Dynamic Privacy Choices

Click here to download the latest version.

Shota Ichihashi*

February 27, 2020

Abstract

I study a dynamic model of consumer privacy and platform data collection. Consumers choose how actively to use a platform in each period. The activity generates information about consumers, which benefits the platform but hurts consumers. In the long-run, consumers lose privacy and obtain a low payoff, but choose a high activity level. To induce this outcome, the platform adopts a privacy policy that gradually expands the scope of data collection. Platform competition is less likely when consumers are receiving lower payoffs from an incumbent. Regulating data collection can hurt consumers in the long-run.

*Bank of Canada, 234 Wellington Street West, Ottawa, ON K1A 0G9, Canada. Email: shotaichihashi@gmail.com. Website: http://shotaichi.weebly.com/. I thank Byung-Cheol Kim, David Miller, Michael Sockin (discussant), Takuo Sugaya, Catherine Tucker, and seminar and conference participants at University of Rochester (Simon Business School) and Privacy Workshop at Princeton for valuable comments. The opinions expressed in this article are the author's own and do not reflect the views of the Bank of Canada.

1 Introduction

Online platforms, such as Amazon, Facebook, Google, and Uber, analyze the activities of consumers and collect a large amount of data. While this data collection may improve their services and benefit consumers, it raises concerns among consumers and policymakers (Crémer et al., 2019; Furman et al., 2019; Morton et al., 2019).

As an example, consider a consumer (she) and a social media platform (it). The consumer reads posts and interacts with friends on the platform. The platform analyzes her profile and browsing behavior, and collects data such as her race, location, and political preferences. If the consumer uses the platform actively, it can collect more data and earn higher revenue, by, say, improving targeted advertising. The consumer faces a trade-off: On the one hand, she enjoys reading posts and connecting with friends on the platform. On the other hand, the consumer may value her privacy or be concerned about the risk of data leakage, identity theft, and price or non-price discrimination based on collected data.[1] This is a "privacy cost" for the consumer of using the platform. If the consumer anticipates a high privacy cost, she may use the platform less actively. The platform can influence this trade-off through a privacy policy. For example, as Facebook did in 2004, the platform may commit to not use cookies to track consumers.[2]

I formalize this example in a dynamic model. In each period, a consumer chooses her activity level. Then, a platform observes a signal about her time-invariant type X. The informativeness of the signal is increasing in the activity level, but decreasing in the platform's privacy level, which specifies the amount of noise added to the signal. The platform's payoff in each period is increasing in the amount of information it has collected. The consumer benefits from the service but incurs a privacy cost, which is increasing in the amount of collected information.

The key economic mechanism is that, if the consumer has less privacy, then she faces a lower marginal privacy cost of using the platform. For example, if Google or Amazon already knows a lot about a consumer, she might not care about letting Google maps track her location or putting Alexa in the living room. In an extreme case, if the platform knows everything, then the marginal privacy cost is zero, because the consumer's activity no longer affects what information the platform has.

[1] Such concerns are highlighted by, for example, the *Cambridge Analytica* scandal.

[2] In 2004, Facebook's privacy policy stated that "we do not and will not use cookies to collect private information from any user." https://web.archive.org/web/20050107221705/http://www.thefacebook.com/policy.php

Thus, data collection may lower consumer welfare, but increases their marginal incentive to use the platform.

In equilibrium, the consumer chooses higher activity levels but receives lower payoffs over time. In the long-run, the consumer loses privacy and incurs a high privacy cost, which can more than offset the benefit from the service; however, the consumer acts as if there is no privacy cost. To induce such an outcome, the platform chooses high privacy levels in early periods to encourage consumer activity, but decreases privacy levels in the long-run to accelerate data collection. As a result, the consumer loses privacy even when she values it arbitrarily highly ex ante. I first show this for a myopic consumer, but the result extends to a forward-looking consumer.

Platform competition is unlikely to mitigate the problem, because data collected by an incumbent becomes an entry barrier: If an incumbent (e.g. Google) has a lot of consumer data, the consumer faces a lower marginal privacy cost of using the incumbent than an entrant (e.g. DuckDuckGo). Thus, the consumer is unlikely to switch to the entrant. The result offers a novel antitrust implication: Market entry is less likely when consumers are suffering from privacy loss and receiving lower payoffs from an incumbent.

I explore various privacy regulations. For example, mandating the platform to adopt a stricter privacy policy may lower consumer welfare in the long-run. Enforcing the long-term commitment of the platform to its privacy policy has no impact on the equilibrium. In contrast, the "right to be forgotten," which enables the consumer to erase past information, may enhance consumer welfare and induce market entry.

The paper has implications on consumer privacy and data collection by digital platforms. First, the consumer's long-run behavior seems consistent with the so-called privacy paradox: Consumers express concerns about data collection, but many of them actively share data with third parties (Acquisti et al., 2016). Second, the platform's equilibrium strategy rationalizes how online platforms, such as Facebook, have relaxed its privacy policy over time. Third, the analysis on competition poses a challenge to firms that offer privacy-friendly alternatives to dominant platforms. Finally, the paper offers dynamic implications of privacy regulations.

This paper contributes to the literature on markets for data and the economics of privacy. First, the paper is closely related to Acemoglu et al. (2019), Bergemann et al. (2019), and Choi et al. (2019). They consider static models in which a platform collects consumer data in exchange for monetary compensation. If some consumers sell their data, the platform can also learn about other consumers. This "data externality" lowers the incentive of each consumer to protect privacy, leading to low compensation and excessive data sharing in equilibrium. The key economic force of this paper is analogous to theirs: A consumer's data provision today lowers her incentive to protect privacy in the future. However, there are two important differences. First, the dynamic model enables me to study new issues, such as market entry, commitment to privacy policies, the right to be forgotten, data retention, consumer myopia, and the evolution of a platform's privacy policy. Second, economic applications are different. The previous works assume that consumers hold data at the outset, and platforms buy data in exchange for monetary compensation. In my model, data arise as a byproduct of an activity from which the consumer enjoys benefits. This formulation directly applies to online platforms that do not pay consumers for data. Moreover, it allows me to study a new question such as a platform's incentive to invest in quality.

This paper also relates to the recent work on competition and data. Hagiu and Wright (2020) study "data-enabled learning" whereby firms can improve their products and services through learning from the data they obtain from their customers. Prufer and Schottmüller (2017) assume that the cost of investing in quality is decreasing in the firm's past sales, and greater investment in quality leads to higher demand in the current period. In contrast to this literature, I assume that data collection can lower consumer welfare, which is necessary to study the value of privacy. Another important difference is whether a consumer regards collected data as sunk. For example, in Hagiu and Wright (2020), regardless of how much data a firm has, a consumer's payoff from an outside option (i.e. not buying a product) is zero. In my paper, once a platform collects data, the consumer incurs the associated privacy cost even when she does not use the platform. This assumption is important for the key mechanism where data collection lowers a consumer's payoff but increases her marginal incentive to use the platform. Hagiu and Wright (2020) allow price competition and consider a rich learning dynamics incorporating "within-user" and "across-user" learning. In contrast, I abstract from pricing, and exclusively focus on within-user learning.

Bonatti and Cisternas (Forthcoming) study consumer privacy in a continuous-time dynamic model. They consider a long-lived consumer with short lived sellers. Sellers can learn about consumer preferences based on scores that aggregate purchase histories, and sellers use information for price discrimination. In contrast, I consider long-lived platforms and abstract away from how platforms use consumer information. Fainmesser et al. (2019) study the optimal design of a platform to store data and invest in information security. They consider a platform that cares about both the activity levels of consumers and the amount of data it can extract. They study how different objectives lead to different platform designs. I adopt simpler preferences for consumers and platforms, but consider a dynamic environment.

The rest of the paper is as follows. Section 2 presents a model of a monopoly platform. Section 3 presents the long-run equilibrium outcome and characterizes the equilibrium privacy policy of the platform. Section 4 considers platform competition and shows that the data collected by an incumbent works as an entry barrier. Section 5 studies the incentive of the consumer or platforms to erase past information. Section 6 considers extensions, including a forward-looking consumer and general payoff functions.

2 Model

Time is discrete and infinite, indexed by $t = 1, 2, \ldots$. There are a consumer (she) and a platform (it). The consumer's type X is drawn from a normal distribution $\mathcal{N}(0, \sigma_0^2)$. X is realized before $t = 1$ and fixed over time. The consumer does not observe X.[3] The platform does not observe X, but receives signals about X.

In each period t, the consumer publicly chooses an *activity level* $a_t \geq 0$. Then, the platform privately observes a signal $s_t = X + \varepsilon_t + z_t$, where $\varepsilon_t \sim \mathcal{N}\left(0, \frac{1}{a_t}\right)$ and $z_t \sim \mathcal{N}(0, \gamma_t)$. A greater a_t reduces the variance of ε_t and makes s_t more informative about X, whereas a greater γ_t increases the variance of z_t and makes s_t less informative about X.[4] $\gamma_t \geq 0$ is the *privacy level* of the platform in period t. All the random variables, $X, \varepsilon_1, z_1, \varepsilon_2, z_2, \ldots$, are mutually independent.

The payoffs are as follows. Suppose that the consumer has chosen activity levels $\boldsymbol{a}_t = (a_1, \ldots, a_t)$ and the platform has chosen privacy levels $\boldsymbol{\gamma}_t = (\gamma_1, \ldots, \gamma_t)$ up to period t. Then, at the end of period t, the platform receives a payoff of $\sigma_0^2 - \sigma_t^2(\boldsymbol{a}_t, \boldsymbol{\gamma}_t) \geq 0$. $\sigma_t^2(\boldsymbol{a}_t, \boldsymbol{\gamma}_t)$ is the variance of the conditional distribution of X given $(\boldsymbol{a}_t, \boldsymbol{\gamma}_t)$, derived from Bayes' rule on and off equilibrium

[3] This is to simplify exposition. If the consumer observes X, then her activity level may become a signal of X. However, all the results of this paper hold with respect to a pooling equilibrium in which the consumer of all types chooses the same activity level after any history.

[4] If $a_t = 0$, then I define s_t as a pure noise, which is independent of X.

paths.[5] A lower $\sigma_t^2(\boldsymbol{a}_t, \boldsymbol{\gamma}_t)$ means that the platform has a more accurate estimate of X, or in other words, the consumer has less privacy. $\sigma_t^2(\boldsymbol{a}_t, \boldsymbol{\gamma}_t)$ is decreasing in a_τ and increasing in γ_τ for any $\tau \leq t$, and independent of realized signals $s_1, \ldots, s_t$. Where it does not cause confusion, I write $\sigma_t^2(\boldsymbol{a}_t, \boldsymbol{\gamma}_t)$ as σ_t^2. The platform discounts future payoffs with a discount factor $\delta_P \in (0, 1)$.

The consumer's payoff in period t is $U(\boldsymbol{a}_t, \boldsymbol{\gamma}_t) := u(a_t) - v\left[\sigma_0^2 - \sigma_t^2(\boldsymbol{a}_t, \boldsymbol{\gamma}_t)\right]$. The first term $u(a_t)$ is the gross benefit of using the platform. $u : \mathbb{R}_+ \to \mathbb{R}$ is strictly concave, continuously differentiable, maximized at $a^* \in (0, \infty)$, and satisfies $u(0) = 0$. The second term $v\left[\sigma_0^2 - \sigma_t^2(\boldsymbol{a}_t, \boldsymbol{\gamma}_t)\right]$ is the *privacy cost*, which captures the negative impact of data collection on the consumer. $v \geq 0$ is an exogenous parameter that captures the value of privacy. The consumer is myopic and chooses a_t to maximize $U(\boldsymbol{a}_t, \boldsymbol{\gamma}_t)$ in each period t (Section 6 considers a forward-looking consumer). The payoffs are normalized so that if $a_t = 0$ for all t, then the platform and the consumer obtain zero payoffs in all periods. The primitives, σ_0^2, $u(\cdot)$, and v, are commonly known to the consumer and the platform.

The timing of the game is as follows. Before $t = 1$, the platform commits to a *privacy policy* $\boldsymbol{\gamma} = (\gamma_1, \gamma_2, \ldots) \in \mathbb{R}_+^\infty$. After observing $\boldsymbol{\gamma}$, the consumer chooses an activity level in each period. I take $(\sigma_t^2(\cdot, \cdot))_{t \in \mathbb{N}}$ as a primitive, and consider pure-strategy subgame perfect equilibrium in which the consumer chooses the highest activity level among all the optimal activity levels. I will later show that the equilibrium outcome remains the same even if the platform has a weaker commitment power.

2.1 Discussion of Modeling Assumptions

As I have discussed in the introduction, the model captures the situation in which a consumer uses a platform and derives utility from its service, and the platform analyzes her activity and collects data. The platform earns higher revenue from a greater amount of data, but data collection reduces the consumer's welfare. Here, I provide further discussion on modeling assumptions.

Data Generation In practice, a consumer may not have a verifiable piece of information about her political preferences or friends' networks. However, as the consumer spends time on browsing

[5]The equivalent formulation is that the platform observes (a_t, s_t), chooses $b_t \in \mathbb{R}$, and obtains an ex post payoff of $-(X - b_t)^2$, which the platform does not observe. Writing the payoffs in terms of σ_t^2 simplifies exposition.

news and "liking" friends' posts, the platform can learn about these characteristics. The model captures this by assuming that the informativeness of the signal is increasing in the activity level. The model excludes a consumer who strategically manipulates (say) browsing history to influence the platform's learning, which seems to be less relevant. The current formulation distinguishes this paper from the recent literature on markets for data, which typically assumes that consumers are exogenously endowed with data (Acemoglu et al., 2019; Bergemann et al., 2019; Choi et al., 2019; Ichihashi, 2019).

It is crucial that the consumer cannot delete past information (σ_t^2 is weakly decreasing over time). In other words, a privacy cost from the data collected in the past is sunk. This is motivated by "data persistence," where it can be difficult for a consumer to delete collected data completely (Tucker, 2018). Section 5 considers an extension in which the consumer or a platform can erase past information.

