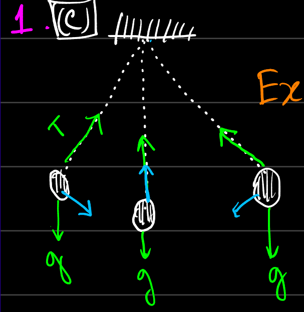
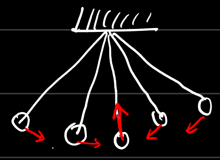
#GR0177

1. $\boxed{C}$



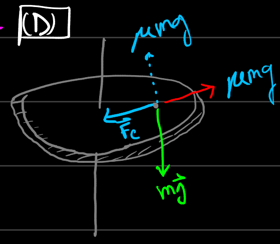

The total Force acting on the bob is $\vec{F} = \vec{T} + m\vec{g}$

Explicity draw the resultant force at all extreme points.



which gives c.

2. $\boxed{D}$




Applying centripetal force law gives

$$\frac{\cancel{m}v^2}{r} = \mu \cancel{m} g \Rightarrow \frac{\omega^2 r^{\cancel{2}\, r}}{\cancel{r}} = \mu g$$

$$\Rightarrow r = \frac{\mu g}{\omega^2}$$

We are given $f = 33.3\,rpm$

$$= \frac{33.3}{60}\,Hz.$$

And $\omega = 2\pi f = \left(2\pi \times \frac{33.3}{60}\right)$

$$= \frac{0.3 \times 10}{4\pi^2 \left(\frac{33.3}{60}\right)^2}$$

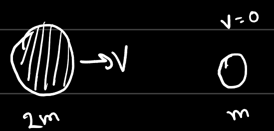
$$\Rightarrow r = \frac{3}{\cancel{4} \times 10 \times \left(\frac{1}{\cancel{2}}\right)^2} \approx 0.3\,m$$

(D) 0.242 m is the closest.

3. $\boxed{D}$ Kepler's 3rd law $\Rightarrow$ The time period T of a planet squared is proportional to the cube of the semi-major axis.

$$T^2 \propto r^3 \Rightarrow T \propto r^{3/2}$$

4. $\boxed{C}$




Momentum conservation

$$2\cancel{m}V = 2\cancel{m}v + \cancel{m}v$$

$$\Rightarrow 2V = 3v$$

$$\Rightarrow v = \frac{2V}{3}$$

Initially, $K.E_i = \frac{1}{\cancel{2}} \cdot \cancel{2} mV^2 = mV^2$

Finally, $K.E_f = \frac{1}{2} m \cdot v^2 + \frac{1}{2} \cdot 2mv^2$

$$= \frac{1}{2} m \left(\frac{2V}{3}\right)^2 + m\left(\frac{2V}{3}\right)^2$$

$$= \frac{\cancel{3}m}{\cancel{2}} \cdot \frac{\cancel{4}^{\,2} V^2}{\cancel{9}_{\,3}} = \frac{2mV^2}{3}$$

Kinetic energy lost, $\Delta E = \left(m - \frac{2m}{3}\right)V^2$

$$= \frac{m}{3} V^2 = \boxed{\frac{1}{3} E_i}$$

5. $\boxed{D}$ 3D harmonic oscillator Hamiltonian:

$$H = \left(\frac{P_x^2}{2m} + \frac{P_y^2}{2m} + \frac{P_z^2}{2m}\right) + \frac{1}{2} m\omega^2 (x^2 + y^2 + z^2)$$

Total 6 quadratic terms. So, each term will recieve a $\frac{1}{2}kT$ contribution to the energy. So, $E = \cancel{6}^{\,3} \cdot \frac{1}{\cancel{2}} kT = 3kT$.

Each quadratic term in the Hamiltonian recieves a contribution of $\frac{1}{2}kT$ to the average energy of a system.

6. $\boxed{(E)}$ We are going to use → two facts to solve this problem.

1. Work done $W = \int P\,dV \Rightarrow$ Area under the P-V curve.
2. The P-V curve for adiabatic processes are more steeper than the P-V curve for isothermal processes.

So, in general, $W_i$ will be greater than $W_a$. $\boxed{0 < W_a < W_i}$

7. $\boxed{(B)}$ Both are N-pole. So they will repel one another.

8. $\boxed{(D)}$ Total charge induced on the plate is equal to the image charge $-Q$.

9. $\boxed{(A)}$ Symmetrically arranged charge have zero field at the center.

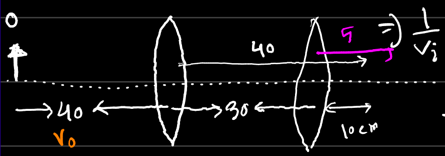
10. $\boxed{(A)}$ $C_1 = 3\times10^{-6}\,F$ & $C_2 = 6\times10^{-6}\,F$ ; $V = 300V$

The capacitors are connected in series. So, $C = \frac{C_1C_2}{C_1+C_2} = \frac{\cancel{18}^{2}\times10^{-12}}{\cancel{9}\times10^{-6}} = 2\times10^{-6}\,F$

Total stored energy, $U = \frac{1}{2}CV^2 = \frac{1}{\cancel{2}}\times\cancel{2}\times10^{-6}\times(300)^2 = 10^{-6}\times9\times10^{4} = 0.09J$

11. $\boxed{(A)}$ Multiple lenses: $\frac{1}{v_o}+\frac{1}{v_i}=\frac{1}{f}$

$v_o = +40$ cm
$f = +20$ cm
$v_i = ?$




$\Rightarrow \frac{1}{v_i} = \frac{1}{f} - \frac{1}{v_o}$

$= \frac{1}{20} - \frac{1}{40}$

$= \frac{2-1}{40} = \frac{1}{40}$cm

$\Rightarrow v_i = +40$ cm

For the second lens, the object (image formed by the first lens) is behind the mirror. So,

$v_o = -10$cm
$f = 10$ cm

$\frac{1}{v_o}+\frac{1}{v_i}=\frac{1}{f} \Rightarrow \frac{1}{v_i} = \frac{1}{f} - \frac{1}{v_o}$

$= \frac{1}{10} + \frac{1}{10}$

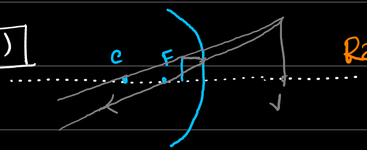
$= \frac{1}{5}$

$\Rightarrow \boxed{v_i = 5cm}$

So, the final image is 5cm on the right of the second lens.

* Convex lens & concave mirrors always have POSITIVE focal length.
* $v_i$ is POSITIVE when image is on the opposite side of the object.

12. $\boxed{(E)}$




Ray tracing / Ray diagrams.

13. $\boxed{(B)}$ Rayleigh criterion: $\boxed{D\sin\theta = 1.22\lambda}$

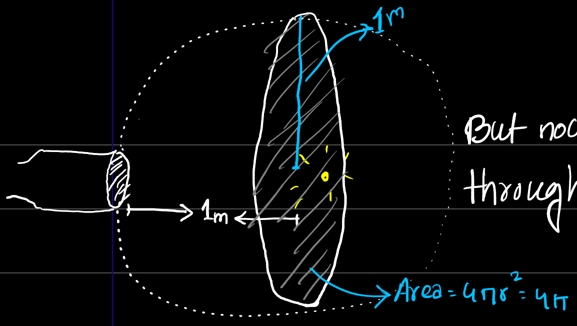
Given, $\theta = 3\times10^{-5}$ rad. So, $\sin\theta \approx \theta$
& $\lambda = 600$ nm $= 6\times10^{-7}$ m.

