \overline{P}_\epsilon$ implies $\hat{p}(s_L^*) = 2(\overline{P}_\epsilon - 0.5\hat{p}(s_M^*))$. The value $\hat{p}(s_M^*)$ is uniquely determined by the condition that the crime rates for signals L and M are equalized, i.e.,

$$F_L\left(2(\overline{P}_\epsilon - 0.5\hat{p}(s_M^*))\right) = F_M(\hat{p}(s_M^*)), \tag{29}$$

where F_L and F_M are the type distributions conditional on signals L and M, respectively. At $\hat{p}(s_M^*) = 0$, the LHS is strictly greater than the RHS. If $\hat{p}(s_M^*)$ is large enough, the RHS is strictly greater than the LHS. Also, F_L and F_H are continuous. By the intermediate value theorem, there exists a unique value of $\hat{p}(s_M^*)$ that solves equation (29). The resulting $(\hat{p}(s_L^*), \hat{p}(s_M^*))$ and the agents' (unique) best responses constitute a unique equilibrium. Note also that the solution $\hat{p}(s_M^*)$ to equation (29) is continuous with respect to ϵ at $\epsilon = 0$. Therefore, even if the search capacity is slightly misspecified, the resulting game has a unique equilibrium, and the crime rate remains close to $1/3$, which is the level associated with (S^*, π^*) when $\overline{P} = 1/2$.

Appendix D: A Model with a Continuum of Officers

In this appendix, I formalize the idea that the simultaneous-move assumption arises when a continuum of individual officers searches agents. Suppose that instead of the police, there is a unit mass of *officers*, $j \in [0,1]$. All officers face the same signal structure (S, π) and observe the same realized signal for each agent.

The timing of the game is as follows. First, each officer chooses a search strategy, $p_j : S \to [0,1]$. The profile of search strategies, $(p_j)_{j\in[0,1]}$, is publicly observed by all agents. Second, nature draws the type and the signal for each agent. Finally, each agent observes their type but not their realized signal, and decides whether to commit a crime.

Each officer j chooses a search strategy p_j to maximize the mass of successful searches, defined as

$$\sigma_j \triangleq \int_0^1 \int_S p_j(s)\, \mathrm{d}\pi(s|x) a(x)\, \mathrm{d}F(x), \tag{30}$$

where $a(x)$ is the probability with which type x commits a crime. As in the baseline model, we obtain identical results if each officer's payoff is strictly increasing in σ_j and arbitrarily dependent on the crime rate. Note that the probability $a(x)$ of type x committing a crime may depend on the chosen profile of search strategies, but I do not write this dependency explicitly. Each officer faces the same search capacity constraint, $\overline{P} < \int_0^1 x \, \mathrm{d}F(x)$, which is analogous to Assumption 1. Identical results hold if there is a mass $\overline{P}$ of officers, each of whom has a search capacity of 1.

Given the search strategies $(p_j)_{j\in[0,1]}$, define *the aggregate search rate for signal s* as

$$\hat{p}(s) \triangleq \int_0^1 p_j(s) \, \mathrm{d}j. \tag{31}$$

Abusing notation, define *the aggregate search rate for type x* as

$$\hat{p}(x) \triangleq \int_S \hat{p}(s) \, \mathrm{d}\pi(s|x). \tag{32}$$

An agent's expected payoffs of committing a crime and not are $x - \hat{p}(x)$ and 0, respectively.

Extending Theorem 0

Claim 7. *Suppose that the officers have full information. An equilibrium exists, and in any equilibrium, almost every agent commits a crime with probability* 1.

Proof. A unilateral deviation by an individual officer does not affect the aggregate search rate (32) for each type. Thus, for any fixed profile of search strategies, the set of best responses by each agent does not change with or without an individual officer's deviation. Then we can construct an equilibrium in which all officers adopt the same search strategy p^* as the police in the first half of the proof of Theorem 0.