Consumer The privacy cost $v(\sigma_0^2 - \sigma_t^2)$ captures monetary or non-monetary reasons for which a consumer wants a platform to have less information. For instance, a consumer may have intrinsic preferences for privacy, which is empirically supported by Kummer and Schulte (2019), Lin (2019), and Tang (2019).[6] For another instance, a consumer may consider the risk of data breach, identity theft, and price or non-price discrimination by platforms and third parties who have access to consumer data.

I acknowledge that preferences on data collection should be more complicated than what is assumed. For example, a consumer may prefer data collection if it leads to better services. Also, the benefit $u(a)$ of using a platform may depend on the amount of data collected. While Section 6 considers more general payoffs, I mainly focus on $u(a_t) - v[\sigma_0^2 - \sigma_t^2]$ for two reasons. First, the above extensions seem to push the equilibrium to more data collection. Because the equilibrium involves full privacy loss (i.e. Proposition 2) even if the consumer wants the platform to have no information , I conjecture that the main insight is robust to those extensions. Second, the current payoff specification is a tractable benchmark where I can characterize the equilibrium.

[6] Lin (2019) considers an experiment that empirically distinguishes intrinsic and instrumental preferences for privacy.

Platform I comment on the platform's strategy and payoffs. First, this paper considers a platform that either chooses a privacy policy at the outset (as in the baseline model) or sets a privacy level at the beginning of each period. They are not fully general. For example, I could consider a mechanism that maps each history of activity levels $(a_s)_{s=1}^{t}$ to a privacy level γ_t in period t. However, such a mechanism seems impractical and prohibitively costly to communicate to consumers.

Second, the platform cannot use monetary transfer. This is consistent with online platforms such as Facebook and Google. If the platform could commit to a mechanism mapping $\boldsymbol{a}_t$ to monetary transfer in each period t, then the platform may commit to compensate the consumer in exchange for a higher activity level. However, I conjecture that positive compensation disappears in the long-run, because the consumer chooses the highest activity level and the platform learns full information even without transfer.

As to payoffs, the platform's payoff can be any decreasing function of $(\sigma_t^2)_{t\in\mathbb{N}}$. All the results and proofs continue to hold without modification (see Section 6 for details).

3 Monopoly Platform

I begin with studying the consumer's behavior, taking the platform's strategy as given. Then, I present the long-run equilibrium outcome. After that, I characterize the platform's equilibrium privacy policy.

3.1 Consumer Behavior

Bayes' rule implies[7]

$$\sigma_t^2(\boldsymbol{a}_t, \boldsymbol{\gamma}_t) = \frac{1}{\frac{1}{\sigma_{t-1}^2(\boldsymbol{a}_{t-1}, \boldsymbol{\gamma}_{t-1})} + \frac{1}{\frac{1}{a_t} + \gamma_t}}. \tag{1}$$

[7]This follows from the fact that if $x|\mu \sim N(\mu, \sigma^2)$ and $\mu \sim N(\mu_0, \sigma_0^2)$, then $\mu|x \sim N\left(\frac{\sigma_0^2}{\sigma^2+\sigma_0^2}x + \frac{\sigma^2}{\sigma^2+\sigma_0^2}\mu_0, \left(\frac{1}{\sigma_0^2} + \frac{1}{\sigma^2}\right)^{-1}\right)$.

Thus, in each period t, the (myopic) consumer chooses a_t to maximize

$$
\begin{aligned}
&u(a_t) - v\left[\sigma_0^2 - \sigma_t^2(\boldsymbol{a}_t, \boldsymbol{\gamma}_t)\right] \\
=&u(a_t) - v\left[\sigma_0^2 - \frac{1}{\frac{1}{\sigma_{t-1}^2(\boldsymbol{a}_{t-1}, \boldsymbol{\gamma}_{t-1})} + \frac{1}{\frac{1}{a_t}+\gamma_t}}\right],
\end{aligned}
$$

taking $\sigma_{t-1}^2(\boldsymbol{a}_{t-1}, \boldsymbol{\gamma}_{t-1})$ and γ_t as given. Define the privacy cost function as $C(a, \gamma, \sigma^2) := v\left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma^2}+\frac{1}{\frac{1}{a}+\gamma}}\right)$. The following lemma summarizes the key properties of the privacy cost C and the marginal privacy cost $\frac{\partial C}{\partial a}$.

Lemma 1. *The following holds.*

1. *$C(a, \gamma, \sigma^2)$ is decreasing in γ and σ^2, and increasing in a.*
2. *$\dfrac{\partial C}{\partial a}(a, \gamma, \sigma^2)$ is decreasing in γ and increasing in σ^2.*

Proof. Point 1 follows from equation (1). Point 2 follows from

$$
\frac{\partial C}{\partial a} = v \cdot \frac{\frac{\frac{1}{a^2}}{\left(\frac{1}{a}+\gamma\right)^2}}{\left(\frac{1}{\sigma^2}+\frac{1}{\frac{1}{a}+\gamma}\right)^2} = \frac{v}{\left(\frac{1}{\sigma^2}\left(1+\gamma a\right)+a\right)^2}. \tag{2}
$$

□

The privacy cost C is decreasing but the marginal cost $\frac{\partial C}{\partial a}$ of using the platform is increasing in σ^2. Thus, if the consumer has less privacy, then she obtains a lower payoff but has a higher incentive to use the platform. Intuitively, if the platform has more information about the consumer, then information generated today affects the platform's learning less, which implies a lower marginal privacy cost. In particular, if the consumer has no privacy ($\sigma^2 \to 0$), then the marginal cost is zero, because a change in the activity level has no impact on the platform's information.

The rest of Lemma 1 is intuitive. The privacy cost C is increasing in the activity level a and the amount of information the platform has collected, $\frac{1}{\sigma^2}$. A higher privacy level γ makes the signal less informative and lowers the privacy cost. A higher γ also reduces the marginal cost, because adding noise makes the platform's learning less sensitive to the consumer activity.

Let $a^*(\gamma, \sigma^2)$ denote the optimal activity level given a privacy level γ in the current period and the conditional variance σ^2 from the previous period:

$$a^*(\gamma, \sigma^2) := \max \left\{ \arg\max_{a \geq 0} u(a) - v \left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma^2} + \frac{1}{\frac{1}{a} + \gamma}} \right) \right\}. \tag{3}$$

Lemma 1 implies that the consumer chooses a higher activity level when the platform sets a higher privacy level or she has less privacy (see Appendix A for the proof). Recall $a^* = \arg\max_{a \geq 0} u(a)$.

Lemma 2. *$a^*(\gamma, \sigma^2)$ is increasing in γ and decreasing in σ^2. For any (γ, σ^2), $\lim_{\hat{\gamma} \to \infty} a^*(\hat{\gamma}, \sigma^2) = \lim_{\hat{\sigma}^2 \to 0} a^*(\gamma, \hat{\sigma}^2) = a^*$.*

The next result presents the long-run outcome when the platform adopts a stationary privacy policy (see Appendix B for the proof).

Proposition 1. *Suppose that the platform chooses a privacy level $\gamma_t = \gamma$ for all $t \in \mathbb{N}$. Let $(a_t^*)_{t \in \mathbb{N}}$ denote the equilibrium activity levels of this subgame. There is a cutoff value $v^*(\gamma) \in (0, \infty)$ such that:*

1. *If $v < v^*(\gamma)$, then a_t^* increases in t, $\lim_{t \to \infty} a_t^* = a^*$, and $\lim_{t \to \infty} \sigma_t^2 = 0$. The consumer's per-period payoff decreases over time.*
2. *If $v > v^*(\gamma)$, then $a_t^* = 0$ and $\sigma_t^2 = \sigma_0^2$ for all $t \in \mathbb{N}$.*

Moreover, $v^*(\gamma)$ is increasing in γ.

The intuition is as follows. If the value of privacy is low, then it is optimal for the consumer to choose a positive activity level $a_1^* > 0$ in $t = 1$. The consumer activity generates some information, which reduces her payoff but increases the marginal net benefit of using the platform. Thus, in $t = 2$, the consumer chooses $a_2^* \geq a_1^*$. Repeating this argument, we can conclude that a_t^* increases over time. This also implies that the platform can perfectly learn the consumer's type X as $t \to \infty$. This is associated with $a_t^* \to a^*$, because the platform's perfectly learning X implies that the marginal privacy cost goes to zero. Thus, in the long-run, the consumer loses her privacy but behaves as if there is no privacy cost. In contrast, the consumer with a high v does not use the platform in $t = 1$, which implies $a_t = 0$ in all $t \geq 2$.

Proposition 1 has a policy implication: Suppose that a regulator, who cares about consumer privacy, mandates a stricter privacy policy ($\gamma_t = \gamma$ becomes $\gamma_t = \bar{\gamma} > \gamma$ for all $t \in \mathbb{N}$). Since $v^*(\cdot)$ is increasing, the regulation increases the cutoff and expands the range of v's under which the consumer experiences the long-run privacy loss (Point 1). To see the welfare implication, suppose that v satisfies $u(a^*) - v\sigma_0^2 < 0$. For a small γ, the consumer might choose $a_t^* = 0$ and obtain a payoff of zero in all periods. If the regulator enforces a large γ, then the consumer chooses $a_1^* > 0$ in $t = 1$ because it is less costly to use the platform. However, this leads to $(a_t^*, \sigma_t^2) \to (a^*, 0)$, and thus the consumer's per-period payoff converges to $u(a^*) - v\sigma_0^2 < 0$. Thus, the regulation can increase the consumer's per-period payoffs in the short-run but decrease them in the long-run. If the regulator puts a large weight on the long-run welfare, it might consider the regulation as detrimental.[8]

3.2 Equilibrium

I now turn to the equilibrium of the entire game (see Appendix C for the proof, which uses the characterization result in Proposition 3).

Proposition 2. *Take any v, and let $(a_t^*, \gamma_t^*, \sigma_t^2)_{t\in\mathbb{N}}$ denote the outcome of any equilibrium. Then,*

$$\lim_{t\to\infty} a_t^* = a^*, \lim_{t\to\infty} \gamma_t^* = 0, \textit{ and } \lim_{t\to\infty} \sigma_t^2 = 0. \tag{4}$$

Moreover, for any $T \in \mathbb{N}$, there is a $\underline{v}$ such that, for any $v \geq \underline{v}$, any equilibrium privacy policy satisfies $\gamma_t^* > 0$ for all $t \leq T$.

Even if the value v of privacy is arbitrarily high, in the long-run, the consumer becomes an active user with no privacy. This contrasts with Proposition 1, where the consumer with a high v does not use the platform. The result also shows that if v is high, the equilibrium privacy policy is nonstationary: In early periods, the platform sets positive privacy levels. In the long-run, the platform offers a vanishing level of privacy, fully monetizing information generated by consumer

[8] The caveat "if the regulator puts a large weight on the long-run welfare" is important. This is because a higher γ increases the consumer's ex ante sum of discounted payoffs from her optimal policy. This is true even if we consider a forward-looking consumer. Thus, we can argue that a higher γ can be undesirable, only if we evaluate the consumer's behavior under an alternative objective that exhibits more patience than the consumer.

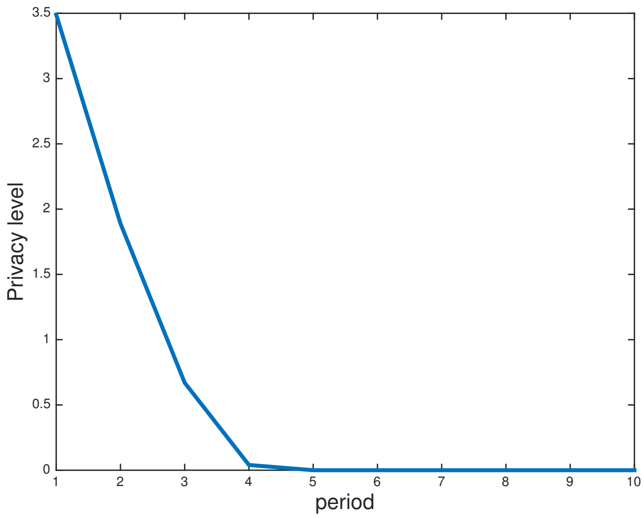

Figure 1(a): Privacy level γ_t

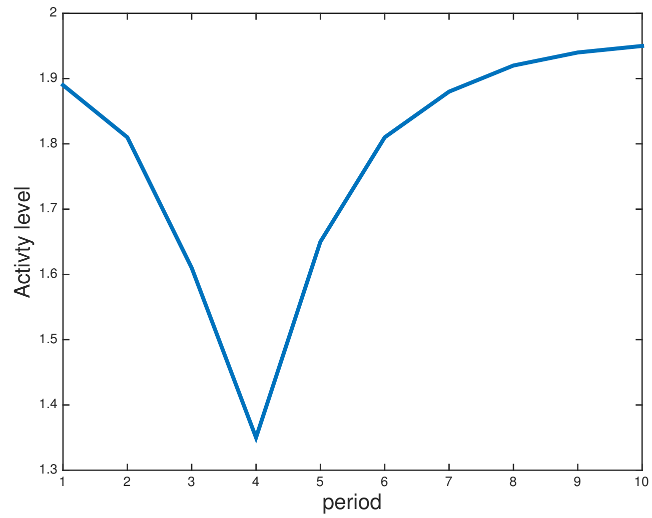

Figure 1(b): Activity level a_t

Figure 1: Equilibrium under $u(a) = 2a - \frac{1}{2}a^2$, $v = 10$, and $\sigma_0^2 = 1$.

activity. The long-run payoff of the consumer is $u(a^*) - v\sigma_0^2$, which can be arbitrarily low if we consider a large v.

The intuition is as follows. In early periods, the platform does not know much about the consumer's type. Then, consumer activity has a large impact on what the platform learns about her type. Thus, the consumer faces a high marginal privacy cost, which discourages her from raising the activity level. To reduce the marginal cost, the platform sets a high privacy level. Thus, in early periods, the platform offers high privacy and slowly learns the consumer type. After a long period of interaction, the platform accurately knows the consumer type, which implies that she faces a low marginal cost. Then, the platform can lower a privacy level to accelerate learning.

Figure 1 depicts the equilibrium dynamics in a numerical example.[9] Figure 1(a) shows that the platform offers a decreasing privacy level, hitting zero in $t = 5$. Figure 1(b) shows that the equilibrium activity level first decreases but eventually approaches $a^* = 2$. This nonmonotonicity contrasts with the case of a stationary privacy policy in Proposition 1.[10]

Proposition 2 has several implications. First, it gives a potential economic explanation of the

[9] I compute the equilibrium strategy profile using Proposition 3.

