A REPORT

ON

KINEMATIC SYNTHESIS AND VISULAIZATION OF A FLAP ACTUATION MECHANISM USING ANALYTICAL METHODS

BY

Muralidhara Samarth 2023A4PS0418P

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Abstract:

The design of high lift devices such as flaps in aircraft is of crucial importance in the aerodynamic efficiency and safety of civil and transport aircraft. Motivated by this, this report focuses on the kinematic synthesis and visualization of a four-bar mechanism for the deployment of single slotted Fowler Flaps, a configuration commonly used in regional civil and transport aircraft. The mechanism was synthesized using analytical methods to guide a specified coupler point through three critical positions corresponding to takeoff, cruise, and landing configurations. The resulting link dimensions and orientations were computed using position synthesis techniques and validated against standard references. The mechanism's motion was then visualized using a kinematic analysis software called MotionGen to verify its feasibility and hence ensure smooth and continuous flap deployment.

Signature of Student

Date

Signature of PS Faculty

Date

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INTRODUCTION

Civil and transport aircraft in the modern world are required to perform efficiently and safely across a broad flight operational range, balancing both the need of high-speed cruise and the need of low-speed maneuverability across takeoff and landing. To satisfy such broad aerodynamic requirements, aircraft wings are equipped with High Lift Devices (HLDs) like slats and flaps that are fixed along the leading and trailing edges respectively. Upon deployment, these HLDs significantly increase lift and accompany a rise in drag. This is useful in takeoff which requires a large lift and landing which not only requires high drag for deceleration but also needs high lift for a smooth, steady descent [1]. HLDs exhibit lift and drag enhancement properties as they increase a wing’s camber and effective surface area.

This report deals primarily with the kinematic synthesis of one type of trailing edge HLD, the single slotted Fowler Flap. These flaps feature a contoured slot between the wing airfoil and the flap airfoil though which air from below flows to the upper surface, consequently stabilizing the boundary layer. This increases the coefficients of lift and drag [11]. When deployed, these flaps are to follow Fowler motion, i.e. they are supposed to extend rearward and deflect downwards simultaneously to increase the camber and effective surface area of the wing thereby increasing lift and drag as mentioned earlier. To achieve this special motion, an analytical method is used to synthesize a four-bar mechanism wherein a point on the coupler passes through three precision points, one each for takeoff, landing and cruise configurations. For the aircraft flap mechanism, the flap airfoil is assumed to be attached to this point of the coupler that passes through the aforementioned points. The four-bar mechanism is selected for this application because of its simplicity and lightweight design compared to alternative configurations such as the dropped hinge and link-track mechanisms [8].

By using a kinematic synthesis approach and validating it through visualization with the help of a software named MotionGen, this report contributes to the mechanism design of the single slotted Fowler Flap with potential applications in the Aerospace Industry. This approach can also be used for similar mechanisms where motion through three precision points is required, making it useful beyond just aircraft flap systems.

SYNTHESIS FRAMEWORK

The four-bar mechanism used for the single slotted Fowler Flap consists of four links: a crank, a ternary coupler, a follower and the ground. The crank is connected to a rotary actuator (a motor) which drives the whole mechanism. The crank is connected to a ternary coupler, that is triangular in shape. This in turn is connected to the output link called the follower. The imaginary link that connects the two pivots: O_2 and O_4 as shown in Fig.1, is called the ground. The flap airfoil is assumed to be connected to a point P in Fig. 1 on the ternary coupler. The mechanism is designed in such a way that point P passes through points P_1, P_2 and P_3. These points are together known as precision points and this method of creating mechanisms is formally known as kinematic synthesis by motion generation.

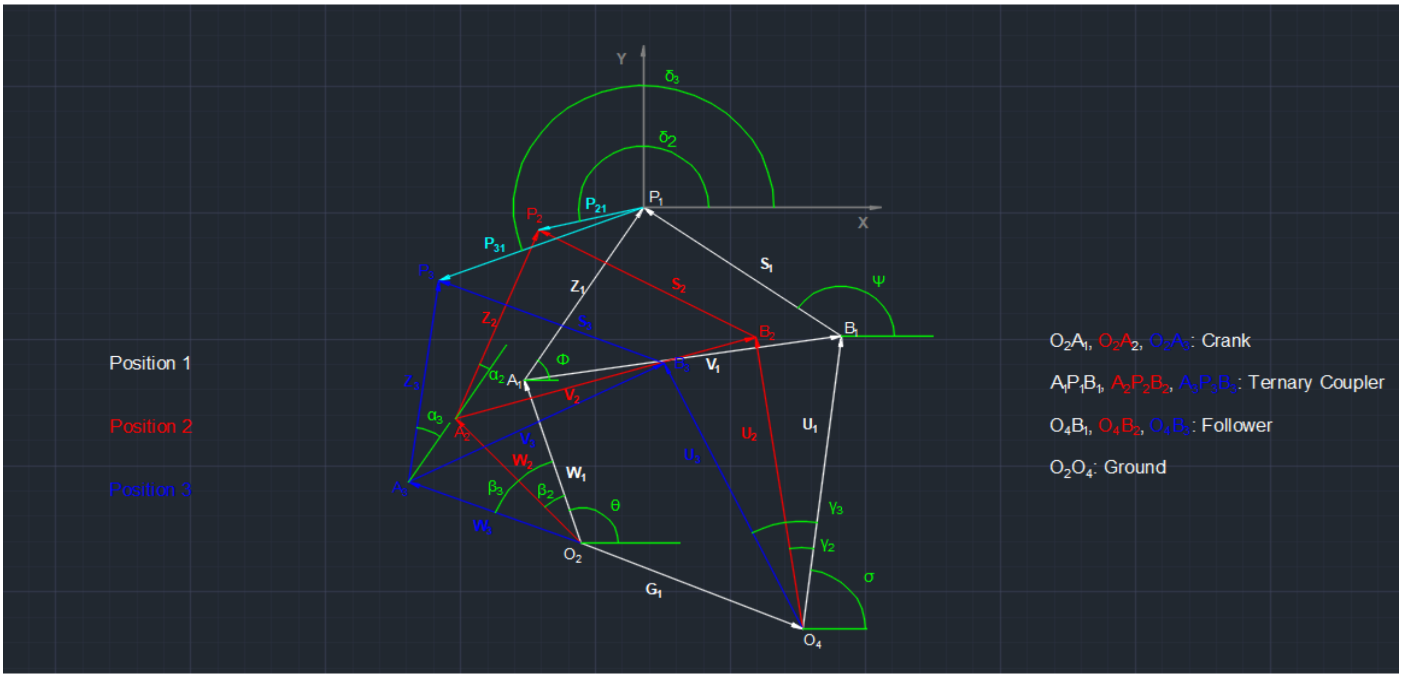

Fig. 1: A generic four-bar mechanism schematic
Adapted from: R. L. Norton, *Design of Machinery*, McGraw-Hill, 2000

