GR9277

1. $\boxed{\text{(C) } \hbar k}$ The wavefunction $e^{i(kx-\omega t)}$ is just the free particle wavefunction with momentum $p = \hbar k$. And it's also one dimensional.

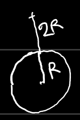
2. $\boxed{\text{(D) } 2d}$ Bragg diffraction: $\boxed{d \sin\theta = n\frac{\lambda}{2}}$

Longest wavelength is when $n=1$. So, $d\sin\theta = \frac{\lambda}{2}$

$\Rightarrow \lambda = 2d\sin\theta$

And the max. value of $\sin\theta = 1$. So, $\lambda_{max} = 2d$.

3. $\boxed{\text{(C) } 4}$ Outside the surface $(r > R)$ the law follows the inverse square law. $\boxed{F(r) \propto \frac{1}{r^2}}$ $(r > R)$



So, $F(R) = \frac{k}{R^2}$; $F(2R) = \frac{k}{4R^2}$

$\Rightarrow F(2R) = \frac{1}{4}F(R) \Rightarrow \frac{F(R)}{F(2R)} = 4$

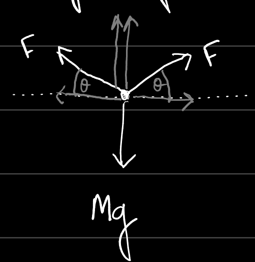
4. $\boxed{\text{(C) } 2}$ Inside Earth, the law doesn't follow the inverse square law anymore. It linearly grows as r is increased $(r < R)$

$\boxed{F(r) \propto r}$ $(r<R)$

So, $F(R) = kR$; $F(R/2) = \frac{kR}{2}$ $\Rightarrow F(R/2) = \frac{F(R)}{2}$

$\Rightarrow \frac{F(R)}{F(R/2)} = 2$.

5. $\boxed{\text{(D) } \frac{2\mu m}{1-\mu}}$ Free-body diagram for the box:



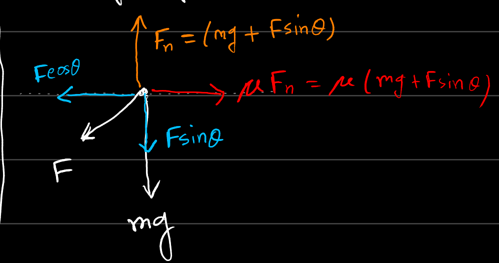
$2F\sin\theta = Mg$

$\Rightarrow 2F\sin 45^\circ = Mg$

$\Rightarrow \cancel{2}F \cdot \frac{\sqrt{2}}{\cancel{2}} = Mg$

$\Rightarrow \boxed{F\sqrt{2} = Mg}$

Free-body diagram for wedge.



$\frac{F\sqrt{2}}{2} = \mu mg + \mu\frac{F\sqrt{2}}{2}$

$\Rightarrow \frac{F\sqrt{2}}{2}(1-\mu) = \mu mg$

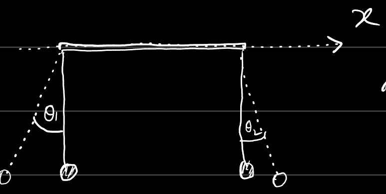
$\Rightarrow M\cancel{g}(1-\mu) = 2\mu m\cancel{g}$

$\Rightarrow \boxed{M = \frac{2\mu m}{1-\mu}}$

IMPORTANT!

[Remember! Whenever you get stuck at a step, either proceed to the next step immediately, you might find a missing clue there, or skip & comeback. But no matter what, DO NOT STAY STUCK!]

7. $\boxed{(A)\ \sqrt{g/L}}$



3 DOF $\Rightarrow$ 3 normal modes.

I. 0 [They oscillate out of phase]

II. $\sqrt{\frac{g(M+2m)}{l\,M}}$ [This depends on x or M]

The only obvious choice is that the mass does not move, and they oscillate with identical frequency but completely out of phase. $\sqrt{g/l}$

Time consuming + tricky.

8. $\boxed{(C)}$ $\vec{\tau} = \vec{r} \times \vec{F}$

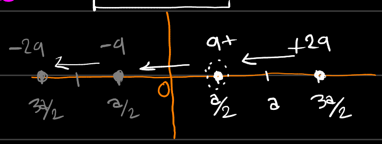
Work out only directions!

$F \propto \hat{k}$ won't work as torque! Eliminate A, B, E.

(C) $(-b\hat{i} - c\hat{k}) \times a\hat{j} = -b\hat{k}$ (Negative in z-dir!)

9. $\boxed{(A)\ \text{Zero.}}$ Induction is M. $\Phi = MI$. But here $B = 0$ so $\Phi = 0$; $M = 0$

10. (E) $\boxed{\frac{1}{4\pi\varepsilon_0}\frac{7q^2}{2a^2}}$



Net force on q, $F = F_{+2q} + F_{-q} + F_{-2q}$

Forces are all in the same direction, so they add.

$$F = \frac{1}{4\pi\varepsilon_0}\left(\frac{2q^2}{a^2} + \frac{q^2}{a^2} + \frac{2q^2}{4a^2}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}\left(2 + 1 + \frac{1}{2}\right) = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}\left(3 + \frac{1}{2}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{7q^2}{2a^2}$$

11. $\boxed{(F)\ \frac{RC\ln 2}{2}}$ For RC circuits, $\boxed{Q = Q_0 e^{-t/RC}}$ Response time $\tau = RC$

Initially, $U_0 = \frac{1}{2}\frac{Q_0^2}{C}$

Finally, $U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q_0^2}{C}e^{-2t/RC}$

But $U = \frac{1}{2}U_0$

$$\Rightarrow \frac{1}{\cancel{2}}\frac{\cancel{Q_0^2}}{\cancel{C}}e^{-2t/RC} = \frac{1}{\cancel{4}_{2}}\frac{\cancel{Q_0^2}}{\cancel{C}}$$

$$\Rightarrow e^{-2t/RC} = \frac{1}{2} \Rightarrow -\frac{2t}{RC} = -\ln 2 \Rightarrow 2t = RC\ln 2 \Rightarrow t = \frac{RC\ln 2}{2}$$

12. $\boxed{(B)\ \frac{V_0\phi}{\alpha}}$ Laplace's equation $\boxed{\nabla^2 V = 0}$ Since $V = V(\phi)$ only,

We can write $\frac{d^2V}{d\phi^2} = 0$

Integrate once $\frac{dV}{d\phi} = A \Rightarrow dV = A\,d\phi$

Integrate twice $V(\phi) = A\phi + B$

Boundary conditions, when $\phi = 0$, $V = 0$

$0 = 0 + B$ So, $B = 0$

$\Rightarrow V(\phi) = A\phi$

when $\phi = \alpha$, $V = V_0$; $V(\alpha) = A\alpha = V_0$

$\Rightarrow A = \frac{V_0}{\alpha}$

So, $\boxed{V(\phi) = \frac{V_0 \phi}{\alpha}}$

Or, you could just say, since the plates are too large, the field $\vec{E}$ is constant. $\vec{E} = -\nabla V$ is constant.

which V gives ∇V = constant? $\frac{d}{d\phi}\frac{V_0\phi}{\alpha} = \frac{V_0}{\alpha}$, a constant.

