

**CS483 - Fundamentals of Artificial Intelligence**  
**Homework Assignment #4**  
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1. Re-calculate the entropy for the feature selection in the example of file “*Gini Impurity Cal in Decision Tree*” rather than Gini impurity method. And then, compare the results from two different criteria

*Hint: taking the reference at the following link for your calculation*

<https://towardsdatascience.com/entropy-how-decision-trees-make-decisions-2946b9c18c8>

## Training Data

Color	Diameter	Label
Green	3	Apple
Yellow	3	Apple
Red	1	Grape
Red	1	Grape
Yellow	3	Lemon

Based on these data, we can compute probability of Label target.

- Since probability is equal to frequency relative, we have  
 $P(\text{Apple}) = \text{Prob}(\text{Apple}) = 2 / 5$   
 $P(\text{Grape}) = \text{Prob}(\text{Grape}) = 2 / 5$   
 $P(\text{Lemon}) = \text{Prob}(\text{Lemon}) = 1 / 5$

So now we calculate the Entropy using the below formula:

$$E(S) = \sum_{i=1}^c -p_i \log_2 p_i$$

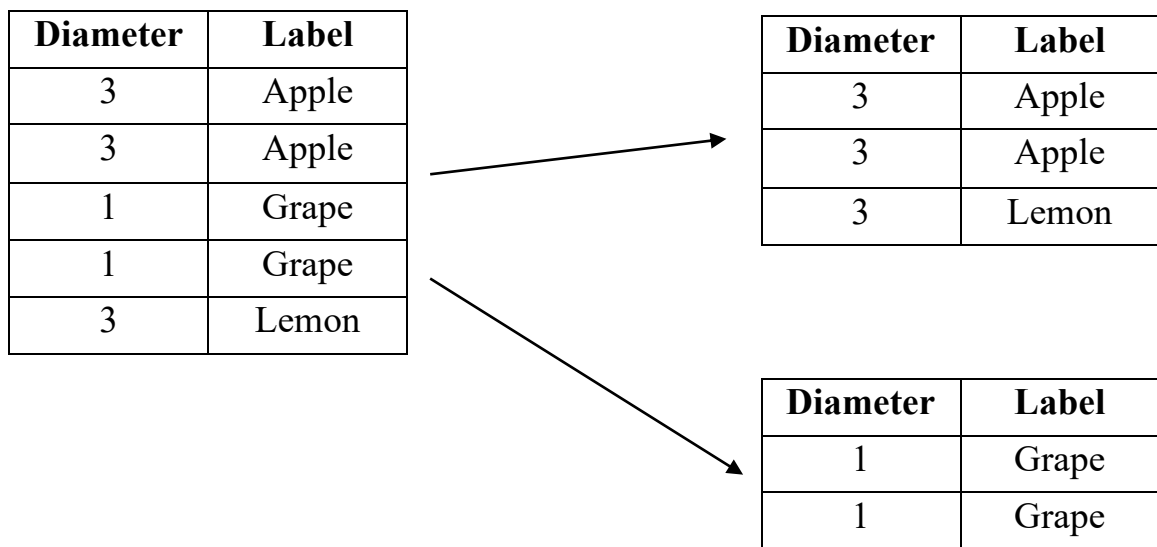
$$= -(2/5) * \log_2(2/5) - (2/5) * \log_2(2/5) - (1/5) * \log_2(1/5)$$

$$= 1.521$$

### 1st Iteration: Find the root of a decision tree

The **Parent Data Table** has classes of 2 Apple, 2 Grape and 1 Lemon which produce entropy of 1.521

- Information Gain for Diameter



$$\text{Entropy of Diameter 3 table} = -(2/3) * \log_2(2/3) - (1/3) * \log_2(1/3)$$

