Terrain traversability prediction with graphical models

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1 Introduction

Terrain-classification from satellite images is highly uncertain. The prior belief of the terrain types needs to be updated with information from the rover which will improve safety of the rover and help in future re-planning.

2 Graphical Modelling

This terrain belief update problem can be analyzed using an undirected graphical model. Consider a graphical model with nodes as the locations in a map. The state of each node is x_i which is a vector of terrain-class probabilities.

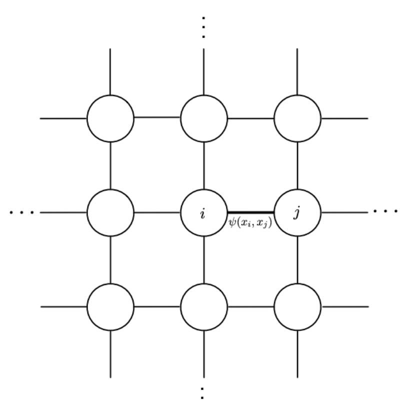

Figure 1: Graphical Model for pair-wise connected nodes

$\mathbf{x}$ is the set of nodes. The joint probability of all the nodes can be written as

$$P(\mathbf{x}) = \frac{1}{Z} \prod_{i} \phi(i) \prod_{i,j} \psi(i, j) \tag{1}$$

$$Z = \sum_{\mathbf{x}} \prod_i \phi(i) \prod_{i,j} \psi(i,j)$$

$\phi(i)$ is the node potential at node i, $\psi(i;j)$ is the edge potential for edge $\{i,j\}$ and Z is the normalizing constant[1].

If some nodes, suppose x_m, x_n are observed, then the distribution of the remaining nodes $x_{\setminus\{m,n\}}$ conditioned on the observed nodes can be written as

$$\begin{aligned} P\left(x_{\setminus\{m,n\}} \mid x_m, x_n\right) &= \frac{P(\mathbf{x})}{\sum_{x_{\setminus\{m,n]}} P(\mathbf{x})} \\ &= \frac{\frac{1}{Z}\prod_i \phi(i)\prod_{i,j}\psi(i,j)}{\sum_{x_{\setminus\{m,n\}}} \frac{1}{Z}\prod_i \phi(i)\prod_{i,j}\psi(i,j)} \end{aligned} \tag{2}$$

In most of the cases, due to large number of nodes, the summation in the denominator is intractable. In those cases, approximate inference techniques like MCMC, Gibbs Sampling and others can be used.But if the number of nodes are small, this summation is tractable and exact inference can be done.

Assume there are 3 types of terrain classes shown in colours yellow, green, blue and the number of nodes is 7(Figure 2). First we start with the belief matrix N whose rows are the states of each node i or the node potentials $\phi(i)$. This is our initial belief of the terrain classes at those locations.(Column 1 corresponds to blue, 2 for green and 3 for yellow.)

$$\begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.7 & 0.2 & 0.1 \\ 0.3 & 0.6 & 0.1 \\ 0.7 & 0.1 & 0.2 \\ 0.3 & 0.6 & 0.1 \\ 0.4 & 0.5 & 0.1 \\ 0.1 & 0.3 & 0.6 \end{bmatrix}$$

Now we write down the edge potential matrices E which is the transition probability between nodes i and j or the edge potential $\psi(i,j)$. There is no straightforward way of knowing the elements , but can be hand-tuned depending on the type of edge. If the edge lies on the boundary connecting the two different types of terrains, then the matrix should show high probability of change, example

$$\begin{bmatrix} 0.3 & 0.6 & 0.1 \\ 0.6 & 0.3 & 0.1 \\ 0.4 & 0.4 & 0.2 \end{bmatrix}$$

which further says that the edge is between terrain type 1 and 2. Similarly, if the edge is between nodes in the same types of terrain, then the edge potential would look like

$$\begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.2 & 0.6 & 0.2 \\ 0.2 & 0.2 & 0.6 \end{bmatrix}$$

