AM5630 Foundations of Computational Fluid Dynamics

Assignment 2

Study of the Convection-Diffusion equation in 2-Dimensions

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Chapter 1

Hybrid Differencing Scheme

The hybrid differencing scheme [1] combines both the central and upwind differencing schemes. The second-order accurate central differencing scheme is used for small Peclet numbers ($Pe < 2$) and upwind scheme is used for $Pe >= 2$. The Peclet number is defined as the ratio of convection to diffusion, and is evaluated for a face (say, west) as:

$$Pe_w = \frac{(\rho u)_w}{\frac{\Gamma_w}{\delta x_{WP}}}$$

The neighbour coefficients for hybrid differencing scheme can be expressed as:

$$a_W = max\left[F_w, (D_w + F_w f_{xw}), 0\right]$$
$$a_E = max\left[-F_e, (D_e - F_e f_{xe}), 0\right]$$
$$a_S = max\left[F_s, (D_s + F_s f_{y_s}), 0\right]$$
$$a_N = max\left[-F_n, (D_n - F_n f_{y_n}), 0\right]$$
$$a_P = a_W + a_E + a_N + a_S + (F_e - F_w + F_n - F_s)$$

where $F_a = (\rho u A)_a$, $D_a = \frac{(\Gamma A)_a}{\delta x_{AP}}$, $f_{xa} = \frac{0.5\Delta x}{\delta x_{AP}}$ is the interpolation factor for face a and adjacent grid point A and P.

These coefficients can be used to express the discretized equation [2]:

$$a_P T_P = a_E T_E + a_W T_W + a_N T_N + a_S T_S + S_u$$

where S_u is source term.

Chapter 2

Problem Statement

Solve the 2-dimensional convection-diffusion equation on the following domain:

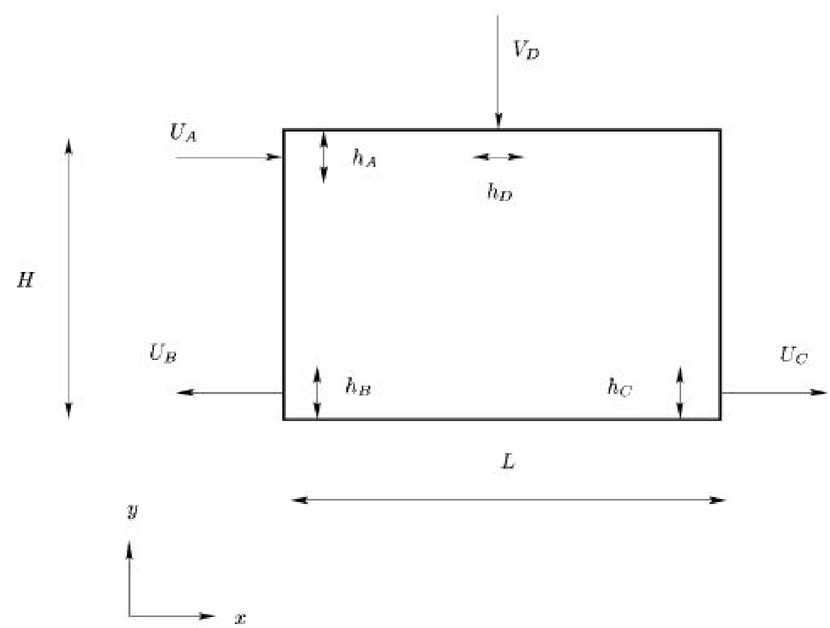

Figure 2.1: Integration domain. The extent in the third coordinate direction is 1. Physical data: $\rho = 1$; $k/cp = 1/500$; $hA/H = hC/H = 0.068$. Boundary conditions: $U_A = 1$; $U_B = 0$; $U_C = 1$; $V_D = 0$; $T_A = 20°C$. At $x = L$; $T = 10°C$.

$\partial T/\partial \eta = 0$ (where, η denotes the coordinate normal to the wall) is to be used at walls if no boundary condition is given.

To determine the convergence, we use the following criterion:

$$\epsilon = \frac{1}{f}\left(\sum_{allcells} |a_E T_{i+1,j} + a_W T_{i-1,j} + a_N T_{i,j+1} + a_S T_{i,j-1} + S - a_P T_{i,j}|\right) < 0.001$$

where $f = (\rho U h)_A \Delta T$, ($\Delta T$ is temperature difference between inlet and outlet) is temperature flux used to normalize the residual.