$\Rightarrow D\theta = 1.22\lambda$

$\Rightarrow D = \frac{1.22\times\cancel{6}^{2}\times\cancel{10^{-7}}}{\cancel{3}\times\cancel{10^{-7}}} \approx 2.5$cm.

14. The NaI/Tl detector, when pressed right next to the radioactive source, recieves photons directly through the surface area of $\pi r^2 \Rightarrow \pi\frac{d^2}{4}$ [d = diameter of the detector)

$\Rightarrow \pi\left(\frac{64\,cm^2}{4}\right) = 16\pi\,cm^2$




But now, the source is dispersing the photons spherically through a circular cross section of $4\pi(100\text{ cm})^2 = 40{,}000\,\pi\text{ cm}^2$

So, initially, $16\pi\text{ cm}^2$. When 1 m away, $40{,}000\pi\text{ cm}^2$

$$\frac{\cancel{16}^{4}\cancel{\pi}}{\cancel{4}\times 10^4\,\cancel{\pi}}\text{ cm}^2 = 4\times 10^{-4}$$

15. $\boxed{(A)}$ Basic lab methods fact: Most precise measurement $\Rightarrow$ Least spread/peaked around a value.
Most accurate measurement $\Rightarrow$ Closest to the true value.

16. Poisson's statistics: Radioactive decays are Poisson's process $\Rightarrow$ the number of counts $N$ in a given time interval $t$ is Poisson-distributed with mean $\boxed{\mu = rt}$
$r$ is the true mean count per unit time interval.

We can approximate $r$ by taking $r = \frac{N}{t}$. The uncertainty in $r$ is then

$$\sigma_r = \frac{\sigma_N}{t} = \frac{\sqrt{\mu}}{t} = \frac{\sqrt{rt}}{t} = \sqrt{\frac{r}{t}} = \frac{1}{\sqrt{N}}$$

Mean $= \mu$
Variance $\sigma_N^2 = \mu$
$\Rightarrow \sigma_N = \sqrt{\mu}$

Fractional uncertainty, $\frac{\sigma_r}{r} = \frac{1}{\sqrt{N}} \Rightarrow \sigma_r = \frac{r}{\sqrt{N}}$

Total counts in 10 s $\to N = 3+0+2+1+2+4+0+1+2+5 = 20$

Sample mean rate, $r = \frac{N}{t} = \frac{20}{10} = 2$

Current relative uncertainty, $\frac{\sigma_r}{r} = \frac{1}{\sqrt{N}} \times 100\% = 22.4\%$

We want this to be 1%. So, $\frac{\sigma_r}{r} = 0.01 = \frac{1}{\sqrt{N}}$

$$\Rightarrow N = \frac{1}{(10^{-2})^2} = 10^4$$

$$t = \frac{N}{r} = \frac{10000}{2} = 5000\text{ s}.$$

More concisely

Total counts, $N = 20$ time $= 10$, sample mean, $r = \frac{N}{t} = 2$

Relative uncertainty $\frac{\sigma_r}{r} = \frac{1}{\sqrt{N}}$ This should be 0.01

So, $N = 10^4$. and $t = \frac{N}{r} = \frac{10000}{2} = 5000$

17. $\boxed{(B)}$ Ground state electron configuration of Phosphorous ($Z=15$)

$1s^2 2s^2 2p^6 3s^2 3p^3 \cancel{3d} \longrightarrow 1s^2 2s^2 2p^6 3s^2 3p^3$

Screening effect doesn't take place until $Z=18$ (Argon)

18. [(A)] For Helium atom ground state, there are two electrons. Removing the first electron requires less work (due to the Coulomb repulsion of the remaining electron). If removing both electrons require 79.0eV, then removing the first one would require less than half of it obviously. The only such option is (A) 24.6eV

19. [(B)] Sun's fusion reaction => It takes 4 hydrogen atoms to create Helium nucleus.

20. [(E)] "Bremstrahlung" => Energy loss of deccelerating electrons in the form of X-rays. So, (E)

21. [(B)] Lyman-α $[n_i = 2; n_f = 1] \Rightarrow$ $\frac{1}{\lambda_L} = R\left(1 - \frac{1}{4}\right) \Rightarrow \lambda_L = \frac{4}{3R}$

Balmer-α $[n_i = 3; n_f = 2] \Rightarrow$ $\frac{1}{\lambda_B} = R\left(\frac{1}{4} - \frac{1}{9}\right) = R\left(\frac{9-4}{36}\right) \Rightarrow \lambda_B = \frac{36}{5R}$

So, $\frac{\lambda_L}{\lambda_B} = \frac{4}{3\cancel{R}} \times \frac{5\cancel{R}}{\cancel{36}_{9}} = \frac{5}{27}$ [Repeat question]

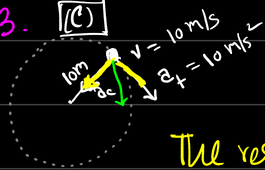
22. [(A)] Only the mass of the moving object gets cancelled out everywhere. Here, we already know r & v.

Centripetal Force formula. $\frac{\cancel{m}v^2}{\cancel{r}} = \frac{GM\cancel{m}}{\cancel{r^2}_{r}}$

$\Rightarrow v^2 = \frac{GM}{r}$

We can determine the planet's mass M, minimum speed v, time period $T^2 \propto r^3$. One thing we cannot calculate is the moon's mass.

23. [(C)]




There's a centripetal acceleration $\vec{a}_c$ & a tangential acceleration $\vec{a}_t$.

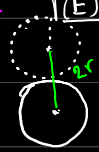
$a_c = \frac{v^2}{r} = \frac{10^2}{10} = 10\,m/s^2$ $\quad \vec{a}_c = \vec{a}_t = 10\,m/s^2$

The resultant acceleration vector $\boxed{\vec{a} = \vec{a}_c + \vec{a}_t}$ Velocity $\vec{v}$ points radially. $\vec{v} \parallel \vec{a}_t$. As $\vec{a}_t = \vec{a}_c$, the angle between them will be 45°.

24. [(C)] $\boxed{V_x \text{ vs } t}$ No acceleration along x-direction. So, it's uniform velocity. II

$\boxed{V_y \text{ vs } t}$ Uniform acceleration along y-direction. $v_y = -\frac{1}{2}gt^2$ III.

25. [(E)] 7 pennies. 1 central penny + 6 pennies adjacent to each other.




$I_c = \frac{1}{2}mr^2$. For the central penny.

$I_t = I_c + 6I'$

$= \frac{1}{2}mr^2 + \frac{54}{2}mr^2$

$I_t = \frac{55}{2}mr^2$

For each surrounding penny,

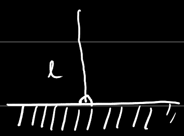
$I' = I_c + m(2r)^2 = \frac{1}{2}mr^2 + 4mr^2$

$= \left(\frac{1}{2} + 4\right)mr^2 = \frac{9}{2}mr^2$

So,

Moment of inertia for a rod, $I = I_c + M\ell^2/4 = \frac{1}{12}m\ell^2 + \frac{1}{4}m\ell^2 = \frac{1}{3}m\ell^2$

26. $\boxed{C}$ Conservation of energy, $mgh = \frac{1}{2}I\omega^2 \Rightarrow \omega^2 = \frac{2mg\ell/2}{I} \Rightarrow \omega = \sqrt{\frac{\cancel{m}g\cancel{\ell}\times 3}{\cancel{m}\ell^{\cancel{2}}}}$




$\Rightarrow \omega = \sqrt{\frac{3g}{\ell}}$ | Linear velocity, $v = \omega\ell$

$= \sqrt{\frac{3g}{\ell}}\cdot\ell = \sqrt{3g\ell}$

Physical pendulum, $\omega = \sqrt{\frac{mg\ell/2}{I}}$

27. $\boxed{A}$ Basic quantum mechanics Fact: Hermitian operators are chosen as the operators to represent physical observables because they have the property that their eigenvalues are always real.

