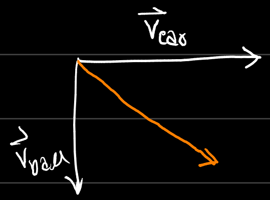
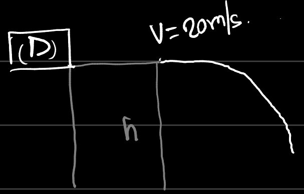
#GIR0877

1. [(B)] Most important! IGNORE THE AIR RESISTANCE!
This becomes a simple velocity addition rule! Car moving to the right, ball thrown out of the window.



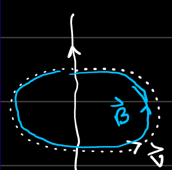

2. [(D)]




$h=(v_0\sin\theta_0)t+\frac{1}{2}gt^2$ $[\theta=0^\circ] \Rightarrow h=\frac{1}{2}gt^2=\frac{1}{\cancel{2}}\times 9.8\times \cancel{2}^{2}=19.6\,m.$

3. [(E)] $P=1W$. We have the same resistance (because we are talking about a resistor) then the relationship is $P=\frac{V^2}{R}$ [R is constant] [Always express in terms of whatever is being held constant] So, doubling the voltage will result in 4 times new power, which is 4W.

4. [(E)] Magnetic force on the loop = Due to the moving charges in the loop.




$F=qvB\sin\theta$ $\quad \vec{v}=v_0\hat{\phi}$ $\quad \vec{B}=B_0\hat{\phi}$ So, $\theta=0$

$\Rightarrow F=0$.

5. [(A)] De-Broglie relation $\boxed{\lambda=\frac{h}{p}}$ So, p and $\lambda$ are related by h.

6. [(E)] Filled n=1 $\quad 1s^2$ | Total electrons $=2+2+6=10$

Filled n=2 $\quad 2s^2 2p^6$

7. [(C)] Basic thermodynamics formula ⇒ R.M.S speed, $\boxed{V_{rms}=\sqrt{\frac{3kT}{m}}=\sqrt{\frac{3RT}{M}}}$

8. [(D)] Stefan-Boltzmann law: $\boxed{Q \propto T^4}$ If absolute temperature increases by a factor of 2, energy radiation increases by a factor of 16.

9. [(E)] There are just all three of the Kepler's laws.

10. [(B)] Conservation of energy ⇒ $\cancel{\frac{1}{2}}mv^2=\cancel{\frac{1}{2}}kx^2$

$\Rightarrow x^2=\frac{mv^2}{k}$

$\Rightarrow x=v\sqrt{\frac{m}{k}}$

11. $\boxed{(C)}$ Ground state of harmonic oscillator is $\frac{1}{2}\hbar\omega$.

12. $\boxed{(C)}$ Bohr's postulate : Angular momentum, $\boxed{L = mvr = n\hbar}$ $\Rightarrow mv = \frac{n\hbar}{r}$

13. $\boxed{\text{Log-log plot} \Rightarrow \text{Power law relation}}$ $\boxed{y = cx^m}$ $\Leftarrow$ Should look for this type of equation.

* C is the y at x = 1.

$\log y = m\log x + \log c$

$y = mx + c$

From the graph, when $x = 1$, $y = 6\times 1 = 6$

$\begin{bmatrix} \log 10 = 1 \\ \log 1 = 0 \end{bmatrix}$

* m is the slope.

$$m = \frac{\log\left(\frac{100}{10}\right)}{\log\left(\frac{300}{3}\right)} = \frac{\log 10}{\log(100)} = \frac{1}{\log 10^2} = \frac{1}{2\log 10} = \frac{1}{2}$$

So, $y = 6x^{1/2}$ $\Rightarrow$ $\boxed{y = 6\sqrt{x}}$

14. $\boxed{(B)}$ Measuring the same quantity in different techniques $\Rightarrow$ Weighted average.

$\sigma_1 = 1\,kg$

$\sigma_2 = 2\,kg$

$$\sigma_T = \frac{1}{\sqrt{\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}}} = \frac{1}{\sqrt{\frac{\sigma_1^2+\sigma_2^2}{\sigma_1^2\sigma_2^2}}} = \frac{\sigma_1\sigma_2}{\sqrt{\sigma_1^2+\sigma_2^2}} = \frac{1\times 2}{\sqrt{1^2+2^2}} = \frac{2}{\sqrt{5}}\,kg$$

15. $\boxed{(E)}$ General lensmaker's equation : $\boxed{\frac{1}{F} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$

Conventions $\Rightarrow$ $R_1$ = The radius of the surface closer to the light source.

$R_2$ = The radius of the surface farther from the light source.

Radius is +(ve) if the center lies to the right from the lens

Radius is −(ve) if the center lies to the left from the lens.

(A) 

$R_1 = -R$

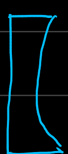
$R_2 = R$

So, $\frac{1}{F} \sim -\frac{2}{R}$ $\Rightarrow F \sim -R/2$

(D) 

$R_1 = +R$ $F \sim R_D/2$

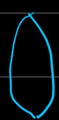
$R_2 = -R$

(B) 

$R_1 = \infty$

$R_2 = R$

$\frac{1}{F} \sim \left(\frac{1}{\infty} - \frac{1}{R}\right) \Rightarrow F \sim -R$

(E) 

$F \sim R_E/2$

(C) 

$R_1 = \infty$

$R_2 = -R$

$F = R$

But $R_E < R_D$

So (E) has the lowest focal length.

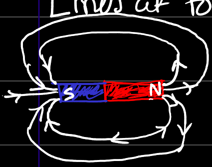
16 $\boxed{D}$ Unpolarized falls on the 1st polarizer $\Rightarrow I_1 = \frac{I_0}{2}$

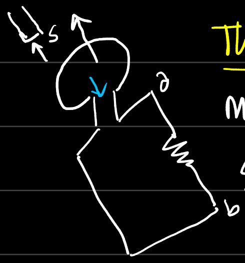
Unpolarized falls on the 2nd polarizer $\Rightarrow I_2 = I_1 \cos^2\theta = \frac{I_0}{4} \approx 25\%$ of $I_0$.

17 $\boxed{A}$ If you can't remember it's $\vec{E} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}$, remember that $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$ and $\lambda = \frac{Q}{r}$.

18. $\boxed{C}$

Lines of forces for bar magnets.




The bar magnet approaches the loop: Magnetic flux out of the loop and INCREASING. So, the induced magnetic field is INTO THE LOOP. Current antidockwise. $b \to a$


The bar magnet leaves the loop: Magnetic flux OUT OF THE LOOP & decreasing. Induced magnetic field will be OUT OF THE LOOP. Current still anti-clockwise. $b \to a$

19. $\boxed{A}$ Wein's displacement law: $\boxed{\lambda_{max} = \frac{2.9\times10^{-3}}{T}} = \frac{\cancel{3}\times10^{-3}}{\cancel{3}\times10^{2}} = 10^{-5}\,m$

$= 10\times10^{-6}\,m = 10\,\mu m$.

