q_1 & q_2 $\{|0\rangle_1 + |1\rangle_1\}$ $\{|0\rangle_2, |1\rangle_2\}$ be standard orthonormal basis

$|\psi\rangle_1 = a_1|0\rangle_1 + b_1|1\rangle_1$ with $|a_1|^2 + |b_1|^2 = 1$

$|\psi\rangle_2 = a_2|0\rangle_2 + b_2|1\rangle_2$ with $|a_2|^2 + |b_2|^2 = 1$

$|0\rangle_1 \otimes |0\rangle_2$, $|0\rangle \otimes |1\rangle_2$, $|1\rangle_1 \otimes |0\rangle_2$, $|1\rangle_1 \otimes |1\rangle_2$

are a basis for combined state space $\mathbb{C}^2 \otimes \mathbb{C}^2$ for q_1 & q_2

$$|\psi\rangle_1 \otimes |\psi\rangle_2 = a_1 a_2 |0\rangle_1 \otimes |0\rangle_2 + a_1 b_2 |0\rangle_1 \otimes |1\rangle_2 + b_1 a_2 |1\rangle_1 \otimes |0\rangle_2 + b_1 b_2 |1\rangle_1 \otimes |1\rangle_2$$

$$= a_1 a_2 |0\rangle_1 |0\rangle_2 + a_1 b_2 |0\rangle_1 |1\rangle_2 + b_1 a_2 |1\rangle_1 |0\rangle_2 + b_1 b_2 |1\rangle_1 |1\rangle_2$$

$$= a_1 a_2 |00\rangle + a_1 b_2 |01\rangle + b_1 a_2 |10\rangle + b_1 b_2 |11\rangle$$

$|00\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ $|01\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ $|10\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ $|11\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$|00\rangle = |0\rangle \otimes |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ 0 \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

ℓ

$$|01\rangle = |0\rangle \otimes |1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ 0 \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$|10\rangle = |1\rangle \otimes |0\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ 1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$|11\rangle = |1\rangle \otimes |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ 1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Matrix for CNOT

$$\begin{array}{cccc} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \\ |00\rangle \quad |01\rangle \quad |10\rangle \quad |11\rangle \end{array}$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

	Qiskit
$CNOT\,|00\rangle = |00\rangle$	$CNOT\,|00\rangle = |00\rangle$
$CNOT\,|01\rangle = |01\rangle$	$CNOT\,|01\rangle = |11\rangle$
$CNOT\,|10\rangle = |11\rangle$	$CNOT\,|10\rangle = |10\rangle$
$CNOT\,|11\rangle = |10\rangle$	$CNOT\,|11\rangle = |01\rangle$