
FROM DETERMINISTIC TO BAYESIAN AUTOENCODERS

A PREPRINT

Feng Geng
shouldsee@qq.com *

Hua Zong Ban

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ABSTRACT

Since the famous paper on denoising autoencoder (DAE), AutoEncoder mainly refers to usage of neural networks to extract meaningful hidden representations, with its performance measured with reconstruction error or generative capabilities, enhanceable with denoising or entropic regularisation. Here we show modern autoencoder objectives connect to traditional approximation objectives like PCA through a simple framework, which unifies the deterministic and probabilistic views.

1 Introduction

Ideas of autoencoders trace back to restricted boltzmann machines. Later research showed denoising objectives prevent overfitting when latent space is bigger than the original space in feedforward neural nets. The addition of noise in latent space is further formalised under the framework of variational inference. The core idea is to approximate the intractable posterior latent distribution with a neural network, so that we do not have to invert the decoder using Bayes theorem.

$$z = f(x) \tag{1}$$

$$L(\theta, \phi) = E_{z \sim q_\theta(z|x)}[p(x|z)] + KL(q_\theta(z|x)||p_\phi(z|x)) \tag{2}$$

On the other hands, non-probabilistic algorithms exist for producing linear and nonlinear embeddings under an error minimisation framework, such as principal components analyses (PCA), locally linear embeddings, Isomap, tSNE, UMAP. Many of the nonlinear embedding algorithms, however, do not include a decoding process, and is thus a one-way process constrained to preserve information. For the two way algorithms, such as PCA, the sample-wise objective can be written as

$$\begin{aligned} L(m) &= \sum_b l(m, x_b) \\ &= \sum_b \max_{z_b} (-||g_m(z_b) - x_b||^2) \\ l(m, z_b, x_b) &= (-||g_m(z_b) - x_b||^2) \end{aligned}$$

note we denote autoencoding process as finding the z that optimise the reconstruction loss. To summarise the autoencoding process and model fitting in a single expression, consider the sum of all possible per-sample decoding errors.

*<http://www.catsmile.info>

$$\begin{aligned}
L(m, Z) &= \sum_x l(m, z_b, x_b) \\
&\leq \sum_x \max_{z_b} l(m, z_b, x_b) \\
&\leq \max_m \sum_x \max_{z_b} l(m, z_b, x_b) \\
&= \max_{m, Z} L(m, Z)
\end{aligned}$$

where the max over m means fitting the model.

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See Section 1.

1.1 Headings: second level

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$$\xi_{ij}(t) = P(x_t = i, x_{t+1} = j | y, v, w; \theta) = \frac{\alpha_i(t) a_{ij}^{w_t} \beta_j(t+1) b_j^{v_{t+1}}(y_{t+1})}{\sum_{i=1}^N \sum_{j=1}^N \alpha_i(t) a_{ij}^{w_t} \beta_j(t+1) b_j^{v_{t+1}}(y_{t+1})} \quad (3)$$

1.1.1 Headings: third level

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2 Examples of citations, figures, tables, references

2.1 Citations

Citations use natbib. The documentation may be found at

<http://mirrors.ctan.org/macros/latex/contrib/natbib/natnotes.pdf>

Here is an example usage of the two main commands (citet and citep): Some people thought a thing [Kour and Saabne, 2014a, Hadash et al., 2018] but other people thought something else [Kour and Saabne, 2014b]. Many people have speculated that if we knew exactly why Kour and Saabne [2014b] thought this...



Figure 1: Sample figure caption.

Table 1: Sample table title

Part		
Name	Description	Size (μm)
Dendrite	Input terminal	~ 100
Axon	Output terminal	~ 10
Soma	Cell body	up to 10^6

2.2 Figures

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2.3 Tables

See awesome Table 1.

The documentation for booktabs (‘Publication quality tables in LaTeX’) is available from:

<https://www.ctan.org/pkg/booktabs>

2.4 Lists

- Lorem ipsum dolor sit amet
- consectetur adipiscing elit.
- Aliquam dignissim blandit est, in dictum tortor gravida eget. In ac rutrum magna.

References

George Kour and Raid Saabne. Real-time segmentation of on-line handwritten arabic script. In *Frontiers in Handwriting Recognition (ICFHR), 2014 14th International Conference on*, pages 417–422. IEEE, 2014a.

Guy Hadash, Einat Kermany, Boaz Carmeli, Ofer Lavi, George Kour, and Alon Jacovi. Estimate and replace: A novel approach to integrating deep neural networks with existing applications. *arXiv preprint arXiv:1804.09028*, 2018.

²Sample of the first footnote.

George Kour and Raid Saabne. Fast classification of handwritten on-line arabic characters. In *Soft Computing and Pattern Recognition (SoCPaR), 2014 6th International Conference of*, pages 312–318. IEEE, 2014b. doi:10.1109/SOCPAR.2014.7008025.