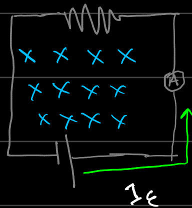
#GR9677

1. $\boxed{(B)}$ Capacitors discharge following exponential behaviour $V(t) = V_0 e^{-t/RC}$
Here r does not matter. So the curve should be B.

2. $\boxed{(B)}$ Here the contribution to current comes from two sources. the 5V cell & the induced emf.



$$\text{Area } A = \left(\frac{10}{100}\right)^2 m^2 = 10^{-2}\, m^2$$

Magnetic field INTO THE LOOP + DECREASING

Current from the 5V battery runs anti-clockwise.

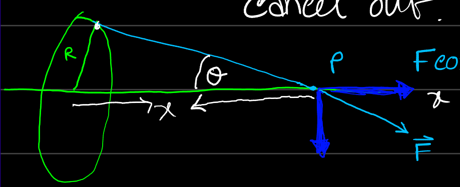
Current from the induced emf runs clockwise.

$$\varepsilon = -\frac{d\Phi}{dt} = \left(\frac{dB}{dt}\right) A = \cancel{150}^{3} \times \frac{1}{\cancel{100}_{2}} = 1.5\,V$$

$$\text{So, } V - \varepsilon = IR \Rightarrow I = \frac{5-1.5}{10} = 0.35\,A$$

3. $\boxed{(B)}$ [Limiting case!] As $R \to 0$, the potential assumes the form for a point charge $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x}$. Only option (B) does that.

4. $\boxed{(A)}$ For the force $\vec{F}$, all x-component stay but the y-components cancel out. For this type of problems, our target is to express the force in a form $\boxed{F = -kx}$.



$$F_T = F\cos\theta$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{-Q}{(R^2+x^2)}\cos\theta$$
$$= \frac{-Q}{4\pi\varepsilon_0 R^2}\cdot\frac{x}{R}$$

As $R >> x$ & $x^2 + R^2 \approx R^2$

$\cos\theta \approx \frac{x}{R}$

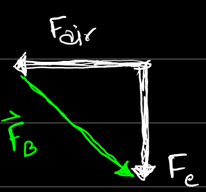
$$\Rightarrow F_T = -\frac{Q}{4\pi\varepsilon_0 R^3}\,x$$

This is our k.

Carefully note the condition! it's $R >> x$, doesn't look like it in the diagram so don't be fooled!

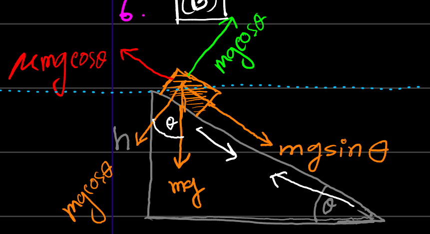
$$\text{So, } \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{Q}{4\pi\varepsilon_0 m R^3}}$$

5. $\boxed{(B)}$ Circular road on level ground $\Rightarrow$ Something must supply a centripetal force.



We know there's air resistance $\vec{F}_{air}$. So something must be added to that so that the resulting force act towards the center to supply the centripetal force. That something is the horizontal force of the road on the car's tires. Only $\vec{F}_B$ gives a resultant centripetal force.

6. $\boxed{(B)}$

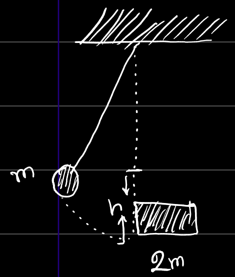


The force acting on the block is the horizontal component of its weight $F = mg\sin\theta$. Again, from friction consideration,

$$mg\sin\theta = \mu mg\cos\theta$$

So, work done by the block, $W = F \times \frac{h}{\sin\theta} = mg\cancel{\sin\theta} \times \frac{h}{\cancel{\sin\theta}}$
$= mgh$.

7. $\boxed{(A)}$



$\boxed{\text{I have to get what's } v_1 \text{ compared to } v_0.}$

Momentum conservation

$$\cancel{m}v_0 = -\cancel{m}v_1 + 2\cancel{m}v_2$$
$$\Rightarrow v_0 = 2v_2 - v_1$$
$$\Rightarrow v_0 = 4v_1 - v_1$$
$$\Rightarrow v_0 = 3v_1 \Rightarrow v_1 = \frac{v_0}{3}$$

K.E conservation

$$\cancel{\tfrac{1}{2}}\cancel{m}v_0^2 = \cancel{\tfrac{1}{2}}\cancel{m}v_1^2 + \cancel{\tfrac{1}{2}}\cdot 2\cancel{m}v_2^2$$
$$\Rightarrow v_0^2 = v_1^2 + 2v_2^2$$
$$\Rightarrow 4v_2^2 - 4v_2v_1 + \cancel{v_1^2} = \cancel{v_1^2} + 2v_2^2$$
$$\Rightarrow 4v_2^2 - 2v_2^2 = 4v_2v_1$$
$$\Rightarrow \cancel{2}v_2^{\cancel{2}} = \cancel{4}^{\,2}\cancel{v_2}v_1$$
$$\Rightarrow v_2 = 2v_1$$
$$\Rightarrow$$

Initial velocity

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v_0^2$$
$$\Rightarrow v_0^2 = 2gh$$

Elastic collisions SCREAM at you to use conservation of K.E.

For the first object,

$$mgh' = \frac{1}{2}mv_1^2$$
$$= \frac{1}{2}m\left(\frac{v_0}{3}\right)^2$$
$$= \frac{1}{2}mv_0^2/9$$
$$\cancel{mg}h' = \cancel{mg}h/9$$

So, $\boxed{h' = \frac{h}{9}}$

8. $\boxed{(A)}$ For damped oscillations in general T becomes larger.

9. $\boxed{(A)}$ Longest wavelength corresponds to α-transitions (transition from the immediate upper level).

Lyman series $\Rightarrow n_f = 1$
Balmer series $\Rightarrow n_f = 2$
Pashchen series $\Rightarrow n_f = 3$
Bracket series $\Rightarrow n_f = 4$.

So, for λ_1, $\boxed{n_f = 1 \text{ (Lyman)}; n_i = 2}$

$$\frac{1}{\lambda_1} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = R\left(1 - \frac{1}{4}\right) = \frac{3R}{4}$$
$$\Rightarrow \lambda_1 = \frac{4}{3R}$$

For λ_2, $\boxed{n_f = 2 \text{ (Balmer)}; n_i = 3}$

$$\frac{1}{\lambda_2} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = R\left(\frac{1}{4} - \frac{1}{9}\right) = R\left(\frac{9-4}{36}\right) = \frac{5R}{36}$$
$$\Rightarrow \lambda_2 = \frac{36}{5R}$$

So, $\frac{\lambda_1}{\lambda_2} = \frac{4}{3\cancel{R}} \times \frac{5\cancel{R}}{\cancel{36}_{9}} = \frac{5}{27}$.

10. $\boxed{(B)}$ Atomic transitions occur in the x-ray region.