The main objective is to calculate the link lengths of the mechanism that deploys the aircraft flap such that the coupler passes through specific predetermined points. For instance, according to [2], the flaps in ATR 72, a 72-passenger aircraft, have preset deployment angle settings 0°, 15° and 30° for cruise, takeoff and landing respectively. To synthesize such a complex four bar mechanism, we first derive a set of equations that determine all the link lengths W, Z, V, S, U and G of a generic four bar mechanism as shown in Fig.1, provided we know the precision points. We then validate a MATLAB code that computes the link lengths (using the derived equations) by cross verifying the results with an example presented in [3]. Finally, we apply all the equations obtained to synthesize the four-bar mechanism that drives the single slotted Fowler Flap used in a Boeing 777. In all the text that follows, the notation X_j refers to the j^{th} position of the point X on the mechanism.

i. <u>Derivation of the link lengths</u>:

We first divide the mechanism into two parts; each called a dyad. From Fig. 1, point P on the ternary coupler is supposed to pass through the precision points P_2 and P_3 whilst initially being located at P_1. This is achieved by the rotation of the crank by angles β_2 and β_3 respectively. The line AP of the ternary coupler is rotated the line through an angle α_2 between P_1 and P_2 and through an angle α_3 between P_1 and P_3. As stated earlier, we wish to derive all four link lengths and angles and the internal dimensions of the coupler (AP and BP). Note that the following steps of the derivation are according to those given in [3].

As seen from Fig. 1, the coordinate system XY is placed with its origin as the first precision point P_1. The position vectors $\mathbf{P_{21}}$ and $\mathbf{P_{31}}$ define the displacements of P_2 and P_3 with respect to P_1. They have angles δ_2 and δ_3, and magnitudes p_{21} and p_{31}, respectively. The dyad WZ defines the left half of the linkage, and the dyad US defines the right half of the linkage. Since both Z and S are part of the rigid coupler, both undergo the same rotation of α_2 and α_3 while P moves through the precision points. Therefore, the known quantities include α_2, α_3, p_{21}, p_{31}, δ_2 and δ_3.

For the ternary coupler, we use triangle law of vector addition and rearrange the terms to obtain:

$$\boldsymbol{V_1} = \boldsymbol{Z_1} - \boldsymbol{S_1} \tag{1}$$

Similarly, the ground link $\mathbf{G_1}$ can be defined as follows:

$$\boldsymbol{G_1} = \boldsymbol{W_1} + \boldsymbol{V_1} - \boldsymbol{U_1} \tag{2}$$

We first solve for the left dyad of the mechanism, consisting of the vectors $\mathbf{W_1}$ and $\mathbf{Z_1}$ and use the same procedure to solve for the right dyad. Going in the clockwise direction, the two loop closure equations for P_1-P_2 and P_1-P_3 respectively are:

$$\begin{aligned} \boldsymbol{W_2} + \boldsymbol{Z_2} - \boldsymbol{P_{21}} - \boldsymbol{Z_1} - \boldsymbol{W_1} = \mathbf{0} \\ \boldsymbol{W_3} + \boldsymbol{Z_3} - \boldsymbol{P_{31}} - \boldsymbol{Z_1} - \boldsymbol{W_1} = \mathbf{0} \end{aligned} \tag{3}$$

With Fig. 1 as reference, substituting the complex equivalents of all the vectors, we obtain:

$$\begin{aligned} we^{j(\theta+\beta_2)} + ze^{j(\phi+\alpha_2)} - p_{21}e^{j\delta_2} - ze^{j\phi} - we^{j\theta} = 0 \\ we^{j(\theta+\beta_3)} + ze^{j(\phi+\alpha_3)} - p_{31}e^{j\delta_3} - ze^{j\phi} - we^{j\theta} = 0 \end{aligned} \tag{4}$$

Rearranging the terms,

$$\begin{aligned} we^{j\theta}\left(e^{j\beta_2} - 1\right) + ze^{j\phi}(e^{j\alpha_2} - 1) = p_{21}e^{j\delta_2} \\ we^{j\theta}\left(e^{j\beta_3} - 1\right) + ze^{j\phi}(e^{j\alpha_3} - 1) = p_{31}e^{j\delta_3} \end{aligned} \tag{5}$$

It is important to note that the magnitude of $\mathbf{W_1}$, $\mathbf{W_2}$ and $\mathbf{W_3}$ are all equal to w as they all represent the same line in a rigid link. The same can be said about the vectors $\mathbf{Z_1}$, $\mathbf{Z_2}$ and $\mathbf{Z_3}$ whose common magnitude is z.

Each of the vector equations in (5) consists of 2 independent scalar equations that can be obtained by using Euler's identity. Therefore, the independent scalar equations that we obtain from (5) are:

Along the x-axis:

$$\begin{aligned} w\cos\theta\,(\cos\beta_2 - 1) - w\sin\theta\sin\beta_2 + z\cos\phi\,(\cos\alpha_2 - 1) - z\sin\phi\sin\alpha_2 = p_{21}\cos\delta_2 \\ w\cos\theta\,(\cos\beta_3 - 1) - w\sin\theta\sin\beta_3 + z\cos\phi\,(\cos\alpha_3 - 1) - z\sin\phi\sin\alpha_3 = p_{31}\cos\delta_3 \end{aligned} \tag{6.1}$$

Along the y-axis:

$$\begin{aligned} w\sin\theta\,(\cos\beta_2 - 1) + w\cos\theta\sin\beta_2 + z\sin\phi\,(\cos\alpha_2 - 1) + z\cos\phi\sin\alpha_2 &= p_{21}\sin\delta_2 \\ w\sin\theta\,(\cos\beta_3 - 1) + w\cos\theta\sin\beta_3 + z\sin\phi\,(\cos\alpha_3 - 1) + z\cos\phi\sin\alpha_3 &= p_{31}\sin\delta_3 \end{aligned} \quad (6.2)$$

In the equations (6.1) and (6.2), the variables we wish to solve for are w, θ, z and Φ. As stated earlier, the known quantities in the above equations include α_2, α_3, p_{21}, p_{31}, δ_2 and δ_3. This leaves us with two variables β_2 and β_3 that we must assume as free choices. This assumption is based on the premise that we may want to specify the angular displacement of the crank to suit some design constraint. In addition, by assuming two free choices, the number of equations equals the number of unknowns and hence the system becomes solvable. The only unknown quantities in (6.1) and (6.2) are the magnitudes and angles of the vectors $\mathbf{W_1}$ and $\mathbf{Z_1}$.