Second, take any equilibrium. There is no profile of search strategies such that the induced aggregate search rate satisfies $\hat{p}(x) \geq x$ for (almost) every type x, because it would violate the search capacity constraint of a positive mass of officers. Thus, the set $X \triangleq \{x \in [0,1] : x > p(x)\}$ has a positive mass, and any type in X commits a crime with probability 1. By the same logic as the proof of Theorem 0, the set Y of types that commit crimes with probability strictly below 1 must have measure zero. □

Extending Theorem 1

I characterize the crime-minimizing signal structure with a continuum of officers. First, to extend Lemma 1, we define the relaxed problem in this new setup: The information of the agents and the officers is now determined by a joint signal structure, (S_P, S_A, π). The timing of the game is as follows. First, each officer chooses a search strategy, $p_j : S_P \to [0, 1]$. The profile of search strategies, $(p_j)_{j\in[0,1]}$, is publicly observed by all agents. Second, nature draws the type x_i and signals (s_i^P, s_i^A) for each agent i. Finally, each agent observes s_i^A but not (x_i, s_i^P), and decides whether to commit a crime.

Claim 8. *In the relaxed problem, the following joint signal structure minimizes a crime rate: The officers learn no information, e.g., $S_P = \{\phi\}$, and each agent learns whether their type exceeds cutoff $\hat{c} \in (0, 1)$ that uniquely solves*

$$\mathbb{E}_F[\tilde{x}|\tilde{x} \le \hat{c}] = \overline{P}. \tag{33}$$

In equilibrium, each officer search every agent with probability $\overline{P}$, and each agent commits a crime if and only if their type exceeds $\hat{c}$.

Proof. The proof follows that of Lemma 1. Take any joint signal structure (S_P, S_A, π) and equilibrium. Let $\hat{p} : S_P \to [0, 1]$ be the aggregate search rate function, and let r be the crime rate in equilibrium. First, replace the agents' signals with action recommendations so that the obedience constraints hold (see the proof of Lemma 1). This step is valid even though the agents move after observing the chosen search strategies, because a single officer's deviation does not change the aggregate search rates and thus does not affect the agents' incentives. In other words, after replacing the agents' signals with action recommendations, the agents' behavior remains the same both on-path and after an officer's deviation. Second, by the same logic as Lemma 1, providing the officers with no information only relaxes the obedience constraints. The resulting problem is to choose a disclosure policy from which the agents receive action recommendations. The rest of the proof is the same as that of Lemma 1. ∎

Claim 9. *In the model with a continuum of officers, Theorem 1 holds verbatim.*

The proof of this claim is identical to that of Theorem 1 and thus omitted.

Appendix E: Proof of Claim 4

I prove Claim 4. Note that if $r = 1$, any equilibrium under full information has crime rate 1 and the search mass on innocents equal to 0 (see Theorem 1), which satisfies the condition in the claim. For $r \in (\hat{r}, 1)$, the following result presents a signal structure and an equilibrium that have the properties described in Claim 4.

Claim 10. *Let $\hat{r}$ be the crime rate under the crime-minimizing outcome in Theorem 1. Take any $r \in (\hat{r}, 1)$. Define $x(r) \triangleq F^{-1}(1-r)$. There exists some $\hat{x} \in (x(r), 1)$ such that the following signal structure and equilibrium attain crime rate r and the minimal search mass on innocents in (19) i.e., $\int_0^{F^{-1}(1-r)} x \, \mathrm{d}F(x)$. If $x \le x(r)$, the signal realization equals x with probability 1. If $x \in (x(r), \hat{x})$, the signal is independent of the agent's type and drawn from $F(\cdot|\tilde{x} \le x(r))$. If $x > \hat{x}$, the signal equals x with probability $\alpha(x) = \frac{x - \mathbb{E}[\tilde{x}|\tilde{x} < x(r)]}{1 - \mathbb{E}[\tilde{x}|\tilde{x} < x(r)]}$ or independent of x and drawn from $F(\cdot|\tilde{x} \le x(r))$ with probability $1 - \alpha(x)$. The police's equilibrium search strategy is as follows:*

$$p(s) = \begin{cases} s & \text{if} \quad s \le x(r) \\ 1 & \text{if} \quad s > \hat{x}. \end{cases} \tag{34}$$

In this equilibrium, the agents commit crimes if and only if $x > x(r)$.