11. $\boxed{D}$ Stern-Gerlach experiment $\Rightarrow$ A beam of neutral hydrogen atoms get splitted into two beams vertically.

12. $\boxed{C}$ Ground state of hydrogen, $E_0 \approx \mu$ $\quad [\mu \approx m_e]$

$\approx -13.6\, eV$

For positronium, the ground state gets halved, because $\mu \approx m_e/2$.

$$E_+ = -6.8\, eV$$

13. $\boxed{B}$ The energy $W = pt$

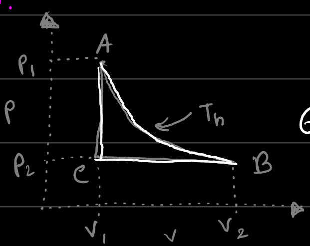
$$\Rightarrow ms\Delta\theta = pt$$

$$\Rightarrow 4200 \times 1 \times 1 \times 1 = 100 \times t \Rightarrow t \approx 42\, s.$$

14. $\boxed{D}$ Say, the final temperature is T_f. The heat lost by one is the heat gained by the other. $\quad \cancel{m}\cancel{c}(T_f - 0) = \cancel{m}\cancel{c}(100 - T_f) \Rightarrow T_f = 50K$

So, heat exchanged, $Q = mc\,(100-50) = 0.1 \times 1 \times 50 = 5\, kcal$.

15.



$\boxed{A \text{ to } B}$ Isothermal So, $dU = 0$

$$Q = W$$

$$Q_1 = W = \int P dV = \int_{V_1}^{V_2} \frac{RT_h}{V} dV = RT_h \int_{V_1}^{V_2} \frac{1}{V} dV = RT \ln(V_2/V_1)$$

$\boxed{B \text{ to } C}$ $P = P_2$ fixed. $Q = W + U$

$$Q_2 = P_2(V_1 - V_2) + C_v(T_h - T_c)$$

$\boxed{C \text{ to } A}$ No volume change, $dW = 0$ So, $Q = U$

$$Q_3 = C_v(T_c - T_h)$$

For formulas related to volume, $\boxed{V_f - V_i}$

For formulas related to temperature, $\boxed{T_i - T_f}$

So, net heat added to the system, $Q = Q_1 + Q_2 + Q_3 = RT_h \ln(V_2/V_1) + P_2(V_1 - V_2) + \cancel{C_v(T_h - T_c)} - \cancel{C_v(T_h - T_c)}$

$$= RT_h \ln\left(\frac{V_2}{V_1}\right) + R(T_c - T_h)$$

$$= RT_h \ln\left(\frac{V_2}{V_1}\right) - R(T_c - T_h)$$

16. $\boxed{B}$ Use common sense sometimes! $10^{-4}\, m \rightarrow$ visible range

$10^{-10}, 10^{-11}, 10^{-16}$ all are subatomic range.

So only suitable option is $10^{-7}\, m$

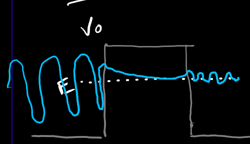
17. $\boxed{E}$ You get probability by squaring the wavefunction $|\Psi|^2$.

Total possibility, 0 to $1 \Rightarrow 1^2$; 1 to $2 \Rightarrow 1^2$; 2 to $3 \Rightarrow 2^2$; 3 to $4 \Rightarrow 3^2$; 4 to $5 \Rightarrow 1^2$

5 to $6 \Rightarrow 0^2$ | So, $1+1+4+9+1+0 = 16$.

Probability between $x=2$ and $x=4 \Longrightarrow 2^2 + 3^2 = 13$. So, the probability is $\frac{13}{16}$.

18. $\boxed{C}$ Whenever a particle approaches a finite potential barrier, there's a tunnelling phenomena.



On the left of the barrier, $V = 0$, so Ψ should be oscillatory.

Inside the barrier, tunnelling phenomenon takes place, Ψ should decrease exponentially.

On the right, $V = 0$ again. Ψ should be oscillatory again but with decreased amplitude.

19. $\boxed{(B)}$ Rutherford scattering $\Rightarrow$ We are given the scattering angle is 180°, so the collision is head-on collision and the particle returns from the point where all of its kinetic energy is counteracted by the repulsive potential energy.

For α-particles, $Z_1 = 2$

For silver, $Z_2 = 50$

$$V = 5\,\text{MeV} = \frac{Z_1 q_2}{r}\frac{1}{4\pi\varepsilon_0} = \frac{50\times(1.6\times10^{-19})^2\times2\times9\times10^9}{r}$$

$$\Rightarrow 5\times10^6\times1.6\times\cancel{10^{-19}} = \frac{100\times(1.6\times\cancel{10^{-19}})^2\times9\times10^9}{r}$$

$$\Rightarrow r = \frac{\overset{20}{\cancel{100}}\times1.6\times10^{-19}\times9\times10^9}{\cancel{5}\times10^6} = 180\times1.6\times10^{-16} = 18\times16\times10^{-16}$$

$$\approx 20\times15\times10^{-16} \approx 3\times10^{-14}$$

20. $\boxed{(D)}$ Momentum conservation

$$mv = -0.6mv + MV$$

$$\Rightarrow 1.6\,mv = MV$$

$$\Rightarrow V = \frac{1.6mv}{M}$$

K.E. conservation

$$\cancel{\tfrac{1}{2}}mv^2 = \cancel{\tfrac{1}{2}}m(0.6v)^2 + \cancel{\tfrac{1}{2}}MV^2$$

$$\Rightarrow mv^2 = 0.36\,mv^2 + MV^2$$

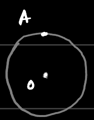
$$\Rightarrow 0.64\,mv^2 = MV^2$$

$$\Rightarrow 0.64\,mv^2 = \cancel{M}\cdot\frac{(1.6mv)^2}{\cancel{M^2}_{M}}$$

$$\Rightarrow M = \frac{(1.6)^2\,\overset{m}{\cancel{m^2v^2}}}{(0.8)^2\cancel{mv^2}} = \left(\frac{1.6}{0.8}\right)^2 m = 4m = 4.4u \quad [m = 4u]$$

$$= 16u$$

21. $\boxed{(C)}$ Circular hoop



Moment of inertia about center, $I_c = MR^2$

Moment of inertia about $A \Rightarrow I = I_c + MR^2 = 2MR^2$

$$T = 2\pi\sqrt{\frac{I}{mgr}} = 2\pi\sqrt{\frac{2\cancel{M}R^{\cancel{2}}}{\cancel{M}g\cancel{R}}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{2R}{g}}$$

$$\Rightarrow T^2 = 4\times\cancel{10}\times\frac{0.4}{\cancel{10}} = 1.6\,s \approx 1.3\,s$$

22. $\boxed{(C)}$ $\Delta x = 3600$ m

$\Delta y = 2$ m

$a = 0.4\times g = 4\,m/s^2$

$$\Delta y = \tfrac{1}{2}at^2$$

$$t = \sqrt{\frac{2\Delta y}{a}}$$

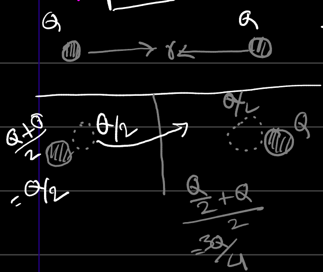
$$\Delta x = vt$$

$$\Rightarrow v = \frac{\Delta x}{t} = \frac{\Delta x\sqrt{a}}{\sqrt{2\Delta y}} = \frac{3600\times\cancel{2}}{\cancel{2}} = 3600\,m/s$$

$$= 3.6\,km/s$$

23. $\boxed{(E)}$ Bertrand's theorem: Stable, non-circular orbit can only occur for inverse square force or harmonic oscillator potential. So, (E) is not true.