28. Given, $|\psi_1\rangle = 5|1\rangle - 3|2\rangle + 2|3\rangle$
$|\psi_2\rangle = |1\rangle - 5|2\rangle + x|3\rangle$
| $|1\rangle, |2\rangle$ & $|3\rangle$ are orthonormal
$\langle m|n\rangle = \delta_{mn}$

For $\langle\psi_1|\psi_2\rangle = 0$

$\Rightarrow [5|1\rangle - 3|2\rangle + 2|3\rangle][|1\rangle - 5|2\rangle + x|3\rangle] = 0$

$\Rightarrow 5\langle 1|1\rangle + 15\langle 2|2\rangle + 2x\langle 3|3\rangle = 0$ $\quad[\langle 1|2\rangle = \langle 2|3\rangle = \langle 3|1\rangle = 0]$

$[\langle 1|1\rangle = \langle 2|2\rangle = \langle 3|3\rangle = 1]$

$\Rightarrow 20 + 2x = 0 \Rightarrow \boxed{x = -10}$

29. $\boxed{C}$ Given, $\psi = \frac{1}{\sqrt6}\psi_{-1} + \frac{1}{\sqrt2}\psi_1 + \frac{1}{\sqrt3}\psi_2$. Eigenvalues $-1, 1, 2$.

Expectation value of $\hat{O} = \langle\psi|\hat{O}|\psi\rangle$

For discrete eigenstates, Expectation value = sum of probability (square of the coefficient) times the eigenvalue.

$= \left(\frac{1}{\sqrt6}\right)^2\langle\psi_{-1}|\hat{O}|\psi_{-1}\rangle + \left(\frac{1}{\sqrt2}\right)^2\langle\psi_1|\hat{O}|\psi_1\rangle + \left(\frac{1}{\sqrt3}\right)^2\langle\psi_2|\hat{O}|\psi_2\rangle$

$= \frac{1}{6}\times(-1)\langle\psi_{-1}|\psi_{-1}\rangle + \frac{1}{2}\times 1\times\langle\psi_1|\psi_1\rangle + \frac{1}{3}\times 2\langle\psi_2|\psi_2\rangle$

$= \frac{1}{2} + \frac{2}{3} - \frac{1}{6} = \frac{3+4-1}{6} = \frac{6}{6} = 1.$

30. $\boxed{A}$ Radial wavefunction of hydrogen atom has the form $\boxed{\psi(r) = Ae^{-br}}$

31. $\boxed{A}$ For positronium, $\mu = m_e/2$. So binding energy $E_0^P = \frac{E_0}{2} = \frac{-13.6}{2} = -6.8$ eV

$\Delta E = E_0^P\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = 6.8\left(1 - \frac{1}{9}\right) = \frac{6.8\times 8}{9} = \frac{54}{9} \approx 6$ eV.

32. $\boxed{D}$ Total energy E is twice its rest energy. $E = 2mc^2$

Energy-momentum relation, $\boxed{E^2 = p^2c^2 + m^2c^4} \Rightarrow 4m^2c^{\cancel{4}} = \cancel{c^2}(p^2 + m^2c^2)$

$\Rightarrow 4m^2c^2 = p^2 + m^2c^2 \Rightarrow \boxed{p = \sqrt3\, mc}$

33. $\boxed{(D)}$ $\Delta\tau = 10^{-8}$ s

$\Delta x_\ell = 30$ m

Pion's speed, $v = \frac{\Delta x_\ell}{\Delta t} = ?$

$\eta = \frac{\Delta x_\ell}{\Delta\tau} = \frac{30}{10^{-8}} = 3\times10^{8}\times10 = 10c$

In natural unit $(c=1) \Rightarrow \Delta\tau = \frac{\Delta x_\ell}{10}$

Invariant interval

$\Delta\tau^2 = \Delta t^2 - \Delta x^2 \Rightarrow \Delta t^2 = \Delta x_\ell^2 + \Delta\tau^2$

$\Rightarrow \Delta t^2 = \Delta x^2 + \frac{\Delta x^2}{100} = \frac{101\,\Delta x_\ell^2}{100}$

$\Rightarrow \frac{\Delta x_\ell}{\Delta t} = \sqrt{\frac{100}{101}} = 0.99c = 2.9\times10^{8}$ m/s.

34. $\boxed{(C)}$ Frame S

$\Delta s^2 = \Delta x^2 - \Delta t^2$

Frame S'

For two events to occur at the same time, $\Delta t' = 0$

$\Delta s'^2 = \Delta x'^2$

So, $\Delta x^2 - \Delta t^2 = \Delta x'^2 \Rightarrow \frac{\Delta x^2}{\Delta t^2} - 1 = \frac{\Delta x'^2}{\Delta t^2} \Rightarrow \frac{\Delta x}{\Delta t} = \sqrt{1 + \frac{\Delta x^2}{\Delta t^2}}$

$\Rightarrow \frac{\Delta x}{\Delta t} = \sqrt{\frac{c^2\Delta t^2 + \Delta x^2}{c^2\Delta t^2}}$ So, $\frac{\Delta x}{\Delta t} > c$

35. $\boxed{(E)}$ Black body radiation $\Rightarrow$ Radiated power is proportional to $T^4$.

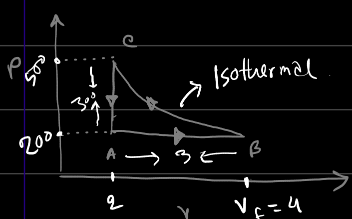
So if $T$ becomes $3T$, $P$ will become $81P$.

36. $\boxed{(E)}$ Adiabatic expansion $\Rightarrow \boxed{dQ = 0}$ So, $\boxed{dW = -dU}$

So, (A) is true $[dQ = 0]$ ; (B) is true $[\Delta S = \int \frac{dQ}{T} = 0]$ (C) $dU = -dW = -\int p\,dV$ true.

(D) $dW = \int p\,dV$ is true (E) $T$ is not constant, so this is NOT true.

37 $\boxed{(D)}$ (i) Counter-clockwise P-V diagram $\rightarrow$ Negative (-ve) work.




(ii) Work done, $W = \int p\,dV$ = Area under the P-V curve.

The only thing I need is $V_B$. I can determine this from CB.

$P_C V_C = P_B V_B \Rightarrow V_B = \frac{P_C V_C}{P_B} = \frac{500\times2}{200} \approx 4$

Area under the curve is [Consider ABC to be roughly a triangle]

$\frac{1}{\cancel{2}}\times\cancel{2}\times300 \Rightarrow$ So, $\boxed{W = -300\,\mathrm{J}}$.

38. LC circuit, Resonance frequency:

Given, $L = 25\times10^{-3}$ H

Resonant Frequency, $\omega = 1000$ rad/s

Capacitance, $C = ?$

$\omega^2 = \frac{1}{LC} \Rightarrow C = \frac{1}{\omega^2 L} = \frac{1}{10^6\times25\times10^{-3}}$

$\Rightarrow C = \frac{1}{25\times10^{3}} = \frac{\cancel{100}^{\,4}}{\cancel{25}\times10^{5}} = 4\times10^{-5}$ F $= 40\times10^{-6}$ F

$\Rightarrow C = 40\,\mu$F.