Proof. For each $x > x(r)$, define $\alpha(x)$ as the solution to the equation

$$x = \alpha(x) + (1 - \alpha(x))\mathbb{E}[\tilde{x}|\tilde{x} < x(r)], \tag{35}$$

or

$$\alpha(x) = \frac{x - \mathbb{E}[\tilde{x}|\tilde{x} < x(r)]}{1 - \mathbb{E}[\tilde{x}|\tilde{x} < x(r)]} \in [0, 1]. \tag{36}$$

Equation (35) means that type x is indifferent between committing and not committing a crime if they face search rate 1 with probability $\alpha(x)$ and search rate $\mathbb{E}[\tilde{x}|\tilde{x} < x(r)]$ with probability $1 - \alpha(x)$.

Fix $\hat{x} > x(r)$ and consider the signal structure described in the statement. The set of possible signals is $[0, x(r)] \cup [\hat{x}, 1]$. Suppose that the police adopt the search strategy (34). Then, any type $x \in [0, x(r)]$ faces search probability x, so they are indifferent between committing and not committing a crime; hence, it is optimal for types below $x(r)$ to abstain from committing crimes. Any type $x \in (x(r), \hat{x})$ faces a search rate of at most $x(r)$ and strictly prefers to commit a crime. Finally, any type $x \geq \hat{x}$ faces search rate 1 with probability $\alpha(x)$ and the expected search rate of $\mathbb{E}[\tilde{x}|\tilde{x} < x(r)]$ with probability $1 - \alpha(x)$. By (35), such types are indifferent between committing and not committing a crime, so it is optimal to commit crimes.

To sum up, given the signal structure and the police's strategy (34), there exist the agents' best responses such that all agents below type $x(r)$ abstain from committing crimes and thus the resulting crime rate is r.

It remains to show that the police's strategy is optimal. To begin with, note that the police's search strategy described above satisfies their search capacity constraint with equality if $\hat{x}$ satisfies

$$\mathbb{E}[\tilde{x}|\tilde{x} < x(r)] \left[F(\hat{x}) + \int_{\hat{x}}^{1} (1 - \alpha(t)) f(t) \, \mathrm{d}t \right] + \int_{\hat{x}}^{1} \alpha(t) f(t) \, \mathrm{d}t = \overline{P}. \tag{37}$$

I show that there is an $\hat{x} \in (x(r), 1)$ that solves (37). If $\hat{x} = 1$, the right-hand side (RHS) becomes weakly greater, because the left-hand side (LHS) is

$$\mathbb{E}[\tilde{x}|\tilde{x} < x(r)] \leq \mathbb{E}[\tilde{x}|\tilde{x} < \hat{c}] = \overline{P},$$

where the equality comes from equation (6). If $\hat{x} = x(r)$, then the LHS exceeds the RHS in (37) for the following reason. In this case, there is no type in $(\hat{x}, x(r))$, so all types are indifferent between committing and not committing crimes under the search strategy described above. Such a search strategy must violate Assumption 1, which means that the LHS exceeds the RHS.

The intermediate value theorem implies that there is some $\hat{x} \in (x(r), 1]$ that solves (37). Therefore, for such an $\hat{x}$, the police's search strategy satisfies their search capacity constraint.

It remains to show that the police have no profitable deviations. The posterior crime rate for signal $s \in [\hat{x}, 1]$ is 1. For signals in $[0, x(r)]$, the posterior crime rate is less than 1 and equalized across all signals. Thus, the police's strategy is optimal so long as it satisfies $p(s) = 1$ for all $s \in [\hat{x}, 1]$ and exhausts search capacity $\overline{P}$. By construction, the police's strategy described above satisfies these conditions.

Finally, in this equilibrium, every type below $x(r)$ chooses not to commit a crime and faces a search rate x. The corresponding search mass on innocents is $\int_0^{x(r)} x \, \mathrm{d}F(x)$. $\square$