Using the substitutions:

$$W_{1x} = w\cos\theta\,; \quad Z_{1x} = z\cos\phi$$

$$W_{1y} = w\sin\theta\,; \quad Z_{1y} = \mathrm{z}\sin\phi$$

And from (6.1) and (6.2), we obtain:

$$\begin{aligned} W_{1x}(\cos\beta_2 - 1) - W_{1y}\sin\beta_2 + Z_{1x}(\cos\alpha_2 - 1) - Z_{1y}\sin\alpha_2 &= p_{21}\cos\delta_2 \\ W_{1x}(\cos\beta_3 - 1) - W_{1y}\sin\beta_3 + Z_{1x}(\cos\alpha_3 - 1) - Z_{1y}\sin\alpha_3 &= p_{31}\cos\delta_3 \\ W_{1y}(\cos\beta_2 - 1) + W_{1x}\sin\beta_2 + Z_{1y}(\cos\alpha_2 - 1) + Z_{1x}\sin\alpha_2 &= p_{21}\sin\delta_2 \\ W_{1y}(\cos\beta_3 - 1) + W_{1x}\sin\beta_3 + Z_{1y}(\cos\alpha_3 - 1) + Z_{1x}\sin\alpha_3 &= p_{31}\sin\delta_3 \end{aligned} \quad (7)$$

After solving the system of equations presented in (7),

$$
\begin{aligned}
w &= \sqrt{W_{1x}^2 + W_{1y}^2} \\
\theta &= \tan^{-1}\left(\frac{W_{1y}}{W_{1x}}\right) \\
z &= \sqrt{Z_{1x}^2 + Z_{1y}^2} \\
\phi &= \tan^{-1}\left(\frac{Z_{1y}}{Z_{1x}}\right)
\end{aligned}
\tag{8}
$$

Similarly, the final four equations for the right dyad of the mechanism are:

$$
\begin{aligned}
(\cos\gamma_2 - 1)U_{1x} - \sin\gamma_2\ U_{1y} + (\cos\alpha_2 - 1)S_{1x} - \sin\alpha_2\ S_{1y} &= p_{21}\cos\delta_2 \\
(\cos\gamma_3 - 1)U_{1x} - \sin\gamma_3\ U_{1y} + (\cos\alpha_3 - 1)S_{1x} - \sin\alpha_3\ S_{1y} &= p_{31}\cos\delta_3 \\
\sin\gamma_2\ U_{1x} + (\cos\gamma_2 - 1)U_{1y} + \sin\alpha_2\ S_{1x} + (\cos\alpha_2 - 1)S_{1y} &= p_{21}\sin\delta_2 \\
\sin\gamma_3\ U_{1x} + (\cos\gamma_3 - 1)U_{1y} + \sin\alpha_3\ S_{1x} + (\cos\alpha_3 - 1)S_{1y} &= p_{31}\sin\delta_3
\end{aligned}
\tag{9}
$$

And we can therefore compute the magnitudes and angles of $\mathbf{U_1}$ and $\mathbf{S_1}$.

$$
\begin{aligned}
u &= \sqrt{U_{1x}^2 + U_{1y}^2} \\
\sigma &= \tan^{-1}\left(\frac{U_{1y}}{U_{1x}}\right) \\
s &= \sqrt{S_{1x}^2 + S_{1y}^2} \\
\psi &= \tan^{-1}\left(\frac{S_{1y}}{S_{1x}}\right)
\end{aligned}
\tag{10}
$$

Now that we have calculated all the initial orientations and length of the links, we can use (1) and (2) to compute $\mathbf{V_1}$ and $\mathbf{G_1}$. This completes the kinematic synthesis of a generic four-bar mechanism by motion generation through three precision points. These equations can hence be used in the kinematic synthesis of the mechanism used for single slotted Fowler Flaps that have to move through three different points during cruise, takeoff and landing. The next section will consist of the validation of a MATLAB code that calculates the link length by applying the derived equations with the help of an example presented in [3].

ii. <u>Validation of MATLAB code:</u>

In this section, we use the derived equations to compute the link lengths of a four-bar mechanism with the help of a MATLAB program, using data presented in an example in [3]. We then compare the synthesized link lengths to the actual link lengths of the example mechanism. This cross verification will validate the use of the MATLAB program to synthesize any generic four-bar mechanism by motion generation through three precision points such as the one used in the single slotted Fowler Flap.

