MATHEMATICS AA SL

Internal Assessment

Analyzing the effect of different downforce level circuits in **Formula 1** on the tyres (Hards)and Medium performance

Session: May 2025

Page Count: 21

Introduction:

Formula 1 (F1) is the pinnacle of motorsports and is the highest class of single-seater racing. It is governed by the Federation Internationale de l'Automobile or FIA. Formula 1 has been around for quite a while, since it was introduced in 1950. Since then, it has significantly evolved into a global spectacle by demonstrating the fastest and most technologically advanced race cars. Each season consists of a series of races, known as the Grand Prix, that are held on various circuits across the globe, which include spectacular street circuits and specific race circuits. Where various automobile constructors such as Scuderia Ferrari, Mercedes, or Red Bull Racing participate to compete with each other by racing. Each of the constructors' teams has to sign up two drivers to represent their teams and win the drivers' and constructors' championship. The F1 cars are extremely fast and are capable of reaching speeds up to 350 km per hour and consist of powerful hybrid V6 engines and highly sophisticated aerodynamic features such as the car's floor, front, and rear wings to generate high downforce, allowing the cars to stay grounded while getting high grip at incredible speeds.

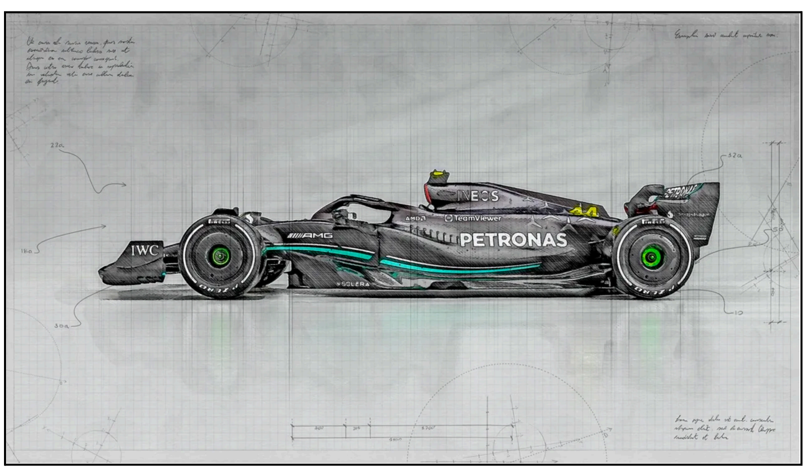


***Figure 1*:** A Formula 1 car

Background Information:

Tyres are one of the most critical components of an F1 car, as they significantly impact the performance and strategies in each race. The tyres used in Formula 1 are supplied by Pirelli and are specifically designed to operate at the extreme speeds and forces generated by F1 cars. These tyres are categorized under "Slick tyre" and "wet tyres", where the slick tyres are typically used in dry weather conditions. The slick tyres are available in different compounds, ranging from soft, medium and hard. The slick tyres provide varying levels of grip and durability across the span of a race. The softer sets of tyres offer higher levels of traction (grip) but wear out and degrade quickly, whereas the Hard sets of tyres tend to last longer and are more durable but provide less traction (grip) initially. The medium sets of tyres are an intermediate between the soft and Hard tyres, which provides an excellent balance between traction and durability. Additionally, Pirelli provides wet tyres designed specifically for racing in rainy conditions. There are two types of wet tyres: intermediate and full-wet. The intermediate tyres are specifically designed to be used on a wet track with no standing water or a drying track. Lastly are the full wet tyres used in heavy rain conditions.

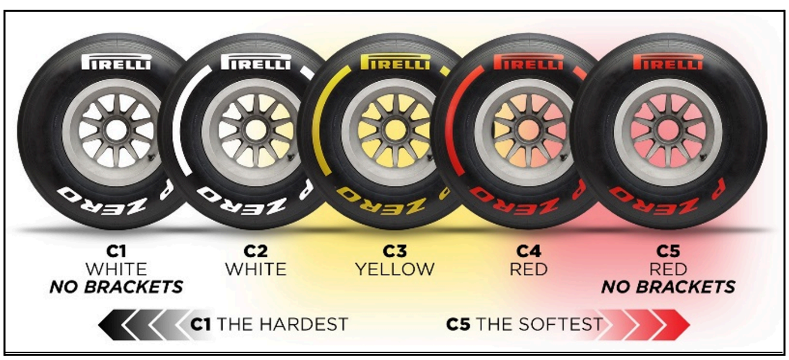


***Figure 2*:**The Pirelli range of tyres

Tyre degradation is a phenomenon that is an extremely important factor in Formula 1. Tyre degradation refers to the deterioration or degradation in a tyre's performance throughout a

Formula 1 Grand Prix race due to various factors such as tyre compound, track temperatures, tyre pressure, wind, driving style, and circuit-specific downforce levels. However, when tyre degradation occurs, it significantly influences lap times and overall race results, meaning that it reduces the tyre grips and stability and decreases lap time. Hence, tyre degradation becomes a crucial factor for teams and drivers because the drivers have to make their tyres last longer, while on the other hand, teams have properly timed their pit stops for new sets of tyres. However, for this exploration, I will be comparing the effect of varied downforce levels of different circuits on the tyre degradation of hard and Medium tyres, as soft tyres are mostly used in the qualifying session and not often in races.

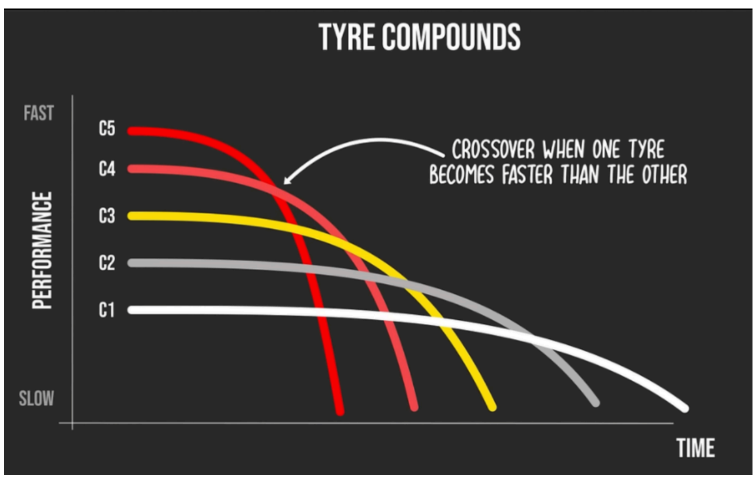


Figure 3**:** Tyre degradation graph for all Pirelli tyre compounds

Lastly, downforce is one of the important factors that lead to tyre degradation. Downforce is an aerodynamic force that pushes down the car on the track and provides increased traction(grip) on the track while allowing the cars to go faster through corners and straights. Downforce in Formula cars is generated by their aerodynamic components such as the rear and front wings, the underbody or the floor of the car. The downforce provides more traction(grip) by pushing the car towards the ground, however, this causes greater friction between the tyres and track surface, leading to tyre degradation. Additionally, the level of downforce varies across different circuits, based on the circuit's layout, characteristics such as corners and straights, and altitude, due to which the circuits on Formula 1 calendars are categorised based on downforce levels into max-high downforce, medium downforce and low downforce circuits.

Rationale:

From a very young age, I have always been amazed by the world of cars, be it the movies, toy cars, or actual cars owned by my family. My parents nurtured my passion for cars by frequently gifting various remote-controlled cars, Hot Wheels cars, each one being more exciting to play with than the last. However, among all those cars, there is one special car that was gifted to me by my late grandfather. The one he gifted me was a miniature Formula 1 car. It made a little boy aware of the pinnacle of motorsports. After learning more, I began following Formula 1 regularly because I was fascinated by the raw speed of the F1 cars. As I grew older alongside regularly followed the sport, my understanding of Formula 1 also gradually deepened. I realized that races are not just a driver, driving a very fast car around a circuit, but it's a team sport where intricate strategies, engineering marvels, and mathematics are combined.

Figure 4**:** A miniature Formula 1 car gifted by my grandfather

Today, with a more analytical mindset, I don't just see Formula 1 only as a motorsport but also as a highly mathematically driven sport with complex collaboration between different fields such as physics, aerodynamics, chemists, and engineers. The journey from childhood obsession of just watching Formula 1 to appreciating the diverse principles the sports incorporate in it, has inspired me to explore the relationship between a circuit's downforce levels and tyre degradation in Formula 1 using mathematics.

Aim:

The aim of this exploration is to analyse and compare the effect of circuits with different downforce levels on the tyre degradation of particular tyre compounds (Medium and Hard) of Pirelli tyres throughout the stints in a race.

Important formula 1 terminologies :

Tyre degradation: It is the loss in tyre performance over the time of race that may result in increased lap times because of the tyre wear and reduced traction (grip). A positive value for tyre degradation means that the tyres are degraded over the laps, in contrast, a negative value for tyre degradation means that the tyres are performing better and are degraded less over the laps.

Downforce: It is an aerodynamic force created by the Formula 1 car's front wing, rear wing and underbody that pushes the car down onto the track for better grip on the track and while cornering.

Pit stops: They are the stops during the race where the worn-out or degraded sets of tyres are replaced with fresh sets of tyres.

Stint: It is a continuous period in which a Formula 1 car runs on the same sets of tyres in a race between the pit stops.

Outlaps: They are the first lap completed, specifically after pit stops or the start of a race and are usually quite slower as the tyres need to warm up till their optimal temperature for best performance; hence, out laps are considered as outliers.

Average (mean) race pace: It is the mean lap time (in seconds) of a driver over a stint or in a race that does not include the out laps or any laps with racing incidents.

Methodology :

To begin with the investigation, I first categorized Formula 1 circuits into three distinct categories based on their downforce level.

Low Downforce Circuits: These types of circuits require low levels of downforce to maximize the straight-line speeds of the car.

Medium Downforce Circuits: These types of circuits consist of a balance between the downforce and speed.

High Downforce Circuits: These types of circuits require higher levels of downforce to help in cornering and maintaining traction or grip on the track.