13. $\boxed{\text{(D) II \& IV}}$

Maxwell's equations:

Gauss's law $\Rightarrow \vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$ .... (1)

No magnetic charge $\Rightarrow \vec{\nabla}\cdot\vec{B} = 0$ .... (2)

Faraday's law $\Rightarrow \vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$ .... (3)

Ampere's law $\Rightarrow \vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \frac{\partial \vec{E}}{\partial t}$ ...(4)

(1) says static charge distribution generates $\vec{E}$ field.

(2) says there are no magnetic charge/monopole.

[If there were, this would be false & it would have been $\vec{\nabla}\cdot\vec{B} = \mu_0\rho_B$]

(3) says changing magnetic field generates $\vec{E}$ field.

[If there was a magnetic monopole, there would have been a magnetic current term there like (4).]

(4) says changing electric field generates $\vec{B}$ field

[As there exists electric charges, there is a current term $\mu_0\vec{J}$]

So, if there were a magnetic charge, (2) & (3) would've been wrong.

14. $\boxed{\text{(C)}}$

Blackbody radiation

$\boxed{Q \propto T^4} \Rightarrow Q_1 = kT^4$ $\quad kT^4 = mc\Delta\theta = mc \times 0{\cdot}5$

Heat

$\boxed{Q = mc\Delta\theta}$

If T were doubled, $Q_2 = 16kT^4$ $\quad 16kT^4 = mc\,\Delta\theta'$

$\Rightarrow \cancel{16}^{8} \times \cancel{mc} \times \frac{1}{\cancel{2}} = \cancel{mc}\,\Delta\theta' \Rightarrow \Delta\theta' = 8$

So the temperature would have been gone from $20^\circ C$ to $(20+8) = 28^\circ C$!

For diatomic ideal gases:

15. $\boxed{(C)\ \frac{7}{2}R}$

Low temperature ($T < 100\,K$) $\Rightarrow$ 3 translational DOF

$$\boxed{C = \frac{3}{2}R}$$

Moderate temperature ($100\,K < T < 500\,K$) $\Rightarrow$ 3 translational + 2 rotational = 5 DOF

$$\boxed{C = \frac{5}{2}R}$$

High temperature ($T > 500\,K$) $\Rightarrow$ 3 translational + 2 rotational + 2 vibrational = 7 DOF

$$\boxed{C = \frac{7}{2}R}$$

[This appeared again on #GIR9677 with slight variation]

16. $\boxed{(A)\ 400\,J}$

$T_H = 727°C = (727+273)K = 1000\,K$

$T_c = 527°C = (527+273)K = 800\,K$

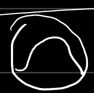
Heat input, $Q_H = 2000\,J$; Wasted heat Q_c ; So, output $W = Q_H - Q_c$

So, Efficiency, $\boxed{\eta = \frac{\text{Output}}{\text{Input}} = \frac{W}{Q_H} = \frac{Q_H - Q_c}{Q_H}}$

Again, $\eta = \frac{T_H - T_c}{T_H}$ $\Rightarrow 0.2 = \frac{W}{2000}$

$= \frac{1000 - 800}{1000}$ $\Rightarrow W = \frac{2 \times 200\cancel{0}}{1\cancel{0}} = 400\,J.$

$= \frac{20\cancel{0}}{100\cancel{0}} = 0.2$

17. (A)  What we are looking for here is a superposition of two trig functions (sine waves). Only (A) looks like it could be a superposition of two sine waves.

18. $\boxed{(C)}$ In coaxial cables, impedance matching is necessary. So the cable is terminated at one end with characteristic impedance in order to stop reflections.

19. $\boxed{A}$ Mass of the Earth is always 6×10^{24} kg. $\boxed{\text{Duh!}}$

20. $\boxed{\text{(D) } 2d = 3w}$ Missing interference maxima ⇒ Constructive interference coincide with single-slit diffraction minima.

Interference maxima	Diffraction minima
$d \sin\theta = m\lambda$	$w \sin\theta = n\lambda$

$$\frac{d}{w} = \frac{m}{n} \Rightarrow nd = mw$$

This narrows our answers to C, D, E.

From the figure it is obvious that $\boxed{w < d}$

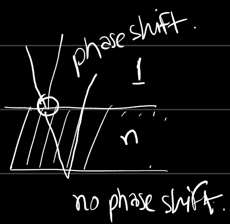
So, $d = \left(\frac{m}{n}\right) w$ $\quad \frac{m}{n}$ must be a fraction $\left(\frac{m}{n} < 1\right)$

(C) $2d = w \Rightarrow d < w$ Eliminated!

(D) $2d = 3w \Rightarrow w < d$ Correct!

(E) $3d = 2w \Rightarrow d < w$ Eliminate!

21. $\boxed{\text{(E) II, III \& IV}}$



I. Absolutely illogical

II. Constructive interference $\Rightarrow 2t = \lambda/2 \Rightarrow 4t = \lambda$

Destructive interference $\Rightarrow 2t = \lambda$

Thickness is in general less than λ $[t < \lambda]$

} True in general

III. There is a phase shift at the front surface $(n > 1)$ True

IV. No phase shift on the back surface $(1 < n)$ True

22. For telescope ⇒ Two lenses: 1. Objective (bigger) 2. Eyepiece.

Focal length f_o $\qquad$ Focal length f_e

Magnification $\boxed{M = \frac{f_o}{f_e}}$

Distance $\boxed{d = f_o + f_e}$

[Important! A beam expander is the opposite of a telescope. Telescope converges, expander diverges. See #58 #GR9677. For expander $f_o < f_e$. But the formula stays the same.]