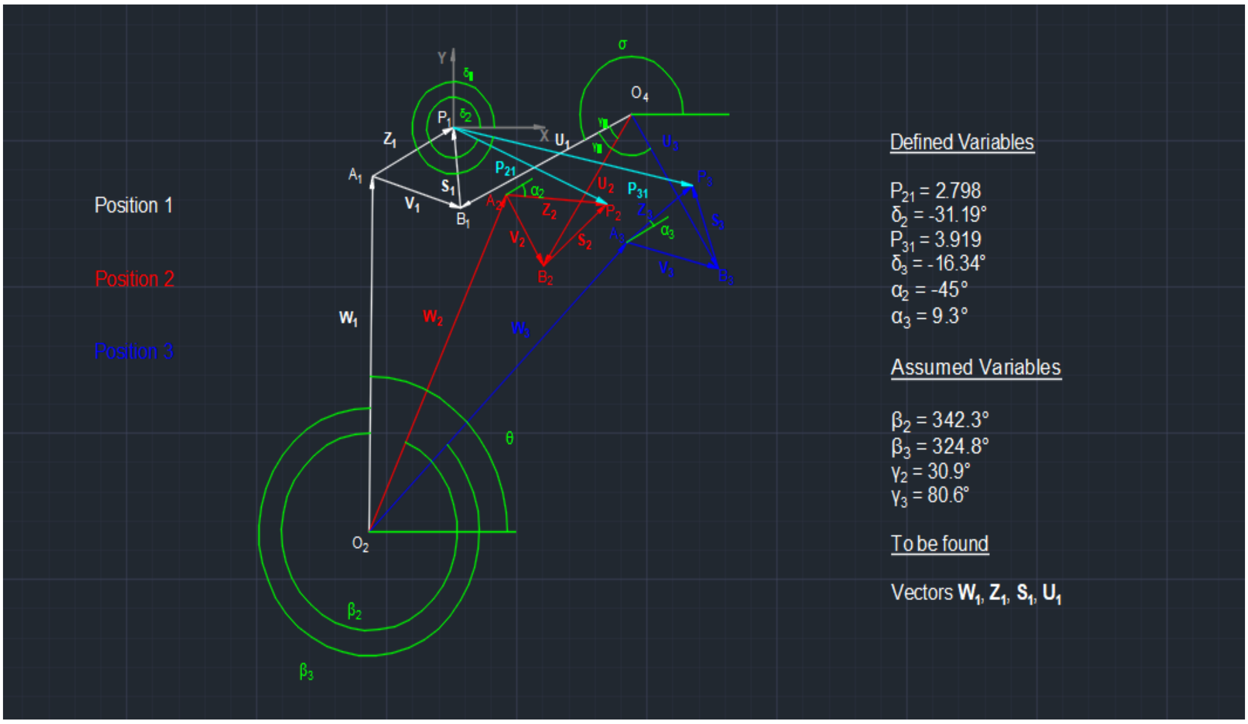

Fig. 2: Example Mechanism schematic
Adapted from R. L. Norton, *Design of Machinery*, McGraw Hill, 2000

Fig. 2 shows the schematic of the example mechanism as presented in [3]. As stated above, we wish to calculate the link lengths of the mechanism given only the points through which the coupler passes through.

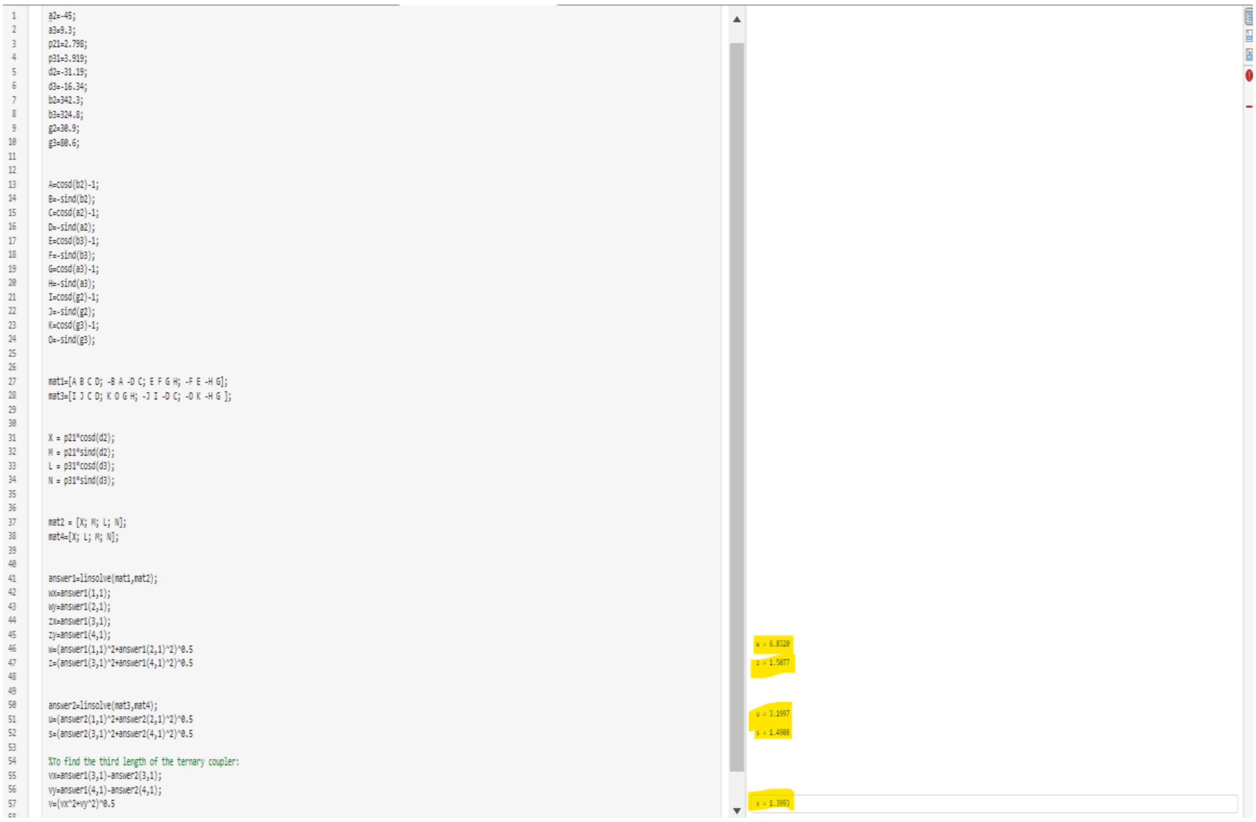

Fig. 3: MATLAB code that computes link lengths of the example mechanism

Fig. 3 shows the MATLAB code that incorporates all the derived equations to calculate the link lengths of the example mechanism. The linear system of equations are solved by using the linsolve() function. This gives us the x and y components **W**, **Z**, **U** and **S**. We then use equations (8) and (10) to compute the respective magnitudes which signify the corresponding link lengths. The computed link lengths are the highlighted valued shown on the right side of Fig. 3.

A comparison between the synthesized and actual link lengths given in [3] is as follows:

Link	Actual Lengths (mm)	Synthesized Lengths (mm)	% Error
U	3.2	3.1997	-0.009
V	1.399	1.3993	0.02
W	6.832	6.832	0

Table 1: Comparison between synthesized and actual link lengths for example mechanism

From the above table, we observe that only a marginal error exists between the actual and synthesized link lengths. This indicates that the MATLAB code is valid and can be used for the kinematic synthesis of the mechanism used in the single slotted Fowler Flap accurately.

Fig. 4 shows the visualization of the example mechanism using an open-source software MotionGen according to the synthesized link lengths and the expected motion was observed. The blue curves in this figure show the path of each of the links when the crank is rotated using a rotary actuator.