20 $\boxed{A}$ Temperature dependence of cosmic scale: $\boxed{T \propto \frac{1}{a}}$ As temperature decreases, the cosmic scale increases (things get further apart). If the temperature were 12K (4 times than now) the scale would be one-quarter (1/4) of what they are now.

21. $\boxed{C}$ For adiabatic processes $\rightarrow$ $\boxed{PV^{\gamma} = \text{constant}}$ $\boxed{TV^{\gamma-1} = \text{constant}}$

22. $\boxed{C}$ Total energy $E = 4mc^2$. Energy-momentum relation $\Rightarrow E^2 = p^2c^2 + m^2c^4$

$\Rightarrow 16m^2\cancel{c^4}^{\,c^2} = (p^2 + m^2c^2)\cancel{c^2} \Rightarrow 15m^2c^2 = p^2 \Rightarrow p = \sqrt{15}\,mc$

23. $\boxed{B}$ $\omega$ is the speed of one spaceship as measured by the other one.

So, $L_0 = \gamma L \Rightarrow \frac{1}{\gamma} = \frac{L}{L_0} = \frac{60}{100} = 0.6$

$\Rightarrow 1 - \frac{\omega^2}{c^2} = 0.36 \Rightarrow \omega^2 = 0.64c^2 \Rightarrow \omega = 0.8c$

We know, $\omega = \frac{u+v}{1+uv/c^2}$ [Since $u = v$ from earth]

$\Rightarrow \omega = \frac{2v}{\frac{c^2+v^2}{c^2}} \Rightarrow 0.8 = \frac{2(v/c)}{1+(v/c)^2} \Rightarrow 0.8 = \frac{2\beta}{1+\beta^2} \Rightarrow 8 = \frac{20\beta}{1+\beta^2}$

$\Rightarrow 8 + 8\beta^2 = 20\beta \Rightarrow 8\beta^2 - 20\beta + 8 = 0$

$\Rightarrow 8\beta^2 - 16\beta - 4\beta + 8 = 0$

$\Rightarrow 8\beta(\beta-2) - 4(\beta-2) = 0$

$\Rightarrow \beta = 2$ or $\beta = 0.5$

$\Rightarrow \boxed{v = 0.5c}$

24. $\boxed{B}$ Velocity of the meter stick $\beta = 0.8$. Again, $L_0 = \gamma L \Rightarrow L = \frac{L_0}{\gamma}$

According to the observer, $v = \frac{L}{\tau} \Rightarrow t = \frac{L}{v} = \frac{L_0}{\gamma v} = \frac{1}{5/3 \times 0.8c}$

$$\gamma = 1/\sqrt{1-0.64} = \frac{1}{0.6} = \frac{5}{3}$$

$$= \frac{\cancel{3} \times 10^{-8}}{5 \times 0.8 \times \cancel{3}} = \frac{1}{4} \times 10^{-8} = 0.25 \times 10^{-8} = 2.5\,\text{ns}.$$

25. $\boxed{E}$ Orthonormality condition $\langle \psi_m | \psi_n \rangle = \delta_{nm}$

$$\Rightarrow \int_{-\infty}^{+\infty} \psi_i^*(x)\psi_j(x)\,dx = \delta_{ij}$$

26. $\boxed{D}$ Repeating question: The most likely distance of the electron is the Bohr radius.

27. $\boxed{C}$ Energy-time uncertainty relation $\Rightarrow$ $\boxed{\Delta E\, \Delta t \sim h}$ $\Rightarrow \cancel{h}\Delta\nu\,\Delta t \sim \cancel{h}$

$$\Rightarrow \Delta\nu \sim \frac{1}{\Delta t} = \frac{1}{1.6\times10^{-9}}\,\text{Hz} = \frac{\cancel{10}^{5}}{\cancel{16}_{8}} \times 10^{-9}\,\text{Hz} = 0.6\times10^{-9}\,\text{Hz} = 600\,\text{MHz}$$

In MHz scale.

8) $\frac{50}{\frac{48}{2}}$ (0.6

28. $\boxed{D}$ $U_0 = \frac{1}{2}kx^2$ $\quad U_1 = \frac{1}{2}k'\left(\frac{x}{2}\right)^2 = 2U_0 \Rightarrow \frac{1}{2}k'\frac{\cancel{x^2}}{4} = k\cancel{x^2} \Rightarrow k' = 8k$

29. $\boxed{C}$ Elastic collisions always scream at you to use conservation of K.E!

$$\cancel{\tfrac{1}{2}}\cancel{M}v^2 = \cancel{\tfrac{1}{2}}\cancel{M}\cdot\frac{v^2}{4} + \cancel{\tfrac{1}{2}}\cancel{M}V^2 \Rightarrow V^2 = \left(1-\frac{1}{4}\right)v^2 = \frac{3}{4}v^2 \Rightarrow V = \frac{\sqrt{3}}{2}v$$

30. $\boxed{D}$ $\boxed{\text{Hamilton's equation of motion}}$ $\boxed{\dot{q} = \frac{\partial H}{\partial p}}$ $\boxed{\dot{p} = -\frac{\partial H}{\partial q}}$

31. $\boxed{C}$ $\rho \cancel{V}\cancel{g} = \rho_w \cdot \frac{3}{4}\cancel{V}\cancel{g} + \rho_o \frac{1}{4}\cancel{V}\cancel{g}$ $\Leftarrow$ Archimedes's principle If an object floats, its weight must be equal or less than the weight of the liquid displaced by the object.

$$\Rightarrow \rho = \cancel{1000}^{250} \times \frac{3}{\cancel{4}} + \cancel{800}^{200} \times \frac{1}{\cancel{4}}$$

$$= 950\,\text{kg/m}^3$$

32. $\boxed{A}$ Bernoulli's principle $\Rightarrow$ $\boxed{\frac{V_1^2}{2} + \frac{P_1}{\rho} + gy_1 = \frac{V_2^2}{2} + \frac{P_2}{\rho} + gy_2}$ Fluid conservation $\Rightarrow \boxed{V_1A_1 = V_2A_2}$

For our case, $\frac{V_0^2}{2} + \frac{P_0}{\rho} = \frac{V'^2}{2} + \frac{P'}{\rho}$ & $V_0 \cancel{\pi} \cancel{r^2} = V' \cancel{\pi} \frac{\cancel{r^2}}{4} \Rightarrow V' = 4V_0$

$$\Rightarrow \frac{V_0^2}{2} + \frac{P_0}{\rho} = \frac{16V_0^2}{2} + \frac{P'}{\rho} \Rightarrow \frac{P'}{\rho} = \frac{P_0}{\rho} - \frac{15V_0^2}{2}$$

$$\Rightarrow \boxed{P' = P_0 - \frac{15}{2}\rho V_0^2}$$

33 $\boxed{(E)}$ $dQ = mc\,dT$ and $\Delta S = \int \frac{dQ}{T} = mc\int_{T_1}^{T_2} \frac{dT}{T} = mc\ln\left(\frac{T_2}{T_1}\right)$