Also verify:

1. Sensitivity to boundary conditions. Make (an interesting) change of one boundary condition.
2. Sensitivity to convergence. Does the temperature change if you use another convergence criterion? Say $\epsilon = 0.01$ and $\epsilon = 0.0001$.
3. Check if you have global conservation, i.e. does the heat flux through all boundaries (inlet, outlet and walls) sum up to zero (as it should if you don't have any source term).

Chapter 3

Results

The mesh used for this assignment is shown in Figure 3.1

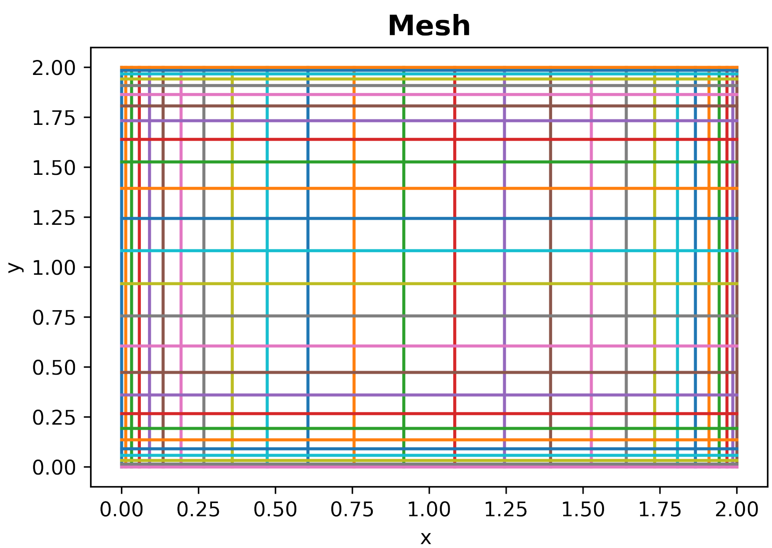

Figure 3.1: Mesh used for this assignment

The velocity field prescribed in the question is given in Figure 3.2

First, we solve for the temperature distribution with given boundary conditions. We use the Gauss-Seidel scheme to iteratively solve the set of discretized equations. The result is shown in Figure 3.3.

Here, the flow is predominantly convective. This is the reason why the temperature of the bulk of the domain is same as that of inlet. Since the wall on the right has a constant temperature lower than that of the inlet flow, the temperature near this wall suddenly decreases. The upwind scheme is used for most parts of the domain, since the flow is majorly convective. Near the