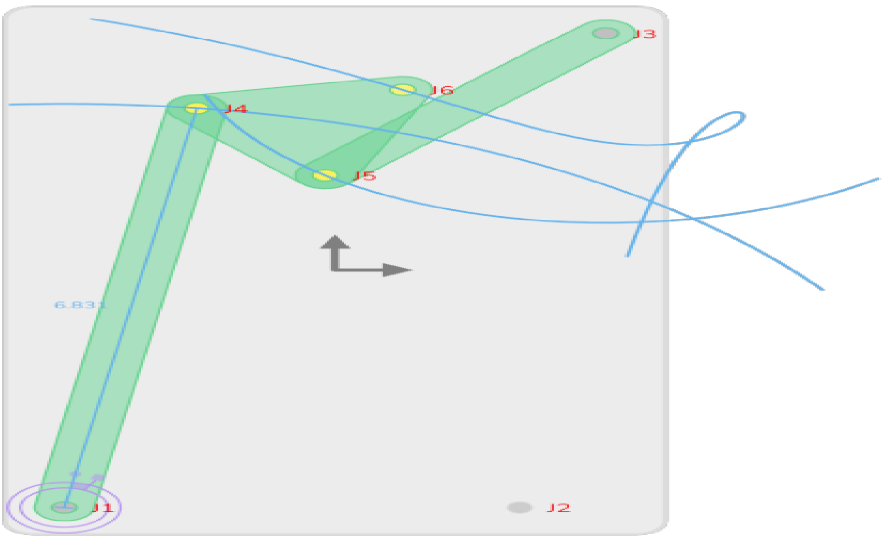

Fig. 4: Visualizing the example mechanism

iii. <u>Kinematic Synthesis of the Fowler Flap used in a Boeing 777 aircraft</u>:

In this section, we calculate the link lengths of the four-bar mechanism that is used in the deployment of the single slotted Fowler Flap used in a Boeing 777 aircraft. The main objective of this is to see if it's possible to produce the link dimensions according to the original design given in [7]. We will follow the same approach that was used in sections i. and ii. to carry out the same.

Fig. 5 below shows the mechanism schematic of the four-bar mechanism that drives the flap. Just like the mechanism from Fig. 1, this mechanism consists of four distinct links, namely: crank, ternary coupler, follower and ground. Note that unlike the mechanism from Fig.1, this mechanism has its ground pivots O_2 and O_4 located on opposite sides.

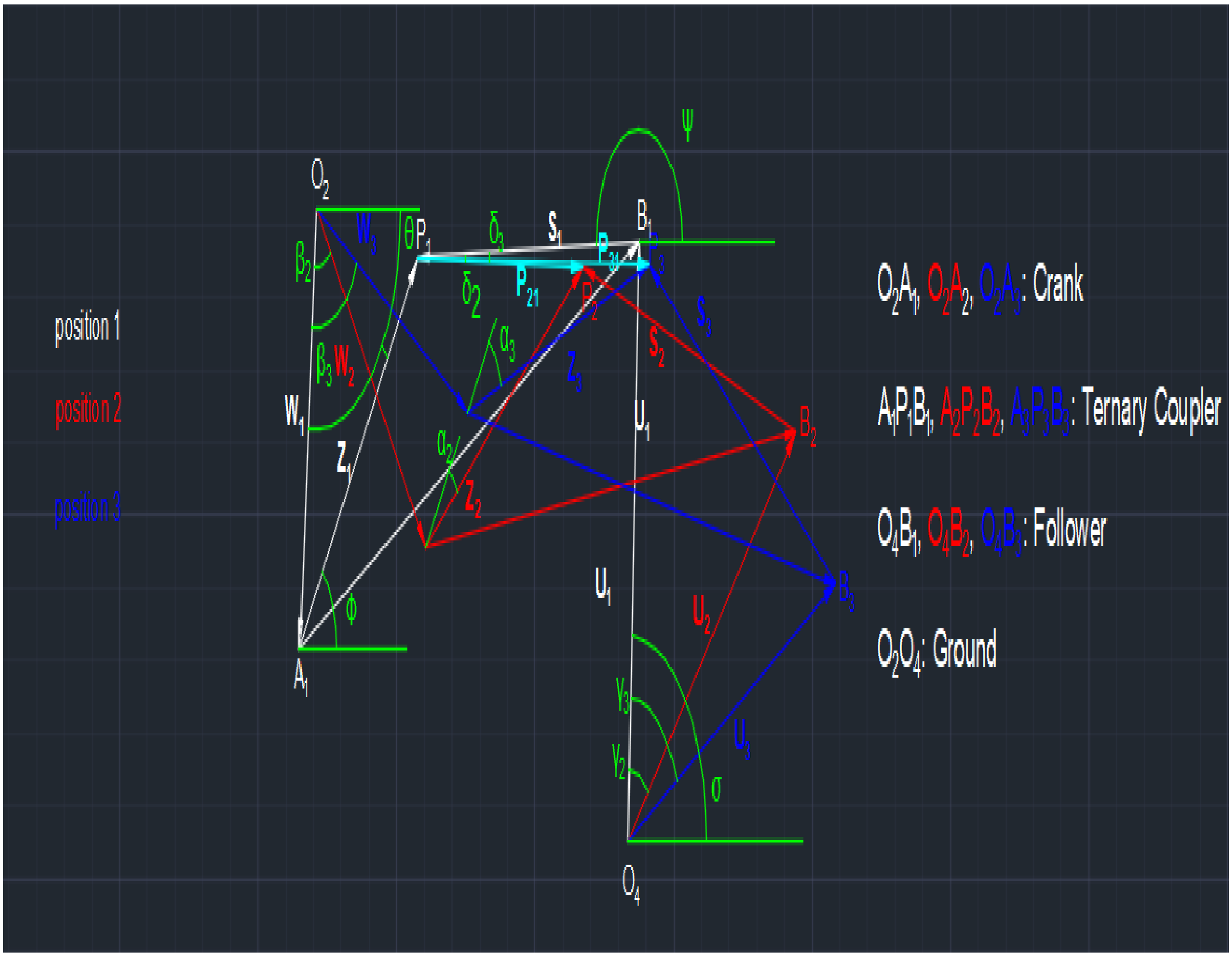

Fig. 5: Mechanism Schematic used in a Boeing 777 aircraft
Adapted from: Lima, D. Z., Aguiar, J. B., & Ferreira, W.G., *Preliminary structural design of a Fowler flap high-lifting device,* 2021

The flap airfoil (not shown in Fig. 5) is attached to point P of the mechanism that moves through the precision point P_1, P_2 and P_3. The flap is in the nested position, at point P_1 during cruise. It is in a slightly extended position at P_2 during takeoff and is in the fully extended position at P_3 during landing, because this flight stage requires the most lift and drag for a steady and smooth descent. Furthermore, according to [5], the ground pivot O_2 is attached to the bottom surface of wing and the ground link O_4 is attached to a slanted, fixed rack structure which in turn is again attached to the bottom side of the wing (not shown in Fig. 5). The rack structure is kept inside an enclosing known as a fairing that is aerodynamically designed.