[10] While I have not manged to prove that the nonmonotonicity always occurs, a numerical exercise suggests that, for a wide range of parameters such that the equilibrium privacy level is strictly decreasing in early periods, the activity level also decreases.

so-called privacy paradox: Consumers seem to casually share their data with online platforms, despite their demand for privacy and concerns about data collection.[11] One may view this puzzle as the long-run equilibrium outcome, where the consumer faces a high privacy cost and zero marginal cost. In contrast to the explanation based on information externalities among consumers, the long-run outcome in Proposition 2 occurs without a priori market imperfection. Indeed, a similar outcome arises even if the consumer is forward-looking and arbitrarily patient (Proposition 8).

Second, observing consumers' intertemporal privacy choices may be useful for applying the revealed preference argument to infer one's preferences for privacy. Indeed, the loss of privacy arises as a long-run outcome of the optimal policy even when a consumer is patient and values privacy highly (Proposition 11). Thus, observing a consumer's privacy choice in a single period may not tell much about one's preferences for privacy (v) if the consumer has already revealed much information.

Third, the consumer's payoff is decreasing but her incentive to use the platform is increasing in past activity levels. This complementarity between past and present consumption is what Becker and Murphy (1988) call rational addiction. One difference from a typical model of rational addiction is that the platform can dynamically adjust the degree of addiction through a privacy policy. As a result, even if the consumer values privacy arbitrarily highly ex ante, she becomes addicted to the platform and loses privacy.

At an anecdotal level, the equilibrium strategy of the platform is also consistent with how the privacy policies of some online platforms have evolved. In 2004, Facebook's privacy policy stated that they would not use cookies to collect consumer information. However, in 2020, it states that they use cookies to track consumers on and possibly off the website.[12] Srinivasan (2019) describes how Facebook has acquired dominance in the social media market as follows:

> "When Facebook entered the market, the consumer's privacy was paramount. The company prioritized privacy, as did its users—many of whom chose the platform over

[11] Acquisti et al. (2016) contains an insightful review of research on the privacy paradox. Recent empirical work, for example, includes Athey et al. (2017).

[12] In 2020, Facebook's privacy policy states that "we use cookies if you have a Facebook account, use the Facebook Products, including our website and apps, or visit other websites and apps that use the Facebook Products (including the Like button or other Facebook Technologies)." https://www.facebook.com/policies/cookies

others due to Facebook's avowed commitment to preserving their privacy. Today, however, accepting Facebook's policies in order to use its service means accepting broad-scale commercial surveillance."

Relatedly, Fainmesser et al. (2019) describe how the business models of online platforms have changed from the initial phase where they expand a user base to the mature phase where they monetize collected information. Proposition 2 rationalizes the described pattern as a platform's optimal policy given the decreasing marginal incentive of consumers to protect their privacy.

3.3 Characterizing Equilibrium Privacy Policy

The following result characterizes the platform's optimal privacy policy. Recall that $a^*(\gamma, \sigma^2)$ is the activity level a_t chosen by the consumer given $\gamma_t = \gamma$ and $\sigma^2_{t-1} = \sigma^2$.

Proposition 3. *The equilibrium privacy policy $(\gamma_1^*, \gamma_2^*, \dots)$ is recursively defined as follows.*

$$\gamma_t^* \in \arg\min_{\gamma \geq 0} \frac{1}{a^*(\gamma, \hat{\sigma}^2_{t-1})} + \gamma, \forall t \in \mathbb{N}, \tag{5}$$

$$\hat{\sigma}_0^2 = \sigma_0^2, \tag{6}$$

$$\hat{\sigma}_t^2 = \frac{1}{\frac{1}{\hat{\sigma}^2_{t-1}} + \frac{1}{\frac{1}{a^*(\gamma_t^*, \hat{\sigma}^2_{t-1})} + \gamma_t^*}}, \forall t \in \mathbb{N}. \tag{7}$$

For any privacy policy $\boldsymbol{\gamma}$, the conditional variances $(\sigma_t^2)_{t \in \mathbb{N}}$ induced by $\boldsymbol{\gamma}$ and the consumer's best responses satisfy $\hat{\sigma}_t^2 \leq \sigma_t^2$ for all $t \in \mathbb{N}$.

Proof. Take any privacy policy $(\gamma_t)_{t \in \mathbb{N}}$, and let $(\sigma_t^2)_{t \in \mathbb{N}}$ denote the sequence of the conditional variances induced by $a^*(\cdot, \cdot)$. I show $\hat{\sigma}_t^2 \leq \sigma_t^2$ for all $t \in \mathbb{N}$. The inequality holds with equality for $t = 0$. Take any $\tau \in \mathbb{N}$. Suppose that $\hat{\sigma}_t^2 \leq \sigma_t^2$ for $t = 0, \dots, \tau - 1$. Then, it holds

$$\sigma_\tau^2 = \frac{1}{\frac{1}{\sigma^2_{\tau-1}} + \frac{1}{\frac{1}{a^*(\gamma_\tau, \sigma^2_{\tau-1})} + \gamma_\tau}} \geq \frac{1}{\frac{1}{\hat{\sigma}^2_{\tau-1}} + \frac{1}{\frac{1}{a^*(\gamma_\tau, \hat{\sigma}^2_{\tau-1})} + \gamma_\tau}} \geq \frac{1}{\frac{1}{\hat{\sigma}^2_{\tau-1}} + \frac{1}{\frac{1}{a^*(\gamma_\tau^*, \hat{\sigma}^2_{\tau-1})} + \gamma_\tau^*}} = \hat{\sigma}_\tau^2.$$

The first inequality follows from the inductive hypothesis and decreasing $a^*(\gamma, \cdot)$. The second inequality follows from (5). $\forall t \in \mathbb{N}, \hat{\sigma}_t^2 \leq \sigma_t^2$ implies that the strategy described by (5), (6), and

(7) is optimal, because it gives the platform a weakly greater profit than any other privacy policy in all periods. $\square$

The objective of the minimization problem (5), $\frac{1}{a^*(\gamma,\hat{\sigma}_{t-1}^2)} + \gamma$, is the variance of the noise term $\varepsilon_t + z_t$ in the signal $s_t = X + \varepsilon_t + z_t$ given the consumer's best response. The minimization problem captures the platform's trade-off. On the one hand, a higher privacy level γ leads to a higher activity level, which leads to a lower variance $\frac{1}{a^*(\gamma,\hat{\sigma}_{t-1}^2)}$ of ε_t. On the other hand, given any activity level, a higher γ lowers the informativeness of the signal. This cost is captured by the second term γ, a variance of z_t. The platform chooses γ_t^* by resolving this trade-off. As the platform solves (5) in each period, the conditional variance evolves according to (7) with the initial condition (6).

The platform chooses its strategy to maximize the sum of discounted payoffs. However, the equilibrium policy is as if it chooses each γ_t to *statically* maximize the informativeness of the signal. The reason is follows. In principle, the platform chooses (say) γ_1 to maximize the sum of period-1 profit and the continuation value. The period-1 profit is increasing in the informative of the signal in $t = 1$ by construction. As more information is generated in $t = 1$, the consumer faces lower marginal costs and chooses higher activity levels in the future. Thus, the continuation value is also increasing in the informativeness of the signal in $t = 1$. As a result, the platform can maximize the sum of discounted profits by maximizing the informativeness of signal in $t = 1$. A similar argument implies that the equilibrium privacy level in any period statically maximizes the informativeness of the signal in that period.

Proposition 3 implies that the platform does not require as strong a commitment power as currently assumed.[13] To state the result formally, I say that the platform has the *short-run commitment power* if it chooses a privacy level γ_t at the beginning of each period t (before the consumer chooses a_t) without committing to future privacy levels.

Corollary 1. *Let $(\boldsymbol{a}^*, \boldsymbol{\gamma}^*)$ denote the equilibrium outcome of the baseline model, in which the platform can commit to any privacy policy. The same outcome $(\boldsymbol{a}^*, \boldsymbol{\gamma}^*)$ can arise in an equilibrium when the platform has the short-run commitment power.*

[13]The proof of the following result also reveals that the equilibrium outcome is independent of the platform's discount factor or time horizon.

Proof. Consider the short-run commitment power. Consider the following strategy profile: For each node with the conditional variance σ^2, the platform sets $\gamma \in \arg\min_{\gamma \geq 0} \frac{1}{a^*(\gamma, \sigma^2)} + \gamma$. The consumer acts according to $a^*(\cdot, \cdot)$ at any node. By construction, $(\boldsymbol{a}^*, \boldsymbol{\gamma}^*)$ arises on the path of play. If the platform deviates at any node, it weakly increases the conditional variances in all periods (Proposition 3). Thus, the platform has no profitable deviation. $\square$

The result shows that the long-run commitment has no value relative to the short-run commitment. In contrast, the platform could be strictly worse off if it has no commitment power: If the platform sets γ_t after observing a_t, then in any equilibrium, the platform sets $\gamma_t = 0$ whenever $a_t > 0$. Anticipating this, the consumer chooses a weakly lower activity level than under the short-run or long-run commitment. In practice, the short-run commitment seems a reasonable assumption, because a platform could be sanctioned for the outright violation of its privacy policy.

4 Platform Competition

I now explore the effect of competition among platforms. There are two platforms, an incumbent (I) and an entrant (E). I is in the market from the beginning of $t = 1$. In period $t^* \geq 2$, E enters the market. t^* is exogenous and known to I at the outset, and thus there is no issue of strategic entry.[14]

Before the entry ($t < t^*$), the consumer chooses her activity level $a_t^I \geq 0$ for I. After the entry ($t \geq t^*$), the consumer chooses $(a_t^I, a_t^E) \in \mathbb{R}_+^2$, where a_t^E is the activity level for E. I assume single-homing: The consumer can choose (a_t^I, a_t^E) if and only if $\min(a_t^I, a_t^E) = 0$. This is natural if two platforms offer substitutable services such as search engines.

I consider two games that differ in the timing of moves. One is *competition with long-run commitment:* I publicly commits to $(\gamma_1^I, \gamma_2^I, \dots)$ at the beginning of $t = 1$, and E publicly commits to $(\gamma_{t^*}^E, \gamma_{t^*+1}^E, \dots)$ at the beginning of period t^*. The other is *competition with short-run commitment:* In each period, each platform chooses a privacy level, after which the consumer chooses an activity level. In particular, I and E set privacy levels γ_t^I and γ_t^E simultaneously in each period $t \geq t^*$.

As in the baseline model, activity on platform $k \in \{I, E\}$ generates a signal $s_t^k = X + \varepsilon_t^k + z_t^k$

[14]Exogenous entry is to simplify the description of equilibrium. If E can choose to enter in any period $t \geq 2$ at a positive entry cost, then we obtain a similar result where E does not enter in equilibrium.

with $\varepsilon_t^k \sim \mathcal{N}\left(0, \frac{1}{a_t^k}\right)$ and $z_t^k \sim \mathcal{N}(0, \gamma_t^k)$. Each platform k privately observes s_t^k. Thus, there is no information spillover between platforms. All the noise terms (ε_t^k, z_t^k) are independent across (t, k).

The payoff of platform $k \in \{I, E\}$ in period t is $\sigma_0^2 - \sigma_{t,k}^2$, where $\sigma_{t,k}^2$ is the conditional variance of X given activity levels $a_1^k, \ldots, a_t^k$ on platform k. The consumer's payoff in period t is given by

$$u(a_t^I) - v\left(\sigma_0^2 - \sigma_{t,I}^2\right) + \mathbf{1}_{\{t \geq t^*\}} \cdot \left[u(a_t^E) - v\left(\sigma_0^2 - \sigma_{t,E}^2\right)\right], \tag{8}$$

where $\mathbf{1}_{\{t \geq t^*\}}$ is the indicator function that equals 1 or 0 if $t \geq t^*$ or $t < t^*$, respectively. Payoff (8) implies that even if the consumer switches to E and never uses I from some period on, she continues to incur a privacy cost based on the information collected by I in the past (and vice versa). For example, suppose that I collects and shares consumer data with an online retailer in period t. Then, consumers may face the risk of price discrimination on the retailer's website in the future, even after they migrate to E. I will later consider how a regulation such as the "right to be forgotten" alters this assumption and changes the equilibrium.

To ensure the existence of an equilibrium, I impose an upper bound on the feasible privacy levels. In practice, the bound might reflect the minimum amount of data that a platform needs to offer services, or the maximum privacy level that a platform can credibly enforce.

Assumption 1. There is a $\bar{\gamma} \in (0, +\infty)$ satisfying $a^*\left(\bar{\gamma}, \sigma_0^2\right) > 0$ such that each platform can choose a privacy level of at most $\bar{\gamma}$.

$a^*\left(\bar{\gamma}, \sigma_0^2\right) > 0$ implies that if a platform chooses $\bar{\gamma}$, then the consumer chooses $a_t^I > 0$ or $a_t^E > 0$. This restriction is necessary for a non-trivial equilibrium in which the consumer uses a platform.

4.1 Equilibrium under Competition

I present an equilibrium in which the consumer never switches to the entrant, and the long-run outcome equals the one under monopoly. If the entry is sufficiently late, then the equilibrium outcome exactly equals the monopoly outcome. Moreover, there is no equilibrium in which the consumer permanently switches to the entrant.

