A f^n is convex if for every $x, y \in \mathbb{R}$ and every $t \in [0,1]$ the following holds:

$$f(tx+(1-t)y) \leq tf(x) + (1-t)f(y)$$

Even though both f & g may be convex, their difference $h(x) = f(x) - g(x)$ need not to be convex

Eg: 1 $f(x) = 2|x|$ and $g(x) = x^2$

$f(x) = 2|x| \rightarrow$ the abs. value of $f(x)$ is convex bcz for any x, y and $t \in [0,1]$:

$$|tx+(1-t)y| \leq t|x| + (1-t)|y|$$

multiply by a (+)ve constant (2) preserve convexity, Hence $f(x) = 2|x|$ is convex

For $g(x) = x^2$

The f^n x^2 is a standard convex f^n (it's second derivative 2 is positive for all x) Thus $g(x) = x^2$ is convex.

Now, their diff is not convex

$h(x) = f(x) - g(x) = 2|x| - x^2$

Choose $x = 0$ $y = 2$ $t = \frac{1}{2}$

$h\left(\frac{0+2}{2}\right) = h(1) = 2|1| - 1^2 = 1$

Weighted avg : $\frac{1}{2}h(0) + \frac{1}{2}h(2) = \frac{1}{2}\cdot 0 + \frac{1}{2}\cdot 0 = 0$

Comparing $h(1) = 1$ vs $\frac{1}{2}h(0) + \frac{1}{2}h(2) = 0$

The convexity condition would req. $h(1) \leq 0$ but $1 \nleq 0$

so $h(x)$ not convex

Eg-2. $f(x) = x^2$ and $g(x) = 2x^2$

$f(x) \longrightarrow$ convex

$2f(x) = g(x) \longrightarrow$ convex

$h(x) = f(x) - g(x) = -x^2$

$h(x) = -x^2$

show that $h(x)$ not convex

$h''(x) = -2 < 0$ so $h(x)$ is concave

Since the second derivative of the difference is less than 0 so this function $h(x)$ is concave & not convex

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