$M = 10 = \frac{f_o}{f_e}$ $\qquad f_o = 1\,m$

$\Rightarrow f_e = \frac{1}{10} = 0.1\,m$

So, $d = (1 + 0.1)\,m = 1.1\,m$

$\boxed{(E)\ 2\times 10^6}$

23. Average speed of conduction electron is given by $v = \sqrt{\frac{2E_f}{m}}$
E_f being Fermi energy $E_f = kT$

$$\Rightarrow v = \sqrt{\frac{2kT}{m}} = \sqrt{\frac{2\times 10^{-23} \times 8 \times 10^4}{10\times 10^{-31}}} = \sqrt{\frac{16\times 10^{-19}}{10^{-30}}}$$

$$= 4\sqrt{10^{-19+30}} = 4\times 10^{11/2} \simeq 10^6$$

24. Argon is a noble gas. Eliminate all but (E)

25. Deep underground Neutrino Experiment (DUNE) $\Rightarrow$ (D) Muons & neutrinos.

26. $\boxed{(B)\ 7m}$ Graph reading $\Rightarrow$ At $t=0$, the counts per minute was 6×10^3. [6 ticks after 10^3]

Half-time $\Rightarrow$ The time elapsed till 6×10^3 reduces to 3×10^3 counts per min.
From the graph, the time is close to 10.

27. (B) $\boxed{\Delta k = \frac{1}{\Delta x_0}}$ Uncertainty relation $\Delta x\, \Delta p \simeq \hbar$

But $\Delta p = \hbar \Delta k \Rightarrow \cancel{\hbar}\Delta k \simeq \frac{\cancel{\hbar}}{\Delta x}$

$$\Rightarrow \Delta k \simeq \frac{1}{\Delta x}$$

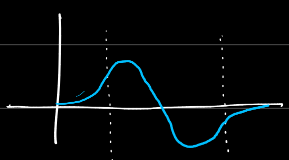
28. $\boxed{(E)\ 13/15}$ $\Psi(\theta,\phi) = \left[\frac{5}{\sqrt{30}} Y_4^3 + \frac{1}{\sqrt{30}} Y_6^3 - \frac{2}{\sqrt{30}} Y_6^0\right]$ $Y_l^m \Rightarrow$ Spherical harmonics.

Probability at $m=3$ is $\left(\frac{5}{\sqrt{30}}\right)^2 + \left(\frac{1}{\sqrt{30}}\right)^2$

$$= \frac{26}{30} = \frac{13}{15}$$

29. $\boxed{B}$ Potential well $\Rightarrow$ Inside the well $\rightarrow$ Oscillatory
Outside the well $\rightarrow$ Decays.

Only (B) satisfies this condition.



30. $\boxed{(E)\ E_0/8}$ Hydrogen ground state $E_0 = 13.6 \approx \boxed{\mu \approx m_e}$

For positronium, $\boxed{\mu \approx m_e/2}$ So ground state is $E_0/2$

And for n, $\boxed{E = \frac{E_0'}{n^2}}$ So, $E = \frac{E_0}{2 \times 2^2} = \frac{E_0}{8}$

31. $\boxed{(B)\ 1\ \text{only}}$ Spectroscopic notation $^{2S+1}L_J$ [For S, $l=0$; For P, $l=1$; For D, $l=2$ ...]

^3S

Here, $2S+1 = 3$

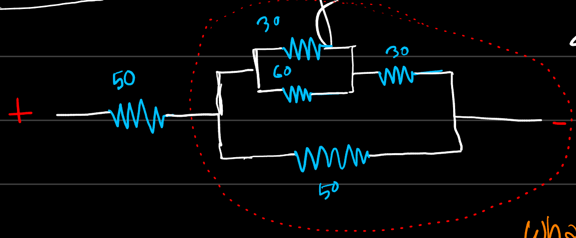
$\Rightarrow S = 1$ & $l = 0$

So, Total angular momentum $J = |l-s|, \ldots\ldots |l+s|$

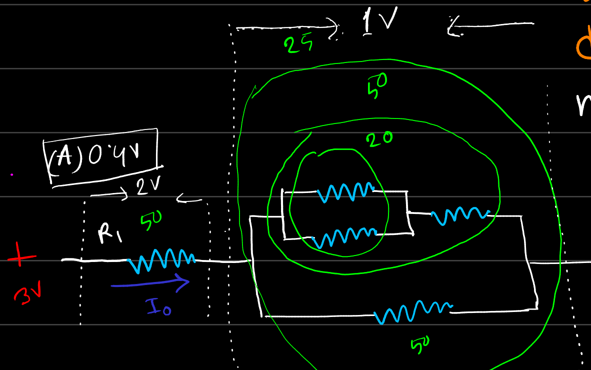
$|l-s| = 1$ So, $J = 1$

$|l+s| = 1$

32. $\boxed{(A)\ R_1}$ Reorienting the circuit. The whole red marked part actually shares the initial current I_0 passing through R_1, because this whole part is in parallel connection. whatever current this part recieves is divided into all the branches. So R_1 has the max. current flowing through it.



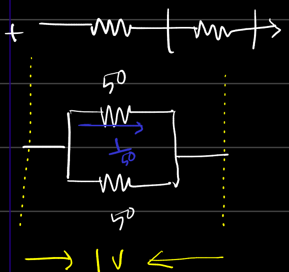
33. $\boxed{(A)\ 0.4V}$



Equivalent resistance $R_{eq} = 75\,\Omega$

So, Initial current $I_0 = \frac{V}{R_{eq}} = \frac{3}{75} = \frac{1}{25}$

Voltage drop across $R_1 \Rightarrow V_{R_1} = I_0 R_1 = \frac{1}{25} \times 50 = 2V$

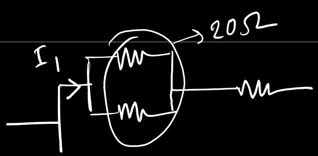


So, voltage drop across the whole parallel part is 1V

Voltage drop same, different current.

Current through the upper arm, $I_1 = \frac{V}{50} = \frac{1}{50}$

This current gets divided again.



$I_1 = \frac{1}{50}$ $R_{eq} = 20\Omega$

So, voltage drop across R_3 & R_4, $V = I_1 R$

$= \frac{20}{50} = 0.4V$

34. (D) Boundary conditions for perfect conductors: $\boxed{\Delta E^{\parallel} = 0}$ $\boxed{\Delta B^{\perp} = 0}$

35. $\boxed{B}$ 6° 2000 lines per cm $\longrightarrow$ Slit width is $a = \dfrac{1}{2000} = 0.5\times10^{-3}$ cm $= 5\times10^{-4}$ cm

$\Rightarrow a = 5\times10^{-6}$ m

$\lambda = 5200\times10^{-10}$ m

$$a\sin\theta = m\lambda$$

$$\Rightarrow \sin\theta = \frac{1\times5200\times10^{-10}}{5\times10^{-6}} = \frac{1\times5.2\times10^{3-7}}{5\times10^{-6}} \approx 0.1$$

So, $\sin\theta \approx \theta \approx 0.1$

Convert from radian to degree: π rad $\rightarrow 180°$

1 rad $\rightarrow \dfrac{180}{\pi}$

0.1 rad $\rightarrow \dfrac{180\times1}{\pi\times10} = \dfrac{\cancel{180}^{\,6}}{\cancel{30}} = 6°$

36. Upon reflections from a conductor, the electric field $\vec{E}$ gets reversed, but magnetic field $\vec{B}$ does not.