To ensure consistency in analyzing the effect of downforce on tyre performance, I selected Charles Leclerc due to his consistent performance across circuits and the availability of his lap-by-lap timings. His driving style provides a reliable model, minimizing variability across different downforce categories. Lap-by-lap timings and tyre compounds for the categorized circuits were collected from Pitwall.com and F1-tempo.com.

***Figure 5*:** Charles Leclerc

Additionally, the lap-by-lap data will be divided into stints (segments) based on their tyre compound (Hard or medium) and the number of laps the tyres were on the car for any particular period in a race. Any unusually high or low lap times within a stint must be checked and removed, as they may result from racing incidents such as tyre lockups, pit stops, safety cars, or accidents. These outliers can introduce inconsistencies or errors in the analysis, affecting the accuracy of tyre degradation trends.

***Table 1*:** Circuit details

Downforce level	Circuit	Length (km)	Number of Race Laps	Number of Turns	Temperature at race day (°C)
LOW	Autodromo Nazionale Monza	5.79	53	11	31
	Spa-Francorchamps	7.004	44	20	20
MEDIUM	Emilia Romagna	4.909	63	19	
	Circuit of The Americas	5.513	56	20	26-27
HIGH	Marina Bay Street Circuit	4.940	62	19	29-32
	Hungaroring	4.381	70	14	33

Relevant Formula – Equations :

1. $$Average\ Race\ Pace = \frac{Sum\ of\ Lap\ Times\ per\ stint}{Number\ of\ Laps\ in\ the\ Stint}$$

2. $$Instantaneous\ rate\ of\ degradation = \frac{dT}{dN} = \frac{dTime}{dLaps}$$

3. $$Tyre\ Degradation\ (D_{Tyre}) = Final\ Lap\ Time - Initial\ Lap\ Time$$

4.0 Data Collection (complete lap-by-lap data can be seen in the appendix)

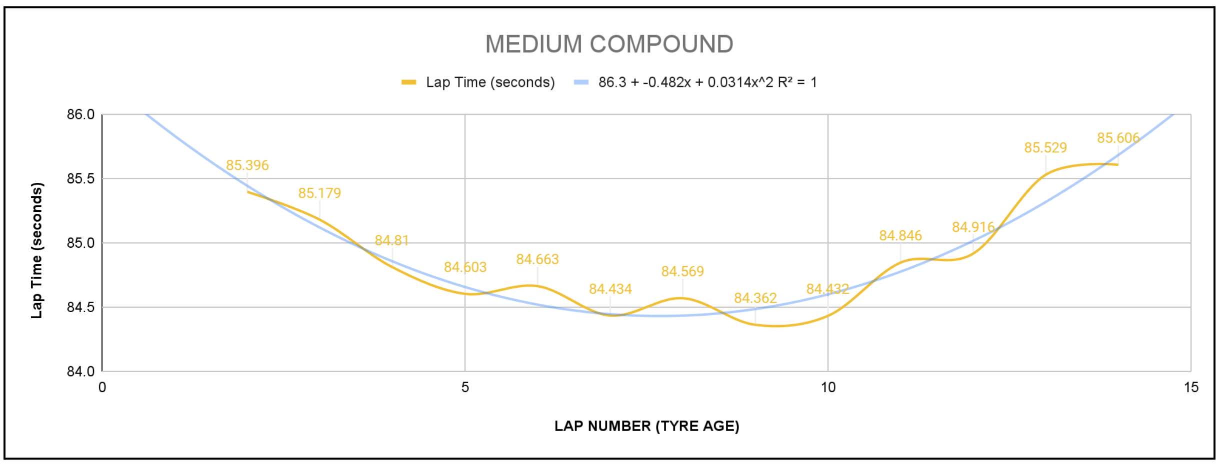
Table 2: Data collection

Circuits / Lap number	Lap Times (seconds)					
	Autodromo Nazionale Monza	Spa-Francorchamps	Circuit of The Americas	Circuit of The Americas	Marina Bay Street Circuit	Hungaroring
1	88.179	107.982	119.522	84.58	97.09	102.028
2	85.396	107.981	98.662	80.926	99.775	83.071
3	85.179	107.971	99.067	81.279	99.14	83.296
4	84.81	108.346	99.034	81.122	99.31	82.892
5	84.603	108.279	98.667	81.043	99.041	82.359
6	84.663	108.38	98.825	81.025	98.797	82.182
7	84.434	108.356	99.032	81.116	98.645	82.7
8	84.569	108.449	98.633	81.215	98.727	82.776
9	84.362	108.444	98.897	81.387	98.691	82.795
10	84.432	112.358	98.643	81.147	98.582	83.066

Medium Tyre Compound
Hard Tyre Compound
Outlier Lap times

Mathematical calculations :

Figure 7: Circuit map of Autodromo Nazionale Monza



Graph 1: Stint on medium tyres

$$Average\ Race\ Pace\ =\ \frac{85.396+85.179+84.81+84.603+84.663+84.434+84.362+84.432+84.846+84.916}{11}$$

$$Average\ Race\ Pace\ =\ 84.746\ seconds/Lap$$

$$y\ =\ 86.3\ -\ 0.482x\ +\ 0.0314x^{2}$$

The function is written in terms of Lap Times (T) and Lap Number (N)

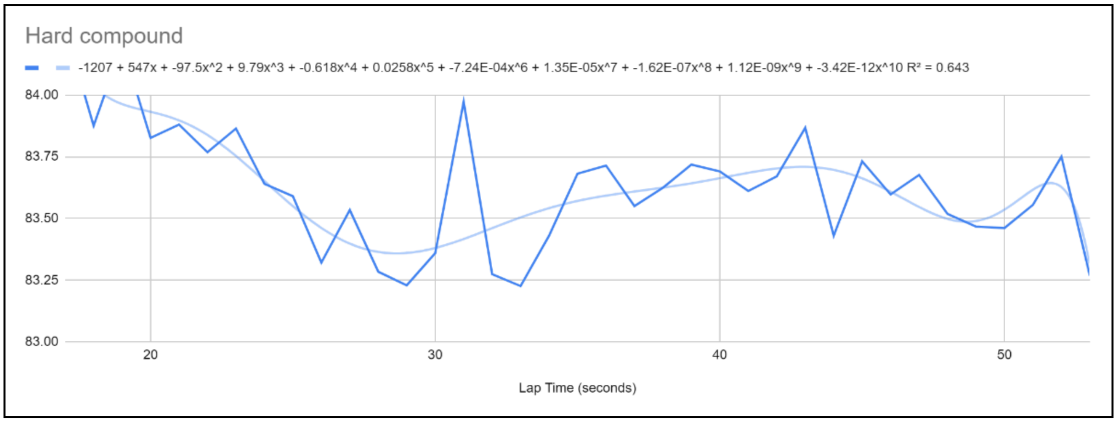
$$T\ =\ 86.3\ -\ 0.482N\ +\ 0.0314N^{2}$$

$$\frac{dT}{dN}\ =-\ 0.482\ +\ 0.0628N$$

$$\frac{dT}{dN}\Big|_{N=1}\ =-\ 0.482\ +\ 0.0628(1)\ =-\ 0.419\ seconds/Lap$$

$$\frac{dT}{dN}\Big|_{N=13}\ =\ Final\ rate\ of\ degradation\ =-\ 0.482\ +\ 0.0628(13)\ \ =+\ 0.334\ seconds/Lap$$

$$Tyre\ Degradation\ = 85.606 - 85.396 =+ 0.210\ seconds$$



Graph 2: Stint on Hardtyres

$Average\ Race\ Pace\ = \frac{84.27+83.875+84.203+...+83.268}{37}$ (Refer to appendix for complete calculation of Average Race Pace)

$Average\ Race\ Pace\ = 83.624\ seconds/Lap$

$$T = 1205 + 547N - 97.5N^2 + 9.79N^3 + \left(-0.618N^4\right) + 0.0258N^5 + \left(-7.24\times10^{-4}N^6\right) + 1.35\times10^{-5}N^7 + \left(-1.62\times10^{-7}N^8\right) + 1.12\times10^{-9}N^9 + (-3.42\times10^{-12}N^{10})$$

$$\frac{dT}{dN} =- 547 - 195N + 29.37N^2 - 2.472N^3 + 0.129N^4 - 4.344\times10^{-3}N^5 + 9.45\times10^{-5}N^6 - 1.296\times10^{-6}N^7 + 1.008\times10^{-8}N^8 - 3.42\times10^{-11}N^9$$

$$\frac{dT}{dN}\Big|_{N=1} =- 547 - 195(1) + 29.37(1)^2 - 2.472x^3 + 0.129x^4 - 4.344\times10^{-3}x^5 + 9.45\times10^{-5}x^6 - 1.296\times10^{-6}x^7 + 1.008\times10^{-8}x^8 - 3.42\times10^{-11}x^9 =+ 379.020\ seconds/Lap$$

$$\frac{dT}{dN}\Big|_{N=37} =- 547 - 195(37) + 29.37(37)^2 - 2.472x^3 + 0.129(37)^4 - 4.344\times10^{-3}(37)^5 + 9.45\times10^{-5}(37)^6 - 1.296\times10^{-6}(37)^7 + 1.008\times10^{-8}(37)^8 - 3.42\times10^{-11}(37)^9 =- 525.36\ seconds/Lap$$