The only information that is given is the location of the precision points P_1, P_2 and P_3. Given this, we wish to find the link lengths w, z, u, s and v. Due to the change in orientation of this mechanism, the derivation of the link lengths is done again.

We derive the governing kinematic equations of the left dyad WZ first and then move on to the right dyad US. Using the same loop closure equations for the left dyad WZ:

$$\begin{aligned} \boldsymbol{W_2} + \boldsymbol{Z_2} - \boldsymbol{P_{21}} - \boldsymbol{Z_1} - \boldsymbol{W_1} = \boldsymbol{0} \\ \boldsymbol{W_3} + \boldsymbol{Z_3} - \boldsymbol{P_{31}} - \boldsymbol{Z_1} - \boldsymbol{W_1} = \boldsymbol{0} \end{aligned} \tag{3}$$

Writing all the vectors in their complex equivalents, we obtain:

$$\begin{aligned} we^{-j(\theta-\beta_2)} + ze^{j(\phi-\alpha_2)} - p_{21}e^{j\delta_2} - ze^{j\phi} - we^{-j\theta} = 0 \\ we^{-j(\theta-\beta_3)} + ze^{j(\phi-\alpha_3)} - p_{31}e^{j\delta_3} - ze^{j\phi} - we^{-j\theta} = 0 \end{aligned} \tag{11}$$

Rearranging the terms we obtain,

$$we^{-j\theta}\left(e^{j\beta_2}-1\right)+ze^{j\phi}\left(e^{-j\alpha_2}-1\right)=p_{21}e^{j\delta_2}$$
$$we^{-j\theta}\left(e^{j\beta_3}-1\right)+ze^{j\phi}\left(e^{-j\alpha_3}-1\right)=p_{31}e^{j\delta_3} \tag{12}$$

Both above vector equations consist of two independent scalar equations that can be obtained by using Euler's identity. Therefore, the independent scalar equations that we obtain from (12) are:

Along the x-axis:

$$w\cos\theta\,(\cos\beta_2-1)+w\sin\theta\sin\beta_2+z\cos\phi\,(\cos\alpha_2-1)+z\sin\phi\sin\alpha_2=p_{21}\cos\delta_2$$
$$w\cos\theta\,(\cos\beta_3-1)+w\sin\theta\sin\beta_3+z\cos\phi\,(\cos\alpha_3-1)+z\sin\phi\sin\alpha_3=p_{31}\cos\delta_3 \tag{12.1}$$

Along the y-axis:

$$-w\sin\theta\,(\cos\beta_2-1)+w\cos\theta\sin\beta_2+z\sin\phi\,(\cos\alpha_2-1)-z\cos\phi\sin\alpha_2=p_{21}\sin\delta_2 \tag{12.2}$$
$$-w\sin\theta\,(\cos\beta_3-1)+w\cos\theta\sin\beta_3+z\sin\phi\,(\cos\alpha_3-1)-z\cos\phi\sin\alpha_3=p_{31}\sin\delta_3$$

The unknown quantities in equations (12.1) and (12.2) are w, θ, z and Φ. As discussed in section i., the known quantities include α_2, α_3, p_{21}, p_{31}, δ_2 and δ_3. Therefore, we take the remaining two quantities β_2 and β_3 as free choices whose values are decided based on design constraints [3]. In the case of an aircraft flap such as the one used in a Boeing 777 aircraft, the design constraint is the fact that the crank is physically restrained from making a complete revolution as the flap airfoil is required to move along an open path of points not a closed loop. This forms the basis for choosing specific values of β_2 and β_3 to solve for the unknowns.

Using the substitutions:

$$W_{1x} = w\cos\theta\,; \quad Z_{1x} = z\cos\phi$$

$$W_{1y} = w\sin\theta\,; \quad Z_{1y} = \mathrm{z}\sin\phi$$

And from (12.1) and (12.2), we obtain:

$$\begin{aligned}
W_{1x}(\cos\beta_2 - 1) + W_{1y}\sin\beta_2 + Z_{1x}(\cos\alpha_2 - 1) + Z_{1y}\sin\alpha_2 &= p_{21}\cos\delta_2 \\
W_{1x}(\cos\beta_3 - 1) + W_{1y}\sin\beta_3 + Z_{1x}(\cos\alpha_3 - 1) + Z_{1y}\sin\alpha_3 &= p_{31}\cos\delta_3 \\
-W_{1y}(\cos\beta_2 - 1) + W_{1x}\sin\beta_2 + Z_{1y}(\cos\alpha_2 - 1) - Z_{1x}\sin\alpha_2 &= p_{21}\sin\delta_2 \\
-W_{1y}(\cos\beta_3 - 1) + W_{1x}\sin\beta_3 + Z_{1y}(\cos\alpha_3 - 1) - Z_{1x}\sin\alpha_3 &= p_{31}\sin\delta_3
\end{aligned} \tag{13}$$

The above system of equations can easily be solved with the help of MATLAB as presented in Fig. 6. After computing the values of W_{1x}, W_{1y}, Z_{1x} and Z_{1y}, we use (8) to compute w, θ, z and Φ.

$$\begin{aligned}
w &= \sqrt{W_{1x}^2 + W_{1y}^2} \\
\theta &= \tan^{-1}\left(\frac{W_{1y}}{W_{1x}}\right) \\
z &= \sqrt{Z_{1x}^2 + Z_{1y}^2} \\
\phi &= \tan^{-1}\left(\frac{Z_{1y}}{Z_{1x}}\right)
\end{aligned} \tag{8}$$