39 $\boxed{(D)}$ High-pass Filter

$X_L = \omega L$

$X_C = \frac{1}{\omega C}$

Inductors are high-frequency choke. So I is a low-pass filter. In II, inductor is connected to the ground. So this is a high-pass filter. In III, Capacitance are low-frequency choke. So III is also a high-pass filter.

40. $\boxed{(D)}$ RL circuit: Voltage decays exponentially following $\boxed{V = V_0 e^{-t/_R}}$

So the answer must either be (D) or (E). But notice that the decay time in (E) is insanely huge, 200 sec! Highly unlikely.

41. $\boxed{(E)}$ Maxwell's equations if magnetic monopoles existed:

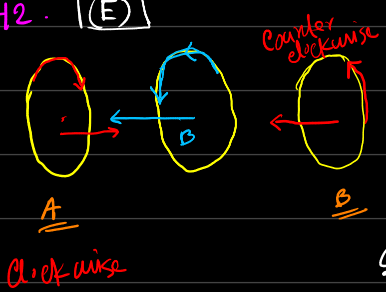
(i) $\vec{\nabla}\cdot\vec{E} = \rho_e/\varepsilon_0$

(ii) $\vec{\nabla}\cdot\vec{B} = \mu_0\rho_B$ → This wouldn't have been $\vec{\nabla}\cdot\vec{B}=0$, it would've been like (i)

(iii) $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} + \mu_0 J_B$ → There would've been a magnetic current term here, like (iv)

(iv) $\vec{\nabla}\times\vec{B} = \mu_0 J_e + \frac{\partial\vec{E}}{\partial t}$

42. $\boxed{(E)}$




For loop A: Magnetic field is INTO THE LOOP & increasing.
So, the induced magnetic field will be OUT OF THE LOOP.
So, the induced current will be clockwise.

For loop B: Magnetic field OUT OF THE LOOP & decreasing.
So, induced magnetic field will point OUT OF THE LOOP.
So the induced current will be anti-clockwise.

43. $\boxed{(D)}$ Commutation relation formula: (i) $[AB,C] = A[B,C] + [A,C]B$

First C will act on B, keeping A to the left. $A[B,C]$

then C will act on A, keeping B to the right. $[A,C]B$.

(ii) $[A,BC] = [A,B]C + B[A,C]$

First A will act on B, keeping C to the right $[A,B]C$

then A will act on C, keeping B to the left. $B[A,C]$

So, $[L_xL_y, L_z] = L_x[L_y,L_z] + [L_x,L_z]L_y = L_x\cdot i\hbar L_x - i\hbar L_y\cdot L_y = \boxed{i\hbar(L_x^2 - L_y^2)}$

44. $\boxed{(D)}$ Wavefunction $\phi_n = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$ & energies, $E_n = \frac{n^2\hbar^2\pi^2}{2mL^2}$. $E_1 = \frac{\hbar^2\pi^2}{2mL^2}$

Possible result of measurement of energy = One of the energy eigenvalues.

Eigenvalues are $E_2 = 4E_1$; $E_3 = 9E_1$; $E_4 = 16E_1$ . . .

45. $\boxed{B}$ $|\psi\rangle = \frac{1}{\sqrt{14}}|1\rangle - \frac{2}{\sqrt{14}}|2\rangle + \frac{3}{\sqrt{14}}|3\rangle$

Expectation value of $\hat{H}$ operator is the sum of the product of the square of the coefficients and the eigenvalues.

Eigenvalues.

$\langle 1|H|1\rangle = \frac{3\hbar\omega}{2}$

$\langle 2|H|2\rangle = \frac{5\hbar\omega}{2}$

$\langle 3|H|3\rangle = \frac{7\hbar\omega}{2}$

$$\langle\hat{H}\rangle = \frac{1}{14}\times\frac{3\hbar\omega}{2} + \frac{4}{14}\times\frac{5\hbar\omega}{2} + \frac{9}{14}\times\frac{7\hbar\omega}{2} = \frac{\hbar\omega}{2}\left[\frac{3}{14}+\frac{20}{14}+\frac{63}{14}\right] = \frac{\hbar\omega}{2}\left(\frac{3+20+63}{14}\right)$$

$$\Rightarrow \langle\hat{H}\rangle = \frac{86}{28}\hbar\omega = \boxed{\frac{43}{14}\hbar\omega}$$

46 $\boxed{E}$ De-Broglie wavelength of a free particle, $\boxed{\lambda = \frac{h}{p}}$ & $p$ is given by $p=\sqrt{2mE}$

But when the particle enters a region with potential $V$, the momentum is given by $E = \frac{p^2}{2m}+V \Rightarrow p^2 = 2m(E-V) \Rightarrow p = \sqrt{2m(E-V)}$

$$\text{So, } \lambda_{new} = \frac{h}{\sqrt{2m(E-V)}} \qquad \lambda = \frac{h}{\sqrt{2mE}}$$

$$= \frac{h}{\sqrt{2mE(1-V/E)}} = \frac{h}{\sqrt{2mE}}(1-V/E)^{-1/2} = \lambda(1-\tfrac{V}{E})^{-1/2}$$

47 $\boxed{B}$ Entropy, $S = k\ln\Omega$ $\quad$ $\Omega$ = the total number of ways the system can be arranged.

Here, there are 2 separate volumes & $n$ molecules. So, $\Omega = 2^n$

$$\Rightarrow S = k\ln(2^n) = \boxed{nk\ln 2}$$

Alternatively, the container is thermally insulated $\Rightarrow$ The process is an isothermal one.

So, entropy, $\Delta S = \int\frac{dQ}{T}$ $\quad$ But $Q = dU + dW = 0 + nRT\ln\left(\frac{2V}{V}\right)$

$$\Rightarrow Q = nRT\ln 2$$

$$\Rightarrow \Delta S = \frac{1}{\cancel{T}}nR\cancel{T}\ln 2$$

$$\Rightarrow \boxed{\Delta S = nR\ln 2}$$

48. $\boxed{C}$ Root mean square velocity $\boxed{V_{rms} = \sqrt{\frac{3RT}{M}}}$ $\quad$ $M_o = 32u$ ; $M_N = 28u$

T constant.

$$\text{So, } \frac{V_{rms}(N_2)}{V_{rms}(O_2)} = \sqrt{\frac{\cancel{3RT}}{M_N}\times\frac{M_o}{\cancel{3RT}}} = \sqrt{\frac{\cancel{32}^{8}}{\cancel{28}_{7}}} = \sqrt{\frac{8}{7}}$$

49. $\boxed{E}$ Two states, with energies $\varepsilon$ & $2\varepsilon$. Degeneracy, $N=2$

The partition function

$$Z = 2\left[e^{-\varepsilon/kT} + e^{-2\varepsilon/kT}\right]$$

50. $\boxed{B}$ The velocity $v = f\lambda$ $\Rightarrow f_o = \frac{v_o}{\lambda}$ $\quad$ But on a colder day, $v = 0.97v_o$

$$\text{So, } f = 0.97\frac{v_o}{\lambda} = 0.97\times 440 = \frac{97\times 44\cancel{0}}{10\cancel{0}} = \frac{4400-132}{10} = 427\text{ Hz}$$

51. $\boxed{(B)}$ When UNPOLARIZED light falls on a polarizer, the intensity $\boxed{I_1 = \frac{I_0}{2}}$ [Average of $\cos^2\theta$ for all values of $\theta$ is $\frac{1}{2}$].

Second polarizer at 45° with the first. $I_2 = I_1 \cos^2\theta = \frac{I_0}{2} \cdot \frac{1}{2} = \frac{I_0}{4}$

Third polarizer at 90° to the first, thus it is at 45° to the second polarizer.