Proposition 4. *Regardless of the commitment assumption:*

1. *There is an equilibrium in which $a_t^E = 0$ for all $t \in \mathbb{N}$, $\lim_{t\to\infty} a_t^I = a^*$, $\lim_{t\to\infty} \sigma_{I,t}^2 = 0$, and $\lim_{t\to\infty} \gamma_t^I = 0$.*

2. *There is a $\underline{t} \geq 2$ such that for any $t^* \geq \underline{t}$, I's privacy levels $(\gamma_t^I)_{t\in\mathbb{N}}$ in the above equilibrium equal the monopoly strategy.*

3. *Switching never occurs: There is no equilibrium in which $a_t^E > 0$ and $a_t^I = 0$ for all $t \geq t^*$.*

The intuition is as follows. Before the entry, I chooses privacy levels that make it optimal for the consumer to choose positive activity levels. Suppose that, upon entry, E chooses the highest privacy level $\bar{\gamma}$. Since the privacy cost from collected data is sunk, the consumer chooses which platform to use based on the marginal (or more precisely, incremental) cost. Because the incumbent has collected data, the consumer faces a lower marginal cost of using I. Thus, if I also chooses $\bar{\gamma}$, then the consumer strictly prefers to use I. This ensures that the consumer only uses I even after the entry. However, the equilibrium choice of I may not be $\bar{\gamma}$: I chooses a privacy level to maximize the amount of information generated subject to the constraint that the consumer prefers I. As time goes by, the constraint is relaxed, because the consumer's marginal cost of using I goes to zero. This ensures that I can offer a vanishing level privacy over time.

The threat of future entry has no impact on I's strategy: Before the entry, I chooses the same privacy levels as monopoly, regardless of commitment assumption. This is because maximizing the amount of information makes consumer switching least likely.

Point 3 implies that, for switching to occur, E needs some advantage in terms of quality of service or privacy level. For the next result, I say that E can *successfully enter the market* if there exists an equilibrium in which the consumer switches to E upon entry, i.e. $a_t^E > 0$ and $a_t^I = 0$ for all $t \geq t^*$. [15]

Proposition 5. *Suppose that the gross benefit of E's service is given by $u^E(\cdot) = u(\cdot) + \Delta$. There is a $\Delta^* > 0$ such that for any $\Delta \geq \Delta^*$, E can successfully enter the market. The minimum Δ^* with this property is increasing in t^*.*

[15] The result considers the entrant's advantage in terms of service quality. We obtain a similar result by considering the entrant that can choose a higher maximum privacy level $\bar{\gamma} + \Delta$.

4.2 Antitrust Implication of Propositions 4 and 5

Why do some online platforms, such as Google and Facebook, have dominant positions? One answer could be that consumers love them: They offer high quality services for free, and consumers tend to use the same platform because of network effect. In other words, we may be seeing market concentration because other firms cannot offer services of comparable quality. Indeed, Furman et al. (2019) states that:

> "Data can act as a barrier to entry in digital markets. A data-rich incumbent is able to cement its position by *improving its service and making it more targeted for users*, as well as making more money by better targeting its advertising" (italicized by the author).

My results contribute to this discussion with the following argument: We do not see competition because consumers are suffering from a lack of privacy and receiving low payoffs from dominant incumbents. This seemingly counterintuitive claim is rationalized by consumers' decreasing marginal incentive to protect privacy. If consumers perceive collected data as sunk, they choose whether to switch based on marginal costs. Then, dominant platforms holding a large amount of data can reduce consumer welfare but make switching less likely to occur. Both the above quote and this idea indicate that data can be an entry barrier. However, the entry barrier in my model is associated with lower consumer welfare.

The insight is relevant to the market for search engines: The incumbent is Google, and the entrant is a "privacy-friendly" alternative such as DuckDuckGo. The result suggests that it might be difficult for DuckDuckGo to penetrate the market simply by offering better privacy protection. If consumers have already lost privacy against Google, their marginal privacy cost of using Google is nearly zero. Thus, DuckDuckGo may need to offer better privacy protection *and* comparable quality of a search engine.

The result also suggests a potential remedy. If consumers can erase collected data when they switch, then competition is more likely to occur. This is because erasing past information eliminates the incumbency advantage. The next section considers such an extension.

5 Erasing Past Information

So far, the amount of information a platform has on the consumer is weakly increasing over time. I now consider the incentive of the consumer or a platform to erase past information.

5.1 The Right to be Forgotten

I consider the right to be forgotten, whereby the consumer can request a platform to erase past information. Below, I describe the model of competition, but a similar description applies to monopoly.

In each period, the consumer makes two decisions. First, the consumer chooses whether to erase past information of each platform in the market. Second, the consumer chooses a_t^I or (a_t^I, a_t^E), depending on whether it is before or after the entry. If she erases information of platform $k \in \{I, E\}$ in period t, then the conditional variance for platform k at the beginning of t becomes σ_0^2. At the end of the period, the consumer still incurs a privacy cost based on information generated in period t. It is costless for the consumer to erase information.

For example, if the consumer erases information of both platforms in period t and uses platform E, then her payoff is

$$u(a_t^E) - v\left[\sigma_0^2 - \sigma_{1,E}^2(a_t^E, \gamma_t^E)\right], \tag{9}$$

where $\sigma_{1,E}^2(a_t^E, \gamma_t^E)$ is the conditional variance for E given one signal based on (a_t^E, γ_t^E). Thus, the privacy cost from E is only based on the signal of period t. Since the consumer has erased information for I and does not use it, she does not incur a privacy cost from I in period t.

In contrast, suppose that the consumer has never erased information. If she uses platform E in period t, then her payoff in period t is

$$u(a_t^E) - v\left[\sigma_0^2 - \sigma_{t,E}^2(\boldsymbol{a}_t^E, \boldsymbol{\gamma}_t^E)\right] - v\left[\sigma_0^2 - \sigma_{t,I}^2(\boldsymbol{a}_t^I, \boldsymbol{\gamma}_t^I)\right], \tag{10}$$

where $\boldsymbol{a}_t^k = (a_1^k, \ldots, a_t^k)$ and $\boldsymbol{\gamma}_t^k = (\gamma_1^k, \ldots, \gamma_t^k)$ for each $k \in \{I, E\}$. Thus, the consumer incurs a privacy cost from both platforms based on past information. The following result summarizes the impact of the right to be forgotten.

Proposition 6. *If the consumer can costlessly erase past information, then regardless of the com-*

mitment assumption, there is the following equilibrium:

1. *Under monopoly, in all periods, the consumer erases information and the platform sets a privacy level γ_1^* defined in (5).*

2. *Under competition, the consumer erases information in all periods, and both platforms set the highest privacy level $\bar{\gamma}$ in any period after the entry (i.e. in period $t \geq t^*$).*

3. *Suppose that the gross benefit of E's service is given by $u^E(\cdot) = u(\cdot) + \Delta$. For any $\Delta > 0$, E can successfully enter the market.*

The right to be forgotten benefits the consumer in three ways. First, it reduces privacy cost. Second, it incentivizes platforms to choose higher privacy levels: Once the consumer erases information, then she incurs a high marginal privacy cost. This ensures that a monopoly platform always offers a period-1 privacy level in any period (Point 1). Under competition, erasing information makes competing platforms homogeneous. This intensifies competition and incentivizes platforms offer the highest privacy level (Point 2). Finally, erasing past information eliminates the incumbency advantage and makes market entry more likely (Point 3).

5.2 Data Retention Policies

This section considers a platform's choice of data retention policies, which have recently been paid attention by economists and legal scholars (Chiou and Tucker, 2017). For simplicity, I focus on the short-run commitment.[16] If there is one platform in the market, then the platform chooses whether to erase past information and sets a privacy level. Then, the consumer chooses an activity level. If there are two platforms, they simultaneously choose whether to erase information. After observing this, they simultaneously set privacy levels. Finally, the consumer chooses an activity level for each platform.

Erasing information affects the conditional variances and payoffs in the same way as the consumer erasing information (see the previous subsection). The following result shows that a platform's optimal data retention policy is to not erase information at all.

[16]The same result holds for other commitment assumptions. For example, if a platform can commit to a privacy policy and a data retention policy at the outset, then it chooses a privacy policy $(\gamma_1, \gamma_2, \dots)$ and $T \in \mathbb{N}$ before $t = 1$. T means that the platform erases information in every T periods.

Proposition 7. *A platform never erases information:*

1. *A monopoly platform never erases information in any period in any equilibrium. The equilibrium outcome equals Proposition 2.*

2. *Under competition, there is an equilibrium in which platforms never erase information in any period. Among the equilibria with this property, Proposition 4 holds.*

If consumer behavior is exogenous, then a platform has no incentive to raise a privacy level or erase past information, because it lowers the profit by reducing the amount of consumer information. However, if consumer behavior is endogenous, then a platform has different incentives for these choices. A platform may have an incentive to increase a privacy level (at least in early periods) because it reduces the consumer's marginal cost and increases her activity level. In contrast, a platform has no incentive to erase information because it increases the consumer's marginal privacy cost and decreases her activity level.

6 Extensions

6.1 Forward-looking Consumer

Suppose that the consumer is patient with one-period ahead discount factor $\delta_C \in (0, 1)$. As in the baseline model, assume that a monopoly platform can commit to any privacy policy at the outset. The following result shows that the long-run equilibrium outcome exhibits a high activity level and no privacy, as the case of a myopic consumer (see Appendix F for the proof, which also proves the existence of an equilibrium).

Proposition 8. *For any v and $\delta_C, \delta_P \in (0, 1)$, in any equilibrium:*

$$\lim_{t\to\infty} a_t^* = a^* \text{ and } \lim_{t\to\infty} \sigma_t^2 = 0. \tag{11}$$

If the platform adopts a nonstationary privacy policy, then the consumer's problem is a nonstationary dynamic programming, which is difficult to solve. Thus, I first show that there is a stationary privacy policy that induces (11). That is, for any discount factor δ_C of the consumer and her value v of privacy, there is a stationary privacy policy such that she loses privacy in the long-run. I then show that the equilibrium policy, which may be nonstationary, also induces the same long-run outcome. The key lemma is that the consumer's sum of discounted payoffs is supermodular in activity levels and the informativeness of signals. This implies that the optimal activity levels are increasing in the amount of information collected in the past and the amount of information the platform will collect in the future.

6.2 Endogenous Quality

So far, the benefit $u(\cdot)$ from the platform's service has been exogenous. Suppose now that the platform can choose its quality: Before $t = 1$, the platform chooses a quality $q \geq 0$. Then, in each period, the consumer receives a gross benefit of $u_q(a) = qa - \frac{1}{2}a^2$, and the platform receives a payoff of $\sigma_0^2 - \sigma_t^2 - c(q)$ for some strictly increasing $c(\cdot)$. The platform chooses q once, but incurs $c(q)$ in every period.

Proposition 9. *A patient platform does not invest in quality: For any $\delta_P \in (0, 1)$, let $q(\delta_P)$ denote the quality in an (arbitrarily chosen) equilibrium. Then, $\lim_{\delta_P \to 1} q(\delta_P) = 0$. Thus, regardless of the consumer's discount factor, as $\delta_P \to 1$, her ex ante sum of discounted payoffs converges to zero, and her long-run equilibrium payoff converges to $-v\sigma_0^2 < 0$.*

Proof. Given (δ_P, q), Let $\Pi(\delta_P, q)$ denote the platform's ex ante sum of discounted profits. For any $q > 0$, the platform's per-period payoff is at most $\sigma_0^2 - c(q)$. Thus, $(1 - \delta_P)\Pi(\delta_P, q) \leq \sigma_0^2 - c(q)$. Suppose to the contrary that there is a sequence $\delta_n \to 1$ such that for some $q' > 0$, $q(\delta_n) \geq q'$ for infinitely many n's (for some selection of equilibria). Without loss of generality, assume $(1-\delta_n)\Pi(\delta_n, q(\delta_n)) \in [0, \sigma_0^2]$ has a limit. Then, $\lim_{n\to\infty}(1-\delta_n)\Pi(\delta_n, q(\delta_n)) \leq \sigma_0^2 - c(q') < \sigma_0^2 - c(q'/2)$. Proposition 8 implies that there is a $\boldsymbol{\gamma}$ under which $(a_t^*, \sigma_t^2) \to (a^*, 0)$ given quality $q'/2$. If the platform chooses $q'/2$ and $\boldsymbol{\gamma}$, then as $\delta_P \to 1$, its average payoff converges to $\sigma_0^2 - c(q'/2)$. Thus, the platform with a large δ_n strictly prefers $q'/2$ to $q(\delta_n)$, which is a contradiction. This implies $\lim_{\delta_P\to 1} u_{q(\delta_P)}(a^*) - v\sigma_0^2 = -v\sigma_0^2$. Also, as the consumer's ex ante payoff is nonnegative but lower than $\frac{u_{q(\delta_P)}(a^*)}{1-\delta_C}$, it converges to 0 as $\delta_P \to 1$. $\square$

6.3 General Privacy Cost Function

Suppose that the consumer's per period payoff is $u(a_t) - C(\sigma_t^2)$. Assume $C(\cdot) : \mathbb{R}_+ \to \mathbb{R}$ is continuously differentiable. This implies that the cost and the marginal cost (i.e. $C(0)$ and $C'(0)$) are bounded. $C(\cdot)$ can be nonmonotone: For example, it can be convex and minimized at some σ^*, which means that the consumer benefits data collection which reduces σ_0^2 to σ^*, but the additional data collection is harmful. The following result shows that the long-run outcome remains the same (see Appendix G).

Proposition 10. *If the consumer is myopic, in any equilibrium:*

$$\lim_{t\to\infty} a_t^* = a^* \text{ and } \lim_{t\to\infty} \sigma_t^2 = 0. \tag{12}$$

6.4 General Payoffs for Platform

All the results of this paper continue to hold if the platform's payoff from any induced sequence $(\sigma_t^2)_{t\in\mathbb{N}}$ of variances is $\Pi((\sigma_t^2)_{t\in\mathbb{N}})$, where $\Pi : \mathbb{R}_+^\infty \to \mathbb{R}$ is bounded and coordinate-wise strictly decreasing. This is because, in the equilibrium for each result, any deviation by a platform weakly increases σ_t^2 for all $t \in \mathbb{N}$. This generalization, for example, permits the following specification. Suppose that the platform sells information to a sequence of short-lived data buyers, and any data sold in period t can be freely replicable in the future and thus have a price of zero in any period $s \geq t+1$. Then, the platform's payoff in period t equals the value of information newly generated in period t. Thus, the ex ante payoff is $\sum_{t=1}^{\infty} \delta_P^{t-1}(\sigma_{t-1}^2 - \sigma_t^2)$. This objective is strictly decreasing in each σ_t^2, because the coefficient of each σ_t^2 equals $-\delta_P^{t-1} + \delta_P^t < 0$.