Writing the loop closure equations for the right dyad US and splitting them into real and imaginary parts, we obtain the following linear system of equations:

$$(\cos\gamma_2 - 1)U_{1x} + \sin\gamma_2\ U_{1y} + (\cos\alpha_2 - 1)S_{1x} + \sin\alpha_2\ S_{1y} = p_{21}\cos\delta_2$$

$$(\cos\gamma_3 - 1)U_{1x} + \sin\gamma_3\ U_{1y} + (\cos\alpha_3 - 1)S_{1x} + \sin\alpha_3\ S_{1y} = p_{31}\cos\delta_3 \quad (14)$$

$$\sin\gamma_2\ U_{1x} - (\cos\gamma_2 - 1)U_{1y} + \sin\alpha_2\ S_{1x} - (\cos\alpha_2 - 1)S_{1y} = p_{21}\sin\delta_2$$

$$\sin\gamma_3\ U_{1x} - (\cos\gamma_3 - 1)U_{1y} + \sin\alpha_3\ S_{1x} - (\cos\alpha_3 - 1)S_{1y} = p_{31}\sin\delta_3$$

Just like in the left dyad, the two free choices in this system are γ_2 and γ_3, which are the angles by which the follower moves with respect to its initial position. These values are decided in accordance with design constraints. The choice of these angles is highly dependent on β_2 and β_3 which were the free choices taken in the left dyad

And we can now use (10) to calculate the magnitudes and angles of $\mathbf{U_1}$ and $\mathbf{S_1}$.

$$u = \sqrt{U_{1x}^2 + U_{1y}^2}$$

$$\sigma = \tan^{-1}\left(\frac{U_{1y}}{U_{1x}}\right) \quad (10)$$

$$s = \sqrt{S_{1x}^2 + S_{1y}^2}$$

$$\psi = \tan^{-1}\left(\frac{S_{1y}}{S_{1x}}\right)$$

We have derived the link lengths again specifically for the mechanism of the Boeing 777 flap because of the change in orientation of links compared to the generic four-bar mechanism presented in section i.. It was observed that using the same equations that were derived in section i. produced erroneous link lengths that did not match with the data obtained from the original design given in [7]. These equations were derived once again to solve this problem.

The Fig. 6 below shows a schematic of the flap mechanism used in a Boeing 777 aircraft wherein the three different positions have been constructed by only knowing the position of the three precision points, i.e. p_{21}, p_{31}, δ_2 and δ_3. Moreover, the values given to the free choices (β_2, β_3, γ_2 and γ_3) are based on the design constraint discussed previously. Note that the assignment of all the above values has been done in accordance with data given in [7] and [8].

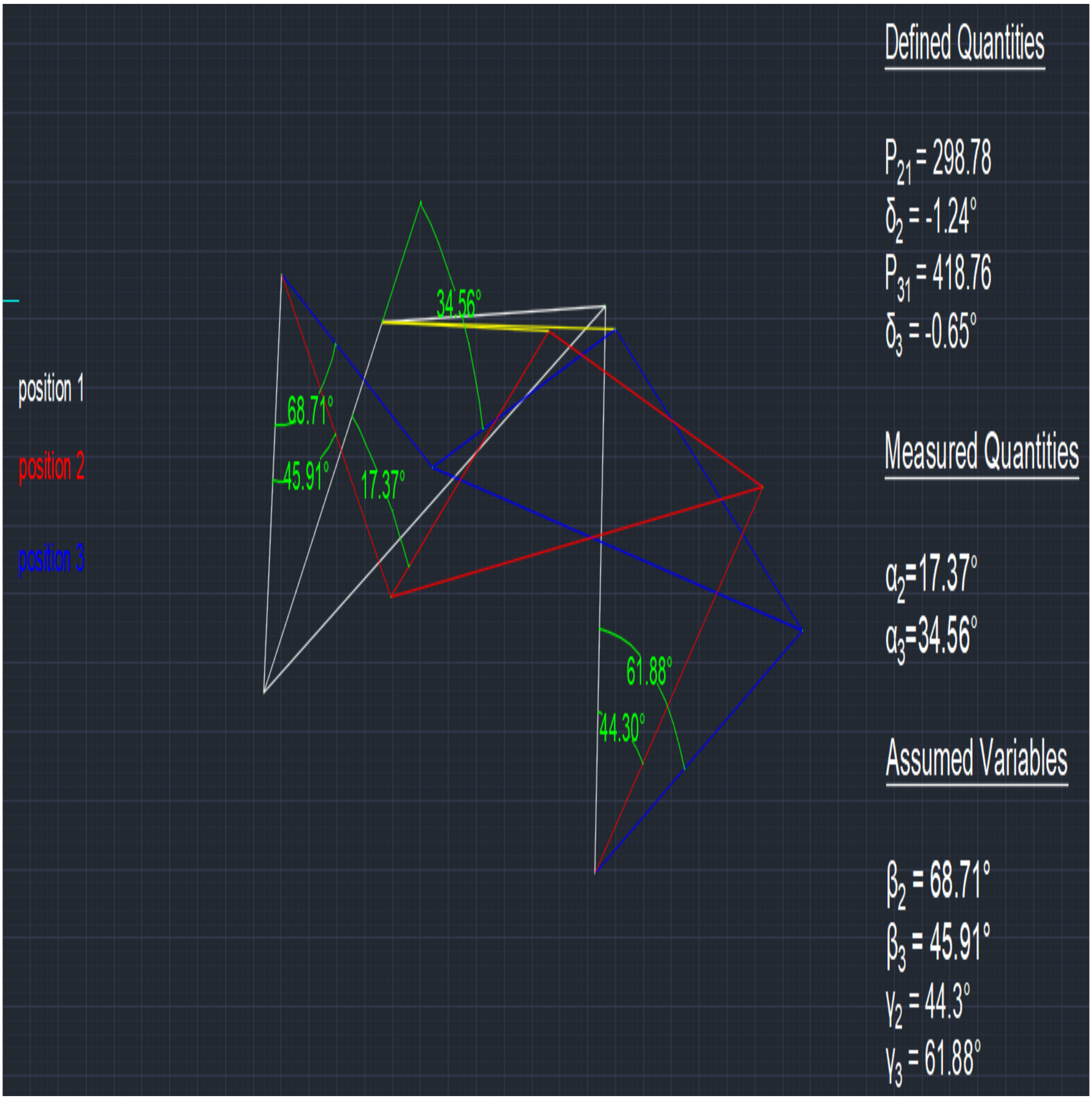

Fig. 5: Mechanism Schematic used in a Boeing 777 showing all variables
Adapted from: Lima, D. Z., Aguiar, J. B., & Ferreira, W.G., *Preliminary structural design of a Fowler flap high-lifting device,* 2021