34 $\boxed{(C)}$ Constant volume, $dW = 0$

$$Q = dU$$
$$\Rightarrow Q = c_v\,dT$$

For monoatomic ideal gases, $C_v = \frac{3}{2}nR$

$$Q = \frac{3}{2}nR\,dT = dU$$

Constant pressure

$$dQ' = dU + dW$$
$$= \frac{3}{2}nR\,dT + P\,dV$$
$$= \frac{3}{2}nR\,dT + nR\,dT$$
$$= \frac{5}{2}nR\,dT = \frac{5}{3}\,\frac{3}{2}nR\,dT = \frac{5}{3}Q$$

35. $\boxed{(B)}$ Efficiency, $\eta = \frac{\text{Output}}{\text{Input}} = \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H} = 1 - \frac{Q_C}{Q_H} = 1 - \frac{T_C}{T_H} = 1 - \frac{280}{300}$

$7^\circ C = 280\,K$

$27^\circ C = 300\,K$

$$\Rightarrow \eta = \frac{\cancel{20}}{\cancel{300}_{15}} = \frac{1}{15}$$

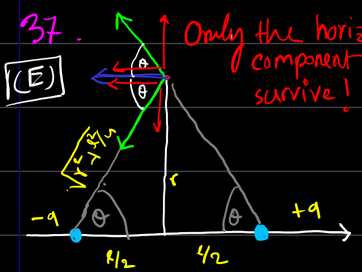
So, $\frac{W}{Q_H} = 1/15 \Rightarrow W = \frac{15000}{15} = 1000\,J$.

36. $\boxed{(A)}$ Magnetic energy at an inductor, $\boxed{U = \frac{1}{2}LI^2}$ For a capacitor, $\boxed{Q = Q_0\cos(\omega t)}$

So, $I = \frac{dQ}{dt} = -Q_0\omega\sin(\omega t)$ & $I^2 = Q_0^2\omega^2\sin^2(\omega t)$ So, $U \sim \frac{1}{2}LQ_0^2\omega^2\sin^2(\omega t)$

So it's a sinewave

37. $\boxed{(E)}$




Only the horizontal component survive!

$$\cos\theta = \frac{l/2}{\sqrt{(l/2)^2 + r^2}}$$

Field due to $-q$, $E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2 + (l/2)^2}$

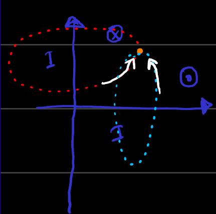
Field due to $+q$, $E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2 + (l/2)^2} = E_1$

The resultant field $E = 2E_1\cos\theta$ [Because only the horizontal components survive]

$$\Rightarrow E = \cancel{2}\cdot\frac{1}{4\pi\varepsilon_0}\,\frac{q}{r^2 + (l/2)^2}\cdot\frac{l/\cancel{2}}{\left[r^2 + (l/2)^2\right]^{1/2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\,\frac{ql}{\left[r^2 + (l/2)^2\right]^{3/2}}$$ and since $r \gg l$, $E = \frac{1}{4\pi\varepsilon_0}\frac{ql}{r^3}(-\hat{x})$

38. $\boxed{(E)}$




The $\vec{B}$ field from the vertical wire points into the page and the $\vec{B}$ field from the horizontal wire points out of the page. They are in opposite direction. And since the point is 'a' distance away from both wires, the total magnetic field is 0.

39. [(C)] Velocity $\frac{\Delta x}{\Delta t} = \frac{4}{5}c$   We know, $\Delta t = \gamma \Delta \tau$   $\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{5}{3}$

$\beta = \frac{4}{5}$

$\Delta \tau = 2.2\times10^{-6}$ s

$\Rightarrow \Delta t = \frac{5}{3}\Delta\tau$

So, $\Delta x = \frac{4c}{5}\times \Delta t = \frac{4c \times \cancel{5} \times \Delta\tau}{3\times\cancel{5}} = 4\times10^{8}\times\Delta\tau$

$\Rightarrow \Delta x = 4\times10^{8}\times 2.2\times10^{-6} = 8.8\times10^{2} = 880$ m.

40. [B] Initial energy = Rest energy of the particle $= Mc^2$

Final energy = Photon energy, $pc + \sqrt{p^2c^2+m^2c^4}$ [Conservation of momentum requires that if the photon has a momentum p, the other particle must also have a momentum p in the opposite direction]

Conservation of energy requires

$Mc^2 = pc + \sqrt{p^2c^2+m^2c^4}$

$\Rightarrow M\cancel{c}^{2} = p\cancel{c} + \cancel{c}\sqrt{p^2+m^2c^2}$

$\Rightarrow Mc = p + \sqrt{p^2+m^2c^2}$

$\Rightarrow (Mc-p)^2 = p^2+m^2c^2$

$\Rightarrow M^2c^2 - 2pMc + \cancel{p^2} = \cancel{p^2} + m^2c^2$

$\Rightarrow (M^2-m^2)\cancel{c^2} = 2pM\cancel{c}$

$\Rightarrow p = \frac{(M^2-m^2)c}{2M}$

41. [B] Photoelectric effect: The energy of the incident photon, one part is spent to eject electron (work function) and the other part is carried of by the ejected electron as kinetic energy.

$\boxed{h\nu = \phi + K}$

K in terms of the stopping potential $V \Rightarrow K = eV$

So, $h\nu = eV + \phi$

$\Rightarrow eV = h\nu - \phi$

$\Rightarrow \underbrace{V}_{y} = \underbrace{\frac{h}{e}\nu}_{mx} - \underbrace{\phi/e}_{c}$

So, the slope is $m = \frac{h}{e}$

42. [E] Visually from the graph, the two waves have a phase difference approximately close to $2\pi$.

43. [D] Basic CMP fact $\Rightarrow$ In diamond structure, the nearest-neighbors of each C atom lie at the corners of a TETRAHEDRON.

44. [D] Basic superconductivity fact $\Rightarrow$ The attraction in Cooper pair is due to the electron-phonon interaction. And phonons are quanta of ionic lattice vibration.

45. [C] General Doppler shift formula:

$\boxed{f_o = \frac{c+v_o}{c+v_s} f_s}$   Here $v_o = v_s$   So, $f_o = f_s$.

Here, $f_o$ = Observed frequency

$f_s$ = Source frequency.

$c$ = Speed of waves in the medium

$v_o$ = Velocity of the observer relative to the medium

$v_s$ = Velocity of the source relative to the medium.