$$Tyre\ Degradation\ = 83.268 - 84.270 =- 1.002\ seconds$$

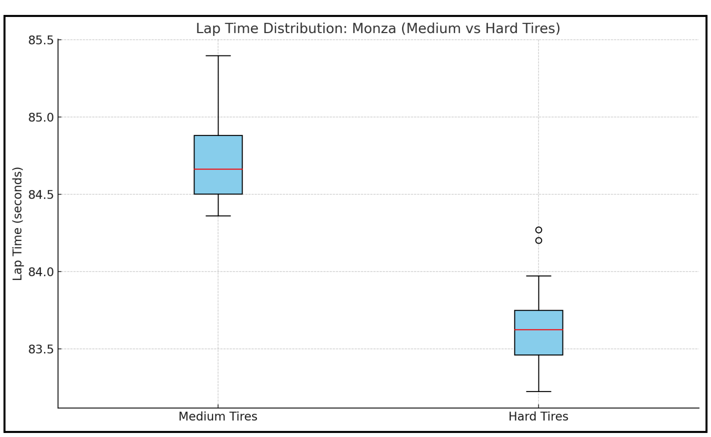
4.2 Processed data table

Table 3: Final Processed Data Table

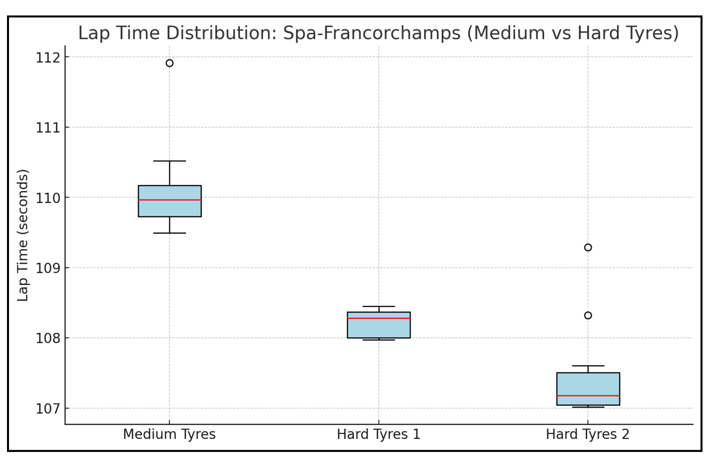
Circuit	Tyre compound	Tyre Age (Laps)	Average Race Pace (seconds)	Initial Rate of Instantaneous Degradation (seconds/lap)	Final Rate of Instantaneous Degradation (seconds/lap)	Tyre Degradation (seconds/lap)
Autodromo Nazionale Monza	Medium	13	84.872	-0.419	+0.334	+0.210
	Hard	37	83.642	+379.020	-525.36	-1.002
Circuit de Spa-Francorchamps	Medium	11	110.090	-14.44	-10372.40	-1.945
	$Hard_1$	11	107.971	-0.150	-0.041	+0.217
	$Hard_2$	18	107.392	-279890.83	+171432.740	+0.027
Circuit of the Americas	Medium	21	99.530	+0.020	+0.030	+1.260
	Hard	29	98.50	-373136	+18722197.75	+0.856
Emilia Romagna	Medium	23	81.441	-5.720	+1758.790	+0.674
	Hard	37	80.659	78827.43	+26855.54	+1.218
Marina Bay Street Circuit	Medium	36	99.176	-4.750	+2332.73	+2.174
	Hard	25	96.698	+199869.45	+35475.46	+2.170
Hungaroring	$Medium_1$	21	84.668	-8.927	1486.74	+1.842
	Hard	15	83.316	+9361486.55	-204493.48	+1.381
	$Medium_2$	28	83.040	+22968052.04	+41831.72	+0.142

Data Analysis :

Low Downforce Level



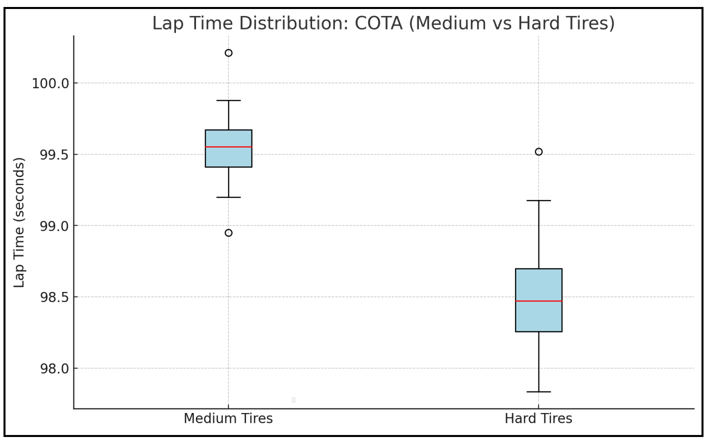
Box-Plot 1. Representing Lap time distribution for Monza



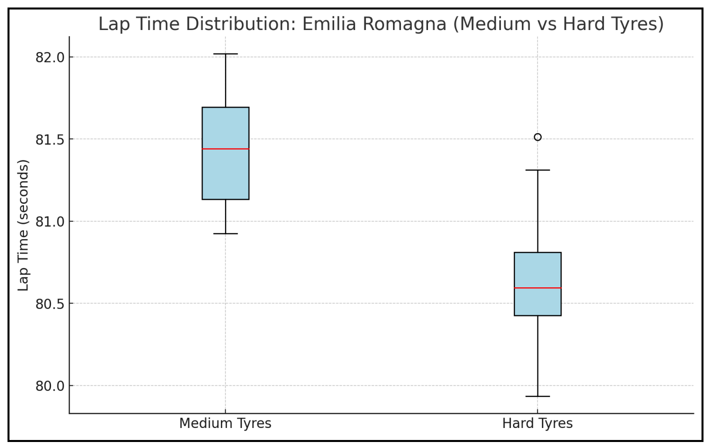
Box-Plot 2. Representing Lap time distribution for SPA

The box plots for Monza and Spa indicate that the Hard tyres have lower medium timings (83.642 and 107.392 seconds) than the Medium tyres (84.872 and 110.9 seconds), showing that Hard tyres have slower degradation and better stability. The Medium tyres have a larger IQR (0.964 seconds in Monza, 0.438 seconds in Spa), indicating greater variability, whereas the Hard tyres have a relatively smaller IQR (0.409 and 0.398 seconds) – giving a more consistent performance. The outliers in Medium tyres are potentially due to external factors such as improving track conditions.

Medium Downforce Level



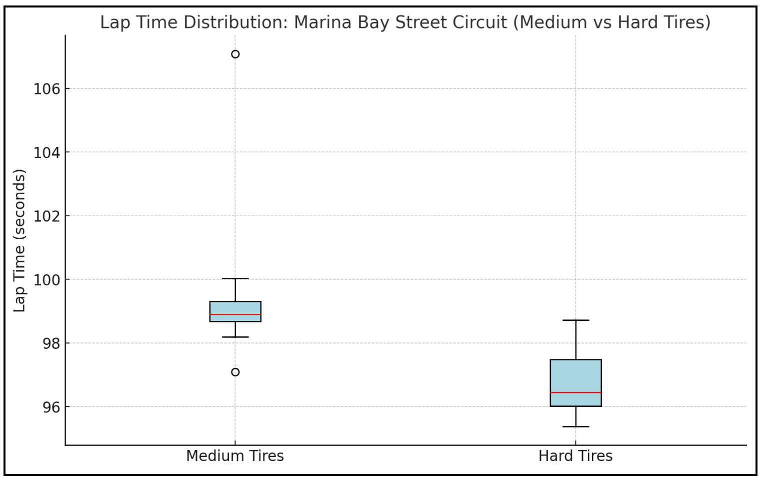
Box-Plot 1. Representing Lap time distribution for COTA



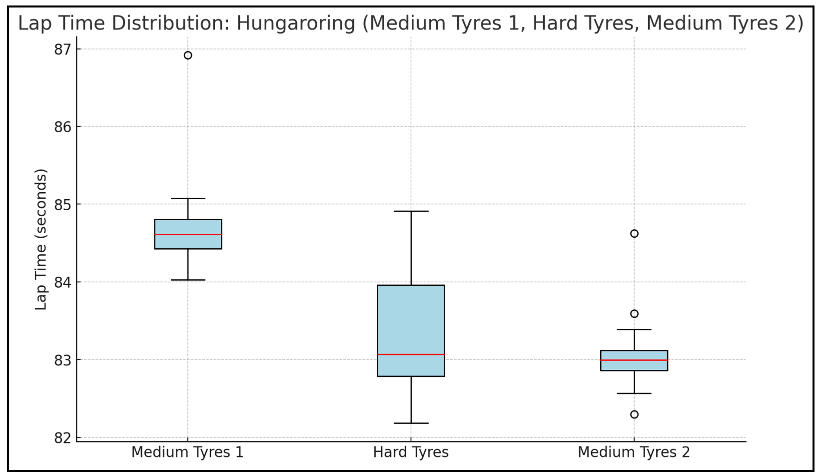
Box-Plot 1. Representing Lap time distribution for Emilia Romagna

The box plots for COTA and Emilia-Romagna indicate that Hard tyres have lower median lap timings (98.50 and 80.659 seconds) than Medium tyres (99.53 and 81.441 seconds), confirming slower degradation and better stability. Unlike the low downforce circuits, the difference in IQR between Medium and Hard tyres is less, which indicates that both tyre compounds show variation in the degradation. The IQR for Medium tyres (0.392 seconds) was smaller than the Hard tyres (0.538 seconds), showing that despite Hard tyres Providing more durability, their lap time has significant variation. While in Emilia Romanga, the Medium tyres have a larger IQR than the Hard tyres –representing more variability and faster degradation; hence Hard tyres are better for long stints, However, the Medium tyres can still compete for shorter stints.

High Downforce Level



Box-Plot 1. Representing Lap time distribution for Marina Bay



Box-Plot 1. Representing Lap time distribution for Hungaroring

The box plots for Marina Bay and Hungaroring show that Hard tyres tend to have lower median timings (96.698 and 83.316 seconds) than Medium tyres (99.176 and 84.668 seconds), similar to what was observed in lower downforce circuits. The IOR for Medium tyres is larger in Marina Bay (0.744 seconds) –indicating more variability, whereas in Hungaroring, the Medium tyres have inconsistent degradation. Hence, after a comparison was done for all the downforce levels, the Hard tyres are the better preferred for long stints in all downforce due to their stability and long-term performance, while the Medium Tyres, despite offering better initial grip, tend to degrade faster.

One-Way ANOVA Test :

I conducted a One-Way ANOVA, a statistical method used to compare the means of three or more independent groups to determine whether differences are statistically significant or due to random variation. In this case, downforce level was the independent variable, and tyre degradation was the continuous dependent variable. Since tyre degradation is a continuous variable, ANOVA helped assess whether degradation varied significantly across circuits with different downforce levels. If a significant difference was found, a post-hoc Tukey's HSD test was performed to identify which downforce levels differed in terms of tyre degradation.