24. $\boxed{(D)}$ Initial charges are Q. Initial force, $F_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{r^2}$



The charges get equally distributed after contact with another conductor. So, for the left ball, new charge is $Q/2$, and new charge on the right ball is $\frac{3Q}{4}$. So, new force, $F_2 = \frac{1}{4\pi\varepsilon_0}\frac{3Q^2}{8r^2} = \frac{3}{8}F_1$

25. $\boxed{(E)}$ Initially, s is open

$Q_0 = \frac{C}{V}$

$C = Q_0 V$

$\Rightarrow U_0 = \frac{1}{2}CV^2$

when s is closed.

(i) The charge Q_0 gets equally distributed. So $Q_0 = Q_1 + Q_2$

& $Q_0 = \frac{1}{2}(Q_1 + Q_2)$; $Q_1 = Q_2$; $U_1 = U_2$

(ii) The capacitors are in parallel, so voltage across them is the same $V_1 = V_2$

(iii) $U_1 = \frac{1}{2}CV^2$; $U_2 = \frac{1}{2}CV^2$ So, $U_1 + U_2 = CV^2$ So, $U_0 \neq U_1 + U_2$

26. $\boxed{(C)}$ Frequency, $f = 103.7\ MHz$

$= 103.7 \times 10^6\ Hz$

$\omega = 2\pi \times 103.7 \times 10^6$

Inductance, $L = 2\ \mu H$

$= 2 \times 10^{-6}\ H$

Resonance frequency, $\omega = \frac{1}{\sqrt{LC}}$

$\Rightarrow \omega^2 = \frac{1}{LC} \Rightarrow C = \frac{1}{\omega^2 L}$

$\Rightarrow C = \frac{1}{4 \times 10 \times 10^9 \times 10^{12} \times 2 \times 10^{-6}} = \frac{1}{8} \times 10^{-11}$

$\approx \frac{10}{8} \times 10^{-12} \approx 1\ pF$

27. $\boxed{(D)}$ [Graph-reading:] * Linear plots: Good for linear relationships.
$[eV_s = hf - W,\ V_s \text{ vs } f]$

* Semi-log plots: Good for exponential relations. $[dN/dt \text{ vs } t]$
* Log-log plots: Good for power-law relations $[P \text{ vs } T]$

Only for V_{out}/V_{in} vs ω, which is a power law relation, linear plot is not best.

28. $\boxed{(D)}$ We are given the velocity of the wave, $v = 0.5\ cm/ms$.
The wavelength of the wave is given in the plot as $\lambda = 6\ cm$.

The frequency of the larger wave is $f = \frac{v}{\lambda} = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12\ ms}$

$= \frac{1}{12 \times 10^{-3}\ s} = \frac{1000}{12} = \frac{\cancel{100}^{9} \times 10}{\cancel{12}}$

$\approx 90\ Hz$

29. Dimensional analysis: * $G = \frac{Nm^2}{kg^2} = \frac{\cancel{M}LT^{-2} \times L^2}{M^{\cancel{2}}} = \frac{L^3}{MT^2}$

Try to make an educated guess first!

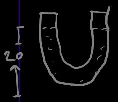
My guess is (E).

* $\hbar = Js = MLT^{-2} \times L^2 T = \frac{ML^2}{T}$

* $c = LT^{-2} = \frac{L}{T^2}$

$$\left(\frac{G\hbar}{c^3}\right)^{1/2} = \left[\frac{\frac{L^3}{\cancel{M}T^2} \times \frac{\cancel{M}L^2}{T}}{L^3T^{-3}}\right]^{1/2} = \left[\frac{L^5 \cancel{T^3}}{L^3 \cancel{T^3}}\right]^{1/2} = L$$

30. $\boxed{(C)}$ Archimedes's principle: Initially, both pipes had a height of 20 cm. After the fluid was poured in the left pipe, there was a depression of x cm, and the fluid itself was 5cm.



So, $h_1 = (20 - x) + 5$ & $h_2 = 20 + x$.

Archimedes's principle $\Rightarrow \rho' \times 5 - \rho \times x = \rho x$

$\Rightarrow x = 10$

So, $h_1 = (20 - 10) + 5 = 15$

$h_2 = 20 + 10 = 30$

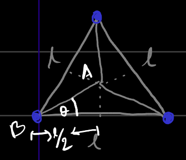
$h_2/h_1 = 2$

31. $\boxed{(E)}$ Objects falling through a viscous medium experiences a retarding force that depends on the velocity. As the velocity increases, so does the retarding force & after a certain point, the retarding force & the gravitational force balances out, leaving the object to fall with a constant terminal velocity. This terminal velocity does depend on b & m. $\boxed{F_{net} = ma - bv}$

32. $\boxed{(B)}$ A is the CM of the system. So, $I_A = 3m(AB)^2 = \frac{3ml^2}{4\cos^2\theta}$

And $I_B = I_A + \frac{3ml^2}{4\cos^2\theta} = \frac{3ml^2}{2\cos^2\theta}$ | So, $\frac{I_B}{I_A} = \frac{2\cancel{I_A}}{\cancel{I_A}} = 2$

Kinetic energy $K.E \propto I$. | So, $I_B : I_A = 2:1$



$\cos\theta = \frac{l/2}{AB}$

$\Rightarrow AB = \frac{l}{2\cos\theta}$

33. $\boxed{(C)}$ $\psi(\theta,\phi) = \left(\frac{5}{\sqrt{38}}Y_1^1 + \frac{3}{\sqrt{38}}Y_5^1 + \frac{2}{\sqrt{38}}Y_5^{-1}\right)$ Spherical harmonics $\Rightarrow Y_l^m$

Probability of finding $l=5 \Rightarrow \left(\frac{3}{\sqrt{38}}\right)^2 + \left(\frac{2}{\sqrt{38}}\right)^2 = \frac{9}{38} + \frac{4}{38} = \frac{13}{38}$

34. $\boxed{(D)}$ Beta-decay $\Rightarrow$ Weak interaction. Weak interaction violates parity symmetry/reflection invariance.