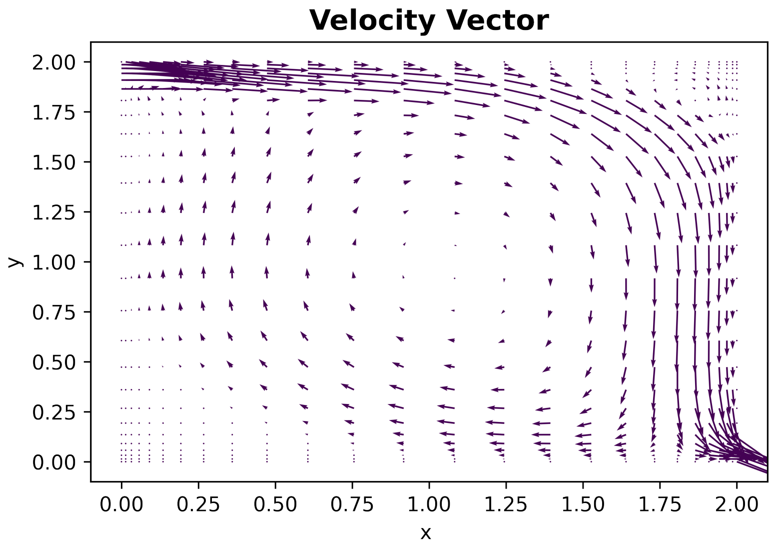

Figure 3.2: Velocity field vector plot

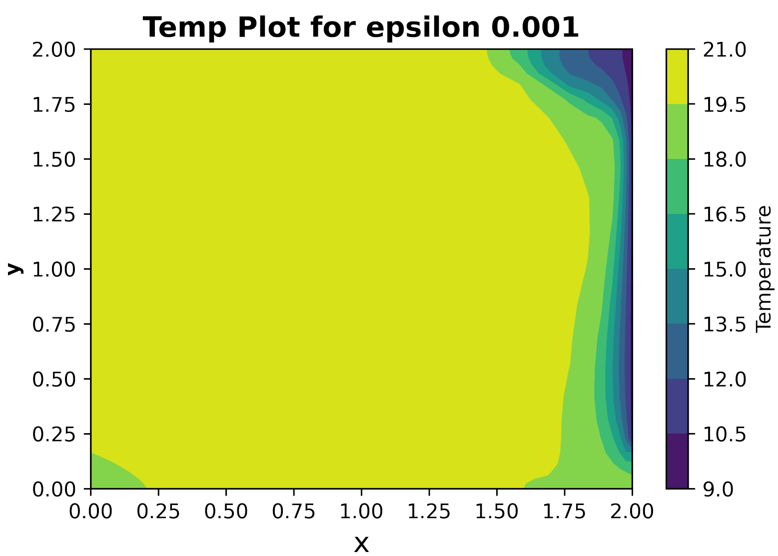

Figure 3.3: Temperature contour plot

right wall, however, CD scheme is used due to predominance of diffusion due to the constant temperature at the wall.

We also investigate the effect of boundary condition changes on the temperature distribution. The boundary conditions are changed as follows:

1. The top wall boundary condition is changed from $\partial T/\partial \eta = 0$ to $T = 0$.

Contour plots are shown in Figure 3.4 for this case. Here we can see that,

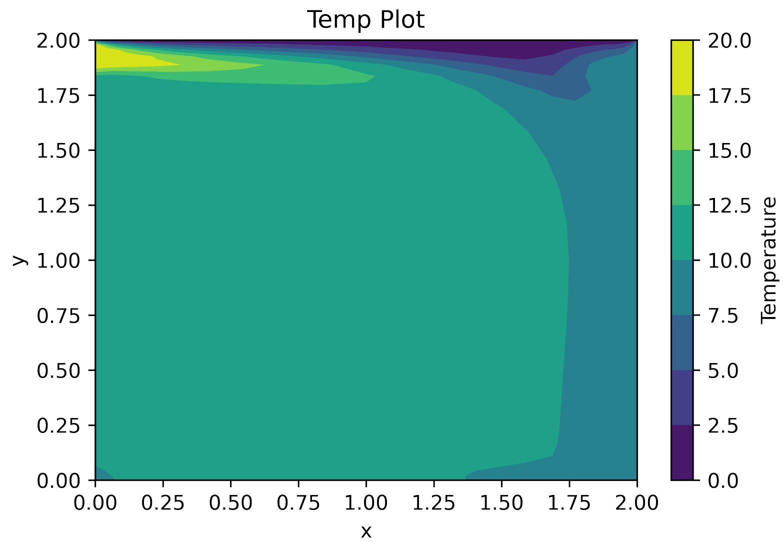

Figure 3.4: Temperature contour plot for $T = 0$ on upper wall

even though the convection of heat flow from the inlet is predominant, the lower temperatures on the boundaries cause the overall temperature in the domain to be lower than the previous case. A plot of temperature along $y = H/2$ as a function of x is shown for $\epsilon = 0.001$ in Figure 3.5.

