A probability distribution analysis of a chain of decay processes :

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1 Introduction

In this paper, we will be studying a chain of probability distribution analysis of some hypothetical decay processes.
We have a particle X that can decay either into Σ (sigma) or Δ (delta). The latter two particles belong to the same family. Therefore, they are sharing many similarities.
The Σ and Δ further decays into one of the three particles- α, β, and γ. These decay chains have different probabilities.
We have established two hypotheses to make a study of these decay processes. The first hypothesis 'H_o' has been constructed on the basis of the particle's mass, spin, and charge which is the same for both Σ and Δ. Therefore, 'H_o' says that these two particles are equiprobable to have appeared in the detector as a product of the decay of X. On the other hand, the second hypothesis 'H_1' prefers to look into the flavor of the particles and it claims that there are more chances for the creation of Σ in the decay process. In the second chain of the decay, the probability of the creation of β from Σ is greater than the production of α, and γ. Whereas, when the Δ decays we see more γ particles in the detector as compared to α and β.

This study is comprised as follows: Sec. 2 explains the two hypothesis and their probability distribution on the basis in n-number of events. A code for data simulations will be provided in Sec. 3, with an analysis of the outputs included in Sec. 4. Finally, the conclusion will be given at Sec. 5.

2 Hypotheses: Detecting a random particle in the experiment

The parent particle X can decay into a Σ or Δ in a single decay.
We assume that these particles have the same mass and charge. Therefore, based on these general properties we assign them an equal probability to get produced in this decay. This is our first assumption or we can consider it as our zero hypotheses–named $H0$. We have assigned "0" to Σ and "1" to Δ. So the counts of "1" and "0" in the data will give us the number of Σ and Δ particles
. Similarly, "1", "2", and "3" has been assigned to α, β, and γ, respectively. Please, note that the "1" of Δ and "1" of α appear in different decays. Therefore, they can be distinguished and can be saved in two different data files.

Although Σ and Δ share great similarities they have different flavors. It might be not wrong if we assign different probabilities for these particles to get in the detector in the decay based on their flavor.

Therefore, our second hypothesis ($H1$) considers that there are more chances that we will hit with more Δ particles if we take some random decays of X. $H1$ assigns a probability of 0.7 to the detection of a Δ particle in a single decay. The probabilities for α, β, or γ remains same in both hypothesis.

3 Code and Experimental Simulation

This experiment is designed in such a way that it will show either "0" or "1" in the decay of the particle X. A program generates these numbers based on $H0$ and $H1$ and also records them in two different text files. We have used the TRandom3 class of ROOT to generate the random numbers.
These secondary particles further decay into the other particles that can be seen as " 1", "2", or "3". These sets of numbers have been generated using TRandom3 class and a function GetNanoSec() of the TTimeStamp class of the ROOT. the total number of decays or trials will be equal to the total number of particles received in the detector. We need to pass an integer as an argument of the main function of the program. This number will be equal to the number of decays we have seen. The simulation part of the experiment is done by the file doDecays.C which can be accessed using the Github link `https://github.com/shoukatphy/Project2`.

4 Analysis

In the first set of decays, we can apply the binomial distribution. This is similar to the two-sided coin toss as particle X can either create a Σ or a Δ particle.
Since These particles further decay into the second set of particles known as α, β, and γ. Therefore, we need to carry out the following integration to go further.

$$p(x|\lambda) = \int d\lambda P(\lambda|H) \tag{1}$$

Where x represents the data of the H-Hypothesis and λ is the data of the first set of the decay. This part has been carried out in the program used to obtain the simulated data.
There are three possible outcomes of the decay of Σ and Δ out of which only one can be seen in a single experiment. Therefore, we consider it will be a categorical distribution. A probabilistic distribution for the production of the above-mentioned two particles is visualized using figure-1 .

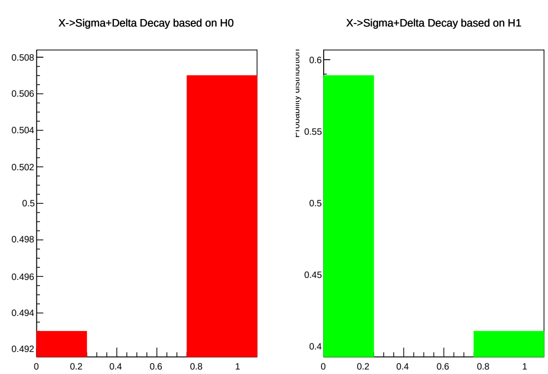

Figure 1: A probability distribution for σ [0] and Δ [1] recorded in the detector

Since $H0$ gives equal probability to each outcome. Therefore, I was expecting to obtain an equal probability for each of the particles. But it shows the opposite of that. Whereas $H1$ makes Σ more probable. Therefore, the corresponding data shows a large probability corresponding to the Σ particles in the right plot in Figure 1.