35. $\boxed{(A)}$ Wavefunctions of identical fermions are anti-symmetric. $\phi_1(x)\phi_2(x) = -\phi_2(x)\phi_1(x)$

And if we construct a state out of two fermions, $\Phi = \frac{1}{\sqrt{2}}(\phi_1(x)\phi_2(x) + \phi_1(x)\phi_2(x))$ the wavefunction becomes identically zero due to this antisymmetric property. So, it is forbidden for two identical fermions to occupy the same quantum state.

$\hookrightarrow$ Pauli exclusion principle.

36. $\boxed{(D)}$ Final rest energy $\Rightarrow Mc^2$. $v = \frac{3c}{5}$

Initial energy $\Rightarrow 2\cdot\frac{mc^2}{\sqrt{1-v^2/c^2}} = \frac{2mc^2}{\sqrt{1-\frac{9}{25}}} = \frac{2mc^2}{\frac{4}{5}} = \frac{10mc^2}{4} = \frac{5mc^2}{2}$

Conservation of energy $\Rightarrow M\cancel{c^2} = \frac{5m\cancel{c^2}}{2} = \frac{5\times 4}{2} = 10$ kg.

Invariant vs conserved

Momentum $\rightarrow$ Conserved, but not invariant.

Total relativistic energy $\rightarrow$ Both conserved & invariant.

Kinetic energy $\rightarrow$ Neither conserved nor invariant.

Rest mass $\rightarrow$ Invariant but not conserved.

37. $\boxed{(D)}$ Velocity addition rule $\Rightarrow w = \frac{u+v}{1+\frac{uv}{c^2}} = \frac{(0.3+0.6)c}{1+\frac{0.18c^2}{c^2}}$

$\Rightarrow w = \frac{0.9c}{1.18} \approx \frac{0.\cancel{9}^{3}c}{1.\cancel{2}_{4}} \approx 0.75c$

38. $\boxed{(D)}$ Momentum, $p = \gamma mv = 5\,MeV/c$ | $\frac{\cancel{\gamma m}v}{\cancel{\gamma m}c^{\cancel{2}}} = \frac{\cancel{5}MeV}{\cancel{10}_{2}MeV\cdot\cancel{c}}$

Total energy, $E = \gamma mc^2 = 10$ MeV. | $\Rightarrow v = \frac{1}{2}c$.

39. $\boxed{(E)}$ Lowest ionization energy ⇒ Valence shell must be almost empty.

(A) ${}^{2}_{4}He$ ⇒ A noble gas. (B) ${}^{7}_{14}N$ ⇒ Valence shell has 5 electrons.

(C) ${}^{8}_{16}O$ ⇒ 6 electrons. (D) ${}^{18}_{40}Ar$ ⇒ A noble gas. (~~F~~ E) ${}^{55}_{133}Cs$ ⇒ A metal.

40. $\boxed{(A)}$ Singly ionized Helium atom ⇒ A one-electron system. $z=2$

Ground state energy, $E_0 = \frac{z^2 \times 13.6\,eV}{n^2} = \frac{4\times 13.6}{n^2} = \frac{54}{n^2}$

So, not all energies are allowed. If $n_f=1$, $E_f = 54$ [Not in the option]

If $n_f = 2$, $E_f = \frac{54}{4} = 13.6\,eV$ [Not in the option]

If $n_f = 3$, $E_f = \frac{54}{9} = 6\,eV$. ✓

41. $\boxed{(A)}$ Electroscopic notation ⇒ ${}^{2S+1}L_J$ ${}^{3}P_{3/2}$ $2S+1=3 \Rightarrow S=1$ | For P, $l=1$ & $n=3$

(A) Transition to ${}^{2}S_{1/2}$. $l=0$ So, $\Delta l = -1$ $\boxed{\text{Allowed}}$ (B) $\Delta l = 0$ [Not allowed]

42. $\boxed{(B)}$ Photoelectric effect ⇒ $K_{max} = h\nu - \phi$ ⇒ $\frac{6.63\times10^{-34}\times3\times10^{8}}{5\times10^{-7}\times1.6\times10^{-19}} - 2.28\,eV$

$c = \nu\lambda$

$\nu = \frac{c}{\lambda}$

$= \frac{hc}{\lambda} - \phi$

$= \frac{\cancel{20}^{5}\times\cancel{10^{-26}}}{\cancel{8}_{2}\times\cancel{10^{-26}}} - 2.28\,eV = (2.5-2.28)\,eV$

$= 0.22\,eV$

43. $\boxed{(C)}$ Stoke's theorem ⇒ The SURFACE integral of the curl of a vector field is equal to the line integral of the vector field. $\oint(\vec\nabla\times\vec u)\cdot d\vec a = \oint \vec u\cdot d\vec l$

According to the question, $\oint(\vec\nabla\times\vec u)\cdot d\vec a = 2\pi R^2$. So, $\oint \vec u\cdot d\vec l = 2\pi R^2$

44. $\boxed{(A)}$ The velocity is given by $v(x) = \beta x^{-n}$

Acceleration, $a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$ | $\frac{dv}{dx} = -n\beta x^{-n-1}$

$\Rightarrow a = \beta x^{-n}\cdot(-n\beta x^{-n-1}) = -n\beta^2 x^{-2n-1}$

45. $\boxed{(E)}$ $V_{out} \approx V_{in}$ at high frequency
$V_{out} \approx 0$ at low frequency } ⇒ High-pass filter!

Remember: $X_L = \omega L$ [Inductors are high frequency choke]

$X_C = \frac{1}{\omega C}$ [For capacitors, we have low resistance at high frequency]

So, a capacitor must be connected in series to the input.

RC HIGH PASS ⇒ C in series, R to the ground] C allows high frequency

RC LOW PASS ⇒ R in series, C to the ground.] blocks low frequency

RL LOW PASS ⇒ L in series, R to the ground. { L allows low frequency

RL HIGH PASS ⇒ R in series, L to the ground } blocks high frequency.

46. [c)] Faraday's law $\Rightarrow \varepsilon = -\frac{d\Phi}{dt}$ | $\Phi = BA\cos(\omega t)$; $\frac{d\Phi}{dt} = -B\pi R^2\omega\sin(\omega t)$

$$\text{So, } \varepsilon = B\pi R^2\omega\cancel{\sin(\omega t)} = \varepsilon_0\cancel{\sin(\omega t)}$$
$$\Rightarrow \omega = \varepsilon_0 / B\pi R^2$$

47. [c)] Faraday's disc : $\Phi = BA = B\pi R^2$

Change in Flux, $\frac{d\Phi}{dt} = NB\pi R^2$

48. [(c)] Half-life of π^+ meson at rest is 2.5×10^{-8} s.

Only half of the π^+ produced reach the detector $\Rightarrow$ This implies the time taken by π^+ meson to reach the detector in its own rest frame, $\Delta\tau = 2.5\times10^{-8}$ s.

Distance in the lab frame, $\Delta x_\ell = 15$ m. | Work in natural units.