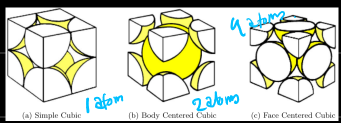
$\Rightarrow I_3 = I_2 \cos^2\theta = I_2 \cdot \frac{1}{2} = \frac{I_0}{4} \times \frac{1}{2} = \frac{I_0}{8}$

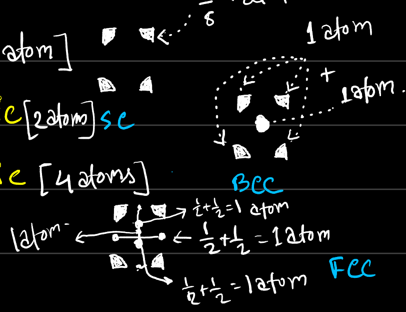
52. $\boxed{(C)}$ 3 generic types of cubic crystals $\Rightarrow$ (i) Simple cubic [1 atom]

(ii) Body centered cubic [2 atoms]

(iii) Face-centered cubic [4 atoms]

Volume of the primitive unit cell $= a^3/2$.

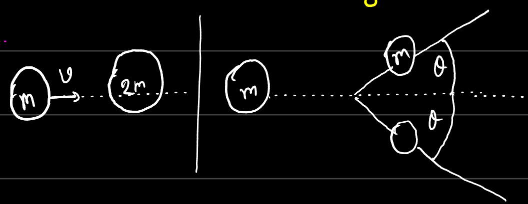



53. $\boxed{(B)}$ Undoped semi-conductor $\rightarrow$ Resistivity decreases with temperature.

54. Impulse, $H = \int F dt$ = Area under F-T curve. $= \frac{1}{\cancel{2}} \times \cancel{2} \times 2 = 2$ kg·m/s

55.




Momentum conservation along x-direction

$\cancel{m}v = 2\cancel{m}v'\cos\theta$

$\Rightarrow v' = \frac{v}{2\cos\theta}$

$\Rightarrow v' = \frac{1}{\cos\theta} \frac{v}{2}$

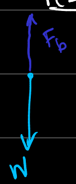
Since $\cos\theta \le 1$

Then $\frac{1}{\cos\theta}$ is always $> 1$.

So, $v' \ge \frac{v}{2}$

So, each particle moves with speed greater than $v/2$.

56. $\boxed{(D)}$




The buoyant force floating the balloon, $F_b = \rho_a V_H g$

weight of the load, $W = mg$.

Fb must be greater than $W + W_H$.

$\Sigma F = F_b - mg = 0$

$\Rightarrow \rho_a V_H \cancel{g} = m\cancel{g}$

$\Rightarrow V_H = \frac{m}{\rho_a} = \frac{300}{1.29} = \frac{\cancel{3} \times 100 \times 10\cancel{0}}{\cancel{130}_{4.3}} = \frac{1000}{4.3} \approx 250 \text{ m}^3$

But the buoyant force must also support the Helium's weight also. So its volume must be slightly larger than 250 m³.

57. $\boxed{(A)}$ Immediately eliminate (D) & (E) because F should not decrease as $\rho$ is increased.

Eliminate (C) because g should not matter, it's the same everywhere.

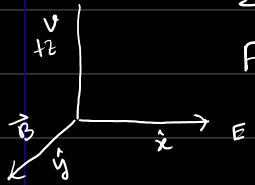
Now, between (A) & (B), use Dimensional analysis. F should have the dimension $[MLT^{-2}]$

✓(A) $\rho \rightarrow \frac{M}{L^3}$ ; $v^2 = \frac{L^2}{T^2}$ ; $A^2 = L^2 \Rightarrow \rho v^2 A = \frac{M \cdot L^2 \cdot L^2}{L^3 T^2} = [MLT^{-2}]$

(B) $\rho v A = \frac{M L \cdot L^2}{L^3 T}$

58. $\boxed{(B)}$ $\vec{E}$ pushes the proton in the $+x$ direction.

$\vec{B}$ pushes the proton in the $-x$ direction $[\hat{z} \times \hat{y} = -\hat{x}]$




As the proton is not deflected, $\vec{E}$ & $\vec{B}$ are balanced.

If the potential V is increased, velocity will increase, so will $\vec{B}$. So ultimately $-x$ wins out.

59. $\boxed{(B)}$ Inductance L is analogous to inertia m

Capacitance is reciprocal to spring constant k

And Q is analogous to $x$.

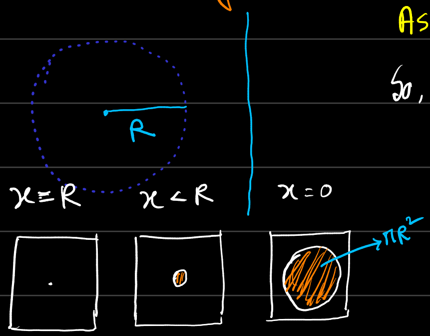
60. Limiting case: As $x = R$, $A = 0$; $\Phi = EA$

As $x = 0$ $A = \pi R^2$ $\Rightarrow \Phi = \frac{\sigma A}{\varepsilon_0}$

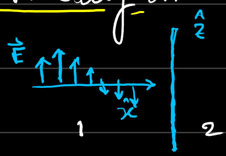
So, only for option (D), $A \to 0$ as $x = R$

& $A = \pi R^2$ as $x = 0$

$E = \frac{\sigma}{\varepsilon_0}$ [On both sides]




61 $\boxed{(C)}$ The $\vec{E}$-field is given by $\vec{E} = E_0 \cos(kx - \omega t)$ and it incident normally on a perfect conductor.




Boundary conditions for electric fields: $\boxed{E_1^{\perp} - E_2^{\perp} = \frac{\sigma}{\varepsilon_0}}$ $\boxed{E_1^{\parallel} - E_2^{\parallel} = 0}$

Let's say the plane wave travelling in the $x$-direction is polarized in the $z$-direction. (the direction of the conductor) So, no perpendicular component.

Now, the parallel component condition says $E_1^{\parallel} = E_2^{\parallel}$

or, $E_1^{i} + E_1^{r} = E_2^{t}$

But since this is a conductor, the transmitted $\vec{E}$ is 0, $E_2^t = 0$. And $E_1^{i} = -E_1^{r}$ $[+\hat{z}$ & $-\hat{z}]$

Thus, on the left, we get 0.

Boundary conditions for magnetic fields: $\boxed{B_1^{\perp} - B_2^{\perp} = 0}$ $\boxed{B_1^{\parallel} - B_2^{\parallel} = k\mu_0}$

As the wave propagates in the $\hat{x}$-direction, $\vec{E}$ is in the $\hat{z}$-direction, $\vec{B}$ will be in the $(-)\hat{y}$-direction (perpendicular to the screen). There is no parallel component. So $B_1^{\parallel} = B_2^{\parallel} = 0$.

$B_1^{\perp} = B_2^{\perp}$ $\Rightarrow B_1^{i} + B_1^{r} = B_2^{t}$ And since $B_2^t = 0$;

and the reflected $\vec{B}$ is also in the negative $-\hat{y}$ direction, we have $2\vec{B}$ in the left.

In short, a conductor sets $\vec{E}$ field too and multiplies a factor of 2 with the $\vec{B}$ field.