Hypotheses

Null hypothesis (H_0): There is no significant difference and relationship in tyre degradation across different downforce levels, and any observed differences in degradation are purely coincidental due to random variation.

Alternative hypothesis (H_1): There is a significant mean difference in tyre degradation across all downforce levels, indicating there is a relationship between downforce levels and tyre degradation.

Table 4: Represents the calculation of the overall mean

Downforce level	**Tyre degradation values**	**Mean ($\bar{x}$)**
Low	+0.210 ,-1.002,-1.945 ,+0.217 ,+0.027	-0.4986
Medium	+1.26, +0.856, +0.674, +1.218	+1.0020
High	+2.174, +2.17, +1.842, +1.381, +0.142	+1.5418
Overall mean		+0.6589

Table 5: Represents summary of One-Way ANOVA result (Refer to Appendix for ANOVA calculations

Source	**DF**	**Sum of Squares (SS)**	**Mean Square (MS)**	**F-statistic**	**p-value**
Between Groups	2	11.067	5.534	9.02	0.0048
Within Groups (Error)	11	6.747	0.613	-	-
Total	13	17.815	-	-	-

Since the p-value is less than 0.05 ($p < 0.05$), we can reject the null hypothesis (**H_0**) and accept the alternative hypothesis(**H_1**), confirming that downforce level has a significant effect on tyre degradation.

Tukey's HSD Test (Refer to Appendix for Tukey's HSD Test calculations)

Table 6:Tukey's HSD Test Results for Tyre Degradation Across Downforce Levels

Comparison	**Mean Difference**	**Standard error (SE)**	**Q Statistics**	**p-value**
2.0404	0.5254	3.88	3.88	0.0025
0.5398	0.5254	1.03	1.03	0.3263
-1.5006	0.5254	-2.86	-2.86	0.0156

Now I will be using the degrees of freedom within groups (11) and number of groups($k=3$), the statistical significance of tyre degradation difference across downforce levels will be determined using p-values. If $p < 0.05$, the difference is statistically significant, meaning that variation in tyre degradation is not due to random chance. Else if $p > 0.05$, then the difference is not statistically significant, indicating that the variation could be due to randomness than the effect of downforce levels on tyre degradation.

High vs Low Downforce

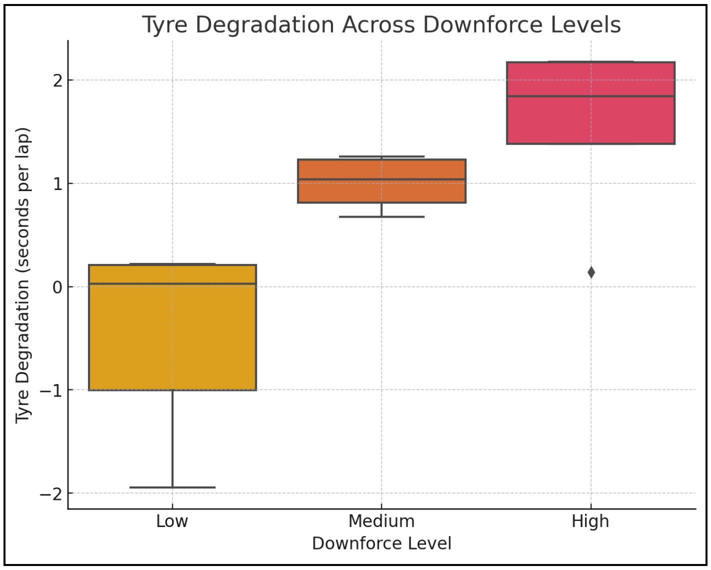
The **p-value** for High vs Low Downforce circuits is 0.0025, which is less than 0.05 ($p > 0.05$), indicating a statistically significant difference in tyre degradation, which confirms that high downforce circuits cause significantly higher tyre degradation compared to low downforce circuits. Since $p < 0.05$, the observed difference is unlikely due to random variation, suggesting that downforce level significantly influences tyre degradation.

High vs Medium Downforce

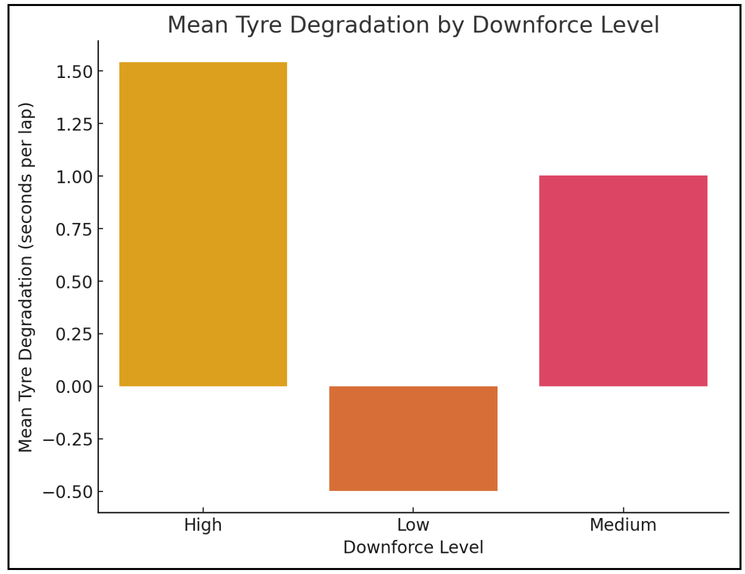
The **p-value** for High vs Medium Downforce circuits is 0.3263, which is greater than 0.05 ($0.3262 > p$), indicating that the difference in tyre degradation between high and medium downforce circuits is not statistically significant. However, high downforce circuits typically consist of greater tyre degradation; the difference is not large enough to confirm a significant impact, suggesting that medium nd high downforce circuits have a similar influence on tyre degradation.

Low vs Medium Downforce

The **p-value** for Low vs Medium Downforce circuits is 0.0156, which is less than 0.05 ($p > 0.05$), indicating a statistically significant difference in tyre degradation—confirming that medium downforce circuits cause greater tyre degradation compared to low downforce circuits. Although this confirms that medium downforce circuits cause greater tyre degradation compared to low downforce circuits. However, the difference is significant, but it is smaller than the High vs Low downforce comparison, which indicates that medium downforce circuits causes significant tyre degradation but not as severe as high downforce circuits.



Box-Plot 7. Representing the combined tyre degradation across all downforce levels

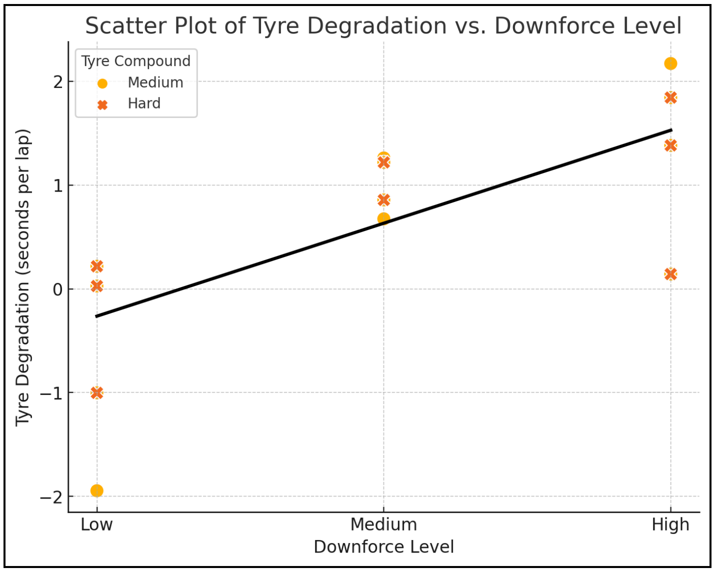


Bar Graph 1. Representing mean tyre degradation by downforce level

As evident in ***Box-Plot** 7*, the lower downforce circuits show a wide spread of tyre degradation value, including some negative values –that indicates that tyres in some stints did not degrade significantly but rather slightly improved over the stint. Moving on, the medium downforce circuits consist of a narrower range of tyre degradation that are all positive, indicating consistent tyre degradation throughout the stints. The high downforce circuits display the highest tyre degradation values, where all the values are greater than 1 second per lap –indicating maximum tyre degradation across all the downforce levels. Therefore, tyre degradation is lowest in low downforce circuits as circuits with lower downforce exert less stress on the tyres, resulting in less tyre degradation. Whereas the medium downforce circuits have consistent tyre degradation, while the high downforce circuits show the highest tyre degradation. This trend can be further verified from ***Bar Graph 1***, where the high downforce circuits have the highest mean tyre degradation (+1.5 seconds per lap), medium downforce level has a moderate mean degradation

(+1.0 seconds per lap) and lastly the low downforce circuits with the least mean degradation (~-0.5 seconds per lap).

Effect of Downforce on Tyre Compounds (Hard and Medium)



Scatter Plot 1. Representing the Tyre Degradation vs Downforce levels

As evident in ***Scatter Plot 1***, there is a positive correlation between the downforce level and tyre degradation (inclusive of both tyre compounds) because as the downforce level increases, the tyre degradation also increases for both tyre compounds. The medium tyres show higher degradation as compared to the Hard tyres across all downforce levels, as the medium tyres wear off and degrade faster. The trend is explicitly evident in the high downforce circuits, where the medium tyres have the highest values for tyre degradation (+2.174 and 1.842 seconds per lap) and they have the higher variability, especially in the low downforce circuits where certain values are negative—indicating instances where tyres maintained performance or even improved the lap times throughout a stint. Hence, medium tyres show inconsistency in the low-downforce circuits, possibly because of varying track conditions, driving style or tyre temperature.

Hard tyres follow a more stable and predictable degradation pattern, specifically in the high-downforce circuit, where they degrade slower than medium tyres. They are also more durable and designed to last for longer stints, making them a better option for longer extended stints because they provide better durability and traction under higher aerodynamic loads.