Solution by using boundary conditions

Since the wave falls normally on the conductor, the $\vec{E}$ fields & $\vec{B}$ fields are parallel to the conductor surface. Parallel components of the $\vec{E}$-fields across a conductor is continuous. So, $\vec{E}_{inside} = \vec{E}_{outside}$

But $\vec{E}_{inside} = 0$ since it's a conductor, and $\vec{E}_{outside} = \vec{E}_{incident} + \vec{E}_{reflected}$.

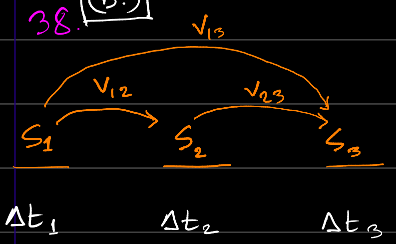
$$\Rightarrow \vec{E}_{incident} + \vec{E}_{reflected} = 0$$

$$\Rightarrow \vec{E}_{incident} = -\vec{E}_{reflected}.$$

For $\vec{B}$ field, we can use the relation $\boxed{\vec{B} = \frac{1}{c}(\hat{k}\times\vec{E})}$ to show that $\vec{B}$ doesn't reverse.

37. $\boxed{(A)\ -c\,\hat{k}}$ Lightspeed is constant for all inertial frames. What matters is the direction. γ_1 moved forward with speed c, so γ_2 will move backwards, but also with speed c.

38. $\boxed{(B.)}$



Time is stretched/elongated in all but the rest frames!

S_1 is the rest frame. Time will be short in all other frames.

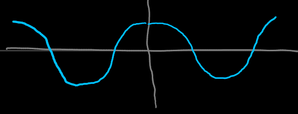
$\Delta t_2 = \gamma\,\Delta t_1$	$\Delta t_3 = \gamma\,\Delta t_1$
$\Rightarrow \Delta t_1 = \Delta t_2\sqrt{1 - V_{12}^2/c^2}$	$\Rightarrow \Delta t_1 = \Delta t_3\sqrt{1 - V_{13}^2/c^2}$
Eliminate (A)	(B)

39. $\boxed{B}$ Looking at the graph, the function is odd (not symmetric about the origin). So the function must be a sine wave!

Eliminate all cosine functions (C), (D), (E)

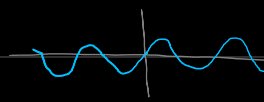
Option (A) gives trivially $v(t) = 0$ because $\sin\left[\frac{2\pi n}{f} t\right] = 0$ for all n

* Cosine waves are even functions $\Rightarrow$ $\boxed{\cos(-x) = \cos(x)}$



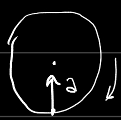
Symmetric about origin.

* Sine functions are odd functions $\Rightarrow$ $\boxed{\sin(-x) = -\sin(x)}$



NOT symmetric about the origin.

40. $\boxed{C}$ Rolling without slipping $\Rightarrow$ No tangential acceleration. Only centripetal acceleration.



41. $\boxed{(D)\ 9600\,J}$ $I = 4\,kg \cdot m^2$ $\quad \omega_i = 80\,rad/s$ $\quad \omega_f = 40\,rad/s$

$$E_i = \frac{1}{2} I \omega_i^2 = \frac{1}{2} \times 4 \times 80^2 \qquad E_f = \frac{1}{2} I \omega_f^2 = \frac{1}{2} \times 4 \times 40^2$$

$$\Delta E = \frac{1}{2} \times \cancel{4}^{\,2} \times (80^2 - 40^2) = 2(80+40)(80-40) = 2 \times 120 \times 40 = 9600\,J$$

42. $\boxed{(D)\ 16\,N \cdot m}$ $t = 10$ $\quad \boxed{\tau = I\alpha}$ $\quad \omega_f = \omega_i - \alpha t \Rightarrow \alpha t = 80 - 40$

$\tau = 4 \times 4 = 16\,Nm.$ $\quad \Rightarrow \alpha = 4\,rad/s^2$

43. $\boxed{B}$ Generalized momentum is $\boxed{p = \frac{\partial L}{\partial q}}$ and $\dot{p} = 0$ so it is a constant.

Not to be confused with the Hamilton's equations $\boxed{(i)\ \dot{q} = \frac{\partial H}{\partial p}}$

$\boxed{(ii)\ \dot{p} = -\frac{\partial H}{\partial q}}$

44. $\boxed{A}$ $y = ax^2 \Rightarrow x^2 = \frac{y}{a} \Rightarrow x = \frac{y^{1/2}}{\sqrt{a}}$ $\quad \dot{x} = \frac{y^{-1/2} \cdot \dot{y}}{2\sqrt{a}} = \frac{\dot{y}}{2\sqrt{ay}} \Rightarrow \dot{x}^2 = \frac{\dot{y}^2}{4ay}$

$T = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} m\dot{y}^2 \qquad V = mgy$

$= \frac{1}{2} m\dot{y}^2\left[1 + \frac{1}{4ay}\right]$ Lagrangian $\boxed{L = T - V} \Rightarrow L = \frac{1}{2} m\dot{y}^2\left(1 + \frac{1}{4ay}\right) - mgy$

45. $\boxed{D}$ $\cancel{m}gh = \frac{1}{2}\cancel{m}v^2 \Rightarrow v = \sqrt{2gh}$ $\quad v_i = 0.8 \times \sqrt{2gh}$ $\quad h' = ?$

$v_f^2 = 0 = v_i^2 - 2gh'$

$\Rightarrow \cancel{2g}h' = 0.64 \times \cancel{2g}h \Rightarrow \boxed{h' = 0.64h}$

46. $\boxed{B}$ $\boxed{\text{Critical isotherm} \Rightarrow \text{A curve in P-V diagram for which } \frac{\partial P}{\partial V} = 0}$

Only curve 2 has a point where taking tangent gives a horizontal line $\left(\frac{\partial P}{\partial V} = 0\right)$.

47. $\boxed{B}$ Find a middle ground where P & V are not at extreme limits.