Speed of π^+ meson, $v = \frac{\Delta x_\ell}{\Delta t} = ?$ | Express $\Delta\tau$ in term of Δx_ℓ.

$$\text{Relativistic four velocity, } \eta = \frac{\Delta x}{\Delta\tau} = \frac{15}{2.5\times10^{-8}} = \frac{\cancel{15}^{2}\times3\times10^8}{\cancel{2.5}\times\cancel{3}} = 2 \quad [\text{Natural unit}, c=1]$$
$$\Rightarrow \Delta\tau = \frac{\Delta x}{2}$$

Rest frame ($\Delta x = 0$)

$\Delta s^2 = \Delta\tau^2$

Lab frame

$$\Delta\tau^2 = \Delta t^2 - \Delta x^2 \Rightarrow \Delta t^2 = \Delta\tau^2 + \Delta x^2 = \frac{\Delta x^2}{4} + \Delta x^2 = \frac{5\Delta x^2}{4}$$
$$\Rightarrow \frac{\Delta x^2}{\Delta t^2} = \frac{4}{5} \Rightarrow \boxed{\frac{\Delta x}{\Delta t} = \frac{2}{\sqrt5} = v}$$

49. [c)] Electric field of an infinite conducting plane, $\vec{E} = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2\varepsilon_0 XY}$

Now, for an observer moving in the x-direction, $\boxed{X = \gamma x'}$ & $\boxed{Y = Y'}$

$$\text{So, } E' = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2\varepsilon_0 x'Y'} = \frac{Q\gamma}{2\varepsilon_0 XY} = \frac{\sigma}{2\varepsilon_0\sqrt{1-v^2/c^2}}$$

The LAWS of electrodynamics are invariant under Lorentz transformation, but the fields themselves are not.

50. [(c)] Frame S

Two events occur at the same time, $\Delta t = 0$

3e minutes apart. $\Delta x = 3$

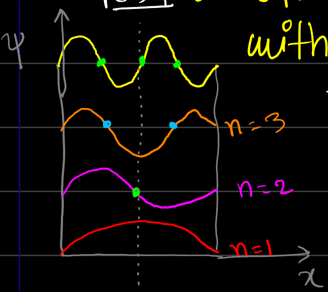
$$\Delta s^2 = \Delta x^2 - \Delta t^2 = 9$$

Frame S'

5e minutes apart in space. $\Delta x' = 5$

$$9 = \Delta x'^2 - \Delta t'^2 \Rightarrow \Delta t'^2 = 25 - 9$$
$$\Rightarrow \Delta t' = 4 \text{ minutes}.$$

51. [(B)] Wavefunction for infinite square well, $\psi_n = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$ with $n = 1, 2, 3, \ldots$



The probability in the middle of the well vanishes only for $n = 2, 4, 6, \ldots$ [even]

52. [(C)] First few spherical harmonics

$\boxed{Y_0^0 = \sqrt{\frac{1}{4\pi}}}$; $\boxed{Y_1^0 = \sqrt{\frac{3}{4\pi}} \cos\theta}$, $\boxed{Y_1^{\pm1} = \mp\sqrt{\frac{3}{8\pi}} \sin\theta\, e^{\pm i\phi}}$

So, $m = \pm 1$. and $m_z = +\hbar, -\hbar$

* The only spherical harmonic proportional to $\sin\theta$ is $Y_1^{\pm1}$
* The only spherical harmonic proportional to $\cos\theta$ is Y_1^0.

53. [(C)] Positronium is a spin singlet state. So, $S = 0$. To conserve spin, the decay product should be two photons (with spin ± 1) so that $S = 1 - 1 = 0$.

54. [(B)] Draw the diagram as you'd view them. The waves are

$\vec{E}_1 = E_0 \cos(kz - \omega t)\hat{x}$; $\vec{E}_2 = E_0 \cos(kz - \omega t + \pi)\hat{y}$

So, $\vec{E} = \hat{x} E_0 \cos(kz - \omega t) - \hat{y} E_0 \cos(kz - \omega t) \Rightarrow \vec{E}_2 = -E_0 \cos(kz - \omega t)$

The two waves are always in phase & equal in magnitude.

So the resultant field is oscillating at 135° from $+x$-direction.

55. [(A)] After decoupling two waves, the intensity is proportional to the sum of the squares of individual fields $\vec{E}_1$ & $\vec{E}_2$. $\boxed{I \propto E_1^2 + E_2^2}$

56. [(C)] Internal reflection: $\boxed{n \sin\theta > 1}$

For critical angle, $n \sin\theta_c = 1$. If $\theta > \theta_c$, there will be reflection.

$n = 1.3$ So, $1.3 \sin\theta_c = 1 \Rightarrow \sin\theta_c = \frac{1}{1.3} = \frac{10}{13} \approx 0.76$

13)100 (0.76
91
90

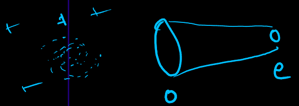
We know $\sin 45^\circ = 0.707$. So, θ_c should be a bit bigger than 45°. Perhaps 50°? YES!

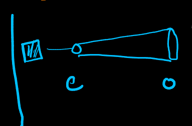
57. [(C)] Minima condition $\boxed{d \sin\theta = m\lambda}$ For first minima, $m = 1$ & $\sin\theta \approx \theta$.

$$\Rightarrow d\theta = \lambda$$

$$\Rightarrow d = \frac{\lambda}{\theta} = \frac{\cancel{4} \times 10^{-7}}{\cancel{4} \times 10^{-3}} = 1 \times 10^{-4}\ \text{m}$$

58. [(E)] [Telescope] vs [beam expander] The initial beam diameter of 1mm gets expanded into 10 mm. So, $M = 10$. A lens of $f_e = 1.5$ cm is put at the output of the laser. $f_o = ?$





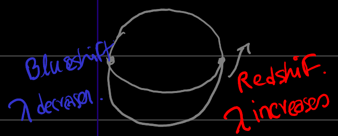
$M = \frac{f_o}{f_e} \Rightarrow f_o = M f_e = 10 \times 1.5 = 15$ cm. & $d = f_o + f_e = (15 + 1.5) = 16.5$ cm.

59. $\boxed{B}$ Energy of photons, $E = nh\nu = n\frac{hc}{\lambda}$; n = number of photons.

Power, $P = \frac{E}{t} = \frac{nhc}{\lambda t} \Rightarrow n = \frac{P\lambda t}{hc} = \frac{10\times10^{3}\times\cancel{6}\times10^{-7}\times10^{-15}}{\cancel{6.63}\times10^{-34}\times3\times10^{8}}$

$\Rightarrow n = \frac{10^{4-22}}{3\times10^{-26}} = \frac{1}{3}\times10^{-18+26} \approx 10^{7}$

60. $\boxed{B}$ Relativistic doppler shift $\Rightarrow \lambda_r = \sqrt{\frac{1\pm\beta}{1\mp\beta}}\,\lambda_s$ $\quad \lambda_s = 122$ nm.