Evaluation :

Strengths	Limitations	Further extensions
In my exploration, I utilized a combination of statistical methods, such as ANOVA and Tukey's HSD test to determine the statistical significance between the variables and calculus-based instantaneous degradation to provide a comprehensive analysis.	Downforce level is just one factor among many that affect tyre degradation. External factors such as weather conditions and track temperature were not accounted for in my exploration.	Including data from multiple drivers across different teams to obtain a more comprehensive and clear understanding of trends in tyre degradation.
I categorized the circuits based on the downforce levels, which allowed me to make structural comparisons of tyre degradation. Additionally, I also divided the race into stints to focus on the effect of downforce on specific tyre compounds and, importantly, using the timings of a single driver to reduce variability.	In my exploration, only one driver's data was considered, which potentially may not be completely representative of other drivers' tyre degradation trends.	Considering other external variables such as track temperature, fuel load and pit stop strategy and their impact on tyre degradation.
Throughout my exploration, I used plenty of graphical representations such as box plots, bar graphs and scatter plots, which were able to support my findings and make the trends in tyre degradation more evident.	In my exploration, the analysis was only limited to the Medium and Hard tyres; however, including soft tyres could have also provided further trends in degradation patterns.	Investigating how tyre degradation changes with a change in fuel load by comparing the degradation at the start of the stint and towards the end of the stint.

Table 8. Strengths, limitations and further extensions of the exploration

Conclusion :

The aim of my exploration was to analyse and compare how circuit downforce levels affect tyre degradation for the Medium and Hard Pirelli tyres over a race stint. Throughout the exploration, I applied calculus to calculate the instantaneous rate of degradation, along with One-Way ANOVA and Tukey's HSD test to understand that higher downforce results in higher tyre degradation. During my exploration, I observed that the low downforce circuits, Monza and Spa, consist of minimal degradation, with values like -1.002 seconds/Lap for Monza on Hard tyres and -1.945 seconds/Lap for Spa on Medium tyres, indicating improved lap timings over the stint because of less aerodynamic load. Moving on, the medium downforce circuits, COTA and Emilia Romagna, consist of a moderate degradation +1.260 seconds/Lap for COTA Medium tyres and +1.218 seconds/Lap for Emilia Hard. This is because of the moderate aerodynamic load and a balance between straights and corners.

While the high downforce circuits, Marina Bay and Hungaroring exhibit the highest degradation, where the Medium degrades the fastest, +2.174 seconds/Lap at Marina Bay and +1.842 seconds/Lap at Hungaroring, making them unsuitable for longer stints in higher downforce, while the Hard tyres degraded at a slower rate, +2.170 seconds/Lap at Marina Bay and +1.381 seconds /Lap at Hungaroring.

From the instantaneous rate of degradation, I found that tyre degradation is higher at the beginning but stabilises over the stint. As at Lap = 1, the degradation was extremely high for Hard tyres on Emilia (+78,827.43 seconds/Lap) and Medium2 at Hungaroring (-204492.48 seconds/Lap). However, later, as the stint progresses, the degradation significantly decreases for both Hungaroring Hards and Hungaroring Medium2, suggesting that due to fuel burn-off and better tyre management improves performance over a stint. Furthermore, the ANOVA test (p=0.00911) confirmed that downforce levels significantly affect tyre degradation. Additionally, from Tukey's HSD test it was deduced that Low vs High downforce circuits and Low vs Medium downforce circuits have significant differences, whereas the High vs Medium downforce circuits were not statistically significant, this is further supported by the scatter plot–indicating that Medium tyres degrade faster than Hard tyres across all conditions.

In conclusion, through this exploration, I was able to analyse and compare the trends in tyre degradation across different downforce circuits. Through my exploration, I found that Hard tyres are the best options for longer endurance stints due to their durability and consistent degradation, while Medium tyres, despite better initial grip tend to degrade fast, hence, they are more suited for shorter aggressive push stints. The insights from my exploration also align with the actual Formula 1 tyre management methods, where the teams strategically select tyres based on the circuit's characteristics for better performance.

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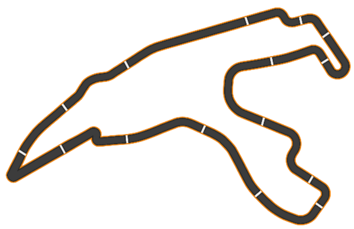
9.0 Appendices

Lap-by-Lap Timings

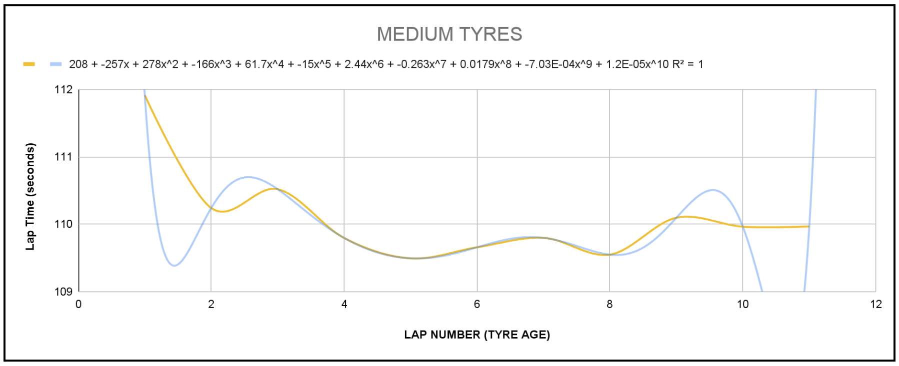
Lap number / Circuit name	Lap Times (in seconds)					
	Autodromo Nazionale Monza	Spa-Francorchamps	Circuit of The Americas	Circuit of The Americas	Marina Bay Street Circuit	Hungaroring
1	88.179	111.912	99.987	84.58	97.09	85.076
2	85.396	110.24	100.341	80.926	99.775	85.076
3	85.179	110.519	122.873	80.931	99.14	84.647
4	84.81	109.796	151.204	81.279	99.31	84.546
5	84.603	109.494	168.785	81.122	99.041	84.112
6	84.663	109.658	98.951	81.043	98.797	84.119
7	84.434	109.798	99.57	81.025	98.645	84.025
8	84.569	109.549	99.307	81.116	98.727	84.381
9	84.362	110.096	99.257	81.215	98.691	84.308
10	84.432	109.963	99.421	81.387	98.582	84.604
11	84.846	109.967	99.597	81.147	98.913	84.632
12	84.916	//114.091	99.448	81.349	98.776	84.411
13	85.529	//122.667	99.399	81.442	98.671	84.642
14	85.606	108.227	99.2	81.661	98.707	84.835
15	90.192	108.021	99.672	81.464	98.515	84.846
16	104.888	107.982	99.782	81.964	98.904	84.894

17	84.27	107.981	99.621	81.714	99.561	84.853
18	83.875	107.971	99.435	81.719	98.814	84.707
19	84.203	108.346	99.553	81.672	98.734	84.625
20	83.826	108.279	99.756	81.757	99.087	84.513
21	83.88	108.38	99.506	81.669	99.315	84.531
22	83.768	108.356	99.411	82.018	99.008	84.471
23	83.864	108.449	99.878	81.943	99.688	86.918
24	83.64	108.444	99.699	81.6	99.372	//102.028
25	83.59	112.358	99.583	86.023	100.029	83.071
26	83.321	121.972	100.211	105.006	98.867	83.296
27	83.534	107.034	119.522	79.935	99.153	82.892
28	83.284	107.039	98.662	80.928	99.795	82.359
29	83.229	107.517	99.067	80.502	98.371	82.182
30	83.359	107.311	99.034	80.426	99.543	82.7
31	83.972	107.018	98.667	80.944	99.214	82.776
32	83.274	107.064	98.825	80.645	98.505	82.795
33	83.226	107.013	99.032	80.589	98.187	83.066
34	83.43	107.023	98.633	80.726	98.494	83.207
35	83.681	108.321	98.897	80.812	99.264	83.845
36	83.714	109.289	98.643	80.562	107.084	84.123
37	83.55	107.601	98.477	80.36	117.168	84.077
38	83.624	107.142	98.57	80.194	96.556	84.912
39	83.718	107.539	97.834	80.507	96.287	84.452
40	83.69	107.354	98.287	80.582	97.48	87.274
41	83.611	107.46	98.395	80.928	96.166	100.816
42	83.67	107.216	98.179	80.24	97.266	-
43	83.867	107.071	98.257	80.942	95.849	82.967

44	83.43	107.061	98.697	80.702	95.639	83.186
45	83.731		98.349	80.431	95.716	83.042
46	83.597		99.177	80.596	95.371	82.923
47	83.676		98.471	81.313	95.758	83.379
48	83.518		98.313	81.513	95.792	83.051
49	83.467		98.326	80.798	96.252	82.674
50	83.461		97.951	80.414	97.53	82.726
51	83.555		97.875	80.364	96.424	82.91
52	83.75		98.092	80.358	96.02	82.834
53	83.268		98.365	80.889	96.099	82.826
54			98.175	80.398	96.454	82.299
55			97.902	80.636	96.56	82.964
56			99.518	80.415	96.497	83.019
57				80.752	96.802	84.626
58				80.553	97.51	83.592
59				80.506	98.218	82.569
60				80.693	98.166	82.58
61				80.783	98.326	82.894
62				81.307	98.726	83.122
63				81.153		82.977
64						83.391
65						83.063
66						83.387
67						83.03
68						82.87
69						83.118
70						83.109