Using the equations (8), (10), (13) and (14) and the data obtained from Fig. 5, we present a MATLAB code in Fig. 6 below that calculates the link lengths of the four-bar mechanism responsible for the deployment of the single slotted Fowler Flap used in a Boeing 777 aircraft.

```
a2=17.34;
a3=34.56;
p21=298.78;
p31=418.76;
d2=-1.24;
d3=-0.65;
b2=45.91;
b3=68.71;
g2=44.3;
g3=61.88;
A=cosd(b2)-1;
B=sind(b2);
C=cosd(a2)-1;
D=sind(a2);
E=cosd(b3)-1;
F=sind(b3);
G=cosd(a3)-1;
H=sind(a3);
mat1=[A B C D; E F G H; B -A -D C; F -E -H G];
R=p21*cosd(d2);
X=p31*cosd(d3);
Y=p21*sind(d2);
N=p31*sind(d3);
mat2=[R; X; Y; N];
R1=p21*cosd(-d2);
X1=p31*cosd(-d3);
Y1=p21*sind(-d2);
N1=p31*sind(-d3);
mat4=[R1; X1; Y1; N1];
I=cosd(g2)-1;
J=sind(g2);
K=cosd(a2)-1;
L=sind(a2);
M=cosd(g3)-1;
O=sind(g3);
P=cosd(a3)-1;
Q=sind(a3);
mat3=[I J K L; M O P Q; J -I L -K; O -M Q -P];
answer1=linsolve(mat1,mat2);
answer2=linsolve(mat3,mat4);
w=(answer1(1,1)^2+answer1(2,1)^2)^0.5
z=(answer1(3,1)^2+answer1(4,1)^2)^0.5
u=(answer2(1,1)^2+answer2(2,1)^2)^0.5
s=(answer2(3,1)^2+answer2(4,1)^2)^0.5
vx=answer1(3,1)-answer2(3,1);
vy=answer1(4,1)-answer2(4,1);
v=(vx^2+vy^2)^0.5
```

```
w = 306.0474
z = 342.9067
u = 410.2031
s = 374.3290

v = 651.6037
```

Fig. 6: MATLAB code that computes link lengths of the B777 flap mechanism

Just like the example mechanism, we present Table 2 that compares the synthesized link lengths calculated from the above MATLAB code with that given in the original design, obtained from [7] and [8].

Link	Actual Lengths (mm)	Synthesized Lengths (mm)	% Error
W	305.56	306.0474	0.16
Z	343.54	342.9067	-0.18
U	413.15	410.2031	-0.71
S	400.94	374.329	-6.63
V	673.98	651.6037	-3.32

Table 2: Comparison between synthesized and actual link lengths for B777 flap mechanism

From Table 2, most of the synthesized dimensions closely resemble the actual lengths used in the Boeing 777 flap actuation mechanism, with deviations remaining within acceptable boundaries. The error in most links is below 1% magnitude wise, which proves the effectiveness of the kinematic synthesis approach to calculate the link lengths.

We now visualize the synthesized mechanism using the MotionGen software to ensure smooth Fowler motion is carried out by the point P of the ternary coupler that is attached to the flap airfoil. The links are constrained to move along the blue curves shown in the below figure.

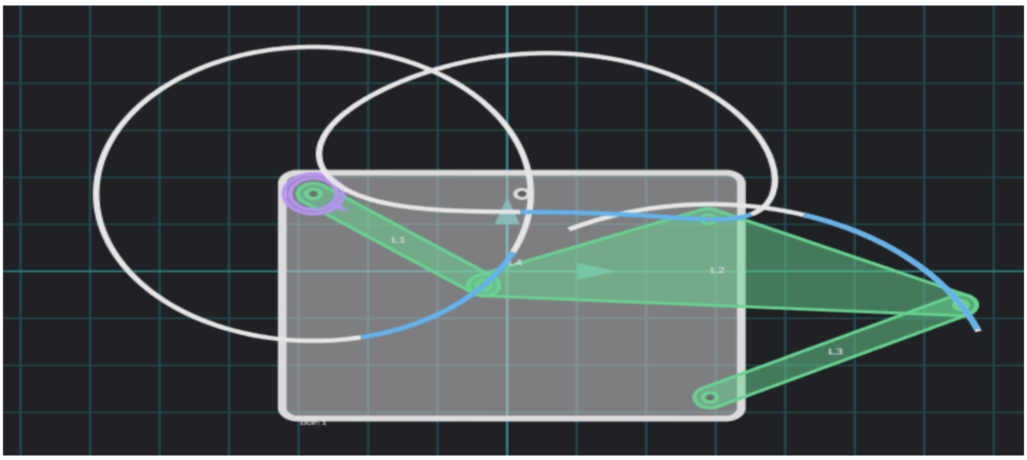

Fig. 7: Visualizing the B777 Flap mechanism

CONCLUSION

In this report, we have derived all the equations necessary for the kinematic synthesis of a generic four-bar mechanism by motion generation through three precision points. These equations were implemented and validated with a standard example mechanism using a MATLAB program. Most importantly, the kinematic synthesis approach was used and applied to calculate the link lengths of the four-bar mechanism that is used in the deployment of single slotted Fowler Flaps used in Boeing 777 aircraft.

To reduce the dimensional error arising in the synthesized link lengths, we can increase the number of precision points the coupler has to pass through to ensure Fowler motion. In such a case, the point P follows a smooth, continuous trajectory, thereby giving more accurate link lengths.

Another concept that can be explored in the future is to attempt automatically synthesize the appropriate mechanism for the single slotted Fowler Flap provided the wing and flap dimensions. This would involve parameterizing all the link lengths to adapt according to the dimensions provided whilst also obeying the kinematic equations derived.

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GLOSSARY

Four-bar mechanism – A basic planar linkage consisting of four rigid bars connected by joints, used to produce desired motion.

Coupler link – The moving link that connects the input and output links in a four-bar mechanism.

Kinematic Synthesis- Kinematic synthesis refers to the process of designing the geometry of a mechanism to achieve a desired motion. It involves selecting the type of mechanism, determining the number of links, and defining the link dimensions to meet specific motion requirements.

High Lift Devices- High lift devices are components on an aircraft wing designed to increase lift, during takeoff and landing, by altering the wing's camber and effective surfaces area. HLDs include flaps and slats.

Precision points – Specific positions through which a mechanism must pass during its motion.