Exercise – II – A

Test of the following series:

1. $\sum_{n=1}^{\infty} \sqrt{\left(\frac{n}{n+1}\right)}$

2. $\sum_{n=1}^{\infty} \cos\frac{1}{n}$

3. $\sum_{n=1}^{\infty} \frac{n}{n+1}$

4. $\sum_{n=1}^{\infty} 3^{n+1}$

5. $\sum_{n=1}^{\infty} \frac{2^n}{\sqrt{4^n+1}}$

6. $\frac{1}{1+2^{-1}} + \frac{2}{1+2^{-2}} + \frac{3}{1+2^{-3}} + \ldots$

7. $1^3 + 2^3 + \ldots + n^3 + \ldots$

8. $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{2.4} + \ldots. + \frac{1}{n(n+1)} + \ldots$

9. $\frac{2}{3} + \frac{8}{5} + \frac{24}{9} + \ldots$

10 Test the convergence of series $\sum_{n=0}^{\infty} (-1)^n$

Answers – II A

1. divergence
2. divergence
3. divergence
4. divergence
5. divergence
6. divergence
7. divergence
8. convergence
9. divergence
10 finitely oscillatory

Exercise – IV

1. Test of the following series :

(i) $\Sigma\left(\frac{n^3+a}{2^n+a}\right)$

(ii) $\frac{1}{x-1}+\frac{x}{x^2-1}+\frac{x^2}{x^3-1}+\ldots(x>1)$

(iii) $1+\frac{x}{2}+\frac{x^2}{5}+\frac{x^3}{10}+\ldots\frac{x^n}{n^2+1}+\ldots$

(iv) $1+\frac{x^2}{2}+\frac{x^3}{4}+\frac{x^6}{6}+\ldots$

(v) $\frac{1}{1+2}+\frac{1}{1+2^2}+\frac{1}{1+2^3}+\ldots$

(vi) $\Sigma\left(\frac{n+1}{n^2}x^n\right)$

(vii) $\Sigma\left(\frac{n^3+a}{2^n+a}\right)$

(viii) $\Sigma(3n-1)2^n$

(ix) $1+3x^2+5x^3+7x^4+\ldots$

2. Show that the series:

$1+\frac{2^p}{2!}+\frac{3^p}{3!}+\frac{4^p}{4!}+\ldots$ is convergence for all values of p.

3. Test for convergence or divergence the series whose n^{th} term is

$(\sqrt{n^2+1}-n)x^{2n}$

Answers – IV

1. **(i)** Convergence

(ii) Divergence

(iii) divergence when $x^2=1$

(iv) Divergence when $x^2=1$

(v) Convergence

(vi) Divergence when $x=1$

(vii) Convergence

(viii) Convergence

(ix) Divergence

3. Divergence when $x^2=1$

Exercise-III

1. Test convergence or divergence of the following series

(i) $\frac{\sqrt{3}}{1.2}+\frac{\sqrt{5}}{3.4}+\frac{\sqrt{7}}{5.6}\ldots$

(ii) $1+\frac{1+2}{1+2^2}+\frac{1+3}{1+3^2}+\ldots$

(iii) $\frac{2}{1^p}+\frac{3}{2^p}+\frac{4}{3^p}+\ldots$

(iv) $\frac{1}{2}+\frac{\sqrt{2}}{5}+\frac{\sqrt{3}}{10}+\ldots\frac{\sqrt{n}}{n^2+1}+\ldots$

(v) $\frac{1}{1+\sqrt{2}}+\frac{2}{1+2\sqrt{3}}+\frac{3}{1+3\sqrt{4}}+\ldots$

(vi) $\log_e 2+\log_e\frac{3}{2}+\log_e\frac{4}{3}+\log_e\frac{5}{4}+\ldots$

(vii) $\frac{1}{n}+\frac{1}{n-1}+\frac{1}{n+1}+\frac{1}{n-2}+\frac{1}{n+2}+\ldots$

(viii) $\frac{1}{(x-k)^2}+\frac{1}{(x-2k)^2}+\frac{1}{(x-3k)^2}+\ldots$

2. Test of the following series whose general terms are given by

(i) $\sqrt{n^2+1}-n$

(ii) $\frac{\sqrt{n+1}-\sqrt{n}}{n^p}$

(iii) $\Sigma\frac{1}{n}\sin\left(\frac{1}{n}\right)$

(iv) $\Sigma\tan^{-1}\left(\frac{1}{n}\right)$

(v) $\left[\sqrt{n^2+1}-n\right]x^{2n}$

(vi) $u_n=\frac{1}{(n^2+n)}$

(vii) $\sum_{n=1}^{\infty}\left[\frac{1}{n}(\sqrt{n^2+n+1}-\sqrt{n^2-n+1}\right]$

3. Test convergence of $\sum_{n=0}^{\infty}u_n$ where $u_n=\frac{\sqrt{n}}{n^2+1}$

Answers – III

1. **(i)** Convergence **(ii)** Divergence **(iii)** Convergence if $p>2$, divergnce $p<2$
(iv) Convergence **(v)** Divergence **(vi)** Divergence **(vii)** divergence
(viii) Convergence

2. **(i)** Divergence **(ii)** Convergence if $p>1$, divergnce $p\le 1$
(iii) Convergence **(iv)** Convergence
(v) Convergence if $x^2>1$, divergnce $x^2\le 1$ **(vi)** Divergence
(vii) Divergence