Circuit 2. Circuit de Spa-Francorchamps



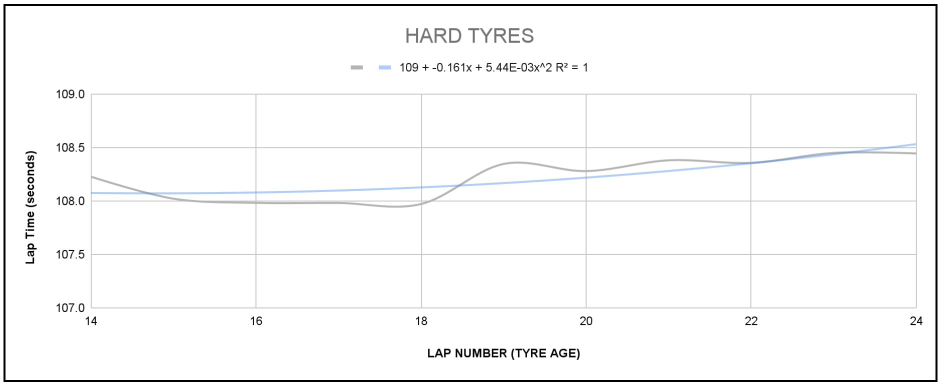
$$Average\ Race\ Pace\ = \frac{84.27+83.875+84.203+...+83.268}{}$$

$$y\ = 208\ +\ -257x\ +\ 278x^2\ +\ -166x^3\ +\ 61.7x^4\ +\ -15x^5\ +\ 2.44x^6\ +\ -0.263x^7$$
$$+\ 0.0179x^8\ +\ -7.03\times10^{-4}x^9\ +\ 1.2\times10^{-5}x^{10}$$

$$\frac{dT}{dN} =-257\ +\ 556x\ -\ 498x^2\ +\ 246.8x^3\ -\ 75x^4\ +\ 14.64x^5\ -\ 1.841x^6\ +\ 0.1432x^7\ -\ 0.006327x^8$$
$$+\ 0.00012x^9$$

$$\frac{dT}{dN}\Big|_{N=1} =-257\ +\ 556(1)-\ 498(1)^2\ +\ 246.8(1)^3\ -\ 75(1)^4\ +\ 14.64(1)^5\ -\ 1.841(1)^6\ +\ 0.1432(1)^7$$
$$-\ 0.006327(1)^8\ +\ 0.00012(1)^9\ =\ -\ 14.44\ seconds/Lap$$

$$\frac{dT}{dN}\Big|_{N=11} =-257\ +\ 556(11)-\ 498(11)^2\ +\ 246.8(11)^3\ -\ 75(11)^4\ +\ 14.64(11)^5\ -\ 1.841(11)^6$$
$$+\ 0.1432(11)^7\ -\ 0.006327(11)^8\ +\ 0.00012(11)^9\ =\ -\ 10372.40\ seconds/Lap$$

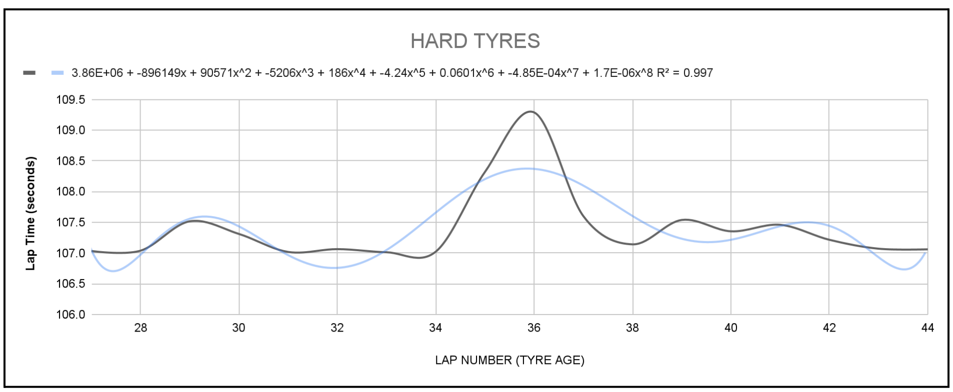


$$y = 109 - 0.161x + 5.44\times10^{-3}x^{2}$$

$$\frac{dT}{dN} = -0.161 + 0.01088x$$

$$\left.\frac{dT}{dN}\right|_{N=1} = -0.161 + 0.01088(1) = -0.150\, seconds/Lap$$

$$\left.\frac{dT}{dN}\right|_{N=11} = -0.161 + 0.01088(11) = -0.041\, seconds/Lap$$

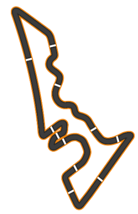


$$y = 3.86\times10^{6} - 896149x + 90571x^{2} - 5206x^{3} + 186x^{4} - 4.24x^{5} + 0.0601x^{6}$$
$$- 4.85\times10^{-7}x^{7} + 1.7\times10^{-6}x^{8}$$

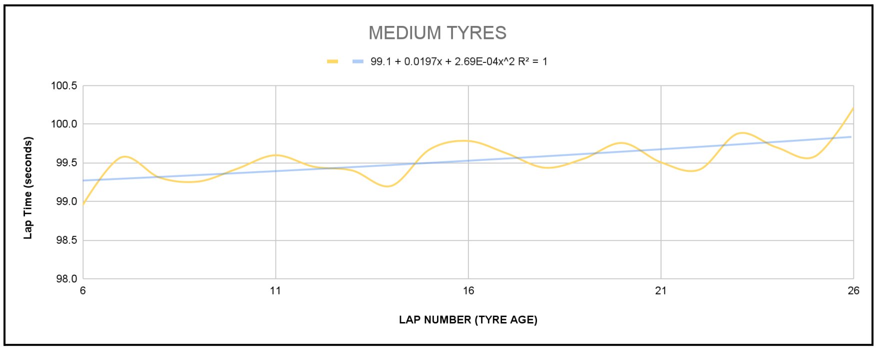
$$\frac{dT}{dN} = -896149 + 181142x - 15618x^{2} + 755x^{3} - 21.2x^{4} + 0.3606x^{5} - 3.395\times10^{-6}x^{6}$$
$$+ 1.36\times10^{-5}x^{7}$$

$$\left.\frac{dT}{dN}\right|_{N=1} = -896149 + 181142(1) - 15618(1)^{2} + 755(1)^{3} - 21.2(1)^{4} + 0.3606(1)^{5}$$
$$- 3.395\times10^{-6}(1)^{6} + 1.36\times10^{-5}(1)^{7} = -279890.83\, seconds/Lap$$

$$\left.\frac{dT}{dN}\right|_{N=18} = -896149 + 181142(18) - 15618(18)^{2} + 755(18)^{3} - 21.2(18)^{4} + 0.3606(18)^{5}$$
$$- 3.395\times10^{-6}(18)^{6} + 1.36\times10^{-5}(18)^{7} = 171432.74 seconds/Lap$$



Circuit 3. Circuit Of The Americas

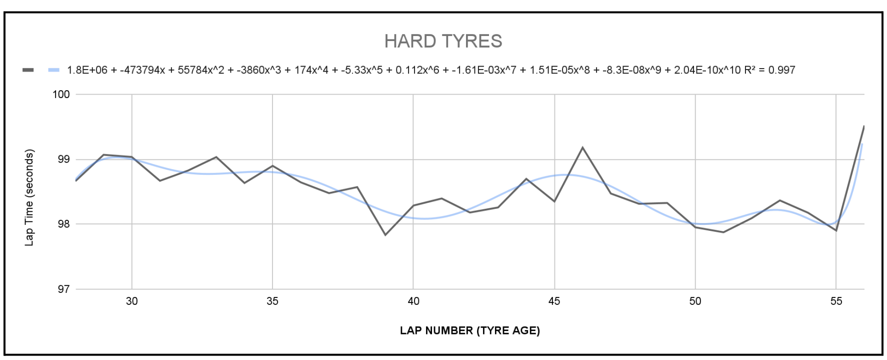


$$y = 99.1 + 0.0197x + 2.69 \times 10^{-4}x^2$$

$$\frac{dT}{dN} = 0.0197 + 5.38 \times 10^{-4}x$$

$$\frac{dT}{dN}(1) = 0.0197 + 5.38 \times 10^{-4}(1) =+ 0.020\, seconds/Lap$$

$$\frac{dT}{dN}(21) = 0.0197 + 5.38 \times 10^{-4}(21) =+ 0.030\, seconds/Lap$$



$$y = 1..8 \times 10^6 - 473794x + 55784x^2 - 3860x^3 + 174x^4 - 5.33x^5 + 0.112x^6$$

$$- 1.61 \times 10^{-3}x^7 + 1.51 \times 10^{-5}x^8 - 8.3 \times 10^{-8}x^9 + 2.04 \times 10^{-10}x^{10}$$

$$\frac{dT}{dN} =- 473794 + 111568x - 11580x^2 + 696x^3 - 26.65x^4 + 0.672x^5 - 0.01127x^6$$

$$+ 0.001208x^7 - 7.47\times10^{-7}x^8 + 2.04\times10^{-9}x^9$$

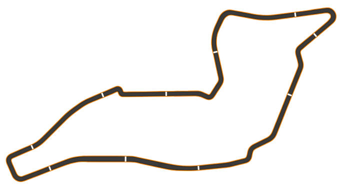
$$\frac{dT}{dN}(1) =- 473794 + 111568(1) - 11580(1)^2 + 696(1)^3 - 26.65(1)^4 + 0.672(1)^5$$

$$- 0.01127(1)^6 + 0.001208(1)^7 - 7.47\times10^{-7}(1)^8 + 2.04\times10^{-9}(1)^9 =- 373136\ seconds/Lap$$

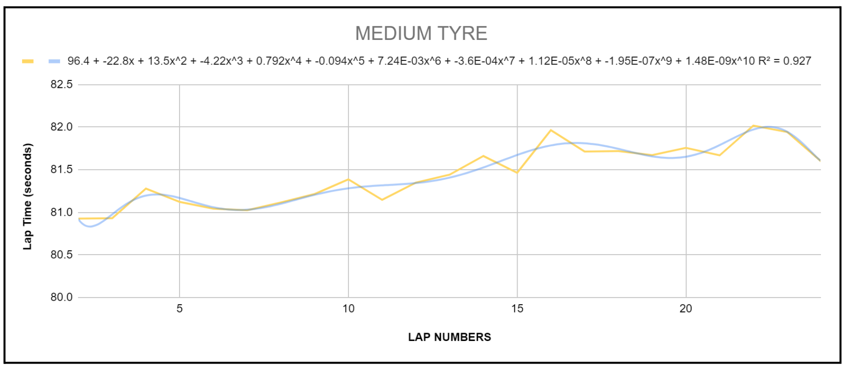
$$\frac{dT}{dN}(29) =- 473794 + 111568(29) - 11580(29)^2 + 696(29)^3 - 26.65(29)^4 + 0.672(29)^5$$

$$- 0.01127(29)^6 + 0.001208(29)^7 - 7.47\times10^{-7}(29)^8 + 2.04\times10^{-9}(29)^9$$

$$=+ 18722197.74\ seconds/Lap$$



Circuit 4. Emilia Romagna



$$y = 96.4 - 22.8x + 13.5x^2 - 4.22x^3 + 0.792x^4 - 0.094x^5 + 7.24 \times 10^{-3}x^6$$