7 Conclusion

I consider a dynamic model in which a consumer chooses how actively to use a platform in each period. The activity generates information about the consumer's type, which benefits the platform but hurts the consumer. Thus, the consumer balances the benefit of using the service and the cost of losing privacy. I show that the long-run equilibrium outcome exhibits no privacy and a high activity level. This is driven by the consumer's decreasing marginal incentive to protect privacy.

Such an outcome occurs even if the consumer is patient and values privacy arbitrarily highly, because the platform adopts a nonstationary privacy policy that offers high privacy levels in early periods. I show that competition is unlikely to occur in particular when the consumer suffers from low privacy. Allowing the consumer to erase past information mitigates the problem, although platforms have no incentive to erase information.

We may view the current model as a new information design problem: A designer commits to a mechanism specifying what information to collect, in order to influence the behavior of an agent whose action generates some information. It would be interesting to consider a general design problem with this feature.

References

Acemoglu, Daron, Ali Makhdoumi, Azarakhsh Malekian, and Asuman Ozdaglar (2019), "Too much data: Prices and inefficiencies in data markets." Working Paper 26296, National Bureau of Economic Research.

Acquisti, Alessandro, Curtis R Taylor, and Liad Wagman (2016), "The economics of privacy." *Available at SSRN 2580411*.

Athey, Susan, Christian Catalini, and Catherine Tucker (2017), "The digital privacy paradox: Small money, small costs, small talk." Technical report, National Bureau of Economic Research.

Becker, Gary S and Kevin M Murphy (1988), "A theory of rational addiction." *Journal of political Economy*, 96, 675–700.

Bergemann, Dirk, Alessandro Bonatti, and Tan Gan (2019), "The economics of social data." Cowles foundation discussion paper 2203, Yale University.

Bonatti, Alessandro and Gonzalo Cisternas (Forthcoming), "Consumer scores and price discrimination." *Review of Economic Studies*.

Chiou, Lesley and Catherine Tucker (2017), "Search engines and data retention: Implications for privacy and antitrust." Technical report, National Bureau of Economic Research.

Choi, Jay Pil, Doh-Shin Jeon, and Byung-Cheol Kim (2019), "Privacy and personal data collection with information externalities." *Journal of Public Economics*, 173, 113–124.

Crémer, Jacques, Yves-Alexandre de Montjoye, and Heike Schweitzer (2019), "Competition policy for the digital era." *Report for the European Commission*.

Fainmesser, Itay P, Andrea Galeotti, and Ruslan Momot (2019), "Digital privacy." *Available at SSRN*.

Furman, Jason, D Coyle, A Fletcher, D McAules, and P Marsden (2019), "Unlocking digital competition: Report of the digital competition expert panel." *HM Treasury, United Kingdom*.

Hagiu, Andrei and Julian Wright (2020), "Data-enabled learning, network effects and competitive advantage." Technical report, Working paper.

Ichihashi, Shota (2019), "Non-competing data intermediaries."

Kummer, Michael and Patrick Schulte (2019), "When private information settles the bill: Money and privacy in googles market for smartphone applications." *Management Science*.

Lin, Tesary (2019), "Valuing intrinsic and instrumental preferences for privacy." *Available at SSRN*, 3406412.

Milgrom, Paul, Chris Shannon, et al. (1994), "Monotone comparative statics." *Econometrica*, 62, 157–157.

Morton, Fiona Scott, Theodore Nierenberg, Pascal Bouvier, Ariel Ezrachi, Bruno Jullien, Roberta Katz, Gene Kimmelman, A Douglas Melamed, and Jamie Morgenstern (2019), "Report: Committee for the study of digital platforms-market structure and antitrust subcommittee." *George J. Stigler Center for the Study of the Economy and the State, The University of Chicago Booth School of Business*.

Prufer, Jens and Christoph Schottmüller (2017), "Competing with big data." *Tilburg Law School Research Paper*.

Srinivasan, Dina (2019), "The antitrust case against facebook: A monopolist's journey towards pervasive surveillance in spite of consumers' preference for privacy." *Berkeley Business Law Journal*, 16, 39.

Tang, Huan (2019), "The value of privacy: Evidence from online borrowers." Technical report, Working Paper, HEC Paris.

Tucker, Catherine (2018), "Privacy, algorithms, and artificial intelligence." *The Economics of Artificial Intelligence: An Agenda*.

Appendix

A Properties of the Consumer's Best Response: Proof of Lemma 2

Proof. Define $U(a,\gamma,\sigma^2) := u(a) - v\left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma^2}+\frac{1}{\frac{1}{a}+\gamma}}\right)$. Lemma 1 implies that $\frac{\partial U}{\partial a}$ is increasing in γ and decreasing in σ^2. The standard argument of monotone comparative statics implies that $a^*(\gamma,\sigma^2)$ is increasing in γ and decreasing in σ^2 (e.g. Milgrom et al. (1994)).

Suppose to the contrary that $\lim_{\sigma^2\to 0} a^*(\gamma,\sigma^2) = a^*$ fails. As $a^*(\gamma,\sigma^2) \leq a^*$ for all (γ,σ^2), there are $\varepsilon > 0$ and $(\sigma_n^2)_{n\in\mathbb{N}}$ such that $\lim_{n\to\infty}\sigma_n^2 = 0$, $a^*(\gamma,\sigma_n^2) \leq a^* - \varepsilon$ for all $n \in \mathbb{N}$, and $\lim_{n\to\infty} a^*(\gamma,\sigma_n^2) = a^* - \varepsilon$. Suppose that the consumer chooses a^* instead of $a < a^*$. Then, the payoff difference is

$$\Delta(a,\sigma^2) := u(a^*) - u(a) - v\left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma^2}+\frac{1}{\frac{1}{a^*}+\gamma}}\right) + v\left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma^2}+\frac{1}{\frac{1}{a}+\gamma}}\right). \tag{13}$$

Note that $\lim_{n\to\infty}\Delta(a^*(\gamma,\sigma_n^2),\sigma_n^2) = u(a^*) - u(a^*-\varepsilon) > 0$. This implies that $\Delta(a^*(\gamma,\sigma_n^2),\sigma_n^2) > 0$ for a large n. Thus, for a large n, the consumer strictly prefers a^* to $a^*(\gamma,\sigma_n^2)$, which is a contradiction. A similar argument implies that $\lim_{\hat{\gamma}\to\infty} a^*(\hat{\gamma},\sigma^2) = a^*$. $\square$

B The Long-run Outcome Under a Stationary Privacy Policy: Proof of Proposition 1

Proof. To emphasize that the optimal activity level depends on v, I write $a^*(\gamma, \sigma^2)$ as $a^*(v, \gamma, \sigma^2)$. Define $v^*(\gamma)$ as follows:

$$v^*(\gamma) = \sup\left\{v \in \mathbb{R} : a^*\left(v, \gamma, \sigma_0^2\right) > 0\right\}. \tag{14}$$

Note that $\frac{\partial U}{\partial a} = u'(a) - v\frac{1}{\left(\frac{1}{\sigma_0^2}(1+\gamma a)+a\right)^2}$. This implies that

$$\begin{aligned}
\frac{\partial U}{\partial a}\Big|_{a=0} &= u'(0) - v \cdot (\sigma_0^2)^2 \\
\frac{\partial U}{\partial a}\Big|_{a=a'} &\leq u'(0) - v \cdot \frac{1}{\left(\frac{1}{\sigma_0^2}\left(1 + \gamma a^*\right) + a^*\right)^2}, \forall a' \in [0, a^*].
\end{aligned}$$

The second inequality holds because $u(\cdot)$ and the privacy cost function are increasing and concave. For a sufficiently small v, the right hand side of the first equality is positive. Thus, $v^*(\gamma)$ is well-defined and positive. For a sufficiently large v, the right hand side of the second inequality is negative. Thus, $v^*(\gamma)$ is finite.

Suppose $v < v^*(\gamma)$. By the construction of $v^*(\gamma)$, $a^*(v, \gamma, \sigma_0^2) > 0$. σ_t^2 decreases in t for any sequence of activity levels. Thus, if $\gamma_t = \gamma$ for any $t \in \mathbb{N}$, then $a^*(v, \gamma, \sigma_t^2)$ is increasing in t and greater than $a_1^* > 0$ for all t. This implies that $\lim_{t\to\infty} \sigma_t^2 = 0$, because

$$0 \leq \sigma_t^2 \leq \frac{1}{\frac{1}{\sigma_0^2} + \frac{t}{\left(\frac{1}{a_1^*}+\gamma\right)}} \to 0 \quad \text{as} \quad t \to \infty.$$

Lemma 2 implies $\lim_{t\to\infty} a_t^* \to a^*$. For $v > v^*(\gamma)$, note that $a^*(v, \gamma, \sigma_0^2) = 0$, which implies that $a_t^* = 0$ for all t. Finally, $v^*(\gamma)$ is increasing in γ, because $a^*\left(v, \gamma, \sigma_0^2\right)$ is increasing in γ. □

C Properties of Equilibrium: Proof of Proposition 2

Proof. Let $(a_t^*, \hat{\sigma}_t^2)_{t\in\mathbb{N}}$ denote the equilibrium activity levels and conditional variances. First, I prove $\lim_{t\to\infty} \hat{\sigma}_t^2 = 0$. By Lemma 2 and Proposition 1, there is a stationary privacy policy $\gamma_t \equiv \gamma$

such that $\lim_{t\to\infty}\sigma_t^2=0$. By Proposition 3, $\hat{\sigma}_t^2\le\sigma_t^2$ for all $t\in\mathbb{N}$, which implies $\lim_{t\to\infty}\hat{\sigma}_t^2=0$.

To show $\lim_{t\to\infty}a_t^*=a^*$, suppose to the contrary that there is an $\varepsilon>0$ such that $a_t^*\le a^*-\varepsilon$ for infinitely many t's. Without loss of generality, suppose $a_t^*\le a^*-\varepsilon$ for all t. Following the proof of Lemma 2, we can conclude that, for a large t, the consumer strictly prefers a^* to a_t^*. Indeed, if the consumer chooses a^* instead of a_t^*, $u(\cdot)$ increases by at least $u(a^*)-u(a^*-\varepsilon)>0$ whereas the increment of privacy cost goes to zero. This is a contradiction.

Next, suppose to the contrary that there is a $\underline{\gamma}>0$ such that $\gamma_t^*\ge\underline{\gamma}$ for infinitely many t's. To simplify exposition, suppose $\gamma_t^*\ge\underline{\gamma}$ for all $t\in\mathbb{N}$. Take any $\varepsilon\in(0,\underline{\gamma})$. Then, as $\lim_{t\to\infty}a_t^*=a^*>0$, for a sufficiently large t, the minimized value in (5) is weakly greater than $\frac{1}{a^*}-\varepsilon+\underline{\gamma}$. To show a contradiction, let T denote the first period such that $a^*(0,\hat{\sigma}_{T-1}^2)>0$. Then, $a^*(0,\hat{\sigma}_{t-1}^2)>0$ for any $t\ge T$. If the platform chooses $\gamma_t=0$ instead of γ_t^* in period $t\ge T$, then the minimand in (5) equals $\frac{1}{a^*(0,\hat{\sigma}_t^2)}$, which converges to $\frac{1}{a^*}<\frac{1}{a^*}-\varepsilon+\underline{\gamma}$ for a sufficiently large t. This implies that for a sufficiently large t, the platform can strictly increase its payoff in period t by setting $\gamma_t=0$, which is a contradiction.

To show the final part, I write $\gamma_t^*(v)$ to emphasize the dependence of the equilibrium privacy level on v. Suppose to the contrary that there is a T such that, for any $\underline{v}$, there is some $v\ge\underline{v}$ such that $\gamma_t^*(v)=0$ for some $t\le T$. Then, we can find $v_n\to\infty$ and $t^*\le T$ such that $\gamma_{t^*}^*(v_n)=0$ for all n. However, for a sufficiently large v_n, $a_{t^*}^*=0$ if $\gamma_{t^*}^*(v_n)=0$. This follows from the proof of Proposition 1, where I show that the consumer with a sufficiently large v chooses $a_1^*=0$ for a fixed γ_1. This contradicts the optimality of $\boldsymbol{\gamma}^*$ because if the platform sets a sufficiently large γ_{t^*}, then the consumer chooses a positive activity level and the minimand in (5) becomes finite (i.e. $\lim_{\hat{\gamma}\to\infty}a^*(\hat{\gamma},\sigma^2)=a^*$). □

D Equilibrium under Competition: Omitted Proofs in Section 4

D.1 Proof of Proposition 4

Proof. For each $k\in\{I,E\}$, I use $-k$ to mean E or I if $k=I$ and $k=E$, respectively. Suppose that, at the beginning of period $t\ge t^*$, the conditional variance for platform k is $\sigma_{t-1,k}^2$. Let γ_t^k denote the privacy level of platform k in period t. The consumer weakly prefers to use platform k

(i.e. $a_t^{-k} = 0$ maximizes her period-t payoff) if

$$\begin{aligned}
&\arg\max_{a \geq 0} u(a) - v[\sigma_0^2 - \sigma_{t,k}^2(\gamma_t^k, a|\sigma_{t-1,k}^2)] - v[\sigma_0^2 - \sigma_{t-1,-k}^2] \\
\geq &\arg\max_{a \geq 0} u(a) - v[\sigma_0^2 - \sigma_{t,-k}^2(\gamma_t^{-k}, a|\sigma_{t-1,-k}^2)] - v[\sigma_0^2 - \sigma_{t-1,k}^2],
\end{aligned}$$

where $\sigma_{t,k}^2(\gamma, a|\sigma_{t-1,k}^2)$ is the conditional variance at the end of period t when platform k chooses γ, the consumer chooses a, and the conditional variance from the previous period is $\sigma_{t-1,k}^2$. Arranging this inequality, we obtain

$$\arg\max_{a \geq 0} u(a) - v[\sigma_{t-1,k}^2 - \sigma_{t,k}^2(\gamma_t^k, a|\sigma_{t-1,k}^2)] \geq \arg\max_{a \geq 0} u(a) - v[\sigma_{t-1,-k}^2 - \sigma_{t,-k}^2(\gamma_t^k, a|\sigma_{t-1,-k}^2)]. \tag{15}$$

The above inequality implies that the consumer prefers to use k if and only if the gross benefit from the service net of the increment of privacy cost is greater for k than $-k$.