48. $\boxed{C}$ [Combining uncertainties:]

1. $f = aA$; $\sigma_f = a\sigma_A$
2. $f = A \pm B$; $\sigma_f = \sqrt{\sigma_A^2 + \sigma_B^2}$
3. $f = AB$; $\frac{\sigma_f}{f} = \sqrt{\frac{\sigma_A^2}{A^2} + \frac{\sigma_B^2}{B^2}}$

$F = ma$

$$\frac{\sigma_F}{F} = \sqrt{\left(\frac{\sigma_m}{m}\right)^2 + \left(\frac{\sigma_a}{a}\right)^2}$$

Weighted average: We measure the same quantity X in two different techniques that yields values x_1 & x_2. The measurements can be combined to give a single value using weighted average. $X = \dfrac{\frac{x_1}{\sigma_{x_1}^2} + \frac{x_2}{\sigma_{x_2}^2}}{\frac{1}{\sigma_{x_1}^2} + \frac{1}{\sigma_{x_2}^2}}$

& the single uncertainty $\sigma_T = \dfrac{1}{\frac{1}{\sigma_{x_1}^2} + \frac{1}{\sigma_{x_2}^2}}$

49. $\boxed{B}$ Muons are really fast-moving particles. Approximate their speed to be $v_\mu \approx 3 \times 10^8$ m/s.

So, $\Delta t = \frac{\Delta x_\mu}{v_\mu} = \frac{\cancel{3}}{\cancel{3} \times 10^8} = 10^{-8}$ s $= 10 \times 10^{-1} \times 10^{-8} = 10$ ns.

So the time-scale is in nanoseconds.

50. $\boxed{B}$ Basic Fact: All wavefunctions can be expanded in a set of basis which are simultaneous eigenfunctions of two operators A & B $\boxed{\text{if the operators commute.}}$ In other words, if two operators commute, they share a set of simultaneous eigenfunctions in which all wavefunctions can be expanded. The two operators can be measured simultaneously & measuring one doesn't disturb the other.

51. $\boxed{A}$ 0 Momentum expectation value vanishes for infinite square well.

More rigorously, $\langle p \rangle = \int -i\hbar \frac{\partial}{\partial x} |\psi|^2 dx$

$$= -i\hbar \frac{2}{a} \int \frac{\partial}{\partial x} \sin^2\left(\frac{n\pi x}{a}\right) dx$$

$$= \frac{-2i\hbar}{a} \int \frac{n\pi}{a} \cdot 2\sin\left(\frac{n\pi x}{a}\right) \cos\left(\frac{n\pi x}{a}\right) dx$$

$$= \frac{-4in\pi\hbar}{a^2} \int \sin\left(\frac{n\pi x}{a}\right) \cos\left(\frac{n\pi x}{a}\right) dx = 0$$

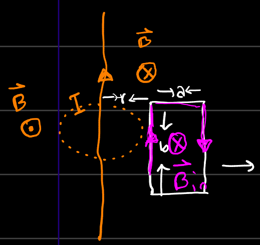
& since sine & cosine functions are orthogonal, $\int \sin\left(\frac{n\pi x}{a}\right)\cos\left(\frac{n\pi x}{a}\right) dx = 0$.

52. [B] This is just the orthonormality condition $\langle\psi_n|\psi_\ell\rangle = \delta_{n\ell}$

54. [B] Energies at infinite square well $\Rightarrow \boxed{E_n = \frac{n^2\hbar^2\pi^2}{2ma^2}}$

The ground state is $E_0 = \frac{\hbar^2\pi^2}{2ma^2}$. So, $E_n \geq \frac{\hbar^2\pi^2}{2ma^2}$

55. [E] Direction of $\vec{B}$ due to a straight wire $\Rightarrow$ Right-hand rule!



Due to the wire with current I, there will be $\vec{B}$ pointing INTO THE SCREEN on the left.

As the loop moves AWAY, the $\vec{B}$ will decrease. So, $\vec{B} \otimes \downarrow$

The induced current in the loop will oppose this decrease by maintaining the original $\vec{B}$. So the induced $\vec{B}_{in}$ will point INTO THE SCREEN. So induced current will be CLOCKWISE!

Eliminate all the anti-clockwise options (A) (B), (C).

Now, the force. Use the RIGHT HAND RULE again to find the force direction!

To the left	To the right
B $\otimes$	B $\otimes$
v $\uparrow$	v $\downarrow$
F $\leftarrow$	F $\rightarrow$

55. [D] $\vec{B} = \frac{\mu_0 I}{2\pi r}$ at the left arm | $\vec{B} = \frac{\mu_0 I}{2\pi(r+a)}$ At the right arm.

$F = i\,dl\,B = \frac{\mu_0 I i b}{2\pi r}$ | $F = -\frac{\mu_0 I i b}{2\pi(r+a)}$

And $F_{top} + F_{bottom} = 0$

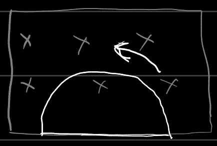
So, net force $F_N = \frac{\mu_0 I i b}{2\pi}\left(\frac{1}{r} - \frac{1}{r+a}\right)$

$$= \frac{\mu_0 I i b}{2\pi}\left(\frac{\cancel{r}+a-\cancel{r}}{r(r+a)}\right) = \frac{\mu_0 I i}{2\pi}\,\frac{ab}{r(r+a)}$$

56. $\boxed{C}$ Ground state energy of a harmonic oscillator is $E = \frac{1}{2}\hbar\omega$

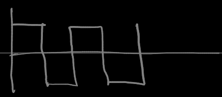
We are given the frequency ν. $\omega = 2\pi\nu$. So, $E = \frac{1}{2}\frac{h}{\cancel{2\pi}} \cdot \cancel{2\pi}\nu = \frac{1}{2}h\nu$

57. $\boxed{A}$ At $t=0$, ε is maximum. $\varepsilon \propto \Phi \propto BA$



As the semi circle rotates, A decreases until its zero. Φ decreases.
As it enters again from the opposite side, ε rises again, but in the opposite direction. Φ increases.

As first Φ decreased & later Φ increased, the emf will rise in value, but have opposite directions.



58. $\boxed{C}$ Electron configuration will proceed as usual all the way to $Z = 18$ [Argon]

So, for $Z = 11$, electron config is $1s^2 2s^2 2p^6 3s^1 3p^0 3d^0$ $[2+2+6+1 = 11]$

59. $\boxed{A}$ Basic fact, should be memorized. The ground state of the Helium atom is a spin singlet.