The difference, $\Delta\lambda = \lambda_s\left[\sqrt{\frac{1+\beta}{1-\beta}} - \sqrt{\frac{1-\beta}{1+\beta}}\right] \Rightarrow \frac{\Delta\lambda}{\lambda} = \frac{\cancel{1}+\beta-\cancel{1}+\beta}{\sqrt{1-\beta^2}}$

As the movement is planetary, $\beta << 1$. $\frac{\cancel{1.8}^{3}\times10^{-12}}{\cancel{1.2}_{2}\times10^{-7}} = 2\beta \Rightarrow \frac{3}{4}\times10^{-5} = \beta = \frac{v}{c}$

$\Rightarrow \frac{9}{4}\times10^{-5+8} = v \quad \Rightarrow v = 2.2\times10^{3}$ m/s

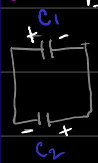
$\Rightarrow v = 2.2$ km/s.

61. $\boxed{B}$ Charge density $\rho(r) = Ar^2$

Applying Gauss's law, $\oint \vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon_0}\int_0^{R/2}\rho\,(4\pi r^2)\,dr$ $\quad | \quad dV = 4\pi r^2 dr$

$\Rightarrow \vec{E}\cdot\cancel{4\pi}\left(\frac{R}{2}\right)^2 = \frac{1}{\varepsilon_0}\cancel{4\pi}A\int_0^{R/2} r^4\,dr \quad \Rightarrow \vec{E} = \frac{4}{\varepsilon_0 R^2}\cdot A\cdot\frac{1}{5}\left[r^5\right]_0^{R/2} = \frac{\cancel{4}A}{5\varepsilon_0\cancel{R^2}}\left[\frac{\cancel{R^5}^{R^3}}{\cancel{2^5}_{8}}\right] = \frac{AR^3}{40\varepsilon_0}$

62. $\boxed{C}$ $\boxed{C_1 = 1\times10^{-6}\,F}$ $\boxed{C_2 = 2\times10^{-6}\,F}$



$V = 5$ V

$\boxed{Q_1 = C_1V = 5\times10^{-6}\,C}$ $\boxed{Q_2 = C_2V = 10\times10^{-6}\,C}$

Now, the charges would redistribute.

$Q_{new} = \frac{(5-10)\times10^{-6}}{2} = -\frac{5}{2}\times10^{-6}$ C in each plate.

Capacitance, $C = C_1 + C_2 = 3\times10^{-6}$ F

Total charge $Q = 2\times Q_{new} = 5\times10^{-6}$ C [Explicitly $(+\frac{5}{2}-\cancel{\frac{5}{2}}+\cancel{\frac{5}{2}}+\frac{5}{2})\times10^{-6} = 5\times10^{-6}$ C]

So, $V = \frac{Q}{C} = \frac{5\times\cancel{10^{-6}}}{3\times\cancel{10^{-6}}} \approx 1.7$ V

63. $\boxed{A}$ Basic Standard Model fact: Muon is a lepton, a fundamental particle.
All the others are baryons, composite particles.

64. $\boxed{C}$ Symmetric fission ⇒ One heavy nucleus get divided into 2 medium-weight nucleus.

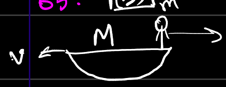
Suppose the heavy nuclei had N nucleons.

$\Delta E = 2E_{medium} - E_{heavy}$

$= (\cancel{2}\times\frac{N}{\cancel{2}}\times8 - N\times7)$ MeV $= N$ MeV.

For heavy nucleus, $N \approx 200$
So, liberated energy is
$\Delta E = 200$ MeV.

65. $\boxed{D}$



Momentum conservation

$mV = -Mv$

$\Rightarrow V = -\frac{Mv}{m}$

Work done by the man = Kinetic energy

$= \frac{1}{2}mV^2 = \frac{1}{2}\cancel{m}\cdot\frac{M^2v^2}{\cancel{m^2}_{m}} = \frac{1}{2}\frac{M^2}{m}v^2$

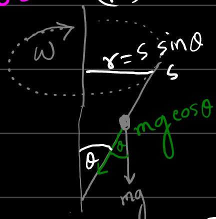
Work done by the boat $= \frac{1}{2}Mv^2$

Total work, $W = \frac{1}{2}\left(M + \frac{M^2}{m}\right)v^2$

66. $\boxed{E}$ A mission to the outer planet ⇒ The spacecraft must have escape velocity.
So, it must have a hyperbolic trajectory.

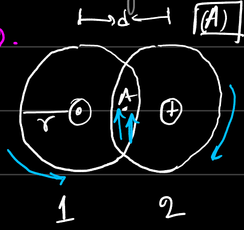
67. [C] Trivia: If the Earth were to be compressed to construct a blackhole, it would have the size of a grape ≈ 1cm.

68. (E).



Lagrangian of a system, $L = T - V$. For this system, $V = mgs\cos\theta$. T would have one contribution from $\dot{s}^2$ & one contribution from the rotational motion. $T = \frac{1}{2}m\dot{s}^2 + \frac{1}{2}I\omega^2$. The answer must be (E).

69.



[A] The net magnetic field at A will be twice the magnetic field due to one of the conductors.

For conductor 1, $\oint \vec{B}_1\cdot d\vec{l} = \mu_0 \int J\cdot da$

$\Rightarrow \vec{B}_1 2\pi \cancel{\frac{d}{2}} = \mu_0 J\pi \left(\cancel{\frac{d}{2}}\right)^2 \;\; d/2$

$\Rightarrow \vec{B}_1 = \frac{\mu_0}{2\pi} J\pi \frac{d}{2}$

So, net $\vec{B}$ field at A is

$\vec{B}_{Net} = \left(\frac{\mu_0}{2\pi}\right) J\pi d\; \hat{y}$

70. [D] Larmor's formula for power radiation $\boxed{P = \frac{q^2a^2}{6\pi\epsilon_0 c^3}}$ More importantly, $\boxed{P \propto q^2a^2}$

$P_A = k\, q_A^2 a_A^2$; $P_B = k\, 2^2 q_A^2 \cdot 4^2 a_A^2 \Rightarrow P_B = 64 P_A \Rightarrow P_B/P_A = 64$.

* Oscillating point charge $\Rightarrow P = \frac{q^2a^2}{6\pi\epsilon_0 c^3}$ $\boxed{P \propto q^2a^2}$
* Oscillating electric dipole $\Rightarrow P_E = \frac{\mu_0 p_0^2\omega^4}{12\pi c}$ $\boxed{P_E \propto p_0^2\omega^4}$ p_0 = Dipole moment

$I = \frac{\mu_0 p_0^2\omega^4}{32\pi c}\frac{\sin^2\theta}{r^2}$ [$\sin\theta$ indicates no radiation occurs along the dipole axis ($\theta = 0°$)]

*Magnetic dipole radiation

$\boxed{P_B = \frac{\mu_0 m_0^2\omega^4}{12\pi c^3}}$ $\boxed{P_B \propto m_0^2\omega^4}$

71. [A] Dimension analysis whatever goes inside the arctan must be dimensionless. (A) $\frac{L}{d}\frac{Vq}{mv^2} \Rightarrow \cancel{\frac{L}{L}} \times \frac{\cancel{ML^2T^{-2}}}{\cancel{ML^2T^{-2}}} = [1]$

72. [A] Positive feedback → Increased amplitude

Negative feedback → Diminished amplitude.

