CS6140 Machine Learning

HW2 - Gradient Descent Learning

Make sure you check the syllabus for the due date. Please use the notations adopted in class, even if the problem is stated in the book using a different notation.

Make sure to read the notes on Gradient Descent for Regression, and chapter 5 of DHS, up to (including 5.6 Relaxation Procedures)

PROBLEM 1 [50 points]

A) Train Linear Regression using Gradient Descent (and then test) on Spambase and Housing datasets form HW1.

B) Run Gradient Descent Logistic Regression on Spambase data.
Note: Normalization matters. When you normalize data features (one feature at a time), you need to normalize all data (train, test, validation) together, as opposed to normalize separately training and testing sets.

Compare for each dataset training and testing performance across all four learning algorithms by making a table like below

	Decision or Regression Tree	**Linear Regression (Normal Equations)**	**Linear Regression(Gradient Descent)**	**LogisticRegression(Gradeint Descent)**
Spambase	Train ACC: Test ACC:	Train ACC: Test ACC:	Train ACC: Test ACC:	Train ACC: Test ACC:
Housing	Train MSE: Test MSE:	Train MSE: Test MSE:	Train MSE: Test MSE:	N/A . - WHY?

C) For classification (Spambase), produce Confusion Matrices (TruePos, FalsePos, TrueNeg, FalseNeg) for Decision Trees, Linear Regression and Logistic Regression - three 2x2 matrices. You will have to use a fixed threshold for each regression algorithm.
D) For classification (Spambase), produce ROC plots comparison between linear regression and logistic regression - two curves. Compute the AUC for each curve.

PROBLEM 2 [40 points] Perceptron Algorithm (Gradient Descent for a different objective)

Step 1: Dowload the perceptron learning data set that I have created. The data set is tab delimited with 5 fields, where the first 4 fields are feature values and the last field is the {+1,-1} label; there are 1,000 total data points.
Step 2: Create a perceptron learning algorithm, as described in class.
Step 3: Run your perceptron learning algorithm on the data set provided. Keep track of how many iterations you perform until convergence, as well as how many total updates (corresponding to mistakes) that occur through each iteration. After convergence, your code should output the raw weights, as well as the normalized weights corresponding to the linear classifier
w1 x1 + w2 x2 + w3 x3 + w4 x4 = 1

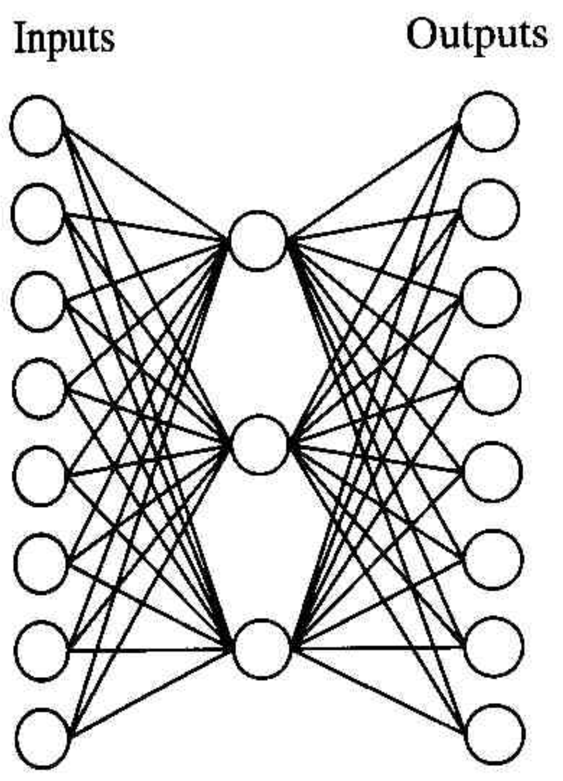
(You will create the normalized weights by dividing your perceptron weights w1, w2, w3, and w4 by -w0, the weight corresponding to the special "offset" feature.)