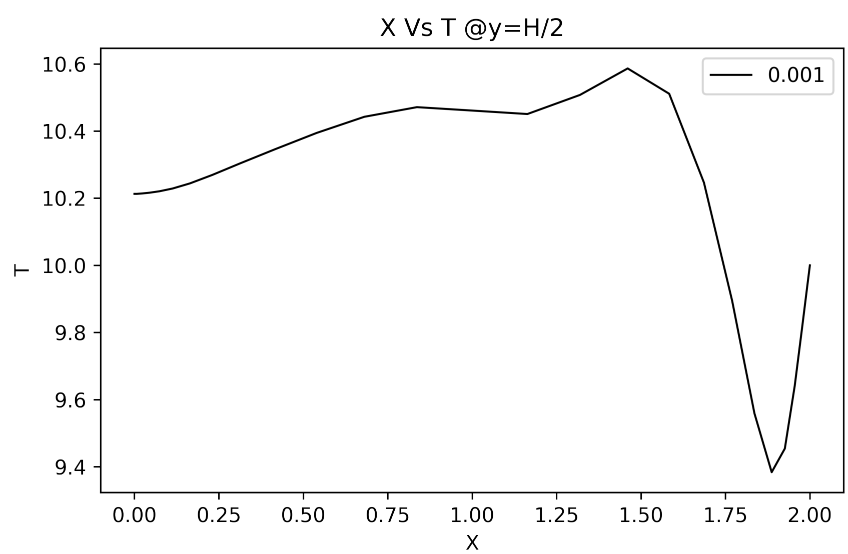

Figure 3.5: Temperature as a function of x at $y = H/2$ for $T = 0$ on upper wall

2. The temperature gradient along the left boundary (except at location of inlet) is 2, i.e., there is a non-zero heat flux into the domain. Contour plots are shown in Figure 3.6 for this case. Here, we can see that due to

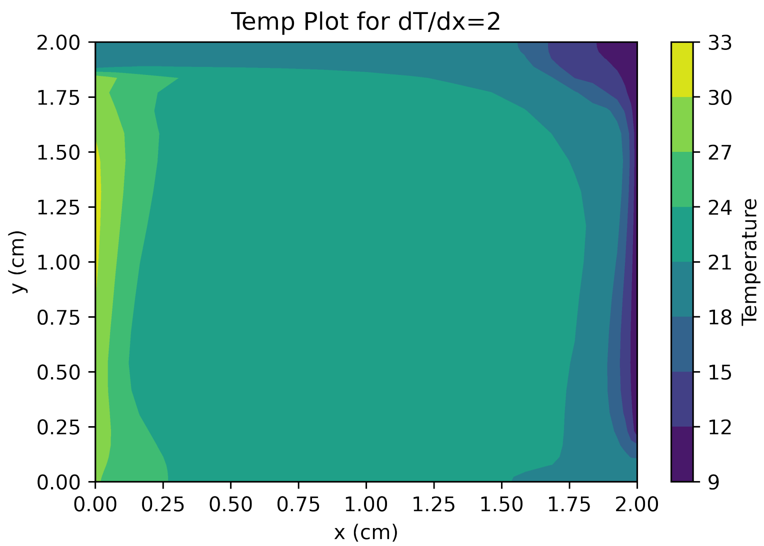

Figure 3.6: Temperature contour plot for $\partial T/\partial \eta = 2$ at left wall

the positive heat flux from the left end, the overall temperature in the domain is now greater than the case with $\partial T/\partial \eta = 0$ on the boundary. A plot of the variation of temperature with x at $y = H/2$ is shown for this case in Figure 3.7. We can observe that the temperature increases slightly due to convection of inlet temperature. But due to the cold region at the top right corner bounded by the two walls at lower temperature, the temperature decreases closer to the right wall due to convection. The temperature increases to $10°C$ again at the boundary to satisfy boundary conditions.

3. The temperature gradient along the left boundary (except at location of inlet) is -2, i.e., there is a non-zero heat flux out of the domain at left boundary. Contour plots are shown in Figure 3.8 for this case. Here, we can see that due to the negative heat flux from the left end, the overall temperature in the domain is now lesser than the case with $\partial T/\partial \eta = 0$ on the boundary. We can see that where velocity is large, due to convection from the inlet, the temperature is also increased. A plot of the variation of temperature with x at $y = H/2$ is shown for this case in Figure 3.9.

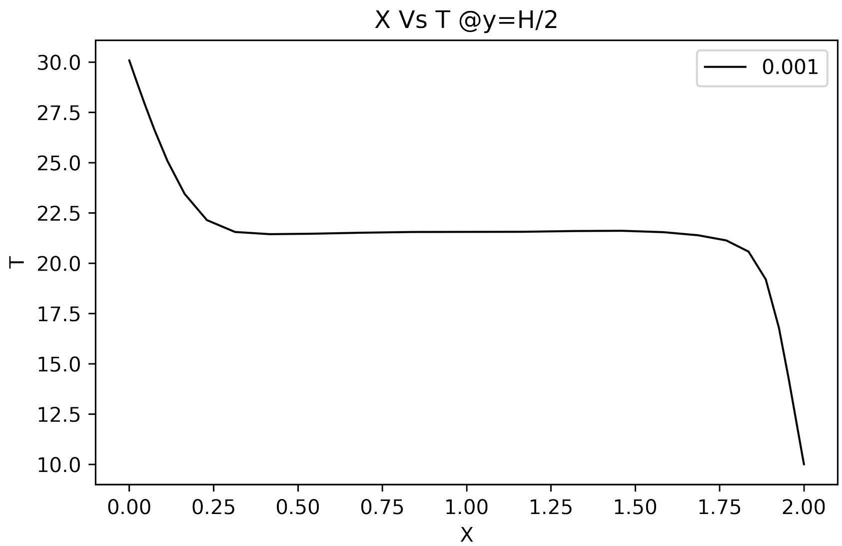

Figure 3.7: Temperature plot as a function of x at $y = H/2$ for $\partial T/\partial \eta = 2$ at left wall

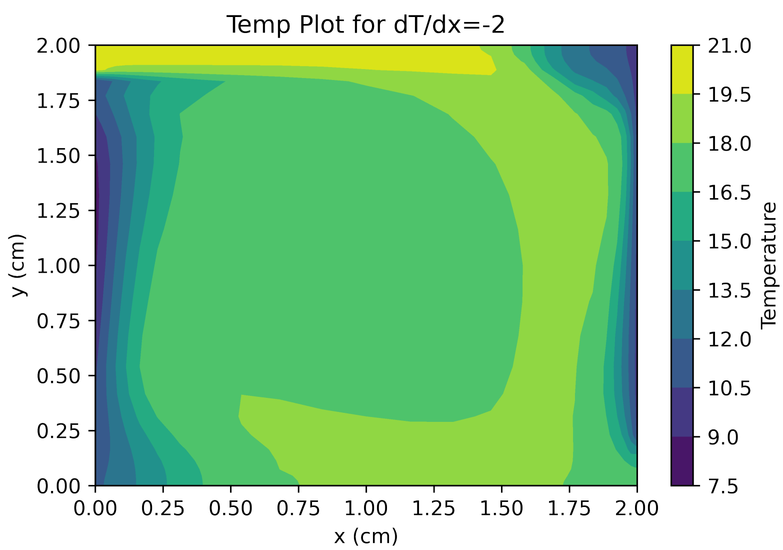

Figure 3.8: Temperature contour plot for $\partial T/\partial \eta = -2$ at left wall

Next we examine the effect of changing ϵ. The contour plots for three cases with $\epsilon = 0.01$, 0.001 and 0.0001 are shown in Figures 3.10, 3.11 and 3.12 respectively.

Here we can see that the solution has not converged for $\epsilon = 0.01$, while for $\epsilon = 0.001$ and 0.0001, it has converged, and gives similar results. This is further illustrated by plotting the temperature as a function of x at $y = H/2$,

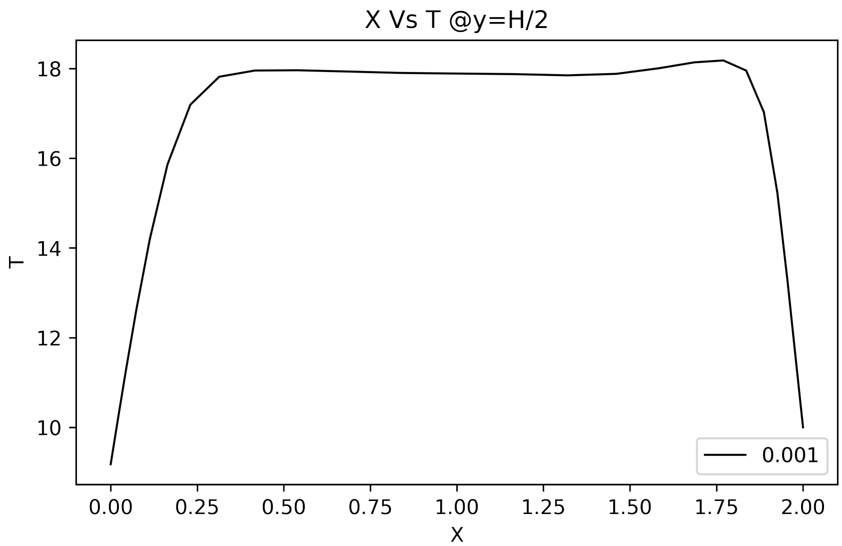

Figure 3.9: Temperature plot as a function of x at $y = H/2$ for $\partial T/\partial \eta = -2$ at left wall

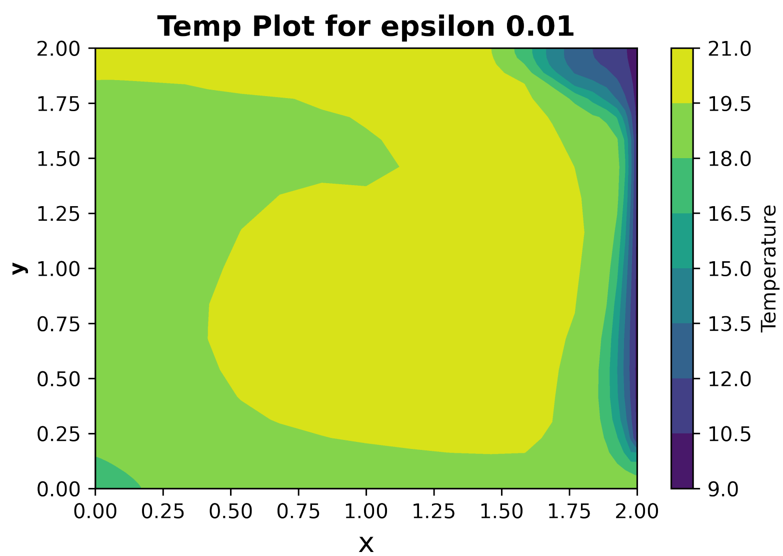

Figure 3.10: Temperature contour plot for $\epsilon = 0.01$

and plotting the residue v/s number of iterations for all three cases. The plots mentioned above can be seen in Figure 3.13 and 3.14 respectively.

Now, we look at the value of global heat flux and heat flux distribution in the domain. The contour plot of heat flux and the heat flux vector plot are shown in Figures 3.15 and 3.16.

Upon checking global conservation, even though we expect to get zero net

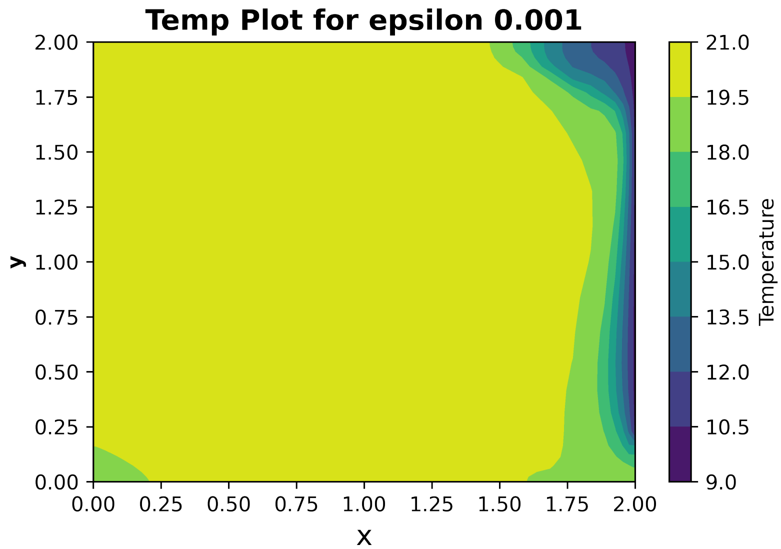

Figure 3.11: Temperature contour plot for $\epsilon = 0.001$

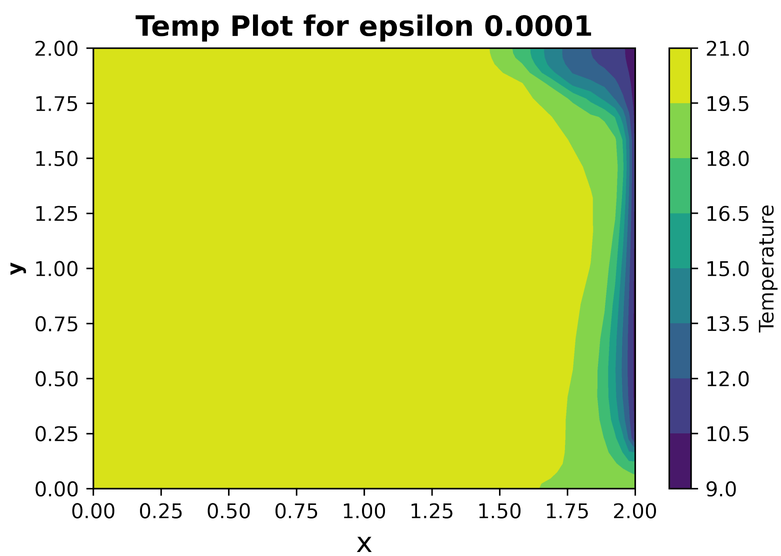

Figure 3.12: Temperature contour plot for $\epsilon = 0.0001$

flux, we get a net flux of -0.37. This could be due to numerical integration errors, which needs to be investigated further.