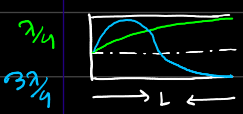
46. [D] $a = 0.14$ m    Sound wave disappear when there's a minimum.
$\theta = 45^\circ$    Diffraction minima $\boxed{a\sin\theta = m\lambda}$

$$\Rightarrow a\sin 45^\circ = \frac{c}{f} \Rightarrow f = \frac{350}{\cancel{0.14 \times \frac{\sqrt{2}}{2}}_{0.07}} = \frac{350 \times 0.7}{0.07} = \frac{3500 \times \cancel{0.7}}{\cancel{0.7}} = 3500\ \text{Hz}$$

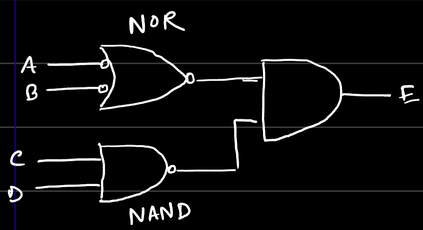
47. [D] Fundamental frequency, $\frac{\lambda}{4} = L \Rightarrow \frac{c}{4f_0} = L \Rightarrow f_0 = \frac{c}{4L}$

Next higher harmonic, $\frac{3\lambda}{4} = L \Rightarrow \frac{3c}{4f} = L \Rightarrow f = 3\cdot\frac{c}{4L} = 3\cdot f_0 = 3\times 131$ Hz

$\Rightarrow f = 393$Hz




48. $E = \overline{\overline{A} + \overline{B}} \cdot \overline{C \cdot D}$




General Boolean algebra ⇒
OR means dot $(\cdot)$
AND means plus $(+)$

49. [D] Types of lasers & how the transfers between energy levels are achieved.

| Type | Example | Level | Medium | How transfer is achieved |
|---|---|---|---|---|
| 1. Solid-state Lasers | Nd:YAG | Four-level system | Crystal/glass. | Atomic energy levels |
| 2. Collisional gas lasers. | He-Ne | Large number of levels. | Gas/Mixture of gases. | Collisions between free atoms. |
| 3. Molecular gas lasers | $CO_2$ | Large number of levels. | Gas | Vibrational energy levels. |
| 4. Dye lasers | | | Liquid/organic dye dissolved in water or alcohol | Electron transfer properties along chains of C atoms. |
| 5. Semiconductor lasers | | | Semiconductor. | Electron-hole annihilation<br>Recombination radiation |
| 6. Free electron lasers. | | Continuous<br>Not discrete | Free electrons | Bremsstrahlung |

50. [C] Energy levels of one electron Bohr atoms: $\boxed{E_n = \mu e^4 \frac{Z^2}{n^2}}$

51 [D] I. true ⇒ Atoms absorb photons with discrete energies. II. False because I is true. III. True
[I & III]

52. $\boxed{(C)}$ Bragg diffraction $\Rightarrow$ $\boxed{d\sin\theta = m\frac{\lambda}{2}}$ Here, $\theta = 14.5^\circ$ & $\sin\theta = 1/4$

$\lambda = 250\times10^{-3}\times10^{-9}\ m = 25\times10^{-11}\ m$

$$\Rightarrow d = \frac{\lambda}{2\sin\theta} = \frac{25\times10^{-11}}{\cancel{2}\times\frac{1}{\cancel{4}_{2}}} = 50\times10^{-11}\ m = 5\times10\times10^{-9}\times10^{-2}\ m = 0.5\ nm.$$

53. $\boxed{(D)}$ Two planets have the same orbit radius R. They have the same angular momentum.

$$L = m_1 V_1 \cancel{R} = m_2 V_2 \cancel{R} \Rightarrow \frac{V_2}{V_1} = \frac{m_1}{m_2}$$

$$T_1 = \frac{2\pi R}{V_1};\ T_2 = \frac{2\pi R}{V_2} \quad\Big|\quad T_1 = 3T_2 \Rightarrow \frac{\cancel{2\pi R}}{V_1} = 3\cdot\frac{\cancel{2\pi R}}{V_2} \Rightarrow \frac{V_2}{V_1} = \frac{m_1}{m_2} = 3$$

54. $\boxed{(E)}$ Solar mass black-hole doesnot increase the gravity!

55. $\boxed{(A)}$ Redshift. Wavelength is increasing.

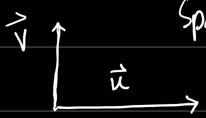
$$\lambda_r = \sqrt{\frac{1+\beta}{1-\beta}}\,\lambda_s \Rightarrow \frac{\lambda_r}{\lambda_s} = \sqrt{\frac{1+\beta}{1-\beta}} \Rightarrow \frac{580}{434} = \sqrt{\frac{1+\beta}{1-\beta}}$$

$$\Rightarrow \left(\frac{4}{3}\right)^2 = \frac{1+\beta}{1-\beta} \Rightarrow \frac{16}{9} = \frac{1+\beta}{1-\beta} \Rightarrow 9+9\beta = 16-16\beta \Rightarrow 9\beta+16\beta = 16-9 \Rightarrow 25\beta = 7$$

$$\Rightarrow \beta = \frac{7}{25} \Rightarrow v = \frac{7}{25}c = 0.28c$$

25) 70 (0.28
50
200

56.



Speed in still air, $\vec{v} = 200$ km/h

Speed of the wind $\vec{u} = 30$ km/h

$\vec{V} \perp \vec{u}$

So, $\vec{V}\cdot\vec{u} = 0$

$\vec{V} = \vec{u} + \vec{v}$

$\Rightarrow (\vec{V} - \vec{u})^2 = \vec{v}^2$

$\Rightarrow \vec{V}^2 - 2\vec{V}\cdot\vec{u} + \vec{u}^2 = \vec{v}^2$

$\Rightarrow \vec{V}^2 + \vec{u}^2 = v^2$

$\Rightarrow V = \sqrt{v^2 - u^2}$

$= \sqrt{(200)^2 - (30)^2}$

$= \sqrt{(200+30)(200-30)}$

$= \sqrt{230\times170}$

$= \sqrt{391}\times10$

$x = 500$ km $\quad x = Vt \Rightarrow t = \frac{x}{V}$

$= \frac{500}{\sqrt{391}\times10}$

$= \frac{50}{\sqrt{391}}$

23
17
161
230
391

57. $\boxed{(B)}$

Figure-1

$F = 3ma$

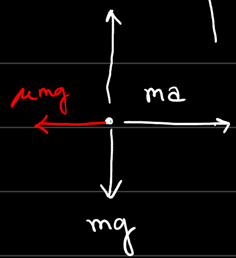
$\Rightarrow a = \frac{F}{3m}$

$F_{12} = \cancel{m}\cdot\frac{F}{3\cancel{m}} = \frac{F}{3}$

Figure-2

$F_{12} = 2m\cdot a = 2\cancel{m}\cdot\frac{F}{3\cancel{m}} = \frac{2F}{3}$

58. $\boxed{(A)}$ For block B,




Frictional force, $F_f = \mu mg = ma = 10\times2 = 20N.$

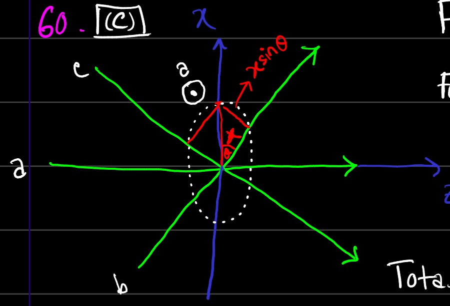
59. $\boxed{(C)}$ Limiting case! If the elevator acceleration were $a=0$, $T = 2\pi\sqrt{\frac{l}{g}}$

If it started to increase, the T would start to decrease, at $a\to\infty$, $T\to0$.