62. [A] Cyclotron frequency:

$q = 2e$

$\vec{B} = \pi/4\ T$

$f = 1600\ Hz$

$\omega = 2\pi f = 2\pi \times 1600$

$\boxed{\omega = \frac{qB}{m}} \Rightarrow m = \frac{qB}{2\pi f}$

$\Rightarrow m = \frac{\cancel{2} \times 1.6 \times 10^{-19} \times \cancel{\pi}}{\cancel{2}\cancel{\pi} \times 1600 \times 4} = \frac{\cancel{1600} \times 10^{-3} \times 10^{-19}}{4 \times \cancel{1600}} = \frac{1}{4} \times 10^{-22} = \boxed{2.5 \times 10^{-23}\ kg}$

63. [D] Wein's displacement law $\Rightarrow$ $\boxed{\lambda_m = \frac{2.9 \times 10^{-3}}{T}}$

Here, $\lambda$ peaks around $2\mu m$. So, $\lambda_m = 2 \times 10^{-6}\ m$.

$$So,\ T = \frac{3 \times 10^{-3}}{2 \times 10^{-6}} = 1.5 \times 10^{3} = 1500K$$

64. [A] (A) is not correct because visible light do not reveal nuclear structure. (B) is true because you emit what you absorb. (C) is true because absorption spectra are unique to each elements. (D) is true for the same reason. (E) is true because for single atom you get energy levels, not band.

65. [D] At high temperatures, $kT \gg h\nu$ & $e^x = 1 + x$

$$So,\ c = 3kN_A \left(\frac{h\nu}{kT}\right)^2 \frac{1 + h\nu/kT}{(1 + h\nu/kT - 1)^2} = 3kN_A \left(\cancel{\frac{h\nu}{kT}}\right)^2 \frac{1}{\cancel{(h\nu/kT)^2}} = 3kN_A$$

66. [D] Can be solved using standard rate problem

Half-life for $\gamma$-emission $\rightarrow$ 24 minutes

Half-life for $\beta$-emission $\rightarrow$ 36 minutes.

Jimmy eats a pie in 24 minutes & Johny eats a pie in 36 minutes. How long would it take to finish the pie if they both were eating at the same time?

24 — 1 pie | 36 — 1 pie

1 — $\frac{1}{24}$ pie | 1 — $\frac{1}{36}$ pie

$\frac{1}{24} + \frac{1}{36}$ pie in 1 m

1 pie in $\frac{1}{\frac{24+36}{24 \times 36}} = \frac{24 \times 36}{24 + 36} = 14.4m$.

67. [E] (A) eliminated. $^{238}U$ can fission spontaneously.

(B) eliminated. Cannot conclude from given information.

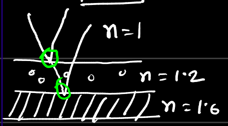
(C) eliminated. It's simply not sensible.

Between (D) & (E), remember that as we move towards the more stable elements, the binding energy per nucleon INCREASES.

68. [E] Process of elimination: Alpha particle is basically $_2^4He^{2+}$ ion, with 2 protons & 2 neutrons. So it's not possible for $_4Be$ to transform into $_3Li$ by emitting $\alpha$-particle.

Losing electron, neutron or positron doesn't alter the type of atom. (B) (C) (D) eliminated.

69. [(B)] $\lambda = 480 \times 10^{-9}$ m. Maxima $2nt = m\lambda$




$$\Rightarrow t = \frac{\lambda}{2n} = \frac{480\times10^{-9}}{2\times1\cdot2} = \frac{\cancel{4\cdot8}^{2}\times10^{2-9}}{\cancel{2\cdot4}} = 2\times10^{-7}$$

Double phase shif. Conditions don't flip.

$$\Rightarrow t = 200 \text{ nm}.$$

70. [(B)] Slit separation, $a = 0\cdot5\times10^{-6}$ m. Fringe separation, $s = 1$ mm.

Maxima, $\boxed{a\sin\theta = m\lambda} \Rightarrow \boxed{a\theta = m\lambda}$

and $s = L\theta \Rightarrow \theta = \frac{s}{L}$ so $\theta \propto s$. and $v = f\lambda \Rightarrow \lambda = \frac{v}{f}$

If $f$ is doubled, $\lambda$ is halved, and since $s \propto \theta \propto \lambda$, $s$ is also halved.

$s' = 0\cdot5$ mm.

71. [(D)] $\lambda_s = 121\cdot5$ nm. $\lambda_r = 607\cdot5$ nm. | Redshift $\Rightarrow$ Wavelength increases. The object is moving away.

$$\lambda_r = \sqrt{\frac{1+\beta}{1-\beta}}\,\lambda_s$$

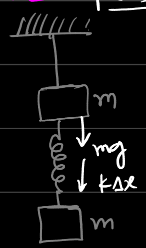
$$\Rightarrow \left[\frac{\lambda_r}{\lambda_s}\right]^2 = \frac{1+v/c}{1-v/c} = \frac{c+v}{c-v}$$

$$\Rightarrow \left(\frac{\cancel{607\cdot5}^{5}}{\cancel{121\cdot5}}\right)^2 = \frac{c+v}{c-v} \Rightarrow 25c - 25v = v + c \Rightarrow 26v = 24c \Rightarrow v = \frac{24}{26}c$$ so closer to $c$!

Increase in wavelength $\rightarrow$ Redshift.

Decrease in wavelength $\rightarrow$ Blueshift.

72. [(E)]



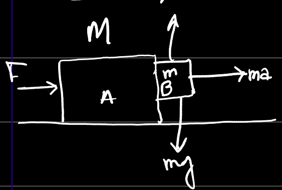

$ma = -mg - k\Delta x$

$\Rightarrow \cancel{m}a = -2\cancel{m}g$

$\Rightarrow a = -2g$.

$mg = -k\Delta x$ [Force exerted on the spring by the lower mass]

73. [(D)] $M = 16$ kg, $m = 4$ kg, $\mu = 0\cdot5$




$F = (M+m)a$

$\Rightarrow a = \frac{F}{M+m}$

$\mu \cancel{m}a = \cancel{m}g$

$\Rightarrow \frac{F\mu}{M+m} = g$

$$\Rightarrow F = \frac{(M+m)g}{\mu} = \frac{(16+4)\times10}{1/2} = 400\text{N}$$

74. [(D)] $L = a\dot{q}^2 + bq^4$

Euler-Lagrange equation of motion $\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = 0$

$\frac{\partial L}{\partial q} = 4bq^3$

$\frac{\partial L}{\partial \dot{q}} = 2a\dot{q}$

$\Rightarrow 4bq^3 - 2a\ddot{q} = 0$

$\Rightarrow 2a\ddot{q} = 4bq^3 \Rightarrow \ddot{q} = \frac{\cancel{4}^{2}b}{\cancel{2}a}q^3$

$\Rightarrow \boxed{\ddot{q} = \frac{2b}{a}q^3}$

75. $\boxed{(E)}$ Rotation matrices $\Rightarrow$ $R_z(\theta) = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $R_y(\theta) = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}$

The given matrix is $\begin{pmatrix} 1/2 & \sqrt{3}/2 & 0 \\ -\sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix}$

So, obviously it's rotation about z-axis.

for what $\theta$ is $\cos\theta = 1/2$ and $\sin\theta = \sqrt{3}/2$ ? $\theta = 60^\circ$. It's a conterclockwise rotation of $60^\circ$ about $\hat{z}$.

76. $\boxed{(C)}$ Eliminate (A) $\Rightarrow$ Electrons do move freely in metals but are more constrained in DOF than a free atom.

Eliminate (B) $\Rightarrow$ Nothing in the problem indicates this. Eliminate (D) No, electrons are slow.

The mean kinetic energy of conduction electrons in metals is much higher than kT because the electrons form a degenerate Fermi gas.