First, assume competition without commitment (a similar proof applies to competition with commitment). Consider the following strategy profile. For each period $t < t^*$, I chooses a monopoly privacy level γ_t^*. Take any period $t \geq t^*$. Let $k^* \in \arg\min_{k=I,E} \sigma_{t-1,k}^2$ denote the platform that has the lower conditional variance (if k^* is not unique, then set $k^* = I$). Then, platform $-k^*$ chooses the highest privacy level $\bar{\gamma}$. Platform k^* chooses a privacy level $\gamma_t^{k^*}$ that solves

$$\begin{aligned}
&\min_{\gamma \in [0,\bar{\gamma}]} \frac{1}{a^*(\gamma, \sigma_{t-1,k^*}^2)} + \gamma \\
\text{s.t.} \quad &\arg\max_{a \geq 0} u(a) - v[\sigma_{t-1,k^*}^2 - \sigma_{t,k^*}^2(\gamma, a|\sigma_{t-1,k^*}^2)] \\
&\geq \arg\max_{a \geq 0} u(a) - v[\sigma_{t-1,-k^*}^2 - \sigma_{t,-k^*}^2(\bar{\gamma}, a|\sigma_{t-1,-k^*}^2)].
\end{aligned} \tag{16}$$

In each period, the consumer myopically chooses a_t^I or (a_t^I, a_t^E) to maximize her per-period payoff. If indifferent, then the consumer uses the platform for which she chose a positive activity level in the most recent period. (If she chose zero activity levels up to period $t-1$, then she sets $a_t^k = 0$ for one of $k \in \{I, E\}$ with equal probability, and chooses a_t^{-k} to maximize her period-t payoff.)

I show that the above strategy profile is an equilibrium. First, the consumer's behavior is optimal by construction. Second, I verify that platforms have no profitable deviation. Without loss of generality, consider a node in period t in which $I = k^*$ and $E = -k^*$. The strategy of E is optimal: By construction, even if E chooses $\bar{\gamma}$ in all periods $s \geq t$, the consumer uses I in any future periods as long as I and the consumer follow the above strategy.

Suppose now that I chooses a privacy level such that the consumer chooses E in period t. If $\sigma^2_{t,E} \leq \sigma^2_{t,I}$, then the consumer uses E in any period $s \geq t+1$. In this case, I's deviation is not profitable. Otherwise, $\sigma^2_{t,E} > \sigma^2_{t,I}$ hold. Note that I obtains a lower payoff in period t, because it is not maximizing the informativeness of the signal. Moreover, at any future period s, I faces an optimization problem

$$\begin{aligned} &\min_{\gamma} \frac{1}{a^*(\gamma, \sigma^2_{s-1,I})} + \gamma \\ \text{s.t.} \quad &\arg\max_{a \geq 0} u(a) - v[\sigma^2_{s-1,I} - \sigma^2_{s,I}(\gamma, a|\sigma^2_{s-1,I})] \\ &\geq \arg\max_{a \geq 0} u(a) - v[\sigma^2_{s-1,E} - \sigma^2_{s,E}(\bar{\gamma}, a|\sigma^2_{s-1,E})]. \end{aligned} \tag{17}$$

After deviation, I faces a strictly lower $\sigma^2_{s-1,E} - \sigma^2_{s,E}(\bar{\gamma}, a|\sigma^2_{s-1,E}) > 0$ because the consumer generated information on E in period t. This means that the set of γ that satisfies the constraint is smaller. Thus, the minimized value in (17) becomes greater for any period $s \geq t+1$ after deviation. This implies that I's payoff is weakly lower for any period $s \geq t$ after the deviation. A similar argument implies that it is not profitable for I to deviate from a monopoly strategy before entry. This is because the deviation lowers I's payoff before and after entry. In particular, the deviation shrinks the set of γ's satisfying the constraint in (17) by increasing $\sigma^2_{s-1,I} - \sigma^2_{s,I}(\gamma, a|\sigma^2_{s-1,I})$.

$\lim_{t\to\infty} \sigma^2_{I,t} = 0$ holds because it holds even if I adopts $\gamma_t = \bar{\gamma}$ for all t, and I chooses each γ^I_t to achieve even lower conditional variances. Given this result, $\lim_{t\to\infty} a^I_t = a^*$ follows the same proof as monopoly.

Suppose that γ^I_t does not converge to 0. Then, there is a convergent subsequence $\gamma^I_{t(n)}$ such that $\lim_{n\to\infty} \gamma^I_{t(n)} = \gamma' > 0$. For a sufficiently large n, both $\gamma = 0$ and $\gamma = \gamma^I_{t(n)}$ satisfy the constraint in (17), because $\sigma^2_{s-1,E} - \sigma^2_{s,E}(\bar{\gamma}, a|\sigma^2_{s-1,E}) = \sigma^2_0 - \sigma^2_{1,E}(\bar{\gamma}, a^*(\bar{\gamma}, \sigma^2_0)|\sigma^2_0) > 0$, but $\lim_{s\to\infty} \sigma^2_{s-1,I} - \sigma^2_{s,I}(0, a^*(0, \sigma^2_{s-1,I})|\sigma^2_{s-1,I}) \leq \lim_{s\to\infty} \sigma^2_{s-1,I} = 0$. As $n \to \infty$, the value of the objective converges to $\frac{1}{a^*}$ and $\frac{1}{a^*} + \gamma'$ for $\gamma = 0$ and $\gamma = \gamma'$, respectively. Thus, for a large n, $\gamma = 0$ achieves a strictly lower value in (17) than $\gamma = \gamma'$. This is a contradiction and thus $\lim_{t\to\infty} \gamma^I_t \to 0$

in the equilibrium.

For a sufficiently large t^*, $\sigma^2_{t^*-1,I} \leq \sigma^2_0 - \sigma^2_{t^*,E}(\bar{\gamma}, a^*(\sigma^2_0, \bar{\gamma})|\sigma^2_0)$. Then, for any period $t \geq t^*$, the constraint (17) holds for any γ. This implies that I's problem is equal to the monopolist's problem after the entry. Combined with the above result, I's choice equals the monopolist's.

Finally, there is no equilibrium in which $a^E_t > 0$ and $a^I_t = 0$ for all $t \geq t^*$. This is because I can then choose $\bar{\gamma}$ for all periods. Given this, the consumer strictly prefers to use I for any period $t \geq t^* \geq 2$, because the consumer has generated information on I in periods $t < t^*$, which leads to a strictly lower marginal privacy cost. □

D.2 Successful Entry: Proof of Proposition 5

Proof. Consider the following strategy profile: In any period $t \leq t^*$, I chooses a monopoly strategy. In any period $t \geq t^*$, I chooses $\bar{\gamma}$, whereas E solves

$$\begin{aligned} &\min_{\gamma \in [0,\bar{\gamma}]} \frac{1}{a^*(\gamma, \sigma^2_{t-1,E})} + \gamma \\ \text{s.t.} \quad &\arg\max_{a \geq 0} u(a) + \Delta - v[\sigma^2_{t-1,E} - \sigma^2_{t,E}(\gamma, a|\sigma^2_{t-1,E})] \\ &\geq \arg\max_{a \geq 0} u(a) - v[\sigma^2_{t-1,I} - \sigma^2_{t,I}(\bar{\gamma}, a|\sigma^2_{t-1,I})]. \end{aligned} \tag{18}$$

Let Δ^* denote the lowest Δ such that the set of γ's that satisfy (18) is nonempty given $t = t^*$, $\sigma^2_{t^*-1,E} = \sigma^2_0$, and the monopoly outcome $\sigma^2_{t-1,I}$. Δ^* is well-defined because the set of all γ's satisfying the constraint is upper hemicontinuous. The rest of the strategy profile is specified analogously to Proposition 4. The same argument as Proposition 4 confirms that this is an equilibrium. Δ^* is increasing in t^*, because a larger t^* decreases $\sigma^2_{t^*-1,I} - \sigma^2_{t^*,I}(\bar{\gamma}, a|\sigma^2_{t-1,I})$.

Finally, suppose $\Delta < \Delta^*$ but there is an equilibrium in which the consumer only uses E in any period $t \geq t^*$. If I adopts a monopoly strategy for any $t < t^*$ and chooses $\gamma^I_t = \bar{\gamma}$ in period t^*, then the consumer strictly prefers to use I in t^*. This weakly increases I's payoff for any period $t < t^*$ and strictly increases I's payoff in period t^*. This is a contradiction. □

E Erasing Past Information: Omitted Proofs from Section 5

E.1 The Right to be Forgotten: Proof of Proposition 6

Proof. Consider monopoly with commitment. Since the consumer's action does not affect a privacy policy, it is optimal for the consumer to erase information in all periods. Anticipating this, the platform maximizes the amount of information generated in each period by solving the problem (5) with $t = 1$. If the platform has only a short-run commitment power, then the platform sets γ_t to maximize the amount of information in each period. Because $\frac{1}{a^*(\sigma^2,\gamma)} + \gamma$ is increasing in σ^2, it is optimal for the consumer to erase information, which leads to a weakly lower amount of information generated. In either case, the platform's problem leads to $\gamma_t = \gamma_1^*$ for all t.

For competition, consider the following strategy profile: Before entry, the consumer erases information in all periods, and chooses the activity level according to $a^*(\cdot,\cdot)$. After entry, the consumer erases information in all periods, and chooses the platform that offers $\gamma_t = \bar{\gamma}$ for all $t \geq t^*$ if there is such a platform (for a node in which both platform have deviated, I assign an equilibrium of that subgame arbitrarily). I sets $\gamma_t^I = \gamma_1^*$ for all $t < t^*$ and $\gamma_t^I = \bar{\gamma}$ for all $t \geq t^*$. E sets $\gamma_t^E = \bar{\gamma}$ for all $t \geq t^*$ upon entry. I can pick any equilibrium in any subgame in which the consumer deviates and chooses to not erase information, because the consumer is worse off relative to no deviation.

Finally, for any $\Delta > 0$, we can construct an equilibrium in which (i) the consumer erases in formation in all periods and sets $a_t^I = 0$ for any $t \geq t^*$, (ii) I sets $\bar{\gamma}$ in any period $t \geq t^*$, and (iii) E sets γ_t^E that makes the consumer indifferent between I and E. This is an equilibrium with successful entry. $\square$

E.2 Data Retention Policies: Proof of Proposition 7

Proof. A monopolists' problem is to solve (5) by choosing a privacy level and whether to erase information. Whenever $\sigma_{t-1}^2 < \sigma_0^2$, erasing information strictly increases the conditional variance, increases the consumer's marginal cost, and shifts $a^*(\cdot,\sigma^2)$ downward. Thus, erasing information strictly lowers the platform's payoff.

In the model of competition, consider the strategy profile in which platforms never erase information on the path of play, and all players behave in the same way as the strategy profile constructed for Proposition 4. The action of each player straightforwardly extends to nodes in which a platform has deleted information, because the relevant state variable in that strategy profile is $(\sigma^2_{I,t-1}, \sigma^2_{E,t-1})$. If a platform erases information, it lowers the payoff and increases the consumer's cost of using the platform. Thus, it is optimal for each platform to not erase information. $\square$

F Forward-looking Consumer: Proof of Proposition 8

This appendix consists of three steps. First, I prove the existence of an equilibrium in which the consumer breaks ties and chooses the "greatest" sequence of activity levels. Second, I prove useful properties of the consumer's value function in the dynamic programming. Finally, I use these results to prove Proposition 8.

I prepare notations. Let $\mathcal{A} := [0, a^*]^{\mathbb{N}}$ denote the set of all sequences of activity levels between 0 and a^*. It is without loss of generality to exclude an activity level strictly above a^*. Let $\boldsymbol{a}$ denote a generic element of $\mathcal{A}$, with the t-th coordinate denoted by a_t. Let $\boldsymbol{\Gamma}$ denote the set of all sequences of non-negative real numbers. Let $\boldsymbol{\gamma}$ denote a generic element of $\boldsymbol{\Gamma}$, with the t-th coordinate denoted by γ_t. I consider product topology for $\mathcal{A}$ and $\boldsymbol{\Gamma}$.

F.1 Existence of an Equilibrium

Take any privacy policy $\boldsymbol{\gamma} \in \boldsymbol{\Gamma}$. The consumer's problem is

$$\max_{\boldsymbol{a}\in\mathcal{A}} \sum_{t=1}^{\infty} \delta_C^{t-1} \left[u(a_t) - v \cdot \left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma_0^2} + \sum_{s=1}^{t} \frac{1}{\frac{1}{a_s}+\gamma_s}} \right) \right]. \tag{19}$$

For any $\boldsymbol{\gamma} \in \boldsymbol{\Gamma}$, let $\mathcal{A}^*(\boldsymbol{\gamma}) \subset \mathcal{A}$ denote the set of all maximizers in (19).

Lemma 3. *$\mathcal{A}^*(\boldsymbol{\gamma})$ is non-empty, compact, and upper hemicontinuous in product topology.*

Proof. First, $\mathcal{A}$ is compact with respect to product topology. Second, the objective function is continuous: To see this, take any sequence of the consumer's choices $(\boldsymbol{a}^n)_{n=1}^{\infty}$ such that $\boldsymbol{a}^n \to \boldsymbol{a}^*$. This implies that, for each $t \in \mathbb{N}$, $\lim_{n\to\infty} a_t^n \to a_t^*$. The consumer's period-$t$ payoff $U_t(\boldsymbol{a}, \boldsymbol{\gamma}) := u(a_t) - v \cdot \left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma_0^2} + \sum_{s=1}^{t} \frac{1}{\frac{1}{a_s}+\gamma_s}} \right)$ is bounded from above and below by $u(a^*) > 0$ and $-v\sigma_0^2 < 0$, respectively. Define $B := \max(u(a^*), v\sigma_0^2) > 0$. Take any $\varepsilon > 0$, and let T^* satisfy $\frac{\delta_C^{T^*}}{1-\delta_C} B < \frac{\varepsilon}{4}$.