So, $T = 2\pi\sqrt{\frac{l}{g+a}}$

$x\sin\theta = x\sin 45^\circ = \frac{x}{\sqrt{2}}$

60. $\boxed{(C)}$




For the wire a, at a distance $x$, $\vec{B}_a = \frac{\mu_0 I}{2\pi x}\hat{y}$

For the wire b, at a distance $\frac{x}{\sqrt{2}}$, $\vec{B}_b = \frac{\mu_0 I}{2\pi x}\sqrt{2}\,\hat{y}$

For the wire c, at a distance $\frac{x}{\sqrt{2}}$, $\vec{B}_c = \frac{\mu_0 I}{2\pi x}\sqrt{2}\,\hat{y}$

Total magnetic field, $\vec{B} = \vec{B}_a + \vec{B}_b + \vec{B}_c = \boxed{\frac{\mu_0 I}{2\pi x}(1+2\sqrt{2})\hat{y}}$

61. (E) Cyclotron motion $\quad qvB = \frac{mv^2}{r}$

$\Rightarrow \frac{q}{m} = \frac{v}{rB} \Rightarrow \frac{q}{m} = \frac{2v}{dB}$ So, $\frac{q}{m} \propto \frac{1}{d}$

If the charge-to-mass ratio is doubled, d is halved.

62. $\boxed{(E)}$ Gauss's law $\Rightarrow \oint \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_0}$. Total flux $\Phi_T = \frac{q}{\varepsilon_0} = \frac{10^{-9}}{9\times10^{-12}} = \frac{1}{9}\times10^3$

$\Rightarrow \Phi_T = 0.1\times10^3 = 100$.

The flux through A, $\Phi_A = -100\ \mathrm{N\cdot m^2/C}$. Then the flux through the rest is

$\Phi = \Phi_T - \Phi_A = 100-(-100) = 200\ \mathrm{Nm^2/C}$

63. $\boxed{(D)}$ If the product involves a photon, the interaction is electromagnetic.

If the product involves a neutrino, the interaction is certainly weak.

64. $\boxed{(D)}$ For angular momentum, $L^2\psi(x) = l(l+1)\hbar^2$. Here $L^2 = 2\hbar^2$

So, $l(l+1) = 2$ or $l = 1$.

For $l=1$, the eigenvalues of $m_z$ are $-\hbar, 0, +\hbar$. [m can have values $-l, -l+1, \ldots 0, \ldots l-1, l$]

65. $\boxed{(C)}$ For harmonic oscillator,

✓ I. A spectrum of evenly spaced energy states. True, they're all $\hbar\omega$ apart.

II. A potential energy function that is linear in position. False. $U = \frac{1}{2}kx^2$.

III. A ground state characterized by zero kinetic energy. False. $E_0 = \frac{1}{2}\hbar\omega$.

✓ IV. A nonzero probability of finding the oscillator outside the classical turning point. True.

66. $\boxed{(D)}$ Hydrogen energy levels $\Rightarrow \boxed{E_n^H \sim \frac{\mu_H}{n^2}}$ Bohr radius, $\boxed{a_0 \sim \frac{1}{\mu}}$

Reduced mass for Hydrogen, $\mu_H = \frac{m_e m_p}{m_e + m_p}$

Reduced mass for muonium: $\mu_m = \frac{m_\mu m_p}{m_\mu + m_p}$

$\Rightarrow \frac{\mu_m}{\mu_H} = \left[\frac{m_\mu \cancel{m_p}}{m_\mu + m_p} \times \frac{m_e + m_p}{m_e \cancel{m_p}}\right] \Rightarrow \mu_m = \mu_H\left[\frac{m_\mu(m_e+m_p)}{m_e(m_\mu+m_p)}\right]$

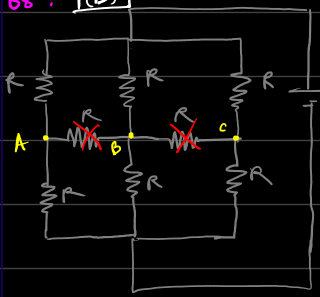
Muon energy levels, $E_n^\mu \sim \frac{\mu_m}{n^2} = \frac{\mu_H}{n^2}\left[\frac{m_\mu(m_e+m_p)}{m_e(m_\mu+m_p)}\right] = \frac{-E_0}{n^2}\left[\frac{m_\mu(m_e+m_p)}{m_e(m_\mu+m_p)}\right]$

67. [D] Parallel plate capacitor. $A = (0.5)^2\ m^2 = 0.25\ m^2$  Capacitance, $C = \frac{\varepsilon_0 A}{d}$

$$\Rightarrow C = \frac{Q}{V} = \frac{Q}{Ed} \quad \text{So, } \frac{\varepsilon_0 A}{\cancel{d}} = \frac{Q}{E\cancel{d}} \Rightarrow E = \frac{Q}{\varepsilon_0 A}$$

$$\Rightarrow \frac{dE}{dt} = \frac{I}{\varepsilon_0 A} = \frac{9}{9\times10^{-12}\times 0.25} = \frac{\cancel{9}\times 4\times10^{12}}{\cancel{9}} = 4\times10^{12}\ \frac{V}{m\cdot s}$$

68. [D]




The circuit is symmetric from left to right. So any point lying on the same height will have no potential difference, so current will not flow through the two horizontal resistors.

Equivalent resistance, $R_{eq}^{-1} = \frac{3}{2R} \Rightarrow R_{eq} = \frac{2R}{3}$

So, current, $I = \frac{V}{R_{eq}} = \frac{3}{2}\frac{V}{R}$

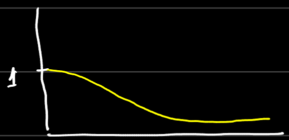
69. [D] The impedance of the circuit, $Z = R + \frac{1}{\omega C}$ | $Z_C = \frac{1}{\omega C}$

The input current, $I = \frac{V_i}{Z}$ $\Rightarrow$

The output, $I = \frac{V_o}{Z_C} \Rightarrow V_o = I Z_C = \frac{V_i}{Z}\cdot Z_C = \frac{V_i}{(R+\frac{1}{\omega C})\omega C} \Rightarrow \frac{V_o}{V_i} = \frac{1}{R\omega C + 1}$

Limiting cases $\Rightarrow$ As $\omega \to 0$ ; $\frac{V_o}{V_i} = 1$.

As $\omega \to \infty$ ; $\frac{V_o}{V_i} = 0$




70. [A] Faraday's law : $A = 10\ cm^2 = \frac{1\cancel{0}}{100\cancel{0}}\ m^2 = 10^{-3}\ m^2$  $\varepsilon = -\frac{\Delta\Phi}{\Delta t}$

$R = 5\Omega$  $\varepsilon = IR = \frac{\Delta Q}{\Delta t}R$

$$\Rightarrow \frac{\Delta Q}{\cancel{\Delta t}} R = -\frac{\Delta B A}{\cancel{\Delta t}}$$

$$\Rightarrow \Delta Q = -\frac{(1-0.5)\times10^{-3}}{5} = \frac{0.5\times10^{-3}}{5}$$

$$\Rightarrow \boxed{\Delta Q = 10^{-4}\ C}$$

71. [B] Cyclotron motion $\Rightarrow qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB}$  Here, $m, q, B$ are constants.