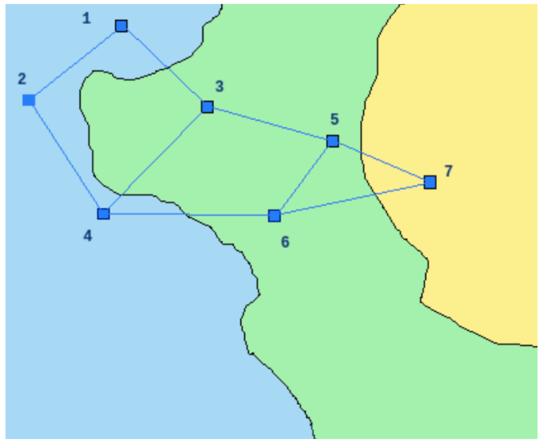

Figure 2: Map showing initial belief with 3 types of terrain and 7 nodes(Locations)

which will tend to keep the terrain types similar. However, these values or parameters need to be tuned depending on how confident we are about the terrain types in that region.Now all the edges can be given their respective potentials depending on their locations in the map. Once the node and edge potentials are ready, inference can be done using openly available software package.

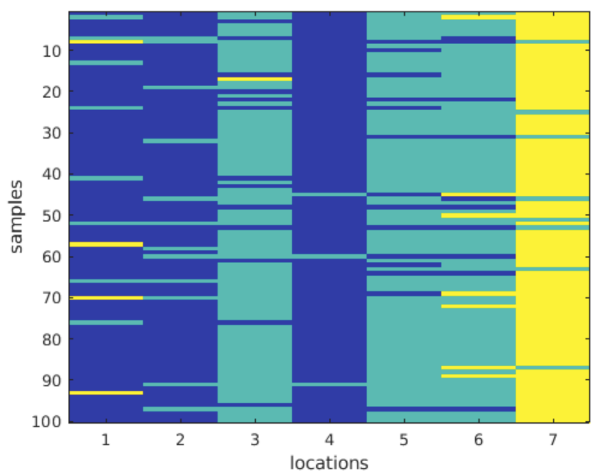

Figure 3: 100 samples drawn from the prior joint distribution. Blue : Terrain 1; Green : Terrain 2; Yellow : Terrain 3

Here, inference on the conditional undirected graphical model has been done using the UGM Matlab Toolbox [2]. Figure (3) shows 100 samples drawn from the prior joint distribution.

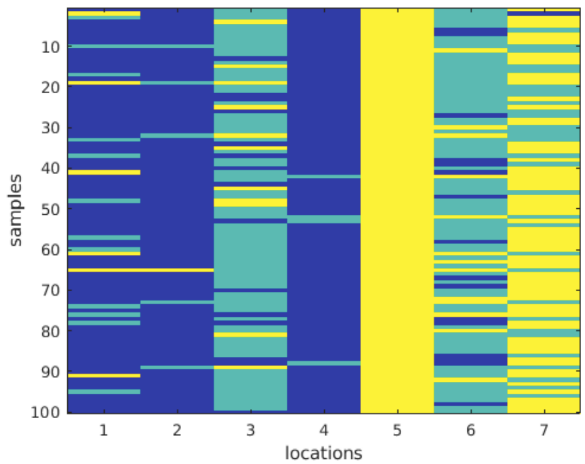

Figure 4: 100 samples drawn from the joint distribution after conditioning on node 5. Blue : Terrain 1; Green : Terrain 2; Yellow : Terrain 3

Now if node 5 is observed to be class 3 (Yellow), the exact conditional inference can be solved. Figure (4) shows 100 samples from the conditional distribution. Due to node 5 being type 3, locations 3 and 6 are more likely to be of type 3 than before.The updated belief matrix is

$$\begin{bmatrix} 0.8494 & 0.1076 & 0.0430 \\ 0.8898 & 0.0956 & 0.0147 \\ 0.1366 & 0.7608 & 0.1027 \\ 0.9308 & 0.0570 & 0.0122 \\ 0.0000 & 0.0000 & 1.0000 \\ 0.2048 & 0.6445 & 0.1508 \\ 0.0126 & 0.3345 & 0.6529 \end{bmatrix}$$

References

[1] Bishop, Christopher M. Pattern recognition and machine learning. springer, 2006.

[2] Schmidt, Mark. "UGM: Matlab code for undirected graphical models." URL http://www. di. ens. fr/mschmidt/Software/UGM. html (2012).

[3] Sridharan, Ramesh. "The Ising model and Markov chain Monte Carlo."