Take a sufficiently large n so that, for each $t \leq T^*$, $\delta_C^{t-1}|U_t(\boldsymbol{a}^n, \boldsymbol{\gamma}) - U_t(\boldsymbol{a}^*, \boldsymbol{\gamma})| < \frac{\varepsilon}{2T^*}$. These inequalities imply that

$$\left| \sum_{t=1}^{\infty} \delta_C^{t-1} U_t(\boldsymbol{a}^n, \boldsymbol{\gamma}) - \sum_{t=1}^{\infty} \delta_C^{t-1} U_t(\boldsymbol{a}^*, \boldsymbol{\gamma}) \right| < \varepsilon.$$

Thus, equation (19) is continuous in $\boldsymbol{a}$. For each privacy policy $\boldsymbol{\gamma}$, let $\mathcal{A}^*(\boldsymbol{\gamma}) \subset \mathcal{A}$ denote the set of all maximizers. Berge maximum theorem implies that $\mathcal{A}^*(\boldsymbol{\gamma})$ is non-empty, compact, and upper hemicontinuous. □

Next, I prove some properties of the consumer's objective $U(\boldsymbol{a}, \boldsymbol{\gamma}) := \sum_{t=1}^{\infty} \delta_C^{t-1} U_t(\boldsymbol{a}, \boldsymbol{\gamma})$. Abusing notation, for any $\boldsymbol{a}, \boldsymbol{a}' \in \mathcal{A}$, write $\boldsymbol{a} \geq \boldsymbol{a}'$ if and only if $a_t \geq a'_t$ for all $t \in \mathbb{N}$. $\geq$ is a partial order on $\mathcal{A}$, and $(\mathcal{A}, \geq)$ is a lattice.

Lemma 4. *For any $\boldsymbol{\gamma}$, $U(\boldsymbol{a}, \boldsymbol{\gamma})$ is supermodular in $\boldsymbol{a}$.*

Proof. Take any $\boldsymbol{\gamma}$. Below, I omit $\boldsymbol{\gamma}$ and write $U(\cdot, \boldsymbol{\gamma})$ as $U(\cdot)$. Take any $\boldsymbol{a}, \boldsymbol{b} \in \mathcal{A}$. For each $n \in \mathbb{N}$, define $(\boldsymbol{a} \vee \boldsymbol{b})^n$ as

$$(\boldsymbol{a} \vee \boldsymbol{b})^n = \begin{cases} \max(a_t, b_t) & \text{if} \quad t \leq n, \\ a_t & \text{if} \quad t > n. \end{cases} \tag{20}$$

Similarly, define $(\boldsymbol{a} \wedge \boldsymbol{b})^n$ as

$$(\boldsymbol{a} \wedge \boldsymbol{b})^n = \begin{cases} \min(a_t, b_t) & \text{if} \quad t \leq n, \\ a_t & \text{if} \quad t > n. \end{cases} \tag{21}$$

Also, define $\boldsymbol{b}^n$ as

$$\boldsymbol{b}^n = \begin{cases} b_t & \text{if} \quad t \leq n, \\ a_t & \text{if} \quad t > n. \end{cases} \tag{22}$$

In product topology, $(\boldsymbol{a} \vee \boldsymbol{b})^n \to \boldsymbol{a} \vee \boldsymbol{b}$, $(\boldsymbol{a} \wedge \boldsymbol{b})^n \to \boldsymbol{a} \wedge \boldsymbol{b}$, and $\boldsymbol{b}^n \to \boldsymbol{b}$ as $n \to \infty$. For each $n \in \mathbb{N}$, $U(\boldsymbol{a})$ is supermodular in the first n activity levels, $(a_1, \ldots, a_n) \in \mathbb{R}_+^n$. Thus, $U((\boldsymbol{a} \vee$

$\boldsymbol{b})^n) + U((\boldsymbol{a} \wedge \boldsymbol{b})^n) \geq U(\boldsymbol{a}) + U(\boldsymbol{b}^n)$. Since $U(\cdot)$ is continuous, we can take $n \to \infty$ and obtain $U(\boldsymbol{a} \vee \boldsymbol{b})) + U(\boldsymbol{a} \wedge \boldsymbol{b}) \geq U(\boldsymbol{a}) + U(\boldsymbol{b})$. □

Lemma 5. *There is an $\bar{\boldsymbol{a}} \in \mathcal{A}^*(\boldsymbol{\gamma})$ such that, for any $\boldsymbol{a} \in \mathcal{A}^*(\boldsymbol{\gamma})$, $\bar{\boldsymbol{a}} \geq \boldsymbol{a}$.*

Proof. First, Corollary 2 of Milgrom et al. (1994) implies that $\mathcal{A}^*(\boldsymbol{\gamma})$ is a sublattice of $\mathcal{A}$. Since $\mathcal{A}^*(\boldsymbol{\gamma})$ is compact, for each $t \in \mathbb{N}$, the projection of $\mathcal{A}^*(\boldsymbol{\gamma})$ on the t-th coordinate, i.e.,

$$\mathcal{A}_t^*(\boldsymbol{\gamma}) := \left\{ a_t \in [0, a^*] : \exists \boldsymbol{a}_{-t} = (a_s)_{s \in \mathbb{N} \setminus \{t\}} \in \mathcal{A}^*(\boldsymbol{\gamma}) \text{ s.t. } (a_t, \boldsymbol{a}_{-t}) \in \mathcal{A}^*(\boldsymbol{\gamma}) \right\}, \tag{23}$$

is compact (here, $(a_t, \boldsymbol{a}_{-t})$ is a sequence of activity levels such that the consumer takes a_t in period t and acts according to $\boldsymbol{a}_{-t}$ in other periods). For each $k \in \mathbb{N}$, let $\boldsymbol{a}^k$ denote an optimal policy such that $\boldsymbol{a}^k = \max \mathcal{A}_k^*(\boldsymbol{\gamma})$. Define $\bar{\boldsymbol{a}}^k := \boldsymbol{a}^1 \vee \cdots \vee \boldsymbol{a}^k$. Since $\mathcal{A}^*(\boldsymbol{\gamma})$ is sublattice, for any each $k \in \mathbb{N}$, $\bar{\boldsymbol{a}}^k$ maximized (19). Also, $\bar{\boldsymbol{a}}^k \to \bar{\boldsymbol{a}}$, where $\bar{a}_t = \max \mathcal{A}_k^*(\boldsymbol{\gamma})$ for any $k \in \mathbb{N}$. Since $\mathcal{A}^*(\boldsymbol{\gamma})$ is compact, $\bar{\boldsymbol{a}} \in \mathcal{A}^*(\boldsymbol{\gamma})$. By construction, for any $\boldsymbol{a} \in \mathcal{A}^*(\boldsymbol{\gamma})$, $\bar{\boldsymbol{a}} \geq \boldsymbol{a}$. □

For each $\boldsymbol{\gamma} \in \boldsymbol{\Gamma}$, let $\bar{\boldsymbol{a}}(\boldsymbol{\gamma}) := (\bar{a}_t(\boldsymbol{\gamma}))_{t \in \mathbb{N}}$ denote the "greatest" strategy of the consumer defined in Lemma 5.

Lemma 6. *For each $t \in \mathbb{N}$, $\bar{\boldsymbol{a}}_t(\boldsymbol{\gamma})$ is upper semicontinuous in $\boldsymbol{\gamma}$.*

Proof. By Lemma 3, $\mathcal{A}^*(\boldsymbol{\gamma})$ is upper hemicontinuous. Thus, the set $\mathcal{A}_t^*(\boldsymbol{\gamma})$ of all activity levels for period t is upper hemicontinuous in $\boldsymbol{\gamma}$. Thus, it is enough to show that for any upper hemicontinuous and compact-valued correspondence $\phi : X \twoheadrightarrow \mathbb{R}$, $f(x) := \max \phi(x)$ is upper semicontinuous. To show this, take any $x_n \to x$. For each n, define $y_n = f(x_n)$. Because there is a subsequence $y_{n(k)}$ of y_n that converges to $\limsup y_n$, it holds that $\limsup y_n = \lim y_{n(k)} = \lim f(x_{n(k)}) \leq f(\lim x_{n(k)}) = f(x)$. The inequality holds because ϕ has a closed graph. Connecting the left and right sides, we establish that $f(\cdot)$ is upper semicontinuous. □

Lemma 7. *There exists an equilibrium.*

Proof. The platform's objective is

$$\sum_{t=1}^{\infty} \delta_P^{t-1} \left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma_0^2} + \sum_{s=1}^{t} \frac{1}{\frac{1}{\bar{a}_s(\boldsymbol{\gamma})} + \gamma_s}} \right). \tag{24}$$

To show it is upper semicontinuous, take $\boldsymbol{\gamma}^n \to \boldsymbol{\gamma}$. Then,

$$\begin{aligned}
&\limsup_{n\to\infty} \sum_{t=1}^{\infty} \delta_P^{t-1} \left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma_0^2} + \sum_{s=1}^{t} \frac{1}{\frac{1}{\bar{a}_s(\boldsymbol{\gamma}^n)} + \gamma_s^n}} \right) \\
&= \lim_{k\to\infty} \sup_{n\geq k} \sum_{t=1}^{\infty} \delta_P^{t-1} \left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma_0^2} + \sum_{s=1}^{t} \frac{1}{\frac{1}{\bar{a}_s(\boldsymbol{\gamma}^n)} + \gamma_s^n}} \right) \\
&\leq \lim_{k\to\infty} \sum_{t=1}^{\infty} \delta_P^{t-1} \left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma_0^2} + \sum_{s=1}^{t} \sup_{n\geq k} \frac{1}{\frac{1}{\bar{a}_s(\boldsymbol{\gamma}^n)} + \gamma_s^n}} \right) \\
&= \sum_{t=1}^{\infty} \delta_P^{t-1} \left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma_0^2} + \sum_{s=1}^{t} \lim_{k\to\infty} \sup_{n\geq k} \frac{1}{\frac{1}{\bar{a}_s(\boldsymbol{\gamma}^n)} + \gamma_s^n}} \right) \\
&= \sum_{t=1}^{\infty} \delta_P^{t-1} \left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma_0^2} + \sum_{s=1}^{t} \frac{1}{\liminf_{n\to\infty} \frac{1}{\bar{a}_s(\boldsymbol{\gamma}^n)} + \gamma_s^n}} \right) \\
&\leq \sum_{t=1}^{\infty} \delta_P^{t-1} \left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma_0^2} + \sum_{s=1}^{t} \frac{1}{\frac{1}{\limsup_{n\to\infty} \bar{a}_s(\boldsymbol{\gamma}^n)} + \lim_{k\to\infty} \inf_{n\geq k} \gamma_s^n}} \right) \\
&\leq \sum_{t=1}^{\infty} \delta_P^{t-1} \left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma_0^2} + \sum_{s=1}^{t} \frac{1}{\frac{1}{\bar{a}_s(\boldsymbol{\gamma})} + \gamma}} \right)
\end{aligned}$$

The second equality comes from the dominated convergence theorem, and the last inequality uses the upper semicontinuity of $\bar{a}_s(\boldsymbol{\gamma})$. Thus, given the consumer's optimal behavior, the platform's objective is upper semicontinuous. Since $\boldsymbol{\Gamma}$ is compact, there is a privacy policy $\boldsymbol{\gamma}^*$ that maximizes the platform's objective. $(\boldsymbol{\gamma}^*, \bar{\boldsymbol{a}}(\cdot))$ is an equilibrium. $\square$

F.2 Properties of Consumer Value Function

For each privacy policy $\boldsymbol{\gamma} \in \boldsymbol{\Gamma}$, define

$$V_{\boldsymbol{\gamma}}(\rho) := \sum_{t=1}^{\infty} \delta_C^{t-1} \left[u(\bar{a}_t(\boldsymbol{\gamma})) - v \cdot \left(\sigma_0^2 - \frac{1}{\rho + \sum_{s=1}^{t} \frac{1}{\frac{1}{\bar{a}_s(\boldsymbol{\gamma})} + \gamma_s}} \right) \right]. \tag{25}$$

$V_{\boldsymbol{\gamma}}(\rho)$ is the consumer's maximum value of the objective starting from the conditional variance $\sigma^2 = \frac{1}{\rho}$.

Lemma 8. *For any $\boldsymbol{\gamma} \in \boldsymbol{\Gamma}$, $V_{\boldsymbol{\gamma}}(\cdot)$ is decreasing and convex. For any $\rho > 0$ and $\Delta > 0$,* $\lim_{\rho\to\infty} V_{\boldsymbol{\gamma}}(\rho) - V_{\boldsymbol{\gamma}}(\rho + \Delta) = 0$.

Proof. Take any privacy policy $\boldsymbol{\gamma}$. Hereafter, I omit $\boldsymbol{\gamma}$ from subscripts (thus, the consumer value function is $V(\cdot)$). Consider the "T-period problem, " in which the consumer's payoff in any period $s \geq T$ is exogenously set to zero. For any $t \leq T$, let $V_t^T(\rho)$ denote the consumer's continuation value in the T-period problem starting from period t given $\frac{1}{\sigma_{t-1}^2} = \rho$:

$$V_t^T(\rho) = \max_{a_t,\ldots,a_T} \sum_{s=t}^{T} \delta_C^{s-t} \left(u(a_s) - v\left(\sigma_0^2 - \frac{1}{\rho_{s-1} + \frac{1}{\frac{1}{a_s}+\gamma_s}} \right) \right).$$

Here, $(\rho_t, \ldots, \rho_{T-1})$ are recursively defined by Bayes' rule given $(a_t, \ldots, a_{T-1})$ and $\rho_{t-1} = \rho$. The standard argument of dynamic programming implies that, for each $t = 1, \ldots, T$,

$$V_t^T(\rho) = \max_{a\geq 0} u(a) - v \cdot \left(\sigma_0^2 - \frac{1}{\rho + \frac{1}{\frac{1}{a}+\gamma_t}} \right) + \delta_C V_{t+1}^T \left(\rho + \frac{1}{\frac{1}{a} + \gamma_t} \right), \tag{26}$$

where $V_{T+1}^T(\cdot) \equiv 0$. I use induction to show that $V_1^T(\rho)$ is decreasing and convex. First, V_{T+1}^T is trivially (weakly) decreasing and convex. Suppose that V_{t+1}^T is decreasing and convex. Since $-v \cdot \left(\sigma_0^2 - \frac{1}{\rho+\frac{1}{\frac{1}{a}+\gamma_t}} \right)$ has the same property, $V_t^T(\cdot)$ is also decreasing and convex. Thus, $V_1^T(\cdot)$ is decreasing and convex.