In the case of the categorical distribution for the three outcomes, the probability distribution for a single event is written as;

$$P(x/Hi) = P_{\alpha}^{x=\alpha} \times P_{\beta}^{x=\beta} \times P_{\gamma}^{x=\gamma} \tag{2}$$

Here, P_i is the assigned probability and the exponent works in such a way that if x gets the particle type it yields 1, otherwise gives 0. For the n-number of decays, the power of each term on the right side of the equation (1) will be raised to n.

We also analyzed the data to obtain the log-likelihood ratio for the two hypotheses that are working based on the equation;

$$LLR = N_{\alpha} log(\frac{P(\alpha|H0)}{P(\alpha|H1)}) \times N_{\beta} log(\frac{P(\beta|H0)}{P(\beta|H1)}) \times N_{\gamma} log(\frac{P(\gamma|H0)}{P(\gamma|H1)}) \tag{3}$$

Where N_{α}, N_{β}, and N_{γ} are the number of particles detected in the N many decays. I was getting big numbers for LLR. Therefore, I got normalized the numbers by dividing a constant

The following figure explains the LLR for two hypotheses.

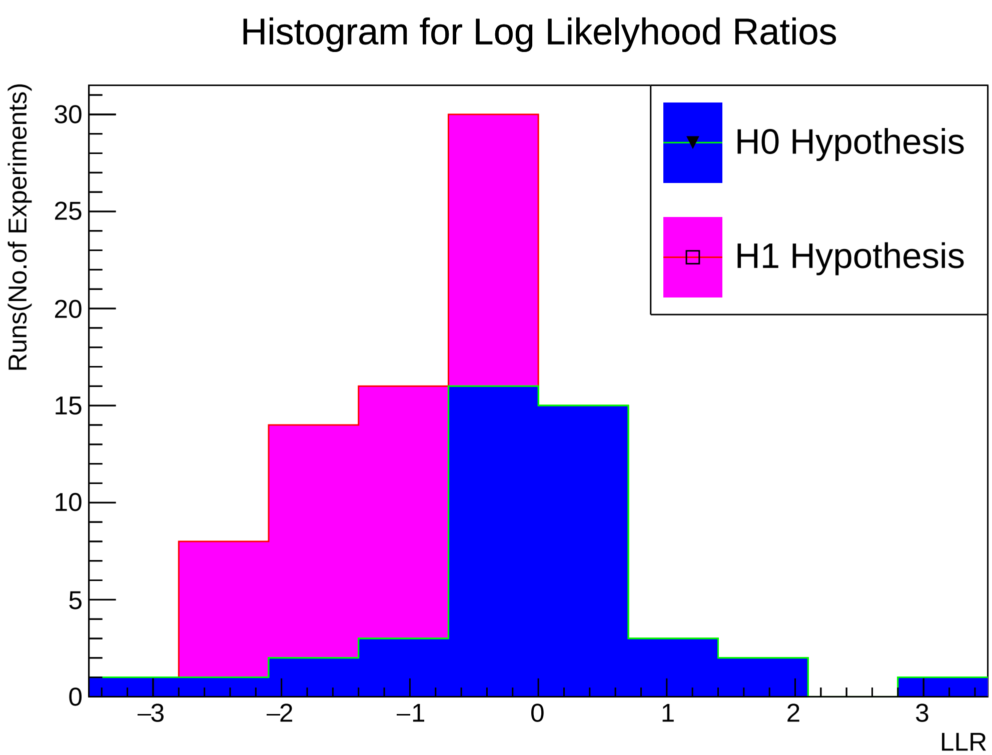

Figure 2: LLR based on H0 and H1 hypothesis

5 Conclusion

We came to the conclusion that the LLR of the two hypotheses makes a difference between the decay products of Σ and Δ otherwise the detectors have no ability to differentiate between the two particles if they are coming from the decay of the Σ or through the decay of Δ. The number of the α, β, and γ particles we have recorded in the detector is different for H_0 and H_1.
If we keep a record of a smaller number of runs, then the two histograms of LLR overlap and can not distinguish between the two hypotheses. But with the increase in the events and runs, it clearly distinguishes between the two hypotheses.