Efficient transformation for identifying global valley locations in 1D data

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Described is a new transformation for finding global valleys in 1D distributions, with particular application to the thresholding of grey-scale images. The applied criterion function estimates the global significance of all the valleys that are located, and thus can cope with multimode distributions. The core transformation can be implemented in just two passes: hence the computational load is O(N) for an N-element distribution, which is optimal.

Introduction: Global thresholding has long been used in image processing applications as a means of locating objects against a plain background. In many real situations it is unreliable, because of the incidence of shadows and other artefacts, and because of natural variations in background illumination [1]. Hence it is not often of direct use in outdoor (e.g. surveillance) applications. However, it can be useful in other situations, especially indoors when the lighting is controlled, and when the camera position is fixed and the objects are flat or the 3D object profile can be ignored. Indeed, it has found widespread use in industrial inspection [1] and in medical (e.g. cytological) applications. In addition, the possibility of rapidly locating objects by thresholding and then scrutinising them more closely by other means must not be ignored. Hence thresholding is not out of date and irrelevant, but rather is one of many tools that may be applied when the occasion warrants it.

There are many approaches to threshold estimation [1–5], including particularly entropy thresholding [2] and its variants, e.g. [3, 4]. However, it is often unclear how such approaches will react to unusual or demanding situations, such as where multiple thresholds have to be found in the same image. Added to this, there is the risk that the more complex approaches will retreat from the original data and miss important aspects of it. Proposed in this Letter is a rigorous means of going back to basics to find global valleys of intensity histograms in such a way as to embody the intrinsic meaning of the data.

New approach: The top trace of Fig. 1 (left) shows the basic situation – where thresholding is effective and the optimum threshold is in principle simple to locate. However, the intensity histogram often contains such a welter of peaks and valleys that even the human eye, with its huge capability for analysis 'at a glance', can be confused – especially when it is necessary to identify global valley positions rather than local minima of lesser significance. The situation is made clearer by the example shown in the top trace of Fig. 1 (right). Here, valley 1 (numbering from the left) is lower than valley 3, but valley 3 is deeper in the sense that it has two high peaks immediately around it; however, valley 1 also lies between the highest two peaks, and in that sense it is the *globally* deepest valley in the distribution.

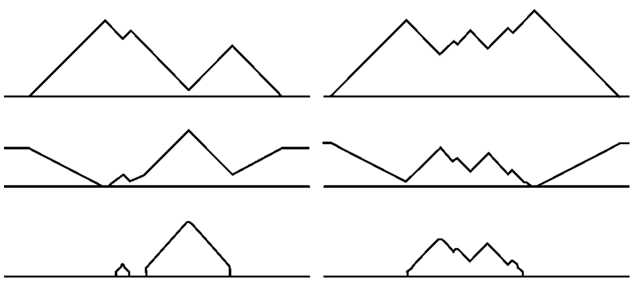

Fig. 1 *Result of applying global minimisation algorithm to 1D datasets*

Left: Basic two-peak structure
Right: Basic multimode structure
Top trace: original 1D datasets
Middle trace: results from (1)
Bottom trace: results from (2)

Clearly, to judge global valley deepness we need a mathematical criterion, so that comparisons between all the valleys can be carried out unambiguously. To proceed, we first apply the following function to the distribution shown in the top trace of Fig. 1 (left):

$$F_j = \max_{i,k}\{1/2[s(h_i - h_j) + s(h_k - h_j)]\} \tag{1}$$

where h_i, h_j, h_k are the heights for three *ordered* sampling points i, j, k in the distribution, and $s(\cdot)$ is a sign function such that $s(u) = u$ if $u > 0$ and $s(u) = 0$ if $u \leq 0$. When this is carried out for the top trace of Fig. 1 (left), the result is a distribution (middle trace of Fig. 1 (left)) which has a maximum at the required valley position. In addition, the values of i and k corresponding to this maximum are the first and third peak positions in the original intensity distribution. The sign function $s(\cdot)$ has the effect of preventing negative responses which would have complicated the situation unnecessarily.

While the function F used above is straightforward to apply and employs linear expressions that are often attractive in permitting in-depth analysis, it results in pedestals at either end of the output distribution: these could complicate the situation when there are many peaks and valleys. Accordingly, we considered other functions, including as the following:

$$G_j = \max_{i,k}\{[s(h_i - h_j)s(h_k - h_j)]^{1/2}\} \tag{2}$$

In contrast with F (which is close in concept to an arithmetic mean), G (which is close in concept to a geometric mean) seems to have all the properties that are required in the present context. In particular, it should be noted that the arithmetic and geometric means are very similar when the two arguments are nearly equal, but are very different when they are highly dissimilar: it is the dissimilar case that applies at the ends of the distribution, and when it is required to suppress a potential valley that has only one peak near to it. These ideas are further confirmed by Fig. 1 (right). For these reasons, we concentrate on function G in the remainder of this Letter.

