Estimation of a Parameter in a Random Decay Process:

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1 Introduction

This work provides an interesting tool to estimate an unknown parameter that is essential to understand the decay process that we have described here. We have been studying three different decay chains of a particle known as η. This particle decays either into α, β, or γ. All the daughter particles share many similarities as they belong to the same family in the standard model of particle physics. But they have different masses. α and γ have slightly different masses, whereas, the mass of β is almost two times greater than the masses of the other two particles.

We have calculated the probabilities to find out these particles in a decay process of η. A model estimates these probabilities based on some differential cross-section and it suggests that the probability to detect α in a single decay process is 0.28. The same goes for the γ. The probability of β is 0.44. Therefore, this model suggests that we will be getting more β in the detector as compared to the other two particles. A parameter describes these probabilities. In a simple way, we can describe the components of this parameter as the probabilities of these particles.

This study will focus on measuring the value of the unknown parameter from the data. In order to do this, We need to determine the Log-Likelihood Ratio (LLR) of the probability distribution function, and then we will write the LLR as a function of the parameter. The maximization of the function will give us the value of the unknown parameter.

End of the study, I will plot the residuals of the parameter using its calculated values from the model and the measured values from the data.

2 Model: Relating the unknown parameter with LLR

The parameter θ that assigns the probabilities for the above mention decays is related to the LLR. Since there are three possible decays, therefore there are three components of the parameter. In this case, LLR can be described using the categorical distribution. The categorical distribution is written as;

$$P(X|p) = \prod_{i}^{N} p_i^{[x=i]} \tag{1}$$

Here "p" is the probability that can also be considered a component of the parameter which is fixed. the "X" is the outcome of the experiment. "x" in the super-script of the "p" is the indicator function. It yields "1" for x=i and "0" otherwise. The LLR is written as;

$$P(p|X) = \prod_{i}^{N} p_i^{[x=i]} \tag{2}$$

Apart from this we also have we have a constraint equation that is ;

$$\Sigma_i^N p_i = 1 \tag{3}$$

We can also think the sum of the components of the parameter is unity.
Using the Lagrange Multiplier method the maximize the LLR we expressed the relations for the three components of this parameter, that is;

$$\theta_j = \frac{\Sigma_i^N \sigma_{j,i}}{N} \qquad , j = 1, 2, 3 \tag{4}$$

The numerator of the Eqn (4) is the number of the particular particles that appear in the detector in N experiments.

3 Code and Experimental Simulation

I have simulated the data based on the probabilities fixed by the components of the parameter. An independent random number has been generated using the TRandom3 and TTimeStamp class of the root. It returns the three possible outcomes in terms of the numbers (1, 2, 3) in a single trial. One can control the number of trials by their own choice by passing an integer to the data simulating function (getDecays.C).
The code is in C++ which can be seen here. `https://github.com/shoukatphy/PHSX815_Project3-`

4 Analysis

Some of the explanation of the analysis part has been already discussed in section 2 "Model: relating the parameter..".
As the LLR is the function of the parameter. Therefore, the parameter value can be estimated using the maximization of the LLR. We also have a constrained equation. Therefore, the maximization of this function is carried out by using the Lagrange multiplier method. We have done this part analytically and found out the relations for the unknown parameter or parameters. We can also relate the parameter with the probability of a single particle instead of the probabilities of all three particles.

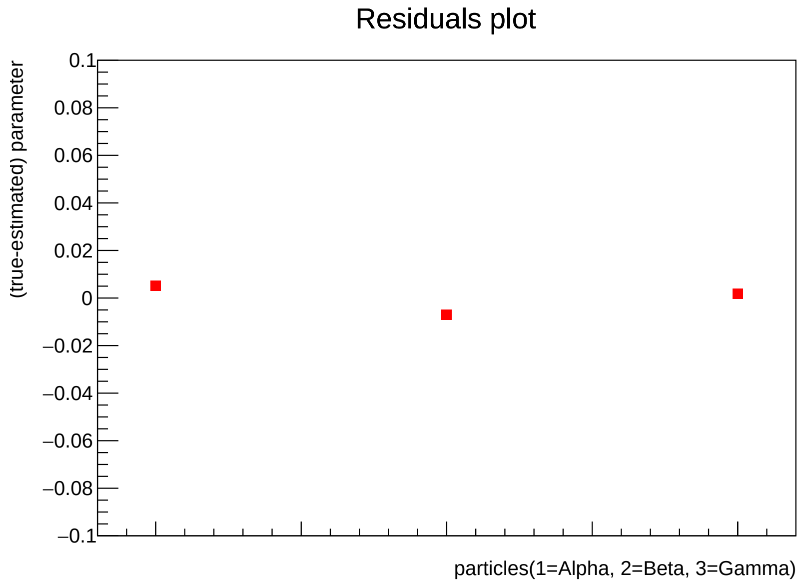

Figure 1: A residual plot between the true values and measured values of the components of the parameter α

My program calculates the parameter by solving the expressions given in Eqn . I also printed out the residual to measure an error in the estimation of these values. The analysis part of the program is carried out by the program (ParamEstimation.C).

5 Conclusion

We have simulated around ten thousand of the experiments based on the given probabilities. The analysis of the data gives us the measured values for the three components of the parameter. We can see that the measured values are close enough to what we have calculated in our model. All the residuals are approaching zero as we increase the number of the in the simulation to generate the data.
Furthermore, it seems to be a great way to estimate the unknown parameters that appear in the models.