77. $\boxed{(E)}$ $E_A - E_B = 0.1\,eV$

The probability of the system being in state A is, $P_A = \dfrac{e^{-E_A/kT}}{Z}$

and of the system being in State B is, $P_B = \dfrac{e^{-E_B/kT}}{Z}$

Then, $\dfrac{P_A}{P_B} = \dfrac{e^{-E_A\beta}}{e^{-E_B\beta}} = e^{-(E_A-E_B)/kT} = e^{-0.1/0.025}$  $\dfrac{0.1}{0.025} = \dfrac{1}{0.25} = \dfrac{1}{1/4} = 4$

So, $\dfrac{P_A}{P_B} = e^{-4}$

78. $\boxed{(E)}$ $\mu \rightarrow e + \nu_\mu + \bar{\nu}_e$ is allowed. Lepton number is conserved. $1 = 1 + 1 - 1$

$\mu \rightarrow e + \nu$ is not, because lepton number is not conserved $1 \neq 1 + 1$

Conservation of lepton number: $\boxed{L = +1}$ $e^-, \mu^-, \nu$ $\boxed{L = -1}$ $\bar{e}, \bar{\mu}, \bar{\nu}$

$\boxed{L = 0}$ All other particles.

79. $\boxed{(D)}$ Total energy, $E = 10$ GeV ; Relativistic momentum $p = 8$ GeV/c

$E^2 = p^2c^2 + m^2c^4$ $\Rightarrow mc^2 = \sqrt{10^2 - 8^2} = \sqrt{(10+8)(10-8)} = \sqrt{18 \times 2} = 6$

$\Rightarrow 10^2 = 8^2 + (mc^2)^2$ $\Rightarrow m = 6\,GeV/c^2$

80. $\boxed{(D)}$ Speed of light in the tube frame, $v = \frac{c}{n}$ ; $n = 4/3$ | Speed of light in lab frame

Speed of the tube, $u = \frac{1}{2}c$. $\Rightarrow v = \frac{3c}{4}$

$w = \dfrac{u+v}{1+uv/c^2} = \dfrac{\frac{3c}{4} + \frac{c}{2}}{1 + \frac{3}{8}} = \dfrac{\frac{3+2}{4}}{11/8} = \frac{5}{4} \times \frac{8}{11} = \frac{10}{11}c$

81. $\boxed{(B)}$ $L^2 Y_l^m(\theta,\phi) = l(l+1)\hbar^2 = 6\hbar^2$ $\Rightarrow l(l+1) = 6$ So, $l = 2$

$L_z Y_l^m(\theta,\phi) = m\hbar = -\hbar$ $\Rightarrow m = -1$ So, it's $Y_2^{-1}(\theta,\phi)$

82. $\boxed{(D)}$ $|\alpha\rangle = |\uparrow\rangle$ ; $|\beta\rangle = |\downarrow\rangle$

So, I. $|\alpha\rangle_1 |\alpha\rangle_2$ valid

II. $\frac{1}{\sqrt{2}}(|\alpha\rangle_1|\beta\rangle_2 - |\alpha\rangle_2|\beta\rangle_1)$

III. $\frac{1}{\sqrt{2}}(|\alpha_1\rangle|\beta\rangle_2 + |\alpha\rangle_2|\beta\rangle_1)$ valid.

Triplet states $(S=1)$: (i) $|\uparrow\rangle|\uparrow\rangle$
(ii) $\frac{1}{\sqrt{2}}(|\uparrow\rangle|\downarrow\rangle + |\downarrow\rangle|\uparrow\rangle)$
(iii) $|\downarrow\downarrow\rangle$
} Symmetric

Singlet state $(S=0)$: (i) $\frac{1}{\sqrt{2}}(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle)$ } Antisymmetric.

83. $\boxed{(C)}$ Components of the spinstates:

$$\psi_{z+} = \begin{bmatrix}1\\0\end{bmatrix} \quad \psi_{x+} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} \quad \psi_{y+} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix}$$

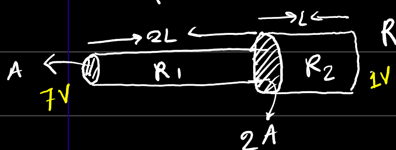
$$\psi_{z-} = \begin{bmatrix}0\\1\end{bmatrix} \quad \psi_{x-} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix} \quad \psi_{y-} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-i\end{bmatrix}$$

So, here, $\psi_{x-} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix} = \frac{1}{\sqrt{2}}\left(\begin{bmatrix}1\\0\end{bmatrix} - \begin{bmatrix}0\\1\end{bmatrix}\right) = \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle)$

84. $\boxed{(D)}$ Selection rules $\Rightarrow$ $\boxed{\Delta l = \pm 1}$ $\boxed{\Delta j = 0, \pm 1}$ $\boxed{\Delta m = 0, \pm 1}$

For A, $\Delta l = 0$, so not allowed. For B, $\Delta l = 1$ $\Delta j = 1$, so allowed. For C, $\Delta j = 0$, so allowed.

85. $\boxed{(A)}$ Both are nichrome wire. So resistivity are the same, $\rho$.




$R_1 = \frac{\rho 2L}{A}$ ; $R_2 = \frac{\rho L}{2A}$ $\Rightarrow \frac{R_1}{R_2} = \frac{\cancel{\rho} 2\cancel{L}}{\cancel{A}} \times \frac{2\cancel{A}}{\cancel{\rho}\cancel{L}} = \frac{4}{1}$ $\Rightarrow V_1 = 4V_2$

Net voltage, $V = 8 - 1 = 7V$.

$V_1 + V_2 = 7V$ $\Rightarrow 4V_2 + V_2 = 7V$ $\Rightarrow 5V_2 = 7$ $\Rightarrow V_2 = \frac{7}{5} = 1.4V$

$V_1 = (7 - 1.4)V = 5.6V$

Potential at the junction $V' = (7 - 5.6)V$
$= 2.4V$

86. (E) Number of turns, $N = 15$

Radius, $r = 10^{-2}$ m

Angular frequency, $\omega = 300$ rad/s

Magnetic field, $\vec{B} = 0.5$ T

When $t = 0$, $\vec{A} \perp \vec{B}$

Faraday's law, $\mathcal{E} = \frac{-d\Phi}{dt}$

At $t = 0$, flux should be zero. $\Phi = 0$

So, $\Phi = \int \vec{B} \cdot d\vec{A} = B\pi r^2 \sin(\omega t)$

$\Rightarrow \mathcal{E} = \frac{-d\Phi}{dt} N = B\pi r^2 \omega N \cos(\omega t)$

$\Rightarrow I = \frac{\mathcal{E}}{R} = \frac{B\pi r^2 \omega N}{R} \cos(\omega t)$

$\Rightarrow I = \frac{\frac{1}{2} \times 10^{-4} \times \cancel{3} \times 10^2 \times \cancel{15}^{5}}{\cancel{9}_{3}} \pi \cos(\omega t) = 0.025\,\pi \cos(\omega t)$ A $= 0.025 \times 10^3 \pi \cos(\omega t)$ mA

$= 25\pi \cos(\omega t)$ mA.

Spherical shell $\rightarrow$ Hollow sphere.

87. $\boxed{(A)}$ The testcharge is inside the left sphere, where potential is constant, so, field is 0.

The only force is from the right sphere $(10d - \frac{d}{2})$ distance away.

$$F = \frac{1}{4\pi\epsilon_0} \frac{qQ}{(10d - \frac{d}{2})^2} = \frac{qQ}{4\pi\epsilon_0 \left(\frac{20d - d}{2}\right)^2} = \frac{qQ}{\cancel{4}\pi\epsilon_0 \frac{19^2 d^2}{\cancel{4}}} = \frac{qQ}{19^2 d^2 \pi\epsilon_0} = \frac{qQ}{361\pi\epsilon_0 d^2}$$

$(20-1)^2$
$= 400 - 40 + 1$
$= 361$

88. (C) Limiting case approach: if $\theta \to 2\pi$, this would simply be a loop of radius R and $\vec{B}$ at the center would've been $\vec{B} = \frac{\mu_0 I}{2R}$.