73. [C] For adiabatic expansion, $\boxed{PV^\gamma = c}$ Work done in expanding the gas from (V_i, P_i) to $(V_f, P_f) \Rightarrow$ $W = \int p\,dV$ $[P = cV^{-\gamma}]$

$= c\int V^{-\gamma}dV = \frac{c}{1-\gamma}\left[V^{1-\gamma}\right]_{V_i}^{V_f}$

$= \frac{c}{1-\gamma}\left\{\frac{V_f}{V_f^\gamma} - \frac{V_i}{V_i^\gamma}\right\} = \frac{1}{(1-\gamma)}\left(\frac{V_f c}{V_f^\gamma} - \frac{V_i c}{V_i^\gamma}\right) = \frac{P_fV_f - P_iV_i}{1-\gamma}$

74. [B] The final temperature is T_f. $\cancel{mc}(500 - T_f) = \cancel{mc}(T_f - 100) \Rightarrow T_f = 300K$.

So, $\Delta S_1 = \int_{500}^{300}\frac{mc\,dT}{T} = mc\ln\left(\frac{3}{5}\right)$; $\Delta S_2 = mc\ln(3)$

$\Delta S = \Delta S_1 + \Delta S_2 = mc\ln\left(\frac{3}{5}\times 3\right) = mc\ln(9/5)$

75. [(D)] Fourier's law of thermal conductivity $\Rightarrow Q = \frac{KA\Delta T}{\Delta x}$

So, $Q_A = \frac{0.8 \times A \times \Delta T}{4}$; $Q_B = \frac{0.025 \times A \times \Delta T}{2}$

$$\Rightarrow \frac{Q_A}{Q_B} = \frac{0.8 \times \cancel{A} \times \cancel{\Delta T}}{4} \times \frac{2}{0.025 \times \cancel{A} \times \cancel{\Delta T}} = \frac{1.6}{0.1} = 16$$

76. [(B)] I. Average momentum of a wave packet is not zero for ALL wave packets.

II. The width of the wave packet increases as $t \to \infty$. This is true for ALL wave packets.

III. As II is true, this can't be true.

IV. Uncertainty principle. True.

77. [(D)] Rewrite the given Hamiltonian in a convenient form.

$$H = -J\, S_1 \cdot S_2 = -\frac{J}{2}\left\{(S_1+S_2)^2 - S_1^2 - S_2^2\right\}$$

So, average energy, $\langle \psi | H | \psi \rangle = -\frac{J}{2}\left[\langle \psi | (S_1+S_2)^2 | \psi \rangle - \langle \psi | S_1^2 | \psi \rangle - \langle \psi | S_2^2 | \psi \rangle\right]$

$$= -\frac{J}{2}\left[(S_1+S_2)(S_1+S_2+1) - S_1(S_1+1) - S_2(S_2+1)\right]$$

78. [(E)] Basic electronics fact: In n-type semiconductors, the dopants modify the conductivity by DONATING electrons to the conduction band, & in p-type semiconductor, the dopants modify the conduction by accepting electrons/ DONATING holes to the conduction band.

79. [(D)] For ideal diatomic gas, $C_v = \frac{3}{2}R$ at very low temperature

$C_v = \frac{7}{2}R$ at very high temperature.

the ratio is $\frac{7}{\cancel{2}} \times \frac{\cancel{2}}{3} = \frac{7}{3}$

80. [(C)] Limiting cases! (i) If $\mu_r >> \mu_e$, the transmitted amplitude would be 0.

(ii) If $\mu_r = \mu_e$, the transmitted amplitude would be 1.

Only (c) fits these limiting conditions.

81. [(B)] Phenomenon of beat occurs when two waves occur at nearly the same frequency.

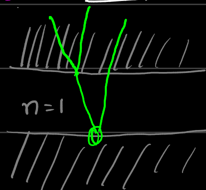
$$f_1 - f_2 = f_{beat}$$

$f_1 = 440\,Hz$ $f_2 = x f_0$ & $f_0 = 73.4\,Hz$ So, $(440.5 - 440) = \boxed{0.5\,Hz} \Rightarrow$ Number of beats per second.

$f_2 = 6f_0 = 440.5\,Hz$

$\hookrightarrow$ 6th harmonic gives the lowest number of beats per second!

82. $\boxed{(E)}$ There's one phaseshift. The extrema condition will be switched.



Maxima $\Rightarrow \boxed{2nt = (m+\frac{1}{2})\lambda} \Rightarrow 2t = (m+\frac{1}{2})\lambda \quad [n=1]$

$\lambda = 488\ nm$. $\Rightarrow t = (m+\frac{1}{2})\frac{\lambda}{2}$. For $m=0$; $t = \frac{\lambda}{4} \Rightarrow \boxed{t = 122\ nm}$ | For $m=1$, $t = \frac{3\lambda}{4}$ $\boxed{t = 366\ nm}$ | For $m=2$, $t = \frac{5\lambda}{4}$ $\boxed{t = 610\ nm}$

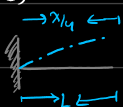
83 $\boxed{(D)}$ [Limiting case!] When $d \to \infty$, every peak & trough will be infinitely high & you won't even need any velocity to make them free fall. Only for (D) $v \to 0$ when $d \to \infty$. So, $v = \sqrt{g/kd^2}$.

84. (D) Lowest normal mode frequency is when the system oscillates together in phase. This is just $\sqrt{g/\ell}$.

Highest normal mode frequency is when the system oscillates out of phase. This might be $\sqrt{g/\ell + \frac{k}{m_1} + \frac{k}{m_2}}$

85. [Limiting case again!] (i) If $M \to \infty$, it would just be a both end fixed wave. So, $\boxed{\lambda = L}$ as $M \to \infty$ & $\frac{\mu}{M} \to 0$

(ii) When $M \to 0$, the left end would be free to move, an open end. So, $\boxed{\lambda = 4L}$ as $M \to 0$ & $\mu/M \to \infty$

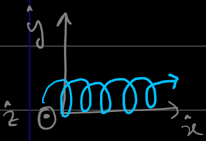


Only (B) is valid under these limiting conditions.

When $M \to \infty$, $\lambda = L \Rightarrow \frac{\mu}{M} = \frac{2\pi}{\lambda}\tan\frac{2\pi\lambda}{\lambda} = 0$

When $M \to 0$, $\lambda = 4L \Rightarrow \frac{\mu}{M} = \frac{2\pi}{\lambda}\tan\frac{2\pi\cancel{\lambda}}{\cancel{\lambda}\cdot 4_2} = \frac{2\pi}{\lambda}\tan\frac{\pi}{2} = \infty$.