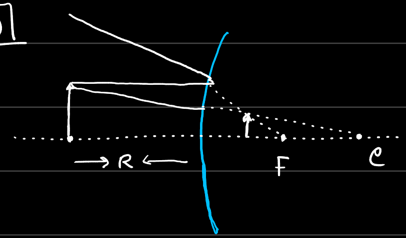
$\Rightarrow r \propto v$. So, $\frac{r_1}{r_2} = \frac{1}{3} = \frac{v_1}{v_2}$

72. [D] Basic spin & wavefunction fact $\Rightarrow$ Fermions have antisymmetric wavefunctions & obey Pauli exclusion principle.

73. [D] From David J. Griffiths $\to$ Introduction to elementary particles.

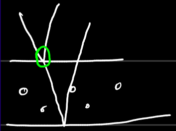
The $J/\psi$ was electronically neutral, extremely heavy meson. It has an unusually long lifetime and hinted at new physics. It was a bound state of a new fourth quark, charm quark, and its antiquark. $\boxed{J/\psi = c\bar{c}}$

74. $\boxed{(E)}$




Ray diagrams! The image appears to be to the right of the mirror, but at a distance less than $F = R/2$. So it must be a distance $R/3$ to the right.

75. $\boxed{(B)}$



$2nt = (m+\frac{1}{2})\lambda \quad m=0$

$2nt = \frac{\lambda_1}{2}$

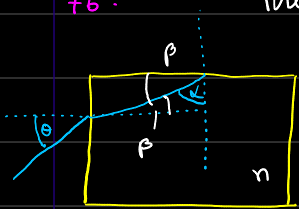
$\Rightarrow t = \frac{\lambda_1}{4n}$

The next wavelength in

$2nt = \frac{3}{2}\lambda$

$\Rightarrow \cancel{2n}\cdot\frac{\lambda'}{\cancel{4n}} = \frac{3\lambda}{\cancel{2}} \Rightarrow \lambda = \frac{\lambda'}{3} = \frac{540}{3} = 180\text{ nm}.$

76. The minimum $\alpha$ is the critical angle $\alpha_c$. So $\alpha > \alpha_c$ for the ray to stay in the medium.




$n \sin\alpha_c = 1$

$\Rightarrow \sin\alpha_c = \frac{1}{n}$

$\Rightarrow \alpha_c = \sin^{-1}(\frac{1}{n})$

So, $\alpha > \alpha_c$

$\Rightarrow \alpha > \sin^{-1}(\frac{1}{n})$

But $\alpha + \beta = \pi/2$

$\Rightarrow \alpha = \pi/2 - \beta$

$\pi/2 - \beta > \sin^{-1}(\frac{1}{n})$

$\Rightarrow \beta < \cos^{-1}(\frac{1}{n})$

$\Rightarrow \sin\beta < \sin\left[\cos^{-1}(\frac{1}{n})\right]$

$\Rightarrow \sin\beta < \sqrt{1-\frac{1}{n^2}}$

$\Rightarrow n\sin\beta < \sqrt{n^2-1}$

And $\sin\theta = n\sin\beta$

$\boxed{\theta < \sin^{-1}(\sqrt{n^2-1})}$

$\boxed{\begin{array}{l}\sin\left[\cos^{-1}(x)\right] = \sqrt{1-x^2} \\ \pi/2 - \sin^{-1}(x) = \cos^{-1}(x)\end{array}}$

77. $\boxed{(C)}$ Average time between collisions, $\boxed{t \propto \frac{1}{v}}$ & $\boxed{v = \sqrt{\frac{3kT}{m}}}$ Finally, $\boxed{t \propto \sqrt{m}}$

78. $\boxed{(B)}$ Partition function, $Z = e^{-E_1\beta} + e^{-E_2\beta}$. Probability, $P(E_2) = \frac{e^{-E_2\beta}}{Z}$

$$\Rightarrow P(E_2) = \frac{e^{-E_2\beta}}{e^{-E_1\beta} + e^{-E_2\beta}}$$

79. $\boxed{(D)}$ $P = \frac{RT}{V-b} - \frac{a}{V^2}$ Work done, $W = \int P\,dV$

$$\Rightarrow W = RT_0\int_{V_1}^{V_2}\frac{dV}{V-b} - a\int_{V_1}^{V_2}\frac{dV}{V^2}$$

$$= RT_0 \ln\left(\frac{V_2-b}{V_1-b}\right) + a\left[\frac{1}{V}\right]_{V_1}^{V_2} = RT_0\ln\left(\frac{V_2-b}{V_1-b}\right) + a\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$$

80 $\boxed{(C)}$ For original spring, $\omega_0 = \sqrt{\frac{k}{m}} = 1$ Hz. $\omega_f = \sqrt{\frac{2k}{\cancel{8}_4 m}} = \frac{1}{2}\sqrt{\frac{k}{m}} = \frac{1}{2}$ Hz

81. $\boxed{B}$ Conservation of energy $\Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$ [Rolling without slipping]

Here, $I = \frac{1}{2}mr^2$

$\omega^2 = \frac{v^2}{r^2}$

$\Rightarrow \frac{1}{2}\cancel{m}v^2 + \frac{1}{2}\cdot\frac{1}{2}\cancel{m}\cancel{r^2}\cdot\frac{v^2}{\cancel{r^2}} = \cancel{m}gh$

$\Rightarrow \left(\frac{1}{2}+\frac{1}{4}\right)\frac{v^2}{g} = h \quad \Rightarrow h = \left(\frac{2+1}{4}\right)\frac{v^2}{g} \quad \Rightarrow h = \frac{3v^2}{4g}$

(Rolling without slipping always means the total kinetic energy is the sum of $\frac{1}{2}mv^2$ and rotational energy $\frac{1}{2}I\omega^2$.

82 $\boxed{D}$ position cordinates $r$ and $\theta$. Relaxed length $s$.

Radial displacement $(r-s)$.

The Lagrangian, $L = T - V$. $\quad T = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2$ ; $V = \frac{1}{2}k(r-s)^2$

$\Rightarrow L = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 - \frac{1}{2}k(r-s)^2$

83 $\boxed{E}$ The Hamiltonian is given by $H = \frac{p_\theta^2}{2mL^2} + \frac{p_\phi^2}{2mL^2\sin^2\theta} - mgL\cos\theta$

Hamilton's equation of motion $\Rightarrow \boxed{\dot{q}_i = \frac{\partial H}{\partial p_i}}$ $\boxed{\dot{p}_i = -\frac{\partial H}{\partial q_i}}$ Here, H is independent of $\phi$.

So, the constant generalized momentum is $p_\phi$, because $\dot{p}_\phi = -\frac{\partial H}{\partial \phi} = 0$.