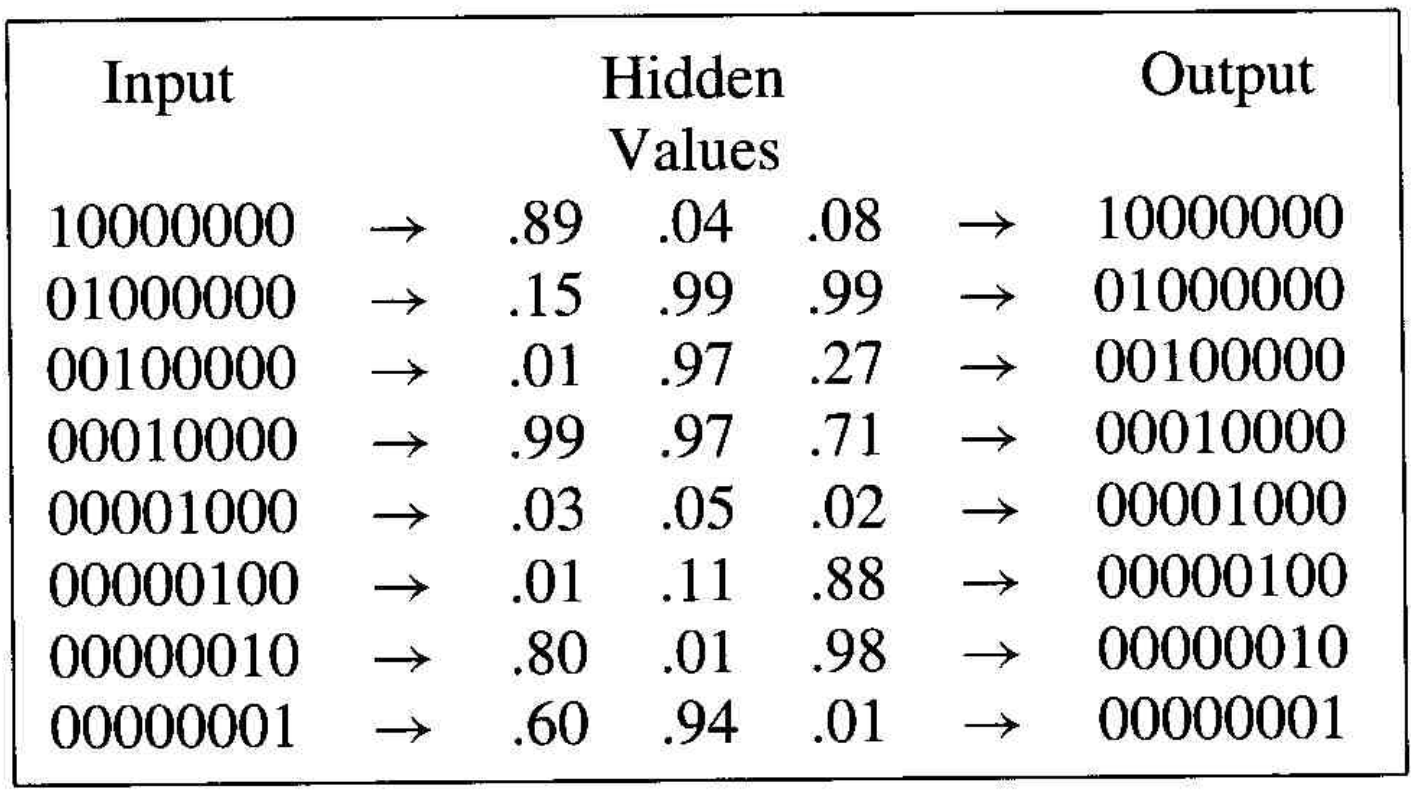
Step 4: Output the result of your perceptron learning algorithm as described above. Your output should look something like the following:

```
[jaa@jaa-laptop Perceptron]$ perceptron.pl perceptronData.txt


Iteration 1 , total_mistake 136
Iteration 2 , total_mistake 68
Iteration 3 , total_mistake 50
Iteration 4 , total_mistake 22
Iteration 5 , total_mistake 21
Iteration 6 , total_mistake 34
Iteration 7 , total_mistake 25
Iteration 8 , total_mistake 0

Classifier weights: -17 1.62036704608359 3.27065807088159 4.63999040888332 6.79421449422058 8.26056991916346
9.36697370729981

Normalized with threshold: 0.0953157085931524 0.192391651228329 0.272940612287254 0.399659676130622 0.485915877597851
0.550998453370577
```
(Note: The output above corresponds to running on a different data set than yours which has six dimensions as opposed to four. Your results will be different, but you should convey the same information as above.)

PROBLEM 3 [70 points] AUTOENCODER Neural Network

Consider the following neural network (left graph), with 8 input units (for data with 8 features), 3 hidden units and 8 output units, and assume the nonlinear functions are all sigmoid.