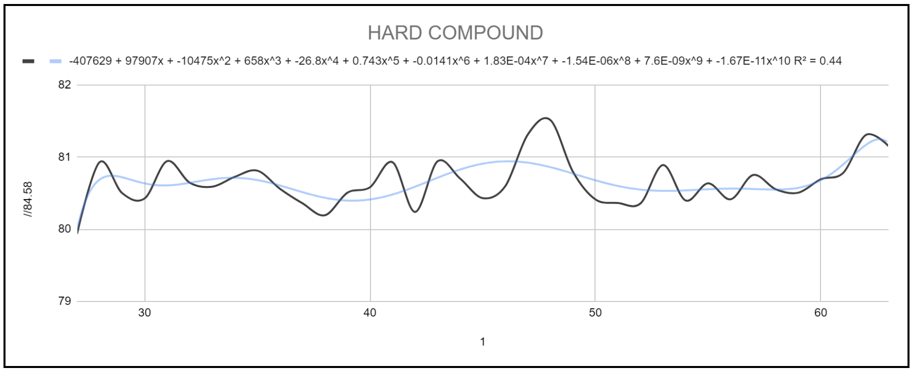
$$- 3.6 \times 10^{-4}x^7 + 1.12 \times 10^{-5}x^8 - 1.95 \times 10^{-7}x^9 + 1.48 \times 10^{-9}x^{10}$$

$$\frac{dT}{dN} =- 22.8 + 27.0x - 12.66x^2 + 3.168x^3 - 0.47x^4 + 0.04344x^5 - 0.00252x^6$$

$$+ 8.96 \times 10^{-5}x^7 - 1.755 \times 10^{-6}x^8 + 1.48 \times 10^{-8}x^9$$

$\frac{dT}{dN}(1) =- 5.720\ seconds/Lap$

$\frac{dT}{dN}(23) =+ 1758.78\ seconds/Lap$



$$y = -\ 407629\ +\ 97907x\ -\ 10475x^2\ +\ 658x^3\ -\ 26.8x^4\ +\ 0.743x^5\ -\ 0.0141x^6$$

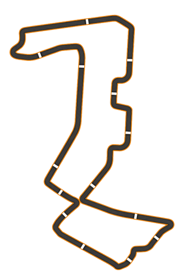
$$1.8\ \times\ 10^{-4}x^7\ -\ 1.54\ \times\ 10^{-6}x^8\ +\ 7.6\ \times\ 10^{-9}x^9\ -\ 1.67\ \times\ 10^{-11}x^{10}$$

$$\frac{dT}{dN} = +\ 97907\ -\ 20950x\ +\ 1974x^2\ -\ 107.2x^3\ +\ 3.715x^4\ -\ 0.0845x^5\ +\ 0.001281x^6$$

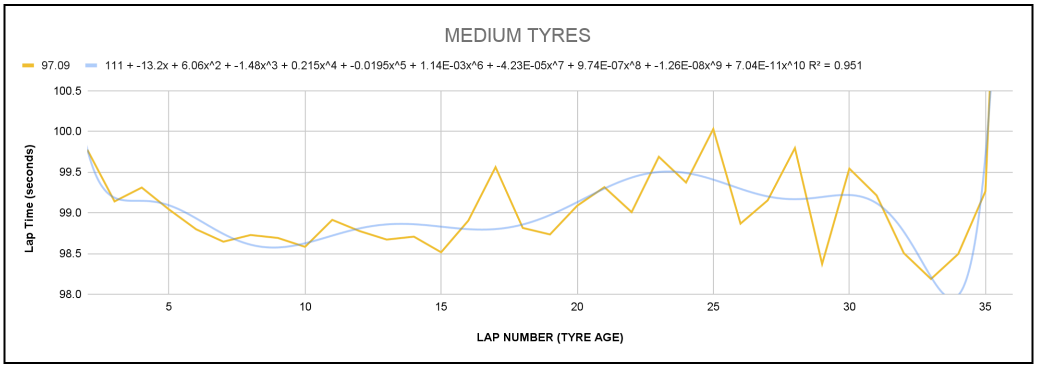
$$-\ 1.232\ \times\ 10^{-5}x^7\ +\ 6.84\ \times\ 10^{-8}x^8\ -\ 1.67\ \times\ 10^{-10}x^9$$

$$\frac{dT}{dN}(1) = -\ 78827.43\ seconds/Lap$$

$$\frac{dT}{dN}(37) = +\ 26855.54\ \ seconds/Lap$$



Circuit 5. Marina Bay Street Circuit

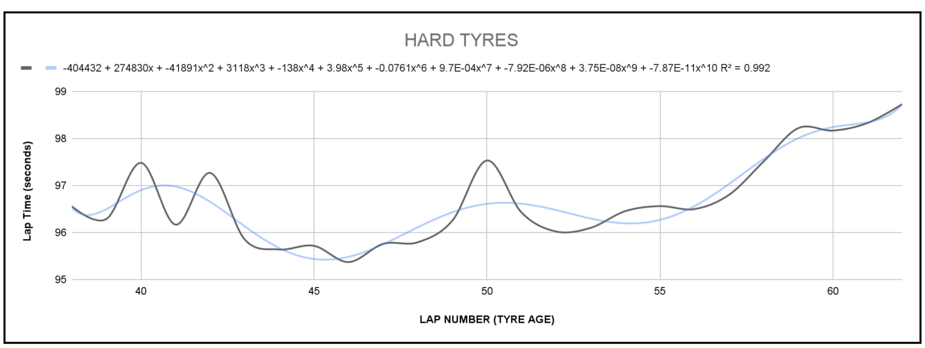


$$y = 111 - 13.2x + 6.06x^2 - 1.48x^3 + 0.215x^4 - 0.0195x^5 + 1.14\times10^{-3}x^6 - 4.23\times10^{-5}x^7$$
$$+ 9.74\times10^{-7}x^8 - 1.26\times10^{-8}x^9 + 7.04\times10^{-11}x^{10}$$

$$\frac{dT}{dN} =- 13.2 + 12.12x - 4.44x^2 + 0.86x^3 - 0.0975x^4 + 6.84\times10^{-3}x^5 - 2.96\times10^{-4}x^6$$
$$+ 7.79\times10^{-6}x^7 - 1.134\times10^{-7}x^8 + 7.04\times10^{-10}x^9$$

$$\frac{dT}{dN}(1) =- 13.2 + 12.12(1) - 4.44(1)^2 + 0.86(1)^3 - 0.0975(1)^4 + 6.84\times10^{-3}(1)^5$$
$$- 2.96\times10^{-4}(1)^6 + 7.79\times10^{-6}(1)^7 - 1.134\times10^{-7}(1)^8 + 7.04\times10^{-10}(1)^9$$
$$=$$

$$\frac{dT}{dN}(36) =- 13.2 + 12.12(36) - 4.44(36)^2 + 0.86(36)^3 - 0.0975(36)^4 + 6.84\times10^{-3}(36)^5$$
$$- 2.96\times10^{-4}(36)^6 + 7.79\times10^{-6}(36)^7 - 1.134\times10^{-7}(36)^8 + 7.04\times10^{-10}(36)^9$$
$$= 2332.73\ seconds/Lap$$



$$y =- 404432 + 274830x - 41891x^2 + 3118x^3 - 138x^4 + 3.98x^5 - 0.0761x^6 + 9.7\times10^{-4}x^7$$
$$- 7.92\times10^{-6}x^8 + 3.75\times10^{-8}x^9 - 7.87\times10^{-11}x^{10}$$

$$\frac{dT}{dN} = 274830 - 83782x + 9354x^2 - 552x^3 + 19.9x^4 - 0.4566x^5 + 6.79\times10^{-3}x^6$$

$$- 6.336\times10^{-5}x^{7} + 3.375\times10^{-7}x^{8} - 7.87\times10^{-10}x^{9}$$

$$\frac{dT}{dN}(1) = 274830 - 83782(1) + 9354(1)^{2} - 552(1)^{3} + 19.9(1)^{4} - 0.4566(1)^{5} + 6.79\times10^{-3}(1)^{6}$$

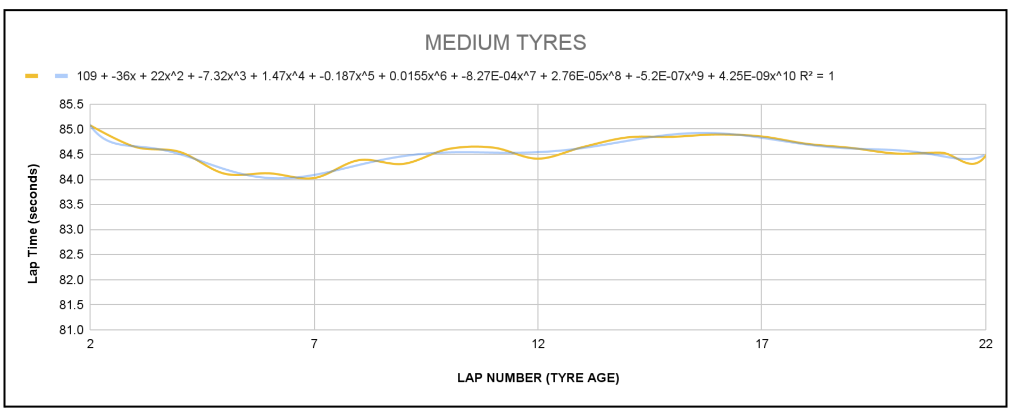
$$- 6.336\times10^{-5}(1)^{7} + 3.375\times10^{-7}(1)^{8} - 7.87\times10^{-10}(1)^{9} =+ 199869.45 second/Lap$$

$$\frac{dT}{dN}(25) = 274830 - 83782(25) + 9354(25)^{2} - 552(25)^{3} + 19.9(25)^{4} - 0.4566(25)^{5}$$

$$+ 6.79\times10^{-3}$$

$$(25)^{6} - 6.336\times10^{-5}(25)^{7} + 3.375\times10^{-7}(25)^{8} - 7.87\times10^{-10}(25)^{9} =+ 35475.47\ second/Lap$$