84. $\boxed{E}$ Center of mass, $x_{cm} = \frac{\int x\,dm}{\int dm}$

First, convert the mass variable to spatial variable. Mass density, $\lambda = \frac{2M}{L^2}x$

So, $dm = \lambda\,dx$

$$x_{cm} = \frac{\int x\lambda\,dx}{\int dm} = \frac{\int \frac{2M}{L^2}x\cdot x\,dx}{M} = \frac{2\cancel{M}\int_0^L x^2\,dx}{L^2\cdot\cancel{M}}$$

$$\Rightarrow x_{cm} = \frac{2\cdot\frac{1}{3}[x^3]_0^L}{L^2} = \frac{\frac{2}{3}L^3}{L^2} = \frac{2}{3}L$$

We could have had an aribtrary constant A insted of $\frac{2M}{L^2}$ in the mass density.

$\lambda = Ax \qquad \int dm = M \quad \Rightarrow \int\lambda\,dx = M = A\int_0^L x\,dx = \frac{A}{2}L^2 \Rightarrow A = \frac{2M}{L^2}$

85 $\boxed{B}$ Infinite square well $\Rightarrow \psi(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{3\pi x}{L}\right)$

Probability of finding the particle between $x = \frac{1}{3}L$ & $x = \frac{2}{3}L$ is

$P = \int |\psi(x)|^2\,dx \qquad |\psi(x)|^2 = \frac{2}{L}\sin^2\left(\frac{3\pi x}{L}\right)$

$= \frac{1}{L}\int\left[1-\cos\left(\frac{6\pi x}{L}\right)\right]dx \qquad = \frac{\cancel{2}}{\cancel{2}L}\cdot 2\sin^2\left(\frac{3\pi x}{L}\right)$

$\Rightarrow P = \frac{1}{L}\int_{L/3}^{2L/3}dx - \frac{1}{L}\int_{L/3}^{2L/3}\cos\left(\frac{6\pi x}{L}\right)dx \qquad = \frac{1}{L}\left[1-\cos\left(\frac{6\pi x}{L}\right)\right]$

$2\sin^2\theta = 1-\cos 2\theta$

$2\cos^2\theta = 1+\cos 2\theta$

$= \frac{1}{L}\left(\frac{2L}{3}-\frac{L}{3}\right) + \frac{6\pi}{L}\left[\sin\frac{6\pi x}{L}\right]_{L/3}^{2L/3}$

$= \frac{1}{\cancel{L}}\left(\frac{\cancel{L}}{3}\right) + \frac{6\pi}{L}\left[\sin\frac{6\pi\cdot 2L}{3L} - \sin\frac{6\pi\cdot L}{3L}\right] = \frac{1}{3}$

86. $\boxed{B}$ Eigenvalue of any matrix $A$ is the solution of the equation

$$A = \begin{pmatrix} 2 & i \\ -i & 2 \end{pmatrix} \qquad \boxed{\mathrm{Det}[A - \lambda I] = 0}$$

$$A - \lambda I = \begin{pmatrix} 2 & i \\ -i & 2 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \begin{pmatrix} 2-\lambda & i \\ -i & 2-\lambda \end{pmatrix}$$

and $\mathrm{Det}(A - \lambda I) = (2-\lambda)^2 + i^2 = 0$ $\Rightarrow (2-\lambda+1)(2-\lambda-1) = 0$

$\Rightarrow (2-\lambda)^2 - 1 = 0$ $\Rightarrow (3-\lambda)(1-\lambda) = 0$

So, $\lambda = 1, 3$.

87. $\boxed{D}$ $[\sigma_x, \sigma_y] = (\sigma_x\sigma_y - \sigma_y\sigma_x)$ $\quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$\sigma_x\sigma_y = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \quad \& \quad \sigma_y\sigma_x = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}$$

$$\sigma_x\sigma_y - \sigma_y\sigma_x = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} - \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} = \begin{pmatrix} 2i & 0 \\ 0 & -2i \end{pmatrix} = 2i\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = 2i\sigma_z$$

Instead of calculating, you could just memorize the commutation relation for Pauli spin matrices $\boxed{[\sigma_i, \sigma_j] = 2i\sigma_k \,\epsilon_{ijk}}$

88. $\boxed{D}$ $\chi = A\begin{pmatrix} 1+i \\ 2 \end{pmatrix}$ Normalization of spin states: $\boxed{\chi^\dagger\chi = 1}$

$\Rightarrow A^2 \begin{pmatrix} 1-i & 2 \end{pmatrix}\begin{pmatrix} 1+i \\ 2 \end{pmatrix} = 1$

$\Rightarrow A^2\left[(1-i^2) + 4\right] = 1$

$\Rightarrow A^2(2+4) = 1 \Rightarrow A = \frac{1}{\sqrt{6}}$

$$\Rightarrow \chi = \begin{pmatrix} \frac{1+i}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \end{pmatrix}$$

Express this in the form $\chi = \alpha\chi_+ + \beta\chi_-$

$$\Rightarrow \chi = \frac{1+i}{\sqrt{6}}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{2}{\sqrt{6}}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

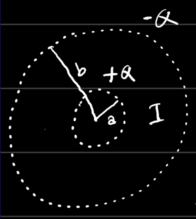
$\alpha^2$ gives the probability of $S_z = +\frac{1}{2}\hbar$ [spin-up] & $\beta^2$ gives the probability of $S_z = -\frac{1}{2}\hbar$ [spin-down]

$$\Rightarrow \chi = \frac{1+i}{\sqrt{6}}\chi_+ + \frac{2}{\sqrt{6}}\chi_-$$

So, $\beta^2 = \frac{4}{6} = \frac{2}{3}$

89. $\boxed{D}$ Limiting case! $k_1$ is the wave number to the left $(x<0)$ and $k_2$ is the wave number to the right $(x>0)$. If $k_2 \to 0$, there will be no transmitted wave, and $R \to 1$. Only $\boxed{D}$ does this.

90. $\boxed{(D)}$ Spherical conducting concentric shells. Gauss's law tells us that field in the region-II is 0. $E_{II} = 0$. So the potential there is constant.




In region-I, $E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$ So, $V_I(r) = -\int_a^r \vec{E}\cdot d\vec{r}$

$$= \frac{-Q}{4\pi\epsilon_0}\int_a^r \frac{dr}{r^2} = \frac{-Q}{4\pi\epsilon_0}\int_a^r r^{-2}\,dr$$

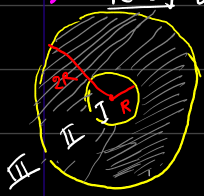
$$\Rightarrow V_I(r) = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r}\right]_a^r = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r}-\frac{1}{a}\right)$$

As potential is continuous, $V_{II}(r) = V_I(r=b) = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{b}-\frac{1}{a}\right)$

Finally, in region-I, $\boxed{V_I(r) = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r}-\frac{1}{a}\right)}$ $\boxed{V_{II}(r) = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{b}-\frac{1}{a}\right)}$

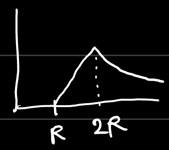
91. $\boxed{(C)}$ If the curl of a vector field is zero, we can write the vector field as a gradient of some vector potential. Since $\vec{\nabla}\times\vec{E} = 0$, we can write it as $\vec{E} = -\nabla\phi$.

