Dirichlet Process Mixture Model

$$\begin{aligned} G \textit{ is a distribution} \\ X \sim G \\ G' \sim DP(G, \alpha) \\ E_{G'}(X') = E_G(X) \\ e.g.:\ G \sim N(0, 1) \end{aligned}$$

Gibbs sampling Updating

Shot from https://www.youtube.com/watch?v=UTW530-QVxo

Slides from http://www.umiacs.umd.edu/~jbg/teaching/CSCI_5622/17a.pdf

Gibbs Sampling for DPMM

$$
\begin{aligned}
& p(z_i = k \mid \vec{z}_{-i}, \vec{x}, \{\theta_k\}, \alpha) && (3)\\
=& p(z_i = k \mid \vec{z}_{-i}, x_i, \vec{x}, \theta_k, \alpha) && (4)\\
=& p(z_i = k \mid \vec{z}_{-i}, \alpha) p(x_i \mid \theta_k, \vec{x}) && (5)\\
=& \begin{cases} \left(\frac{n_k}{n_\cdot + \alpha}\right) \int_\theta p(x_i \mid \theta) p(\theta \mid G, \vec{x}) & \text{existing} \\ \alpha \int_\theta p(x_i \mid \theta) p(\theta \mid G) & \text{new} \end{cases} && (6)\\
=& \begin{cases} \left(\frac{n_k}{n_\cdot + \alpha}\right) \mathcal{N}\left(x, \frac{n\bar{x}}{n+1}, 1\right) & \text{existing} \\ \alpha \mathcal{N}\left(x, 0, 1\right) & \text{new} \end{cases} && (7)
\end{aligned}
$$

Scary integrals assuming G is normal distribution with mean zero and unit variance. (Derived in optional reading.)

Figure 1: