

Homework 12

Code:

```
1 # HW 12 Central Limit Theorem
2 # 65011304 Inthat Sappipat
3
4 # Initialize x.bar to store sample means
5 x.bar = 0
6
7 # Generate a population with a chi-squared distribution and store in variable named popu.x
8 popu.x <- rchisq(n = 700, df = 4)
9
10 # Loop to simulate drawing random samples and calculating sample means
11 for(i in 1:3000){
12   samp.x = sample(popu.x, size = 80, replace = TRUE)
13   x.bar[i] = mean(samp.x)
14 }
15
16 # plot the population distribution
17 hist(popu.x)
18
19 # plot the sampling distribution
20 hist(x.bar)
21
22 # Rule 1 proof: Compare means of sample means and population
23 mean(x.bar)
24 mean(popu.x)
25
26 # Rule 2 proof: Compare standard deviations of sample means and population
27 sd(x.bar)
28 sd(popu.x) / sqrt(80)
```

Result:

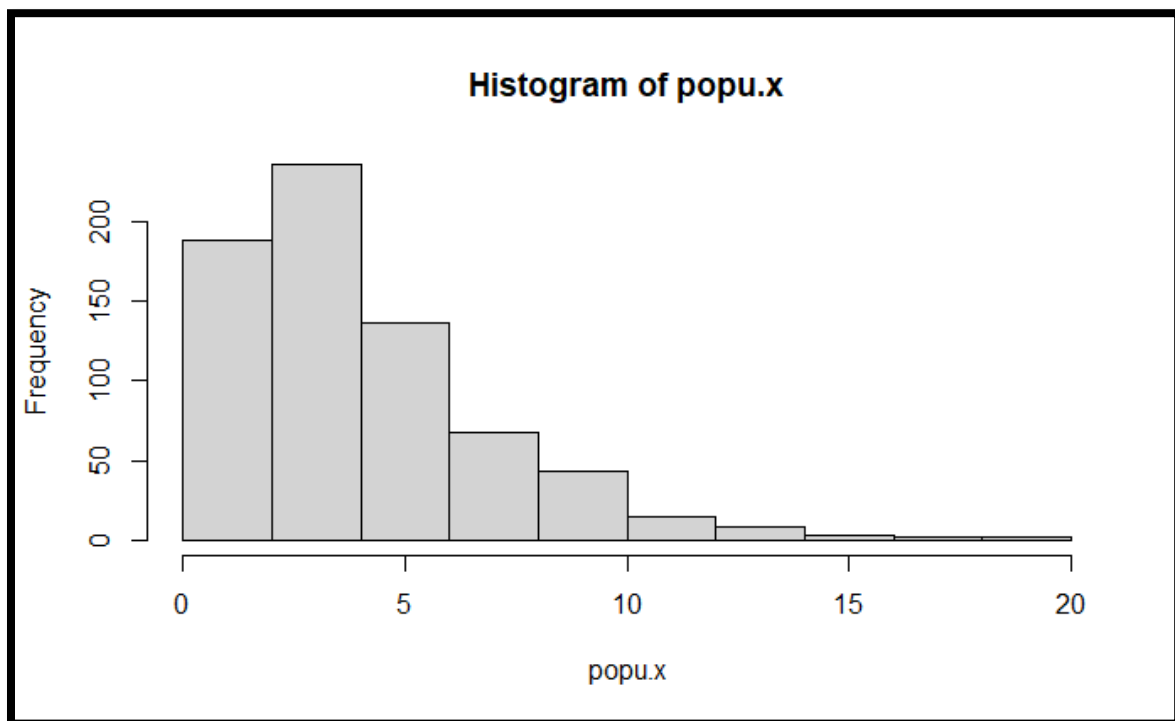
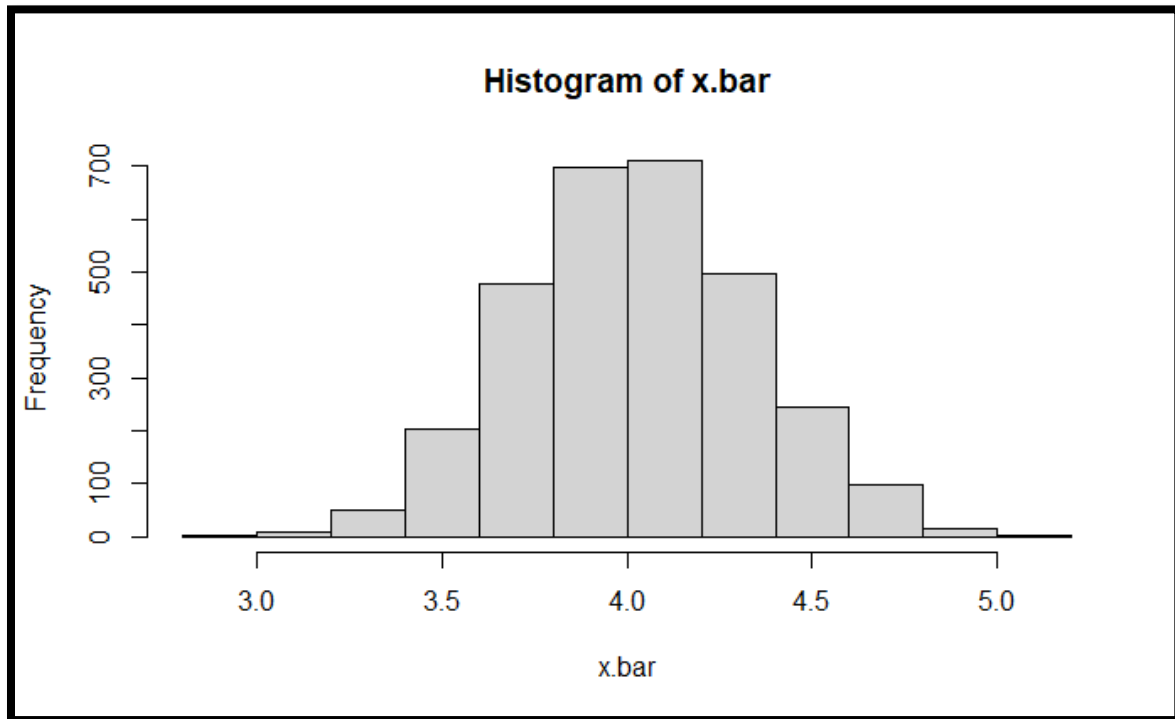
```
> # Rule 1 proof
> mean(x.bar)
[1] 4.021897

> mean(popu.x)
[1] 4.018739

> # Rule 2 proof
> sd(x.bar)
[1] 0.3149702

> sd(popu.x) / sqrt(80)
[1] 0.3101887
```

Histogram:



Conclusion:

From the experiment, in rule 1, the mean of the sample means ($\text{mean}(\bar{x})$) is close to the mean of the population ($\text{mean}(\text{popu.x})$). This is in line with the first rule of the CLT, which states that the mean of the sampling distribution approaches the mean of the population as the sample size increases. In rule 2, the standard deviation of the sample means ($\text{sd}(\bar{x})$) is close to the standard deviation of the population divided by the square root of the sample size ($\text{sd}(\text{popu.x}) / \sqrt{80}$). This is consistent with the second rule of the CLT, stating that the standard deviation of the sampling distribution decreases as the sample size increases. The histograms of the population ($\text{hist}(\text{popu.x})$) and the sampling distribution ($\text{hist}(\bar{x})$) demonstrate the transformation from a chi-squared distribution to a normal distribution. As per the CLT, the sampling distribution tends to be normal, regardless of the shape of the original population distribution.