92. $\boxed{(E)}$ When $0 < r < R$ [Region-I] $\vec{B} = 0$ (No current enclosed)




When $R < r < 2R$ [Region-II], $\vec{B}$ increases linearly (Enclosed current increases)

When $r > 2R$ [Region-III], $\vec{B}$ falls off following $\vec{B} \sim \frac{1}{r}$.




93. $\boxed{(B)}$ Parallel plate capacitor:

Before inserting dielectric.

$C_0 = \frac{\epsilon_0 A}{d}$ $\quad C = \frac{Q}{V_0}$ $\quad V_0 = Ed$

The quantities we are concerned with are

$E_0 = \frac{V_0}{d}$ $\quad U_0 = \frac{1}{2}C_0 V_0^2$

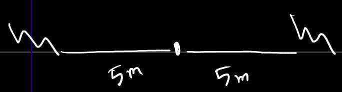
After inserting dielectric

$C = \frac{K\epsilon_0 A}{d} = KC_0$ $\quad V = V_0$ became voltage supply is the same.

$E = \frac{V_0}{d} = E_0$ $\quad$ ~~E~~ $U = \frac{1}{2}KC_0V_0^2$

$= KU_0$

94. $\boxed{(C)}$ Using Lorentz transformation




$$\Delta t' = \gamma\left(\Delta t - \frac{vx}{c^2}\right) \qquad \Delta t = 0 \quad \Delta t' = 13\text{ns} \quad \Delta x = 10$$

$$\Rightarrow 13\times10^{-9} = \gamma\beta\frac{\Delta x}{c} \Rightarrow \gamma\beta = \frac{13\times10^{-9}\times3\times10^{8}}{10}$$

$$\Rightarrow \frac{\beta}{\sqrt{1-\beta^2}} = \frac{40\times10^{-1}}{10} \Rightarrow \frac{\beta^2}{1-\beta^2} = \frac{\cancel{16}\,8\times\cancel{10^{2}}\times\cancel{10^{-2}}}{\cancel{100}\,50} \Rightarrow 50\beta^2+8\beta-8 = 0 \Rightarrow 25\beta^2+8\beta-8=0$$

$$\beta = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-8\pm\sqrt{64+800}}{100} = \frac{-8\pm\sqrt{864}}{100} \approx \frac{-8\pm30}{100} \approx \frac{38}{100}$$

$$\Rightarrow v = 0.38c$$

95. $\boxed{(D)}$ Commutation relation identities: (i) $[A,BC] = [A,B]C + B[A,C]$

Canonical commutation $[J_x, J_y] = i\hbar J_z \varepsilon_{ijk}$ (ii) $[AB,C] = A[B,C] + [A,C]B$

Here, $[J_x J_y, J_x] = J_x[J_y, J_x] + [J_x, J_x]J_y = -J_x[J_x, J_y] = -i\hbar J_x J_z$

96. $\boxed{(D)}$ To create n-type semiconductors, we need excess of electrons, we need PENTAVALENT atoms such as As, P, Sb or N.

97. $\boxed{(E)}$ Compton scattering $\Rightarrow \Delta\lambda = \frac{h}{mc}(1-\cos\theta)$. Here $\theta' = 90^\circ$

$\Rightarrow \Delta\lambda = \frac{h}{mc} \Rightarrow \Delta\lambda = \frac{hc}{mc^2}$ | $\lambda = \frac{hc}{E_i}$ ; $\lambda' = \frac{hc}{E_s}$

$$\Delta\lambda = hc\left(\frac{1}{E_i} - \frac{1}{E_s}\right)$$

$$\Rightarrow \frac{h\cancel{c}}{mc^2} = \cancel{hc}\left(\frac{E_f - E_i}{E_i E_f}\right) \Rightarrow E_i E_f = mc^2(E_i - E_f)$$

$$\Rightarrow E_f(E_i + mc^2) = mc^2 E_i$$

$$\Rightarrow E_f = \frac{E_i mc^2}{E_i + mc^2}$$

98. (D) Use Lepton number conservation!

$L = +1$ for $e^-, \mu^-, \nu$

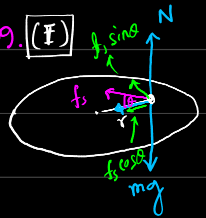
$L = -1$ for $e^+, \mu^+, \bar{\nu}$

$L = 0$ for everything else.

(A) $-1 \neq -1 + 1$ (B) $-1 \neq 0 + 1$ (C) $-1 \neq 0 - 1 + 1$ (D) $-1 = -1 + \cancel{1} \cancel{-1}$

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$$

99. $\boxed{(F)}$




Rotates at constant acceleration $\alpha$.

$m\vec{a} = \vec{f}_s + m\vec{g} + \vec{N}$ Here $\vec{a}$ has a tangential component $\vec{a}_t$ & a radial component $\vec{a}_c$.

$$\vec{a}_t = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\omega r) = r\alpha \ ; \ a_c = \frac{v^2}{r} = \frac{\omega^2 r^2}{r} = \omega^2 r.$$

$$f_s \cos\theta = \frac{mv^2}{r} = \frac{m\omega^2 r^2}{r} = m\omega^2 r \qquad f_s \sin\theta = m\vec{a}_t = m\alpha r$$

$$\tan\theta = \frac{\cancel{m}\alpha \cancel{r}}{\cancel{m}\omega^2 \cancel{r}} \Rightarrow \theta = \tan^{-1}\left(\frac{\alpha}{\omega^2}\right)$$

100. (E) Partition function, $Z = \sum_n e^{-E_n \beta}$. For harmonic oscillator, $E_n = (n+\frac{1}{2})\hbar\omega$.

So,

$$Z = \sum_n e^{-(n+\frac{1}{2})\hbar\omega/kT} = \sum_n e^{-n\hbar\omega/kT} \cdot \underbrace{e^{-\frac{1}{2}\hbar\omega/kT}}_{\text{Pull this out of the sum.}}$$

$$\Rightarrow Z = e^{-\frac{1}{2}\hbar\omega/kT} \sum_n e^{-n\hbar\omega/kT}$$

We know, $\sum_n e^{-nx} = \frac{1}{1-e^{-x}}$ So, $\sum_n e^{-n\hbar\omega/kT} = \frac{1}{1-e^{-\hbar\omega/kT}}$

$$\text{And } Z = \frac{e^{-\frac{1}{2}\hbar\omega/kT}}{e^{-\hbar\omega/kT}\left(e^{\hbar\omega/kT}-1\right)} = \frac{e^{\frac{1}{2}\hbar\omega/kT}}{e^{\hbar\omega/kT}-1}$$

$$\sum_{n=0}^{\infty} e^{-nx} = \frac{1}{e^x - 1}$$

$$\sum_{n=1}^{\infty} e^{-nx} = \frac{1}{1-e^{-x}} = \frac{e^x}{e^x-1}$$