Circuit 6. Hungaroring



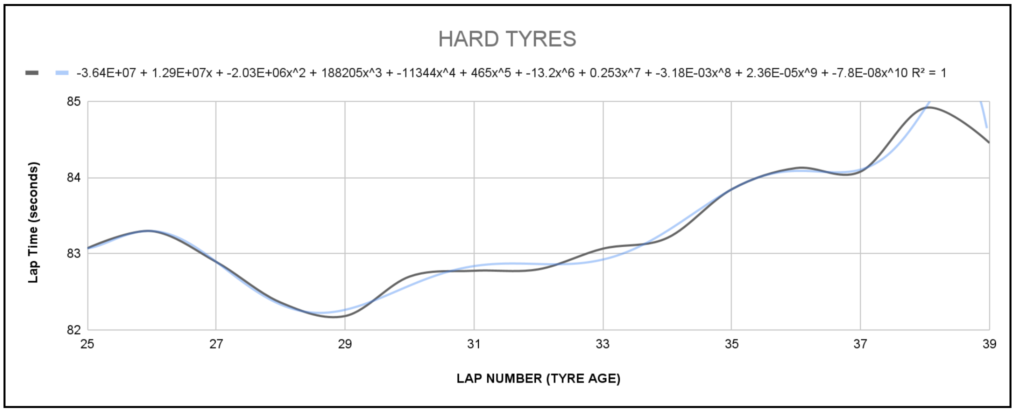
$$y = 109 - 36x + 22x^{2} - 7.32x^{3} + 1.47x^{4} - 0.187x^{5} + 0.0155x^{6} - 8.27\times10^{-4}x^{7} + 2.76\times10^{-5}x^{8}$$
$$- 5.2\times10^{-7}x^{9} + 4.25\times10^{-9}x^{10}$$

$$\frac{dT}{dN} =- 36 + 44x - 21.96x^{2} + 5.88x^{3} - 0.935x^{4} + 0.093x^{4} + 0.093x^{5} - 5.789\times10^{-3}x^{6}$$

$$+ 2.208\times10^{-4}x^{7} - 4.68\times10^{-6}x^{8} + 4.24\times10^{-8}x^{9}$$

$\frac{dT}{dN}(1) =- 8.927\ seconds/Lap$

$\frac{dT}{dN}(21) =+ 1486.74\ seconds/Lap$

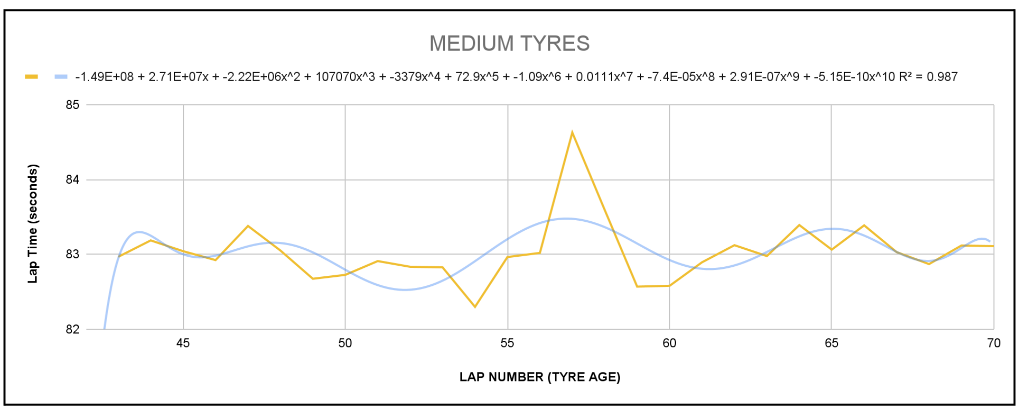


$$y =- 3.64\times10^{7} + 1.29\times10^{7}x - 2.03 \times10^{6}x^{2}188205x^{3} - 11344x^{4} + 465x^{5} - 13.2x^{6} + 0.235x^{7} - 3.18\times10^{-3}x^{8} + 2.36\times10^{-5}x^{9} - 7.8\times10^{-8}x^{10}$$

$$\frac{dT}{dN} = 1.29\times10^{7} - 4.06\times10^{6}x + 564615x^{2} - 45376x^{3} + 2325x^{4} - 79.2x^{5} + 1.645x^{6} - 0.02544x^{7} + 2.124\times10^{-5}x^{8} - 7.8\times10^{-7}x^{9}$$

$\frac{dT}{dN}(1) =+ 9361486.422\ seconds/Lap$

$\frac{dT}{dN}(15) =- 1639712.227\ seconds/Lap$



$$y = 1.49\times10^{8} + 2.71\times10^{7}x - 2.22\times10^{6}x^{2} + 107070x^{3} - 3379x^{4} + 72.9x^{5} - 1.08x^{6} + 0.0111x^{7} - 7.4\times10^{-5}x^{8} + 2.91\times10^{-7}x^{9} - 5.15\times10^{-10}x^{10}$$

$$\frac{dT}{dN} = 27100000 - 4440000x + 321210x^{2} - 13516x^{3} + 364.5x^{4} - 6.54x^{5} + 0.0777x^{6} - 0.000592x^{7} + 2.619\times10^{-6}x^{8} - 5.15\times10^{-9}x^{9}$$

$\frac{dT}{dN}(1) =+ 22968052.04\ seconds/Lap$

$\frac{dT}{dN}(15) =+ 41831.72\ seconds/Lap$

ANOVA Test calculations

$Number\ of\ Groups\ (k)\ =\ 3$

$\bar{x}_{Low} = \frac{0.210+(-1.002)+(-1.945)+0.217+0.027}{5} =-\ 0.4986$

$\bar{x}_{Medium} = \frac{1.26+0.856+0.674+1.218}{4} = 1.0020$

$\bar{x}_{High} = \frac{2.174+2.17+1.842+1.381+0.142}{5} = 1.5418$

$\bar{x}_{Total} = \frac{(0.210-1.002-1.945+0.217+0.027)+(1.26+0.856+0.674+1.218)+(2.174+2.17+1.842+1.381+0.142)}{14}$

$\overline{X}_{Total} = 0.6589$

Sum of Squares (SS)

Between Groups Sum of Squares (SSB)

$SSB = n_1(\overline{X}_{Low} - \overline{X}_{Total})^2 + n_2(\overline{X}_{Medium} - \overline{X}_{Total})^2 + n_3(\overline{X}_{High} - \overline{X}_{Total})^2$

$SSB = 5(-\ 0.4986 - 0.6589)^2 + 4(1.0020 - 0.6589)^2 + 5(1.5418 - 0.6589)^2$

$SSB = 11.067$

Within groups of Squares (SSW)

$SSW = \sum_{i=1}^{k} \sum_{j=1}^{n_i} (X_{ij} - \overline{X}_i)^2$

$SSW = 6.747$

Total sum of Squares (SST)

$SST = SSB + SSW = 11.067 + 6.747 = 17.815$

Degrees of Freedom

Between the groups

$df_{Between} = K - 1 = 3 - 1 = 2$

Errors Within Groups

$df_{within} = n - k = 14 - 3 = 11$

$df_{Total} = n - 1 = 14 - 1 = 13$

Mean square between groups (MSB)

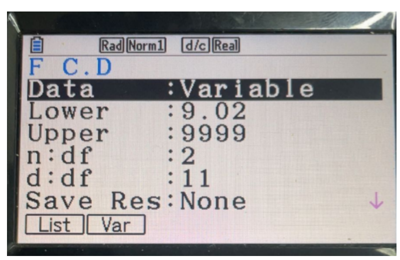
$MSB = \frac{SSW}{df_{Between}} = \frac{11.067}{2} = 5.534$

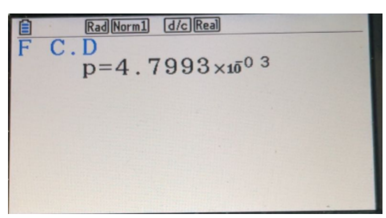
Mean square within groups (MSW)

$MSW = \frac{SSW}{df_{within}} = \frac{6.747}{11} = 0.613$

F-Statisticc

$F = \frac{MSB}{MSW} = \frac{5.534}{0.613} = 9.02$





TUKEY's HSD Test

$$SE = \sqrt{MSW \times (\frac{1}{n1} + \frac{1}{n2})}$$

$$SE = \sqrt{0.613 \times (\frac{1}{5} + \frac{1}{4})}$$

$$SE = \sqrt{0.613 \times (0.2 + 0.25)}$$

$$SE = \sqrt{0.276} = 0.5254$$

Mean Difference ($\overline{X}$) = $\overline{X}_1 - \overline{X}_2$

High vs Low Downforce

$1.5418 - (-0.4986) = 2.0404$

High vs Medium Downforce

$1.5418 - 1.0020 = 0.5398$

Low vs Medium Downforce

$-0.4986 - 1.0020 = 1.5006$

Q-Statistic

$$Q = \frac{Mean\ Difference}{SE}$$

High vs Low Downforce

$$Q = \frac{2.0404}{0.5254} = 3.88$$

High vs Medium Downforce

$$Q = \frac{0.5398}{0.5254} = 1.03$$

Low vs Medium Downforce

$$Q = \frac{-1.5006}{0.5254} = -2.86$$

p values using GDC

$p = 2 \times P(T > |Q|)$

High vs Low Downforce

$p = 2 \times 0.00128 = 0.0025$

High vs Medium Downforce

$p = 2 \times 0.16525 = 0.3251$

Low vs Medium Downforce

$p = 2 \times 0.00775 = 0.0156$