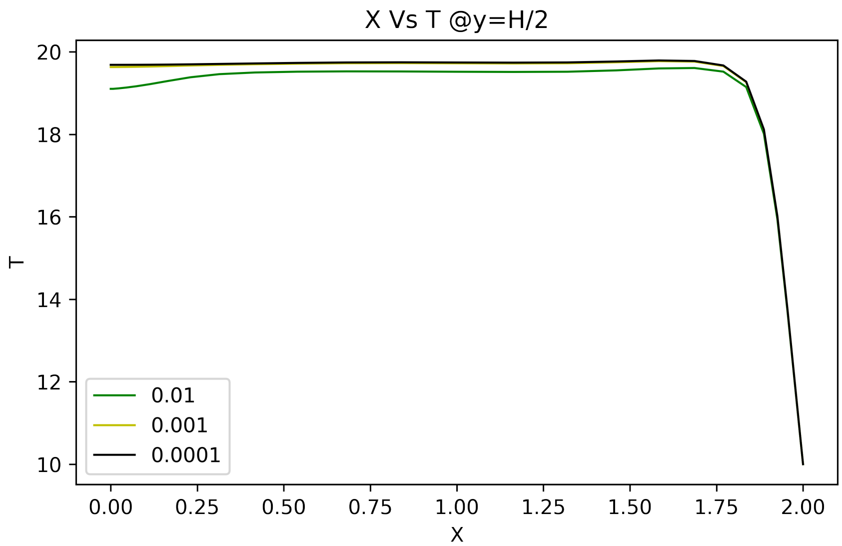

Figure 3.13: Temperature as a function of x at $y = H/2$ for various ϵ

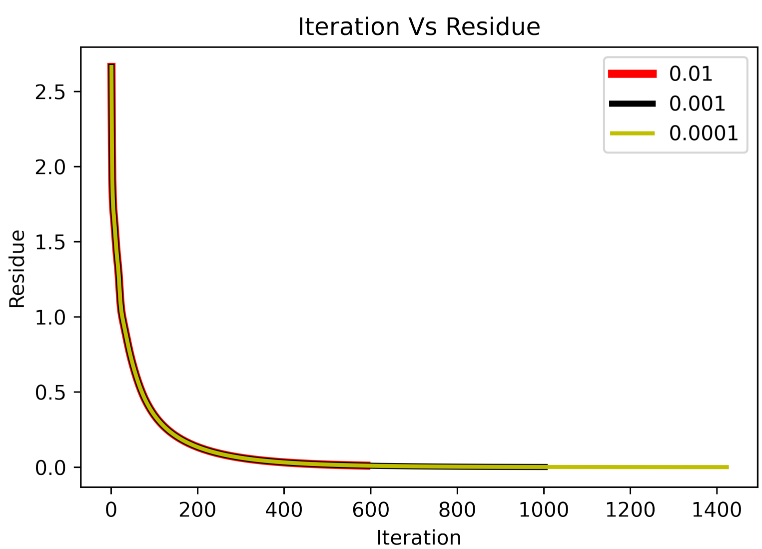

Figure 3.14: Residue v/s number of iterations for various ϵ

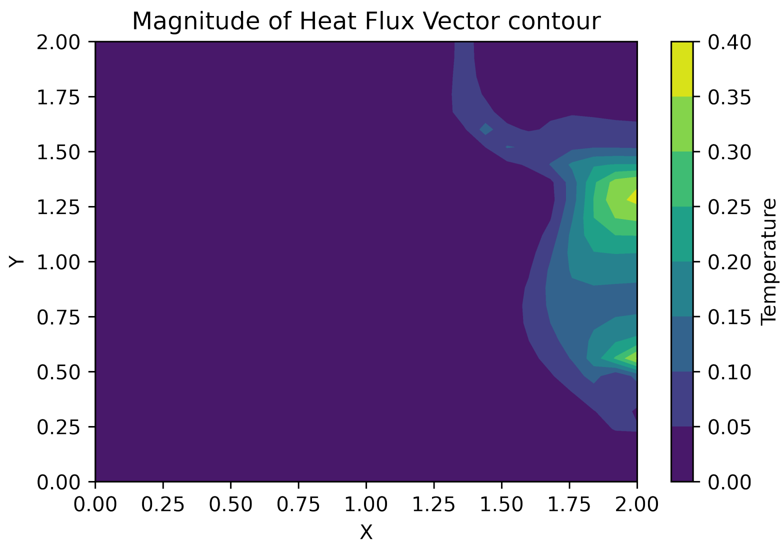

Figure 3.15: Contour plot of heat flux over the domain

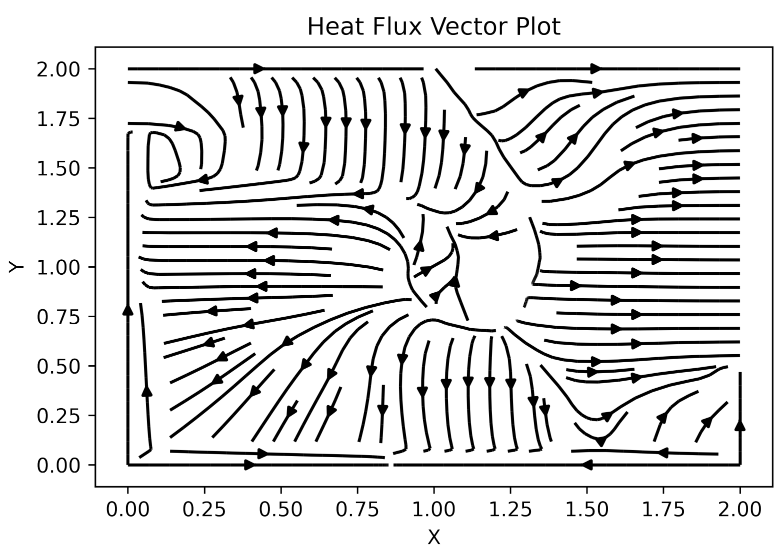

Figure 3.16: Vector plot of heat flux over the domain

Chapter 4

Conclusions

In this study, we have analyzed the problem of 2-dimensional flow with both convection and diffusion processes involved. We have discretized the equations using hybrid differencing scheme, and solved using iterative Gauss-Seidel method. The results of temperature distribution for the case with $\epsilon = 0.001$ show converged solution to be primarily dominated by convection, leading to the bulk of the domain having high temperature due to convection of the inlet flow. Near the boundary where temperature is lower, diffusion dominates, and we observe a region of lower temperature. When the top wall condition is changed to $T = 0$, even though the convection of heat flow from the inlet is predominant, the lower temperatures on the boundaries cause the overall temperature in the domain to be lower than the previous case. When we change the left boundary condition to have non-zero heat flux, we can see a corresponding increase or decrease in the overall flow temperature depending on whether there is positive or negative heat flux. When the value of ϵ is high, we see that the solution doesn't converge. When the value of ϵ is decreased to 0.001 or lower, the solution can be observed to be converged. This observation is supported by plots of temperature profile as a function of x at $y = H/2$, and residue v/s number of iterations. There is a small error in global conservation of mass, which may be due to numerical errors, and needs to be investigated further.

Bibliography

[1] H. K. Versteeg and W. Malalasekera. *An introduction to computational fluid dynamics: the finite volume method*. 2nd ed. OCLC: ocm76821177. Harlow, England ; New York: Pearson Education Ltd, 2007. ISBN: 9780131274983.

[2] Vagesh D Narasimhamurthy. *AM5630 Foundations of Computational Fluid Dynamics Course*. English. Jul - Nov 2022. AM5630 Foundations of Computational Fluid Dynamics Course AM5630. Chennai, 2022.