Input		Hidden Values				Output
10000000	→	.89	.04	.08	→	10000000
01000000	→	.15	.99	.99	→	01000000
00100000	→	.01	.97	.27	→	00100000
00010000	→	.99	.97	.71	→	00010000
00001000	→	.03	.05	.02	→	00001000
00000100	→	.01	.11	.88	→	00000100
00000010	→	.80	.01	.98	→	00000010
00000001	→	.60	.94	.01	→	00000001

a)The 8 training data inputs are identical with the outputs, as shown in the right side table. Implement this network and the backpropagation algorithm to compute all the network weights; you should initialize the weights with nontrivial values (i.e not values that already minimize the erorr).

HINT: on the trained network, you should obtain values for the hidden units somehow similar with the ones shown in the table (up to symmetry). Feel free to make changes to the algorithm that suit your implementation, but briefly document them.

b) Since the outputs and inputs are identical for each datapoint, one can view this network as an encoder-decoder mechanism. (this is one of the uses of neural networks). In this context, explain the purpose of the training algorithm. (I expect a rather nontechnical --but documented-- answer).

c) Can this encoder-decoder scheme work with 1 hidden unit? Can it work with 2 hidden units? What if there are more than 8 inputs and outputs? Justify your answers mathematically.

Hints: Some students worked easier from this algorithm, rather than the one from Mitchell's book

PROBLEM 4 [20 points]

DHS ch6, Pb1
Show that if the transfer function of the hidden units is linear, a three-layer network is equivalent to a two-layer one. Explain why,

therefore, that a three-layer network with linear hidden units cannot solve a non-linearly separable problem such as XOR or n-bit parity.

PROBLEM 5 [30 points]

Read prof Andrew Ng's lecture on ML practice advice. Write a brief summary (1 page) explaining the quantities in the lecture and the advice.

Read prof Pedro Domingos's paper on A Few Useful Things to Know about Machine Learning. Write a brief summary (1 page), with bullet points.

PROBLEM 6 [Extra Credit]

DHS chapter 5 Pb 2 (page 271)

PROBLEM 7 [Extra Credit]

DHS chapter 5 Pb 5, page 271

PROBLEM 8 [Extra Credit]

DHS chapter 5 Pb 6, page 271

PROBLEM 9 [extra credit]

For a function f(x1,x2,..., xn) with real values, the "Hessian" is the matrix of partial second derivatives

$$H(f) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1\,\partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_1\,\partial x_n} \\ \frac{\partial^2 f}{\partial x_2\,\partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \cdots & \frac{\partial^2 f}{\partial x_2\,\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_n\,\partial x_1} & \frac{\partial^2 f}{\partial x_n\,\partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_n^2} \end{bmatrix}$$

Consider the log-likelihood function for logistic regression

$$l(\theta) = \sum_i y_i \log h_\theta(\mathbf{x}_i) + (1 - y_i)\log(1 - h_\theta(\mathbf{x}_i))$$

Show that its Hessian matrix H is negative semidefinite, i.e. for any vector z satisfies

$$z^T H z \leq 0.$$

Remark: This fact is sometimes written and implies the log-likelihood function is concave.

Hint: $\sum_i \sum_j z_i x_i x_j z_j = (\mathbf{x}^T \mathbf{z})^2 \geq 0$

PROBLEM 10 [extra credit]

Run Logistic Regression on the Spambase dataset, but using Newton's numerical method instead of Gradient Descent. An intro to Newton's method can be found in the lecture notes.

PROBLEM 11 [extra credit]

Given a ranking of binary items by prediction score, the ROC is the curve plotted of True Positives vs False Positives for all possible thresholds. The AUC is the area under the ROC curve.
Prove that the AUC is also the percentage of item pairs (i,j) in correct order.

As an illustrative example, the classifier output might be

object	score	truelabel
A	100	1
B	99	1
C	96	0
D	95	1
E	90	1
F	85	0
G	82	1
H	60	0
K	40	0
I	38	0

The ROC curve is obtained by truncating the list above at all ranks, and for each such threshold computing false-positive-rate and true-positive-rate (and plot them).
The problem asks to show that the area under the ROC curve is approximated by the percentage of pairs in correct order. In this example the item pairs in **incorrect order** are (C,D), (C,E), (C,G), (F,G)