Define $V^\infty(\rho) := \lim_{T\to\infty} V_1^T(\rho)$. $V^\infty(\rho)$ is decreasing and convex, because these properties are preserved under pointwise convergence. I show that $V^\infty(\rho)$ is the value function of the original problem, i.e., $V^\infty(\cdot) = V(\cdot)$. Take any ρ, and let $(\bar{a}_1, \bar{a}_2, \ldots) \in \mathcal{A}^*(\boldsymbol{\gamma})$ denote the optimal policy. For any finite T,

$$V_1^T(\rho) \geq \sum_{s=1}^{T} \delta_C^{s-1} \left(u(\bar{a}_s) - v\left(\sigma_0^2 - \frac{1}{\rho_{s-1} + \frac{1}{\frac{1}{\bar{a}_s}+\gamma_s}} \right) \right). \tag{27}$$

By taking $t \to \infty$, we obtain $V^\infty(\rho) \geq V(\rho)$. Suppose to the contrary that $V^\infty(\rho) > V(\rho)$. Then, there is a sufficiently large $T \in \mathbb{N}$ such that $V_1^T(\rho) - \frac{\delta_C^T}{1-\delta_C} v\sigma_0^2 > V(\rho)$. If the consumer in the original infinite horizon problem adopts the T-optimal policy that gives $V_1^T(\rho)$ up to period t, then she can obtain a strictly greater payoff than $V(\rho)$, which is a contradiction. Thus, $V^\infty(\rho) = V(\rho)$.

Suppose that the consumer starting from $\rho + \Delta$ chooses the policy $(\bar{a}_t^{\rho})_{t\in\mathbb{N}}$ that is optimal for ρ. Let $(\hat{\rho}_t)_{t=1}^{\infty}$ denote the induced sequence of the precisions after $\rho+\Delta$, i.e. $\hat{\rho}_t = \rho + \Delta + \sum_{s=1}^{t} \frac{1}{\frac{1}{\bar{a}_s^{\rho}}+\gamma_s}$. Note that $\hat{\rho}_t \geq \rho_t$ for all $t \in \mathbb{N}$. Then, it holds that $0 \leq V(\rho) - V(\rho+\Delta) \leq \sum_{t=1}^{\infty} \delta_C^{t-1}\left(\frac{1}{\rho} - \frac{1}{\rho+\Delta}\right) = \frac{1}{1-\delta_C}\left(\frac{1}{\rho} - \frac{1}{\rho+\Delta}\right)$. Thus, $\lim_{\rho\to\infty} V(\rho) - V(\rho+\Delta) = 0$. $\square$

F.3 Consequences of Previous Lemmas

The following result extends Proposition 1.

Proposition 11. *Take any $\boldsymbol{\gamma} \in \boldsymbol{\Gamma}$ such that $\gamma_t = \gamma$ for all $t \in \mathbb{N}$. Let $(\bar{a}_t)_{t\in\mathbb{N}}$ denote the equilibrium strategy in the subgame following $\boldsymbol{\gamma}$. There is a $v^*(\gamma) > 0$ such that the following holds:*

1. *If $v < v^*(\gamma)$, then $\bar{a}_t$ is increasing in t, $\lim_{t\to\infty} \bar{a}_t = a^*$, and $\lim_{t\to\infty} \sigma_t^2 = 0$.*
2. *If $v > v^*(\gamma)$, then $\bar{a}_t = 0$ for all $t \in \mathbb{N}$.*

Moreover, $v^*(\gamma)$ is increasing and $\lim_{\gamma\to\infty} v^*(\gamma) = \infty$.

Proof. Since $\gamma_t = \gamma$ for all t, the value function $V(\cdot)$ satisfies the Bellman equation

$$V(\rho) = \max_{a\geq 0} u(a) - v\left(\sigma_0^2 - \frac{1}{\rho + \frac{1}{\frac{1}{a}+\gamma}}\right) + \delta_C V\left(\rho + \frac{1}{\frac{1}{a}+\gamma}\right). \tag{28}$$

Lemma 8 implies that $V(\cdot)$ is decreasing and convex. Thus, the maximand in (28) has the increasing differences in (a, ρ). Thus, $\bar{a}(v, \gamma, \rho)$, the greatest maximizer, is increasing in ρ.

Define

$$v^*(\gamma) := \sup\left\{v \in \mathbb{R} : \bar{a}_1(v, \gamma, \rho) > 0\right\}. \tag{29}$$

$v^*(\gamma)$ is increasing because $\bar{a}_1(v, \gamma, \rho)$ is. Suppose to the contrary that there is a sequence $\gamma^n \to \infty$ such that $v^*(\gamma^n) \leq \bar{v}$ for some $\bar{v} < \infty$. Take the consumer with $v > \bar{v}$. Suppose that the consumer takes $a_t = a^* = \arg\max_{a\geq 0} u(a)$ for all $t \in \mathbb{N}$. As, $\gamma^n \to \infty$,the consumer's period-t payoff converges to $u(a^*)$ for each $t \in \mathbb{N}$. As the consumer's objective is continuous in per-period payoffs with product topology, the sum of discounted payoffs converges to $\frac{u(a^*)}{1-\delta_C} > 0$. This contradicts that $v > \bar{v}$ should choose $a_t = 0$ for all t. Thus, $\lim_{\gamma\to\infty} v^*(\gamma) = \infty$.

By the identical argument with the case of the myopic consumer, we can conclude that the consumer's activity level is positive and increasing in t if $v < v^*(\gamma)$. This implies $\lim_{t\to\infty} \sigma_t^2 \to$

0, or equivalently, $\lim_{t\to\infty} \rho_t = \infty$ with $\rho := \frac{1}{\sigma_t^2}$. By Lemma 8, for any $\rho > 0$ and $\Delta > 0$, $\lim_{\rho\to\infty} V(\rho) - V(\rho+\Delta) = 0$. This, combined with $\lim_{t\to\infty} \rho_t = \infty$, implies $\lim_{t\to\infty} \bar{a}_t(v, \gamma, \rho) = a^*$. Finally, $v > v^*(\gamma)$ implies $\bar{a}_1 = 0$. This implies $\bar{a}_t = 0$ for all $t \in \mathbb{N}$ because the conditional variance does not change. □

The following result is specific to the forward looking consumer. It states that whenever the change in a privacy policy leads to more informative signals in some periods, the consumer also chooses greater activity levels in other periods.

Lemma 9. *Take $\boldsymbol{\gamma}, \boldsymbol{\gamma}' \in \boldsymbol{\Gamma}$. Define $\mathcal{T} = \{t \in \mathbb{N} : \gamma_t = \gamma'_t\}$. Suppose that $\frac{1}{\bar{a}_t(\boldsymbol{\gamma})} + \gamma_t \leq \frac{1}{\bar{a}_t(\boldsymbol{\gamma}')} + \gamma'_t$ for all $t \in \mathbb{N} \setminus \mathcal{T}$. Then, $\bar{a}_t(\boldsymbol{\gamma}) \geq \bar{a}_t(\boldsymbol{\gamma}')$ for all $t \in \mathcal{T}$.*

Proof. Let $\boldsymbol{\beta}$ be any one of $\boldsymbol{\gamma}$ and $\boldsymbol{\gamma}'$. I decompose the consumer's problem (19) into two steps. First, given any $(a_t)_{t\notin\mathcal{T}}$, the consumer chooses $(a_t)_{t\in\mathcal{T}}$ to maximize the following hypothetical objective function:

$$\sum_{t=1}^{\infty} \delta_C^{t-1} \left[\mathbf{1}_{\{t\in\mathcal{T}\}} u(a_t) - v \cdot \left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma_0^2} + \sum_{s=1}^{t} \frac{1}{\frac{1}{a_s} + \beta_s}} \right) \right]. \tag{30}$$

Note that the consumer does not receive a benefit of $u(a_t)$ in period $t \notin \mathcal{T}$. This leads to a mapping that maps any $(a_t)_{t\notin\mathcal{T}}$ to the (greatest) optimal choice of $(a_t)_{t\in\mathcal{T}}$. In the second step, the consumer chooses $(a_t)_{t\notin\mathcal{T}}$ to maximize her original objective, taking the mapping $(a_t)_{t\notin\mathcal{T}} \mapsto (a_t)_{t\in\mathcal{T}}$ as given.

For any $t \notin \mathcal{T}$, a_t affects (30) only through $\frac{1}{a_t} + \gamma_t$, because $\mathbf{1}_{\{t\in\mathcal{T}\}} = 0$. Moreover, the same argument as in the proof of Lemma 4 implies that (30) is supermodular in $\left((a_t)_{t\in\mathcal{T}}, \left\{ \left(\frac{1}{a_s} + \gamma_s \right)^{-1} \right\}_{s\notin\mathcal{T}} \right)$. This implies that if $\frac{1}{\bar{a}_t(\boldsymbol{\gamma})} + \gamma_t \leq \frac{1}{\bar{a}_t(\boldsymbol{\gamma}')} + \gamma'_t$ for all $t \in \mathbb{N} \setminus \mathcal{T}$, then $\bar{a}_t(\boldsymbol{\gamma}) \geq \bar{a}_t(\boldsymbol{\gamma}')$ for all $t \in \mathcal{T}$. □

F.4 $\lim_{t\to\infty} a_t^* = a^*$ and $\lim_{t\to\infty} \sigma_t^2 = 0$: Proof of Proposition 8

Proof. Let $\boldsymbol{\gamma}^*$ denote the equilibrium privacy policy, and let $\boldsymbol{a}^*$ denote the equilibrium activity levels. First, I show $\lim_{t\to\infty} \sigma_t^2 = 0$. Suppose to the contrary that $\lim_{t\to\infty} \sigma_t^2 \neq 0$. As σ_t^2 is weakly decreasing, it holds that $\lim_{t\to\infty} \sigma_t^2 = \hat{\sigma}^2 > 0$ exists. This implies that $a_t^* \to 0$, which I prove to be a contradiction.

By Proposition 11, there exists a $\hat{\gamma}$ such that $v^*(\hat{\gamma}) > v$. That is, if the platform chooses $\gamma_t = \hat{\gamma}$ for all t, then the consumer chooses $\bar{a} > 0$ in $t = 1$. Define $B := \frac{1}{\bar{a}} + \hat{\gamma}$. Consider T^* such that, for all $t \geq T^*$, $\frac{1}{a_t^*} > B$. Suppose that the platform replaces γ_t^* with $\hat{\gamma}$ for all $t \geq T^*$, and commits to such a new policy ex ante. Take any period $t \geq T^*$. Since the consumer's activity levels after T^* solve the Bellman equation with the "initial state" of $\rho = \frac{1}{\sigma_{T^*}^2} \geq \frac{1}{\sigma_0^2}$, the consumer chooses an activity level weakly greater than $\bar{a} > 0$ after period T^*. Thus, the variance of the noise $\varepsilon_t + z_t$ in the signal s_t is at most $\frac{1}{\bar{a}} + \hat{\gamma} < \frac{1}{a_t^*} + \gamma_t^*$. Thus, this change strictly increases the platform's profit in any period $t \geq T^*$. By Lemma 9, this change also increases the consumer's activity level for any period $t < T^*$. Thus, as a result of the deviation, the platform's payoffs increase in all periods and strictly increase in some periods, which contradicts $\boldsymbol{\gamma}^*$ being optimal. Thus, in equilibrium, $\lim_{t\to\infty} \sigma_t^2 = 0$ holds. Finally, $\lim_{t\to\infty} \sigma_t^2 = 0$ implies $a_t^* \to a^*$: Otherwise, there is a convergent subsequence $a_{t(n)}^* \to a' < a^*$, however, the consumer could be strictly better off by choosing a^*, due to Lemma 8. $\square$

G General Privacy Cost: Proof of Proposition 10

Proof. Consider the consumer's problem in period t. Given the conditional variance σ^2 at the end of period $t-1$ and the privacy level γ in period t, the consumer chooses a to maximize $U(a, \gamma, \sigma^2) := u(a) - C\left(\frac{1}{\frac{1}{\sigma^2} + \frac{1}{\frac{1}{a}+\gamma}}\right)$. It holds that

$$\frac{\partial U}{\partial a} = u'(a) + C'\left(\frac{1}{\frac{1}{\sigma^2} + \frac{1}{\frac{1}{a}+\gamma}}\right) \cdot \frac{1}{\left(\frac{1}{\sigma^2}(1+\gamma a) + a\right)} \geq u'(a) - B \cdot \frac{1}{\left(\frac{1}{\sigma^2}(1+\gamma a) + a\right)}, \tag{31}$$

where $B := \sup_{x \in [0, \sigma_0^2]} |C'(x)| < \infty$. If $\lim\limits_{t\to\infty} \sigma_t^2 > 0$, then $\lim\limits_{t\to\infty} a_t^* = 0$. This implies that $\lim_{t\to\infty} \frac{1}{a_t^*} + \gamma_t^* = \infty$. Consider a hypothetical payoff function

$$U_B(a, \gamma, \sigma^2) = u(a) - B \cdot \left(\sigma_0^2 - \frac{1}{\frac{1}{\sigma^2} + \frac{1}{\frac{1}{a}+\gamma}}\right).$$

(31) implies that $\frac{\partial U}{\partial a} \geq \frac{\partial U_B}{\partial a}$. Take any γ' such that $a_B^*(\gamma', \sigma^2) := \max\left\{\arg\max_{a\geq 0} U_B(a, \gamma', \sigma_0^2)\right\} > 0$. Then, for any $\sigma^2 \leq \sigma_0^2$, $a^*(\gamma', \sigma^2) \geq a_B^*(\gamma', \sigma^2) \geq a_B^*(\gamma', \sigma_0^2) > 0$. Take T such that for all $t \geq T$, $\frac{1}{a_t^*} + \gamma_t^* \geq \frac{1}{a_B^*(\gamma', \sigma_0^2)} + \gamma'$ Then, the platform can achieve a lower $\frac{1}{a_t} + \gamma_t$ for any $t \geq T$ by

replacing γ_t^* with γ', which is a contradiction. A similar argument implies that $\lim_{t \to \infty} a_t^* = a^*$. $\square$