60. $\boxed{D}$

$m = 0.1 m_e$

$m_e = 9.8 \times 10^{-31}$

$B = 1\,T$

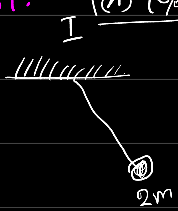
$q = 1.67 \times 10^{-19}$

Cyclotron frequency, $\omega = \frac{qB}{m}$

$\Rightarrow \omega = \frac{1.6 \times 10^{-19} \times 1}{\frac{1}{10} \times 10 \times 10^{-31}} = \frac{1.6 \times 10^{-18}}{10^{-30}} \approx 1.6 \times 10^{12}$ rad/s

Closest to 1.8×10^{12} rad/s

61. $\boxed{(A)\ (6/5)^{1/2}}$



$\omega = \sqrt{\frac{Mgr_{cm}}{I}}$

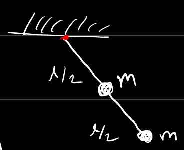
Config-I

$I = 2ml^2$; $M = 2m$

$r_{cm} = l$

$\omega_I = \sqrt{\frac{\cancel{2m}g\cancel{l}}{\cancel{2m}l^{\cancel{2}}}}$

$\Rightarrow \omega_I = \sqrt{g/l}$



Config-II

$I = m\left(\frac{l^2}{4} + l^2\right) = \frac{5ml^2}{4}$

$r_{cm} = \frac{\cancel{m}(l + l/2)}{2\cancel{m}} = \frac{3l}{4}$

$\omega_{II} = \sqrt{\frac{2\cancel{m}g \times 3\cancel{l} \times \cancel{4}}{\cancel{4} \times 5\cancel{m}l^{\cancel{2}}}} = \sqrt{\frac{6}{5}}\sqrt{g/l}$

$\frac{\omega_{II}}{\omega_I} = \frac{\sqrt{6/5}\cancel{\sqrt{g/l}}}{\cancel{\sqrt{g/l}}} = (6/5)^{1/2}$

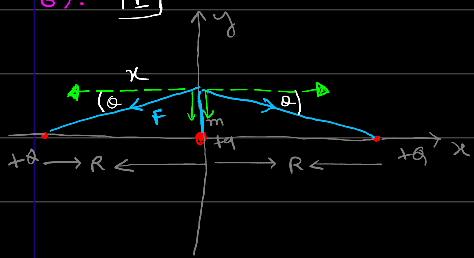
62. $\boxed{E}$ Work done by iso-something process: $W = \int p\,dV$ supplanted by gas equation $PV = nRT$ to solve for P in terms of the constant, for isothermal case, $p = \frac{nRT}{V}$

$$W = \int p\,dV = \int \frac{RT}{V}\,dV = RT\int_{V_1}^{V_2}\frac{dV}{V} = RT\ln(V_2/V_1)$$

63. $\boxed{D}$ Basic fact of thermodynamics ⇒ An arrangement of isolated system which is of maximum probability is the most stable state. So, no spontaneous change.

64. $\boxed{B}$ $\vec{E} = k\hat{z}$ $k \neq 0$. If there's a field, there must be a charge density in the region ⇒ Maxwell's first equation $\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$

65. $\boxed{E}$ Whenever facing this problem, try to express the force as in this form $\boxed{F = -kr}$



In this case $F = -ky$.

F in the y direction is $F = 2F'\sin\theta$

$$F' = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{R^2} \qquad \Rightarrow F = \frac{\cancel{2}}{\cancel{4}\pi\varepsilon_0}\frac{Qq}{R^2}\frac{y}{R} \quad [\sin\theta \approx \frac{y}{R}]$$

[small oscillations → $x \approx R$]

$$\Rightarrow F = \underbrace{\frac{Qq}{2\pi\varepsilon_0 R^3}}_{\rightarrow k} y$$

So, $F = -ky$

$$\text{So, } \omega = \sqrt{\frac{k}{m}} = \left(\frac{Qq}{2\pi\varepsilon_0 m R^3}\right)^{1/2}$$

66. $\boxed{(E)\ 10{,}000\,J}$ $\mu = \frac{m}{l} = 2$ $l = 10\,m$

$\Rightarrow m = \mu l = 20\,kg$ $F(l) = mgl$

$$\text{Work done, } W = \int_0^l F\,dl = \int_0^{10} mgl\,dl = mg\cdot\tfrac{1}{2}[l^2]_0^{10} = \cancel{20}^{10}\times 10\times\tfrac{1}{\cancel{2}}\times 100 = 10{,}000\,J$$

67. $\boxed{C}$ Transmitted intensity varies as $A + B\cos 2\theta$. $I = A + B\cos 2\theta$

For plane polarized light ⇒ $I = I_0\cos^2\theta$ We can re-write $2\cos^2\theta = (1 + \cos 2\theta)$

$$\text{So, } I = I_0(\underbrace{1/2}_{\text{Unpolarized}} + \underbrace{1/2\cos 2\theta}_{\text{plane polarized}})$$

68. $\boxed{(C)\ 10\,cm}$ Rayleigh criterion ⇒ $\boxed{D\sin\theta = 1.22\lambda}$

$\Rightarrow D\theta = 1.22\lambda$

$$\Rightarrow D = \frac{1.22\lambda}{\theta} = \frac{1.22\times 5.5\times 10^3\times 10^{-10}}{8\times 10^{-6}} = \frac{\cancel{12}^{3}\times 10^{(-1+3-10)}\times 5.5}{\cancel{8}_{2}\times 10^{-6}}$$

$$= 7.5\times 10^{-8+6} \approx 8\,cm$$

Closest to 10 cm

69. $\boxed{(D)\ \frac{2}{3}c}$

Light speed through a medium at refractive index n, $\boxed{c' = \frac{c}{n}}$ $\quad n = \frac{3}{2}$ So, $c' = \frac{2}{3}c$