$$= 0.918$$

$$\text{Entropy of Diameter 1 table} = -(2/2) * \log_2(2/2)$$

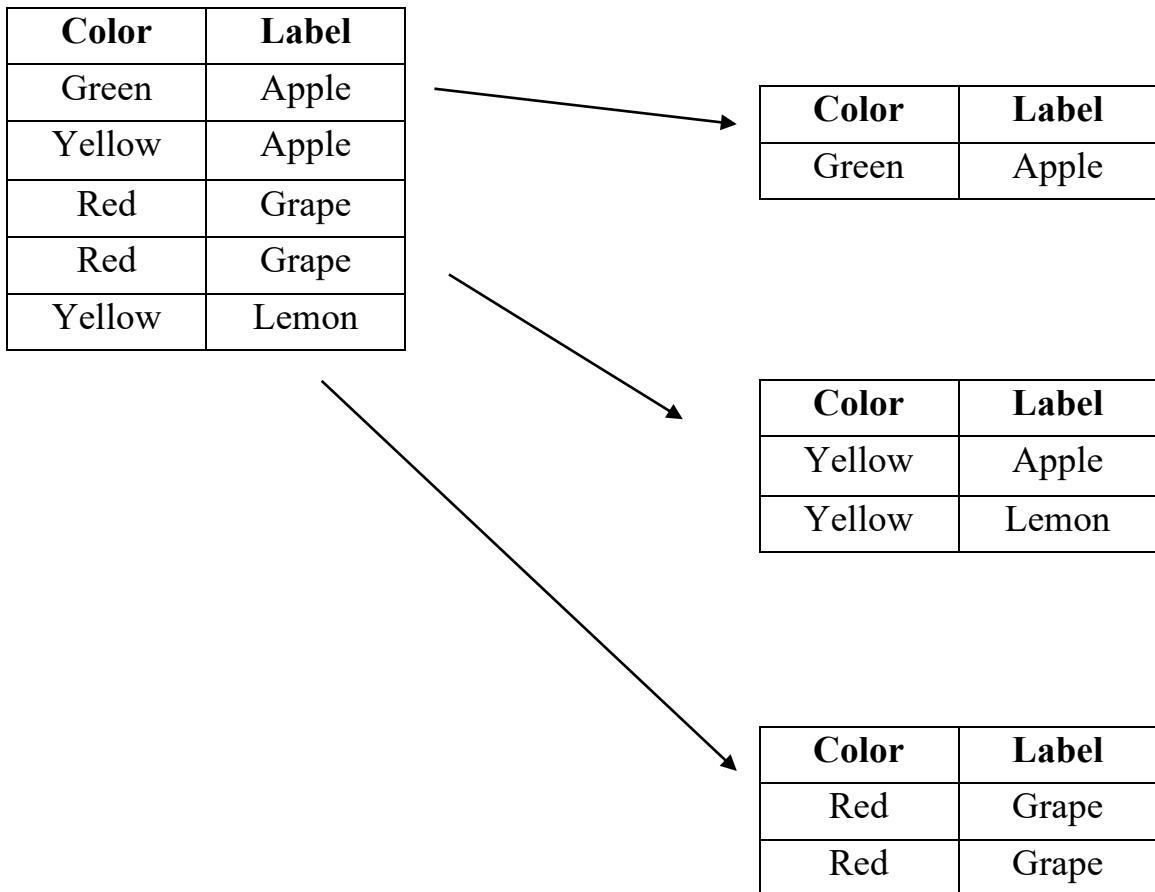
$$= 0$$

The entropy of the Information Gain for Diameter

$$= 1.521 - (3/5 * 0.918 + 2/5 * 0)$$

$$= 0.970$$

- Information Gain for Color



$$\begin{aligned} \text{Entropy of Green table} &= -(1/1) * \log_2(1/1) \\ &= 0 \end{aligned}$$

$$\begin{aligned} \text{Entropy of Yellow table} &= -(1/2) * \log_2(1/2) - (1/2) * \log_2(1/2) \\ &= 1 \end{aligned}$$

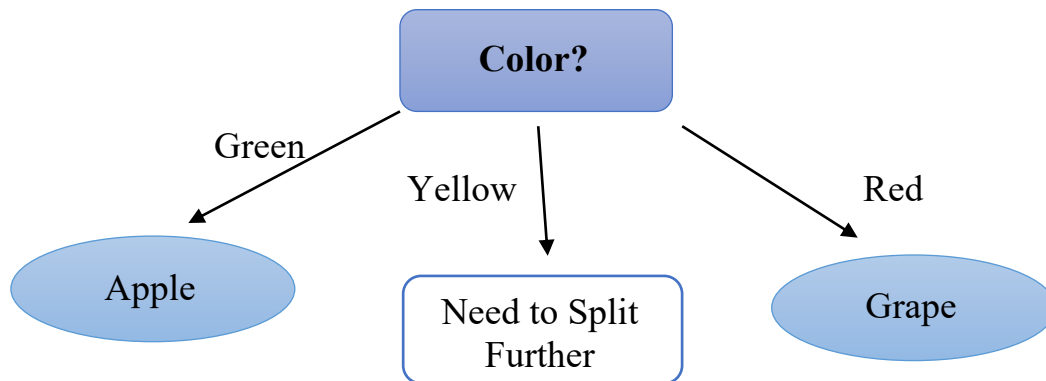
$$\begin{aligned} \text{Entropy of Red table} &= -(2/2) * \log_2(2/2) \\ &= 0 \end{aligned}$$

$$\begin{aligned} \text{The entropy of the Information Gain for Color} &= 1.521 - (1/5 * 0 + 2/5 * 1 + 2/5 * 0) \\ &= 1.121 \end{aligned}$$

Table below summarizes the information gain for all four attributes.  
Result of the First Iteration:

Gain	Diameter	Color
Entropy	0.970	1.121

- Color is selected as the root because it has the highest information gain.



- Since Green and Red Color have been associated with pure class, we do not need these data any longer.
- For second iteration, our data table D come from the Yellow Color because it is not associated with pure class.
- As we have only 2 features Color and Diameter from which color is the root so further will diameter.
- Green color is Apple and Red color is Grape so with diameter 1 and 3, where 1 is grape and 3 is apple or lemon from which apple is gone with color.
- Hence diameter = 3 is lemon associated with yellow color.