86. $\boxed{(B)}$ $\vec{E}$ field in $+y$-direction ; $\vec{B}$ field in the $+z$ direction.

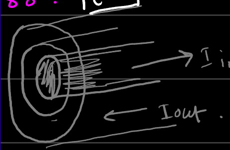


$\vec{F} = q\vec{E}\hat{y} + q(\vec{v}\times\vec{B})(\hat{y}\times\hat{z}) = q\vec{E}\hat{y} + qvB\hat{x}$

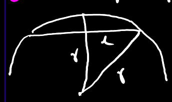
$\vec{B}$ causes cyclotron motion. $\vec{E}$ causes a drift velocity $\vec{v}_d = \frac{\vec{E}\times\vec{B}}{B^2} = \frac{E}{B}\hat{x}$ in the $+x$-direction.

87. $\boxed{(A)}$ $\frac{mv^2}{R} = qvB \Rightarrow \frac{mv}{R} = qB \Rightarrow mvR = L = qBR^2$

88. $\boxed{(B)}$ H goes to 0 at $r=0$ and $r>c$. There are two such options (B) & (D). Note that for (D), $H \to 0$ at $r=a$, which is not true. So, (B).



89. $\boxed{(D)}$



$\frac{mv^2}{r} = qvB \Rightarrow mv = qBr$ | $\cancel{r^2} = \ell^2 + (r-s)^2 = \ell^2 + \cancel{r^2} - 2rs + s^2$. And since $s \ll \ell$; $\ell^2 = 2rs \Rightarrow r = \frac{\ell^2}{2s}$. | $p = qB\frac{\ell^2}{2s}$.

90. WAS NOT SCORED.

91. $\boxed{(E)}$ This would violate second law of thermodynamics.

92. $\boxed{(D)}$ The given potential $V(x) = -ax^2 + bx^4$

Minima of the potential $V' = 0$

$$\Rightarrow -2ax_0 + 4bx_0^3 = 0 \quad \Rightarrow \cancel{4}bx_0^{\cancel{3}} = \cancel{2}a\cancel{x_0}$$

$$\Rightarrow x_0^2 = \frac{a}{2b} \quad \Rightarrow x_0 = \sqrt{\frac{a}{2b}}$$

We get the force $F(x) = -\frac{dV}{dx} = 2ax - 4bx^3$

$$\Rightarrow F(x-x_0) = 2a(x-x_0) - 4b(x-x_0)^3 = 2ax - 2ax_0 - 4b(x^3 - 3x^2x_0 + 3x_0^2x - x_0^3)$$

$$\Rightarrow F \approx 2ax - 4b\cdot 3x_0^2 x = 2ax - 12bx_0^2 x = (2a - 12bx_0^2)x$$

$$= \left(2a - \frac{12a}{2b}\right)x = (4bx_0^2 - 12bx_0^2)x = -\left(8\cancel{b}\cdot\frac{a}{2\cancel{b}}\right)x = -4ax$$

So, $k = 4a$ $\quad \Rightarrow \omega = \sqrt{\frac{k}{m}} = 2\sqrt{\frac{a}{m}}$

93. $\boxed{(D)}$ The potential has two contributions. (i) Harmonic oscillator potential $\frac{1}{2}kx^2$

(ii) $V = mgx$.

One half of T should be then $\pi\sqrt{\frac{m}{k}}$

Only (D) contains this.

94. $\boxed{(D)}$ N subsystems. Two states with energies 0 & ε.

Partition function, $Z = \sum e^{-E_i\beta} = 1 + e^{-\varepsilon/kT}$

Internal energy from partition function $\Rightarrow$ $\boxed{U = \frac{-\partial}{\partial\beta}\ln Z}$ for a single system.

As there are N subsystem, $\boxed{U = -N\frac{\partial}{\partial\beta}\ln Z}$ $\Rightarrow U = -N\cdot\frac{1}{Z}\frac{\partial Z}{\partial\beta}$

$$\frac{\partial Z}{\partial\beta} = \frac{\partial}{\partial\beta}(1 + e^{-\varepsilon\beta}) = -\varepsilon e^{-\varepsilon\beta} \quad \Rightarrow U = \frac{-N(-\varepsilon e^{-\varepsilon\beta})}{1+e^{-\varepsilon\beta}} = \frac{N\varepsilon \cancel{e^{-\varepsilon\beta}}}{\cancel{e^{-\varepsilon\beta}}(1+e^{\varepsilon\beta})}$$

$$\Rightarrow \boxed{U = \frac{N\varepsilon}{1+e^{\varepsilon/kT}}}$$

95. $\boxed{(E)}$ Common sense. When a material becomes superconducting, its C_v should also exhibit a sudden jolt/jump. Only (E) has this sudden jolt.

96. $\boxed{(A)}$ Difference between photons & all other particles is its ability to maintain its constant velocity across all reference frames.

97 $\boxed{(E)}$ Probability current density, $\boxed{j = \frac{-i\hbar}{2m}(\psi^*\nabla\psi - \psi\nabla\psi^*)}$

$$\psi(x,t) = e^{i\omega t}\alpha\cos(kx) + e^{i\omega t}\beta\sin(kx)$$

$$\psi^*(x,t) = e^{-i\omega t}\alpha^*\cos(kx) + e^{-i\omega t}\beta^*\sin(kx)$$

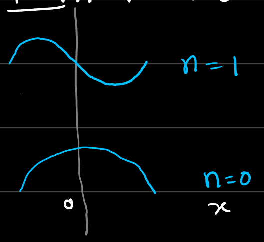
$$\Delta\psi = e^{i\omega t}k[\beta\cos(kx) - \alpha\sin(kx)]$$

$$\Delta\psi^* = e^{-i\omega t}k[\beta^*\cos(kx) - \alpha^*\sin(kx)]$$

So, $j = \frac{\hbar k}{2m}(\alpha^*\beta - \beta^*\alpha)$

Putting an infinite barrier enforces a boundary condition that ψ must go to 0 at that poin

98. $\boxed{\text{(D)}}$ Harmonic oscillator potential $V(x) = \frac{1}{2} m\omega^2 x^2$. If there's an infinite barrier at $x = 0$, $n = 1, 3, 5$ won't be affected. But all $n = 0, 2, 4$ will vanish.



So, $E_n = \frac{3\hbar\omega}{2}$ $(n=1)$ | $E_n = \frac{7\hbar\omega}{2}$ $(n=3)$ | $E_n = \frac{11\hbar\omega}{2}$ $(n=5)$

99. $\boxed{\text{(B)}}$ Metastable state should not include $n=1$ or $n=3$. So, $n=2$ only.

100. $\boxed{\text{(C)}}$ Raising & lowering operators.

I. They commute only with spherically symmetric Hamiltonian. So, not always true.

II. $\hat{A}^\dagger$ & $\hat{A}$ operators are NON-HERMITIAN operators.

III. $\hat{A}^\dagger \neq \hat{A}$ NON-HERMITIVITY.