70. $\boxed{(D)\ 100\,mc}$

Rest energy $E = mc^2$

Total energy $E = 100\, mc^2$

$$E^2 = p^2c^2 + m^2c^4$$

$$\Rightarrow 10000\, m^2c^4 - m^2c^4 = p^2c^2$$

$$\Rightarrow 100^2\, m^2c^2 = p^2$$

$$\Rightarrow \boxed{p = 100\, mc}$$

71. $\boxed{B}$ $\quad E_2 - E_1 = \varepsilon > 0$

Average number of sub-systems in the state E_1 $= \dfrac{N_0 e^{-\beta E_1}}{e^{-\beta E_1} + e^{-\beta E_2}} = \dfrac{N_0 \cancel{e^{-\beta E_1}}}{\cancel{e^{-\beta E_1}}\left[1 + e^{-\beta(E_2 - E_1)}\right]}$

$$= \frac{N_0}{1 + e^{-\varepsilon/kT}}$$

72. $\boxed{A}$ $\quad$ Heat capacity of the system $\boxed{C = \frac{dU}{dT}}$

$$U = E_1 N_0 + \frac{N_0 \varepsilon}{1 + e^{\varepsilon/kT}}$$

$f = \frac{1}{x}$

$x = 1 + e^y$

$y = \frac{\varepsilon}{kT}$

$$\frac{dU}{dT} = \frac{df}{dx}\frac{dx}{dy}\frac{dy}{dT}$$

$$\frac{df}{dx} = -x^{-2} = -\frac{1}{x^2}$$

$$\frac{dx}{dy} = e^y$$

$$\frac{dy}{dT} = \frac{\varepsilon}{k}\cdot\frac{d}{dT}(T^{-1}) = \frac{-\varepsilon}{kT^2}$$

$$\frac{dU}{dT} = \frac{N_0 \varepsilon\, e^{\varepsilon/kT}}{\left[1 + e^{\varepsilon/kT}\right]^2}\cdot\frac{\varepsilon}{kT^2}$$

$$= \frac{N_0 e^{\varepsilon/kT}}{\left[1 + e^{\varepsilon/kT}\right]^2}\,\frac{\varepsilon^2}{k^2T^2}\cdot k$$

$$= N_0 k \left(\frac{\varepsilon}{kT}\right)^2 \frac{e^{\varepsilon/kT}}{\left[1 + e^{\varepsilon/kT}\right]^2}$$

73. $\boxed{C}$

Definition of entropy, $S = Nk \ln Z$

the partition function, $Z = e^{-E_1\beta} + e^{-E_2\beta}$

At high temperature, $Z = 1 + 1 = 2$

So, at arbitrarily high temperature, $S = Nk\ln 2$.

And

Third law of thermodynamics $\Rightarrow S = 0$ at $T = 0$.

For these type of problems, try to write down mass, center of mass, radius and I explicitly.

74.

For hoop X	For hoop Y
Mass = 4M; $r = 2d$	Mass = M $r = d/2$
$I = I_0 + Mr^2 = \frac{1}{2}Mr^2 + Mr^2 = \frac{3}{\cancel{2}} M \cdot \cancel{2}^{2} d^2$	$I = I_0 + Mr^2 = \frac{1}{2}Mr^2 + Mr^2 = \frac{3}{2}Mr^2 = \frac{3}{2}M\frac{d^2}{4}$
$\Rightarrow I = 6 \cdot 4Md^2 = 24Md^2$	$\Rightarrow I = \frac{3}{8}Md^2$

$$T = 2\pi\sqrt{\frac{I}{mg r_{cm}}}$$

$$= 2\pi\sqrt{\frac{\cancel{24}^{3}Md^2}{\cancel{4}Mg \cdot \cancel{2}\cancel{d}}} \propto \sqrt{\frac{3d}{g}}$$

$$T_y = 2\pi\sqrt{\frac{I}{mgr}}$$

$$\propto \sqrt{\frac{3M\cancel{d^2}^{d} \cdot \cancel{2}}{\cancel{8}_4 Mgd}} \propto \sqrt{\frac{3d}{4g}} = \frac{1}{2}\sqrt{\frac{3d}{g}}$$

$$\Rightarrow \boxed{T_y = T/2}$$

75. $\boxed{E}$ Conservation of momentum demands that both the products' momenta are identical. Now, if we write their energies as $P^2/2m$, We immediately see that M_{TH} is much larger, so Helium must have higher kinetic energy.

76. $\boxed{(A)\ 7/2}$

$s \to l = 0$
$p \to l = 1$
$p \to l = 1$

Maximum total angular momentum is when everything simply adds. $J = l + s \Rightarrow 2 + \frac{3}{2} = 7/2$

$l = 0 + 1 + 1 = 2$

$s = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$

77. $\boxed{E}$ Intrinsic magnetic moment, $\boxed{\mu_s = \gamma \vec{s}}$, with $\boxed{\gamma = \frac{eg}{2m}}$

So, $\frac{\mu_n}{\mu_e} = \frac{\gamma_n}{\gamma_e} = \frac{m_e}{m_n}$ | But $m_n >> m_e$. So $\frac{\mu_n}{\mu_e} < 1$.

78. $\boxed{(C)}$ Limiting case! At time $t = 0$, $v_x = 2v$; $\left.\frac{dx}{dt}\right|_{t=0} = 2v$

which of the options gives this?

(B) $\frac{dx}{dt} = v + \frac{1}{2}\cancel{b}\frac{3v}{\cancel{b}}\cos\left(\frac{3vt}{b}\right) = 2.5v$

(C) $\frac{dx}{dt} = \frac{1}{2}v + \frac{1}{2}\cancel{b}\frac{3v}{\cancel{b}} = \frac{v+3v}{2} = \frac{4v}{2} = 2v$

79. $\boxed{(A)}$ Phase velocity, $\boxed{v_p = \frac{\omega}{k}}$

Group velocity $\boxed{v_g = \frac{d\omega}{dk}}$

Between k_1 & k_2, the slope $\frac{d\omega}{dk}$ is actually negative. So, $v_p = +(ve)$ & $v_g = -(ve)$

So they're in opposite direction.

80. $\boxed{(B)\ 0.5\,\text{Å}}$ Energy of photons $\Rightarrow$ $\boxed{E = \frac{hc}{\lambda}}$ $\Rightarrow \lambda = \frac{hc}{E}$

$$\Rightarrow \lambda = \frac{4\times10^{-15}\times3\times10^{8}}{2.5\times10^{4}} = \frac{12\times10^{-11}}{2.5} = 4\times10^{-11} = 0.4\,\text{Å}$$

close to $0.5\,\text{Å}$

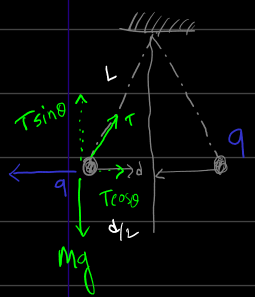
81. $\boxed{(C)\ \frac{1}{\sqrt{LC}}}$ Resonant frequency $\omega = \frac{1}{\sqrt{LC}} \Rightarrow$ Current have its maximum steady-state amplitude.

82. $\boxed{(D)\ \frac{3H}{Md^2}}$ Impulse $\boxed{H = \int \tau\, dt = I\alpha t}$ Here, $I = \frac{1}{3}Md^2$

$\omega = \alpha t$

$\Rightarrow H = I\omega \Rightarrow \omega = \frac{H}{I}$ | So, $\boxed{\omega = \frac{3H}{Md^2}}$

83. $\boxed{(A)}$ Pith balls will be in equilibrium forces are balanced.