Option (c) gives $\vec{B} = \frac{\mu_0 I \cdot 2\pi}{4\pi R} = \frac{\mu_0 I}{2R}$.

Biot-Savart law: $\boxed{\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \hat{r}}{r^2}}$

Here, $dl = r d\theta$; So, $\vec{B} = \frac{\mu_0 I}{4\pi r^2} \int_0^\theta r\, d\theta = \frac{\mu_0 I}{4\pi R} \int_0^\theta d\theta$

$\Rightarrow \boxed{\vec{B} = \frac{\mu_0 I \theta}{4\pi R}}$

89. (E) $M = 200$ kg, $R = 2.5$ m

Conservation of angular momentum

$L_0 = L_f$

$I_M = \frac{1}{2} M R^2$

$L_0 = I_M \omega$

$m = 40$ kg

$I_0 = I_M + I_c$
$= \frac{1}{2} M R^2 + m R^2$
$= (\frac{1}{2} M + m) R^2$
$= (\frac{1}{2} \times 200 + 40) \times 2.5^2$
$= 140 \times \frac{25 \times 25}{100} = 875$

$L_0 = I_0 \omega$
$= 875 \times 2$

$I_f = \frac{1}{2} M R^2 = \frac{1}{2} \times 200 \times 2.5^2 = 100 \times \frac{25^2}{100} = 625$

$L_f = 625 \times \omega_f$

$\Rightarrow 35 \times 25 \times 2 = 25^2 \omega_f$

$\Rightarrow \omega_f = \frac{35 \times 25 \times 2}{25^2} = \frac{14}{5} = 2.8$ rad/s.

90. (A) Time period, $\boxed{T = 2\pi \sqrt{\frac{m}{k}}}$ So, $T \propto \frac{1}{\sqrt{k}}$

For figure 1, $K_1 = 2k$ [Parallel connection]

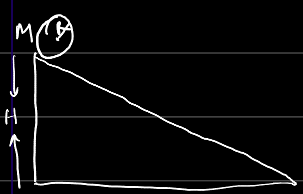
So, $T_1 = \frac{c}{\sqrt{2k}}$

In figure-2, $K_2 = \sqrt{\frac{k}{2}}$

So, $T_2 = \frac{c\sqrt{2}}{\sqrt{k}}$

So, $\frac{T_1}{T_2} = \frac{\cancel{c}}{\sqrt{2}\cancel{\sqrt{k}}} \times \frac{\cancel{\sqrt{k}}}{\cancel{c}\sqrt{2}} = \frac{1}{2}$.

91. (B) Translational speed at the bottom $v = \sqrt{\frac{8gH}{7}}$




$MgH = \frac{1}{2} I \omega^2 + \frac{1}{2} M v^2$

$MgH = \frac{1}{2} I \cdot \frac{v^2}{R^2} + \frac{1}{2} M v^2$

$\Rightarrow MgH = (\frac{1}{2} \frac{I}{R^2} + \frac{1}{2} M) v^2$

$\Rightarrow M\cancel{gH} = (\frac{I}{R^2} + M) \frac{1}{2} \cdot \frac{8\cancel{gH}}{7}$

$v = \omega R \Rightarrow \omega = \frac{v}{R}$

$\Rightarrow M = \frac{4I}{7R^2} + \frac{4M}{7} \Rightarrow \frac{4I}{7R^2} = M - \frac{4M}{7} = \frac{3M}{7} \Rightarrow I = \frac{3}{4} M R^2$

92. [E] Hamiltonian of any system, $\boxed{H = T + V}$ Here, $T = \frac{p_1^2}{2m} + \frac{p_2^2}{2m}$

& $V = \frac{1}{2}k(l-l_0)^2$ ⟶ Careful! This is the potential, not force.

$$\text{So, } H = \frac{1}{2}\left[\frac{p_1^2}{m} + \frac{p_2^2}{m} + k(l-l_0)^2\right]$$

93. [C] Repeating question. This is something you should just memorize.

Ground state of Hydrogen $\psi_0 = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$

Most probable value of $r$ is the Bohr radius $a_0$.

94. (E) Non-degenerate time independent perturbation theory: The first order correction to the $n^{th}$ energy $E_n^0$ is proportional to $\lambda$ & the expectation value of $\Delta H$ in the unperturbed state. If the Hamiltonian is $\boxed{H = H^0 + \lambda\Delta H}$ then

$$E_n = E_n^0 + \lambda\langle n|\Delta H|n\rangle$$

Here, $\Delta H = V(a + a^\dagger)^2 = V\left(a^2 + a^\dagger a + aa^\dagger + a^{\dagger 2}\right) \Rightarrow \Delta H = V(aa^\dagger + a^\dagger a)$

First order shift in $|2\rangle$

$$E_2 = \langle 2|\Delta H|2\rangle = \left[\langle 2|aa^\dagger|2\rangle + \langle 2|a^\dagger a|2\rangle\right]V$$
$$= \left[(\sqrt{2+1})^2\langle 3|3\rangle + (\sqrt{2})^2\langle 1|1\rangle\right]V$$
$$= (3+2)V = 5V.$$

95. [A] The initial field $E_0 = \frac{\sigma}{2\varepsilon_0}$. With $\varepsilon = k\varepsilon_0$, $E = \frac{\sigma}{2\varepsilon} = \frac{\sigma}{2\varepsilon_0}\frac{1}{k} = \frac{E_0}{k}$

96. [E] Larmor's formula ⇒ Power radiated depends on $q$ & acceleration $a$.

97. [E] Limiting case! When $n = 1$, there should be no spread in $\theta$. So $\delta\theta' = \theta$

(A) $\delta\theta' = \delta\lambda$ NO (B) $\delta\theta' = 1$ NO

(E) $\delta\theta' = \theta'$

98. [A] Basic statistical mechanics fact ⇒ The expectation value of any operator $\hat{O}$ is given by $\langle\hat{O}\rangle = \frac{\sum O_i e^{-E_i\beta}}{Z}$.

99. What is happening here? A photon strikes an electron (mass m) at rest. The photon is then destroyed by creating an electron-positron pair. So, after collision we have 3 particles each having a mass of m.

This part is the most crucial part!

Now, conservation of energy ⇒ $\underbrace{E_i = pc}_{\text{photon energy}} + \underbrace{mc^2}_{\text{Electron energy}} = E_f = 3\sqrt{\left(\frac{pc}{3}\right)^2 + (mc^2)^2}$

↳ Conservation of momentum.

$p^2c^2 + 2mc^2pc + m^2c^4 = 9\left(\frac{p^2c^2}{9} + m^2c^4\right)$

$\Rightarrow \cancel{p^2c^2} + 2mc^2pc + m^2c^4 = \cancel{9}\left(\frac{\cancel{p^2c^2} + 9m^2c^4}{\cancel{9}}\right)$

$\Rightarrow \cancel{2}\cancel{mc^2}pc = \cancel{8}^{4}\cancel{m^2}^{m}\cancel{c^4}^{c^2} \Rightarrow pc = 4mc^2$

100. Turn it into a simple ratio problem! Or you could just remember that green lasers have wavelength 510-570 nm.

85865 fringes ⟶ $\lambda_r$

1 fringe ⟶ $\frac{\lambda_r \times 10{,}0000}{85865} = \frac{632 \times 100{,}000}{85{,}000} \approx 540$ nm.

100,000 fringe ↗