Overall, the rationale for this approach is that we are looking for the most significant valley in (typically) an intensity distribution, corresponding to an optimum discriminating point between (for example) dark objects and light background in the original image. While in some cases the situation is obvious (Fig. 1 (left)), in general it is difficult to sort out a confusing set of peaks and valleys and in particular to identify global valleys. So the concept embodied in (2) is that of aiming to guarantee an optimal global solution by automatic means. Clearly, by analysis of the output distribution, it is also possible to find a range of maxima corresponding to global valley positions in the input distribution: to this extent the method is able to cope with multimode distributions and to find multiple threshold positions.

With all histogramming methods, it is necessary to take due account of local noise in the distribution, which could lead to inaccurate results. While this can be achieved by pre-smoothing, e.g. using a 1D mean or median filter, we have found that smoothing the output distribution obtained by applying G can also be highly effective (see also the 'Resuts' Section below).

Fig. 2 *Result of applying new algorithm to multimode intensity distribution*

Left: Original grey-scale image
Right: Reconstituted image after multiple thresholding using seven most significant peaks in output distribution (see Fig. 3)

Minimising computation: Another important factor is the amount of computation required for the new approach, since a complete scan of all possible sets of sampling points i, j, k is required to obtain the optimal solution: the computational load for this complete scan is O(N^3) if the number of elements of the distribution, such as the number of grey levels in an intensity distribution, is N. However, the computation can be reduced to just O(N), which is effectively optimal. To achieve this, we merely apply two passes over the data, the first from the left and the second from the right: in the first pass, we produce a separate distribution which records at each point the current maximum of previous (left) values of h_i; in the second pass we record at each point the current maximum of the previous (right) values of h_k. Then all we need is a final pass which computes the maximum value of the chosen function and records where this occurs. Note also that the three passes can be reduced to two by integrating the second pass with the final pass. As indicated above, single passes of this type involve computational loads of O(N).

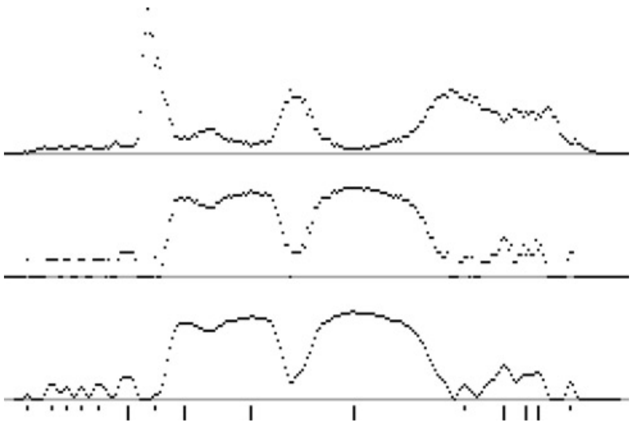

Fig. 3 *1D distributions for Fig. 2 (left)*

Top trace: original intensity histogram for grey-scale image of Fig. 2 (left)
Middle trace: results from (2)
Bottom trace: smoothed results from (2)
Long (short) vertical lines just below bottom trace indicate most (least) significant peaks. (Note: three traces computed 25 times more accurately than rounded values displayed, so peak locations are determined as accurately as indicated)

Results: The ideas presented above are further tested by the image shown in Fig. 2 (left). Starting with this image, the following sequence of operations is applied: (i) an intensity histogram is generated (top trace in Fig. 3); (ii) the function G is applied (middle trace in Fig. 3); (iii) the output distribution is smoothed (bottom trace); (iv) peaks are located (see vertical lines just below the bottom trace); (v) the most significant peaks are chosen as threshold levels; (vi) a new image is generated by applying the mean of the adjacent threshold intensity levels. The result (Fig. 2 (right)) is a reasonably segmented likeness of the original image, albeit with clear limitations in the cloud regions – simply because accurate renditions of these would require a rather full range of grey levels, and thresholding is not appropriate in such regions. However, what is significant is the ease with which the new approach automatically incorporates multi-level thresholding of multimode intensity distributions, a point that has been a distinct difficulty with entropy thresholding for example [3].

Concluding remarks: This Letter describes a new transformation aimed at finding global valleys in 1D distributions, with particular application to thresholding of grey-scale images. It embodies a criterion function, which should be adaptable to differing circumstances, and uses a straightforward two-pass approach for minimising computation: as a result the algorithm runs extremely quickly. In addition, the paradigm can readily be adapted to cope with noise; it can also cope with multimode distributions. Finally, the approach gives an estimate of the significance, and thus a priority order, of all the valleys that are located. Note that the significance is global rather than local, because the relevant arguments (h_i, h_k) are measured over the whole of the appropriate (left and right) regions of the input distribution. In this respect the method emulates to a worthwhile degree the global analysis capabilities of the human visual system.

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