$T\sin\theta = Mg$ & $T\cos\theta = \frac{kq^2}{d^2}$

$$\frac{\sin\theta}{\cos\theta} = \frac{Mg}{\frac{kq^2}{d^2}} = \frac{Mgd^2}{kq^2}$$

For small θ, $\tan\theta \approx \frac{L}{d/2}$; $\frac{2L}{d} = \frac{Mgd^2}{kq^2} \Rightarrow \frac{2kq^2L}{Mg} = d^3$

$$\Rightarrow d = \left(\frac{2kq^2L}{Mg}\right)^{1/3}$$

84. $\boxed{(D)}$ Let's consider the options.

(A) is true $\Rightarrow$ Larmor formula $\boxed{P \propto q^2 a^2}$

(B) is also true

(C) is also true $\Rightarrow$ Poynting theorem $\boxed{\vec{S} = (\vec{E}\times\vec{B})\frac{1}{\mu_0}}$

(D) NOT TRUE $\Rightarrow$

(E) is true $\Rightarrow$ Far from source, EM waves act like plane waves & $E_z = B_z = 0$

85. $\boxed{(C)\ 1.4\,\text{MeV}/c}$ $mc = 0.5$ MeV | Total energy in natural unit

$E = 1.5$ MeV

$$E^2 = p^2 + m^2$$

$$\Rightarrow p^2 = E^2 - m^2 = 1.5^2 - 0.5^2$$

$$= (1.5+0.5)(1.5-0.5)$$

$$= 2$$

So, $p = \sqrt{2}\ \text{MeV}/c = 1.4\ \text{MeV}/c$

86. $\boxed{(B)}$ We don't need $V_0 \Rightarrow$ Eliminate (A), (C)

Capacitor discharges according to $\boxed{V = V_0 e^{-t/RC}}$ So, we need R. Eliminate (C), (E)

We also need t which is found by s. So (B) is the answer.

87. $\boxed{(C)}$ $F = \frac{k}{r^3}$ For total energy, $E = T + V$, we need T & V.

1. $V = \int F\,dr$
$= k\int r^{-3}\,dr$
$= k\left[\frac{r^{-3+1}}{-3+1}\right]$
$\Rightarrow V = \frac{-k}{2r^2}$

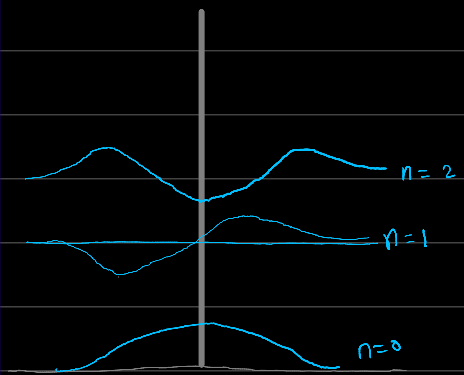
$\frac{mv^2}{\cancel{r}} = \frac{k}{\cancel{r^3}\, r^2}$
$\Rightarrow \frac{1}{2}mv^2 = \frac{k}{2r^2}$
$\Rightarrow T = \frac{k}{2r^2}$

So, $E = T + V$
$= \frac{k}{2r^2} - \frac{k}{2r^2} = 0$

88. $\boxed{(E)}$ * The voltage doesn't depend on dielectrics, it's constant. (A), (B), (D) X

* $Q_0 = CV_0$
$Q_f = kCV_0$ $\Rightarrow$ So, $Q_0 \neq Q_f$. (C) X That leaves only (E)

89. Putting an infinite barrier at the origin means imposing a boundary condition that there must be a node at the origin.



$\Leftarrow$ This is the harmonic oscillator wavefunctions.
So, when there's an infinite barrier at the origin, $n = 0$, $n = 2$, $n = 3$.. states will disapper (because they don't have nodes at the origin. Only $n = 1, 3, 5, \ldots$ states will be allowed because they have nodes at the origin. So, only odd values of n will remain.

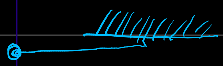
90. $\boxed{(B)}$ Memorize the spacing of different energy levels:

Rotation $\approx 10^{-3}$ eV
Vibration $\approx 10^{-1}$ eV
Electron ≈ 1 eV

91. (E) (A) Eliminate $\Rightarrow$ π^- is not a lepton.
(B) Eliminate $\Rightarrow$ Λ has spin-1/2. It's a baryon.
(C) Eliminate $\Rightarrow$ Weak interactions generally produces neutrinos.
(D) Eliminate $\Rightarrow$ Angular momentum is conserved.
(E) ✓

92. [D] The frequency of the alternating voltage for 3 pairs of magnet must also be 3 times, right?

93. (E) Total acceleration $\vec{a} = \vec{a}_c + \vec{g}$ | Apply limiting case!



When $\theta = 0^\circ$, $\vec{a}_c = 0$, so $\vec{a} = \vec{g}$

Which option gives $\vec{a} = g$ at $\theta = 0^\circ$? E. $g\sqrt{1} = g$.

94. We only need to consider x & t.

$$\boxed{\begin{aligned} x' &= \gamma(x - vt) \\ t' &= \gamma\left(t - \frac{vx}{c^2}\right) \end{aligned}}$$

Coefficient of x must be greater than 1.

Coefficient of t must be greater than 1.

Only option satisfying this is (c)

95. [C] Can be solved using dimensional analysis.

$a = 10^{12}$ protons/sec

$b = 10^{20}$ nuclei/cm^2

$c = 10^{2}$ protons/sec

$d = 10^{-4}$ st.

Combine them into cm^2/st.

$$\frac{c}{abd} = \frac{10^2}{10^{12} \times 10^{20} \times 10^{-4}} = 10^{2-28} = 10^{-26}\ \text{cm}^2/\text{st}.$$

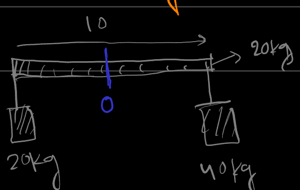
96. [C] Memorize! This setup is used to find the refractive index of air which is 1·00029.

97. [D] Basic CMP fact: Effective mass $m^* = \dfrac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)}$

98.

99. (A) Hydrogen atom is exactly solvable by QM so the correction is 0.

100. (c) Mass get balanced at the COM. $x_{cm} = \dfrac{\Sigma m_i x_i}{\Sigma m_i} = \dfrac{(40\times5) - (20\times5)}{20+20+40} = \dfrac{200-100}{80}$



20 kg at $x = 0$

20 kg at $x = -5$

40 kg at $x = +5$

$$= \frac{100}{80} = \frac{5}{4} = 1{\cdot}25$$